Title: Supermassive black holes in non-spherical galactic nuclei and enhanced rates of star capture events

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Abstract: We consider the stellar-dynamical processes which lead to the capture or tidal disruption of stars by a supermassive black hole, review the standard theory of two-body relaxation and loss-cone repopulation in spherical galactic nuclei, and extend it to the axisymmetric and triaxial nuclear star clusters.

In the absense of symmetry which conserves angular momentum, the orbits of stars experience regular or chaotic changes of angular momentum even in the smooth potential of star cluster, which creates a substantial population of "centrophilic" orbits. We discuss the loss cone draining rates, i.e. rates of capture of stars from these orbits.

Next we consider the relaxation phenomena in non-spherical nuclei, focusing on the differences between spherical, axisymmetric and triaxial cases. It turns out that the rates of repopulation of the loss cone are moderately higher in non-spherical systems, but in the triaxial case an additional, often substantial, increase of capture rates comes from draining of the centrophilic orbit population.

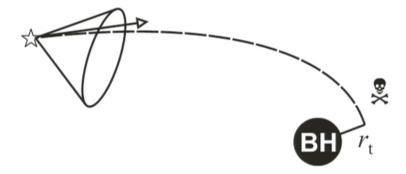
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Capture of stars by a supermassive BH

The black hole captures or tidally disrupts stars passing at a distance closer than $r_t \geq r_{\bullet} \equiv \frac{2GM_{\bullet}}{c^2}$, or, equivalently, with angular momentum

$$L^2 < L_{\bullet}^2 = \max\left[\left(\frac{4GM_{\bullet}}{c}\right)^2, GM_{\bullet}r_t\right]$$

The region of phase space with $L < L_{\bullet}$ is called the loss cone.



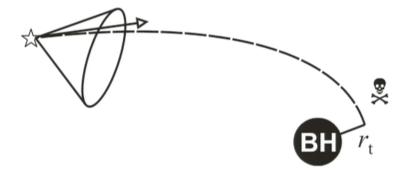
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Nuclear star clusters

- Supermassive black hole M_{bh}
- Stellar cusp (for example, a power law density profile $\rho \sim r^{-\gamma}$)
- Total gravitational potential (non-spherical):

$$\Phi(\overrightarrow{r}) = -rac{GM_{bh}}{r} + \Phi_\star \left(rac{r}{r_0}
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- Consider motion inside radius of influence r_{infl} = GM_{bh}/σ²
 => dominant contribution to potential is from SMBH
 => orbits are perturbed Keplerian ellipses
 which precess due to torques from stellar potential
- Orbital time T_{rad} << precession time $T_{prec} \sim r_{infl}/\sigma$

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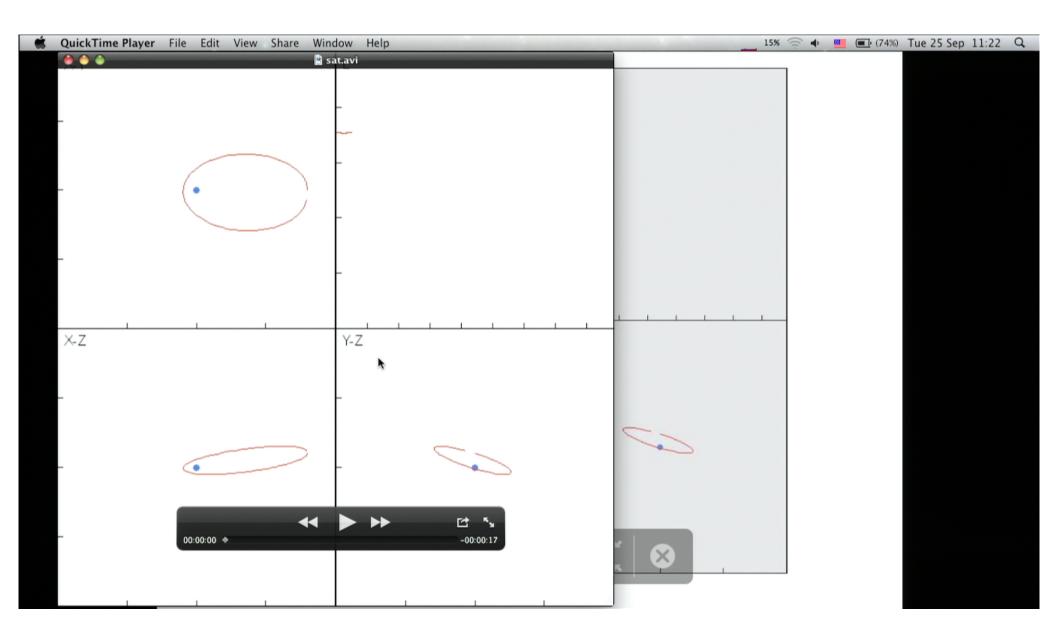
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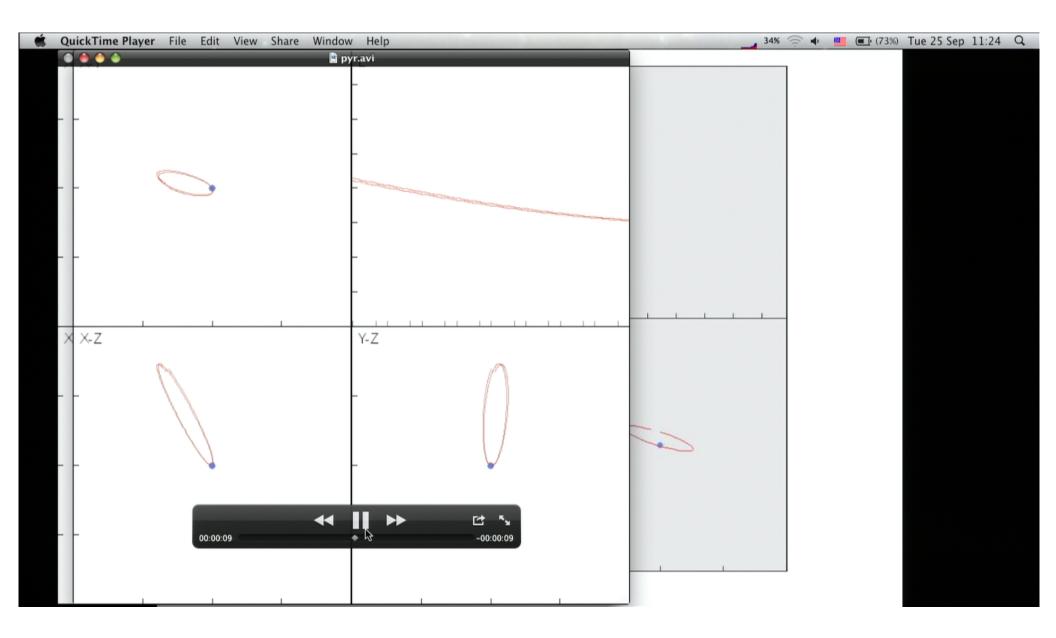
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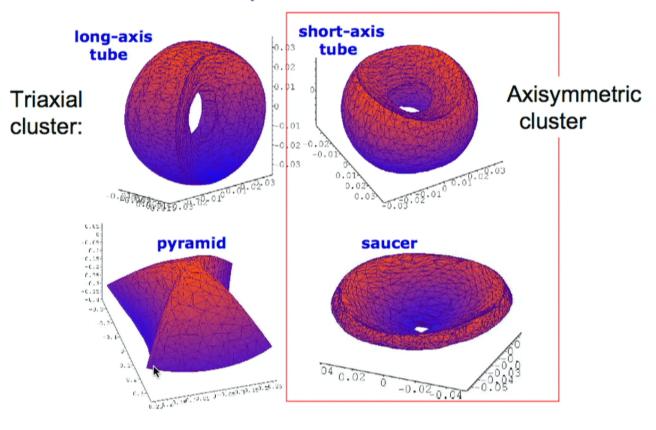


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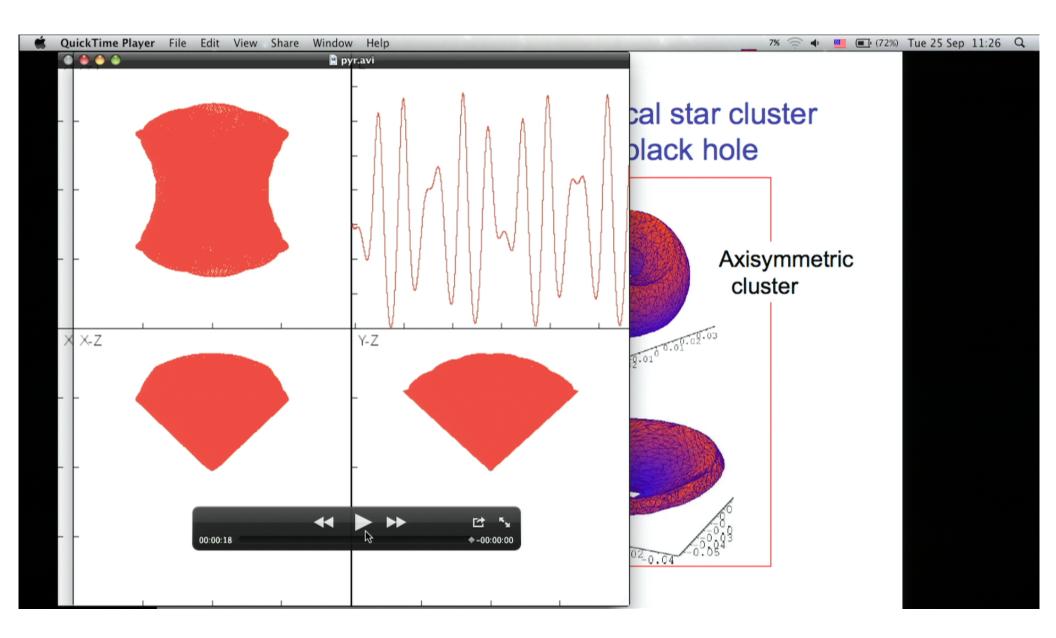


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Types of orbits in non-spherical star cluster around a supermassive black hole



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Motion in a near-keplerian potential

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- "Fast" timescale radial period $T_{\rm rad} = \frac{2\pi r^3}{\sqrt{GM_{\bullet}}}$.
- "Slow" timescale precession period due to distributed mass $T_{\rm prec} = T_{\rm rad} \frac{M_{\bullet}}{M_{\star}(r)}$.

The separation of fast and slow timescales allows for the existence of an additional integral of motion $\mathcal{H} = \oint_{\text{orbit}} \Phi_{\star}(r)$.

In both axisymmetric and triaxial cases the motion is **completely integrable**.

Integrals of motion: **E** (total energy), **H** (secular hamiltonian), **L**_Z (z-component of angular momentum) in axisymmetric case / another integral in triaxial case.

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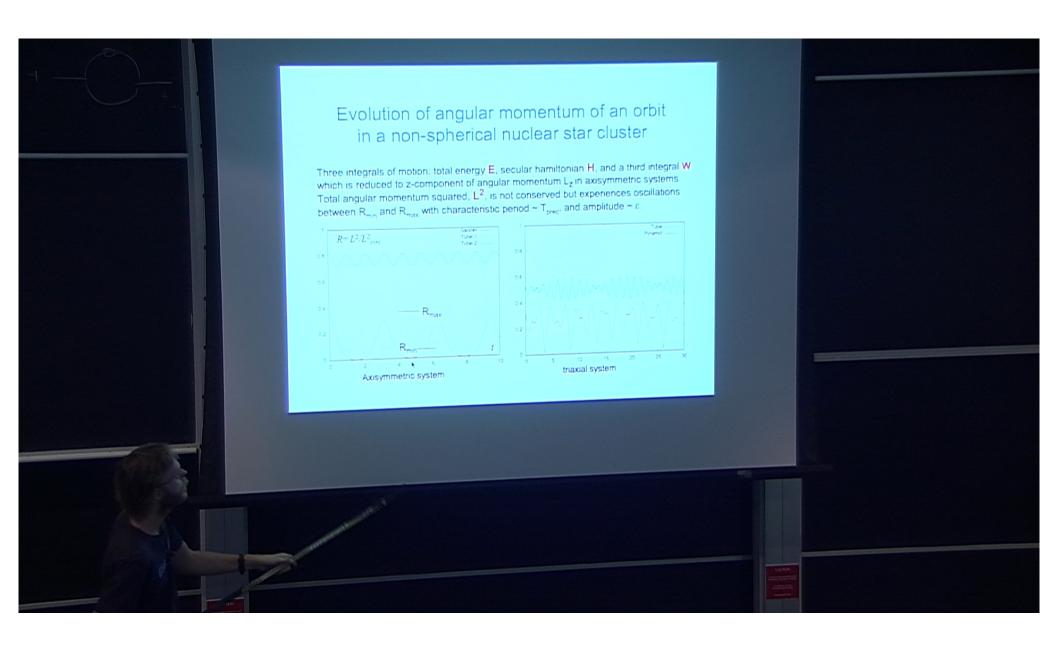
"Extended loss region" in a non-spherical nuclear star cluster

The region of phase space (E, H, W) occupied by orbits for which the squared angular momentum L² may drop below the capture boundary L², is called "extended loss region"

- For axisymmetric systems, the condition of being in the extended loss region is L_Z < L. and (L²) < ε (i.e. only a fraction ~ ε of orbits with z-component of angular momentum below capture boundary may actually be captured).
- For triaxial systems, all pyramid orbits^(*) are centrophylic (i.e. may attain arbitrary low values of angular momentum), their fraction in the total population is ~ ε.
 - (*) relativistic effects change this conclusion for most tightly-bound orbits

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	Spherical	Axisymmetric	Triaxial
Fraction of stars with L^2_{min} < X	∝ X	$\propto \sqrt{X\epsilon}$	∞ ε
Fraction of time that such a star has L ² < X (i.e. capture probability)	1	\sqrt{X}	X
Survival time of such stars (assuming they are captured immediately after reaching L ² < R _{capt})	(10 ¹⁻⁵ yr)	T _{prec} (10 ⁵⁻⁶ yr) MW nucleus)	may be longer than 10 ¹⁰ yr
but that may not be true in the presence of relaxation	fraction of stars increases timescale for draining the loss region		

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Relaxation time $T_{rel} = \frac{0.34\,\sigma(r)^3}{G^2\,\overline{m}_\star\,\rho_\star(r)\,\ln\Lambda}$ – timescale for diffusion in E and L

Definition of "classical" loss cone for the spherical case:

Loss cone is the region in phase space in which an orbit would be captured(*) at the nearest pericenter passage, i.e. at most within 1 radial period, having $L^2 < L^2_{\bullet}$ (*) in the absence of relaxation

The question is how fast the changes in L occur compared to radial period:

$$q = \Delta L^2/L^2$$

q << 1 – empty loss cone regime:

stars are captured as soon as they enter LC; population of stars with L<L, is small

q >> 1 - full loss cone:

stars may move in and out of LC many times before being captured at the end of T_{rad}, distribution function of stars inside LC is the same as outside (near its boundary)

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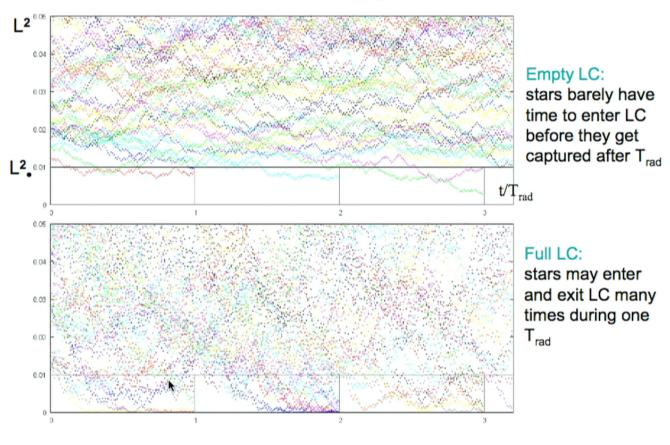
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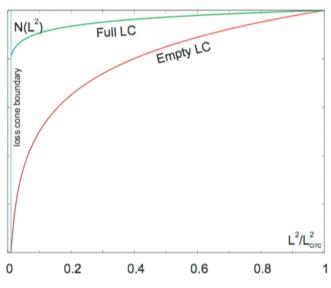
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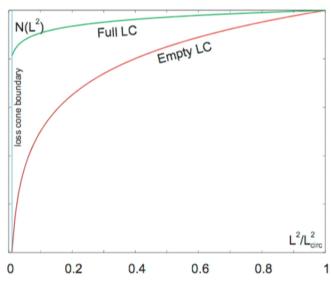
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- Distribution function of stars in angular momentum is a solution to Fokker-Planck (diffusion) equation with a "sink" at loss cone boundary
- The global quasi-stationary solution at given E has a logarithmic profile:
 N(L²) = N₀ + A*log(L)
- In the empty loss cone regime, the capture rate is dominated by the slope of N(L), i.e. limited by the diffusion coefficient
- In the full loss cone regime, capture rate is simply the size of loss cone divided by T_{rad}
- Stars close to BH are in empty regime, outer parts of the galaxy may be in full loss cone regime



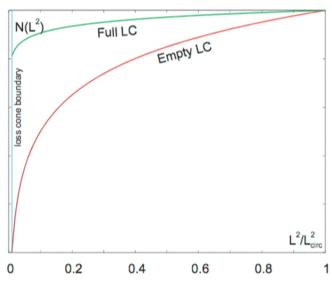
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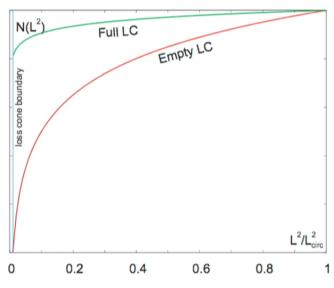
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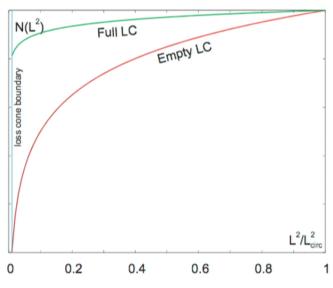
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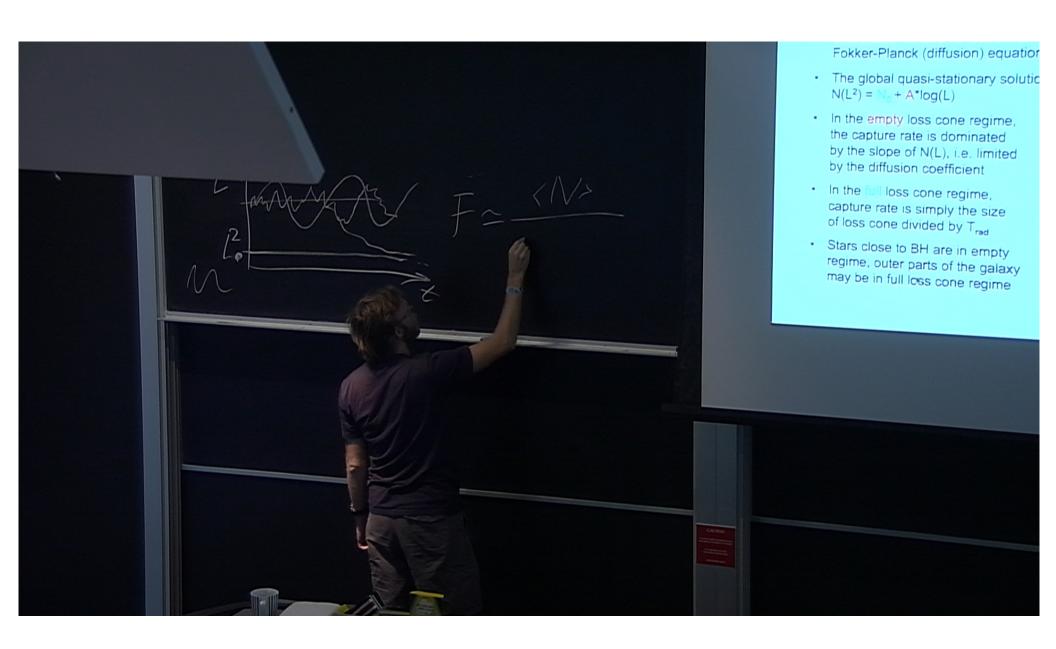


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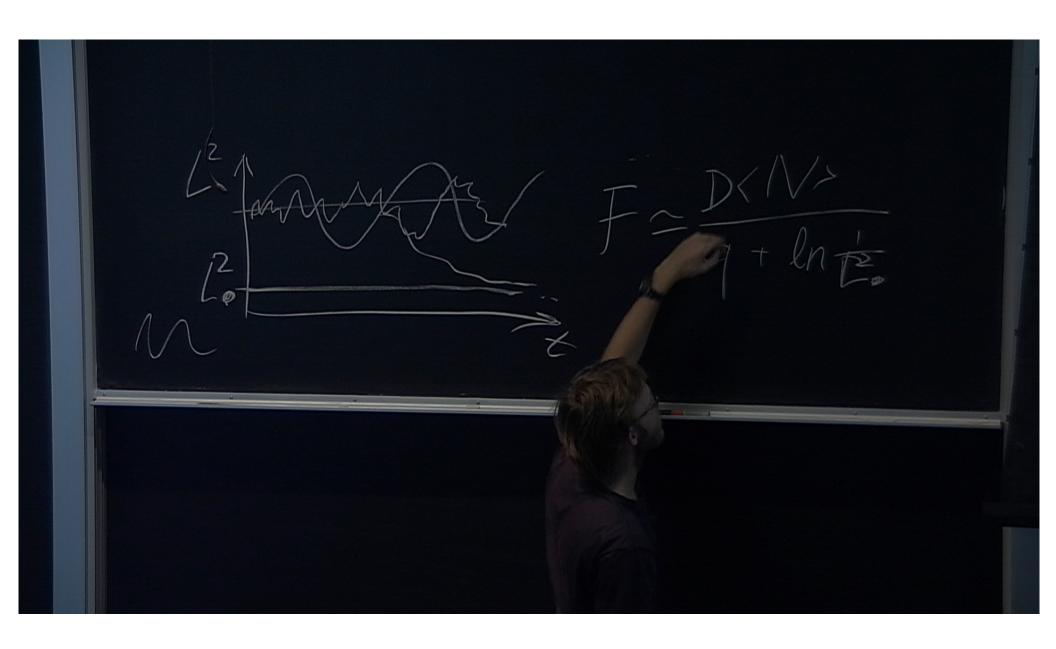
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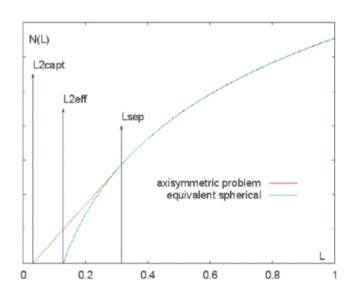
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Non-spherical galaxies: Loss cone draining vs. relaxation

- Draining time of the loss region may be >10¹⁰yr in the triaxial case, and the capture rate from the loss region depends on the efficiency of changing of ang.momentum due to regular precession rather than due to 2-body relaxation.
- After all orbits with L²_{min}<L².
 have been drained, the influx of stars from higher L is still limited by diffusion (relaxation)
- Because the size of loss region is larger that spherical loss cone, flux will be larger in the diffusionlimited regime
- In the full loss cone regime there is almost no difference from the spherical case



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Conclusions

- In non-spherical nuclear star clusters the star angular momentum L is changed not only due to 2-body relaxation, but also due to regular precession in the smooth additional potential of stellar cluster
- This facilitates the capture of stars at low L:
 the "extended loss region" is where L²_{min}<L², not just L²<L².
- Draining time of this region $T_{drain} \sim T_{prec} \sim 10^{5-6}$ yr in axisymmetric case and much longer, comparable to Hubble time, in triaxial case
- For T >> T_{drain}, capture rate is determined by relaxation with a larger effective capture boundary. Compared to the spherical case, the enhancement in total capture rate is moderate (~factor of few) and is important only in the range of energies where the loss cone would be empty in the spherical case.
- For giant elliptical galaxies, which are deeply in the empty loss cone regime, the enhancement may be more dramatic.
- This applies to the rates of tidal disruption events, EMRI, binary SMBH, ...

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