

Title: Introduction to Mathematica - Lecture 2a

Date: Sep 07, 2012 09:00 AM

URL: <http://pirsa.org/12090052>

Abstract:

$$\phi(x, y)$$
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + e^{2\phi} = 0$$

$$\begin{array}{c} \phi(x, y) \\ \text{Liouville: } \nabla_x^2 \phi + \nabla_y^2 \phi + e^{-2\phi} = 0 \end{array}$$

Liouville :

Laplace :

$$\phi(x, y) \quad \phi = c \log \left(\frac{1 - g(x+iy)f(x-iy)}{c' g'(x+iy)f'(x-iy)} \right)$$
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + e^{-2\phi} = 0$$
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \quad \phi = f(x+iy) + g(x-iy)$$

Partial Differential Equations

Analytics

Partial Differential Equations

Analytics

```
D[x|
```

```
D[ $\phi$ [x, y], {x, 2}]
```


Partial Differential Equations

Analytics

In[560]:=

```
D[x Cos[x], x]  
D[x Cos[x], {x, 2}]
```

Out[560]=

```
Cos[x] - x Sin[x]
```

Out[561]=

```
-x Cos[x] - 2 Sin[x]
```

Analysis

In[560]:=

```
D[x Cos[x], x]  
D[x Cos[x], {x, 2}]
```

Out[560]=

```
Cos[x] - x Sin[x]
```

Out[561]=

```
-x Cos[x] - 2 Sin[x]
```

```
D[φ[x, y], {x, 2}]
```


Analytics

In[560]:=

```
 $\partial_x (x \cos[x])$   
 $\partial_{\{x,2\}} (x \cos[x])$ 
```

Out[560]=

```
 $\cos[x] - x \sin[x]$ 
```

Out[561]=

```
 $-x \cos[x] - 2 \sin[x]$ 
```

```
 $\partial_{\{x,2\}} \phi[x, y]$ 
```

Analytics

```
Leq =  $\partial_{\{x,2\}} \phi[x,y] + \partial_{\{x,y\}} \phi[x,y] + \text{Exp}[-2 \phi[x,y]] ;$ 
```

Analytics

$$\text{Leq} = \partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]];$$

$$\phi_0 = c L|$$

Analytics

$$\text{Leq} = \partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]];$$

In[567]:=

$$\phi0 = c \text{Log} \left[\frac{1 - g[x + I y] f[x - I y]}{d g'[x + I y] f'[x - I y]} \right];$$

Analytics

```
Leq =  $\partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]]$ ;
```

In[567]:=

```
 $\phi0 = c \text{Log} \left[ \frac{1 - g[x + I y] f[x - I y]}{d g'[x + I y] f'[x - I y]} \right];$ 
```

In[568]:=

```
Leq
```

Out[568]=

```
 $e^{-2 \phi[x, y]} + \phi^{(2,0)}[x, y] + \phi^{(y,0)}[x, y]$ 
```

Analytics

```
Leq =  $\partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]]$ ;
```

In[567]:=

```
 $\phi0 = c \text{Log} \left[ \frac{1 - g[x + I y] f[x - I y]}{d g'[x + I y] f'[x - I y]} \right];$ 
```

In[568]:=

```
Leq
```

Out[568]=

```
 $e^{-2 \phi[x, y]} + \phi^{(2,0)}[x, y] + \phi^{(y,0)}[x, y]$ 
```


$$\text{Leq} = \partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]];$$

In[569]:=

$$\phi0[x_, y_] = c \text{Log} \left[\frac{1 - g[x + I y] f[x - I y]}{d g'[x + I y] f'[x - I y]} \right];$$

Set::write : Tag Times in

$$\left(c \text{Log} \left[\frac{1 - f[x + \text{Times}[\ll 2 \gg]] g[x + \text{Times}[\ll 2 \gg]]}{d f'[x - i y] g'[x + i y]} \right] \right)[x_, y_]$$

is Protected. >>

blabla = 5

In[568]:=

```
Leq =  $\partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]];$ 
```

In[569]:=

```
 $\phi_0[x_, y_] = c \text{Log} \left[ \frac{1 - g[x + I y] f[x - I y]}{d g'[x + I y] f'[x - I y]} \right];$ 
```

Set::write : Tag Times in

$\left(c \text{Log} \left[\frac{1 - f[x + \text{Times}[\ll 2 \gg]] g[x + \text{Times}[\ll 2 \gg]]}{d f'[x - i y] g'[x + i y]} \right] \right)[x_, y_]$

is Protected. >>

In[570]:=

```
blabla = 5
```

Out[570]=

5

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Friday Morning Part 1.nb

$$\text{Leq} = \partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]];$$

In[573]:=

`ClearAll[ $\phi$ ]`

In[574]:=

$$\phi_0[x_, y_] = c \text{Log} \left[\frac{1 - g[x + I y] f[x - I y]}{d g'[x + I y] f'[x - I y]} \right];$$

In[570]:=

`blabla = 5`

Out[570]=

5

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$\sqrt{[dg'[x + Iy] f'[x - Iy]]}$

In[578]:=

Leq

Out[578]=

$$e^{-2\phi[x,y]} + \phi^{(2,0)}[x,y] + \phi^{(y,0)}[x,y]$$

In[577]:=

Leq /. $\phi \rightarrow f0[x,y]$

Out[577]=

$$e^{-2f0[x,y][x,y]} + f0[x,y]^{(2,0)}[x,y] + f0[x,y]^{(y,0)}[x,y]$$

100%

$$\phi_0[x_, y_] = c \text{Log} \left[\frac{1 - g[x + I y] f[x - I y]}{d g'[x + I y] f'[x - I y]} \right];$$

In[578]:=

Leq

Out[578]=

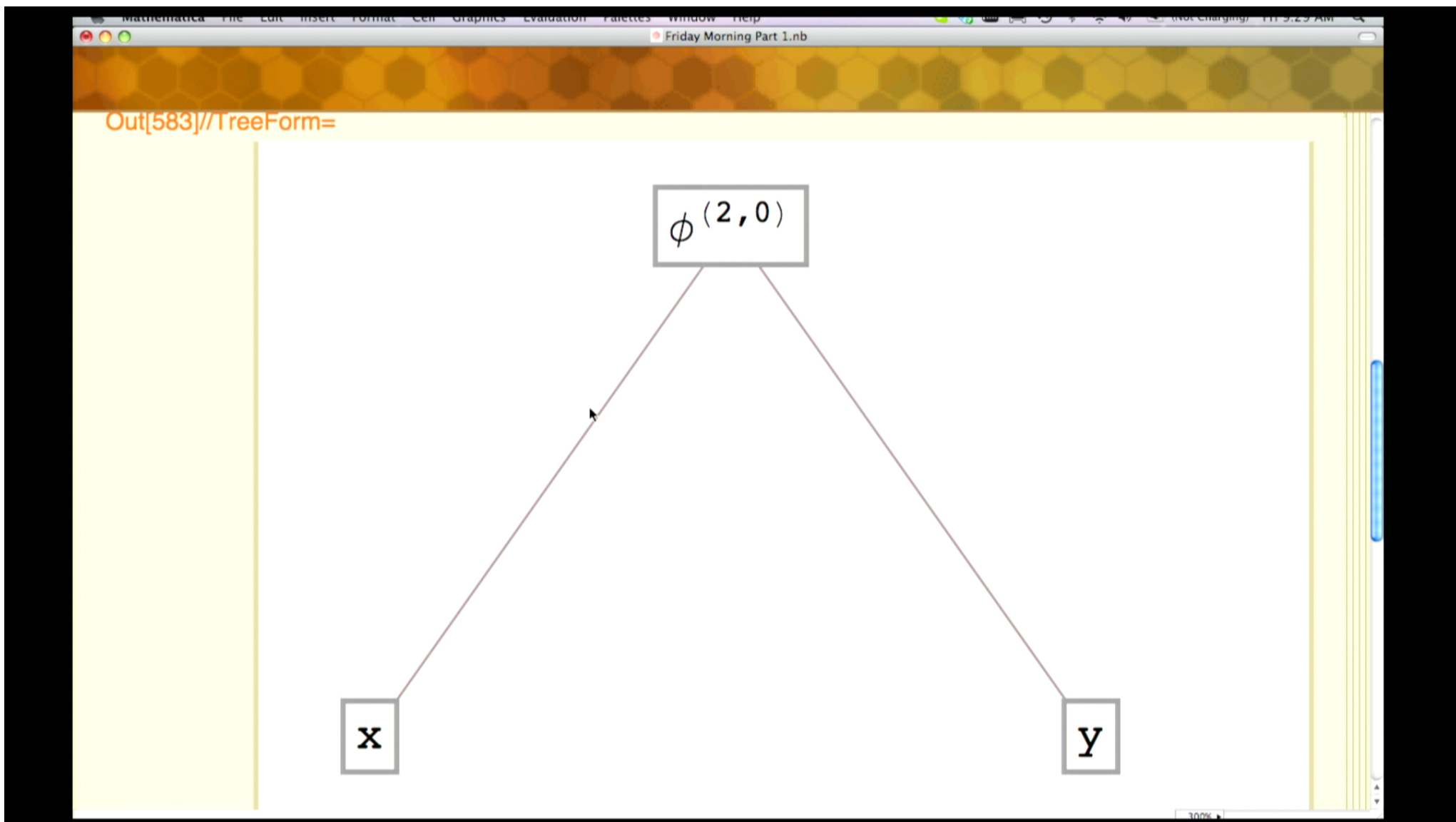
$$e^{-2 \phi[x, y]} + \phi^{(2,0)}[x, y] + \phi^{(y,0)}[x, y]$$

In[581]:=

Leq /. $\phi[x_, y_] \rightarrow \phi_0[x, y]$

Out[581]=

$$\left(\frac{1 - f[x - I y] g[x + I y]}{d f'[x - I y] g'[x + I y]} \right)^{-2 c} + \phi^{(2,0)}[x, y] + \phi^{(y,0)}[x, y]$$



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Leq

In[585]:=

```

 $\phi^{(2,0)}[x, y] // \text{FullForm}$ 
 $\phi[x, y] // \text{FullForm}$ 

```

Out[585]//FullForm=

```

Derivative[2, 0][\[Phi]][x, y]

```

Out[586]//FullForm=

```

\[Phi][x, y]

```

In[581]:=

```

Leq /.  $\phi[x_, y_] \rightarrow \phi0[x, y]$ 

```

Out[581]=

$$\left(\frac{1 - f[x - i y] g[x + i y]}{d f'[x - i y] g'[x + i y]} \right)^{-2c} + \phi^{(2,0)}[x, y] + \phi^{(y,0)}[x, y]$$

200%

```

 $\phi^{(2,0)}[x, y] // \text{FullForm}$ 
 $\phi[x, y] // \text{FullForm}$ 

```

Out[585]//FullForm=

```
Derivative[2, 0][\[Phi]][x, y]
```

Out[586]//FullForm=

```
\[Phi][x, y]
```

In[587]:=

```
Leq /.  $\phi \rightarrow \phi 0$ 
```

Out[587]=

$$\left(\frac{1 - f[x - i y] g[x + i y]}{d f'[x - i y] g'[x + i y]} \right)^{-2 c} -$$

$$\left(c d f'[x - i y] g'[x + i y] (-g[x + i y] f'[x - i y] - f[x - i y] g'[x + i y]) \right.$$

$$\left(\frac{-g[x + i y] f'[x - i y] - f[x - i y] g'[x + i y]}{d f'[x - i y] g'[x + i y]} - \frac{(1 - f[x - i y] g[x + i y]) f''[x - i y]}{d f'[x - i y]^2 g'[x + i y]} - \right.$$

$$\left. \frac{(1 - f[x - i y] g[x + i y]) g''[x + i y]}{d f'[x - i y] g'[x + i y]^2} \right) \Bigg/ (1 - f[x - i y] g[x + i y])^2 +$$

$$\frac{1}{1 - f[x - i y] g[x + i y]} c d g'[x + i y] f''[x - i y] \left(\frac{-g[x + i y] f'[x - i y] - f[x - i y] g'[x + i y]}{d f'[x - i y] g'[x + i y]} - \right.$$

$$\left. \frac{(1 - f[x - i y] g[x + i y]) f''[x - i y]}{d f'[x - i y]^2 g'[x + i y]} - \frac{(1 - f[x - i y] g[x + i y]) g''[x + i y]}{d f'[x - i y] g'[x + i y]^2} \right)$$

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Cell[1112][φ0]

$$\phi0[x_, y_] = c \text{Log} \left[\frac{1 - g[x + I y] f[x - I y]}{d g'[x + I y] f'[x - I y]} \right];$$

In[599]:=

Leq //. φ → φ0 // FullSimplify

Out[599]=

$$\left(\frac{1 - f[x - I y] g[x + I y]}{d f'[x - I y] g'[x + I y]} \right)^{-2 c} - \frac{4 c f'[x - I y] g'[x + I y]}{(-1 + f[x - I y] g[x + I y])^2}$$

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In[592]:=

```
ClearAll[φ0]
φ0[x_, y_] = c Log[ $\frac{1 - g[x + I y] f[x - I y]}{d g'[x + I y] f'[x - I y]}$ ];
```

In[599]:=

```
Leq //. φ → φ0 // FullSimplify
```

Out[599]=

$$\left(\frac{1 - f[x - i y] g[x + i y]}{d f'[x - i y] g'[x + i y]} \right)^{-2c} - \frac{4 c f'[x - i y] g'[x + i y]}{(-1 + f[x - i y] g[x + i y])^2}$$

In[601]:=

```
Leq //. φ → φ0 /. c → 1 // FullSimplify
```

Out[601]=

$$\frac{f'[x - i y] g'[x + i y] (-4 + d^2 f'[x - i y] g'[x + i y])}{(-1 + f[x - i y] g[x + i y])^2}$$

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In[591]:=

$$\text{Leq} = \partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]];$$

In[607]:=

```
ClearAll[phi0]
phi0[x_, y_] = c Log[ (1 - g[x + I y] f[x - I y])^2 /
  d g'[x + I y] f'[x - I y] ];
```

In[609]:=

```
Leq /. phi -> phi0 // FullSimplify
```

Out[609]=

$$\left(\frac{(-1 + f[x - i y] g[x + i y])^2}{d f'[x - i y] g'[x + i y]} \right)^{-2c} - \frac{8 c f'[x - i y] g'[x + i y]}{(-1 + f[x - i y] g[x + i y])^2}$$

In[610]:=

```
Leq /. phi -> phi0 /. c -> 1/2 /. d -> 4 // FullSimplify
```

Out[610]=

0

200%

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In[591]:=

$$\text{Leq} = \partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]];$$

ClearAll[$\phi 0$]

$$\phi 0[x_, y_] = c \text{Log}\left[\frac{(1 - g[x + I y] f[x - I y])^2}{d g'[x + I y] f'[x - I y]}\right];$$

$$\phi 0[x_, y_] = \frac{1}{2} \text{Log}\left[\frac{(1 - g[x + I y] f[x - I y])^2}{4 g'[x + I y] f'[x - I y]}\right];$$

In[609]:=

Leq //. $\phi \rightarrow \phi 0$ // FullSimplify

Out[609]=

$$\left(\frac{(-1 + f[x - I y] g[x + I y])^2}{d f'[x - I y] g'[x + I y]}\right)^{-2 c} - \frac{8 c f'[x - I y] g'[x + I y]}{(-1 + f[x - I y] g[x + I y])^2}$$

In[610]:=

Leq //. $\phi \rightarrow \phi 0$ /. $c \rightarrow 1/2$ /. $d \rightarrow 4$ // FullSimplify

Out[610]=

0

200%

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In[591]:=

$$\text{Leq} = \partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]];$$

In[611]:=

$$\phi_0[x_, y_] = c \text{Log}\left[\frac{(1 - g[x + I y] f[x - I y])^2}{d g'[x + I y] f'[x - I y]}\right];$$

$$\phi_0[x_, y_] = \frac{1}{2} \text{Log}\left[\frac{(1 - g[x + I y] f[x - I y])^2}{4 g'[x + I y] f'[x - I y]}\right];$$

In[614]:=

$$\text{Leq} /. \phi \rightarrow \phi_0 // \text{FullSimplify}$$

Out[614]=

$$0$$

In[610]:=

$$\text{Leq} /. \phi \rightarrow \phi_0 /. c \rightarrow 1/2 /. d \rightarrow 4 // \text{FullSimplify}$$

Out[610]=

$$0$$

200%

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In[617]:=

```
Leq //.  $\phi \rightarrow \phi_0$  // FullSimplify
```

Out[617]=

```
0
```

Out[618]=

```
 $\phi_0[x, y] /. \{f \rightarrow, g \rightarrow\};$ 
```

$$\frac{1}{2} \operatorname{Log} \left[\frac{(1 - f[x - i y] g[x + i y])^2}{4 f'[x - i y] g'[x + i y]} \right]$$

100%

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Friday Morning Part 1.nb

```
Leq //.  $\phi \rightarrow \phi 0$  // FullSimplify
```

Out[617]=

0

f by identity, $f[x]=x$

g by $g[x] = x^2$

```
 $\phi 0[x, y] /. \{f \rightarrow |, g \rightarrow\};$ 
```

Out[618]=

$$\frac{1}{2} \operatorname{Log} \left[\frac{(1 - f[x - i y] g[x + i y])^2}{4 f'[x - i y] g'[x + i y]} \right]$$

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Friday Morning Part 1.nb

Out[621]=

0

In[625]:=

1 '
0 & [x]
0 [x]

Out[625]=

0 &

Out[626]=

0

Out[627]=

0 [x]

100%

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Friday Morning Part 1.nb

In[629]:=

$$\phi 0[x, y]$$

Out[629]=

$$\frac{1}{2} \operatorname{Log} \left[\frac{(1 - f[x - i y] g[x + i y])^2}{4 f'[x - i y] g'[x + i y]} \right]$$

In[628]:=

$$\phi 0[x, y] /. \{f \rightarrow 1\};

100%$$

$$\frac{1}{2} \text{Log} \left[\frac{4 f' [x - i y] g' [x + i y]}{1} \right]$$

`ϕ0[x, y] /. {f[z_] → z}`

In[631]:=

`ϕ0[x, y] /. {f[z_] → z}`

Out[631]=

$$\frac{1}{2} \text{Log} \left[\frac{(1 - (x - i y) g[x + i y])^2}{4 f' [x - i y] g' [x + i y]} \right]$$

In[632]:=

`f' [x - i y] // FullForm`

Out[632]//FullForm=

`Derivative[1][f][Plus[x Times[Complex[0, 1], y]]]`

$$\frac{1}{2} \text{Log} \left[\frac{4 f' [x - i y] g' [x + i y]}{1} \right]$$

`phi0[x, y] /. {f[z_] -> z}`

In[631]:=

`phi0[x, y] /. {f[z_] -> z}`

Out[631]=

$$\frac{1}{2} \text{Log} \left[\frac{(1 - (x - i y) g[x + i y])^2}{4 f' [x - i y] g' [x + i y]} \right]$$

In[632]:=

`f' [x - i y] // FullForm`

Out[632]//FullForm=

`Derivative[1][f][Plus[x Times[Complex[0, 1], y]]]`

$$\frac{1}{2} \text{Log} \left[\frac{4 f' [x - i y] g' [x + i y]}{1} \right]$$

$\phi 0[x, y] /. \{f[z_] \rightarrow z\}$

In[634]:=

$F[x_] = x;$

In[638]:=

$\phi 0[x, y] /. \{f \rightarrow (\# \&), g \rightarrow (\#^2 \&)\}$

Out[638]=

$$\frac{1}{2} \text{Log} \left[\frac{(1 - (x - i y) (x + i y)^2)^2}{8 (x + i y)} \right]$$

In[635]:=

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Friday Morning Part 1.nb

Out[617]=

0

In[648]:=

$\phi 1[x_, y_] = \phi 0[x, y] /. \{f \rightarrow (\# \&), g \rightarrow (\#^2 \&)\}$

Out[648]=

$$\frac{1}{2} \text{Log} \left[\frac{\left(1 - (x - i y) (x + i y)^2\right)^2}{8 (x + i y)} \right]$$

300%

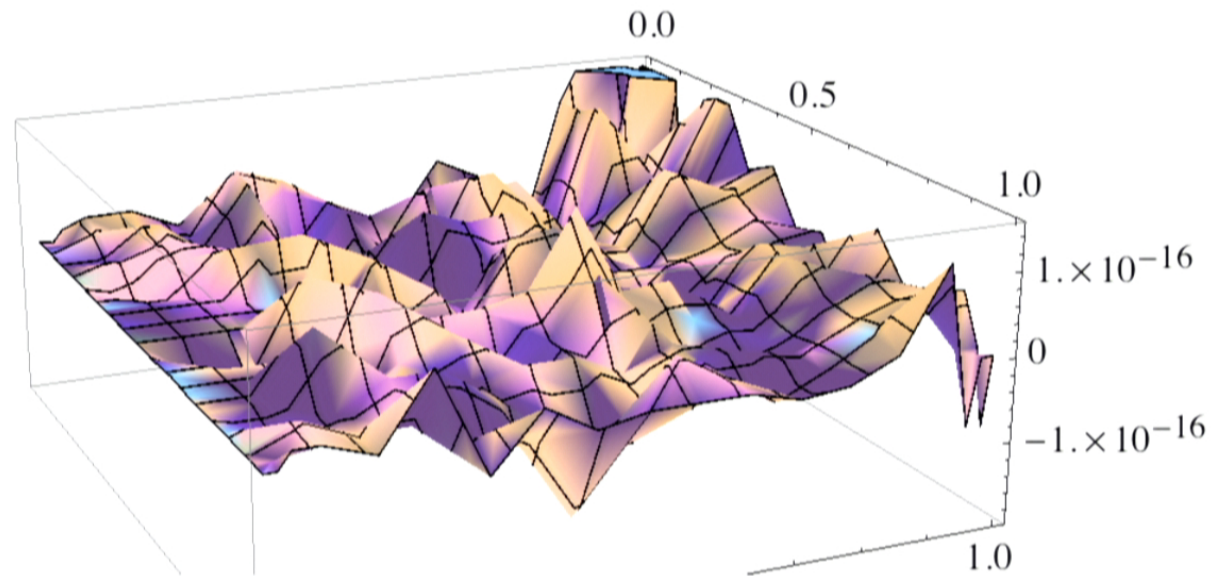
The image is a composite of two overlapping windows. The background window is the Wolfram Mathematica application, displaying the documentation for the 'Plot' function. The title bar reads 'Plot - Wolfram Mathematica'. The search bar contains 'ref/Plot'. The main content area shows a plot of several overlapping colored curves (blue, green, red, purple) on a coordinate system. Below the plot, there are sections for 'Scope', 'Generalizations & Extensions', 'Options', 'Applications', 'Properties & Relations', and 'Neat Examples'. A 'SEE ALSO' section lists related functions like 'DiscretePlot', 'ListLinePlot', 'ParametricPlot', 'PolarPlot', 'Plot3D', 'ContourPlot', 'Graphics', and 'Show'. A 'TUTORIALS' section lists 'Basic Plotting', 'Options for Graphics', and 'Graphics and Sound'. A 'MORE ABOUT' section lists various topics like 'Differential Equations', 'Function Visualization', 'Precollege Education', 'Statistical Visualization', 'Summary of New Features in Mathematica 8', and 'New in 6.0: Function Visualization'. A 'RELATED LINKS' section lists 'Demonstrations with Plot (Wolfram Demonstrations Project)'. At the bottom, it says 'New in 1 | Last modified in 6'. The foreground window is a presentation slide with a yellow background and a blue hexagonal pattern at the top. It contains the text 'x from x_{min} to x_{max} .', 'ots several functions f_i .', and a blue arrow pointing right. There is also a large mathematical expression $y)^2)^2]$ on the slide.

$$\frac{1}{2} \operatorname{Log} \left[\frac{1}{16} e^{-4 i y} \left(1 + e^{2 x + 2 i y} + e^{4 i y} \right)^2 \right]$$

In[659]:=

```
Plot3D[ϕ1[x, y] // Im, {x, 0, 1}, {y, 0, 1}]
```

Out[659]=



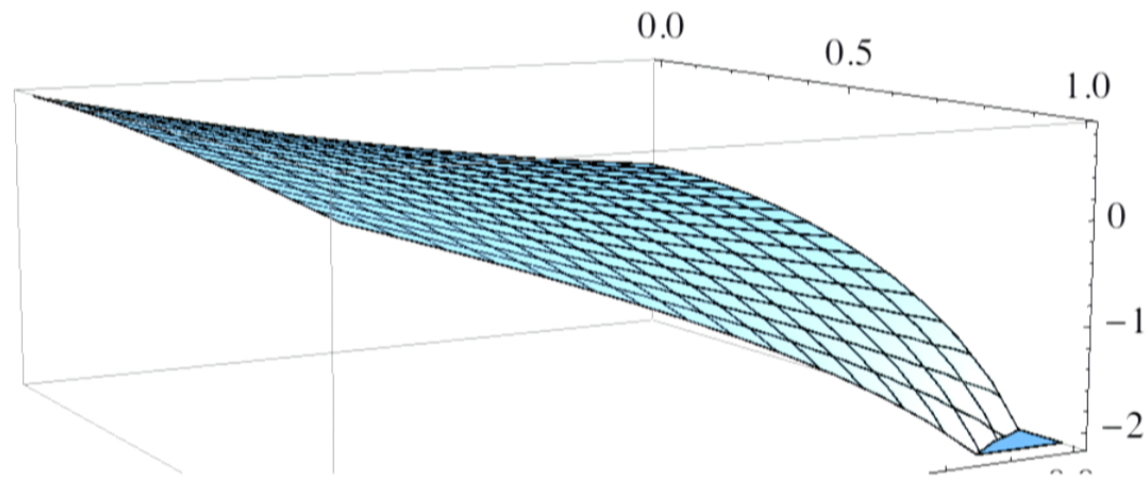
Out[658]=

$$\frac{1}{2} \operatorname{Log} \left[\frac{1}{16} e^{-4 i y} \left(1 + e^{2 x + 2 i y} + e^{4 i y} \right)^2 \right]$$

In[661]:=

```
Plot3D[ϕ1[x, y] // Re, {x, 0, 1}, {y, 0, 1}]
```

Out[661]=

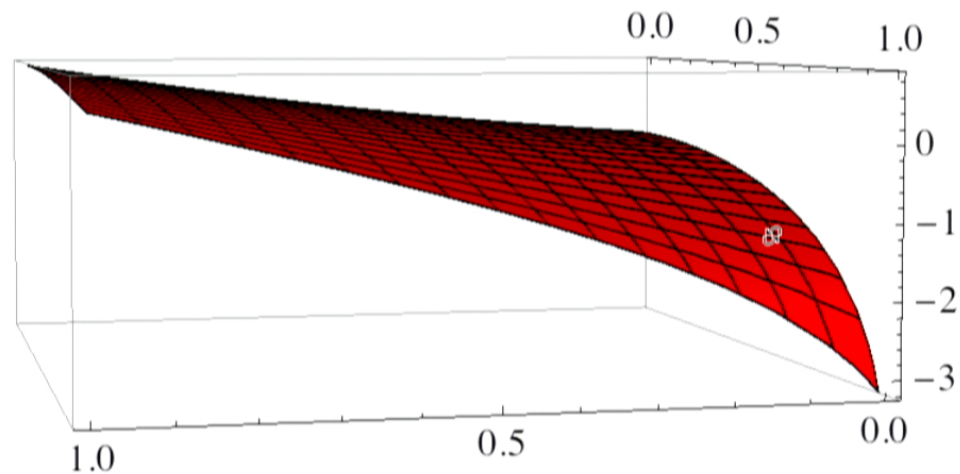


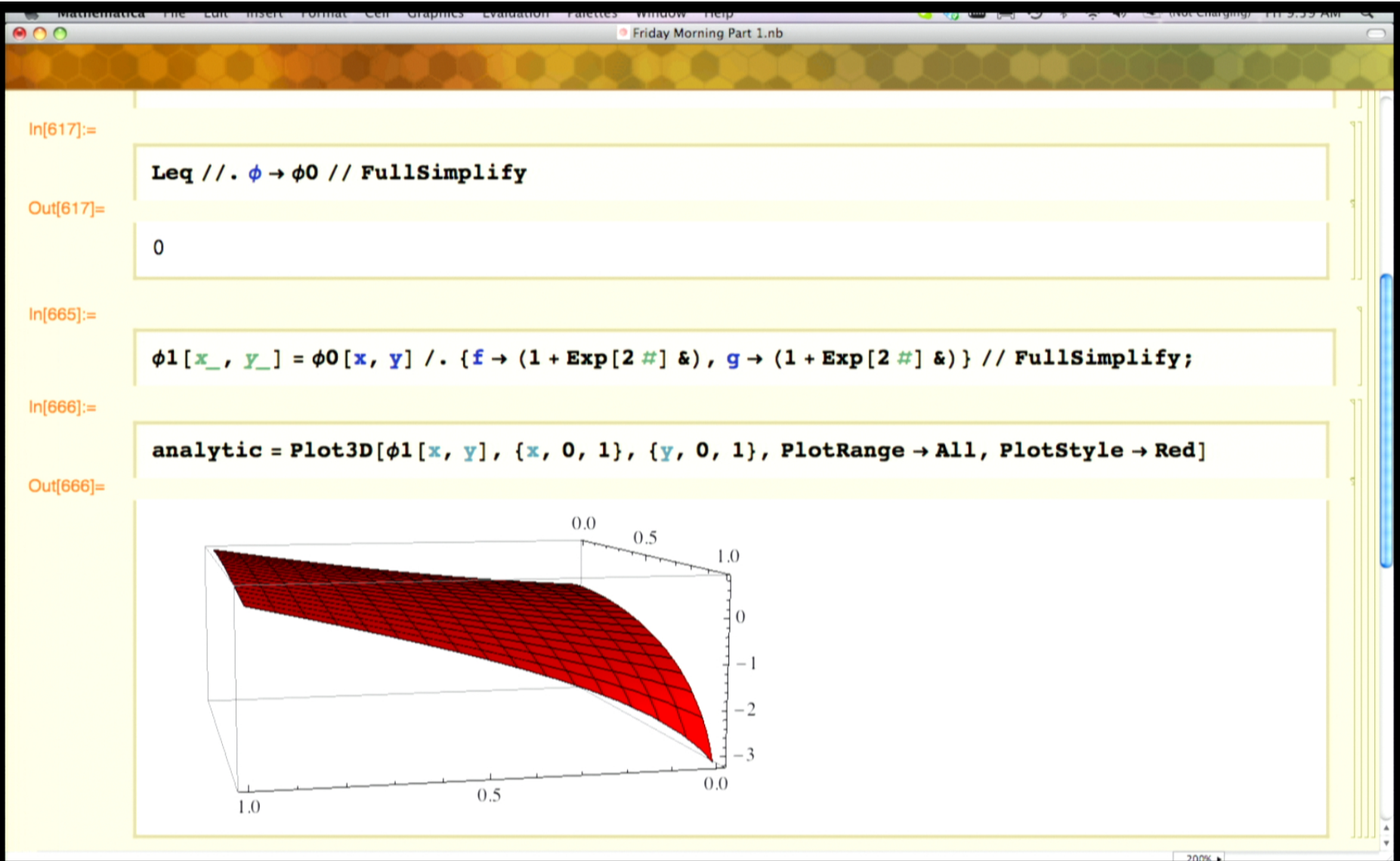
$$\frac{1}{2} \operatorname{Log} \left[\frac{1}{16} e^{-4 i y} \left(1 + e^{2 x+2 i y} + e^{4 i y} \right)^2 \right]$$

In[664]:=

```
analytic = Plot3D[phi1[x, y] // Re, {x, 0, 1},
  {y, 0, 1}, PlotRange -> All, PlotStyle -> Red]
```

Out[664]=





Partial Differential Equations

■ Analytics

In[615]:=

```
Leq =  $\partial_{\{x,2\}} \phi[x, y] + \partial_{\{y,2\}} \phi[x, y] + \text{Exp}[-2 \phi[x, y]]$ ;
```

In[616]:=

```
 $\phi0[x_, y_] = \frac{1}{2} \text{Log} \left[ \frac{(1 - g[x + I y] f[x - I y])^2}{4 g'[x + I y] f'[x - I y]} \right];$ 
```

In[617]:=

```
Leq /.  $\phi \rightarrow \phi0$  // FullSimplify
```

Out[617]=

0

In[665]:=

```
 $\phi1[x_, y_] = \phi0[x, y] /. \{f \rightarrow (1 + \text{Exp}[2 \#] \&), g \rightarrow (1 + \text{Exp}[2 \#] \&)\} // \text{FullSimplify};$ 
```

In[666]:=

```
analytic = Plot3D[ $\phi1[x, v]$ , {x, 0, 1}, {v, 0, 1}, PlotRange -> All, PlotStyle -> Red]
```


Partial Differential Equations

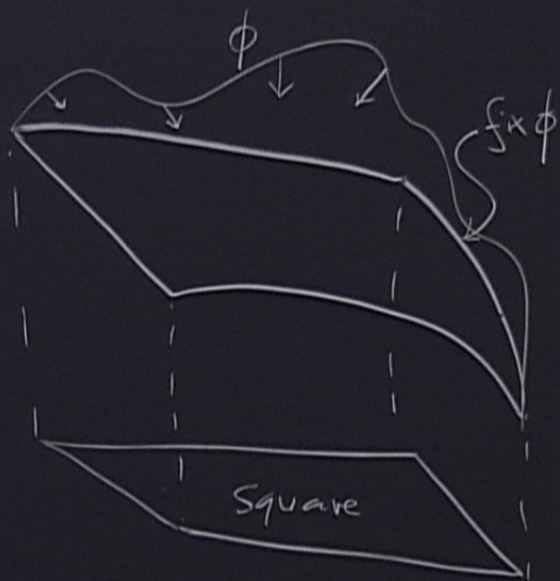
- Analytics
- Numerics

In[667]:=

Leq

Out[667]=

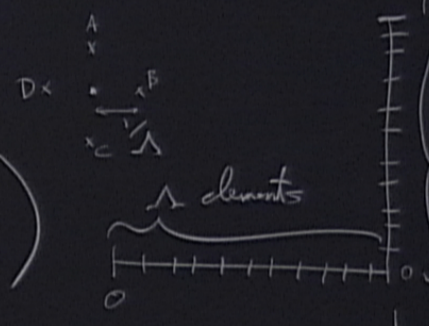
$$e^{-2\phi[x,y]} + \phi^{(0,2)}[x,y] + \phi^{(2,0)}[x,y]$$



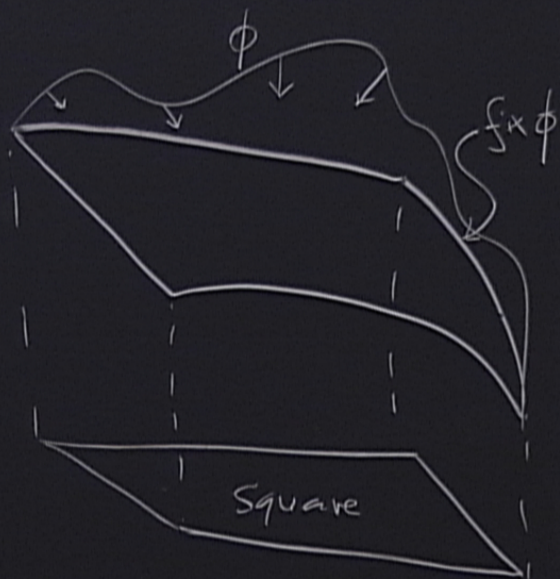
$$\nabla^2 \phi + \nabla^2 \psi$$

$$\downarrow$$

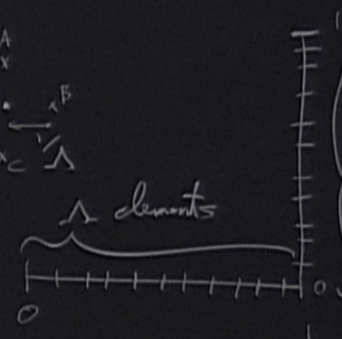
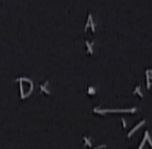
$$\frac{1}{4} (\psi_A + \psi_B + \psi_C + \psi_D - 4\psi_c)$$



1 element



$$\frac{1}{4} \left(\psi_A + \psi_B + \psi_C + \psi_D - 4\psi_{\text{center}} \right)$$



elements

$$I > 0$$

Partial Differential Equations

- Analytics
- Numerics

In[667]:=

Leq

Out[667]=

$$e^{-2\phi[x,y]} + \phi^{(0,2)}[x,y] + \phi^{(2,0)}[x,y]$$

G[n_] :=

Leq

$$e^{-2\phi[\mathbf{x}, \mathbf{y}]} + \phi^{(0,2)}[\mathbf{x}, \mathbf{y}] + \phi^{(2,0)}[\mathbf{x}, \mathbf{y}]$$

```
G[n_] := G[n - 1] + n G[n - 2];
```

G[5]	
------	--

$$\begin{aligned} & (((((\dots(((((G[-249-1]) - 249 G[-249-2]) - 248 G[-248- \\ & \quad 2]) - 247 G[-247-2]) - 246 G[-246-2]) - 245 \\ & \quad G[-245-2]) - 244 G[-244-2]) - 243 \\ & \quad G[-243-2]) - 242 G[-242-2]) - 241 G[\\ & \quad -241-2]) - 240 G[-240-2]) - 239 G[-239- \end{aligned}$$

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Out[667]=

$$e^{-2\phi[x,y]} + \phi^{(0,2)}[x,y] + \phi^{(2,0)}[x,y]$$

In[670]:=

```
G[n_] := G[n - 1] + n G[n - 2];  
G[0] = 1;  
G[1] = 2;
```

In[676]:=

```
G[100]
```

Out[675]=

```
35 696
```

200%

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In[677]:=

```
G[n_] := G[n] = G[n - 1] + n G[n - 2];  
G[0] = 1;  
G[1] = 2;
```

In[682]:=

```
G[100]
```

Out[682]=

```
253 432 098 810 473 036 968 111 090 447 973 896 728 572 522 374 012 678 663 012 251 250 290 473 \n349 127 602 176
```

In[681]:=

```
? G
```

Global`G

```
G[0] = 1  
G[1] = 2  
G[2] = 4  
G[3] = 10
```

200%

■ Analytics

■ Numerics

In[667]:=

Leq

Out[667]=

$$e^{-2\phi[x,y]} + \phi^{(0,2)}[x,y] + \phi^{(2,0)}[x,y]$$

```
G[n_] := G[n] = G[n - 1] + n G[n - 2];
```

```
G[0] = 1;
```

```
G[1] = 2;
```

```
|
```

Out[682]=

```
253 432 098 810 473 036 968 111 090 447 973 896 728 572 522 374 012 678 663 012 251 250 290 473 \
349 127 602 176
```

■ Analytics

■ Numerics

In[667]:=

Leq

Out[667]=

$$e^{-2\phi[x,y]} + \phi^{(0,2)}[x,y] + \phi^{(2,0)}[x,y]$$

In[684]:=

```
G[n_] := G[n] = G[n - 1] + n G[n - 2];  
G[0] = 1;  
G[1] = 2;
```

In[683]:=

G[100]

Out[683]=

```
253 432 098 810 473 036 968 111 090 447 973 896 728 572 522 374 012 678 663 012 251 250 290 473  
349 127 602 176
```


■ Analytics

■ Numerics

In[667]:=

Leq

Out[667]=

$$e^{-2\phi[x,y]} + \phi^{(0,2)}[x,y] + \phi^{(2,0)}[x,y]$$

$$\begin{aligned} \psi[n_][x_ , y_] &:= \\ \psi[n][x, y] &= \\ \frac{1}{4} &\left(\psi[n-1]\left[x + \frac{1}{\Lambda}, y\right] + \psi[n-1]\left[x - \frac{1}{\Lambda}, y\right] + \psi[n-1]\left[x, y + \frac{1}{\Lambda}\right] + \psi[n-1]\left[x, y - \frac{1}{\Lambda}\right] + \right. \\ &\left. \frac{1}{\Lambda^2} \text{Exp}[-2\psi[n-1][x, y]] \right) \end{aligned}$$

In[684]:=

```
G[n_] := G[n] = G[n - 1] + n G[n - 2];
G[0] = 1;
G[1] = 2;
```

In[683]:=

G[100]

■ Analytics

■ Numerics

In[667]:=

Leq

Out[667]=

$$e^{-2\phi[x,y]} + \phi^{(0,2)}[x,y] + \phi^{(2,0)}[x,y]$$

$\Delta = 20;$

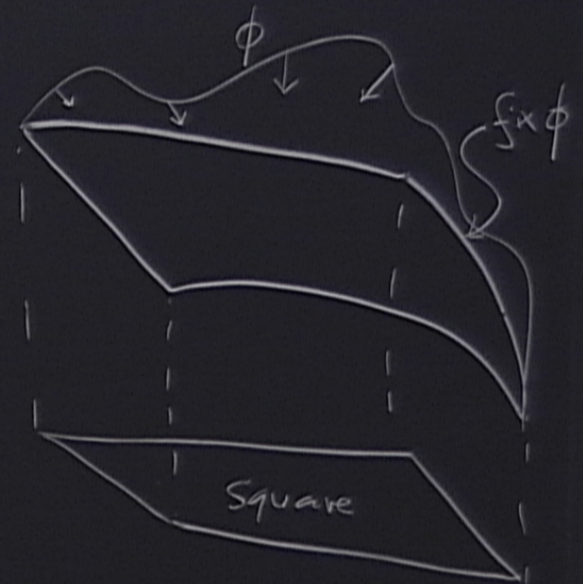
$$\begin{aligned} \psi[n_][x_ , y_] &:= \\ \psi[n][x, y] &= \\ &\frac{1}{4} \left(\psi[n-1]\left[x + \frac{1}{\Delta}, y\right] + \psi[n-1]\left[x - \frac{1}{\Delta}, y\right] + \psi[n-1]\left[x, y + \frac{1}{\Delta}\right] + \psi[n-1]\left[x, y - \frac{1}{\Delta}\right] + \right. \\ &\quad \left. \frac{1}{\Delta^2} \text{Exp}[-2\psi[n-1][x, y]] \right); \end{aligned}$$

In[684]:=

```
G[n_] := G[n] = G[n - 1] + n G[n - 2];
G[0] = 1;
G[1] = 2;
```


$$\psi_n(x,y) \leftrightarrow \psi[n][x,y]$$

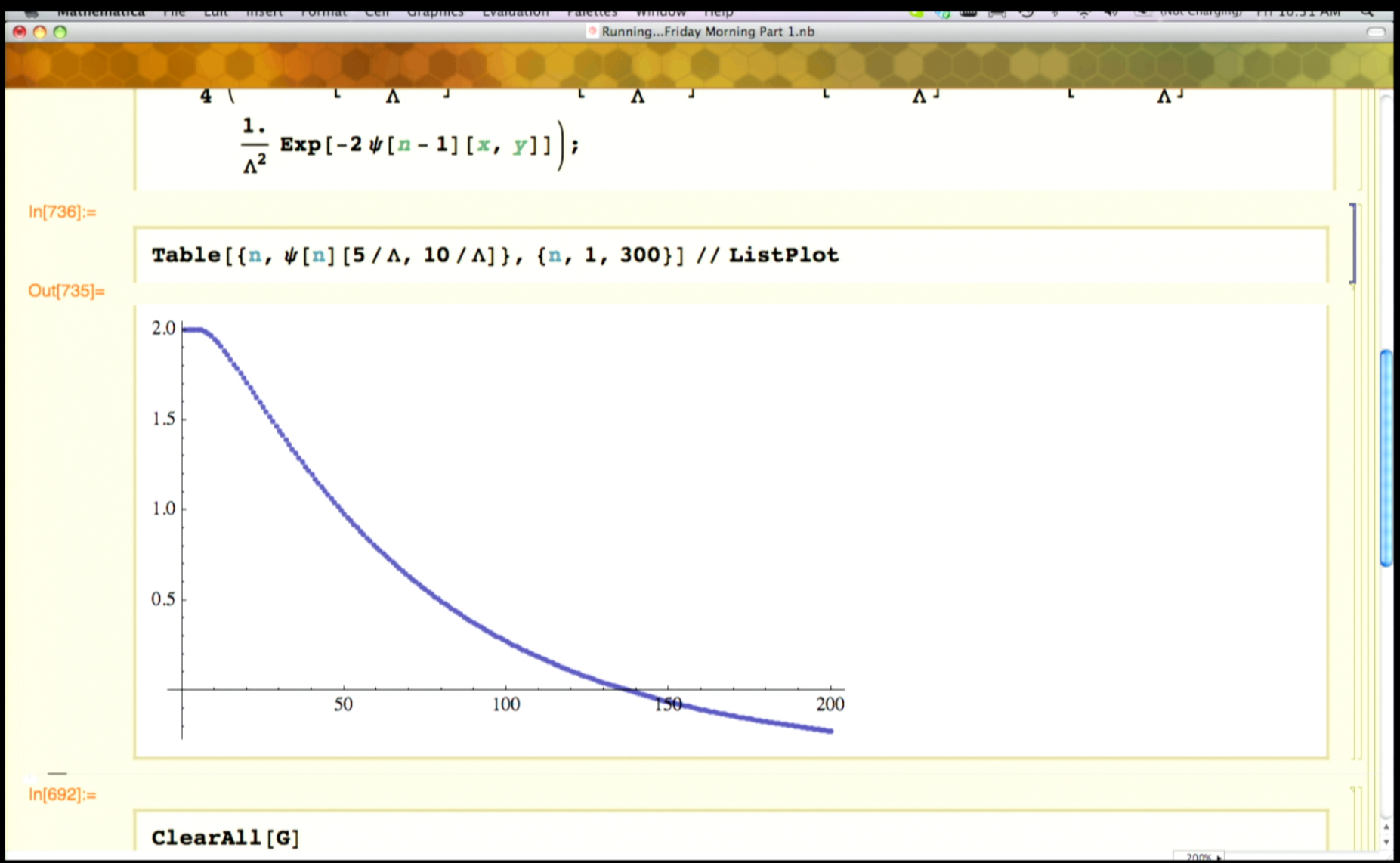
or
 $\psi[n, x, y]$

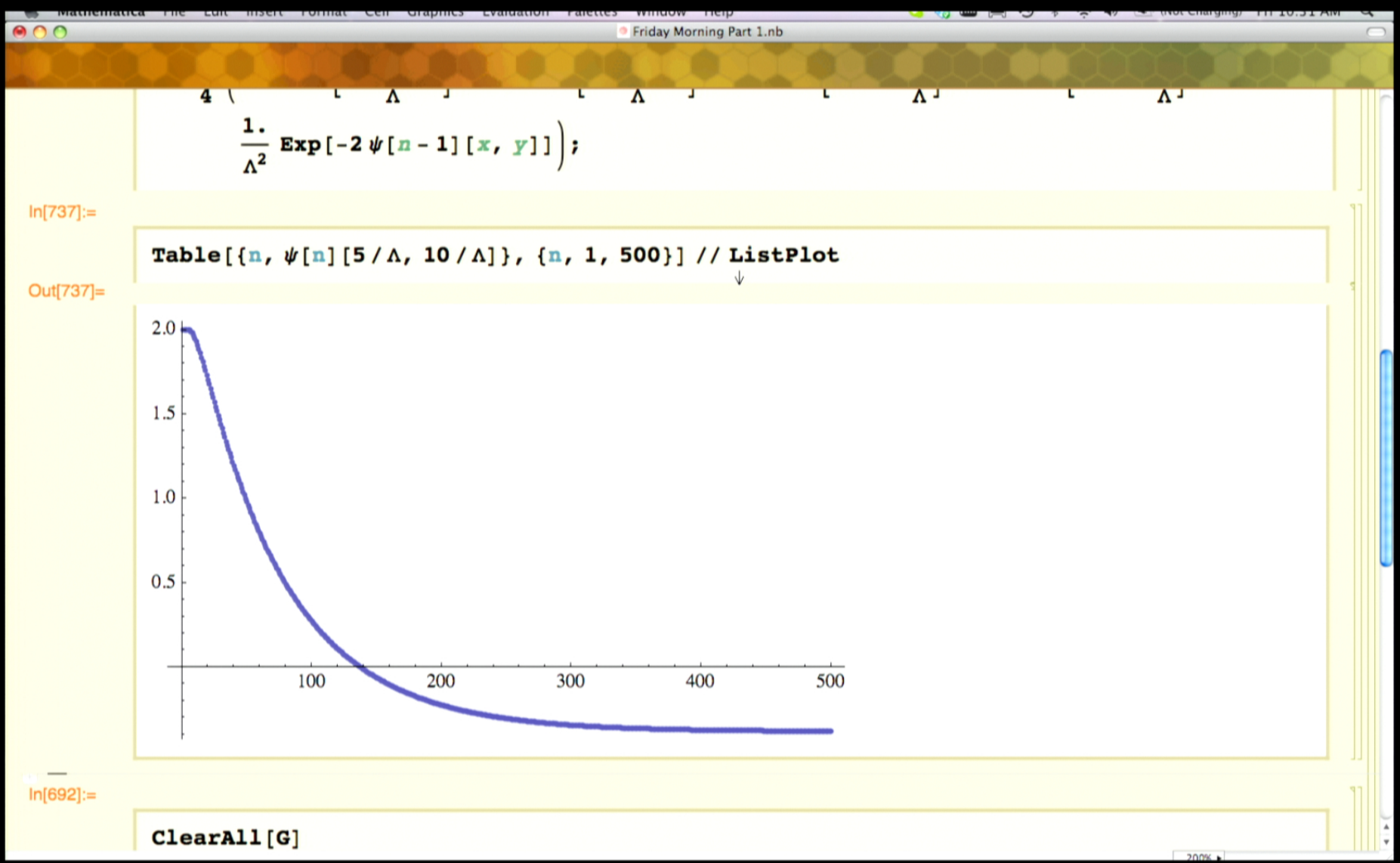


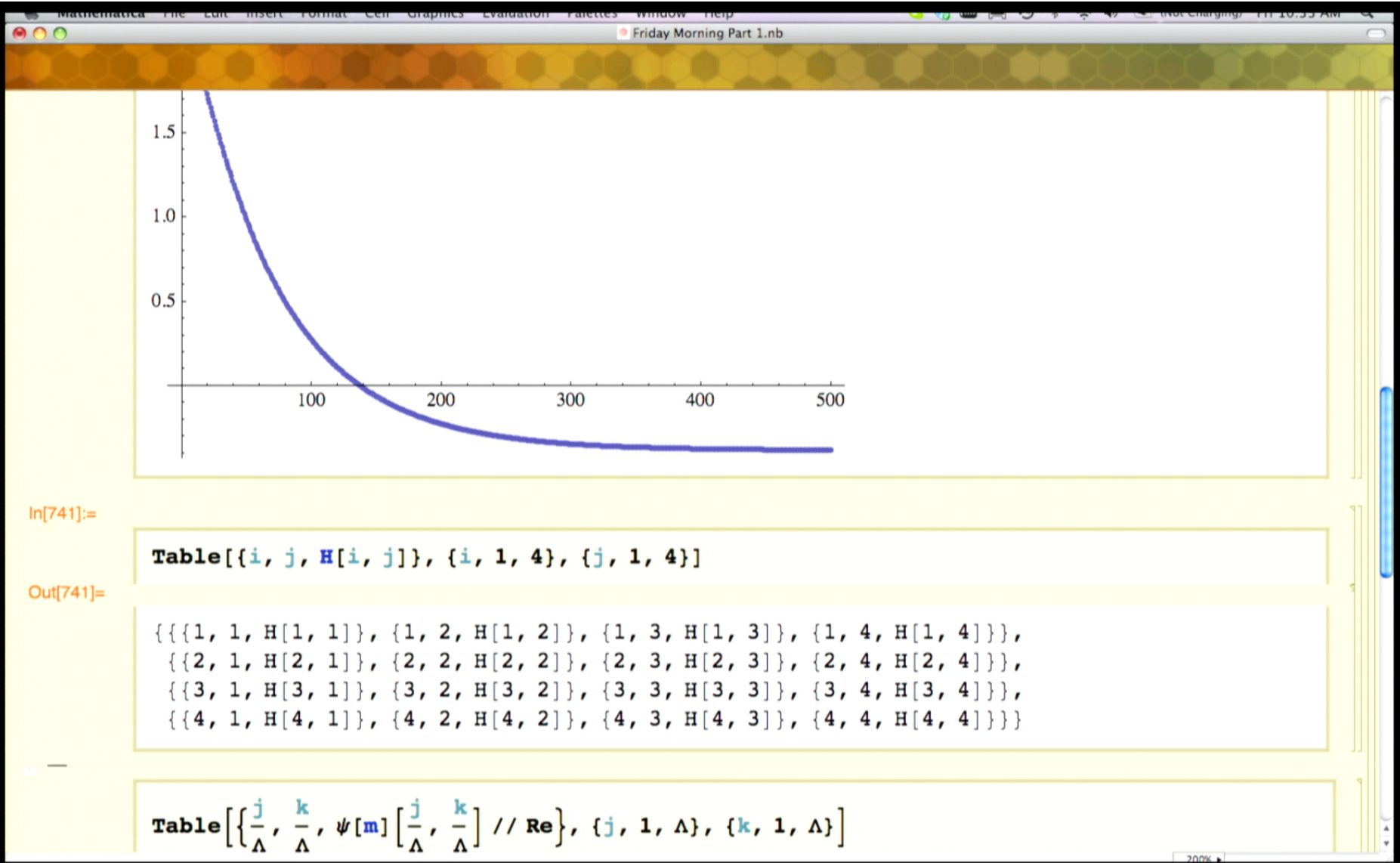
Ψ_1 = whatever ending on
square bc

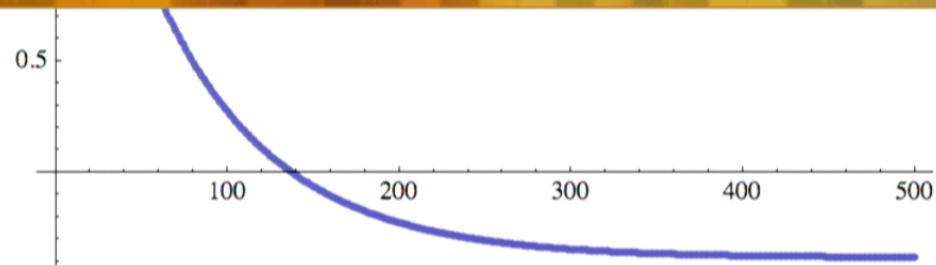
$\Psi_\infty = \Psi_{\text{analytic}}$

$$\Psi_n(x, y) \leftrightarrow \Psi[n][x, y] \text{ or } \Psi[n, x, y]$$









In[743]:=

```
Flatten[Table[{i, j, H[i, j]}, {i, 1, 4}, {j, 1, 4}]]
```

Out[743]=

```
{1, 1, H[1, 1], 1, 2, H[1, 2], 1, 3, H[1, 3], 1, 4, H[1, 4], 2, 1, H[2, 1], 2,
 2, H[2, 2], 2, 3, H[2, 3], 2, 4, H[2, 4], 3, 1, H[3, 1], 3, 2, H[3, 2], 3, 3,
 H[3, 3], 3, 4, H[3, 4], 4, 1, H[4, 1], 4, 2, H[4, 2], 4, 3, H[4, 3], 4, 4, H[4, 4]}
```

In[744]:=

?Flatten

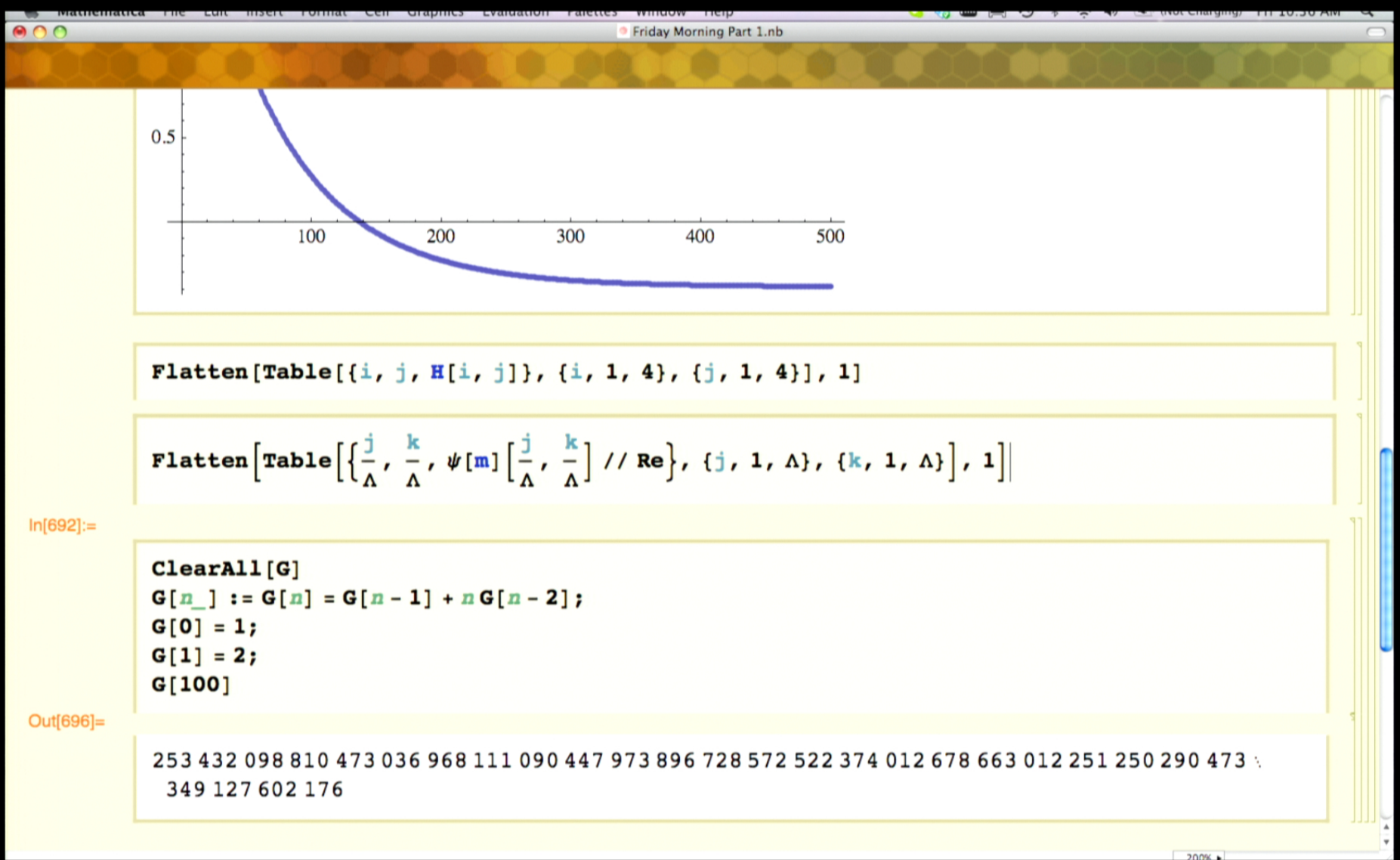
Flatten[list] flattens out nested lists.

Flatten[list, n] flattens to level n.

Flatten[list, n, h] flattens subexpressions with head h.

Flatten[list, {{s₁₁, s₁₂, ...}, {s₂₁, s₂₂, ...}, ...}]

flattens list by combining all levels s_{ij} to make each level i in the result. >>




```

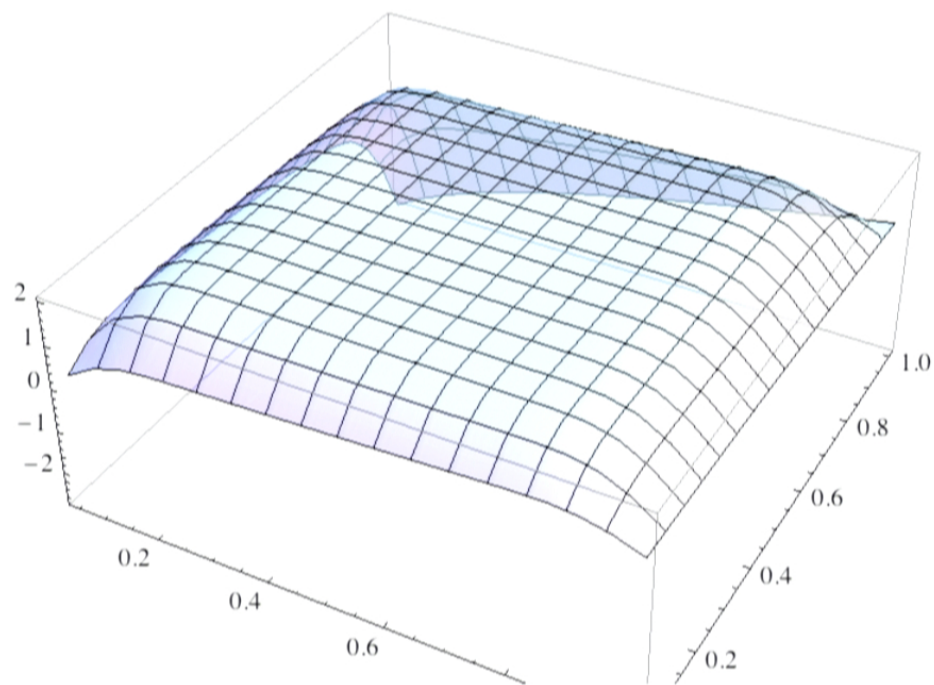
num[m] = ListPlot3D[Flatten[Table[{ $\frac{j}{\Lambda}$ ,  $\frac{k}{\Lambda}$ ,  $\psi[m][\frac{j}{\Lambda}, \frac{k}{\Lambda}] // \text{Re}$ }, {j, 1,  $\Lambda$ }, {k, 1,  $\Lambda$ }], 1],
  PlotRange -> All, PlotStyle -> Opacity[.5]]

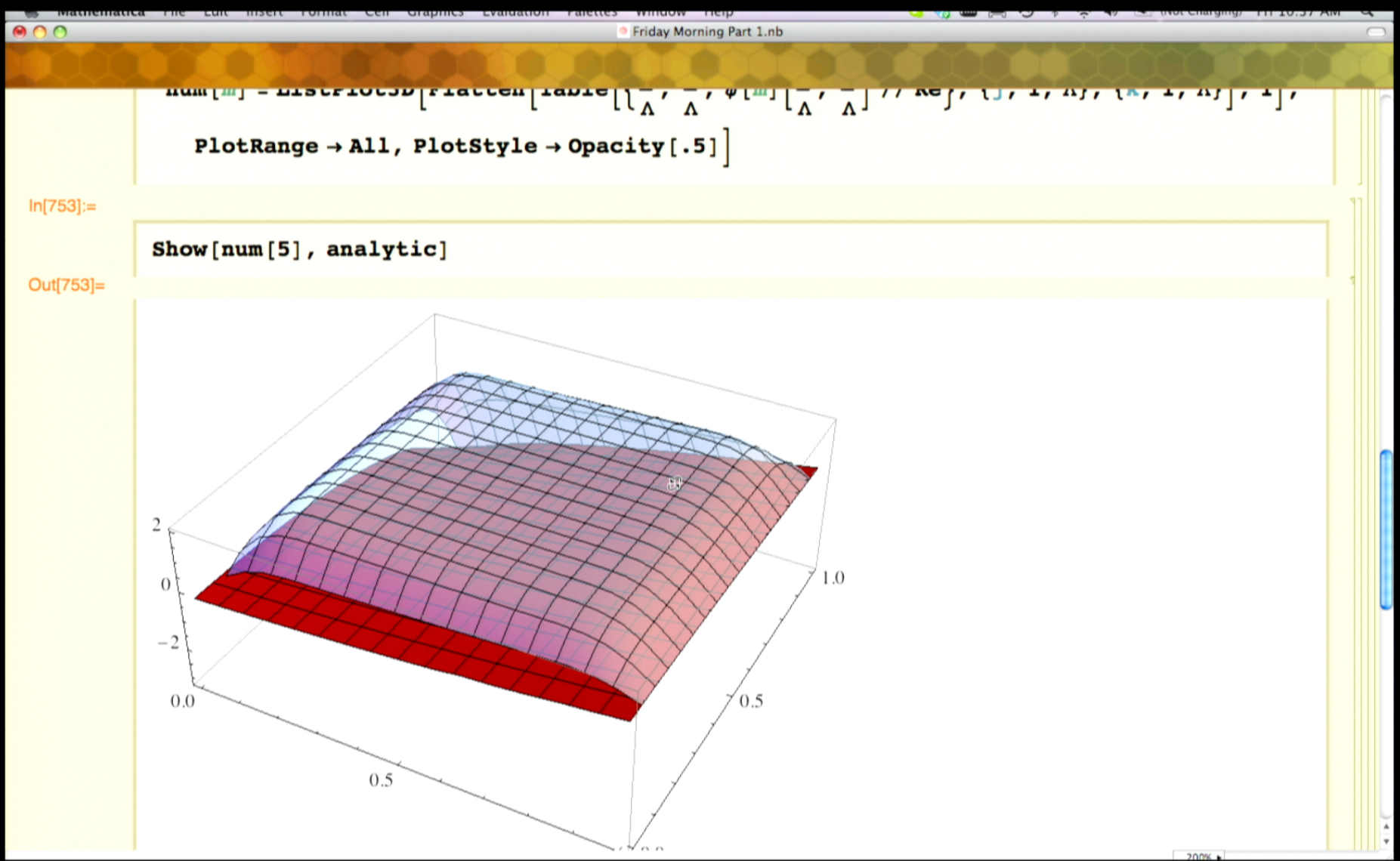
```

In[752]:=

num[5]

Out[752]=



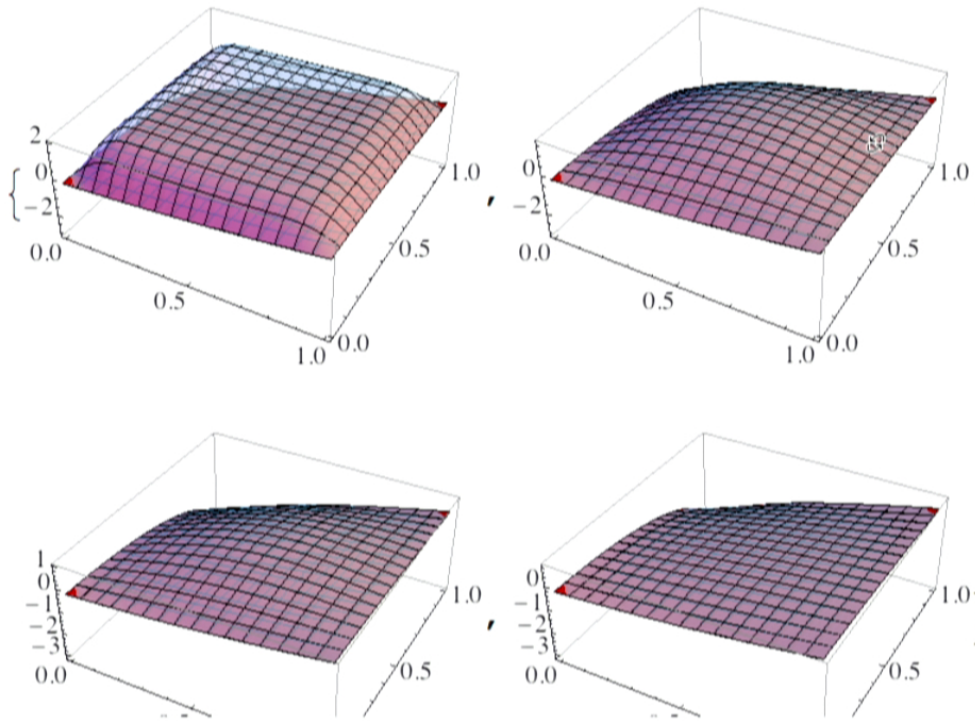


```
num[#] = ListPlot3D[Flatten[Table[{ $\frac{1}{\Delta}$ ,  $\frac{1}{\Delta}$ ,  $\psi[\frac{1}{\Delta}, \frac{1}{\Delta}]$  Re}, { $\frac{1}{\Delta}$ , 0,  $\Delta$ }, { $\frac{1}{\Delta}$ , 0,  $\Delta$ }, {1, 1, 1}], 1],  
PlotRange -> All, PlotStyle -> Opacity[.5]]
```

In[757]:=

```
Show[num[#], analytic] & /@ {5, 50, 100, 200}
```

Out[757]=

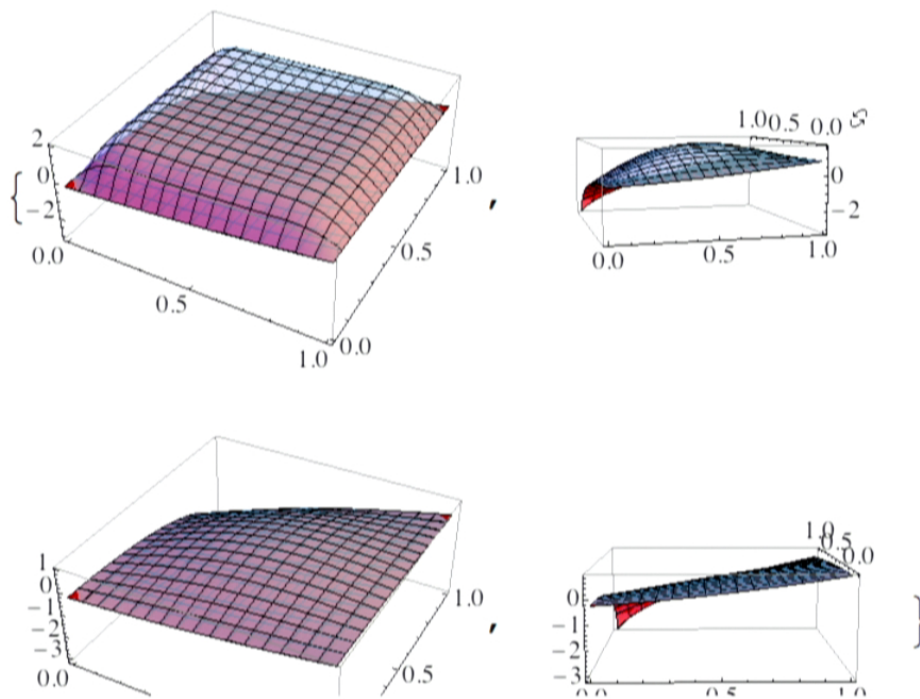



```
num[m] = ListPlot3D[Flatten[Table[{ $\frac{j}{\Lambda}$ ,  $\frac{k}{\Lambda}$ ,  $\psi[m][\frac{j}{\Lambda}, \frac{k}{\Lambda}] // \text{Re}$ }, {j, 0,  $\Lambda$ }, {k, 0,  $\Lambda$ }], 1],  
PlotRange -> All, PlotStyle -> Opacity[.5]]
```

In[757]:=

```
Show[num[#], analytic] & /@ {5, 50, 100, 200}
```

Out[757]=



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In[667]:= **Leq**

Out[667]:=
$$e^{-2\phi[x,y]} + \phi^{(0,2)}[x,y] + \phi^{(2,0)}[x,y]$$

In[697]:= $\Lambda = 20;$

In[722]:= **ClearAll**[ψ]

$\psi[n_][0, y_] = \phi1[0, y];$
 $\psi[n_][1, y_] = \phi1[1, y];$
 $\psi[n_][x_, 0] = \phi1[x, 0];$
 $\psi[n_][x_, 1] = \phi1[x, 1];$

$\psi[1][x_, y_] = 2;$

$\psi[n_][x_, y_] :=$

$$\psi[n][x, y] = \frac{1}{4} \left(\psi[n-1]\left[x + \frac{1}{\Lambda}, y\right] + \psi[n-1]\left[x - \frac{1}{\Lambda}, y\right] + \psi[n-1]\left[x, y + \frac{1}{\Lambda}\right] + \psi[n-1]\left[x, y - \frac{1}{\Lambda}\right] + \frac{1}{\Lambda^2} \text{Exp}[-2\psi[n-1][x, y]] \right);$$

(* Table[{n, $\psi[n][5/\Lambda, 10/\Lambda]$ }, {n, 1, 500}]]//ListPlot; *)

In[755]:= $\text{num}[m_] := \text{num}[m] = \text{ListPlot3D}\left[\text{Flatten}\left[\text{Table}\left[\left\{\frac{j}{\Lambda}, \frac{k}{\Lambda}, \psi[m]\left[\frac{j}{\Lambda}, \frac{k}{\Lambda}\right] // \text{Re}\right\}, \{j, 0, \Lambda\}, \{k, 0, \Lambda\}\right], 1\right],\right.$
 $\left. \text{PlotRange} \rightarrow \text{All}, \text{PlotStyle} \rightarrow \text{Opacity} [.5]\right]$

In[757]:= **Show**[$\text{num}[\#]$, **analytic**] & /@ {5, 50, 100, 200}



100%

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Friday Morning Part 1.nb

In[667]:= **Leq**

Out[667]:= $e^{-2 \phi[x, y]} + \phi^{(0,2)}[x, y] + \phi^{(2,0)}[x, y]$

In[697]:= $\Lambda = 20;$

In[722]:= **ClearAll**[ψ]

$\psi[n_][0, y_] = \phi 1[0, y];$
 $\psi[n_][1, y_] = \phi 1[1, y];$
 $\psi[n_][x_, 0] = \phi 1[x, 0];$
 $\psi[n_][x_, 1] = \phi 1[x, 1];$

$\psi[1][x_, y_] = 2;$

$\psi[n_][x_, y_] :=$
 $\psi[n][x, y] = \frac{1}{4} \left(\psi[n-1]\left[x + \frac{1}{\Lambda}, y\right] + \psi[n-1]\left[x - \frac{1}{\Lambda}, y\right] + \psi[n-1]\left[x, y + \frac{1}{\Lambda}\right] + \psi[n-1]\left[x, y - \frac{1}{\Lambda}\right] + \frac{1}{\Lambda^2} \text{Exp}[-2 \psi[n-1][x, y]] \right);$

(* Table[{n, $\psi[n][5/\Lambda, 10/\Lambda]$ }, {n, 1, 500}]]//ListPlot; *)

In[755]:= $\text{num}[m_]:= \text{num}[m] = \text{ListPlot3D}\left[\text{Flatten}\left[\text{Table}\left[\left\{\frac{j}{\Lambda}, \frac{k}{\Lambda}, \psi[m]\left[\frac{j}{\Lambda}, \frac{k}{\Lambda}\right] // \text{Re}\right\}, \{j, 0, \Lambda\}, \{k, 0, \Lambda\}\right], 1\right],\right.$
 $\left. \text{PlotRange} \rightarrow \text{All}, \text{PlotStyle} \rightarrow \text{Opacity} [.5]\right]$

In[757]:= **Show**[$\text{num}[\#]$, **analytic**] & /@ {5, 50, 100, 200}



100%


```

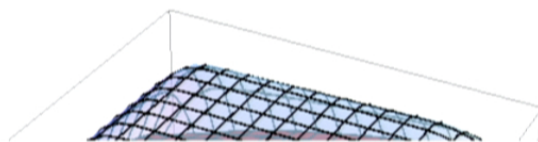
num[m_] :=
num[m] =
ListPlot3D[
  Flatten[Table[{ $\frac{j}{\Lambda}$ ,  $\frac{k}{\Lambda}$ ,  $\psi[m][\frac{j}{\Lambda}, \frac{k}{\Lambda}] // \text{Re}$ },
    {j, 0,  $\Lambda$ }, {k, 0,  $\Lambda$ }], 1], PlotRange → All,
  PlotStyle → Opacity[.5]]

```

In[757]:=

```
Show[num[#], analytic] & /@ {5, 50, 100, 200}
```

Out[757]=



```

ψ[n_][x_, y_] :=
  ψ[n][x, y] =
    1/4 (ψ[n - 1][x + 1/Λ, y] + ψ[n - 1][x - 1/Λ, y] +
      ψ[n - 1][x, y + 1/Λ] + ψ[n - 1][x, y - 1/Λ] +
      1/Λ² Exp[-2 ψ[n - 1][x, y]]);

```

```

(* Table[{n, ψ[n][5/Λ, 10/Λ]}, {n, 1, 500}] //
  ListPlot; *)

```

In[755]:=

```

num[m_] :=
  num[m] =
    ListPlot[

```

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$$\frac{1}{\Lambda^2} \exp[-2\psi[n-1][x, y]];$$

```
(* Table[{n, ψ[n][5/Λ, 10/Λ]}, {n, 1, 500}] //
ListPlot; *)
```

In[755]:=

```
num[m_] :=
num[m] =
ListPlot3D[
Flatten[Table[{j/Λ, k/Λ, ψ[m][j/Λ, k/Λ] // Re},
{j, 0, Λ}, {k, 0, Λ}], 1], PlotRange → All,
PlotStyle → Opacity[.5]]
```

100%