AIMS VIDEO COURSES

SUPPORTING BOOKLET

PROBABILITY & STATISTICS

WITH

PROF DAVID SPIEGELHALTER

AIMS

SOUTH AFRICA

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African Institute for Mathematical Sciences

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AIMS Online Courses

The mission of the AIMS academic programme is to provide an excellent, advanced education in the mathematical sciences to talented African students in order to develop independent thinkers, researchers and problem solvers who will contribute to Africa's scientific development.

Teaching at AIMS is based on the principle of learning and understanding, rather than simply listening and writing, during classes, and on creating an atmosphere of increasing our knowledge through class discussions, through small group discussions, by formulating conjectures and assessing the evidence for them, and sometimes going down wrong paths and learning from the mistakes that led us there. The essential features of the classes at AIMS are that, in contrast to formal lecture courses, they are highly interactive, where the students engage with the lecturer throughout the class time, are encouraged to learn together in a journey of questioning and discovery, and where lecturers respond to the needs of the class rather than to a pre-determined syllabus. AIMS teaching philosophy is to promote critical and creative thinking, to experience the excitement of learning from true understanding, and to avoid rote learning directed only towards assessment.

Leading international and local experts offer the courses at AIMS, which are three weeks long (each module consisting of 30 hrs) and collectively form the coursework for a structured masters degree which also includes a research component. The advertised content is a guide, and the lecturers are encouraged, and indeed expected, to adapt daily to meet the current needs of the students.

Over the past ten years AIMS has achieved international recognition for this innovative and flexible approach. It has been the starting point for the remarkable success of our students and alumni and we all benefit from the support of many who have "witnessed the AIMS-magic and keep coming back for more."

This year we have decided to film selected courses and to make them available to a larger audience as an online facility. African universities may choose to use these courses to supplement and enhance their own postgraduate programmes. We believe this would be best achieved through engagement with AIMS. One way for this to happen, would be for AIMS to suggest or nominate a specialist tutor to spend time at the university, guiding students who follow the online programme. Where possible expert lecturers who have taught at AIMS may visit the university to give a short introduction to the course. We would welcome this interaction as well as the contribution our online courses will make to the growth of the mathematical sciences ecosystem in Africa.

Barry Green
Director & Professor of Mathematics
African Institute for Mathematical Sciences
January 2013

PROBABILITY & STATISTICS

2012

PROF DAVID SPIEGELHALTER

DAY 10

Class Data Collection

This is anonymous - please do not put your name.

We chose a random number between 1 and 100

The number selected and assigned to you is $X =$

1. Do you think that the *percentage* of countries, amongst all those in the United Nations, that are in Europe is **higher** or **lower** than X?
2. Give your best estimate of the *percentage* of countries, amongst all those in the United Nations, that are in Europe
3. How many children does your oldest uncle or aunt have?

NOTE TO THE TUTORS:

Assign half of the question sheets the number 10 and the other half the number 60 (thus not random, but without the students knowing).

Data collection

Please answer questionnaire: (then get tutors to calculate averages)

A lucky escape

SA Time: 30 September 2012 10:52:34 GMT+02:00

Wave-smashing surprise for 2 restaurants

September 29 2012 at 12:48pm
By JUNIOR BESTER

Comment on this story

Customers having lunch at two popular Kalk Bay restaurants got a lot more than they bargained for on Friday when a huge wave shattered four large picture windows which usually offer patrons one of the city's most sought-after views.

Furniture was tossed around and electrical appliances destroyed in the affected eateries, the upstairs Harbour House and Live Bait below.

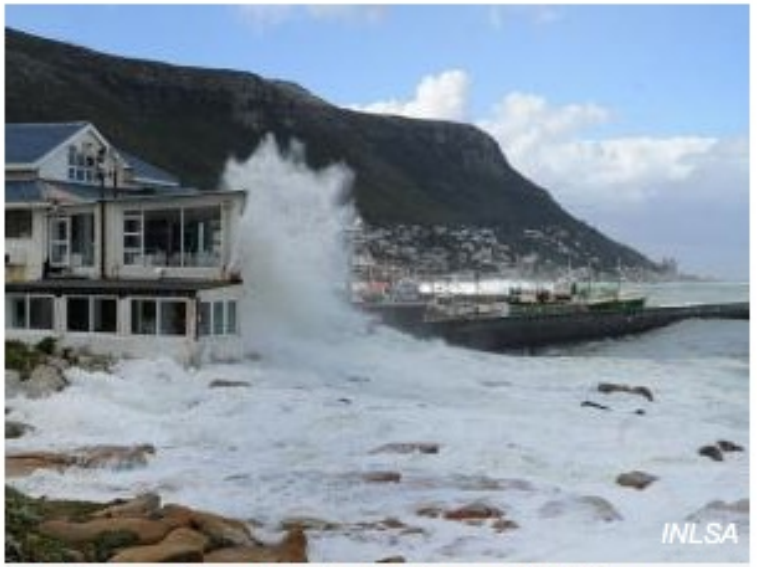
"We were almost hypnotised by the huge waves smashing against the window.

"Then all of a sudden it just smashed through the window," a still shocked Live Bait customer, Michelle Attala, said.

Their waitress had warned them they should move away from the windows before high tide at 4pm, but they had joked that they would "be brave"

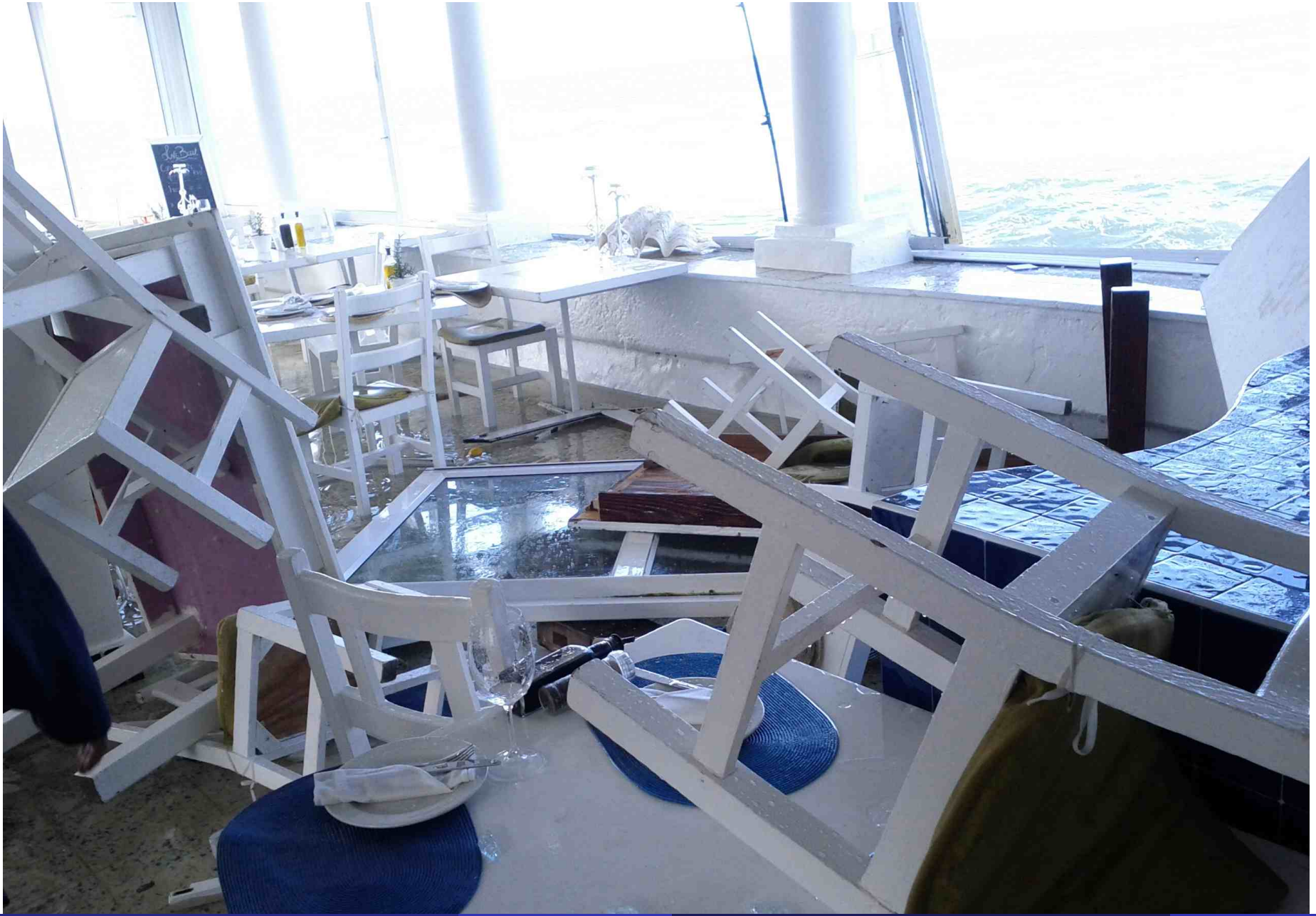
.

The incident occurred at about 1.30pm.



Waves smash against the windows of Kalk Bay harbour restaurants. The large picture windows of both Harbour House and Live Bait were smashed in, resulting in serious damage to the interiors and one minor injury. Photo: Leon Lestrade.

A lucky escape



Statistical discovery

1. Important problem
2. **Good survey / experimental design**
3. Good quality data
4. Data exploration and visualisation
5. Modelling data as random variables arising from some distribution
6. Formal statistical inference about the truth

Broad types of study

- Sample surveys
- Experiments
- Observation of large populations

http://stattrek.com/tutorials/statistics-tutorial.aspx

Bias and 'error'

Want to expect to get the right thing if we take a big enough sample, $\hat{\theta} \to \theta_0$

Bias is when, even with a big sample, we will get the wrong answer. i.e. $\mathbb{E}[\hat{\theta}] - \theta_0 = \text{bias} \neq 0$

Sampling error does not mean a mistake, it is just the standard error $\sqrt{V} = \sigma/\sqrt{n}$ and will get small as n gets large.

Sample surveys

Generally to estimate proportion p

e.g. the percentage of students in your university on Facebook

If unbiassed, then asking $n = 1000$ students will give a proportion $\hat{p} = X/n$ with $\mathbb{E}[\hat{p}] = p$

Standard error for $\hat{p}$ is $\sqrt{V} = \sqrt{\hat{p}(1-\hat{p})/1000}$

This is maximised when $\hat{p} = 0.5$, when $\sqrt{V} = 1/\sqrt{4000} = 0.016$

95% interval for p is $\hat{p} \pm 1.96\sqrt{\hat{p}(1-\hat{p})/1000}$; at most $\hat{p} \pm 0.032$

So with 1000 people, can estimate a true proportion with less than 3% sampling error.

Generally, surveys need only be of 1000 people

Exercise

How would you estimate the proportion of students at your university who are on Facebook?

How would you sample students?

Discuss.

Methods for sample surveys

Non-probability samples, e..g.

- Ask for volunteers
- 'Convenience sample' - ask who is easiest, walking past, your friends etc

But we can't estimate our error with these methods

Probability samples (random selection)

- Simple random sampling (all possible combinations equal likely)
- Stratified sampling (e.g. sampling areas, and then sampling houses)
- Cluster sampling (e.g. sample areas, and then ask every house in that area)

Can adjust confidence intervals for these different methods

Can adjust for known imbalances to estimate proportion in the population

Sampling Bias in surveys

The 'Sampling Frame' should be a *representative sample*

Things go wrong if

- Inadequate coverage: some people are never asked
- Non-response: people are asked but do not answer

What are the problems with

- telephone surveys?
- Internet surveys?
- surveys of listeners to radio shows?

Measurement bias

The questions should be balanced

Things go wrong if

- The question is loaded towards one response
- People cannot understand the question
- People do not feel confident about confidentiality

Example:

* have you stopped beating your wife?

* do you think the police are doing enough about the terrible crime problem?

Experiments

Evaluating interventions / treatments

Suppose we wanted to see whether 'conditional cash transfers' reduced the spread sexually transmitted infections (STIs) in a rural African setting?

('Conditional cash transfers': i.e. couples given cash provided they were tested *and* did not acquire infections)

What might we do?

Discuss

Experiments

Broad options:

1. Observe the outcomes of people who got cash and people who did not ('observational')
2. Observe current outcomes, and then introduce cash transfers and see what changes there were ('before/after')
3. Randomly allocate people to 'cash' and 'no cash', and see what happened

Randomisation should control bias - any resulting difference between the groups must be either due to chance or the intervention.

Use P-values to see whether could be due to chance

Randomised controlled trials of treatments

Open Access Research

BMJ open
accessible medical research

Incentivising safe sex: a randomised trial of conditional cash transfers for HIV and sexually transmitted infection prevention in rural Tanzania

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Randomised controlled trials of treatments

ABSTRACT

Objective: The authors evaluated the use of conditional cash transfers as an HIV and sexually transmitted infection prevention strategy to incentivise safe sex.

Design: An unblinded, individually randomised and controlled trial.

Setting: 10 villages within the Kilombero/Ulanga districts of the Ifakara Health and Demographic Surveillance System in rural south-west Tanzania.

Participants: The authors enrolled 2399 participants, aged 18–30 years, including adult spouses.

Interventions: Participants were randomly assigned to either a control arm (n=1124) or one of two intervention arms: low-value conditional cash transfer (eligible for $10 per testing round, n=660) and high-value conditional cash transfer (eligible for $20 per testing round, n=615). The authors tested participants every 4 months over a 12-month period for the presence of common sexually transmitted infections. In the intervention arms, conditional cash transfer payments were tied to negative sexually transmitted infection test results. Anyone testing positive for a sexually transmitted infection was offered free treatment, and all received counselling.

Randomised controlled trials of treatments

Main outcome measures: The primary study end point was combined prevalence of the four sexually transmitted infections, which were tested and reported to subjects every 4 months: *Chlamydia trachomatis*, *Neisseria gonorrhoeae*, *Trichomonas vaginalis* and *Mycoplasma genitalium*. The authors also tested for HIV, herpes simplex virus 2 and syphilis at baseline and month 12.

Results: At the end of the 12-month period, for the combined prevalence of any of the four sexually transmitted infections, which were tested and reported every 4 months (*C trachomatis*, *N gonorrhoeae*, *T vaginalis* and *M genitalium*), unadjusted RR for the high-value conditional cash transfer arm compared to controls was 0.80 (95% CI 0.54 to 1.06) and the adjusted RR was 0.73 (95% CI 0.47 to 0.99). Unadjusted RR for the high-value conditional cash transfer arm compared to the low-value conditional cash transfer arm was 0.76 (95% CI 0.49 to 1.03) and the adjusted RR was 0.69 (95% CI 0.45 to 0.92). No harm was reported.

STIs tested every 4 months by nucleic acid amplification tests in the group that was eligible for the $20 payments, but no such reduction was found for the group receiving the $10 payments.

- The results suggest that conditional cash transfers used to incentivise safer sexual practices are a potentially promising new tool in HIV and STIs prevention. Additional larger study would be useful to clarify the effect size, to calibrate the size of the incentive and to determine whether the intervention can be delivered cost effectively.

Randomised controlled trials of treatments

Randomisation

Individual-level randomisation took place at the study station after baseline interview and testing, with participants selecting coloured balls from an opaque bag. The randomisation took place in public view and in two stages with participants first randomly selecting one of four balls to determine their allocation to the intervention or the control arm. In order to study potential peer-effects, in randomly selected subvillages, the probably of selection in the intervention arm was 75% (three balls out of four) and in the other subvillages, it was 25% (one ball out of four); based on the distribution of participants across subvillages, we thus expected 48% of the overall sample to be randomised into the control arm. Participants randomised into the intervention arm were further invited to choose one of two balls from a second bag determining in which of the two intervention arms (low-value conditional cash transfers and high-value conditional cash transfers) they would be allocated. These highly transparent procedures were deemed necessary for acceptability of randomisation in a popu-

Experiments

Broad options:

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Randomisation should control bias - any resulting difference between the groups must be either due to chance or the intervention.

Use P-values to see whether could be due to chance

Randomised trials

Good principles of randomised trials

1. A written protocol for design, analysis and reporting
2. Pre-specify primary and secondary outcome measures to avoid 'fishing expeditions'
3. Randomly allocate subjects to treatments
4. If possible, keep both subjects and researchers 'blind' to the treatment given ('double-blind'). This may involve 'placebo' (pretend) treatments
5. Minimise missing data/dropout, and report if it happens
6. Pre-specify reasons for stopping the trial
7. Report results, whatever they are
8. Report according to how people were randomised (even if they did not exactly follow the treatment)
9. Appropriate statistical analysis: estimates, intervals, P-values etc

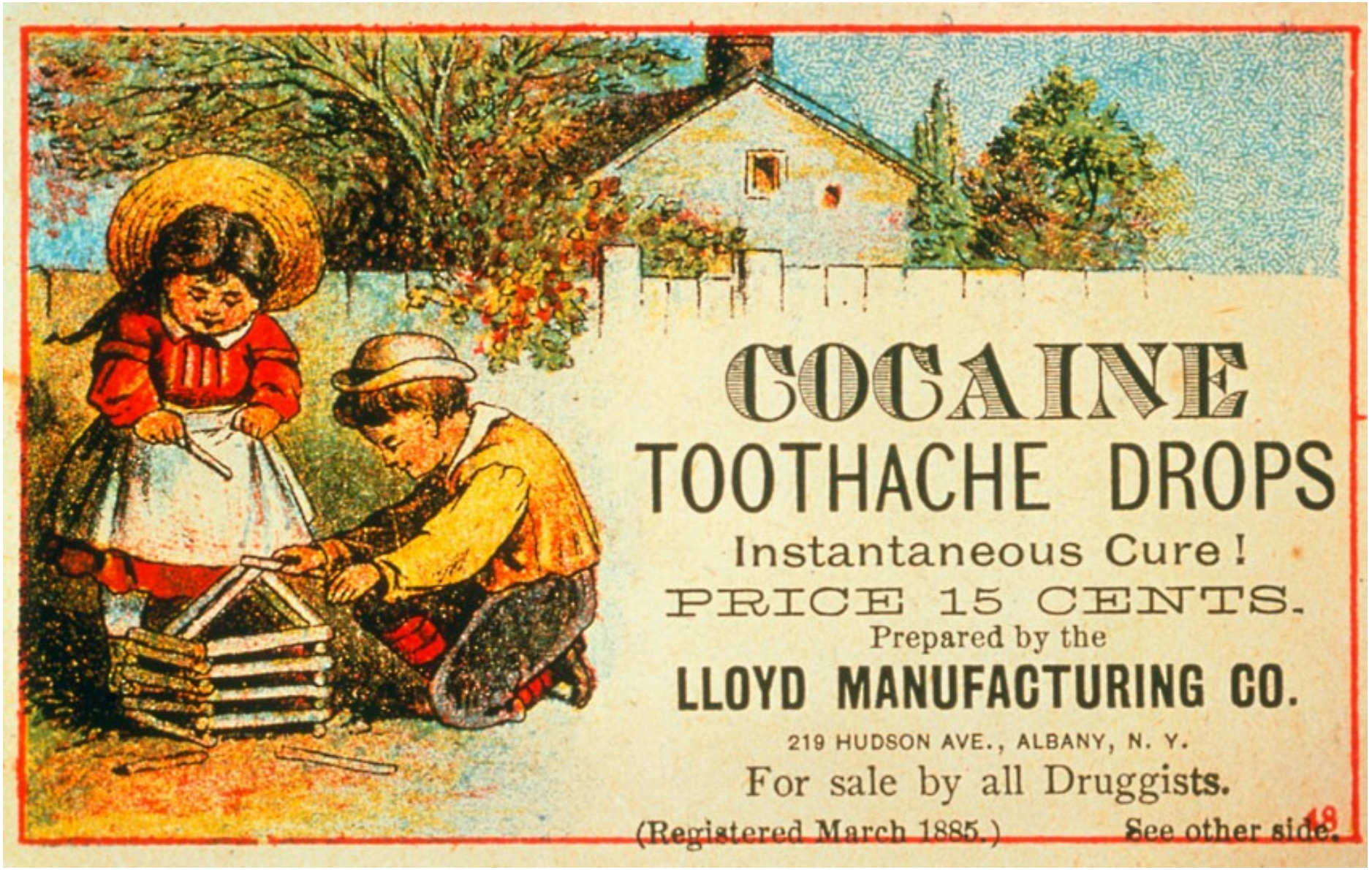
Randomised trials

To have a new pharmaceutical product approved for sale, the drug should be proved to be safe and effective

Usually 2 randomised trials, each with $P < 0.05$

This is to avoid ineffective, and possibly dangerous, treatments.

Like these old American treatments.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TOOTHACHE DROPS
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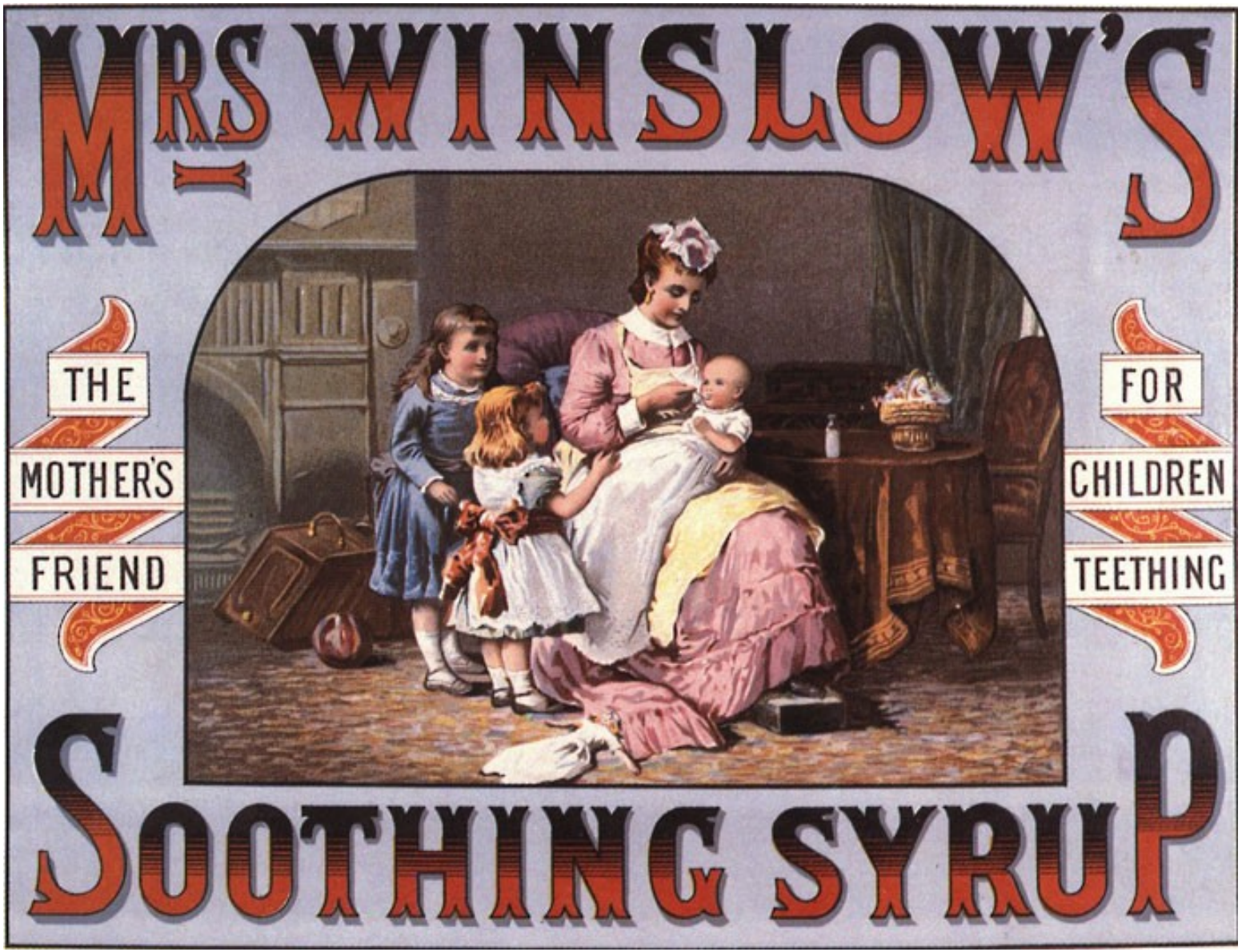
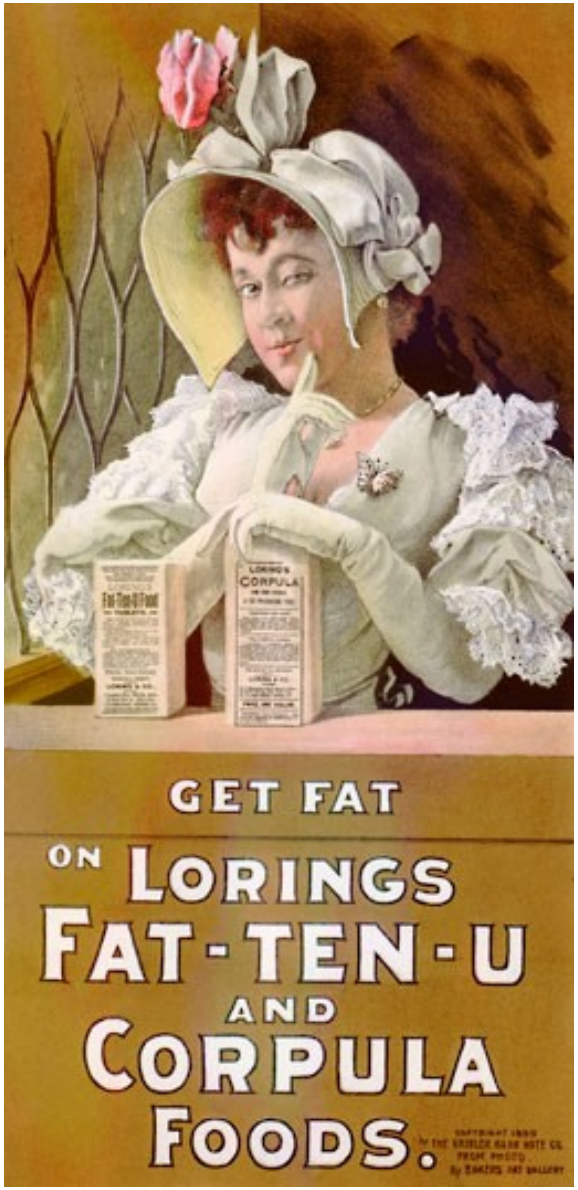
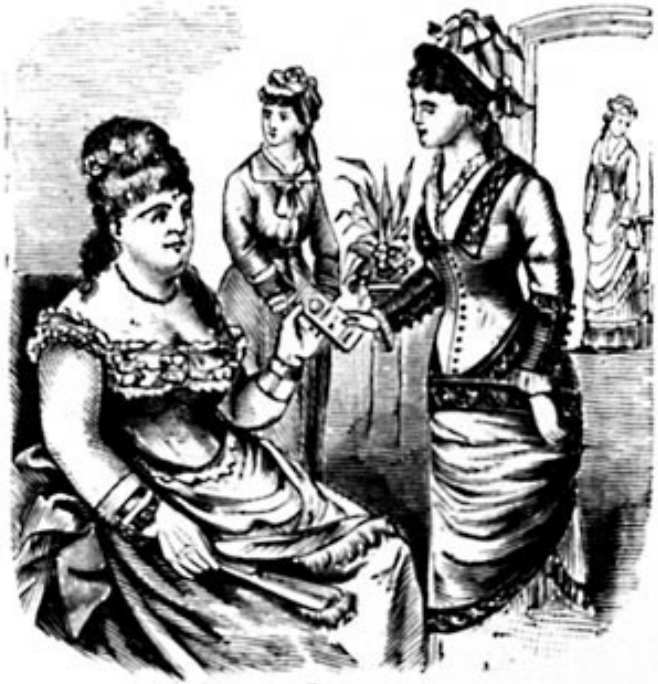
PRICE 15 CENTS.
Prepared by the
LLOYD MANUFACTURING CO.
219 HUDSON AVE., ALBANY, N. Y.
For sale by all Druggists.
(Registered March 1885.)
See other side.
18

Contained morphine




GET FAT
ON LORINGS
FAT-TEN-U
AND
CORPULA
FOODS.

ANTI-FAT



The Great Remedy for Corpulence

ALLAN'S ANTI-FAT

is composed of purely vegetable ingredients, and is perfectly harmless. It acts upon the food in the stomach, preventing its being converted into fat. Taken in accordance with directions, **it will reduce a fat person from two to five pounds per week.**