Title: Homogeneous and Isotropic Universe from Nonlinear Massive Gravity

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Abstract: The question of finite range gravity, or equivalently, whether graviton can have a non-zero mass, has been one of the major challenges in classical field theory for the last 70 years.

Generically, a massive gravity theory contains an extra degree in addition to the 5 polarizations of massive graviton, which turns out to be a ghost. Recently, de Rham, Gabadadze and Tolley constructed a nonlinear theory of massive gravity, which successfully eliminates the ghost. Moreover, the theory has also phenomenological relevance, since the graviton mass may account for the accelerated expansion of the present universe, providing an alternative to dark energy. I will present self-accelerating cosmological solutions in the framework of this theory. The cosmological perturbations around these backgrounds have an interesting behavior: instead of the 5 degrees of freedom expected from a massive spin-2 field, only the 2 gravity wave polarizations are dynamical, at linear level. However, nonlinear analysis of the extra modes reveal the existence of ghost instabilities. This implies that a consistent universe solution in this theory should be inhomogeneous and/or anisotropic.

Why massive gravity?

- Theoretical motivation: Is there a massive gravity theory which reduces smoothly to GR in the massless limit? Are the predictions of GR stable against small graviton mass? $\Rightarrow$ A major challenge for more than 70 years.
- Observational motivation: Supernovae $\Rightarrow$ dark energy. Alternative approach: associate this effect with the gravity sector, by large distance modifications of GR.

Can graviton have a mass/finite range?

Linear theory

- Extending linearized GR with a mass term

$$\mathcal{L}_m = m_g^2 \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right) , \qquad (g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu})$$

`Fierz, Pauli '39`

- Massive spin 2 field $\Rightarrow$ 5 dynamical degrees of freedom
- Discontinuity with GR in the limit $m_g \to 0$ `van Dam, Veltman '70` `Zakharov '70`

- Linear theory breaks down at distances $r < \left(m_g^{-4} \, r_g \right)^{1/5}$
  $\Rightarrow$ Non-linear effects can recover continuity `Vainshtein '72`
- Since mass term breaks diffeomorphism invariance, there are generically 6 degrees of freedom. The additional degree has a wrong sign kinetic term (BD ghost). `Boulware, Deser '72`

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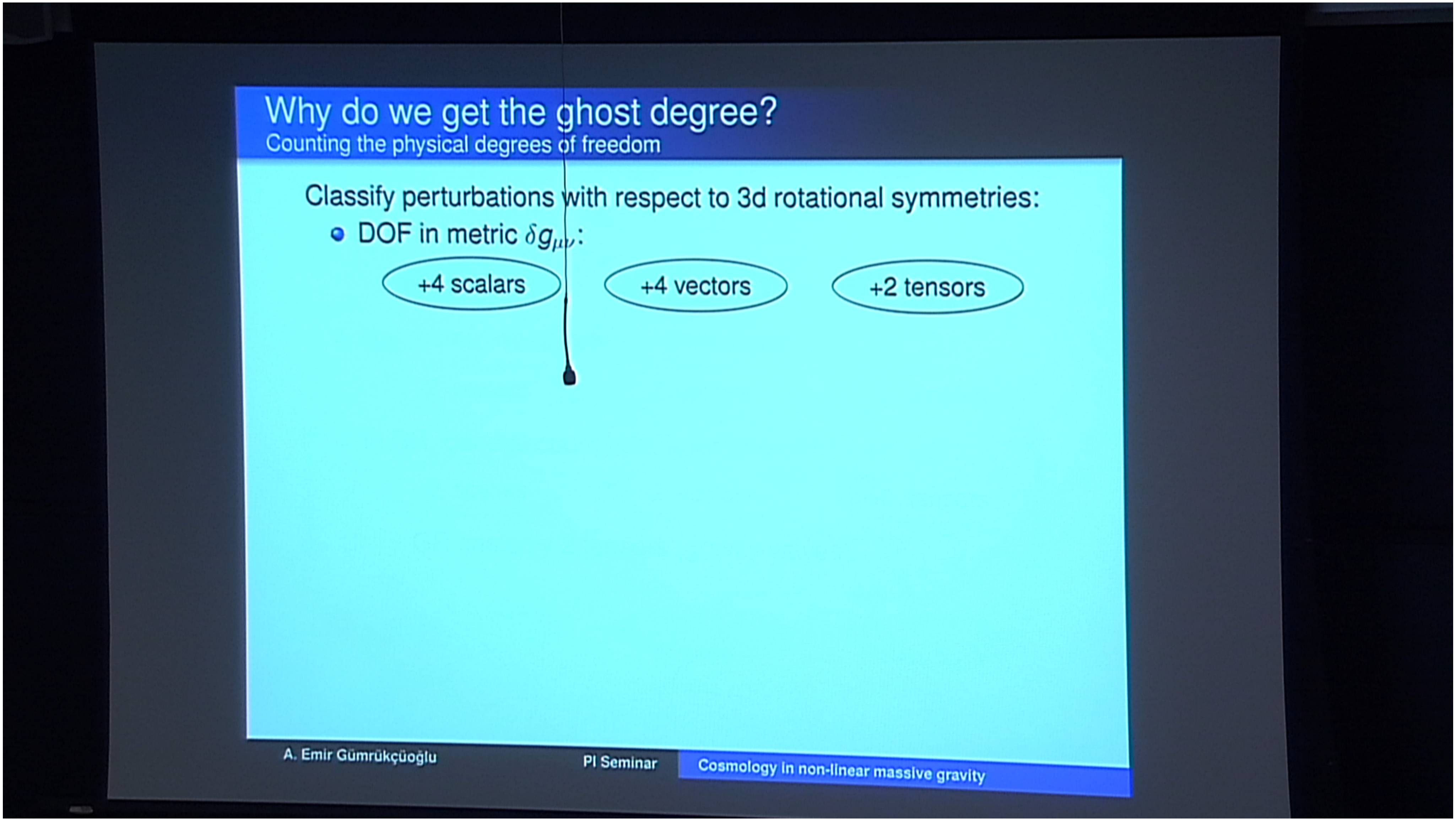
Why do we get the ghost degree?
Counting the physical degrees of freedom
Classify perturbations with respect to 3d rotational symmetries:
DOF in metric $\delta g_{\mu\nu}$:
+4 scalars
+4 vectors
+2 tensors
A. Emir Gümrükçüoğlu
PI Seminar
Cosmology in non-linear massive gravity

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Counting the physical degrees of freedom

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- DOF in metric $\delta g_{\mu\nu}$:

  +4 scalars | +4 vectors | +2 tensors

- $\delta g_{0\mu}$ components are non-dynamical:

  -2 scalars | -2 vectors | -0 tensors

- In GR, general coordinate invariance $x^\mu \to x^\mu + \xi^\mu$:

  -2 scalars | -2 vectors | -0 tensors

  $\Longrightarrow$ GR has only 2 tensors (gravity waves).

- In a generic massive theory, no gauge invariance:

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- However, we expect massive spin–2 particle to have 5 d.o.f. (1 s, 2 v, 2 t). The extra scalar is the BD ghost.

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Covariant EFT formulation

Arkani-Hamed, Georgi, Schwartz '03

- Mass term breaks general coordinate invariance.
- Gauge degrees are redundancies of description. Introduce four scalar (Stückelberg) fields, one for each broken gauge degree: ϕ^a $(a = 0, 1, 2, 3)$
- Introduce covariant analogue of $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$,

$$H_{\mu\nu} = g_{\mu\nu} - \eta_{ab}\,\partial_\mu\phi^a\,\partial_\nu\phi^b\,, \qquad \phi^a = x^a + \pi^a$$

$H_{\mu\nu} \to h_{\mu\nu}$ in the unitary gauge $\phi^a = x^a$.

- Decoupling limit

$$m_g \to 0\,, \qquad M_p \to \infty\,, \qquad \Lambda_5 = \left(m_g^4\,M_p\right)^{1/5} = \text{finite}$$

BD ghost $\Leftrightarrow$ Helicity–0 part of π^a, i.e. $\eta_{ab}\pi^a = \partial_a\pi$

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Removing the instability
de Rham, Gabadadze '10

- Generalization of F-P: EH + zero derivative mass terms

$$S = \frac{M_p^2}{2}\int d^3x\sqrt{-g}\left[R - \frac{m^2}{4}\left(U_2(g,H) + U_3(g,H) + ...\right)\right]$$

$$U_2(g,H) = [H^2] - [H]^2\,,$$
$$U_3(g,H) = c_1[H^3] + c_2[H^2][H] + c_3[H]^3\,,$$
$$\cdots$$

$$\left(g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}=\eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b+H_{\mu\nu}\,,\qquad [Q]\equiv Q_{\mu\nu}g^{\mu\nu}\right)$$

- In the decoupling limit, the mass term is built out of

$$H_{\mu\nu} = 2\,\partial_\mu\partial_\nu\pi - \partial_\mu\partial^\sigma\pi\,\partial_\sigma\partial_\nu\,\pi$$

- F-P term U_2 is a total derivative in decoupling limit
- Tune c_i at each order by requiring mass terms are full derivatives in DL. From quintic order on, the terms vanish identically.

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Nonlinear massive gravity

de Rham, Gabadadze, Tolley '10

- Impose Poincaré symmetry in the Stückelberg field space. Invariant "line element":

$$ds_\phi^2 = \eta_{ab}\, d\phi^a\, d\phi^b$$

- Mass term depends only on $g_{\mu\nu}$ and the *fiducial metric*

$$f_{\mu\nu} = \eta_{ab}\, \partial_\mu\phi^a\, \partial_\nu\phi^b$$

- Requiring that the sixth degree (BD ghost) is canceled at any order, the most general action is:

$$S_m[g_{\mu\nu}, f_{\mu\nu}] = M_p^2 m_g^2 \int d^4x \sqrt{-g}\,(\mathcal{L}_2 + \alpha_3\mathcal{L}_3 + \alpha_4\mathcal{L}_4)$$

$$\left[\begin{array}{l} \mathcal{L}_2 = \frac{\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\rho\sigma}}{2} K^\alpha{}_\mu K^\beta{}_\nu \\ \mathcal{L}_3 = \frac{\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\sigma}}{3!} K^\alpha{}_\mu K^\beta{}_\nu K^\gamma{}_\rho \quad \text{and} \quad K^\mu{}_\nu \equiv \delta^\mu{}_\nu - \left(\sqrt{g^{-1}f}\right)^\mu{}_\nu \\ \mathcal{L}_4 = \frac{\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}}{4!} K^\alpha{}_\mu K^\beta{}_\nu K^\gamma{}_\rho K^\delta{}_\sigma \end{array}\right]$$

- $K_{\mu\nu} \to \partial_\mu\partial_\nu\pi$ in DL $\Longrightarrow \mathcal{L}_i$ become total derivatives.
- Away from the DL, still 5 dof — Hassan, Rosen '11

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Massive cosmology

- A general massive gravity theory with 5 degrees of freedom, built partly to address the dark energy problem.
  $\Rightarrow$ Can we get cosmological solutions?
- Look for simplest solutions in the simplest version of the theory.
  $\Longrightarrow$Does it work?
  (continuity with GR, stability, description of thermal history...)
  - yes $\Rightarrow$ predictions of observables to constrain the theory
  - no $\Rightarrow$ relax the solution and/or theory

Which theory?

Massive gravity zoology in 3+1

1. Drop Poincaré symmetry in the field space

$$f_{\mu\nu} = \bar{\eta}_{ab}\, \partial_\mu \phi^a\, \partial_\nu \phi^b \,,$$

   with generic $\bar{\eta}$. Hassan, Rosen, Schmidt-May '11

2. Ghost-free bigravity: introduce dynamics for the fiducial metric Hassan, Rosen '11
3. Ghost-free trigravity, multigravity etc... Khosravi et al '11; Nomura, Soda '12
4. Quasi-dilaton, varying mass, ... d'Amico et al '12; Huang, Piao, Zhou '12

The list is still growing...

In this talk, I will only allow extensions of the type 1.

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Which cosmology?

Goal

- Homogeneous and isotropic universe solution, which can accommodate the history of the universe.
- Preserved homogeneity/isotropy for linear perturbations

- FRW ansatz for the both physical and fiducial metrics

$$ds^2 = -N(t)^2\, dt^2 + a(t)^2\, \Omega_{ij}\, dx^i\, dx^j$$

$$ds_\phi^2 = -n(\phi^0)^2\, (d\phi^0)^2 + \alpha(\phi^0)^2\, \Omega_{ij}\, d\phi^i\, d\phi^j$$

$$\left[\begin{array}{c} \Omega_{ij} = \delta_{ij} + \dfrac{K\, \delta_{il}\delta_{jm} x^l\, x^m}{1 - K\, \delta_{lm} x^l x^m} \\ \langle \phi^a \rangle = \delta^a_\mu\, x^\mu \end{array}\right]$$

Is this form for $f_{\mu\nu}$ the only choice?

- Case with different $f_{\mu\nu} \longrightarrow$ FRW $g_{\mu\nu}$ d'Amico et al '11; Koyama et al '11; Volkov '11,'12; Kobayashi et al '12
- Although background dynamics homogeneous+isotropic, there *is* a broken FRW symmetry in the Stückelberg sector, which *can* be probed by perturbations.

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Cosmological solutions for Minkowski fiducial metric

AEG, Lin, Mukohyama '11a

- No flat FRW, for Minkowski fiducial. `d'Amico et al '11`
- But open FRW solutions exist

$$ds^2 = -N^2\, dt^2 + a^2\, \Omega_{ij}^{(K<0)}\, dx^i\, dx^j$$

$$ds_\phi^2 = -n^2\, dt^2 + \alpha^2\, \Omega_{ij}^{(K<0)}\, dx^i\, dx^j$$

$$\left[n = \dot{\alpha}/\sqrt{|K|}\right] \Longleftarrow \text{Minkowski in open chart}$$

Minkowski in open coordinates

- Minkowski metric $ds_\phi^2 = -[d\tilde{\phi}^0]^2 + \delta_{ij} d\tilde{\phi}^i d\tilde{\phi}^j$
- After coordinate transformation

$$\tilde{\phi}^0 = \frac{\alpha(\phi^0)}{\sqrt{|K|}} \sqrt{1 + |K| \delta_{ij} \phi^i \phi^j}\,, \qquad \tilde{\phi}^i = \alpha(\phi^0) \phi^i\,.$$

becomes:

$$ds_\phi^2 = -\frac{[\alpha'(\phi^0)]^2}{|K|} [d\phi^0]^2 + [\alpha(\phi^0)]^2\, \Omega_{ij}(\{\phi^i\})\, d\phi^i d\phi^j$$

- No closed FRW chart of Minkowski $\Longrightarrow$ no closed solution

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Equation of motion for $\phi^0 \Longrightarrow$ 3 branches of solutions:

$$\left(\frac{\dot{a}}{N} - \sqrt{|K|}\right) J_\phi\left(\frac{\alpha}{a}\right) = 0$$

- Branch $\mathrm{I} \Longrightarrow \dot{a} = \sqrt{|K|}N \Longrightarrow g_{\mu\nu}$ is also Minkowski (open chart)

  $\Longrightarrow$ *No cosmological expansion!*

- Branch $\mathrm{II}_\pm \Longrightarrow J_\phi(\alpha/a) = 0$

  $$\left[J_\phi(X) \equiv 3 + 3\,\alpha_3 + \alpha_4 - 2\,(1 + 2\,\alpha_3 + \alpha_4)\,X + (\alpha_3 + \alpha_4)\,X^2\right]$$

$$\alpha = a\,X_\pm, \quad \text{with } X_\pm \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4} = \text{constant}$$

<u>For $K = 0$</u>, this branch not present. Only Branch I remains.

Cosmological solutions for Minkowski fiducial metric

AEG, Lin, Mukohyama '11a

Equation of motion for $\phi^0 \Longrightarrow$ 3 branches of solutions:

$$\left(\frac{\dot{a}}{N} - \sqrt{|K|}\right) J_\phi\left(\frac{\alpha}{a}\right) = 0$$

- Branch I $\Longrightarrow \dot{a} = \sqrt{|K|}N \Longrightarrow g_{\mu\nu}$ is also Minkowski (open chart)

  $\Longrightarrow$ *No cosmological expansion!*

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Extension to generic fiducial metric

AEG, Lin, Mukohyama '11b

- Extending the field space metric, the line elements are

$$ds^2 = -N^2\, dt^2 + a^2\, \Omega_{ij} dx^i\, dx^j$$

$$ds_\phi^2 = -n^2\, dt^2 + \alpha^2\, \Omega_{ij} dx^i\, dx^j$$

Generically, we can have spatial curvature with either sign

e.g. at the cost of introducing a new scale (H_f), de Sitter fiducial can be brought into flat, open and closed FRW form.

Equations of motion for $\phi^0 \Rightarrow$ 3 branches of solutions

$$(a\, H - \alpha H_f)\, J_\phi \left(\frac{\alpha}{a}\right) = 0$$

- Branch I : $a\, H = \alpha\, H_f$ $\quad \left[H_f \equiv \frac{\dot{\alpha}}{\alpha\, n} \right]$
- Branch $\mathrm{II}_\pm$: 2 cosmological branches
  $\alpha(t) = X_\pm\, a(t)$

$\Longrightarrow$ same solution as in Minkowski fiducial

- Expansion in Branch I can be determined by the matter content
  $\Rightarrow$ in principle, can have cosmology.
  However, for dS fiducial $\Rightarrow$ Higuchi vs. Vainshtein conflict
  Fasiello, Tolley '12

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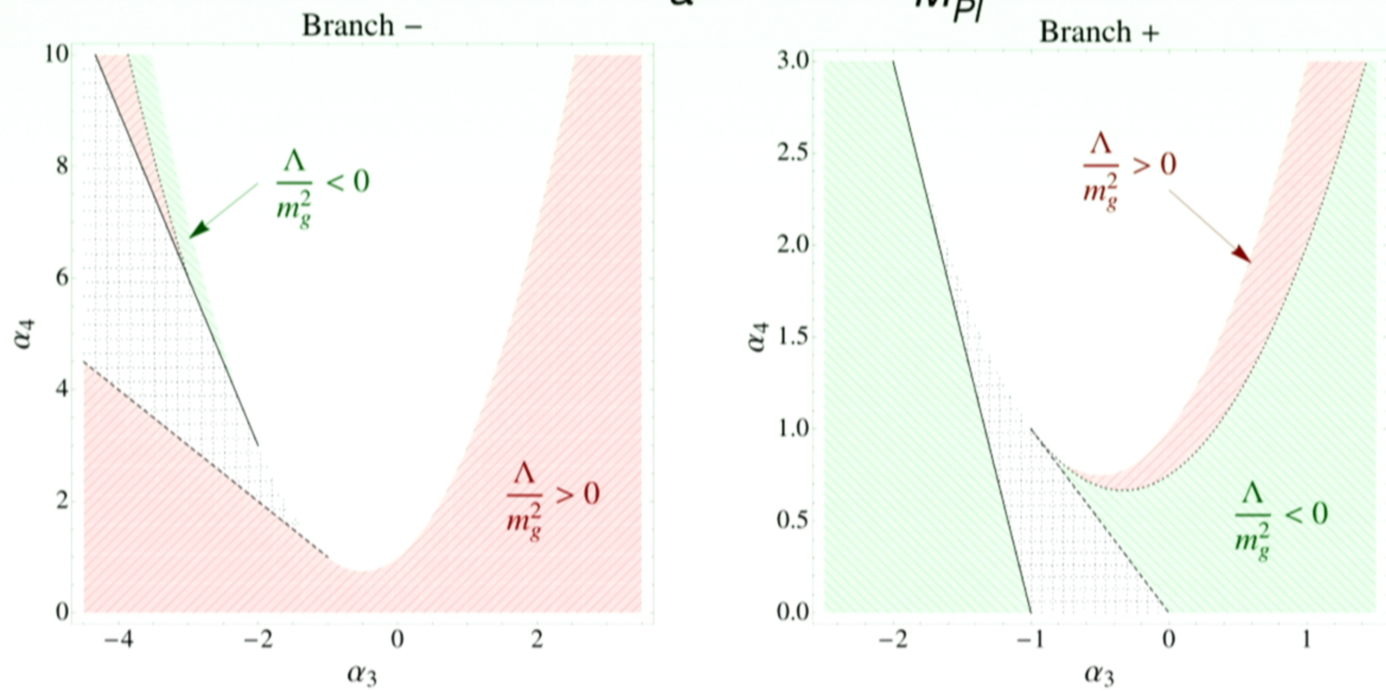
Fasiello, Tolley '12

Branch $\mathrm{II}_\pm$: Self-acceleration

- Evolution of Branch $\mathrm{II}_\pm$, with generic (conserved) matter

$$\left[H \equiv \frac{\dot{a}}{a\,N}\right] \rightarrow 3\,H^2 + \frac{3\,K}{a^2} = \Lambda_\pm + \frac{1}{M_{Pl}^2}\rho$$

→ *independent of H_f*



$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right]$$

Perturbing the solution

- Lack of BD ghost does not guarantee stability. Higuchi '87
- Scalar sector may include additional couplings, giving rise to potential conflict with observations. Does Vainshtein mechanism still work?
- Can we distinguish massive gravity from other models of dark energy/modified gravity?

Perturbations and gauge invariant variables

AEG, Lin, Mukohyama '11b

- Perturbations in the metric, Stückelberg fields and matter fields:

$$g_{00} = -N^2(t)\,[1+2\phi]\,,\quad g_{0i} = N(t)a(t)\beta_i\,,\quad g_{ij} = a^2(t)\left[\Omega_{ij}(x^k) + h_{ij}\right]$$

$$\varphi^a = x^a + \pi^a + \frac{1}{2}\pi^b\partial_b\pi^a + O(\epsilon^3)\,,\qquad \sigma_I = \sigma_I^{(0)} + \delta\sigma_I$$

← *matter sector*

- Scalar-vector-tensor decomposition:

$$\beta_i = D_i\beta + S_i\,,\qquad \pi_i = D_i\pi + \pi_i^T\,,$$

$$h_{ij} = 2\psi\Omega_{ij} + \left(D_iD_j - \tfrac{1}{3}\Omega_{ij}\triangle\right)E + \tfrac{1}{2}(D_iF_j + D_jF_i) + \gamma_{ij}$$

$$D_i \leftarrow \Omega_{ij}\,,\quad \triangle \equiv \Omega^{ij}D_iD_j$$
$$D^iS_i = D^i\pi_i^T = D^iF_i = 0$$
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- Gauge invariant variables without Stückelberg fields:

Originate from $g_{\mu\nu}$ and matter fields $\delta\sigma_I$

$$Q_I \equiv \delta\sigma_I - \mathcal{L}_Z\sigma_I^{(0)}\,,$$
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$$Z^0 \equiv -\tfrac{a}{N}\beta + \tfrac{a^2}{2N^2}\dot E$$
$$Z^i \equiv \tfrac{1}{2}\Omega^{ij}(D_jE + F_j)$$
$$\text{Under } x^\mu \to x^\mu + \xi^\mu:$$
$$Z^\mu \to Z^\mu + \xi^\mu$$

- However, we have 4 more degrees of freedom:

Associated with Stückelberg fields

$$\psi^\pi \equiv \psi - \frac{1}{3}\triangle\pi - \frac{\dot a}{a}\pi^0\,,\qquad E^\pi \equiv E - 2\pi\,,\qquad F_i^\pi \equiv F_i - 2\pi_i^T$$

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Quadratic action

- After using background constraint for Stückelberg fields:

$$S^{(2)} = \underbrace{S^{(2)}_{\rm EH} + S^{(2)}_{\rm matter} + S^{(2)}_{\Lambda_\pm}}_{\text{depend only on } Q_I, \Phi, \Psi, B_i, \gamma_{ij}} + \underbrace{\tilde{S}^{(2)}_{\rm mass}}_{\tilde{S}^{(2)}_{\rm mass} = S^{(2)}_{\rm mass} - S^{(2)}_{\Lambda_\pm}}$$

- The first part is equivalent to GR + $\Lambda_\pm$+ Matter fields σ_I.
- The additional term:

$$M_{GW}^2 \equiv m_g^2 \left(1 - \frac{a\,n}{\alpha\,N}\right) \frac{\alpha^2}{a^2} \times \left[(1 + 2\,\alpha_3 + \alpha_4) - \frac{\alpha}{a}\,(\alpha_3 + \alpha_4)\right]$$

$$\tilde{S}^{(2)}_{\rm mass} = M_p^2 \int d^4x\, N\, a^3 \sqrt{\Omega}\, M_{GW}^2 \times \left[3(\psi^\pi)^2 - \frac{1}{12} E^\pi \triangle(\triangle + 3K) E^\pi + \frac{1}{16} F^i_\pi (\triangle + 2K) F^\pi_i - \frac{1}{8} \gamma^{ij}\gamma_{ij}\right]$$

- The only common variable is γ_{ij}. Dispersion relation of tensor modes:

$$\omega_{GW}^2 = \frac{k^2}{a^2} + M_{GW}^2(t)$$

- E^π, ψ^π, F_i^π have no kinetic term!

Cancellation of kinetic terms

Other examples of cancellation

- Self-accelerating solutions in the decoupling limit
  `de Rham, Gabadadze, Heisenberg, Pirtskhalava '10`
- Inhomogeneous de Sitter solutions
  `Koyama, Niz, Tasinato '11`
- dS and Schwarschild dS solutions in the decoupling limit
  `Berezhiani, Chkareuli, de Rham, Gabadadze, Tolley '11`
- A branch of self-accelerating solutions in bimetric gravity
  `Crisostomi, Comelli, Pilo '12`
- Self-accelerating spherically symmetric, isotropic solutions
  `Gratia, Hu, Wyman '12`
- Branch of self-accelerating solutions in quasi-dilaton massive gravity
  `d'Amico, Gabadadze, Hui, Pirtskhalava '12`

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- dS a
- A br
- Self
- Bran

What is the fate of these degrees?

1. Infinitely strong coupling?
2. Infinitely heavy degrees? Then, they can be integrated out $\Longrightarrow$ same d.o.f. as in GR, Higuchi bound (or its analogue) irrelevant, no need for Vainshtein mechanism. The only signature imprinted in the perturbations is the GW signal.
   AEG, Kuroyanagi, Lin, Mukohyama, Tanahashi '12
   [arXiv:1208.5975]

Need to go beyond linear order to determine which case is realized

Probing the non-linear action with linear tools

de Felice, AEG, Mukohyama '12

- Symmetry of the background $\Rightarrow$ cancellation
- Instead of computing the high order action, we slightly break the isotropy and compute the quadratic terms.
- The small deviation from isotropy in the background is interpreted as a homogeneous perturbation in the FRW solution. This will allow us to obtain information on the high order terms in the exact FRW case.

Introducing small anisotropy

- The simplest anisotropic extension of flat FRW is the degenerate Bianchi type–I metric

$$|\sigma| \ll 1$$

$$ds^2 = -N^2 dt^2 + a^2 \left[e^{4\sigma} dx^2 + e^{-2\sigma} (dy^2 + dz^2) \right]$$

- Different fiducial metric $\Leftrightarrow$ different theory. In order to have continuity with the FRW solutions, we keep $f_{\mu\nu}$ isotropic:

$$ds_\phi^2 = -n^2 \, dt^2 + \alpha^2 \, (dx^2 + dy^2 + dz^2)$$

- Vacuum configuration (with bare Λ)

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Perturbations around the anisotropic background

Strategy

- Make use of the residual symmetry on the *y*–*z* plane; decomposition wrt 2d rotations:
  (2d) $\boxed{3\text{S} + 2\text{V}}$ dof expected to propagate in gravity sector
- Write the quadratic action, define G.I. variables, expand fields in Fourier space, integrate out non-dynamical degrees.
- Expand background around FRW for small σ
- Diagonalize the system $\Longrightarrow$ obtain dispersion relations for energy eigenstates

Kinetic terms and eigenfrequencies

κ: Kinetic term before canonical normalization
ω: Frequency after diagonalization

2d vectors

1. $\kappa_1 = \mathcal{O}(\sigma^0) > 0$ and $\omega_1^2 = \frac{k^2}{a^2} + M_{GW}^2$
   $\Longrightarrow$ 1 of the GW in the isotropic limit
2. $\kappa_2 = \mathcal{O}(\sigma)$ and $\omega_2^2 \propto \frac{k^2}{\sigma}$
   $\Longrightarrow \kappa > 0$ if a time dependent condition satisfied

2d scalars

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   $\Longrightarrow$ 1 of the GW in the isotropic limit
2. $\kappa_2 = \mathcal{O}(\sigma)$ and $\omega_2^2 \propto \frac{k^2}{\sigma}$
3. $\kappa_3 = -C(\vec{k})\,\kappa_2$ and $\omega_3^2 \propto \frac{k^2}{\sigma}$, with $C(\vec{k}) > 0$
   $\Longrightarrow$ Either 2 or 3 has always negative kinetic term!

There is always a ghost in 2d scalar sector. Since $\omega \propto k$, we cannot integrate it out from the low energy effective theory. The solution is unstable!

Kinetic terms and eigenfrequencies

κ: Kinetic term before canonical normalization
ω: Frequency after diagonalization

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Fate of isotropic solutions?

- Quadratic kinetic term for $1 \gg |\sigma| \neq 0$
  $\iff \phi_{k_1}\dot{\phi}_{k_2}\dot{\phi}_{k_3}$ type terms, with one $k_i = 0$.
- Homogeneous and isotropic solutions in massive gravity have ghost instability which arises from the cubic order action.
- This conclusion is valid for $\pm$ cosmological branch solutions of massive gravity with arbitrary fiducial metric.
- Non-linear analysis indicate the kinetic term for the longitudinal degrees reappear at cubic order. d'Amico '12
- Similar solutions in variants of the theory (e.g. in bigravity, quasi-dilaton...) have the vanishing kinetic term behavior.
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Perturbations and gauge invariant variables

AEG, Lin, Mukohyama '11b

- Perturbations in the metric, Stückelberg fields and matter fields:

$$g_{00} = -N^2(t)\,[1+2\phi]\,, \quad g_{0i} = N(t)a(t)\beta_i\,, \quad g_{ij} = a^2(t)\left[\Omega_{ij}(x^k) + h_{ij}\right]$$

$$\varphi^a = x^a + \pi^a + \frac{1}{2}\pi^b\partial_b\pi^a + O(\epsilon^3)\,, \qquad \sigma_I = \sigma_I^{(0)} + \delta\sigma_I$$

← *matter sector*

- Scalar-vector-tensor decomposition:

$$\beta_i = D_i\beta + S_i\,, \qquad \pi_i = D_i\pi + \pi_i^T\,,$$

$$h_{ij} = 2\psi\Omega_{ij} + \left(D_iD_j - \tfrac{1}{3}\Omega_{ij}\triangle\right)E + \tfrac{1}{2}(D_iF_j + D_jF_i) + \gamma_{ij}$$

$$D_i \leftarrow \Omega_{ij}\,, \quad \triangle \equiv \Omega^{ij}D_iD_j$$
$$D^iS_i = D^i\pi_i^T = D^iF_i = 0$$
$$D^i\gamma_{ij} = \gamma^i_i = 0$$

- Gauge invariant variables without Stückelberg fields:

Originate from $g_{\mu\nu}$ and matter fields $\delta\sigma_I$

$$Q_I \equiv \delta\sigma_I - \mathcal{L}_Z\sigma_I^{(0)}\,,$$
$$\Phi \equiv \phi - \tfrac{1}{N}\partial_t(NZ^0)\,,$$
$$\Psi \equiv \psi - \tfrac{\dot a}{a}Z^0 - \tfrac{1}{6}\triangle E\,,$$
$$B_i \equiv S_i - \tfrac{a}{2N}\dot F_i\,,$$

$$\left(\begin{array}{c} Z^0 \equiv -\frac{a}{N}\beta + \frac{a^2}{2N^2}\dot E \\ Z^i \equiv \frac{1}{2}\Omega^{ij}(D_jE + F_j) \\ \text{Under } x^\mu \to x^\mu + \xi^\mu : \\ Z^\mu \to Z^\mu + \xi^\mu \end{array}\right)$$

- However, we have 4 more degrees of freedom:

Associated with Stückelberg fields

$$\psi^\pi \equiv \psi - \frac{1}{3}\triangle\pi - \frac{\dot a}{a}\pi^0\,, \qquad E^\pi \equiv E - 2\pi\,, \qquad F_i^\pi \equiv F_i - 2\pi_i^T$$

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Alternatives

Introduce dynamics for fiducial metric

- Branch I solutions in bigravity, quasi-dilaton? Can dynamics of the fiducial metric resolve Higuchi/Vainshtein conflict?

Find new symmetry to naturally remove these degrees

- Imposing a symmetry to remove the degrees with zero κ?

Breaking the FRW symmetry in the fiducial metric

- It is still possible to have a H&I physical metric, while either H or I is broken in Stückelberg sector.
- Inhomogeneous examples already exist, although `d'Amico '12` showed that cancellation occurs in two such examples (`d'Amico et al '11` and `Koyama, Niz, Tasinato '11`).
- In our analysis, anisotropy introduced only as a technical tool. However, kinetic terms of these polarizations *are* second order. $\Longrightarrow$ A universe with finite anisotropy, which looks isotropic at the background level may have a chance to evade the ghost.

$\Longrightarrow$ *This is our next step*

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Anisotropic FRW

AEG, Lin, Mukohyama '12

- Consider Bianchi I metric, with finite anisotropy

$$ds^2 = -N^2 dt^2 + a^2 \left[e^{4\sigma} dx^2 + e^{-2\sigma} (dy^2 + dz^2) \right]$$

- Fiducial metric is de Sitter

$$ds_\phi^2 = -n^2 dt^2 + \alpha^2 \left(dx^2 + dy^2 + dz^2 \right) \longleftarrow \left[\frac{\dot{\alpha}}{\alpha\, n} = H_f = \text{constant} \right]$$

Vacuum configuration: Fixed points

- Seek solutions with $\dot{H} = \dot{X} = \dot{\sigma} = 0 \longleftarrow \left[H \equiv \frac{\dot{a}}{a\, N} ,\ X \equiv \frac{\alpha}{a} \right]$
- Dropping isotropic F.P., and points that require fine tuning gives

$$e^\sigma = \sqrt{\frac{H_f\, X}{H}}$$

- The remaining equations of motion reduce to algebraic equations on X and H.

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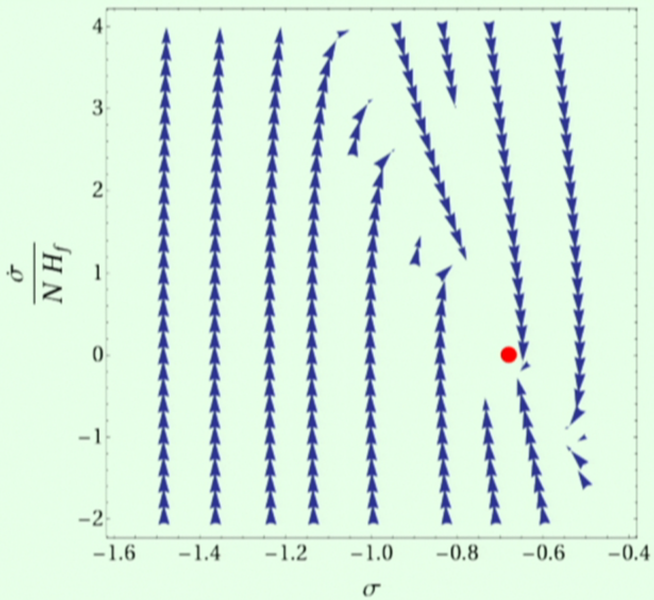
Stability of the anisotropic fixed point

Local stability

- Perturb H, σ and X around the F.P. value
- Can reduce the equations to
$$\delta\sigma'' + 3\,X_0\,e^{-2\,\sigma_0}\,\delta\sigma' + M^2\,\delta\sigma = 0 \longleftarrow \left[{}' \equiv \frac{1}{H_f\,N}\,\frac{d}{dt}\right]$$
- Local stability requirement: $M^2(\frac{m_g}{H_f},\,\alpha_3,\,\alpha_4) > 0$

Global Stability



- Parameters:
  $m_g = 20\,H_f\,,\ \alpha_3 = -\frac{1}{20}\,,\ \alpha_4 = 1$
- Fixed point:
  $X \simeq 4\,,\ e^{\sigma} \simeq \frac{1}{2}\,,\ H \simeq 16\,H_f$
- On F.P., isotropic expansion $\dot\sigma = 0$.
  In GR, this is equivalent to a FRW universe. In MG, a coordinate redefinition renders physical metric isotropic, but now the fiducial metric becomes anisotropic.
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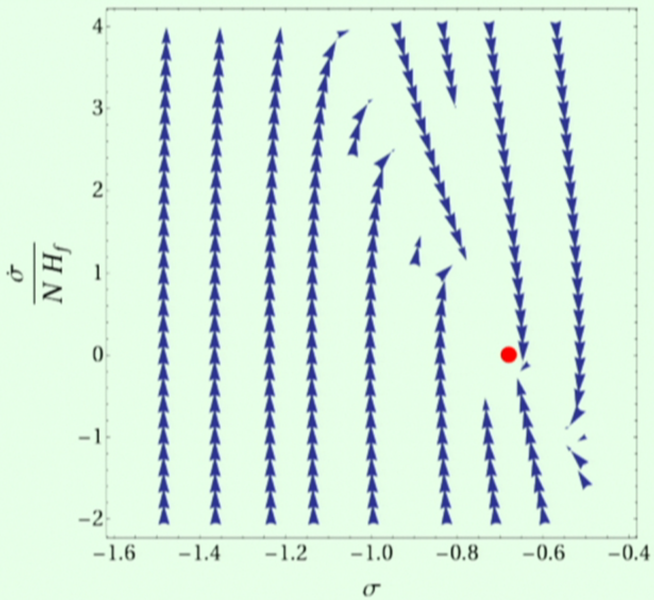

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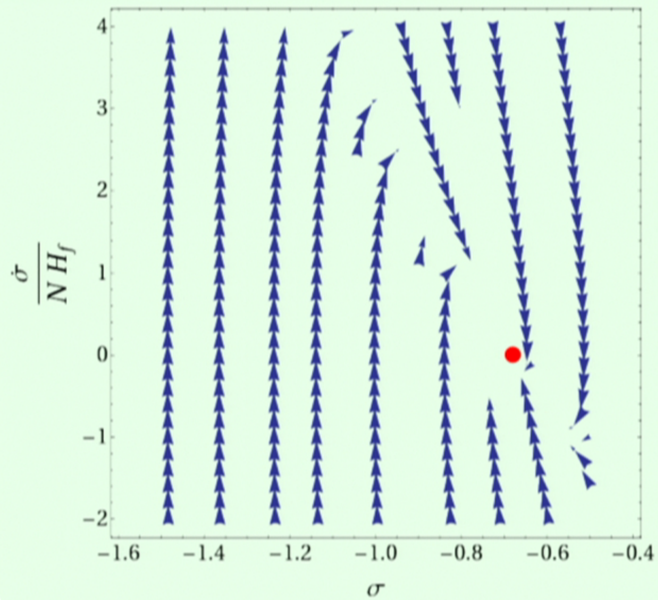

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- Although the theory admits self-accelerating, isotropic and homogeneous universe solutions, these suffer from a non-linear instability. This conclusion is valid for any fiducial metric, and may extend to spherically symmetric solutions, as well as self-accelerating solutions in other versions of the theory.
- We have introduced a new solution with finite anisotropy, while the expansion is purely isotropic. The background dynamics is equivalent to FRW, Anisotropy appears in the Stückelberg sector, can be probed by metric perturbations. We expect the breaking of statistical anisotropy to be subdominant by the smallness of m_g.
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