

Title: Introduction to Mathematica - Lecture 1b

Date: Sep 04, 2012 10:30 AM

URL: <http://pirsa.org/12090051>

Abstract:

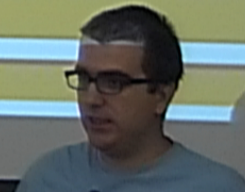
The image shows a Mathematica notebook window with a table of contents. The window title bar includes the Mathematica logo and menu items: File, Edit, Insert, Format, Cell, Graphics, Evaluation, Palettes, Window, Help. The system tray shows the date and time: Tuesday Morning Part 1, 11:11 AM. The table of contents is displayed in a light blue background with dark blue text. The items are: Shift + Enter, Equal signs = vs := vs ==, Lists, Substitutions, Delayed Replacements, Apply, Map, Distribute and Headers, Patterns, and Extra.

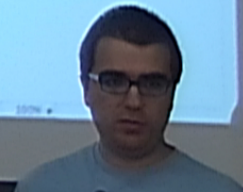
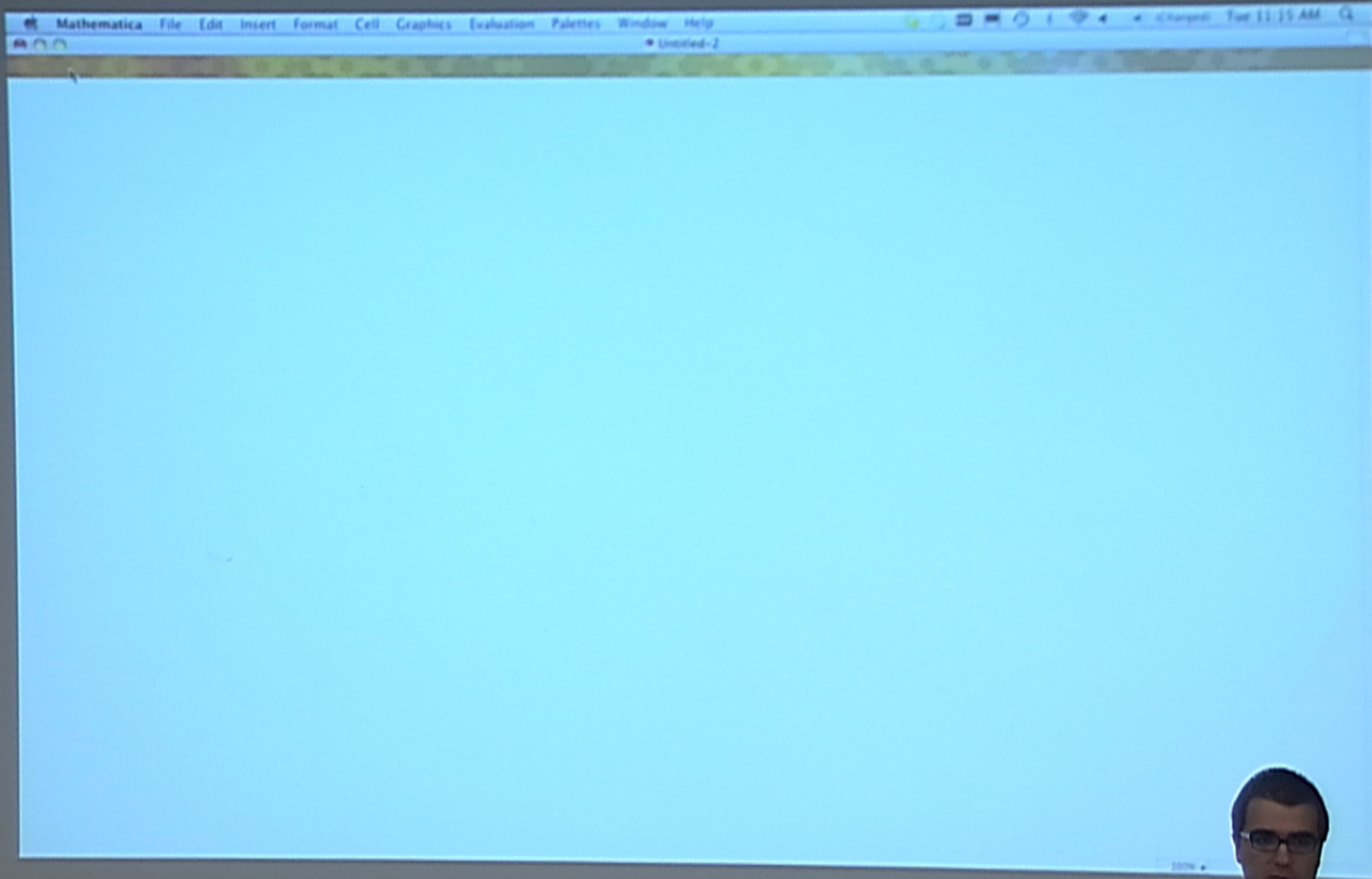
| |
|---------------------------|
| Shift + Enter |
| Equal signs = vs := vs == |
| Lists |
| Substitutions |
| Delayed Replacements |
| Apply, Map |
| Distribute and Headers |
| Patterns |
| Extra |

The image shows a presentation slide within a Mathematica application window. The window title bar includes the Mathematica logo and menu items: File, Edit, Insert, Format, Cell, Graphics, Evaluation, Palettes, Window, Help. The system tray shows the date and time: Tuesday Morning Part 1.nb, Tue 11:13 AM. The slide content is as follows:

Basics of Mathematica

- Shift + Enter
- Equal signs = vs := vs ==
- Lists
- Substitutions
- Delayed Replacements
- Apply, Map
- Distribute and Headers
- Patterns





$$\{\gamma_a, \gamma_b\} = 2\delta_{ab}$$

$$\{\gamma_a, \gamma_b\} = 2\delta_{ab}$$

$$\{\gamma_a, \gamma_b\} = 2 \delta_{ab}$$
$$\gamma_a \gamma_b + \gamma_b \gamma_a$$

$$\delta_{ab} \delta_{bc} = \delta_{ac}$$
$$\delta_{aa} = \dim$$

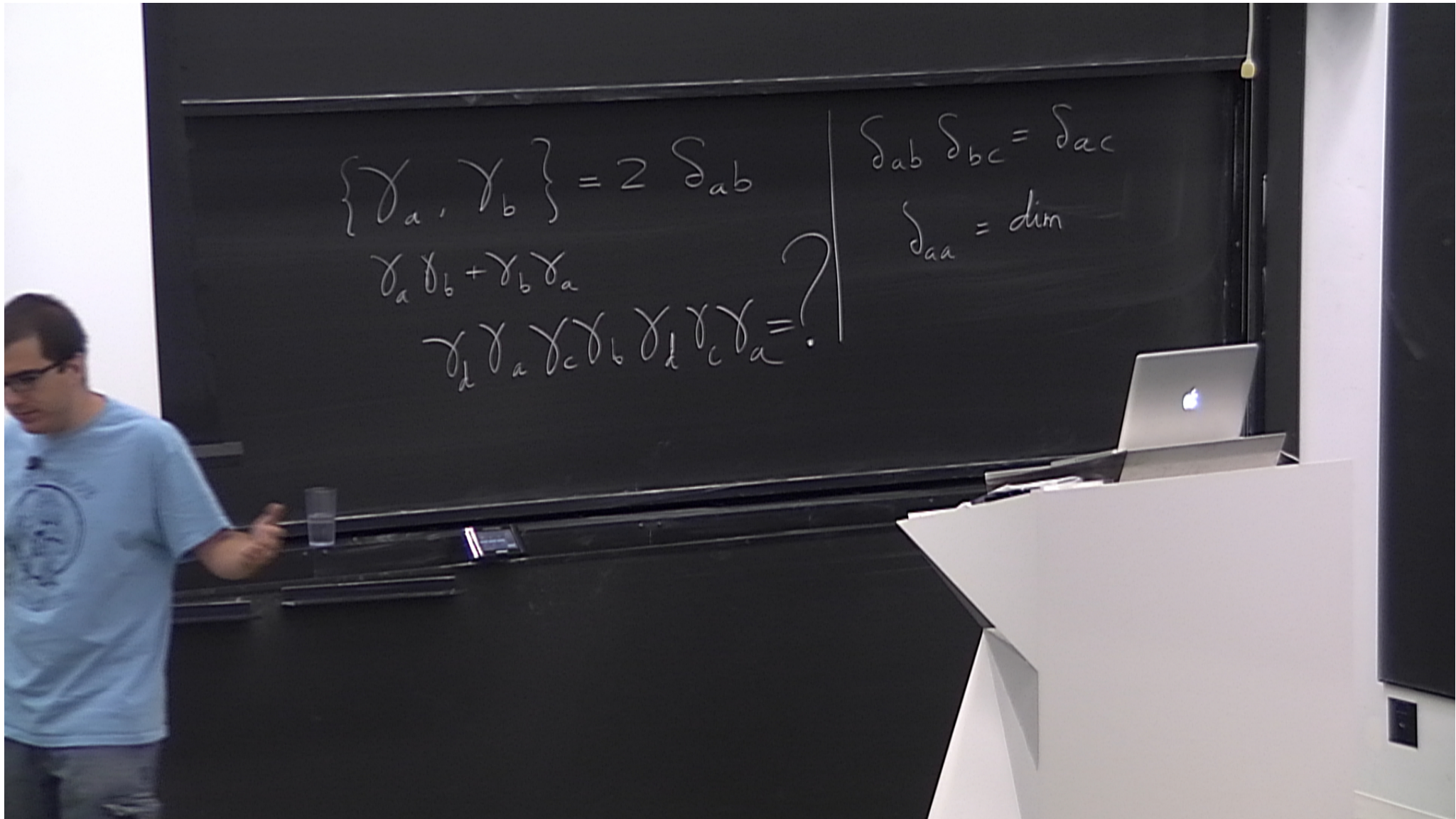
$$\{\gamma_a, \gamma_b\} = 2 \delta_{ab}$$

$$\gamma_a \gamma_b + \gamma_b \gamma_a$$

$$\gamma_a \gamma_a \gamma_c \gamma_b \gamma_d \gamma_c \gamma_a = ?$$

$$\delta_{ab} \delta_{bc} = \delta_{ac}$$

$$\delta_{aa} = \dim$$



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

In[172]:=

```
CenterDot[a, b]
```

Out[172]=

```
a · b
```

? CenterDot :. : -> .

CenterDot[x, y, ...] displays as $x \cdot y \cdot \dots$ >>

```
 $\gamma[d] \cdot \gamma[a] \cdot \gamma[c] \cdot \gamma[b] \cdot \gamma[c] \cdot \gamma[a]$ 
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
 Untitled-2

$a \cdot b$

? CenterDot $\cdot \cdot \rightarrow \cdot$

CenterDot[x, y, ...] displays as $x \cdot y \cdot \dots$ >>

$\{\gamma_a, \gamma_b\} = 2 \delta_{ab}$

$\gamma[d] \cdot \gamma[a] \cdot \gamma[c] \cdot \gamma[b] \cdot \gamma[d] \cdot \gamma[c] \cdot \gamma[a] / \cdot$
 $\gamma[a_] \cdot \gamma[b_] \Rightarrow -\gamma[b] \cdot \gamma[a] + 2 \delta[a, b]$

```
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

In[179]:=

$$\gamma[a] \cdot \gamma[b] /. \gamma[a_] \cdot \gamma[b_] \Rightarrow -\gamma[b] \cdot \gamma[a] + 2 \delta[a, b] /;$$


$$\text{Not@OrderedQ}[\{a, b\}]$$


Out[179]=

$$\gamma[a] \cdot \gamma[b]$$


In[180]:=

$$\gamma[d] \cdot \gamma[a] /. \gamma[a_] \cdot \gamma[b_] \Rightarrow -\gamma[b] \cdot \gamma[a] + 2 \delta[a, b] /;$$


$$\text{Not@OrderedQ}[\{a, b\}]$$


Out[180]=

$$-\gamma[a] \cdot \gamma[d] + 2 \delta[d, a]$$

```



```

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Untitled-2

rules =
{c___ · γ[a_] · γ[b_] · d___ =>
  c · (-γ[b] · γ[a] + 2 δ[a, b]) · d /;
  Not@OrderedQ[{a, b}],
c___ · γ[a_] · γ[a_] · d___ =>
  c · δ[a, a] · d /; Not@OrderedQ[{a, b}],
a1___ · a2|

γ[d] · γ[a] · γ[c] · γ[b] · γ[d] · γ[c] · γ[a] //.

Out[182]=

(-γ[a] · γ[d] + 2 δ[d, a]) · (-γ[b] · γ[c] + 2 δ[c, b]) ·
(-γ[c] · γ[d] + 2 δ[d, c]) · γ[a]

In[176]:=

```

```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

rules =
  {c__ . γ[a_] . γ[b_] . d__ =>
    c . (-γ[b] . γ[a] + 2 δ[a, b]) . d /;
    Not@OrderedQ[{a, b}],
    c__ . γ[a_] . γ[a_] . d__ =>
    c . δ[a, a] . d /; Not@OrderedQ[{a, b}],
    a1__ . a2_ . a3__ => a2 a1 . a3 /; NumberQ[a2]};

γ[d] . γ[a] . γ[c] . γ[b] . γ[d] . γ[c] . γ[a] //.

Out[182]=
(-γ[a] . γ[d] + 2 δ[d, a]) . (-γ[b] . γ[c] + 2 δ[c, b]) .
(-γ[c] . γ[d] + 2 δ[d, c]) . γ[a]

In[176]:=
OrderedQ[{b, a}]

```

```
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

rules =
{c__ ·  $\gamma$ [a_] ·  $\gamma$ [b_] · d__  $\Rightarrow$ 
  c · (- $\gamma$ [b] ·  $\gamma$ [a] + 2  $\delta$ [a, b]) · d /;
  Not@OrderedQ[{a, b}],
c__ ·  $\gamma$ [a_] ·  $\gamma$ [a_] · d__  $\Rightarrow$ 
  c ·  $\delta$ [a, a] · d /; Not@OrderedQ[{a, b}],
a1__ · a2_ · a3__  $\Rightarrow$  a2 a1 · a3 /; NumberQ[a2],
a1__ · (a2_ a4_) · a3__  $\Rightarrow$ 
  a2 a1 · a4 · a3 /; NumberQ[a2]};

 $\gamma$ [a] · 5 ·  $\gamma$ [b] /. rules
 $\gamma$ [a] · 5 ·  $\gamma$ [b] /. rules

Out[187]=

5  $\gamma$ [a] ·  $\gamma$ [b]
```

```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

a1__ . a2_ . a3__ :=> a2 a1 . a3 /; NumberQ[a2] ,
a1__ . (a2_ a4_) . a3__ :=>
a2 a1 . a4 . a3 /; NumberQ[a2] ,
a1__ . (a2_ + a4_) . a3__ :=> a1 . a2 . a3 + a1 . a4 . a3};

In[195]:=
 $\gamma[d] \cdot \gamma[a] \cdot \gamma[c] \cdot \gamma[b] \cdot \gamma[d] \cdot \gamma[c] \cdot \gamma[a] // . \text{rules} //$ 
Simplify

Out[195]=
- (( $\gamma[a] \cdot \gamma[d]$ ) . ( $\gamma[b] \cdot \gamma[c]$ ) . ( $\gamma[c] \cdot \gamma[d]$ ) .  $\gamma[a]) +$ 
2 (( $\gamma[a] \cdot \gamma[d]$ ) . ( $\gamma[b] \cdot \gamma[c]$ ) .  $\delta[d, c] \cdot \gamma[a] +$ 
( $\gamma[a] \cdot \gamma[d]$ ) .  $\delta[c, b] \cdot (\gamma[c] \cdot \gamma[d]) \cdot \gamma[a] -$ 
2 ( $\gamma[a] \cdot \gamma[d]$ ) .  $\delta[c, b] \cdot \delta[d, c] \cdot \gamma[a] +$ 
 $\delta[d, a] \cdot (\gamma[b] \cdot \gamma[c]) \cdot (\gamma[c] \cdot \gamma[d]) \cdot \gamma[a] -$ 
2  $\delta[d, a] \cdot (\gamma[b] \cdot \gamma[c]) \cdot \delta[d, c] \cdot \gamma[a] -$ 
2  $\delta[d, a] \cdot \delta[c, b] \cdot (\gamma[c] \cdot \gamma[d]) \cdot \gamma[a] +$ 

```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
 Untitled-2

$(\gamma[a] \cdot \gamma[d]) \cdot (\gamma[b] \cdot \gamma[c]) \cdot (\gamma[c] \cdot \gamma[d]) \cdot \gamma[a] //$
TreeForm

Out[199]//TreeForm=

```

graph TD
    Root[CenterDot] --> C1[CenterDot]
    Root --> C2[CenterDot]
    Root --> C3[CenterDot]
    Root --> G1[γ]
    C1 --> G1_1[γ]
    C1 --> G1_2[γ]
    G1_1 --> A1[a]
    G1_2 --> D1[d]
    C2 --> G2_1[γ]
    C2 --> G2_2[γ]
    G2_1 --> B1[b]
    G2_2 --> C1_1[c]
    C3 --> G3_1[γ]
    C3 --> G3_2[γ]
    G3_1 --> C2_1[c]
    G3_2 --> D2[d]
    G1 --> A2[a]
  
```

```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2
a1_ . (a2_ + a4_ ) . a3_ => a1 . a2 . a3 + a1 . a4 . a3,
CenterDot[a_] => Flatten[CenterDot[a]]};
CenterDot[a_] => Flatten[CenterDot[a]]};

In[202]:=
In[202]:=
Simplify
Simplify
Out[202]:=
Out[202]:=

-4  $\gamma[a]$  .  $\delta[d, b]$  .  $\delta[c, c]$  .  $\delta[d, a]$  +
4  $\delta[d, a]$  .  $\gamma[b]$  .  $\delta[c, c]$  .  $\delta[d, a]$  -
8  $\delta[d, a]$  .  $\delta[c, b]$  .  $\gamma[c]$  .  $\delta[d, a]$  +
8  $\delta[d, a]$  .  $\delta[c, b]$  .  $\delta[c, a]$  .  $\gamma[d]$  +
8  $\delta[d, a]$  .  $\delta[c, b]$  .  $\delta[d, c]$  .  $\gamma[a]$  +
2  $\gamma[a]$  .  $\gamma[b]$  .  $\gamma[d]$  .  $\delta[c, c]$  .  $\delta[d, a]$  +
4  $\gamma[a]$  .  $\gamma[b]$  .  $\delta[d, c]$  .  $\delta[c, a]$  .  $\gamma[d]$  +
4  $\gamma[a]$  .  $\gamma[d]$  .  $\delta[c, b]$  .  $\delta[c, a]$  .  $\delta[d, a]$  +
4  $\gamma[a]$  .  $\gamma[d]$  .  $\delta[c, b]$  .  $\delta[c, a]$  .  $\delta[d, a]$  +

```

```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

In[201]:=
rules =
{c__ . Y[a_] . Y[b_] . d__ => c . (-Y[b] . Y[a] + 2 δ[a, b]) . d /; Not@OrderedQ[{a, b}],
c__ . Y[a_] . Y[a_] . d__ => c . δ[a, a] . d,
a1__ . a2_ . a3__ => a2 a1 . a3 /; NumberQ[a2],
a1__ . (a2_ a4_) . a3__ => a2 a1 . a4 . a3 /; NumberQ[a2],
a1__ . (a2_ + a4_) . a3__ => a1 . a2 . a3 + a1 . a4 . a3,
CenterDot[a_] => Flatten[CenterDot[a]]};

In[202]:=
Y[d] . Y[a] . Y[c] . Y[b] . Y[d] . Y[c] . Y[a] //. rules // Simplify

Out[202]=
-4 Y[a] . δ[d, b] . δ[c, c] . δ[d, a] + 4 δ[d, a] . Y[b] . δ[c, c] . δ[d, a] -
8 δ[d, a] . δ[c, b] . Y[c] . δ[d, a] + 8 δ[d, a] . δ[c, b] . δ[c, a] . Y[d] +
8 δ[d, a] . δ[c, b] . δ[d, c] . Y[a] + 2 Y[a] . Y[b] . Y[d] . δ[c, c] . δ[d, a] -
4 Y[a] . Y[b] . δ[d, c] . δ[d, c] . Y[a] + 4 Y[a] . Y[d] . δ[c, b] . Y[c] . δ[d, a] -
4 Y[a] . Y[d] . δ[c, b] . δ[c, a] . Y[d] - 4 Y[a] . Y[d] . δ[c, b] . δ[d, c] . Y[a] +
4 Y[a] . δ[d, b] . Y[c] . δ[d, c] . Y[a] + 2 Y[a] . δ[d, b] . δ[c, c] . Y[a] . Y[d] -
4 δ[d, a] . Y[b] . Y[c] . δ[d, c] . Y[a] - 2 δ[d, a] . Y[b] . δ[c, c] . Y[a] . Y[d] -
4 δ[d, a] . δ[c, b] . Y[a] . Y[c] . Y[d] + 2 Y[a] . Y[b] . Y[c] . Y[d] . δ[d, c] . Y[a] -
Y[a] . Y[b] . Y[d] . δ[c, c] . Y[a] . Y[d] + 2 Y[a] . Y[d] . δ[c, b] . Y[a] . Y[c] . Y[d]

In[176]:=
OrderedQ[{b, a}]

```

```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

In[208]:=
NumericQ[δ[a, b]]

Out[208]=
False

In[215]:=
NumQ[_?NumericQ] = True;
NumQ[δ[_, _]] = True;
NumQ[_] = False;

In[218]:=
NumQ[δ[a, b]]

Out[218]=
True

In[205]:=
rules =
{c_ . Y[a_] . Y[b_] . d_ => c . (-Y[b] . Y[a] + 2 δ[a, b]) . d /; Not@OrderedQ[{a, b}],
c_ . Y[a_] . Y[a_] . d_ => c . δ[a, a] . d,
a1_ . a2_ . a3_ => a2 a1 . a3 /; NumericQ[a2],
a1_ . (a2_ a4_) . a3_ => a2 a1 . a4 . a3 /; NumericQ[a2],
a1_ . (a2_ + a4_) . a3_ => a1 . a2 . a3 + a1 . a4 . a3,
CenterDot[-1] & Flatten[CenterDot[-1]]}

```

```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

NumQ[x_] = False;

rules =
{c_ . Y[a_] . Y[b_] . d_ => c . (-Y[b] . Y[a] + 2 δ[a, b]) . d /; Not@OrderedQ[{a, b}],
c_ . Y[a_] . Y[a_] . d_ => c . δ[a, a] . d,
a1_ . a2_ . a3_ => a2 a1 . a3 /; NumQ[a2],
a1_ . (a2_ a4_) . a3_ => a2 a1 . a4 . a3 /; NumQ[a2],
a1_ . (a2_ + a4_) . a3_ => a1 . a2 . a3 + a1 . a4 . a3,
CenterDot[a_] => Flatten[CenterDot[a]]};

In[237]:=
Y[d] . Y[a] . Y[c] . Y[b] . Y[d] . Y[c] . Y[a] //. rules // Simplify // Expand

Out[237]=
-4 Y[a] . Y[c] . Y[d] δ[c, b] δ[d, a] + 8 CenterDot[Y[d]] δ[c, a] δ[c, b] δ[d, a] -
2 Y[a] . Y[b] . Y[d] δ[c, c] δ[d, a] - 4 CenterDot[Y[d]] δ[b, a] δ[c, c] δ[d, a] -
8 CenterDot[Y[c]] δ[c, b] δ[d, a]^2 + 4 CenterDot[Y[b]] δ[c, c] δ[d, a]^2 +
2 CenterDot[Y[d]] δ[a, a] δ[c, c] δ[d, b] - 4 CenterDot[Y[a]] δ[c, c] δ[d, a] δ[d, b] -
2 Y[b] . Y[c] . Y[d] δ[a, a] δ[d, c] + 4 Y[a] . Y[c] . Y[d] δ[b, a] δ[d, c] -
4 Y[a] . Y[b] . Y[d] δ[c, a] δ[d, c] + 8 CenterDot[Y[c]] δ[b, a] δ[d, a] δ[d, c] -
8 CenterDot[Y[b]] δ[c, a] δ[d, a] δ[d, c] + 8 CenterDot[Y[a]] δ[c, b] δ[d, a] δ[d, c] -
4 CenterDot[Y[c]] δ[a, a] δ[d, b] δ[d, c] + 8 CenterDot[Y[a]] δ[c, a] δ[d, b] δ[d, c] +
4 CenterDot[Y[b]] δ[a, a] δ[d, c]^2 - 8 CenterDot[Y[a]] δ[b, a] δ[d, c]^2 +
2 CenterDot[Y[c]] δ[a, a] δ[c, b] δ[d, d] - 4 CenterDot[Y[a]] δ[c, a] δ[c, b] δ[d, d] -
CenterDot[Y[b]] δ[a, a] δ[c, c] δ[d, d] + 2 CenterDot[Y[a]] δ[b, a] δ[c, c] δ[d, d]

```

```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

NumQ[x_] = False;

rules =
{c_ . Y[a_] . Y[b_] . d_ => c . (-Y[b] . Y[a] + 2 δ[a, b]) . d /; Not@OrderedQ[{a, b}],
c_ . Y[a_] . Y[a_] . d_ => c . δ[a, a] . d,
a1_ . a2_ . a3_ => a2 a1 . a3 /; NumQ[a2],
a1_ . (a2_ a4_) . a3_ => a2 a1 . a4 . a3 /; NumQ[a2],
a1_ . (a2_ + a4_) . a3_ => a1 . a2 . a3 + a1 . a4 . a3,
CenterDot[a_] => Flatten[CenterDot[a]]};

In[238]:=
Y[d] . Y[a] . Y[c] . Y[b] . Y[d] . Y[c] . Y[a] //. rules /. CenterDot[Y[a_]] -> Y[a] //
Simplify // Expand

Out[238]=
-4 Y[a] . Y[c] . Y[d] δ[c, b] δ[d, a] + 8 Y[d] δ[c, a] δ[c, b] δ[d, a] +
2 Y[a] . Y[b] . Y[d] δ[c, c] δ[d, a] - 4 Y[d] δ[b, a] δ[c, c] δ[d, a] -
8 Y[c] δ[c, b] δ[d, a]^2 + 4 Y[b] δ[c, c] δ[d, a]^2 + 2 Y[d] δ[a, a] δ[c, c] δ[d, b] -
4 Y[a] δ[c, c] δ[d, a] δ[d, b] - 2 Y[b] . Y[c] . Y[d] δ[a, a] δ[d, c] +
4 Y[a] . Y[c] . Y[d] δ[b, a] δ[d, c] - 4 Y[a] . Y[b] . Y[d] δ[c, a] δ[d, c] +
8 Y[c] δ[b, a] δ[d, a] δ[d, c] - 8 Y[b] δ[c, a] δ[d, a] δ[d, c] +
8 Y[a] δ[c, b] δ[d, a] δ[d, c] - 4 Y[c] δ[a, a] δ[d, b] δ[d, c] + 8 Y[a] δ[c, a] δ[d, b] δ[d, c] -
4 Y[b] δ[a, a] δ[d, c]^2 - 8 Y[a] δ[b, a] δ[d, c]^2 + 2 Y[c] δ[a, a] δ[c, b] δ[d, d] -
4 Y[a] δ[c, a] δ[c, b] δ[d, d] - Y[b] δ[a, a] δ[c, c] δ[d, d] + 2 Y[a] δ[b, a] δ[c, c] δ[d, d]

```

```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

CenterDot[γ[a_]] → γ[a] // Simplify // Expand

Out[243]=

-dim3 γ[b] + 2 dim2 γ[a] δ[b, a] +
2 dim2 γ[c] δ[c, b] - 4 dim γ[a] δ[c, a] δ[c, b] +
2 dim γ[a] · γ[b] · γ[d] δ[d, a] -
4 dim γ[d] δ[b, a] δ[d, a] -
4 γ[a] · γ[c] · γ[d] δ[c, b] δ[d, a] +
8 γ[d] δ[c, a] δ[c, b] δ[d, a] +
4 dim γ[b] δ[d, a]2 - 8 γ[c] δ[c, b] δ[d, a]2 +
2 dim2 γ[d] δ[d, b] - 4 dim γ[a] δ[d, a] δ[d, b] -
2 dim γ[b] · γ[c] · γ[d] δ[d, c] +
4 γ[a] · γ[c] · γ[d] δ[b, a] δ[d, c] -
4 γ[a] · γ[b] · γ[d] δ[c, a] δ[d, c] +
8 γ[c] δ[b, a] δ[d, a] δ[d, c] +
8 γ[b] δ[c, a] δ[d, a] δ[d, c] +
8 γ[a] δ[c, b] δ[d, a] δ[d, c] -

```

```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Untitled-2

a1__ . (a2_a4_) . a3__ :=> a2 a1 . a4 . a3 /; NumQ[a2],
a1__ . (a2_ + a4_) . a3__ :=> a1 . a2 . a3 + a1 . a4 . a3,
CenterDot[a_] :=> Flatten[CenterDot[a]],
δ[c_, c_] → dim,
δ[c_, d_]² → dim,
δ[a1_, a2_] a3_ :=>};

In[243]:=
γ[d] · γ[a] · γ[c] · γ[b] · γ[d] · γ[c] · γ[a] //. rules /.
CenterDot[γ[a_]] → γ[a] // Simplify // Expand

Out[243]=
-dim³ γ[b] + 2 dim² γ[a] δ[b, a] +
2 dim² γ[c] δ[c, b] - 4 dim γ[a] δ[c, a] δ[c, b] +
2 dim γ[a] · γ[b] · γ[d] δ[d, a] -
4 dim γ[d] δ[b, a] δ[d, a] -

```



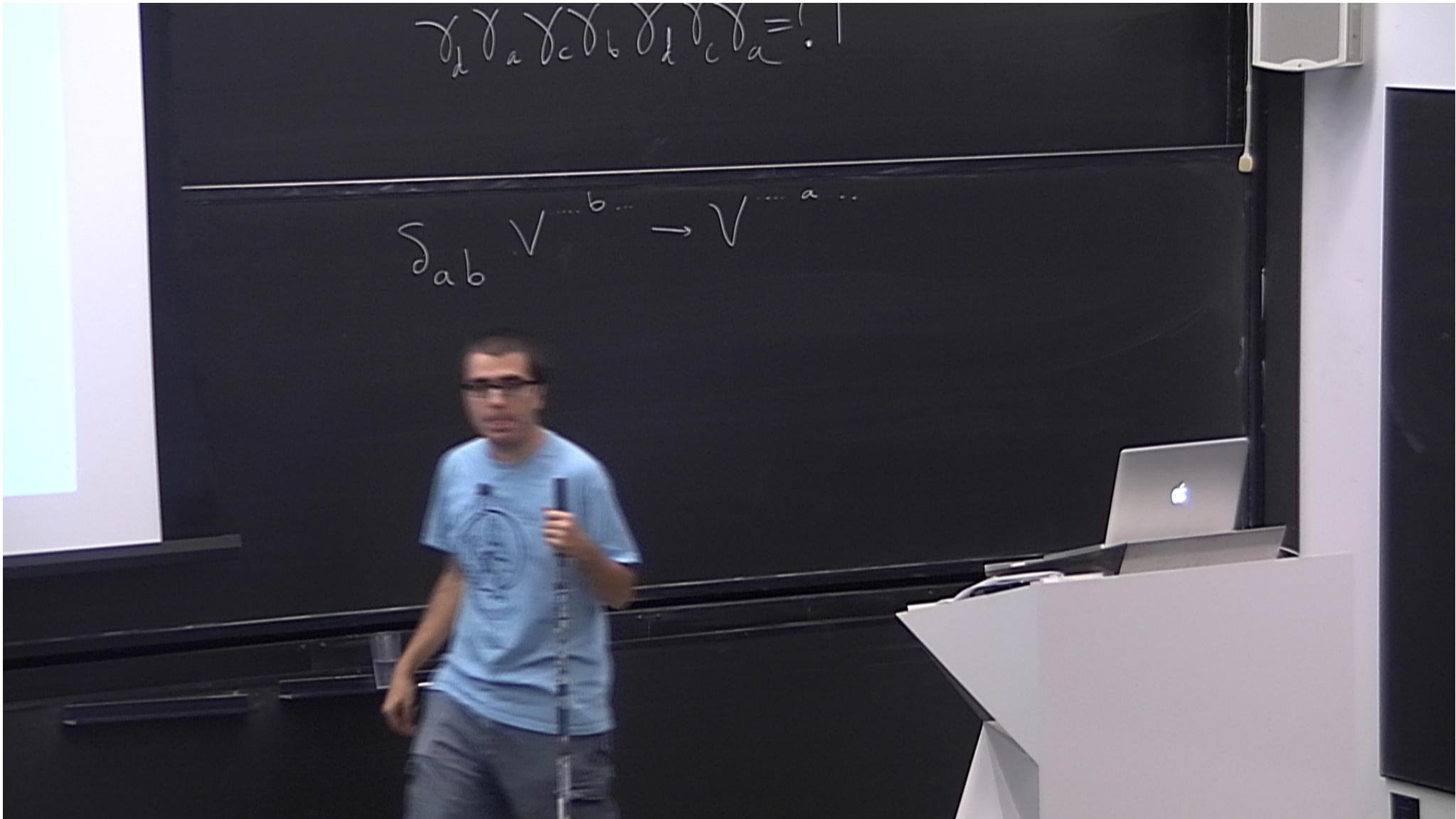
```

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+ Untitled-2
a1___ . (a2_ a4_) . a3___ => a2 a1 . a4 . a3 /; NumQ[a2],
a1___ . (a2_ + a4_) . a3___ => a1 . a2 . a3 + a1 . a4 . a3,
CenterDot[a_] => Flatten[CenterDot[a]],
δ[c_, c_] → dim,
δ[c_, d_]² → dim,
δ[a1_, a2_] a3_ :>};

In[243]:=
γ[d] · γ[a] · γ[c] · γ[b] · γ[d] · γ[c] · γ[a] //. rules /.
CenterDot[γ[a_]] → γ[a] // Simplify // Expand

Out[243]=
-dim³ γ[b] + 2 dim² γ[a] δ[b, a] +
2 dim² γ[c] δ[c, b] - 4 dim γ[a] δ[c, a] δ[c, b] +
2 dim γ[a] · γ[b] · γ[d] δ[d, a] -
4 dim γ[d] δ[b, a] δ[d, a] -

```



$$\gamma_d \gamma_a \gamma_c \gamma_b \gamma_d \gamma_c \gamma_a = 1$$

$$\int_{ab} \underbrace{V \dots b}_{a3-} \rightarrow \int_{a3 \cdot b \rightarrow a} V \dots a \dots$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Tuesday Morning Part 2.nb

```
δ[a1_, a2_] a3_ => (a3 /. a1 -> a2) /; Not@FreeQ[a3, a1];
```

In[256]:=

```
γ[d] · γ[a] · γ[c] · γ[b] · γ[d] · γ[c] · γ[a] //. rules /. CenterDot[γ[_]] -> γ[_] //  
Simplify
```

Out[256]=

$$-(-16 + 24 \text{dim} - 10 \text{dim}^2 + \text{dim}^3) \gamma[b]$$

Deck of cards

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$$-(-16 + 24 \text{dim} - 10 \text{dim}^2 + \text{dim}^3) \gamma[D]$$

Deck of cards

`{Heart[2], Hearts[3], ...}`

`{Ace, Range[2, 10]}`

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$$-(-16 + 24 \text{dim} - 10 \text{dim}^2 + \text{dim}^3) \gamma[B]$$

Deck of cards

```
{Heart[2], Hearts[3], ...}
```

In[257]:=

```
Range[2, 10]
```

Out[257]=

```
{2, 3, 4, 5, 6, 7, 8, 9, 10}
```

```
Flatten[{Ace, Range[2, 10], Jacks, Queen, King}]
```

Out[258]=

```
{Ace, {2, 3, 4, 5, 6, 7, 8, 9, 10}, Jacks, Queen, King}
```

```
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DECK OF CARDS

{Heart[2], Hearts[3], ...}

In[257]:=
Range[2, 10]

Out[257]=
{2, 3, 4, 5, 6, 7, 8, 9, 10}

In[261]:=
# /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King}]

Out[261]=
Hearts /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King}]
Spades /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King}]

Out[262]=
{Hearts[Ace], Hearts[2], Hearts[3], Hearts[4], Hearts[5], Hearts[6], Hearts[7],
Hearts[8], Hearts[9], Hearts[10], Hearts[Jacks], Hearts[Queen], Hearts[King]}

{Spades[Ace], Spades[2], Spades[3], Spades[4], Spades[5], Spades[6], Spades[7],
Spades[8], Spades[9], Spades[10], Spades[Jacks], Spades[Queen], Spades[King]}
```

```
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DECK OF CARDS

{Heart[2], Hearts[3], ...}

In[257]:=
Range[2, 10]

Out[257]:=
{2, 3, 4, 5, 6, 7, 8, 9, 10}

In[264]:=
# /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King]} & /@
{Hearts, Spades, Diamonds, Clubs} // Flatten

Out[264]:=
{Hearts[Ace], Hearts[2], Hearts[3], Hearts[4], Hearts[5], Hearts[6],
Hearts[7], Hearts[8], Hearts[9], Hearts[10], Hearts[Jacks], Hearts[Queen],
Hearts[King], Spades[Ace], Spades[2], Spades[3], Spades[4], Spades[5],
Spades[6], Spades[7], Spades[8], Spades[9], Spades[10], Spades[Jacks],
Spades[Queen], Spades[King], Diamonds[Ace], Diamonds[2], Diamonds[3],
Diamonds[4], Diamonds[5], Diamonds[6], Diamonds[7], Diamonds[8],
Diamonds[9], Diamonds[10], Diamonds[Jacks], Diamonds[Queen], Diamonds[King],
Clubs[Ace], Clubs[2], Clubs[3], Clubs[4], Clubs[5], Clubs[6], Clubs[7],
Clubs[8], Clubs[9], Clubs[10], Clubs[Jacks], Clubs[Queen], Clubs[King]}
```


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Deck of cards

In[270]:=

```
Hearts = Style[♥, {Red, Large}];  
Clubs = Style[♣, Large];  
Diamonds = Style[♦, {Red, Large}];  
Spades = Style[♠, Large];
```

In[276]:=

```
Hearts@Flatten[{Ace, Range[2, 10], Jacks, Queen, King}]
```

Out[276]=

```
♥[{Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jacks, Queen, King}]
```

In[274]:=

```
# /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King]} & /@  
{Hearts, Spades, Diamonds, Clubs} // Flatten
```

Out[274]=

```
{♥[Ace], ♥[2], ♥[3], ♥[4], ♥[5], ♥[6], ♥[7], ♥[8], ♥[9], ♥[10], ♥[Jacks],  
♥[Queen], ♥[King], ♠[Ace], ♠[2], ♠[3], ♠[4], ♠[5], ♠[6], ♠[7], ♠[8], ♠[9],  
♠[10], ♠[Jacks], ♠[Queen], ♠[King], ♦[Ace], ♦[2], ♦[3], ♦[4], ♦[5], ♦[6],  
♣[Ace], ♣[2], ♣[3], ♣[4], ♣[5], ♣[6], ♣[7], ♣[8], ♣[9], ♣[10], ♣[Jacks],  
♣[Queen], ♣[King]}
```

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Deck of cards

In[270]:=

```
Hearts = Style[♥, {Red, Large}];  
Clubs = Style[♣, Large];  
Diamonds = Style[♦, {Red, Large}];  
Spades = Style[♠, Large];
```

In[280]:=

```
deck = # /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King]} & /@  
{Hearts, Spades, Diamonds, Clubs} // Flatten
```

Out[280]=

```
{♥[Ace], ♥[2], ♥[3], ♥[4], ♥[5], ♥[6], ♥[7], ♥[8], ♥[9], ♥[10], ♥[Jacks],  
♥[Queen], ♥[King], ♠[Ace], ♠[2], ♠[3], ♠[4], ♠[5], ♠[6], ♠[7], ♠[8], ♠[9],  
♠[10], ♠[Jacks], ♠[Queen], ♠[King], ♦[Ace], ♦[2], ♦[3], ♦[4], ♦[5], ♦[6],  
♦[7], ♦[8], ♦[9], ♦[10], ♦[Jacks], ♦[Queen], ♦[King], ♣[Ace], ♣[2], ♣[3],  
♣[4], ♣[5], ♣[6], ♣[7], ♣[8], ♣[9], ♣[10], ♣[Jacks], ♣[Queen], ♣[King]}
```

In[281]:=

```
Hearts /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King]}
```

```
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deck = # /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King]} & /@
{Hearts, Spades, Diamonds, Clubs} // Flatten

Out[280]=
{♥[Ace], ♥[2], ♥[3], ♥[4], ♥[5], ♥[6], ♥[7], ♥[8], ♥[9], ♥[10], ♥[Jacks],
♥[Queen], ♥[King], ♠[Ace], ♠[2], ♠[3], ♠[4], ♠[5], ♠[6], ♠[7], ♠[8], ♠[9],
♠[10], ♠[Jacks], ♠[Queen], ♠[King], ♦[Ace], ♦[2], ♦[3], ♦[4], ♦[5], ♦[6],
♦[7], ♦[8], ♦[9], ♦[10], ♦[Jacks], ♦[Queen], ♦[King], ♣[Ace], ♣[2], ♣[3],
♣[4], ♣[5], ♣[6], ♣[7], ♣[8], ♣[9], ♣[10], ♣[Jacks], ♣[Queen], ♣[King]}

In[281]:=
RandomSample[deck]

Out[281]=
{♦[8], ♠[7], ♥[Ace], ♣[7], ♥[King], ♠[Queen], ♥[5], ♠[6], ♥[7], ♠[4], ♥[2],
♠[2], ♠[5], ♣[2], ♥[Jacks], ♣[9], ♠[Jacks], ♦[Ace], ♣[4], ♦[Jacks], ♠[8],
♦[5], ♥[4], ♣[Jacks], ♥[Queen], ♦[3], ♣[Queen], ♣[8], ♦[10], ♥[3], ♦[Queen],
♣[Ace], ♥[10], ♦[King], ♠[King], ♦[6], ♥[8], ♠[Ace], ♣[King], ♦[2], ♦[4],
♥[6], ♣[3], ♣[5], ♠[10], ♣[6], ♥[9], ♦[9], ♠[9], ♦[7], ♣[10], ♠[3]}
```

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Deck of cards

```
In[270]:=
Hearts = Style[♥, {Red, Large}];
Clubs = Style[♣, Large];
Diamonds = Style[♦, {Red, Large}];
Spades = Style[♠, Large];

In[293]:=
deck = # /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King]} & /@
{Hearts, Spades, Diamonds, Clubs} // Flatten;

In[294]:=
Shuffle := (shuffledDeck = RandomSample[deck];)

take[n_] := (temp = shuffledDeck[[1 ;; n]],
shuffledDeck = Drop[shuffledDeck, n]; temp)

In[291]:=
Shuffle

In[292]:=
shuffledDeck

Out[292]=
```

```
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Hearts = Style[♥, {Red, Large}];
Clubs = Style[♣, Large];
Diamonds = Style[♦, {Red, Large}];
Spades = Style[♠, Large];

In[293]:=
deck = # /@ Flatten[{Ace, Range[2, 10], Jacks, Queen, King]} & /@
      {Hearts, Spades, Diamonds, Clubs} // Flatten;

In[294]:=
Shuffle := (shuffledDeck = RandomSample[deck];)

In[295]:=
take[n_] := (
  temp = shuffledDeck[[1 ;; n]];
  shuffledDeck = Drop[shuffledDeck, n];
  temp)

■ Poker

Shuffle
{"Player 1:"
```

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■ Poker

In[302]:=

```
Shuffle  
{{"Player 1:", take[2]}, {"Player 2:", take[2]}} // TableForm
```

Out[303]//TableForm=

| | |
|-----------|-----------|
| Player 1: | ♠ [8] |
| | ♠ [Queen] |
| Player 2: | ♠ [3] |
| | ♦ [King] |

In[304]:=

```
take[3]
```

Out[304]=

```
{♥ [9], ♥ [3], ♦ [9]}
```

In[300]:=

```
shuffledDeck // Length
```

Out[300]=

```
45
```

2004

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■ Poker

In[302]:=

```
Shuffle  
{{"Player 1:", take[2]}, {"Player 2:", take[2]}} // TableForm
```

Out[303]//TableForm=

| | |
|-----------|-----------|
| Player 1: | ♠ [8] |
| | ♠ [Queen] |
| Player 2: | ♠ [3] |
| | ♦ [King] |

In[304]:=

```
take[3]
```

Out[304]=

```
{♥ [9], ♥ [3], ♦ [9]}
```

In[300]:=

```
shuffledDeck // Length
```

Out[300]=

```
45
```


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- Use only patterns and replacements to transform

$$\{\{M, a, \{p, \{\}, \{\{l\}, e\}\}\}, S, \{y, \{r\}\}, u, p\}$$

into

$$\{M, a, p, l, e, S, y, r, u, p\}$$

- Using help find the solution to

$$\frac{k}{k+2}y_{k+1} - y_k + \frac{k+2}{2k}y_{k-1} = 0, \quad y_0 = 1$$

Hint: The function you need is called RS....