Title: New Observations about the Renormalization Group Flow

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Abstract: We review recent breakthroughs in understanding some general features of the Renormalization Group and of Quantum Field Theory. We discuss some applications of these new results and their deep connection to the entanglement of the Quantum Field Theory vacuum.


New Observations about the
Renormalization Group Flow
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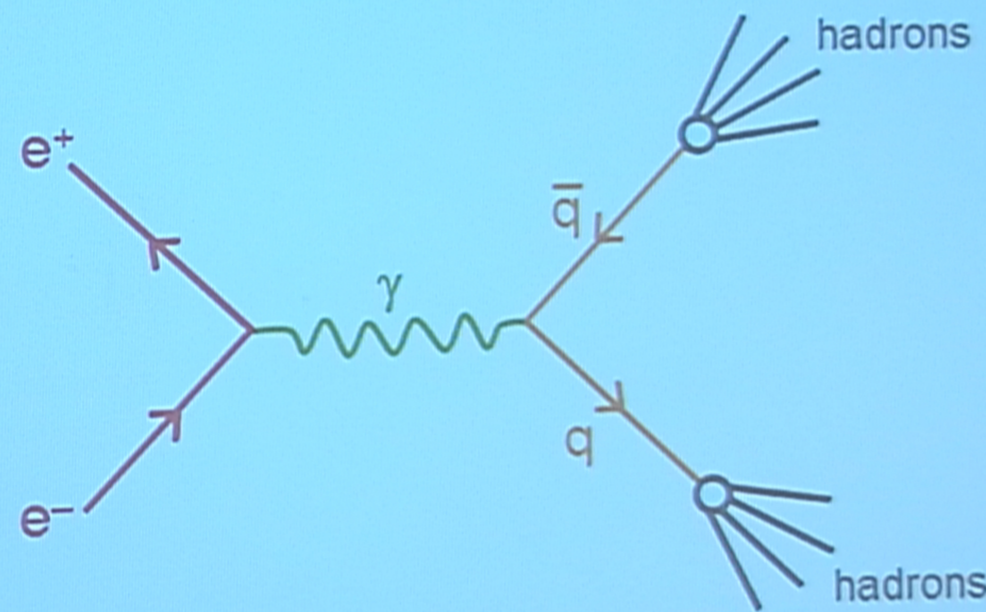
Perimeter Institute, 2012

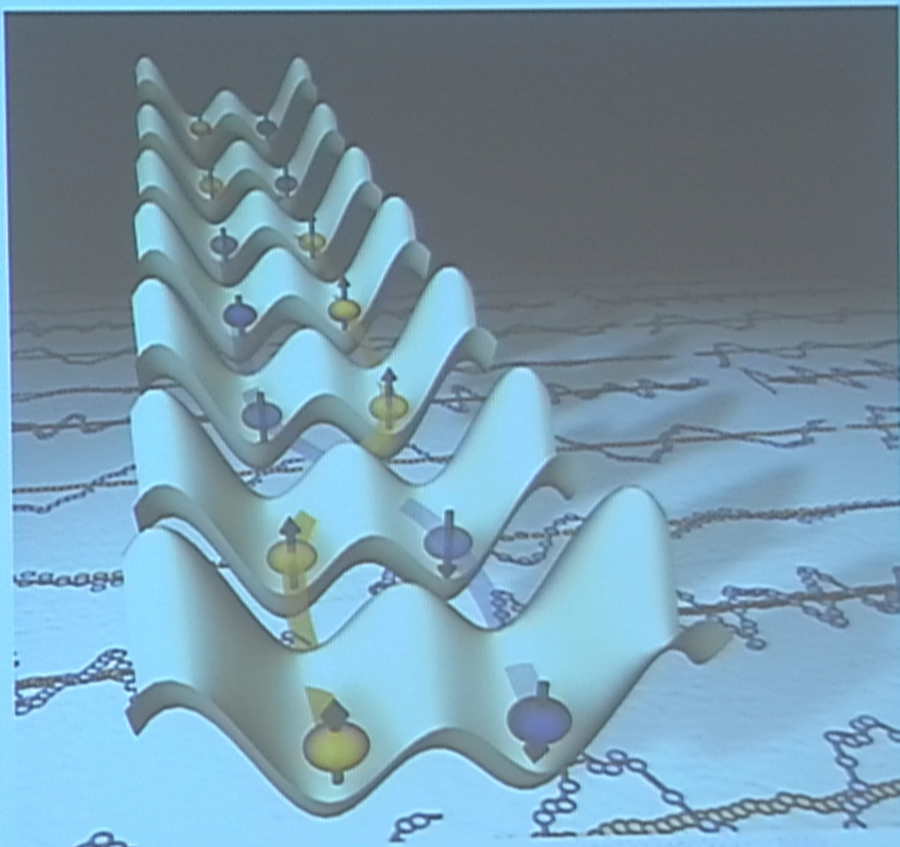
QFT Works!

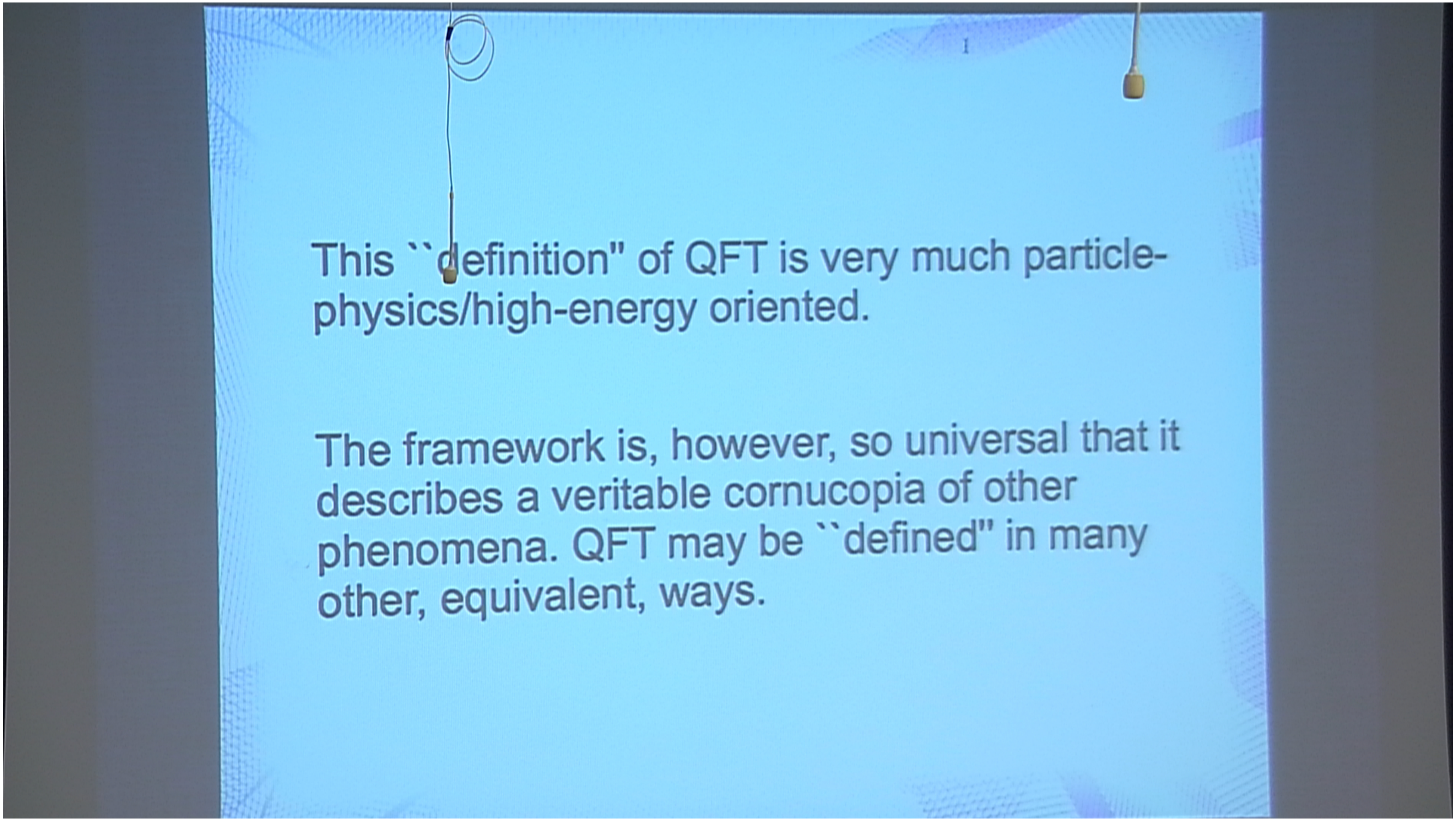
Quantity	Group(s)	Value	Standard Model	pull
M_Z [GeV]	LEP	91.1876 ± 0.0021	91.1874 ± 0.0021	0.1
Γ_Z [GeV]	LEP	2.4952 ± 0.0023	2.4972 ± 0.0011	−0.9
$\Gamma(\text{had})$ [GeV]	LEP	1.7444 ± 0.0020	1.7436 ± 0.0011	—
$\Gamma(\text{inv})$ [MeV]	LEP	499.0 ± 1.5	501.74 ± 0.15	—
$\Gamma(\ell^+\ell^-)$ [MeV]	LEP	83.984 ± 0.086	84.015 ± 0.027	—
σ_{had} [nb]	LEP	41.541 ± 0.037	41.470 ± 0.010	1.9
R_e	LEP	20.804 ± 0.050	20.753 ± 0.012	1.0
R_μ	LEP	20.785 ± 0.033	20.753 ± 0.012	1.0
R_τ	LEP	20.764 ± 0.045	20.799 ± 0.012	−0.8
$A_{FB}(e)$	LEP	0.0145 ± 0.0025	0.01639 ± 0.00026	−0.8
$A_{FB}(\mu)$	LEP	0.0169 ± 0.0013		0.4
$A_{FB}(\tau)$	LEP	0.0188 ± 0.0017		1.4
R_b	LEP/SLD	0.21664 ± 0.00065	0.21572 ± 0.00015	1.1
R_c	LEP/SLD	0.1718 ± 0.0031	0.17231 ± 0.00006	−0.2
$R_{s,d}/R_{(d+u+s)}$	OPAL	0.371 ± 0.023	0.35918 ± 0.00004	0.5
$A_{FB}(b)$	LEP	0.0995 ± 0.0017	0.1036 ± 0.0008	−2.4
$A_{FB}(c)$	LEP	0.0713 ± 0.0036	0.0741 ± 0.0007	−0.8
$A_{FB}(s)$	DELPHI/OPAL	0.0976 ± 0.0114	0.1037 ± 0.0008	−0.5
A_b	SLD	0.922 ± 0.020	0.93476 ± 0.00012	−0.6
A_c	SLD	0.670 ± 0.026	0.6681 ± 0.0005	0.1
A_s	SLD	0.895 ± 0.091	0.93571 ± 0.00010	−0.4
A_{LR} (hadrons)	SLD	0.15138 ± 0.00216	0.1478 ± 0.0012	1.7
A_{LR} (leptons)	SLD	0.1544 ± 0.0060		1.1
A_μ	SLD	0.142 ± 0.015		−0.4
A_τ	SLD	0.136 ± 0.015		−0.8
$A_e(Q_{LR})$	SLD	0.162 ± 0.043		0.3
$A_\tau(\mathcal{P}_\tau)$	LEP	0.1439 ± 0.0043		−0.9
$A_e(\mathcal{P}_\tau)$	LEP	0.1498 ± 0.0048		0.4
Q_{FB}	LEP	0.0403 ± 0.0026	0.0424 ± 0.0003	−0.8

...but it is complicated. Relativity together with quantum theory imply that we cannot fix the number of particles



One needs to study many body quantum systems with an unconstrained number of particles...

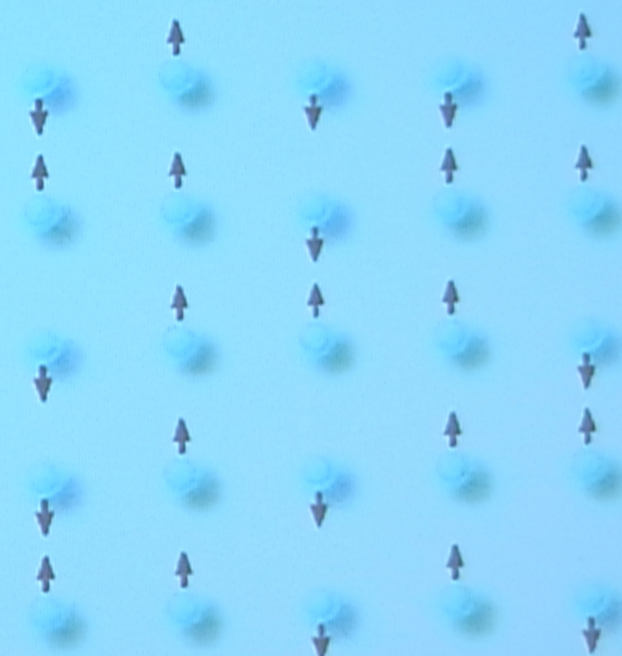



This ``definition" of QFT is very much particle-physics/high-energy oriented.
The framework is, however, so universal that it describes a veritable cornucopia of other phenomena. QFT may be ``defined" in many other, equivalent, ways.

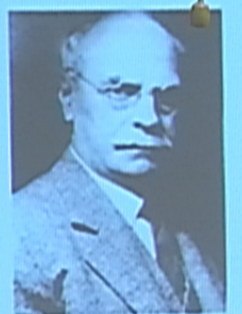
Example 1: Critical points of 2d spin systems. (Including Ising, Potts and many others.) Many of those are described by special QFTs known as Minimal Models.

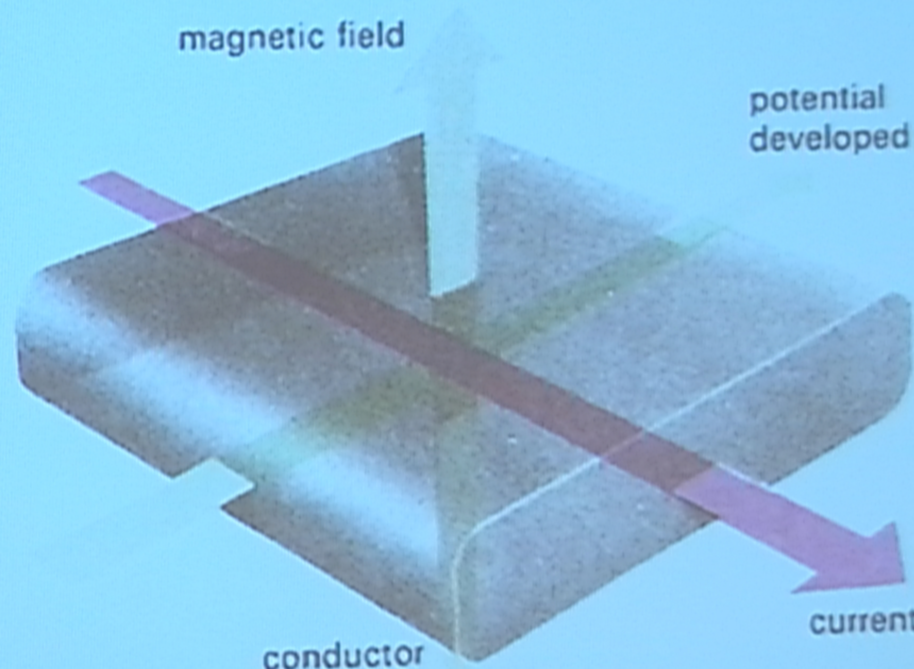
Example 2:

3d statistical systems such as 3d Ising model, superfluid Helium, etc.



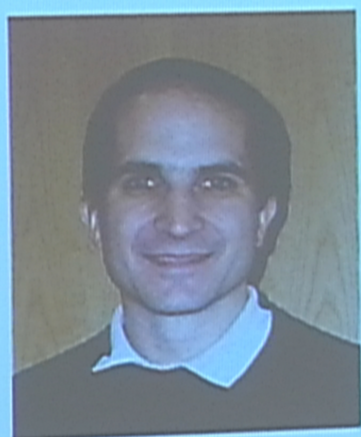
Example 3: Systems that appear in condensed matter physics, for example, the Hall effect, and many others...

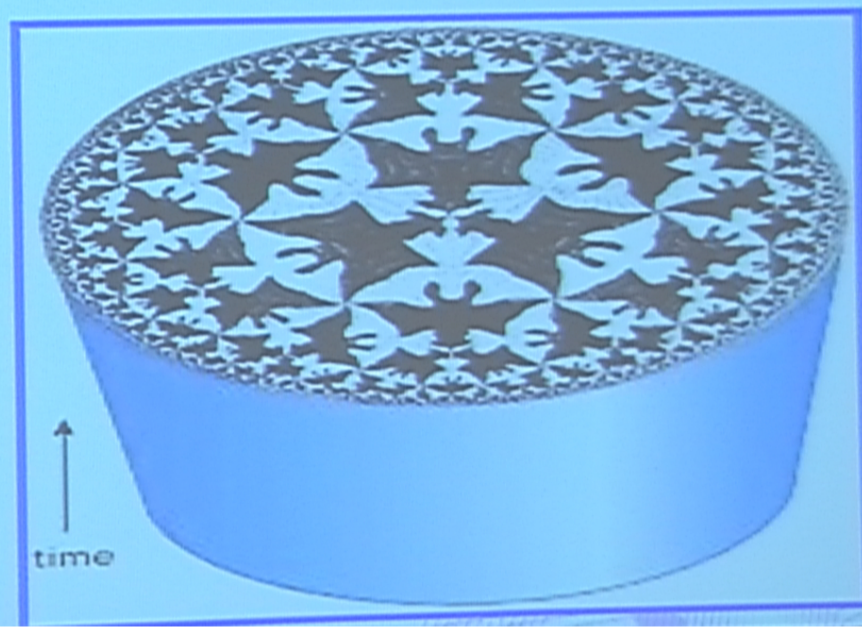




So far we have seen that QFT describes phenomena in particle physics, statistical physics, and condensed matter. As if this is not enough...

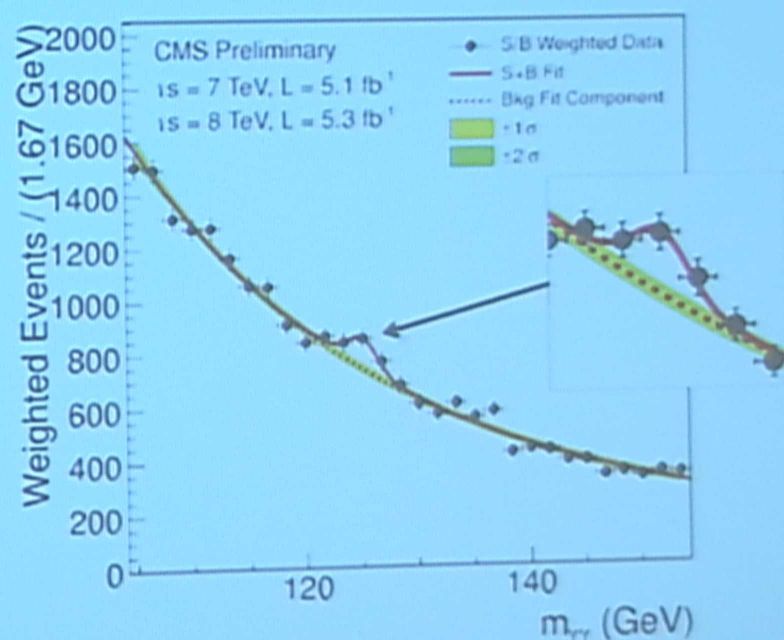
Example 4: QFT is equivalent to quantum gravity in spaces that asymptote to Anti-de Sitter.





However, we don't know what QFT is about!

- We cannot explain QFT to mathematicians.
- This is not "their problem." It is a clear sign we need to understand things better.
- If weakly coupled, can use perturbation theory



But many interesting models are strongly coupled. Then we quickly run out of steam.

- Quantum Chromodynamics, non-Fermi liquids, boiling water, high Tc (?)...
- The problem of Yang-Mills theory, closely related to Quantum Chromodynamics, is one of the celebrated Millennium Problems.

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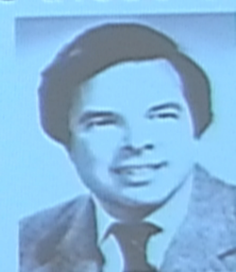
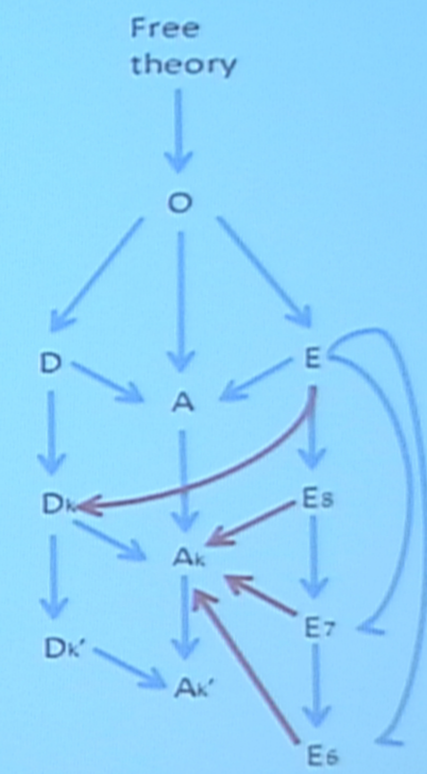
The Paradigm

- Define the microscopic rules for who interacts with whom and how much.
- Gradually zoom out, coarse graining the microscopic information.
- At each typical scale we have some set of collective excitations that we use to describe the system.
- In strongly interacting systems these collective excitations can differ a lot from the microscopic degrees of freedom.



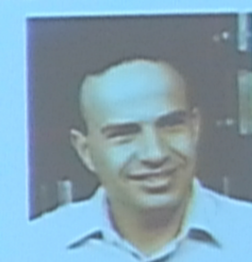
- This procedure of zooming out and forgetting the small irrelevant details is also called the ``Renormalization Group Flow."
- Even if the defining microscopic rules are simple, the interactions can become strong as we coarse grain. Then very nontrivial RG Flows can take place, radically changing the nature of the collective excitations.



The Three Basic Rules

- You will get nowhere by churning equations – the only way to proceed is the Renormalization Group.
- Do not trust arguments based on the lowest order of perturbation theory. (I hope Gross/Politzer/Wilczek are not in the audience...)
- You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry!

- In most cases we cannot control such strongly coupled phenomena.
- One exception is Yang-Mills theory, where nature herself ``solved” the theory for us, showing a dramatic transformation of the microscopic quarks and gluons into a zoo of hadrons.
- Some special QFT that possess extra symmetries can be studied more successfully. Such dramatic RG flows were observed there as well.



Having an utterly different language to describe a system after one zooms out and coarse grains is familiar also from thermodynamics, hydrodynamics, and many fields outside of physics.

Recently, progress has been made concerning this very general phenomenon. It appears that there are very stringent, model independent, constraints on how degrees of freedom can evolve, transmute, and morph.

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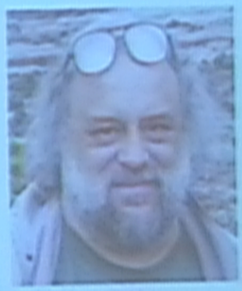
A Little History

- For RG Flows in 2D QFT one can introduce a real number, c .
- It counts the number of collective excitations (i.e. degrees of freedom).
- It is well defined even for strongly interacting systems, generalizing the notion of counting in free theories...
- It was shown [86] to decrease monotonically as we zoom out



$$c_{UV} > c_{IR}$$

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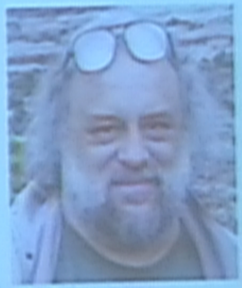

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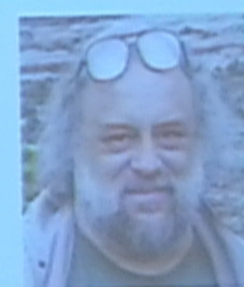


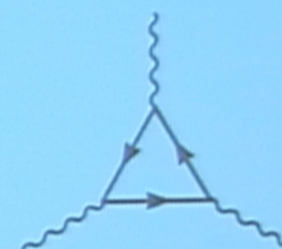
$$c_{UV} > c_{IR}$$

- So our degrees of freedom can be strongly coupled, transmute, distort, as long as their number does not increase!
- WHY? (we still don't know)
- RG Flows are irreversible. Space of theories is foliated by c .
- Countless applications in 2D QFTs, statistical systems, condensed matter, BH physics etc.
- c has a very interesting interpretation in terms of the Entanglement Entropy of the QFT vacuum.

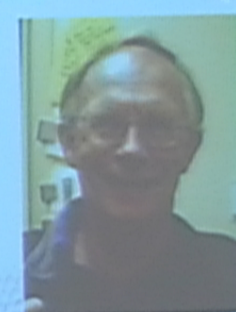
Do such constraints operate in higher-dimensional QFTs? important for high energy physics, condensed matter, statistical physics...

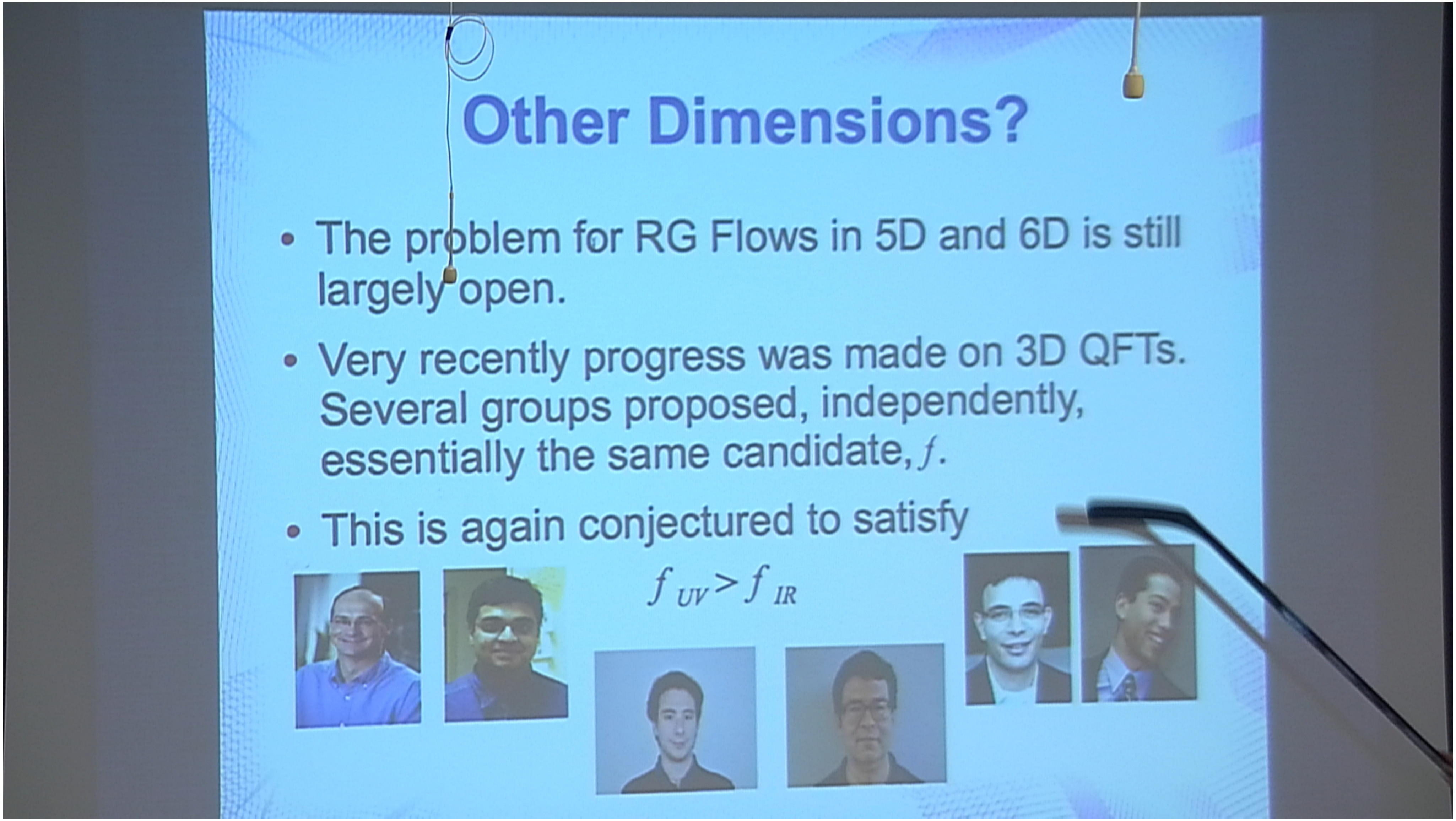
- For 4D QFTs, such as Quantum Chromodynamics, a conjecture generalizing the 2D story was put forward in the 80s. A certain quantity a conjectured to satisfy

$$a_{UV} > a_{IR}$$

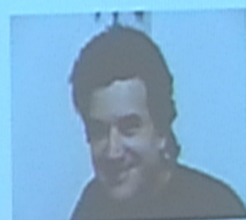


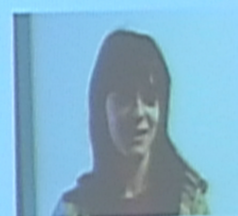
- Proposal appeared correct in examples (there had been occasional counter-examples that were later rebutted)




Other Dimensions?
• The problem for RG Flows in 5D and 6D is still largely open.
• Very recently progress was made on 3D QFTs. Several groups proposed, independently, essentially the same candidate, f.
• This is again conjectured to satisfy
$f_{UV} > f_{IR}$

- If correct, this conjecture would teach us important things about models such as boiling water and superfluid Helium. In addition it would constrain severely the dynamics of many interesting 3D models.
- There is not yet a conventional QFT proof of this inequality!
- Recently, the relation of these inequalities to information theory and Entanglement Entropy was used to argue for this 3D inequality.





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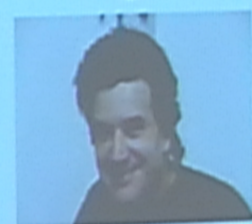
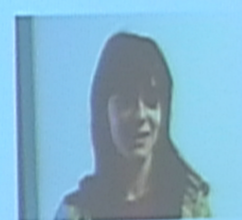


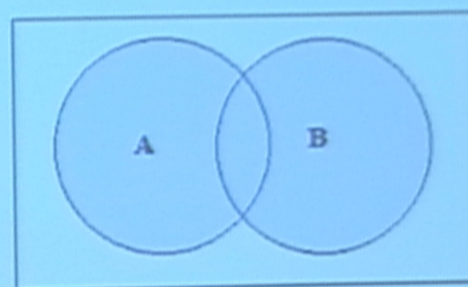


Subadditivity

The Subadditivity inequality in QFT is inherited from its more familiar form in information theory.

In the context of QFT it takes the form

$$S(A)+S(B)\geq S(A\vee B)+S(A\wedge B)$$



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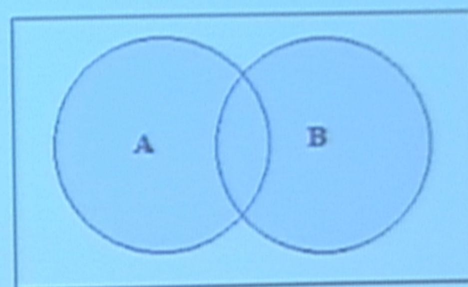

Manipulating this property in complicated ways the inequality $f_{UV}>f_{IR}$ was reached.

However, the story in 3D is not yet complete. Some important properties obeyed in 2D and 4D are not yet established, and it is unclear how to apply this inequality in many of the systems that appear in high energy physics and condensed matter. (This is because of some pathologies that arise with gauge fields.)

Summary

- General inequalities regarding collective excitations in many body quantum systems.
- Connections to entanglement in the QFT vacuum.
- Tantalizing suggestions and ideas in 3D.
- Many previously mysterious dynamical questions now solved.
- Many new questions...

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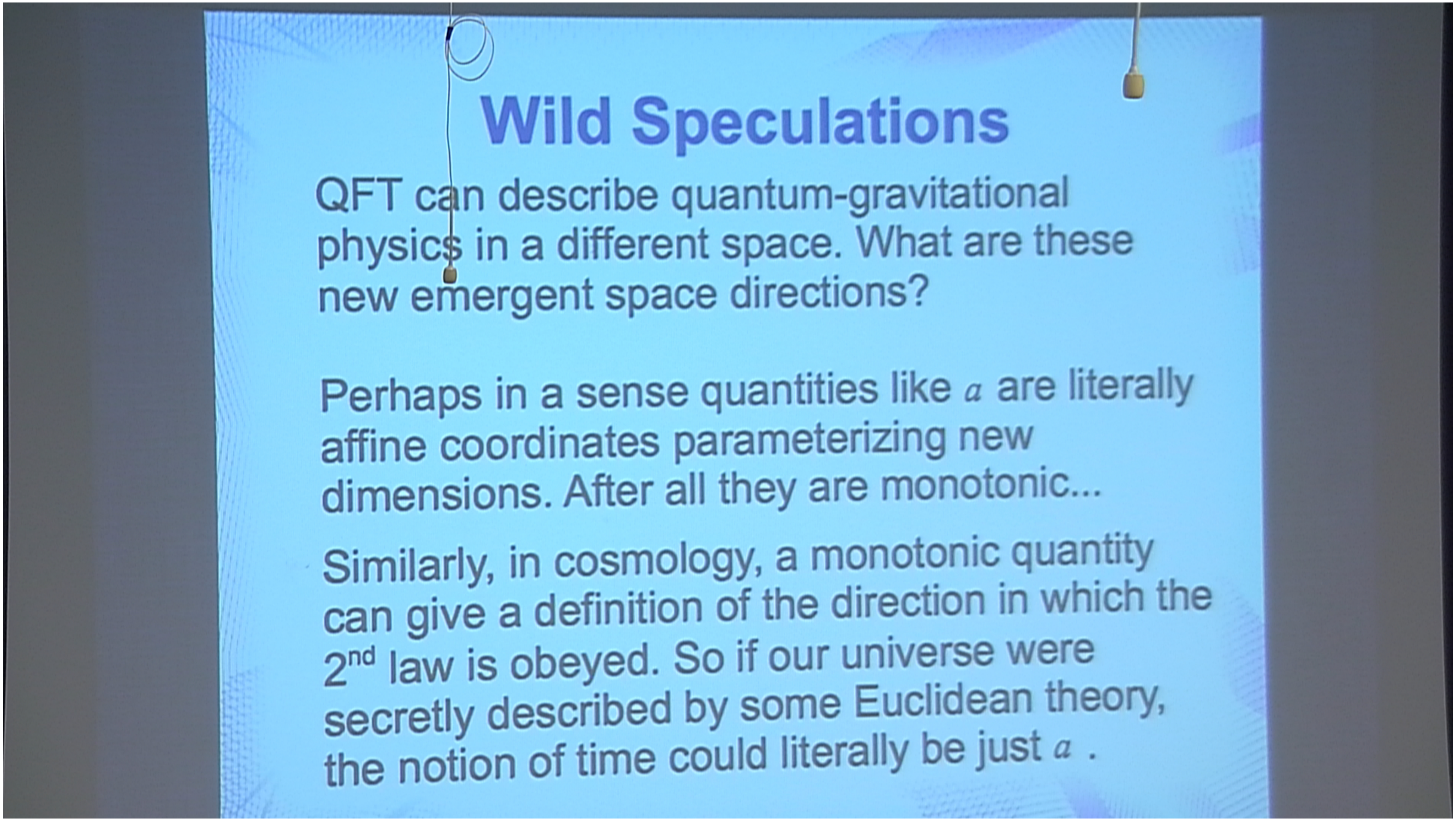
Open Questions

- Are there analogous constraints on general RG Flows without Lorentz/Euclidean Invariance? Should be very interesting for more extensive applications in condensed matter physics!
- Is the Entanglement Entropy of the QFT vacuum a new order parameter of some sort?
- RG Flows in 3D?
- Why do these inequalities exist?!


Wild Speculations
QFT can describe quantum-gravitational physics in a different space. What are these new emergent space directions?
Perhaps in a sense quantities like a are literally affine coordinates parameterizing new dimensions. After all they are monotonic...
Similarly, in cosmology, a monotonic quantity can give a definition of the direction in which the 2nd law is obeyed. So if our universe were secretly described by some Euclidean theory, the notion of time could literally be just a .