

Title: Charge fractionalization and entanglement entropy for compressible phases in holography

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Abstract:

Charge fractionalization and entanglement entropy for compressible phases in holography



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Condensed Matter Seminar - PI

Sept 11, 2012

S. Hartnoll, S. Sachdev, B. Swingle



References:

LH, S. Sachdev, Phys Rev D 84, 026001 (2011)

S. Hartnoll, LH, Class. Quantum Grav. 29, 194001 (2012)

LH, S. Sachdev, B. Swingle, Phys Rev B 86, 035121 (2012)

Motivation

- ◉ Challenge: strongly interacting electrons
- ◉ Holography as a tool?

- ◉ Here: focus on compressible phases ($T=0$)
- ◉ Arise naturally in holography
- ◉ Can we characterize these phases?

Outline

- ⊙ Compressible phases
- ⊙ Compressible phases in holography
- ⊙ Characteristic features
 - Part 1: Charge fractionalization
 - Part 2: Holographic entanglement entropy
- ⊙ Connection to compressible phases in condensed matter
- ⊙ Recent developments and outlook

Compressible phases

- ◉ Key example: Fermi Liquid
- ◉ $d > 1$, translational invariance
- ◉ Globally conserved U(1) charge, Q
- ◉ Expectation value, $\langle Q \rangle$, varies smoothly as a function of chemical potential μ

$$d\langle Q \rangle / d\mu \neq 0$$

Compressible phases

◉ Fermi Liquid

- Interacting many-particle system adiabatically connected to non-interacting Fermi gas
- Fermi Surface (Green's function pole)
- Long-lived quasi-particles
- Signatures: resistivity $\sim T^2$, specific heat $\sim T$

◉ Non Fermi Liquid

- Strongly interacting system; fermions couple to emergent dynamical gauge field
- Fermi Surface (Green's function singularity)
- No long-lived quasi-particles

◉ FL*: both types of Fermi surfaces

Fermi surface

- ◉ Luttinger theorem relates total charge, $\langle Q \rangle$, to area, A , enclosed by Fermi surface:
$$\langle Q \rangle = A$$
- ◉ The Fermi surface is locus in momentum space, $k = k_F$, where Green's function is singular
- ◉ *Compressible phases have Fermi surface*

Powell, Sachdev, Büchler, PRB 72, 024534 (2005);
Coleman, Paul, Rech, PRB 72, 094430 (2005);
LH, Sachdev, PRD 84, 026001 (2011)

Compressible phases in holography

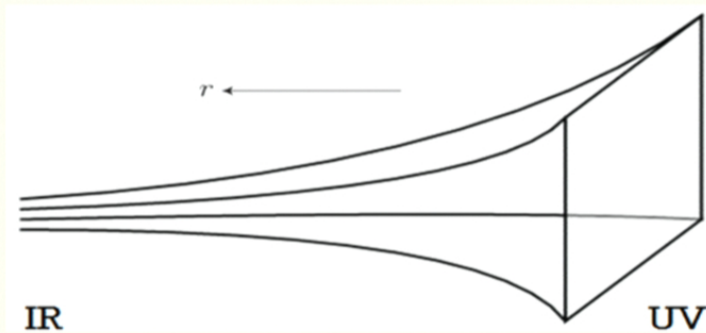
AdS/CFT duality

BULK

- Dimensions: $d+1$
- AdS spacetime

BOUNDARY

- Dimensions: d
- CFT

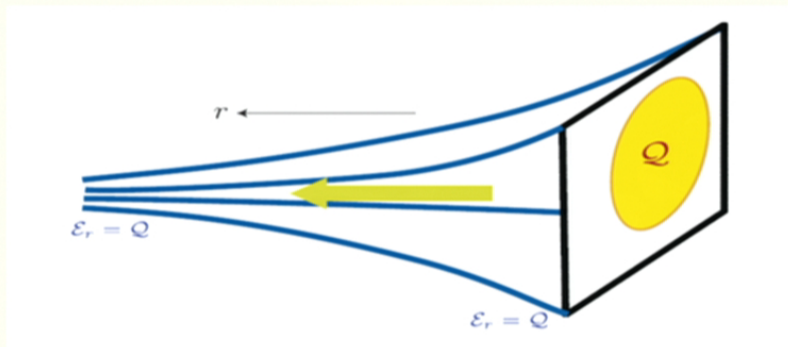


Maldacena, *Adv. Theor. Math. Phys.* 2, 231 (1998)

Finite charge density

BULK

- Dimensions: $d+1$
- AdS spacetime
- Electric field sourced by bulk charges



BOUNDARY

- Dimensions: d
- CFT
- Finite chemical potential



Electric field sources

- ◉ Boundary theory at finite chemical potential is dual to a bulk theory with charge sources
- ◉ Two types of charge sources:
 - Charged fermionic particles in the bulk
 - Charge hidden behind horizon
- ◉ Three possibilities: cohesive, fully fractionalized and partially fractionalized

Electric field sources

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- ◉ Three possibilities: cohesive, fully fractionalized and partially fractionalized

Einstein-Maxwell theory

- Effective field theory in the bulk

$$\mathcal{L} = \frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}$$

- Simplest theory dual to a compressible theory
- Bulk solution is extremal black hole
- Zero temperature entropy
- Solution: include dilaton field

Einstein-Maxwell-dilaton theory

- Effective field theory in the bulk:

$$\mathcal{L} = \frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{\mu\nu} F^{\mu\nu}$$

- Dilaton couples to Maxwell fields: $Z(\Phi)$
- Dilaton potential: $V(\Phi)$
- Dilaton (neutral scalar) mass is related to scaling dimension of dual operator, we choose it relevant
- UV theory at boundary will flow to an IR theory in the bulk under RG

Charged fluid

- Effective field theory in the bulk including fermions explicitly:

$$\mathcal{L} = \frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{\mu\nu}F^{\mu\nu} + p(\mu_{\text{loc}})$$

- Pressure of the fluid: $-p = \rho - \mu_{\text{loc}} \sigma$
- Fluid description allows us to include backreaction

Hartnoll, Tavanfar, PRD 83,046003 (2011)

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Outline

- ◉ Solve eom in the UV and IR
- ◉ UV solution: AdS_4
- ◉ Multiple IR solutions:
 - Lifshitz solution
 - Fractionalized solution
 - Cohesive solution
- ◉ We obtain phase diagram as a function of coupling to relevant operator dual to dilaton

Hartnoll, LH, CQG 29, 194001 (2012)

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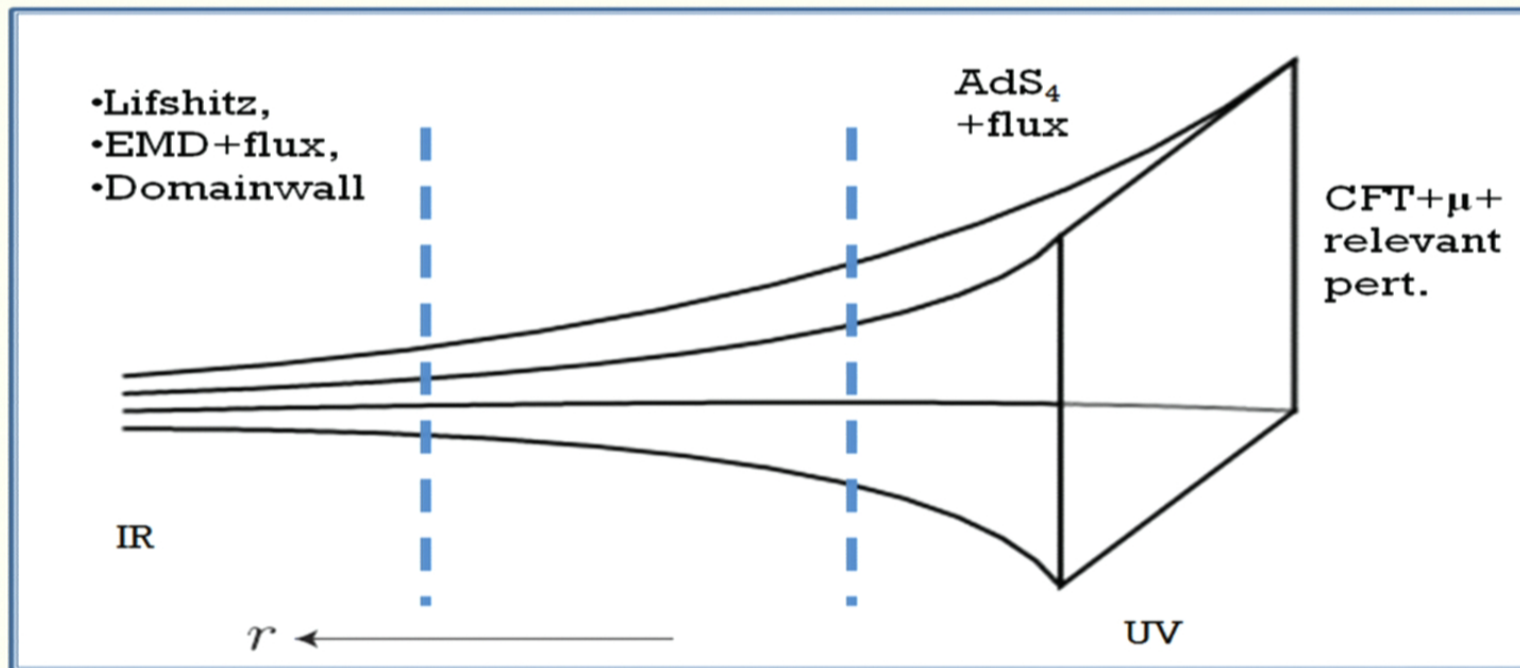
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Phases in holography



Setup

- ◉ Metric:

$$ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right)$$

- ◉ Maxwell potential and local chemical potential:

$$A = \frac{eL}{\kappa} h(r) dt \quad \mu_{\text{loc}} = \frac{e}{\kappa} \frac{h}{\sqrt{f}}$$

- ◉ Dilaton potential:

$$V(\Phi) = -6 \cosh(2\Phi/\sqrt{3}) \quad Z(\Phi) = e^{2\Phi/\sqrt{3}}$$

- ◉ Conventions: $r \rightarrow 0$ in UV, $r \rightarrow \infty$ in IR

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EOM

$$\begin{aligned}\frac{1}{r} \left(\frac{f'}{f} + \frac{g'}{g} + \frac{4}{r} \right) + \frac{gh}{\sqrt{f}} \hat{\sigma} + 2\Phi'^2 &= 0, \\ \frac{1}{r} \left(\frac{f'}{f} - \frac{1}{r} \right) + g \left(\hat{p} - \frac{1}{2}V(\Phi) \right) - \frac{Z(\Phi)h'^2}{2f} + \Phi'^2 &= 0, \\ -\Phi'' + \frac{1}{2} \left(-\frac{f'}{f} + \frac{g'}{g} + \frac{4}{r} \right) \Phi' + \frac{gV'(\Phi)}{4} - \frac{Z'(\Phi)h'^2}{4f} &= 0, \\ \frac{d}{dr} \left(\frac{Z(\Phi)h'}{r^2\sqrt{fg}} \right) - \frac{\sqrt{g}}{r^2} \hat{\sigma} &= 0.\end{aligned}$$

Asymptotic solutions

for:

- ⊙ UV: $r \rightarrow 0$ & $\Phi \rightarrow 0$
- ⊙ IR: $r \rightarrow \infty$ &
 - $\Phi \rightarrow \text{const}$
 - $\Phi \sim \log r \rightarrow +\infty$
 - $\Phi \sim -\log r \rightarrow -\infty$
- ⊙ Full bulk solution numerically

EOM

$$\begin{aligned} \frac{1}{r} \left(\frac{f'}{f} + \frac{g'}{g} + \frac{4}{r} \right) + \frac{gh}{\sqrt{f}} \hat{\sigma} + 2\Phi'^2 &= 0, \\ \frac{1}{r} \left(\frac{f'}{f} - \frac{1}{r} \right) + g \left(\hat{p} - \frac{1}{2}V(\Phi) \right) - \frac{Z(\Phi)h'^2}{2f} + \Phi'^2 &= 0, \\ -\Phi'' + \frac{1}{2} \left(-\frac{f'}{f} + \frac{g'}{g} + \frac{4}{r} \right) \Phi' + \frac{gV'(\Phi)}{4} - \frac{Z'(\Phi)h'^2}{4f} &= 0, \\ \frac{d}{dr} \left(\frac{Z(\Phi)h'}{r^2\sqrt{fg}} \right) - \frac{\sqrt{g}}{r^2} \hat{\sigma} &= 0. \end{aligned}$$

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UV solution

- ◉ Dilaton potential:

$$V(\Phi) = -6 - 4\Phi^2 + \mathcal{O}(\Phi^4), \quad Z(0) = 1$$

- ◉ Dilaton mass: $-2/L^2$,
scaling dimension dual operator: $\Delta=2$

$$\Phi = \phi_0 r + \frac{\langle \mathcal{O} \rangle}{2} r^2 + \dots$$

- ◉ Metric: AdS_4
- ◉ Maxwell potential: $h = c(\hat{\mu} - \hat{Q}r + \dots)$

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relevant coupling

- ◉ Metric: AdS_4
- ◉ Maxwell potential: $h = c(\hat{\mu} - \hat{Q}r + \dots)$

Hartnoll, LH, CQG 29, 194001 (2012)

IR solution I: Lifshitz

- ◉ Dilaton to **constant** in IR ($r \rightarrow \infty$)

$$ds^2 = L^2 \left(-\frac{dt^2}{r^{2z}} + g_L \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \right) \quad A = \frac{eL}{\kappa} h_L \frac{dt}{r^z} \quad \Phi = \phi_L$$

- ◉ Scale invariant solution
- ◉ Flux vanishes in IR, all charge carried by fluid
- ◉ There is a relevant perturbation of Lifshitz fixed point
⇒ **need to tune parameter in UV to hit fixed point in IR**
- ◉ Parameter is **relevant coupling** ϕ_0
- ◉ Lifshitz solution characterizes phase transition between two phases

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IR solution II: Fractionalized

- ◉ Dilaton: $\Phi \sim \log r \rightarrow +\infty$ for $r \rightarrow \infty$
 $\Rightarrow Z \rightarrow \infty$, effective Maxwell coupling small

- ◉ Hyperscaling violation metric

$$ds^2 = L^2 \left(-\frac{dt^2}{r^6} + g_0 \frac{dr^2}{r^4} + \frac{dx^2 + dy^2}{r^2} \right)$$

- ◉ Flux to constant for $r \rightarrow \infty$

\Rightarrow must be sourced by **charge hidden behind horizon**

- ◉ Local chemical potential to zero, so **no fluid in IR**

- ◉ Solution for range of values of ϕ_0

\Rightarrow Entire **class of solutions**

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IR solution II: Fractionalized

- ◉ No fluid in IR
- ◉ Fluid can be present in the bulk if full solution has region with $\hat{\mu}_{\text{loc}} > \hat{m}$
- ◉ **Fully fractionalized**: all charge behind horizon
- ◉ **Partially fractionalized**: total charge is sum of charge behind horizon and charge carried by fluid
- ◉ As we tune relevant coupling ϕ_0 charge leaks out from behind horizon and populates fluid

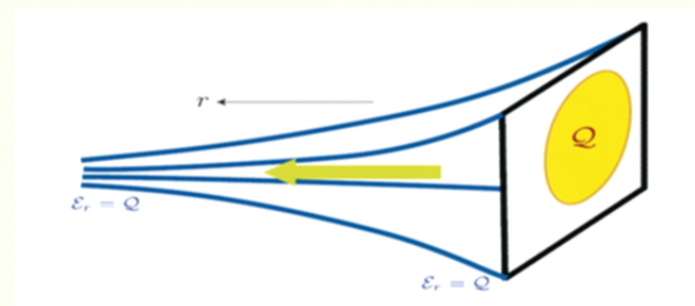
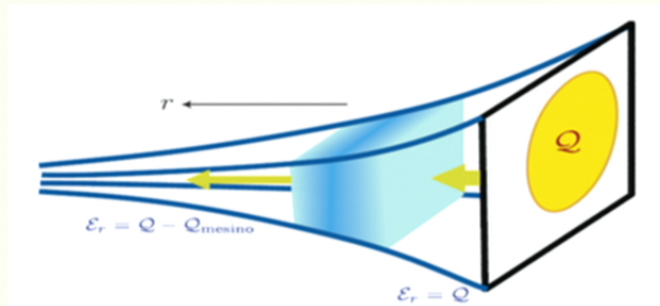
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IR solution II: Fractionalized



Partially fractionalized

charge partially hidden
behind horizon rest carried
by fermions in bulk

Fully fractionalized

all charge hidden behind
horizon

Hartnoll, LH, CQG 29, 194001 (2012)

IR solution III: Cohesive

- ◉ Dilaton: $\Phi \sim -\log r \rightarrow -\infty$ for $r \rightarrow \infty \Rightarrow Z \rightarrow 0$
- ◉ Flux to zero for $r \rightarrow \infty$
 \Rightarrow **all charge sourced by fluid**
- ◉ The metric is domain-wall solution
$$ds^2 = e^{G(r)} [-f(r)dt^2 + dx^2 + dy^2] + \frac{dr^2}{f(r)}$$
- ◉ Local chemical potential:
 $\hat{\mu}_{\text{loc}}(r \rightarrow \infty) = \text{const.} > \hat{m}$
so **fluid all the way to IR**

Hartnoll, LH, CQG 29, 194001 (2012)

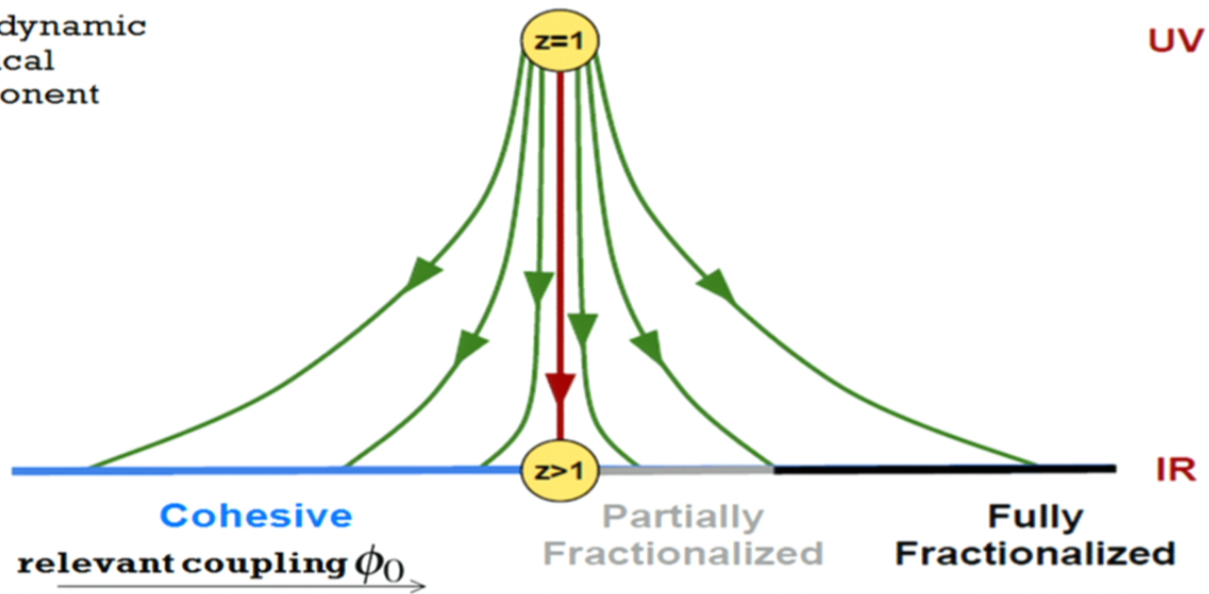
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Hartnoll, LH, CQG 29, 194001 (2012)

Phase diagram

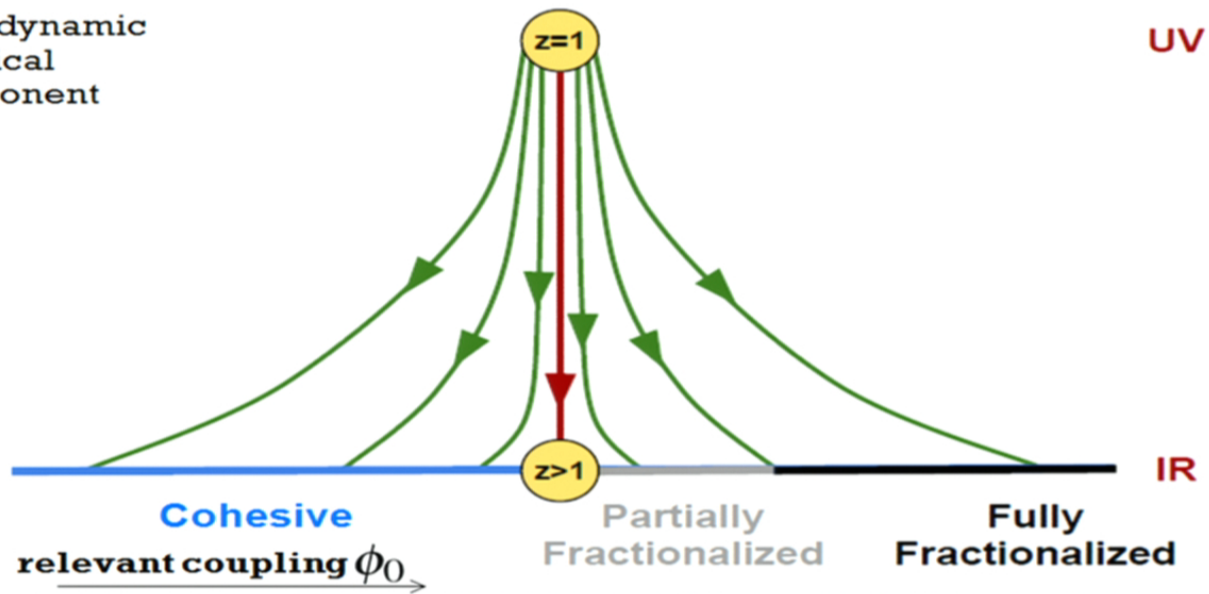
z is dynamic
critical
exponent



Hartnoll, LH, CQG 29, 194001 (2012)

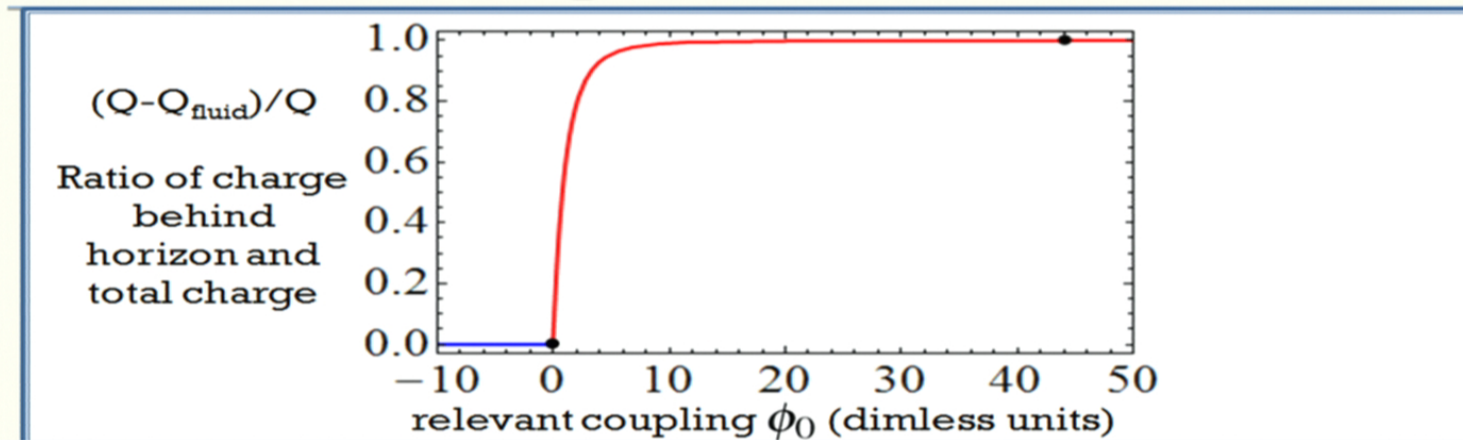
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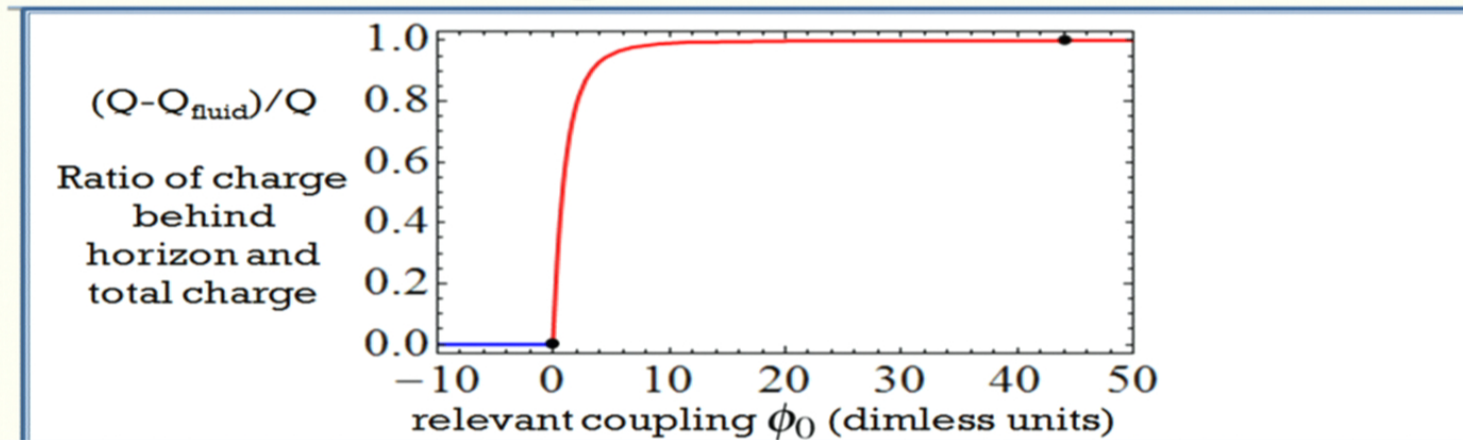
Charge fractionalization



Cohesive	Partially fractionalized	Fully fractionalized
all charge carried by fermions in bulk	charge partially hidden behind horizon rest carried by fermions in bulk	all charge hidden behind horizon

Hartnoll, LH, CQG 29, 194001 (2012)

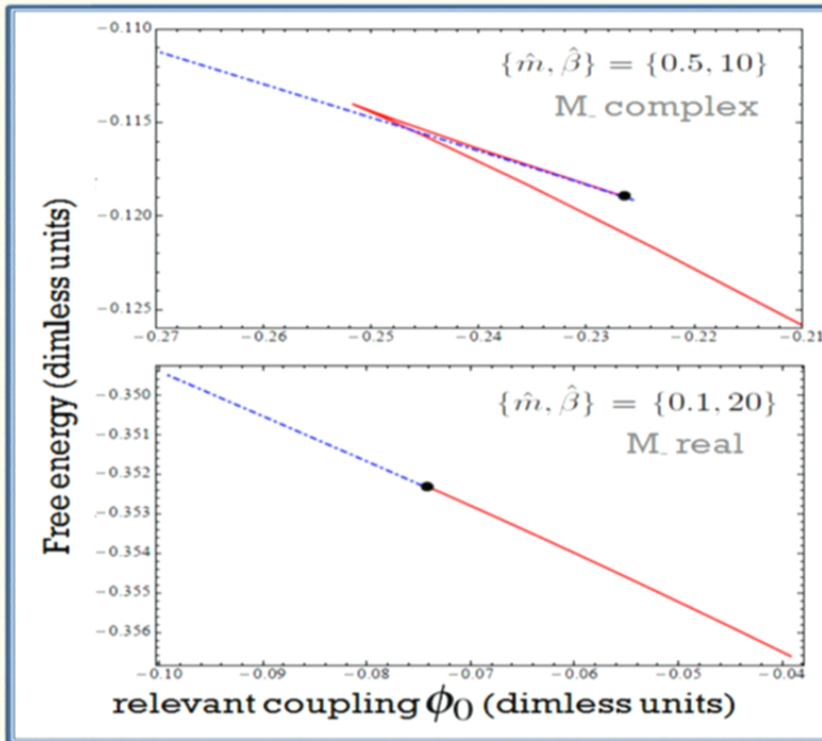
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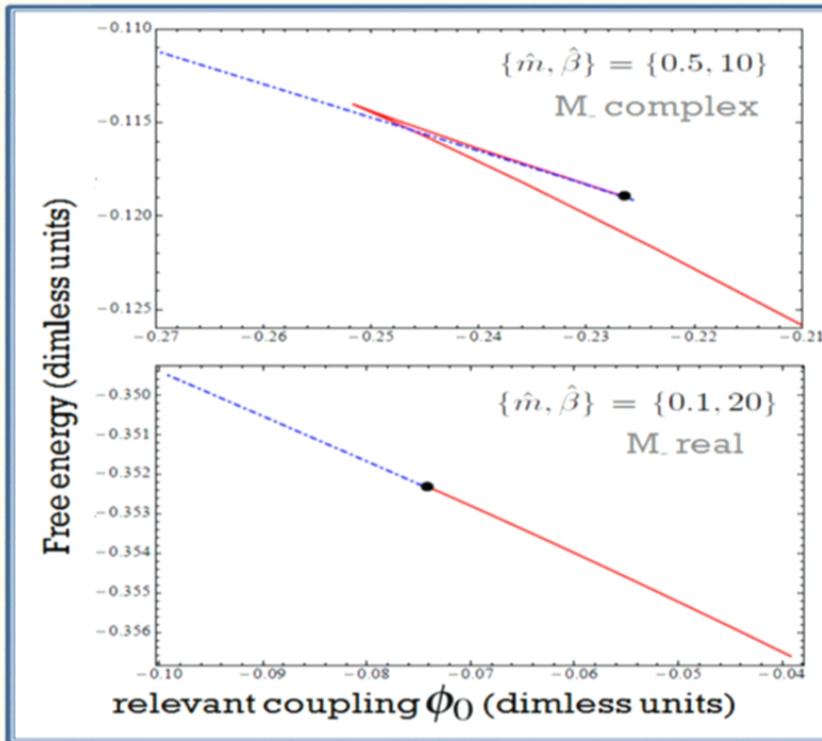
1st order vs 2nd order transition



- ⊙ Lifshitz solution characterizes phase transition
- ⊙ It depends on fluid parameters m and β
- ⊙ Scaling dimension of relevant operator can be complex: indicates **first order** transition

Hartnoll, LH, CQG 29, 194001 (2012)

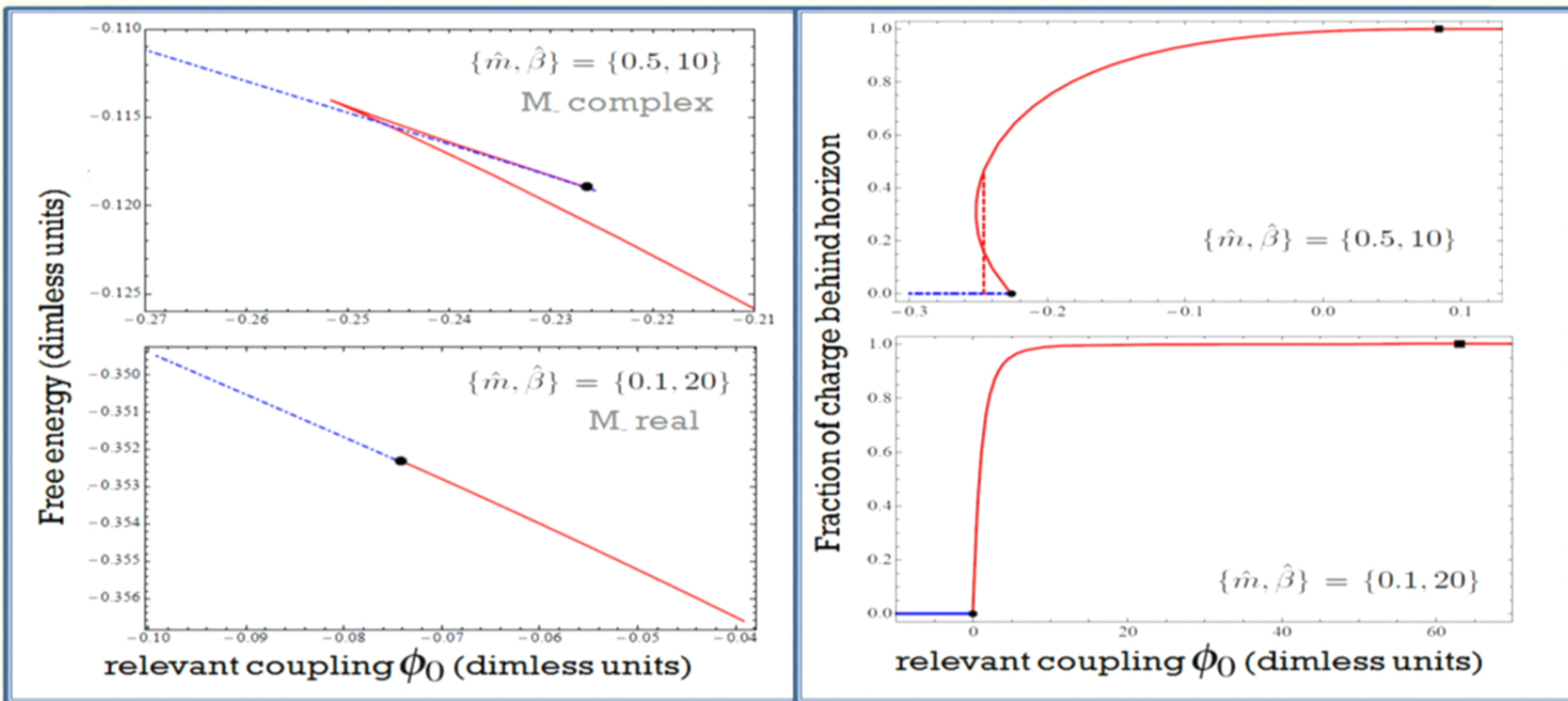
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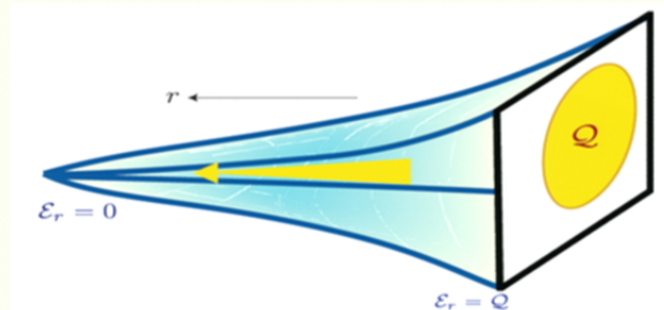
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Fermi Surface?

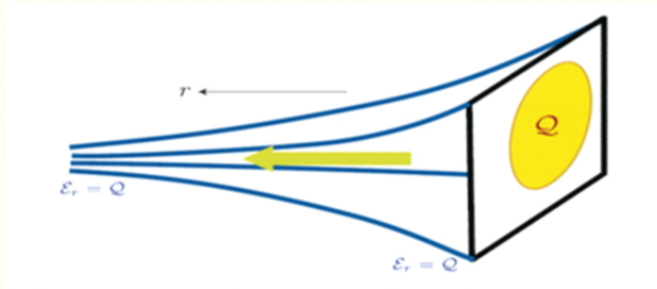
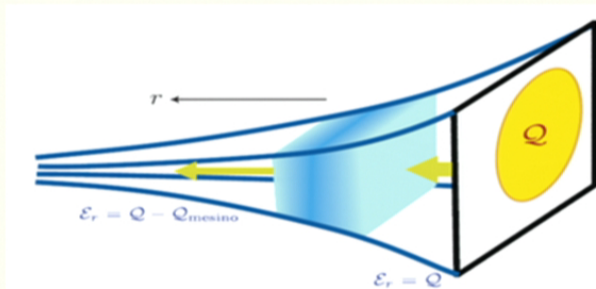
- ◉ Fermions in fluid can be studied in the bulk
- ◉ They obey Luttinger theorem
- ◉ *Cohesive phase is FL-like*



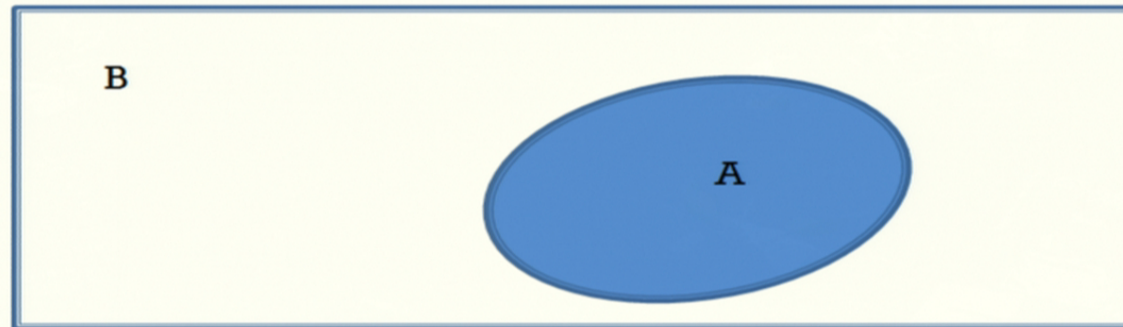
Hartnoll, Hofman, Vegh, JHEP 1108, 096 (2011);
Iqbal, Liu, Mezei, arXiv:1105.4621;
Sachdev, PRD 84, 066009 (2011)

Fermi Surface?

- ⦿ What about fractionalized phases?
- ⦿ Is there a hidden Fermi surface?
- ⦿ Holographic entanglement entropy provides indirect probe of the FS



Entanglement entropy



- ⊙ $\rho_A = \text{Tr}_B \rho$ = density matrix of region A
- ⊙ Entanglement entropy:
$$S_A = -\text{Tr}(\rho_A \log \rho_A)$$
- ⊙ Boundary law for gapped systems:
$$S_A \sim P$$

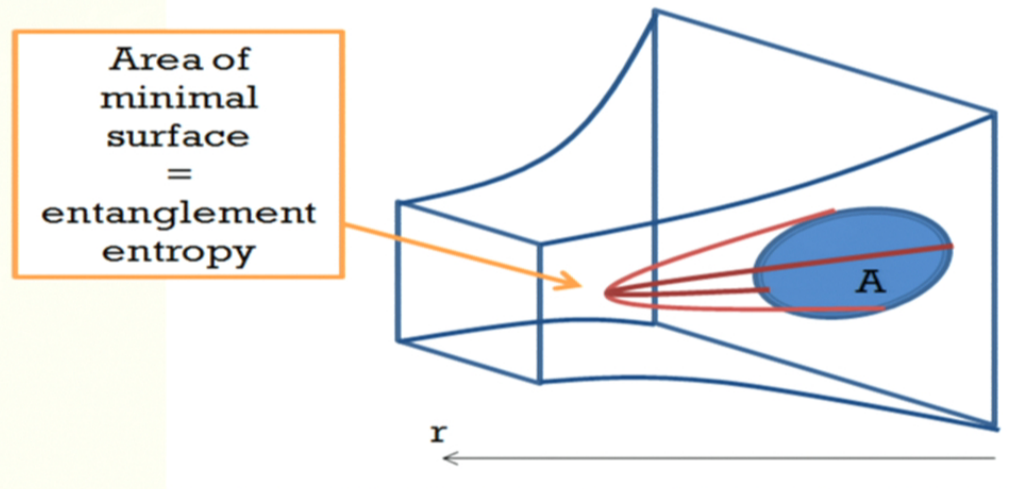
P is perimeter of region A

Entanglement entropy



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Holographic entanglement entropy



Ryu, Takayanagi, PRL 96, 18160 (2006)

Einstein-Maxwell-dilaton theory

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with $V(\Phi) = -V_0 e^{-\beta\Phi}$, $Z(\Phi) = Z_0 e^{\alpha\Phi}$

General solution for fractionalized phase:

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

Scale trafo: $x_i \rightarrow \zeta x_i$, $t \rightarrow \zeta^z t$, $ds \rightarrow \zeta^{\theta/d} ds$

z dynamical critical exponent

θ hyperscaling violation exponent

θ and z depend on d , α and β

Ogawa, Takayanagi, Ugajin, arXiv:1111.1023;
LH, Sachdev, Swingle, PRB 85, 035121 (2012)

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LH, Sachdev, Swingle, PRB 85, 035121 (2012)

Log violation of area law

$$ds^2 = \frac{1}{r^2} \left(- \frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

- ⊙ For a **specific choice** of α and β , s.t. $\theta=d-1$, we find log violation of area law ($d=2$):
with Q charge behind horizon
 $S_A \sim \sqrt{QP} \log[\sqrt{QP}]$
- ⊙ Log violation of area law indicates **hidden FS** with $k_F \sim \sqrt{Q}$
- ⊙ This matches precisely with Luttinger theorem ($d=2$): $Q \sim k_F^2$!

Ogawa, Takayanagi, Ugajin, arXiv:1111.1023;
LH, Sachdev, Swingle, PRB 85, 035121 (2012)



Inequalities

- Local QFT: at most log violation of entanglement entropy: $\theta \leq d-1$
- 'Null energy condition' yields for dynamical critical exponent: $z \geq 1 + \theta / d$
- NB for $d=2$ and $\theta=d-1$ we get $z \geq 3/2$
 $z=3/2$ is state of the art field theory result for NFL!



LH, Sachdev, Swingle, PRB 85, 035121 (2012)

M. A. Metlitski and S. Sachdev, Phys. Rev. B 82, 075127 (2010)



Synthesis

- ◉ We found different compressible phases via charge fractionalization
- ◉ Cohesive phase (all charge in bulk) is FL-like
- ◉ Fractionalized phases (some or all charge hidden) violate Luttinger theorem
- ◉ Holographic EE probe for hidden FS
- ◉ In certain Einstein-Maxwell-dilaton theories Luttinger theorem is restored via hidden FS
 - Partially fractionalized phase:
hidden FS and FL-like FS  FL*
 - Fully fractionalized phase:
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Order parameters

- ◉ Confinement/deconfinement: Polyakov loop, Wilson loop, entanglement entropy
- ◉ Realizations in holography as surfaces in bulk
- ◉ Is there an order parameter for fractionalized/cohesive phases?
- ◉ Holographic proposal: flux through minimal surface
- ◉ Volume scaling in deconfined fractionalized phases and vanishing in confining cohesive phase

Hartnoll, Radičević, ArXiv:1205.5291

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Hidden FS continued

- ⊙ Entanglement entropy in agreement with hidden Fermi surface
- ⊙ BUT no low energy modes at finite momentum observed in spectral density of transverse current operator ...
- ⊙ Probe Friedel oscillations via monopoles (1D)
- ⊙ 1 large ($\sim N$) FS vs $\sim N$ small FSs
- ⊙ Explore connection to dimensionally reduced pure gravity theories

Hartnoll, Shaghoulian, ArXiv:1203.4236
Iqbal, Faulkner, ArXiv:1207.4208
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More boundary law violations

- ◉ Einstein-Maxwell-Dilaton theories can exhibit boundary law violations up to volume scaling
- ◉ Extreme limit (volume scaling) in extremal Reissner-Nordström solution
- ◉ RN solution interpreted as extensive GS degeneracy
- ◉ What about intermediate violations?
- ◉ Anomalous GS degeneracy can lead to anomalous EE scaling
- ◉ BUT (large) GS degeneracy does not imply boundary law violation
- ◉ Fractal Fermi Surface does lead to anomalous EE scaling, but requires infinite range interactions

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Conclusions

- ⊙ Compressible phases in holography
- ⊙ Phase diagram for Einstein-Maxwell-dilaton-fluid theory
- ⊙ Lifshitz solution characterizes phase transition, 1st order vs 2nd order
- ⊙ Cohesive, partially/fully fractionalized phases
- ⊙ Similarities with FL, FL*, NFL
- ⊙ Holographic entanglement entropy indirect probe for FS
- ⊙ For certain dilaton theories we find hidden FS, consistent with Luttinger theorem
- ⊙ Needed: more (direct) evidence for hidden FS, such as finite momentum operators...