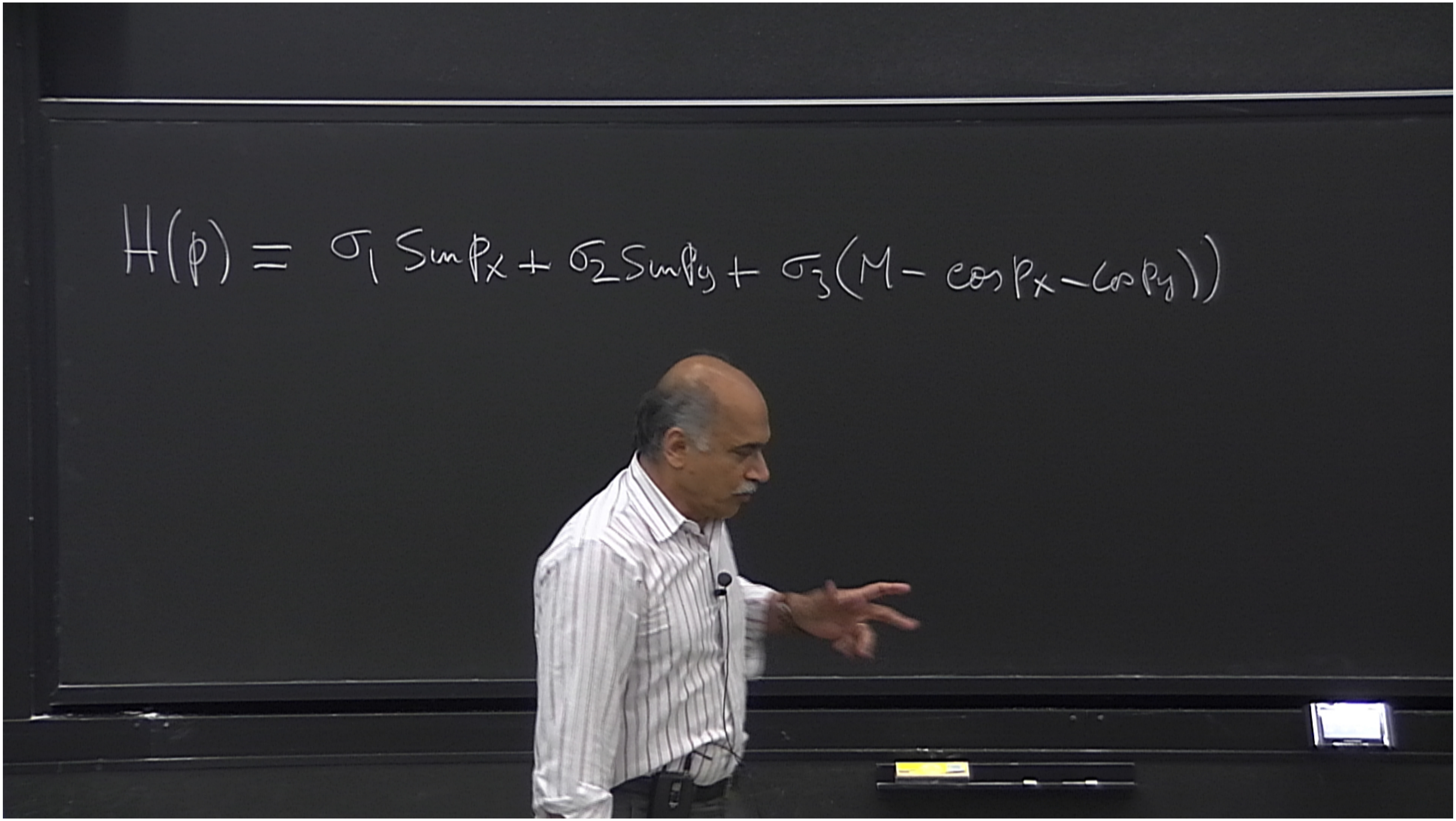
Title: Hamiltonian Theory of Fractional Chern Bands

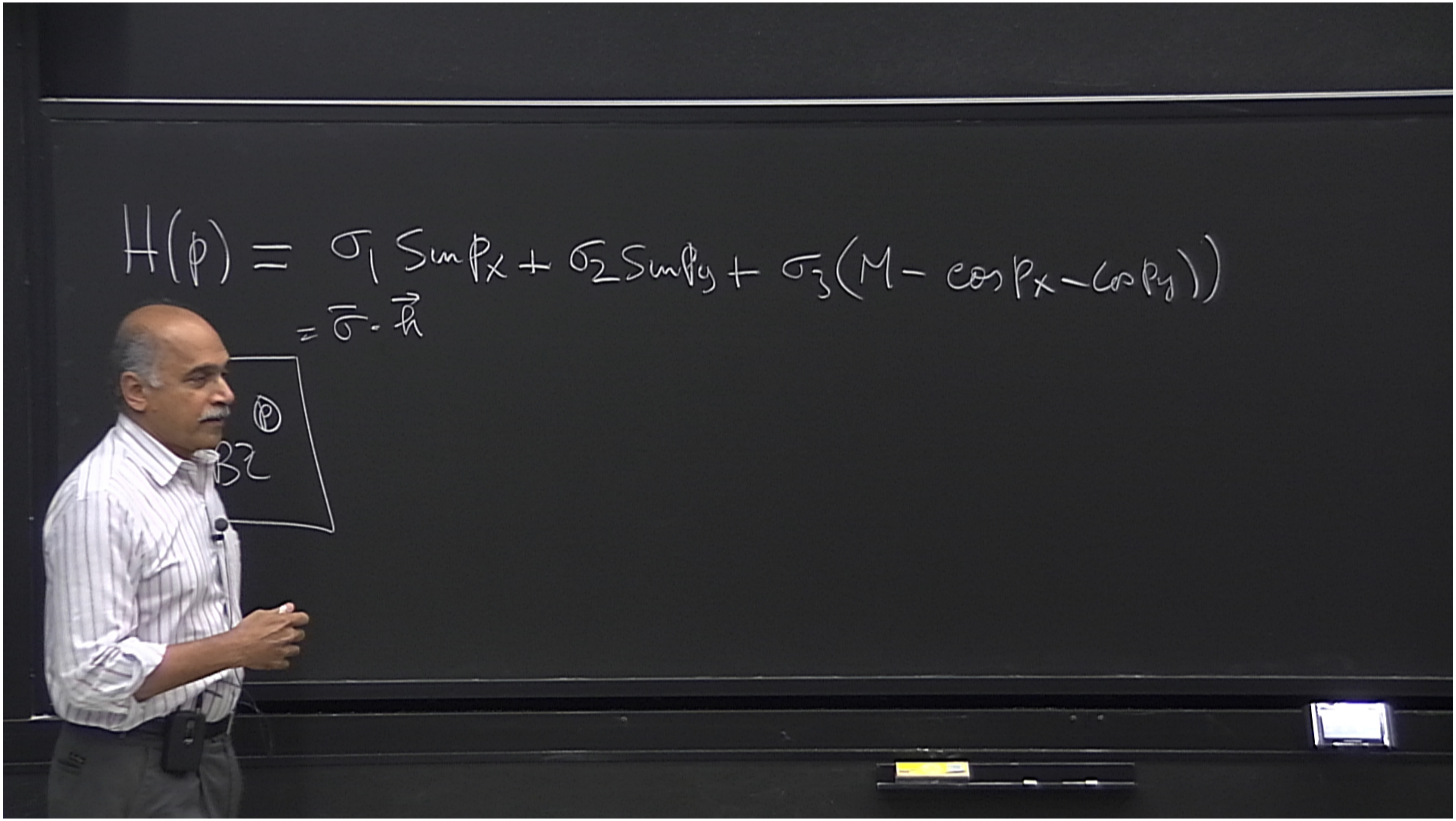
Date: Sep 14, 2012 02:30 PM

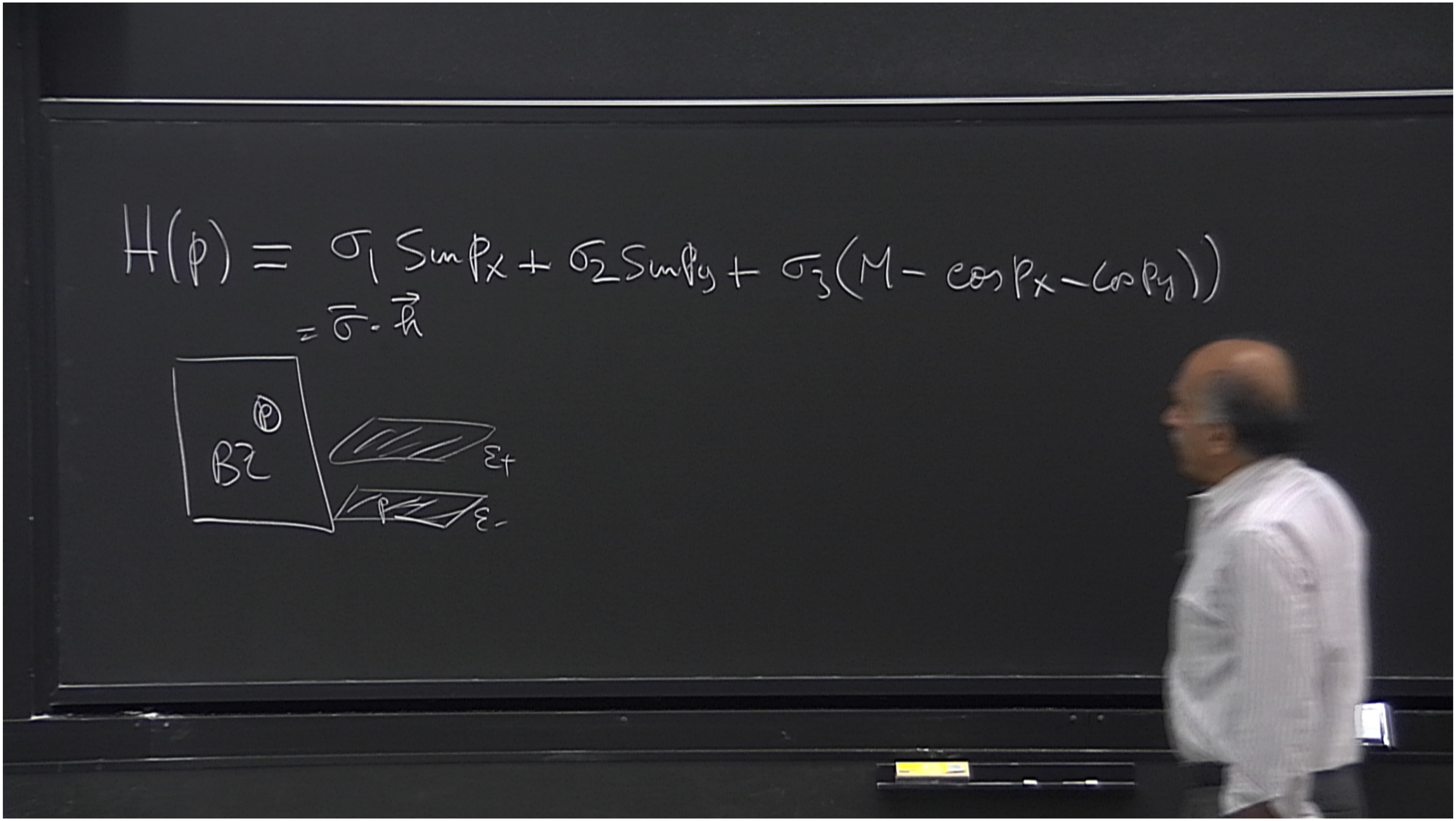
URL: http://pirsa.org/12090047

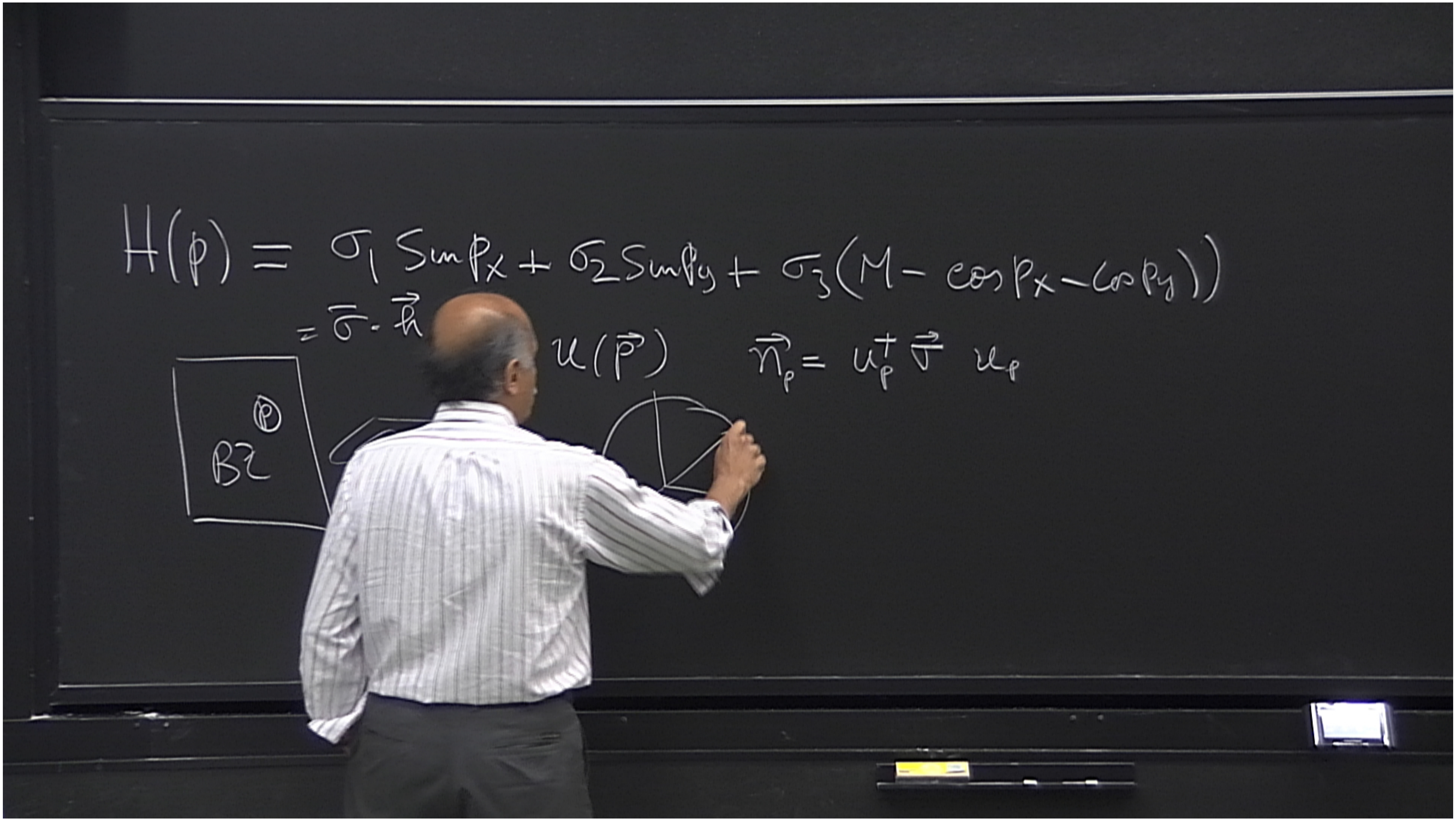
Abstract: It has been known for some time that a system with a filled band will have an integer quantum Hall conductance equal to its Chern number, a toplogical index associated with the band. While this is true for a system in a magnetic field with filled Landau Levels, even a system in zero external field can exhibit the QHE if its band has a Chern number. I review this issue and discuss a more recent question of whether a partially filled Chern band can exhibit the Fractional QHE. I describe the work done with Ganpathy Murthy in which we show how composite fermions, which were so useful in explaining the usual FQHE, can be introduced here and with equal success by adapting our Hamiltonian Theory of CFs developed for the FQHE in the continuum.

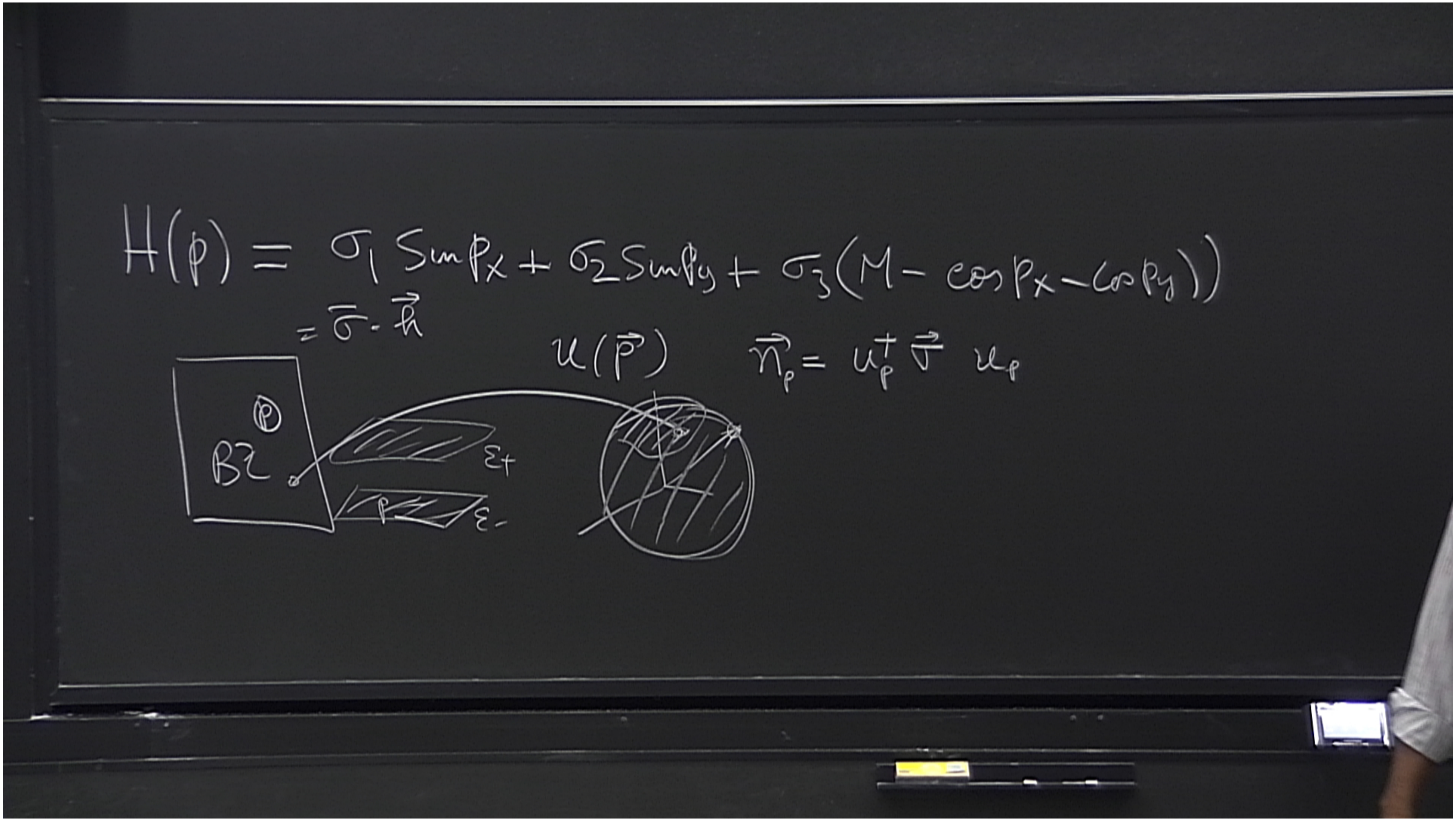

H(p)


$H(p) = \sigma_1 \sin p_x + \sigma_2 \sin p_y + \sigma_3 (M - \cos p_x - \cos p_y))$


$H(p) = \sigma_1 \sin p_x + \sigma_2 \sin p_y + \sigma_3 (M - \cos p_x - \cos p_y))$
$= \vec{\sigma} \cdot \vec{h}$
p
BZ


$H(p) = \sigma_1 \sin p_x + \sigma_2 \sin p_y + \sigma_3 (M - \cos p_x - \cos p_y))$
$= \vec{\sigma} \cdot \vec{h}$
BZ
p
ε_+
ε_-


$H(p) = \sigma_1 \sin p_x + \sigma_2 \sin p_y + \sigma_3 (M - \cos p_x - \cos p_y))$
$= \vec{\sigma} \cdot \vec{h}$
$u(\vec{p})$
$\vec{n}_p = u_p^\dagger \vec{\sigma} \; u_p$
BZ


$H(p) = \sigma_1 \sin p_x + \sigma_2 \sin p_y + \sigma_3 (M - \cos p_x - \cos p_y)$
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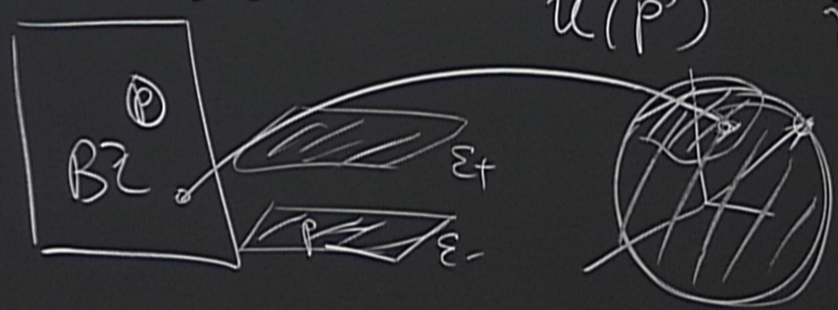
BZ
p
ε_+
ε_-

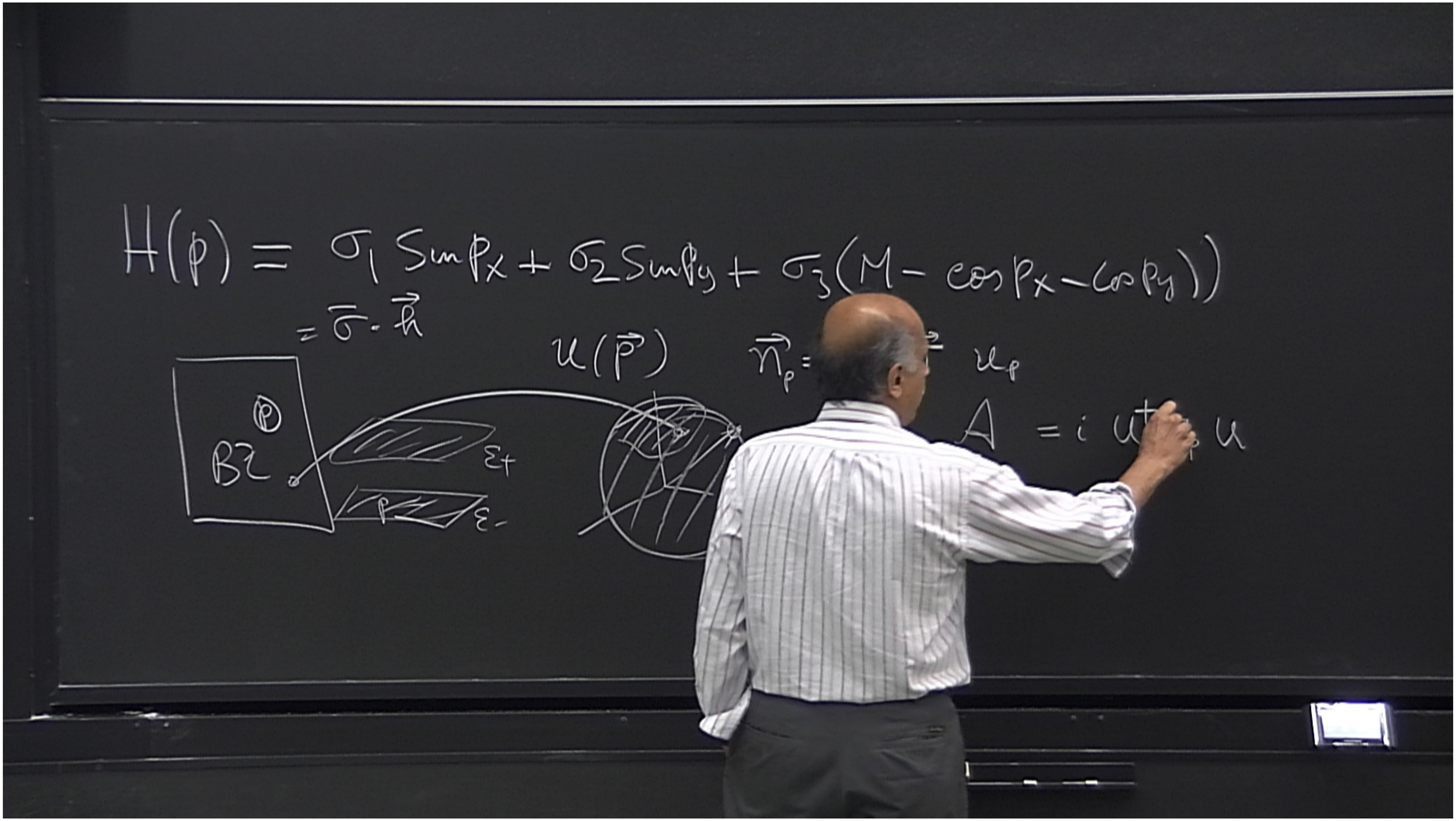
$$H(p) = \sigma_1 \sin p_x + \sigma_2 \sin p_y + \sigma_3 (M - \cos p_x - \cos p_y)$$

$$= \vec{\sigma} \cdot \vec{h}$$

$u(\vec{p}) \qquad \vec{n}_p = u_p^\dagger \vec{\sigma}\, u_p$



$C = -1$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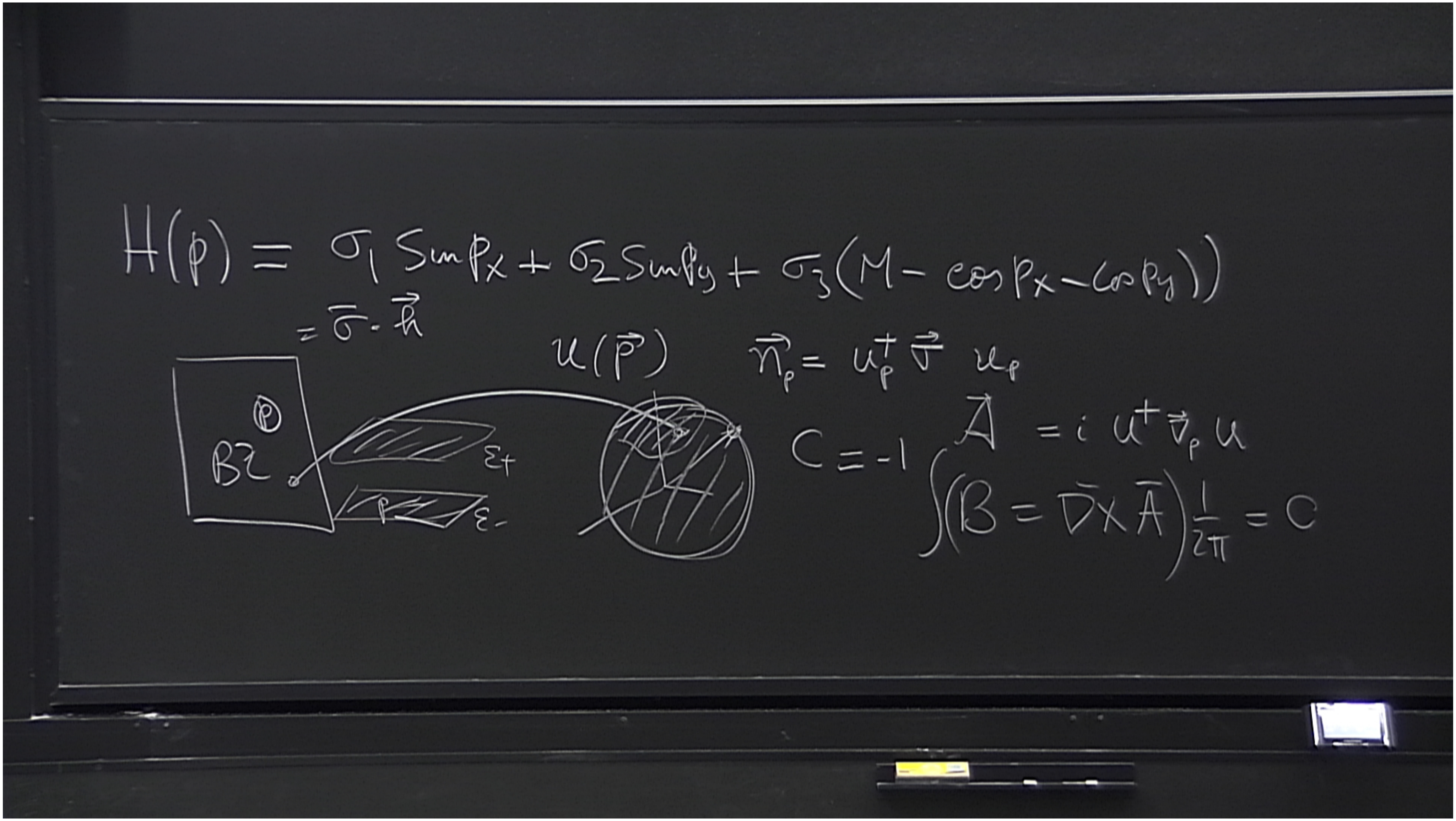
$= \vec{\sigma} \cdot \vec{h}$
$u(\vec{p})$
$\vec{n}_p =$
u_p
$A = i\, u^\dagger$
u
BZ
p
ε_+
ε_-



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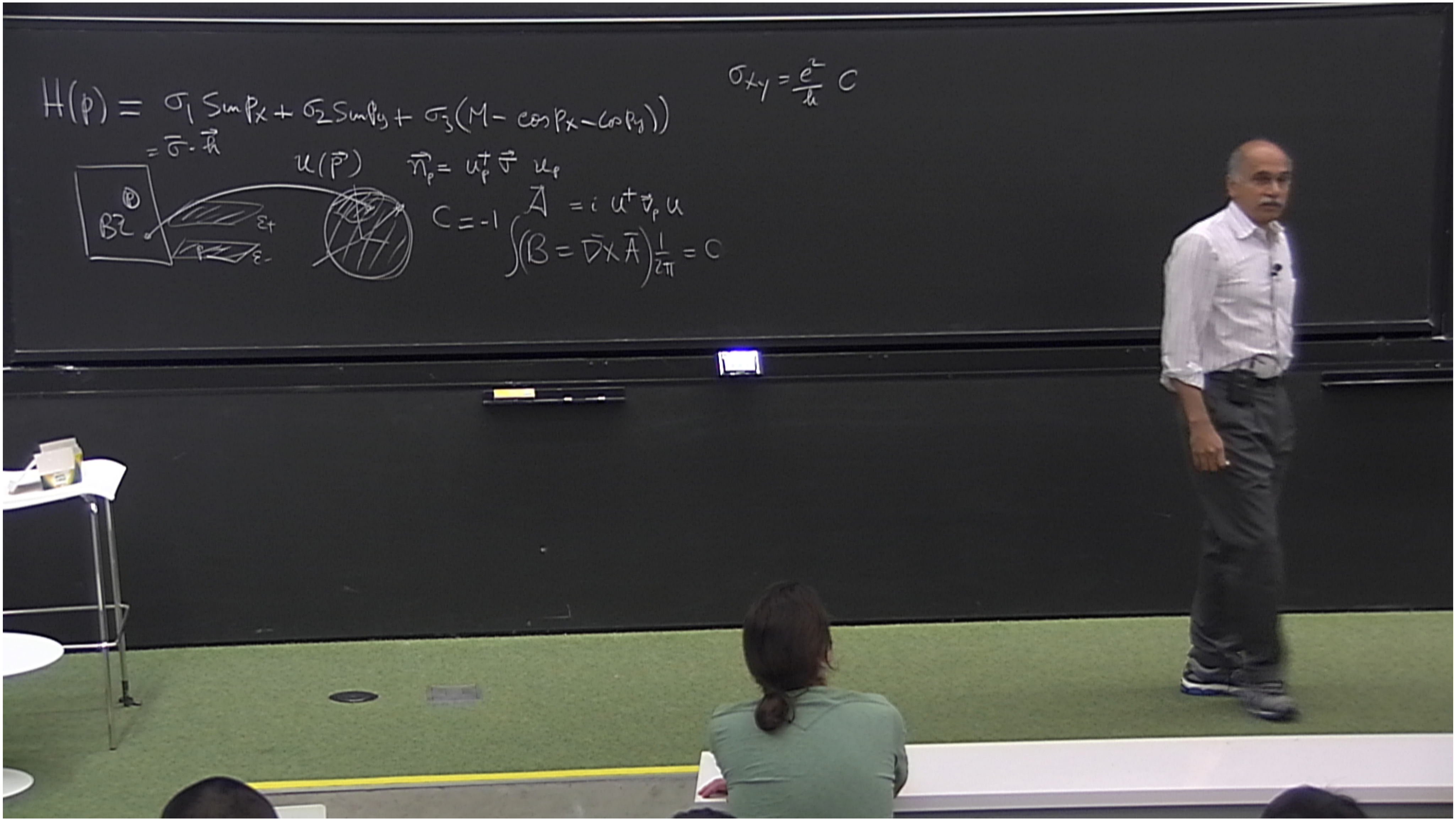
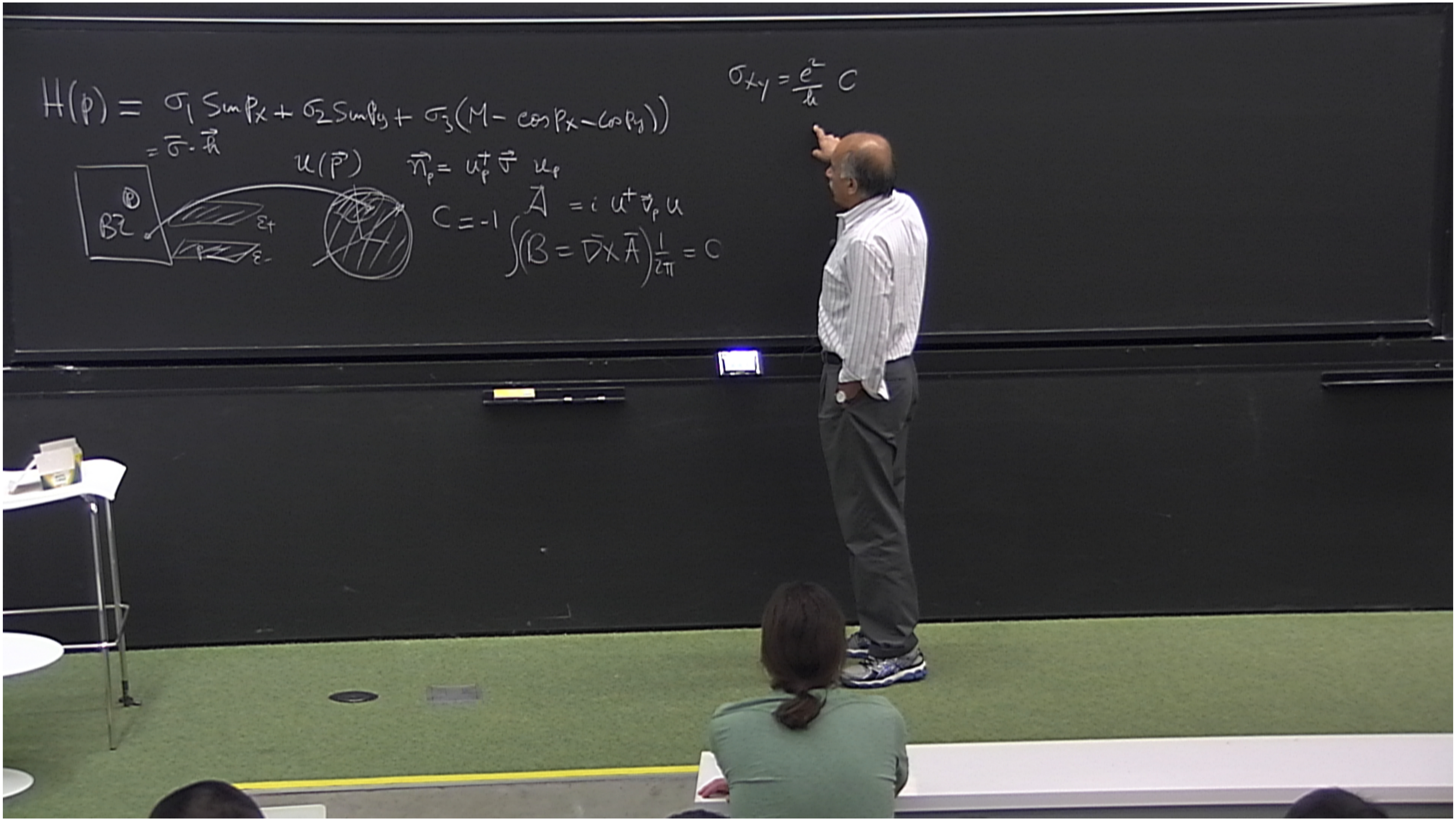
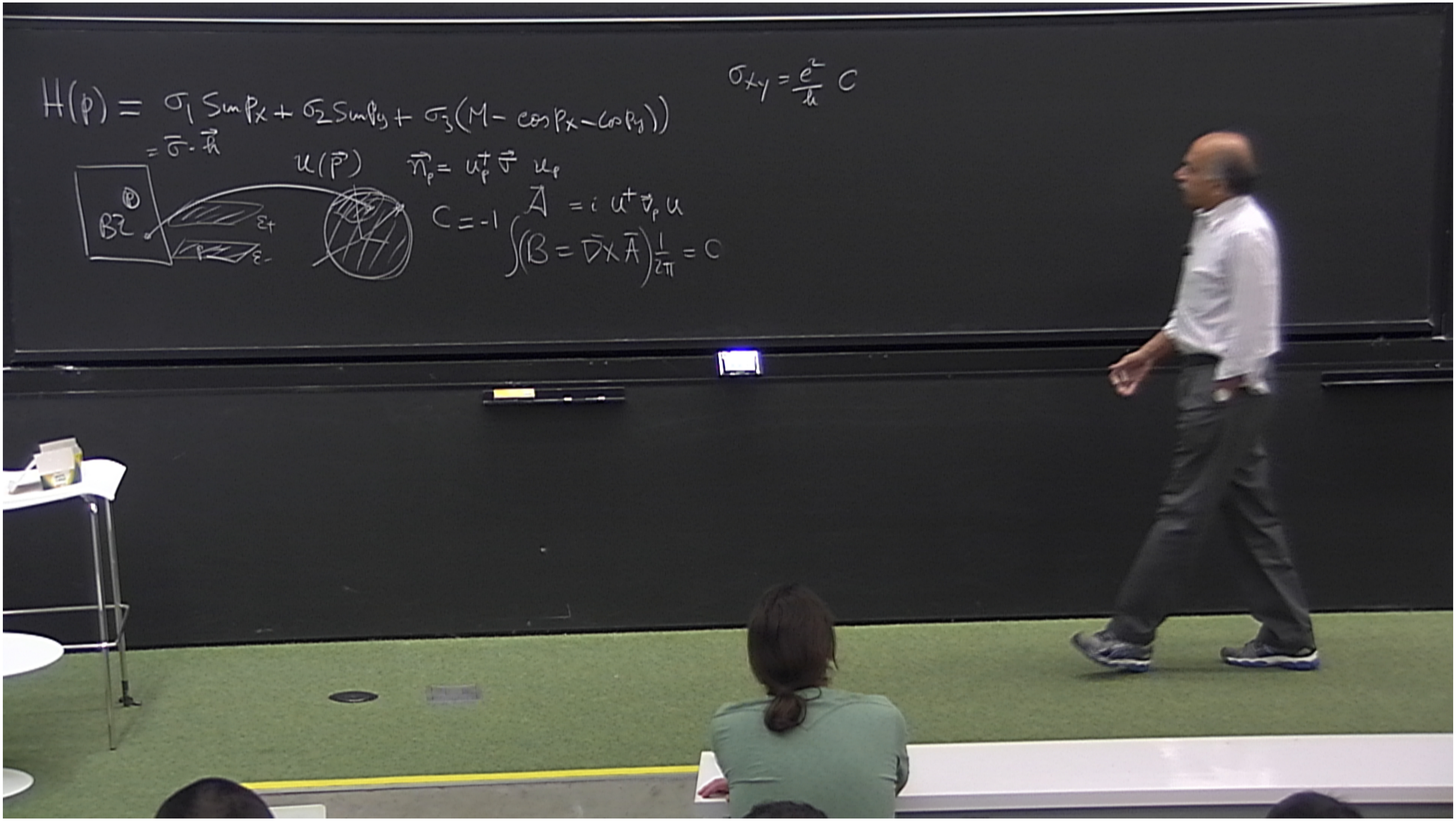
$= \vec{\sigma} \cdot \vec{h}$
$u(\vec{p})$
$\vec{n}_p = u_p^{\dagger} \vec{\sigma} u_p$
BZ
p
ε_+
ε_-
$C = -1$
$\vec{A} = i u^{\dagger} \vec{\nabla}_p u$
$\int (B = \vec{\nabla} \times \vec{A}) \frac{1}{2\pi} = C$



$H(p) = \sigma_1 \sin p_x + \sigma_2 \sin p_y + \sigma_3 (M - \cos p_x - \cos p_y)$
$= \vec{\sigma} \cdot \vec{h}$
$\sigma_{xy} = \frac{e^2}{h} C$
BZ
$u(\vec{p})$
$\vec{n}_p = u_p^+ \vec{\sigma} u_p$
ε_+
ε_-
$C = -1$
$\vec{A} = i u^+ \nabla_p u$
$\int (B = \nabla \times \vec{A}) \frac{1}{2\pi} = C$


H(p) = σ1 Sin px + σ2 Sin py + σ3(M − cos px − cos py))
= σ · h
u(p)
n_p = u_p^+ σ u_p
BZ
ε+
ε−
C = −1
A = i u^+ ∇_p u
∫(B = ∇ × A) 1/2π = C
σxy = e²/h C



$H(p) = \sigma_1 \sin p_x + \sigma_2 \sin p_y + \sigma_3(M - \cos p_x - \cos p_y))$
$= \vec{\sigma} \cdot \vec{h}$
$u(\vec{p})$
$\vec{n}_p = u_p^{+} \vec{\sigma}\, u_p$
BZ
ε_+
ε_-
$C = -1$
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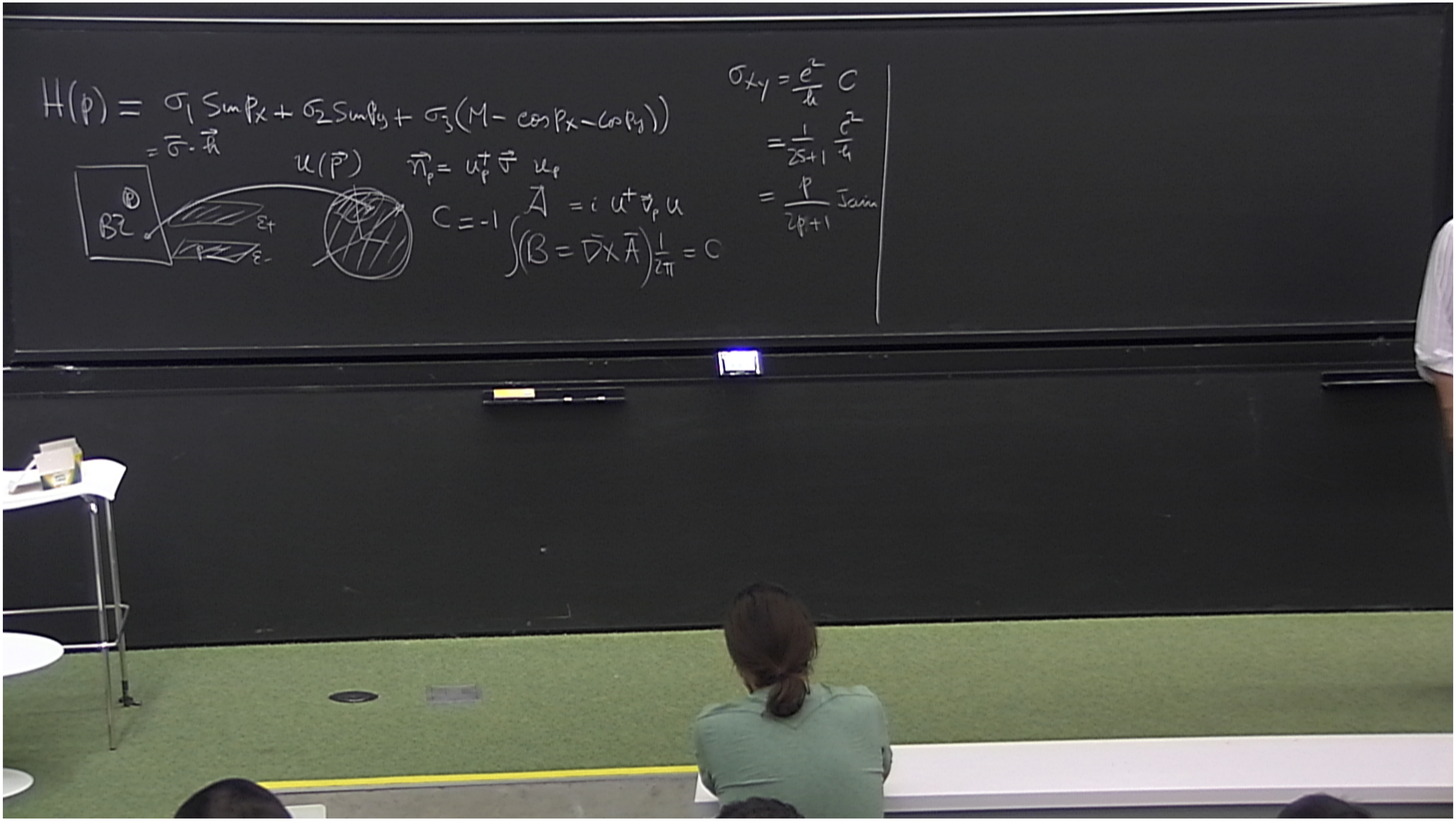
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= σ · h
u(p)
n_p = u_p^+ σ u_p
A = i u^+ ∇_p u
C = −1
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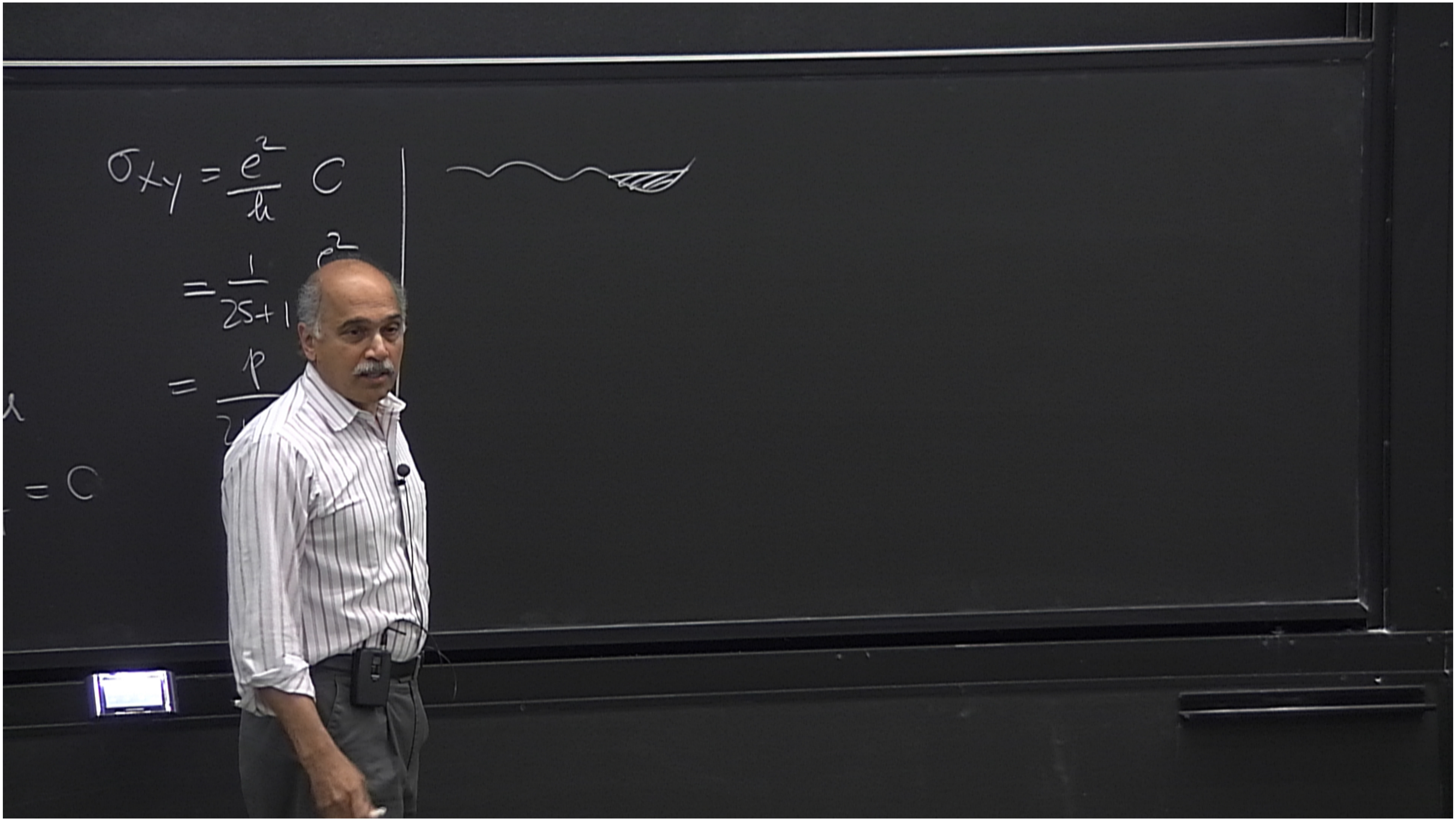
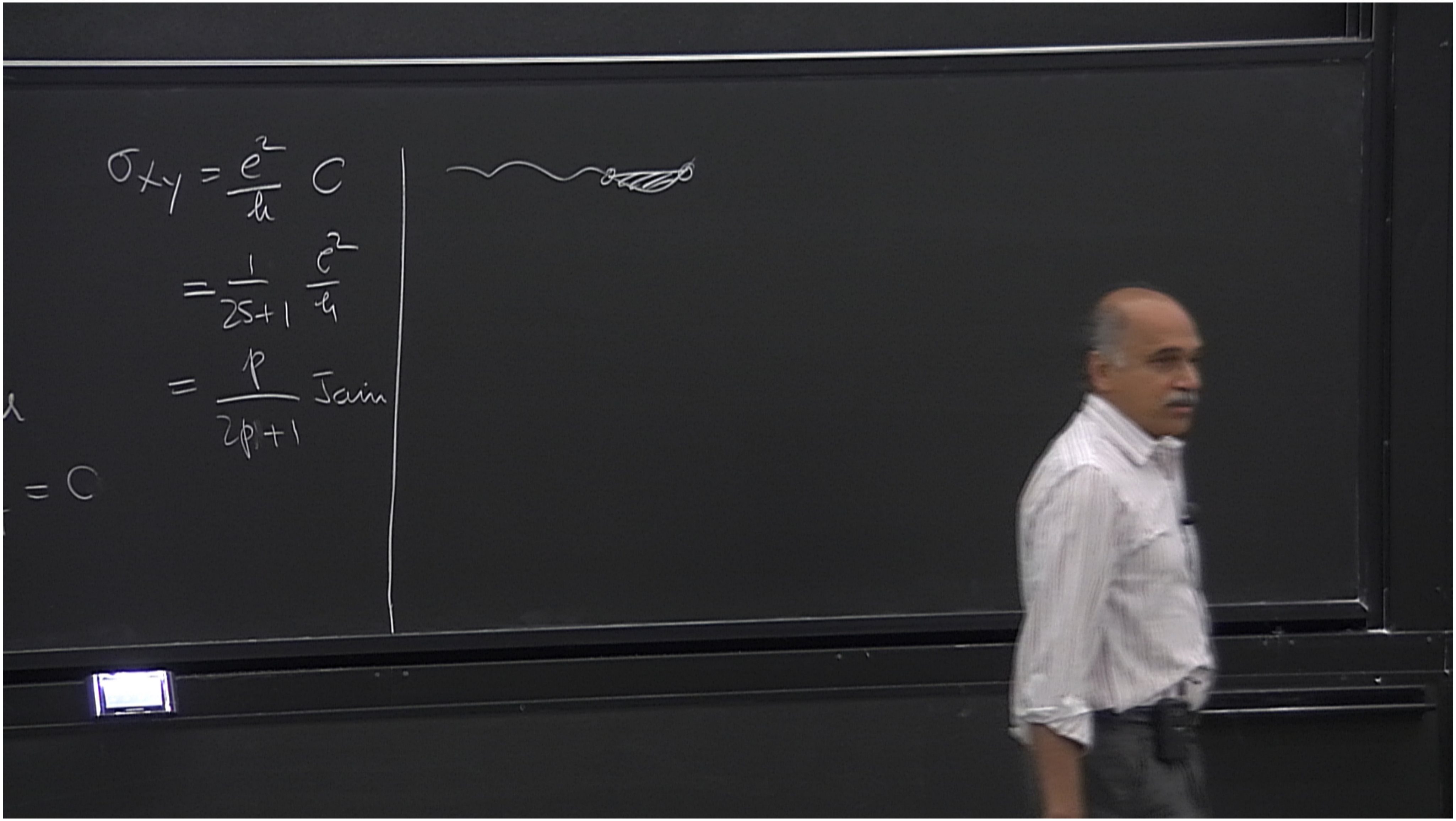
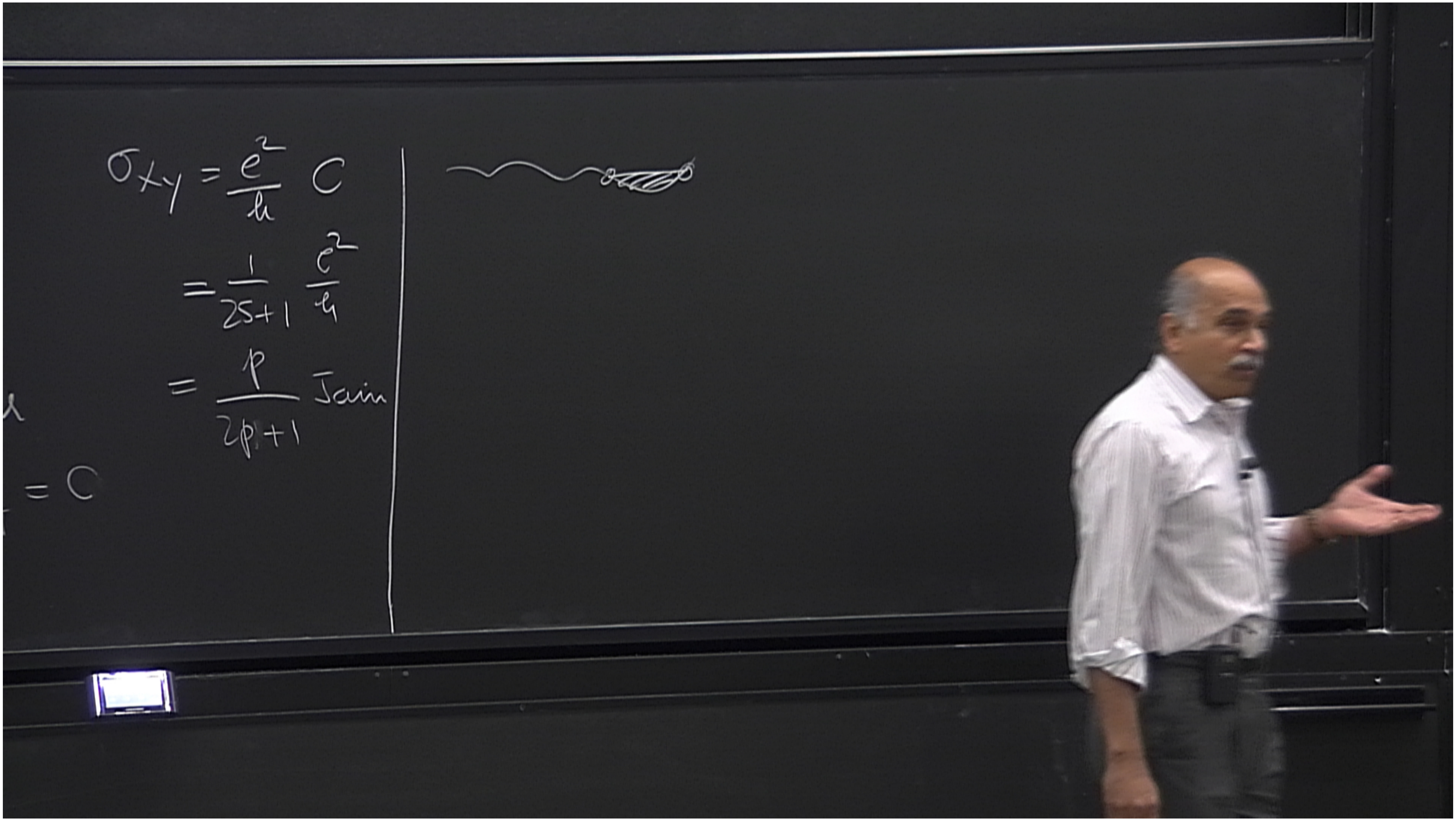
BZ
ε+
ε−
σxy = e²/h C
= 1/(2s+1) e²/h
= p/(2p+1) Jain

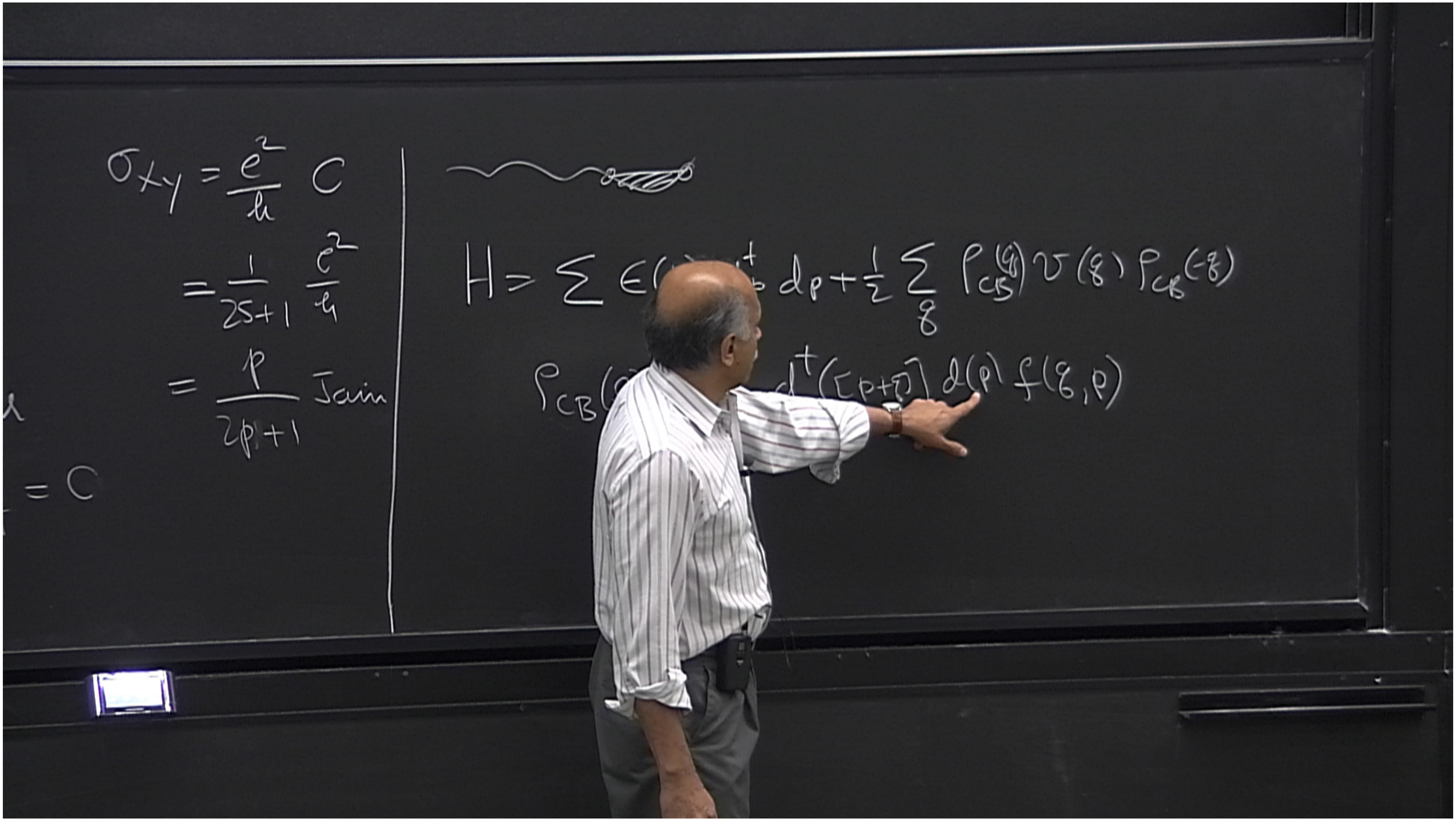

$\sigma_{xy} = \frac{e^2}{h} C$
$= C$


$\sigma_{xy} = \frac{e^2}{h} C$
$= \frac{1}{2s+1} \frac{e^2}{h}$
$= \frac{p}{2p+1}$ Jain
$= C$


σxy = e²/h C
= 1/(2S+1) e²/h
= p/(2p+1) Jain
= C

$$\sigma_{xy} = \frac{e^2}{h} C = \frac{1}{2s+1} \frac{e^2}{h} = \frac{p}{2p+1} \text{ Jain}$$

$$H = \sum \epsilon(p)\, d_p^\dagger d_p + \frac{1}{2} \sum_q \rho_{CS}(q)\, v(q)\, \rho_{CS}(-q)$$



$\sigma_{xy} = \frac{e^2}{h} C$
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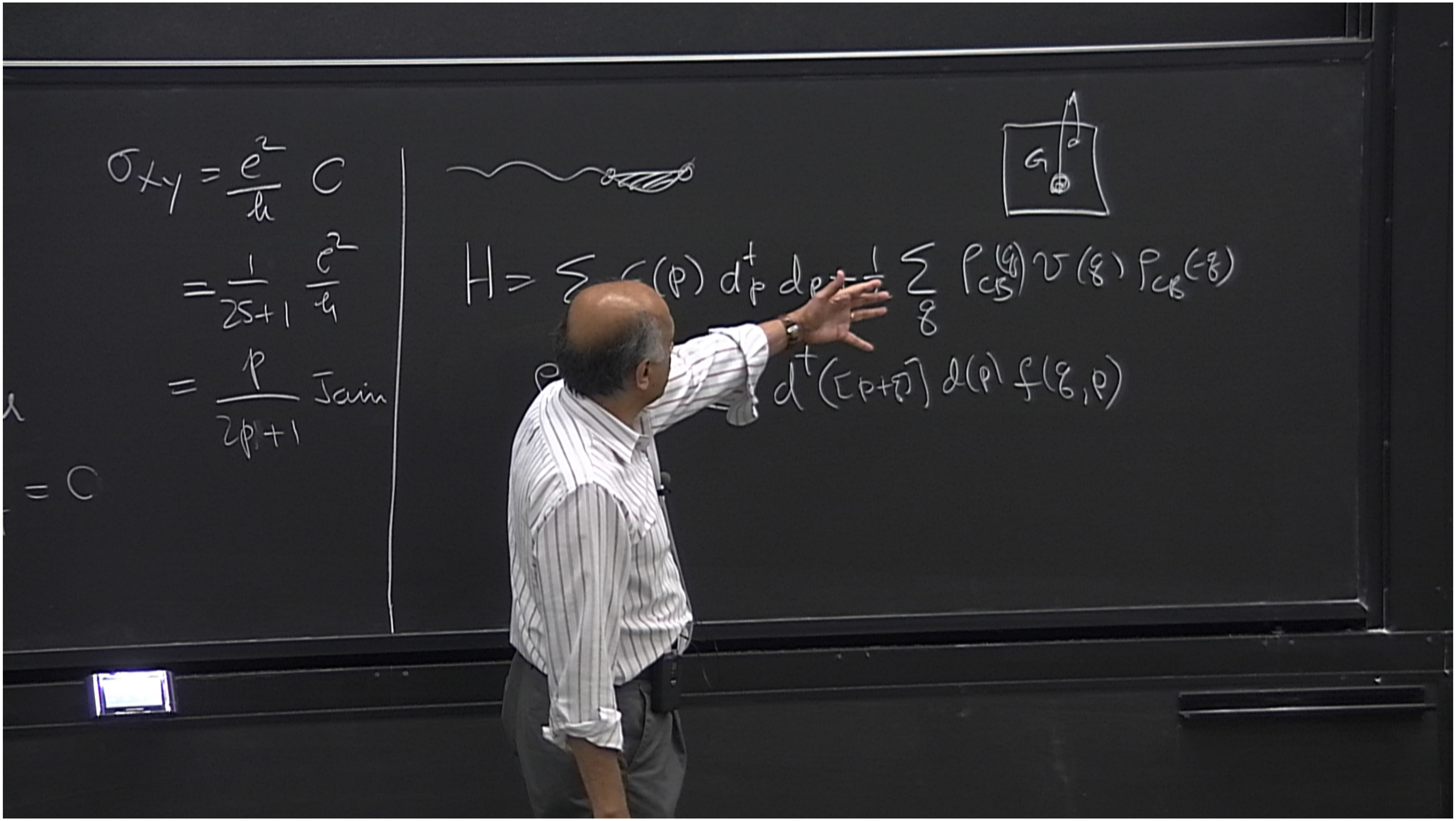
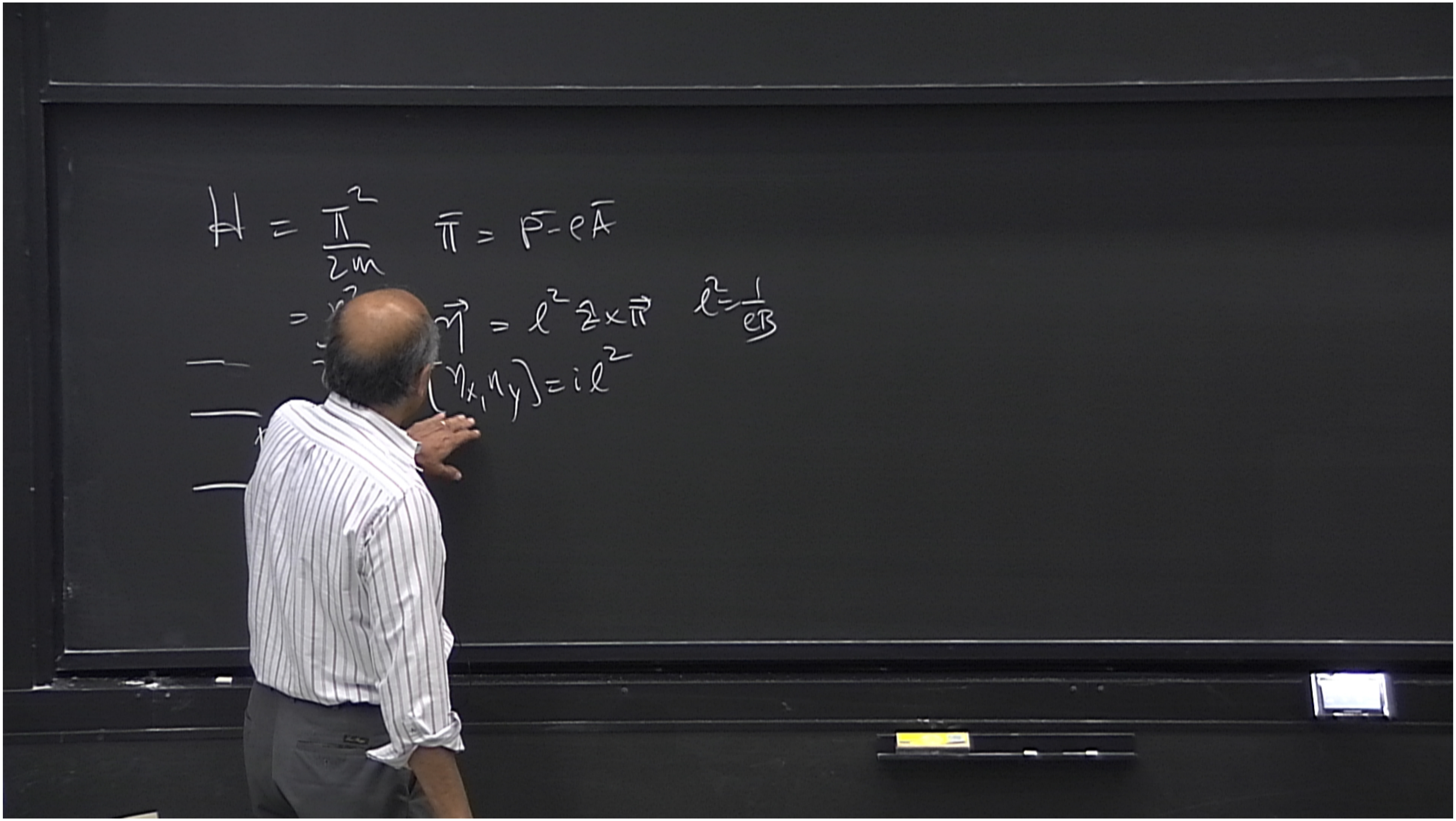
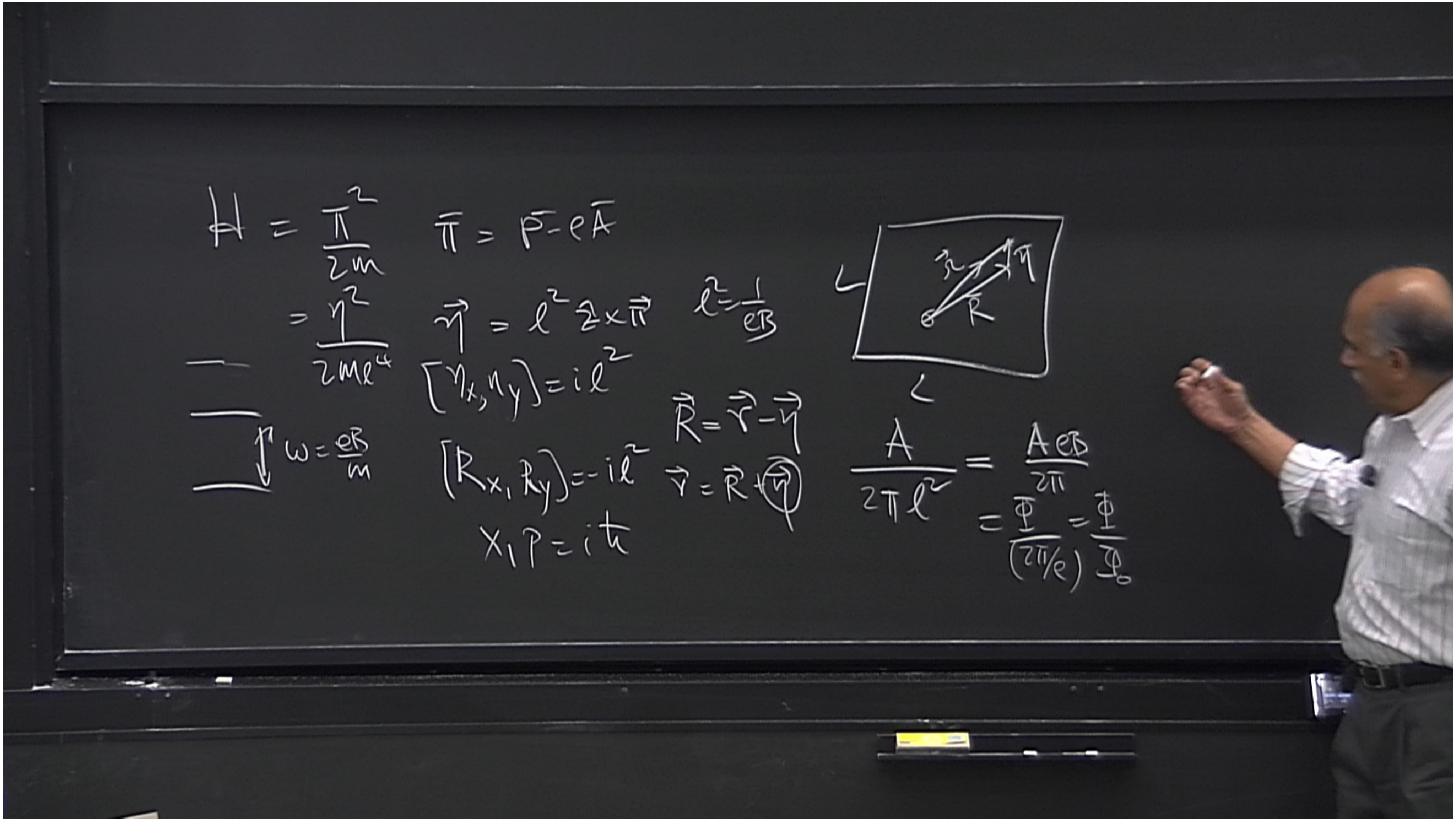
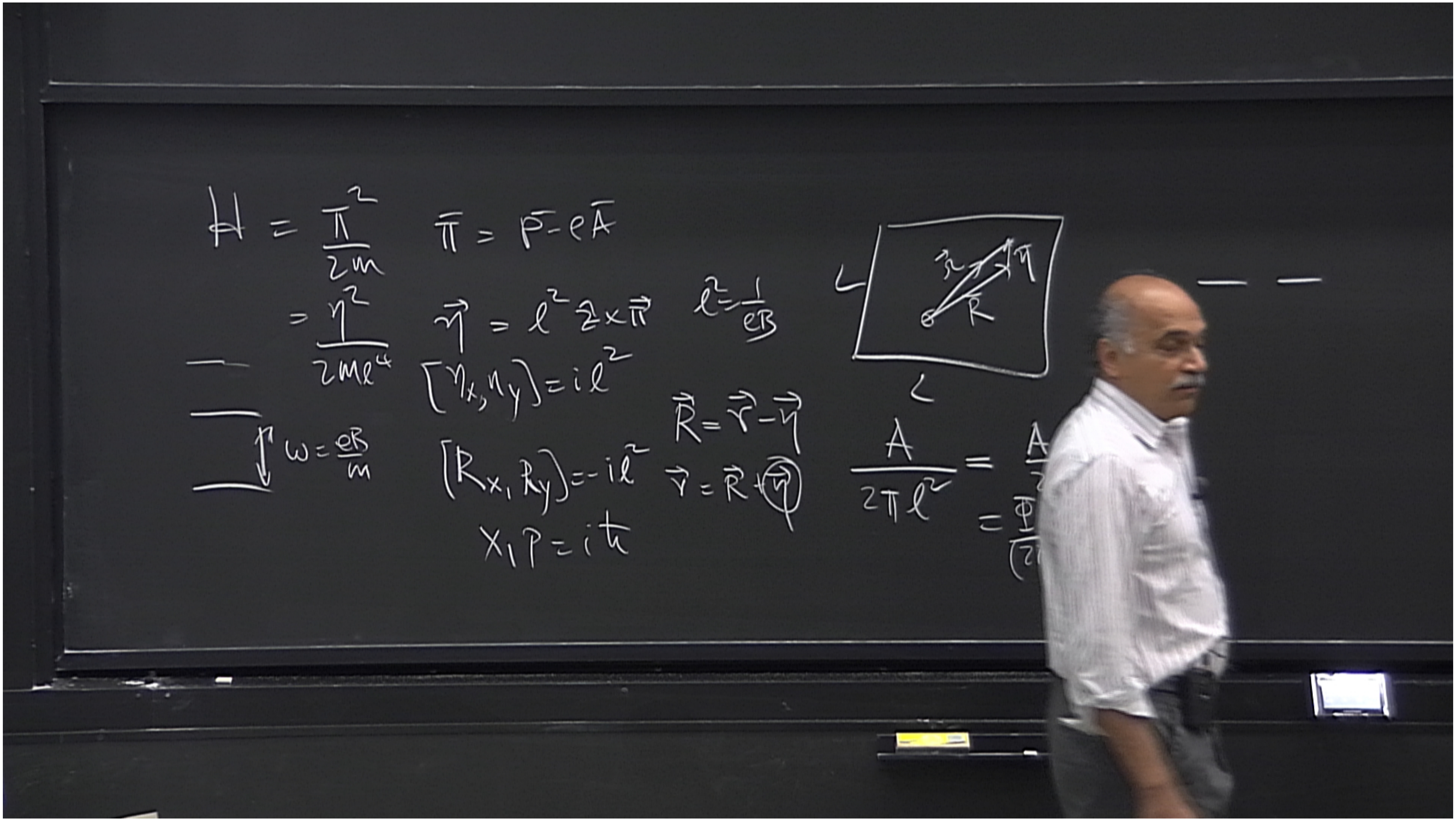
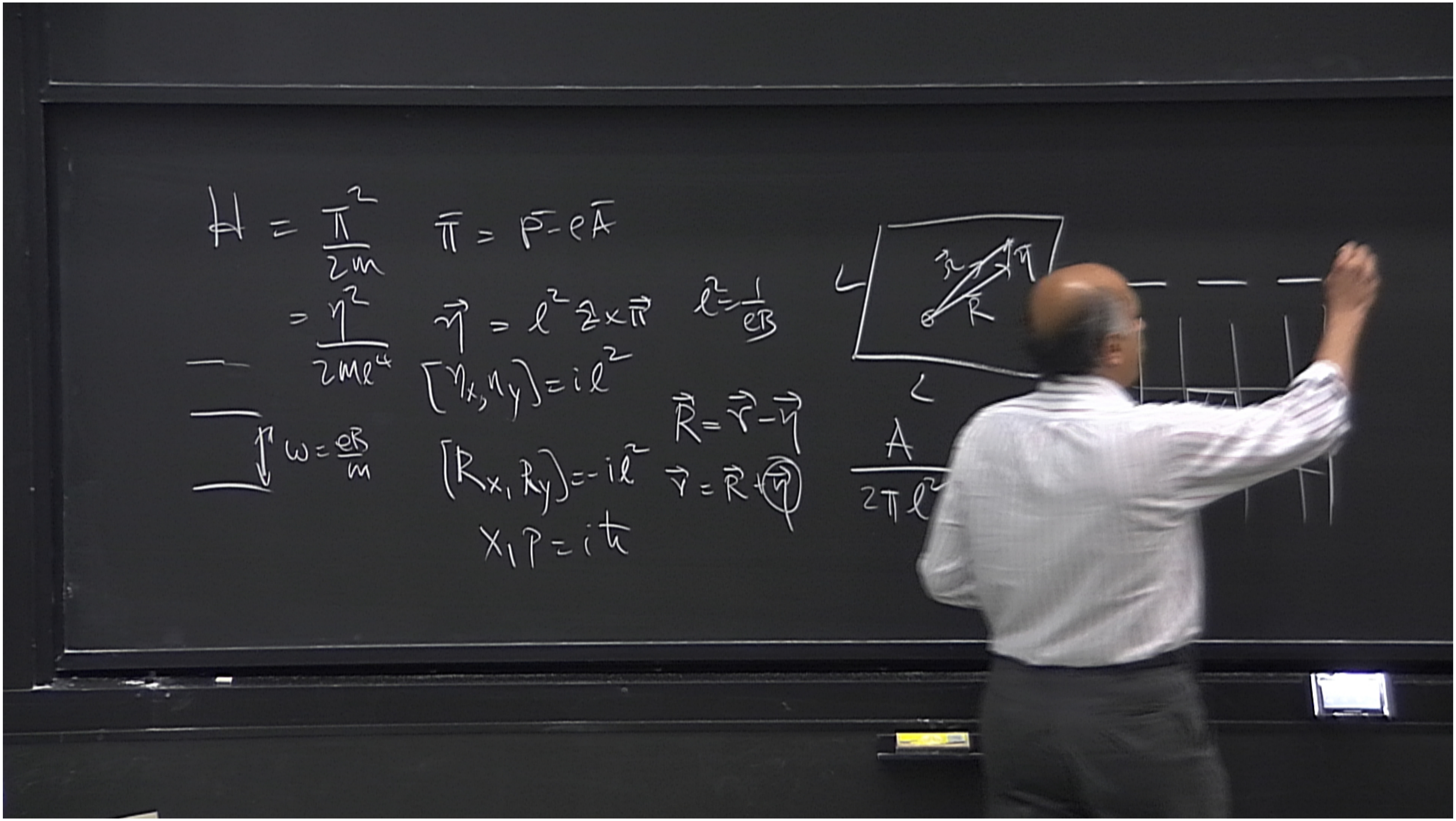
$= \frac{p}{2p+1}$ Jain
$= C$


σxy = e²/h C
= 1/(2s+1) e²/h
= p/(2p+1) Jain
= C


$H = \frac{\pi^2}{2m}$
$\bar{\pi} = \bar{p} - e\bar{A}$
$\vec{\eta} = \ell^2 \hat{z} \times \vec{\pi}$
$\ell^2 = \frac{1}{eB}$
$[\eta_x, \eta_y] = i\ell^2$


$H = \frac{\pi^2}{2m}$
$\vec{\pi} = \vec{p} - e\vec{A}$
$= \frac{\eta^2}{2m\ell^4}$
$\vec{\eta} = \ell^2 \hat{z} \times \vec{\pi}$
$\ell^2 = \frac{1}{eB}$
$[\eta_x, \eta_y] = i\ell^2$
$\omega = \frac{eB}{m}$
$[R_x, R_y] = -i\ell^2$
$\vec{R} = \vec{r} - \vec{\eta}$
$\vec{r} = \vec{R} + \vec{\eta}$
$x, p = i\hbar$
L
L
$\vec{r}$
$\vec{\eta}$
R
$\frac{A}{2\pi\ell^2} = \frac{AeB}{2\pi}$
$= \frac{\Phi}{(2\pi/e)} = \frac{\Phi}{\Phi_0}$


H = π²/2m
π = p − eA
= η²/2ml⁴
η = l² ẑ × π
l² = 1/eB
[ηx, ηy] = il²
ω = eB/m
[Rx, Ry] = −il²
x, p = iħ
R = r − η
r = R + η
A/2πl² =



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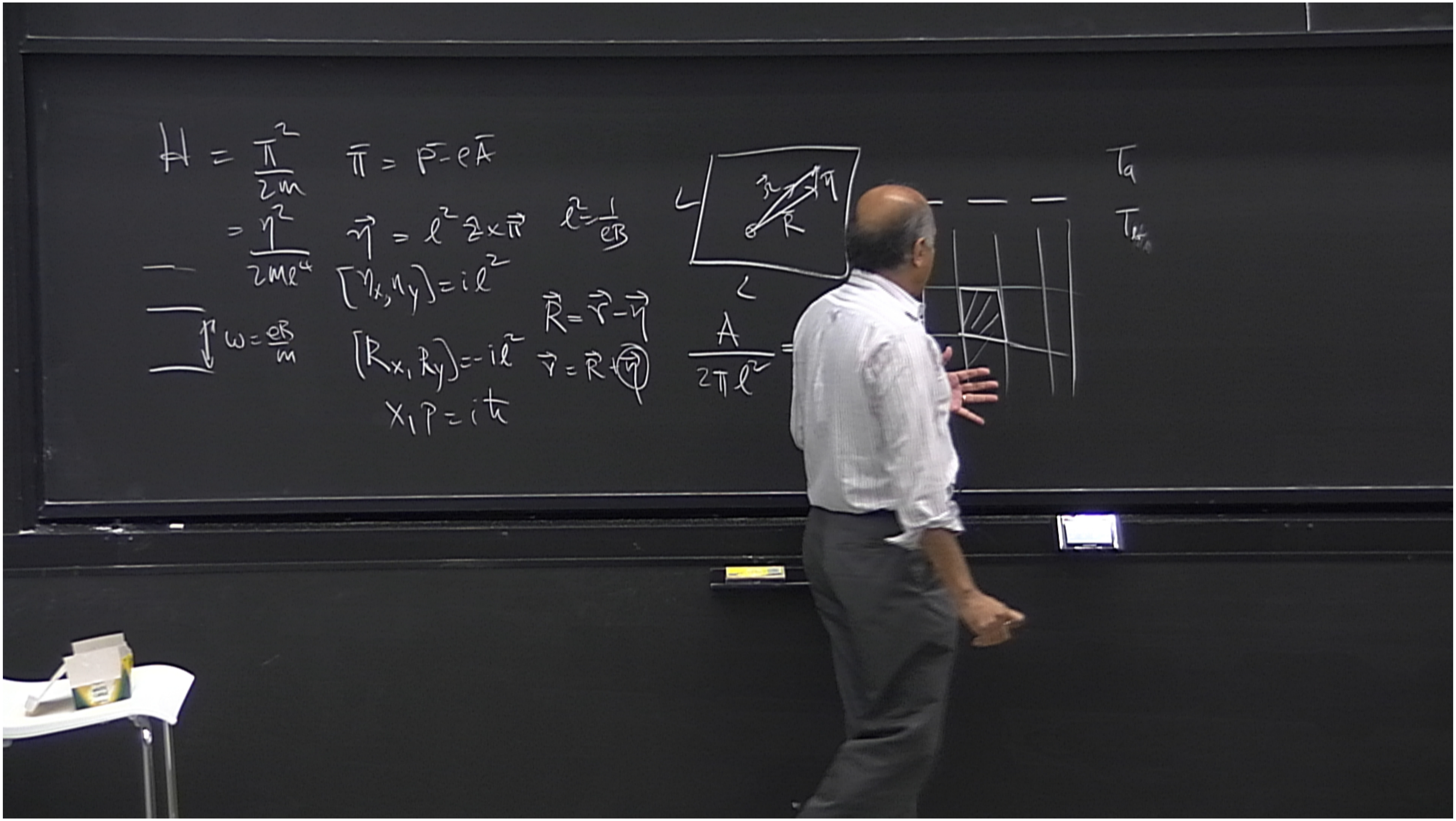
$[R_x, R_y] = -i\ell^2$
$\vec{R} = \vec{r} - \vec{\eta}$
$\vec{r} = \vec{R} + \vec{\eta}$
$x, p = i\hbar$
$\frac{A}{2\pi\ell^2}$
L
L
r
R
η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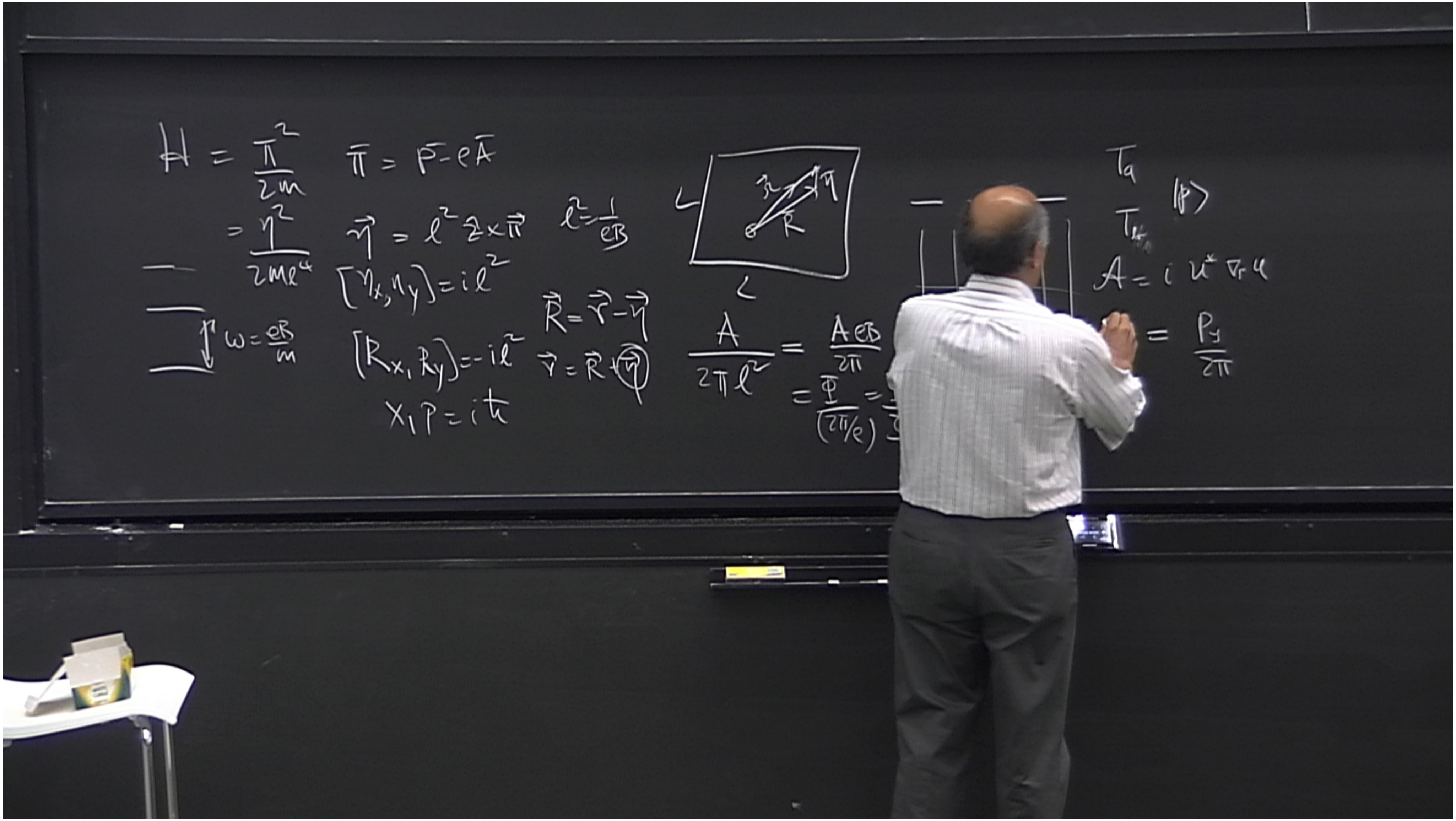
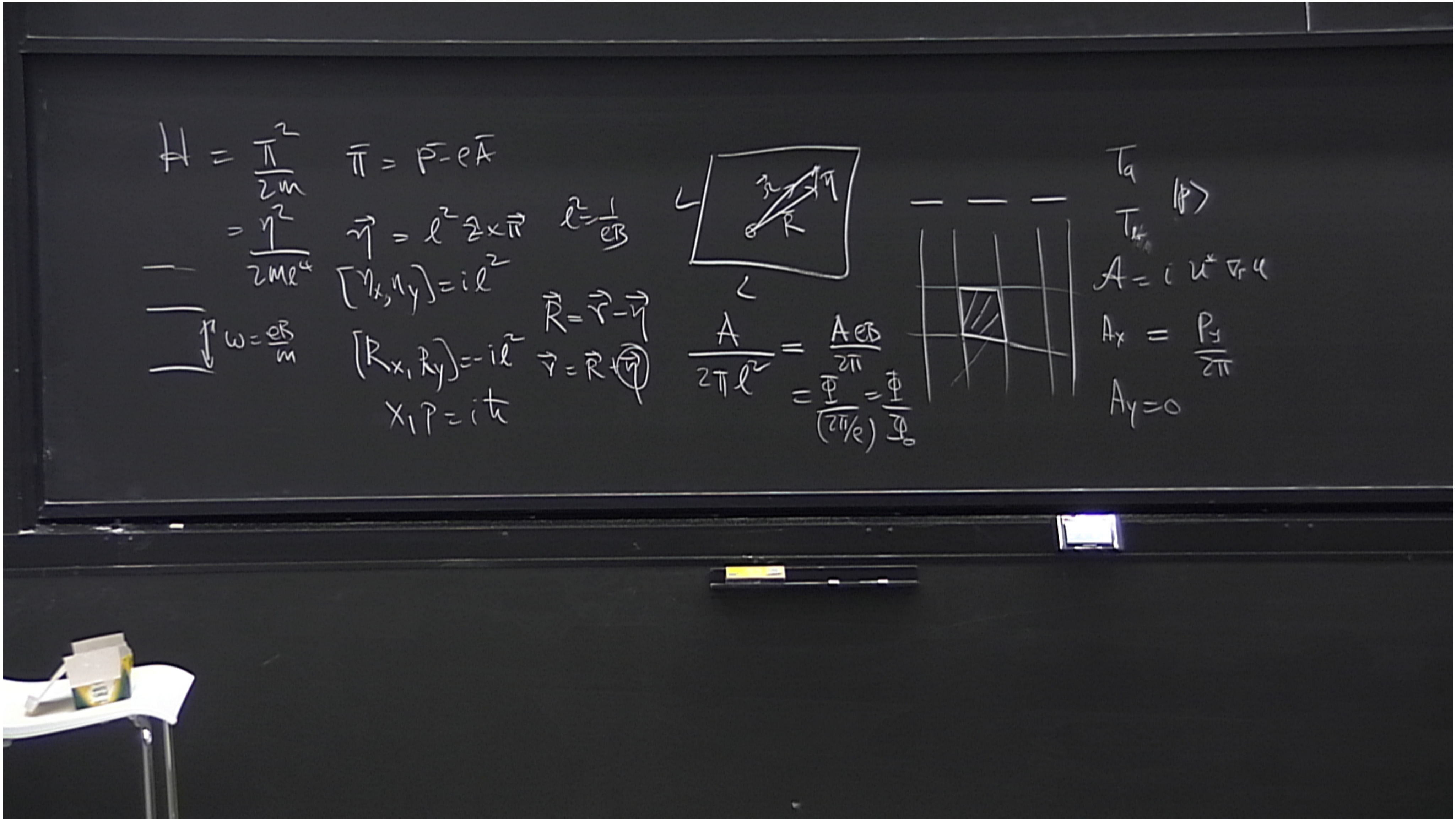
$\vec{\eta} = \ell^2 \hat{z} \times \vec{\pi}$
$\ell^2 = \frac{1}{eB}$
$[\eta_x, \eta_y] = i\ell^2$
$\omega = \frac{eB}{m}$
$[R_x, R_y] = -i\ell^2$
$x, p = i\hbar$
$\vec{R} = \vec{r} - \vec{\eta}$
$\vec{r} = \vec{R} + \vec{\eta}$
$\frac{A}{2\pi\ell^2} =$


H = π²/2m   π = p − eA
= η²/2ml⁴   η = l² ẑ × π   l² = 1/eB
[η_x, η_y] = i l²
ω = eB/m
[R_x, R_y] = −i l²
R = r − η
r = R + η
X, P = iħ
A/2πl² = AeB/2π
= Φ/(2π/e)
T_a
|p⟩
A = i u* ∇ u
= P_y/2π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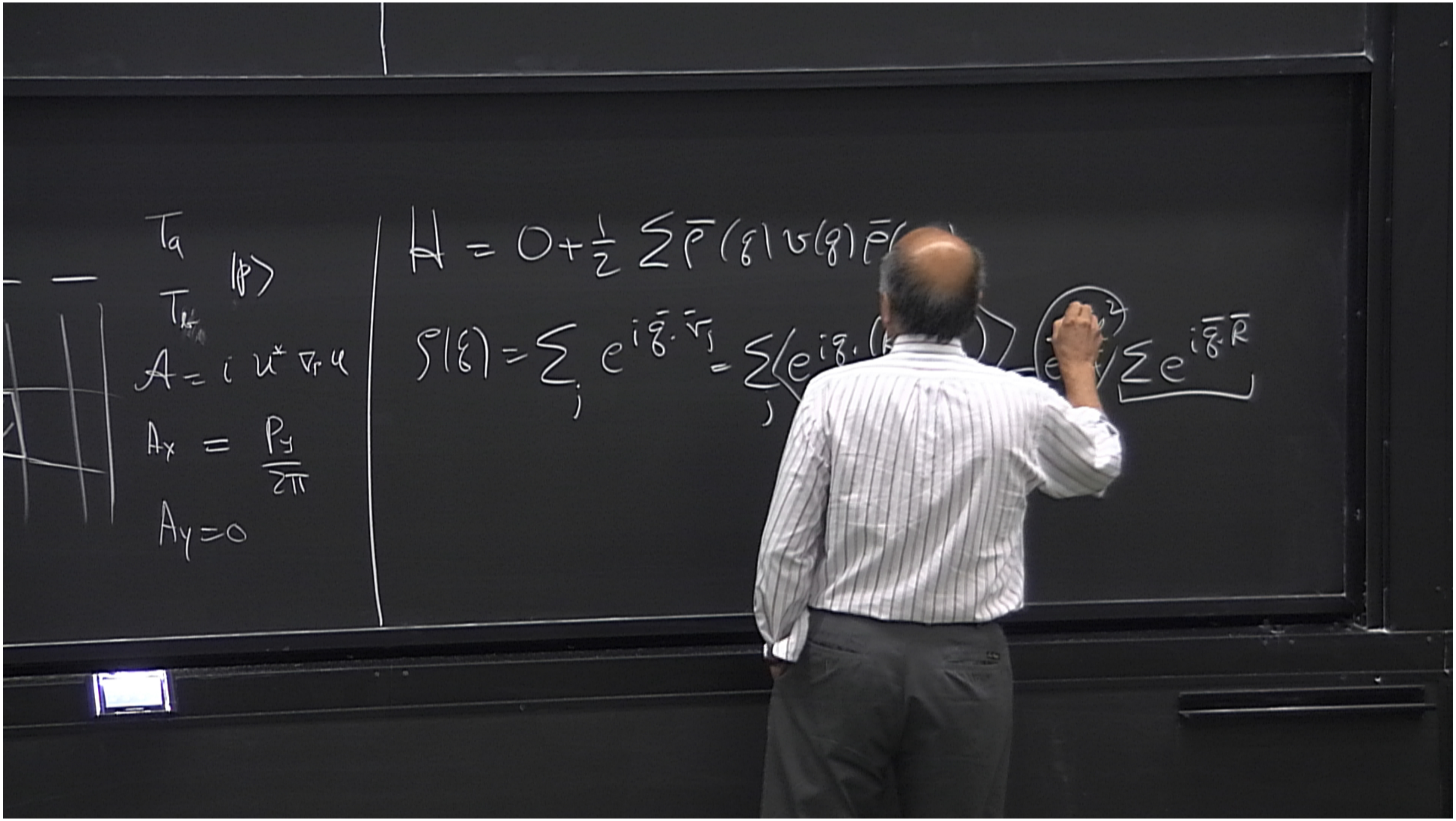
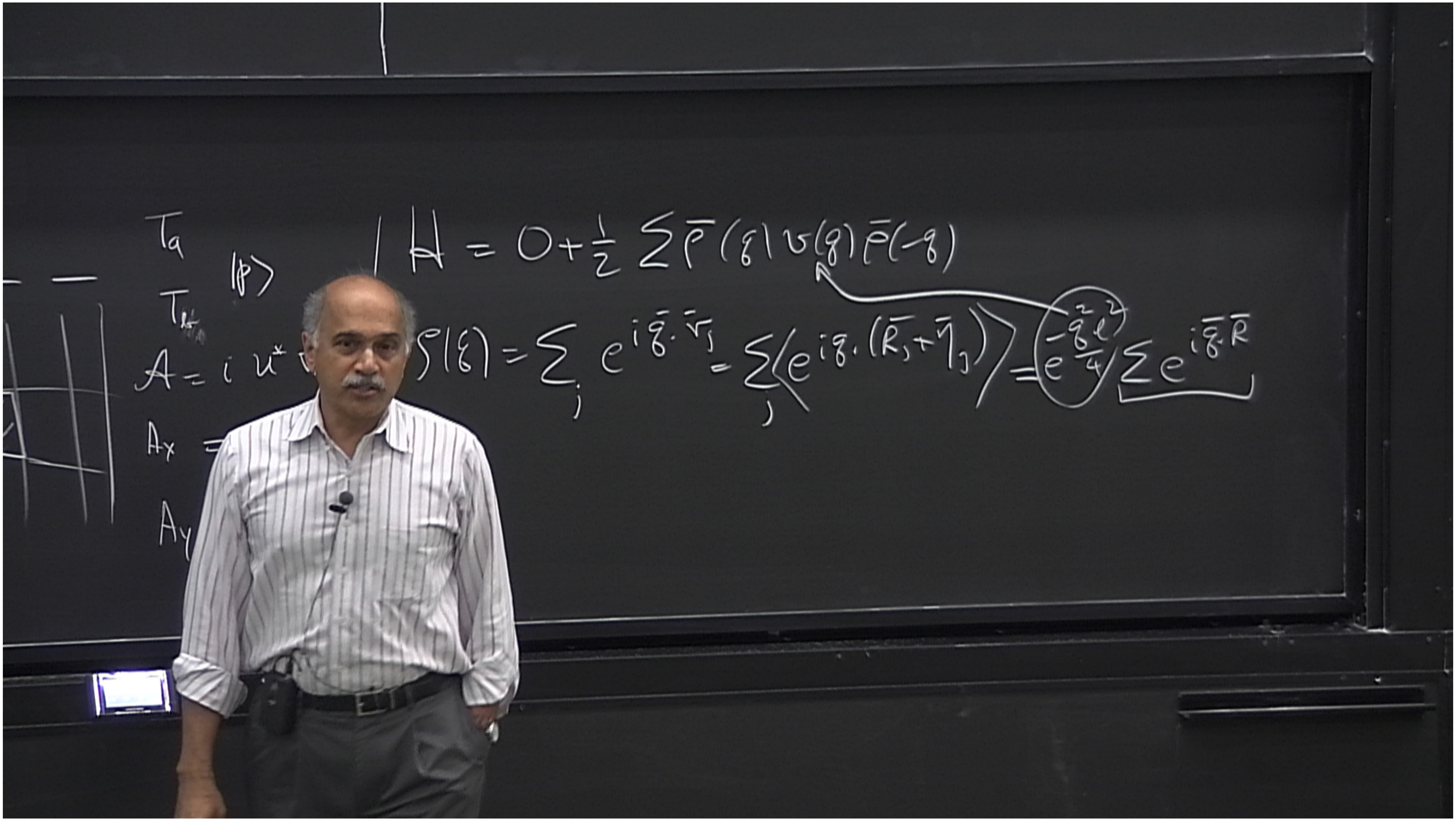
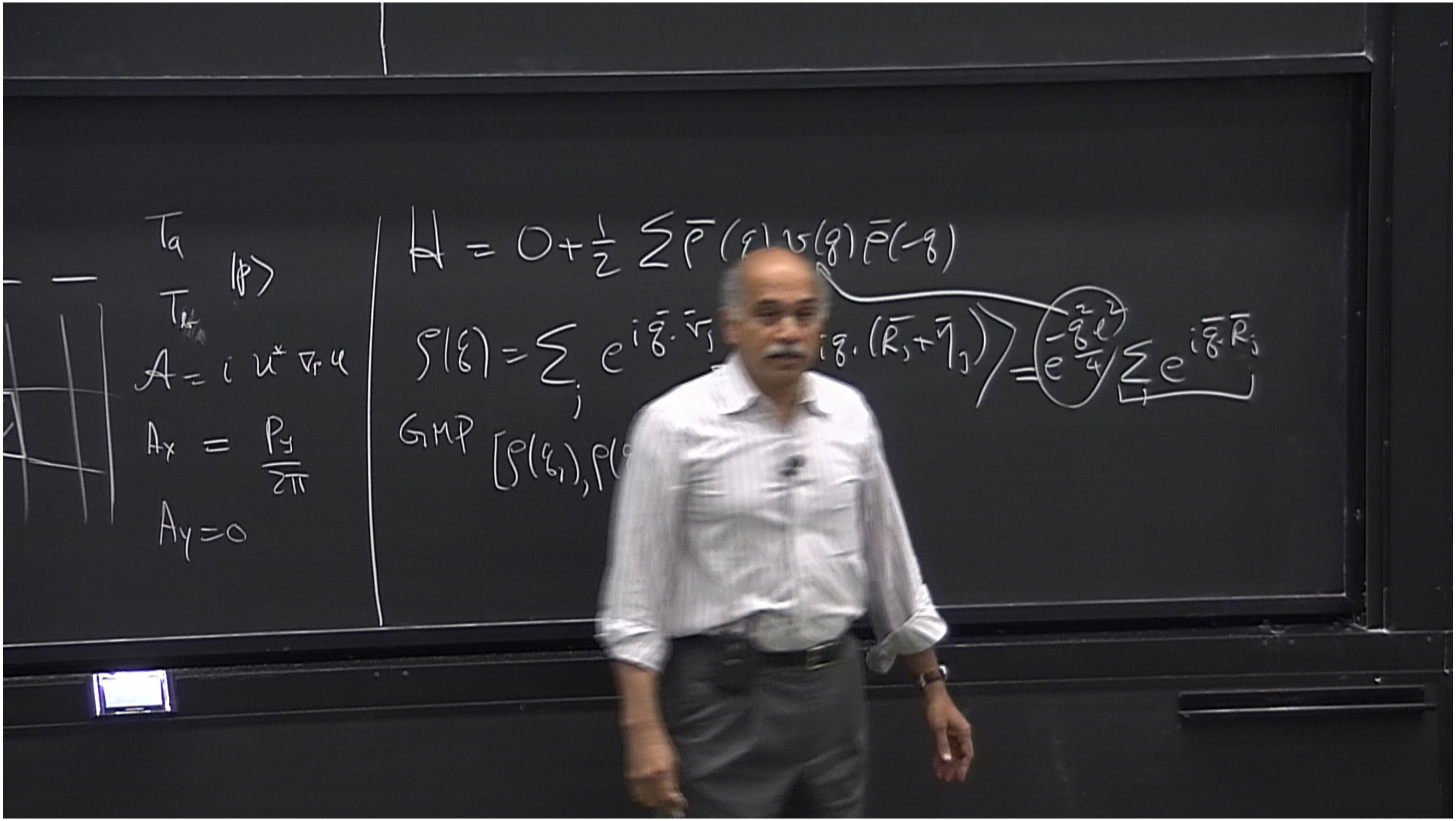
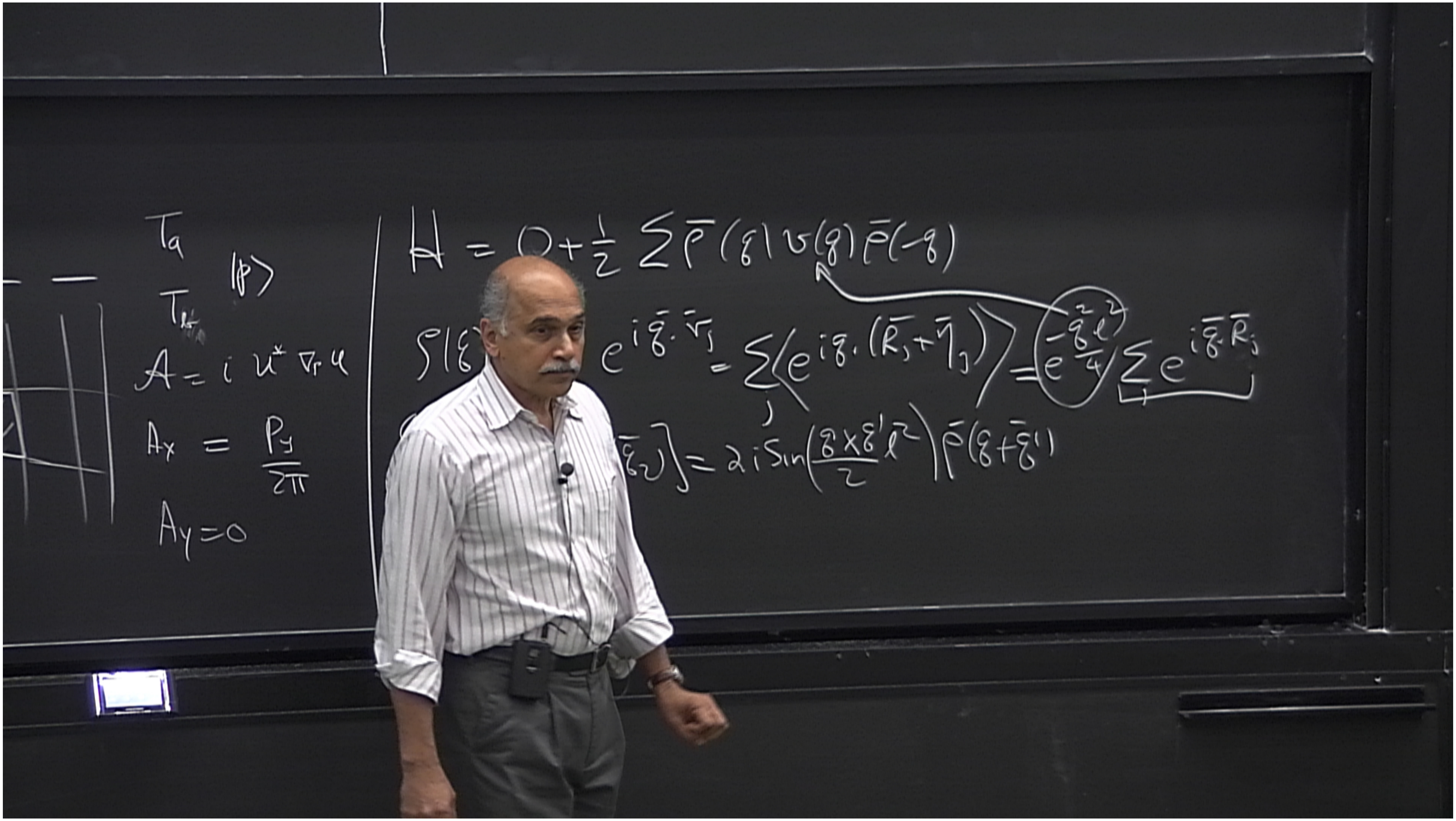
$x, p = i\hbar$
L
r
η
R
L
$\frac{A}{2\pi\ell^2} = \frac{A\,eB}{2\pi}$
$= \frac{\Phi}{(2\pi/e)} = \frac{\Phi}{\Phi_0}$
T_a
$|p\rangle$
$A = i\, u^* \nabla_r u$
$A_x = \frac{P_y}{2\pi}$
$A_y = 0$

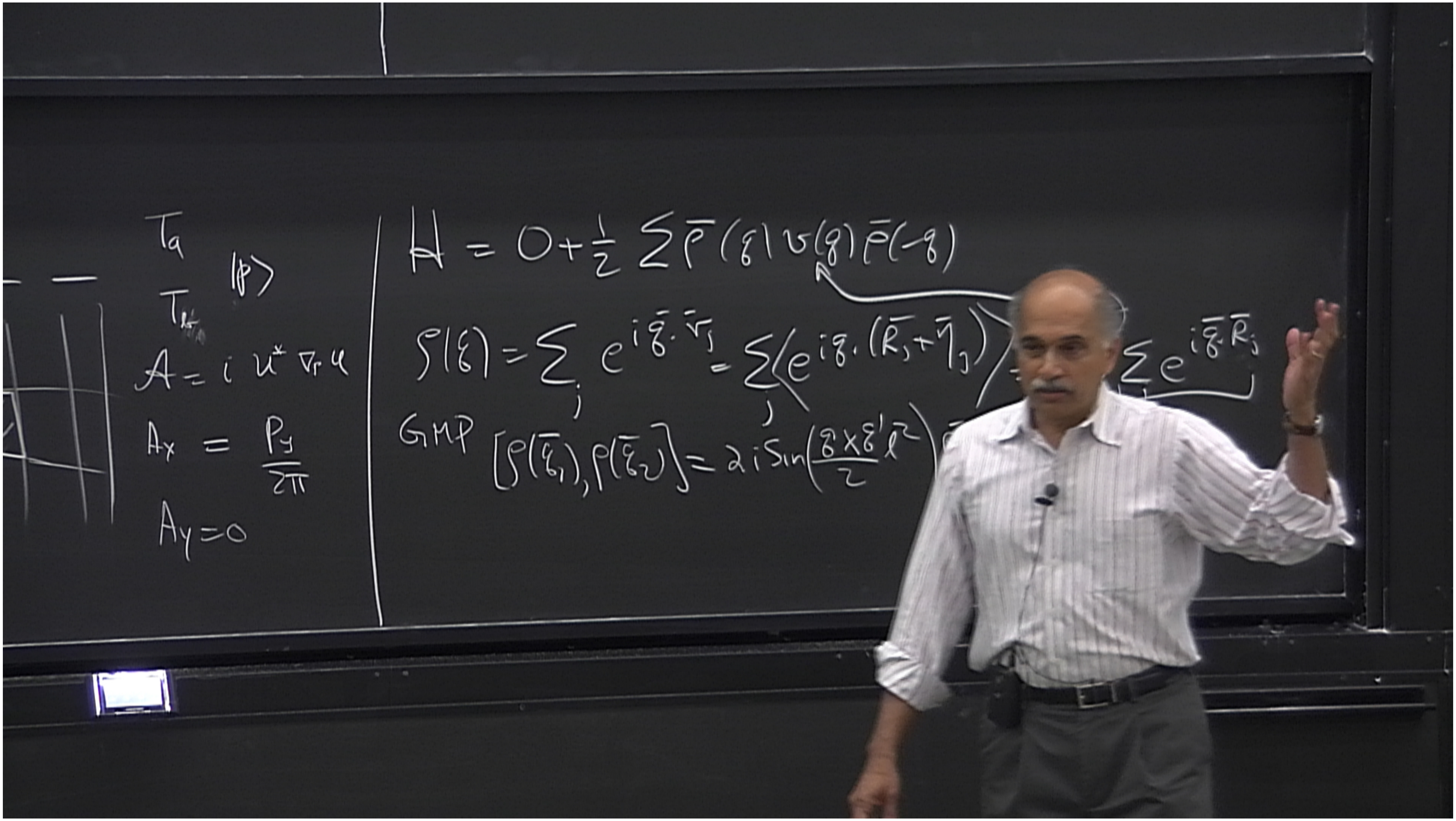


T_a
$|p\rangle$
$A = i\, u^* \nabla_r u$
$A_x = \frac{P_y}{2\pi}$
$A_y = 0$
$H = 0 + \frac{1}{2} \sum \bar{\rho}(q)\, v(q)\, \bar{\rho}($
$\rho(q) = \sum_j e^{i\bar{q}\cdot\bar{r}_j} = \sum_j \langle e^{iq\cdot($
$\sum e^{i\bar{q}\cdot\bar{R}}$


T_a
$|\psi\rangle$
$H = 0 + \frac{1}{2} \sum \bar{\rho}(q) v(q) \bar{\rho}(-q)$
$\rho(q) = \sum_j e^{i\bar{q}\cdot\bar{r}_j} = \sum_j \langle e^{i\bar{q}\cdot(\bar{R}_j+\bar{\eta}_j)} \rangle = e^{-\frac{q^2 \ell^2}{4}} \sum e^{i\bar{q}\cdot\bar{R}}$
$A = i\, u^*$
$A_x =$
A_y


H = 0 + 1/2 Σ ρ̄(q) v(q) ρ̄(−q)
A = i u* ∇ u
Ax = Py/2π
Ay = 0
GMP


|p>
A = i u* ∇ u
Ax = Py / 2π
Ay = 0
H = 0 + 1/2 Σ ρ̄(q) v(q) ρ̄(−q)


GMP
Ay = 0

T_a

T_{ℓ} $\quad |\psi\rangle$

$A = i\, u^* \nabla_r u$

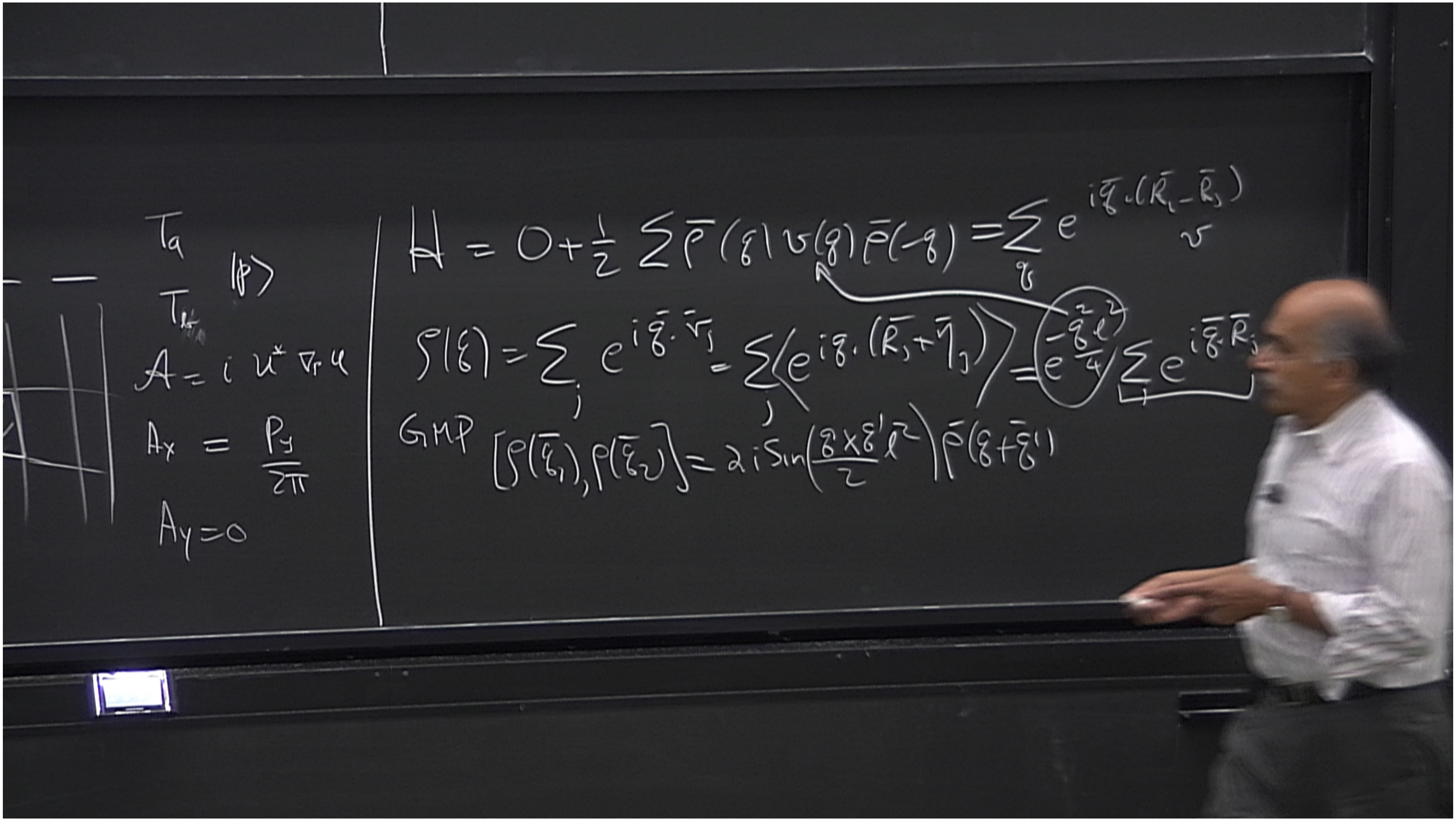
$A_x = \frac{P_y}{2\pi}$

$A_y = 0$

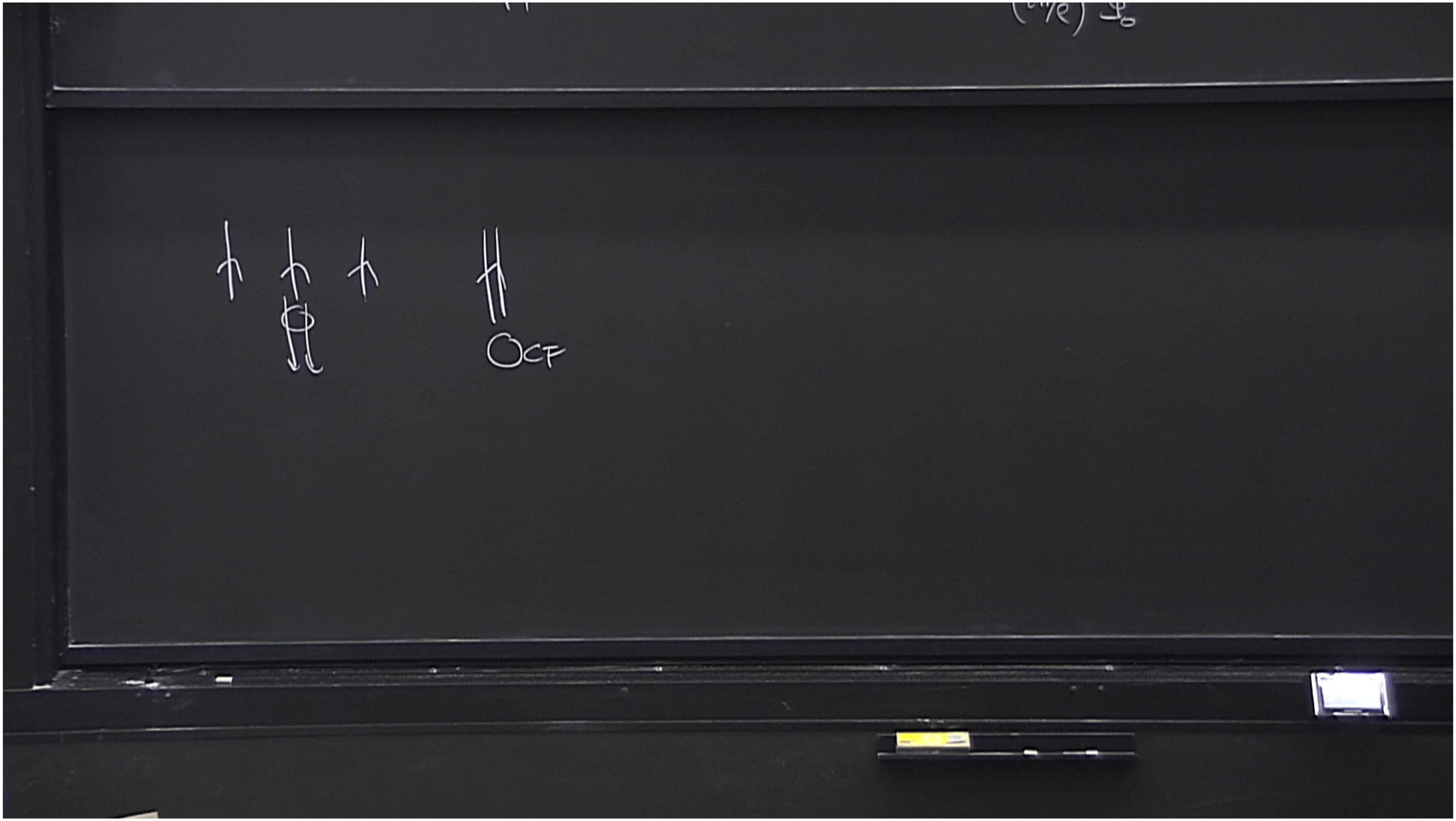
$$H = 0 + \frac{1}{2}\sum \bar\rho(q)\, v(q)\, \bar\rho(-q) = \sum_q e^{i\bar q\cdot(\bar R_i - \bar R_j)}\, v$$

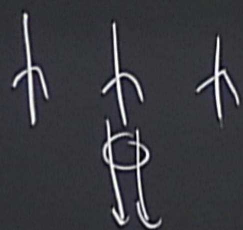
$$\rho(q) = \sum_j e^{i\bar q\cdot\bar r_j} = \sum_j \left\langle e^{i q\cdot(\bar R_j + \bar\eta_j)}\right\rangle = e^{-\frac{q^2\ell^2}{4}} \underbrace{\sum_j e^{i\bar q\cdot\bar R_j}}$$

GMP $[\bar\rho(\bar q_1), \bar\rho(\bar q_2)] = 2i\sin\left(\frac{q\times q'}{2}\ell^2\right)\bar\rho(q+\bar q')$



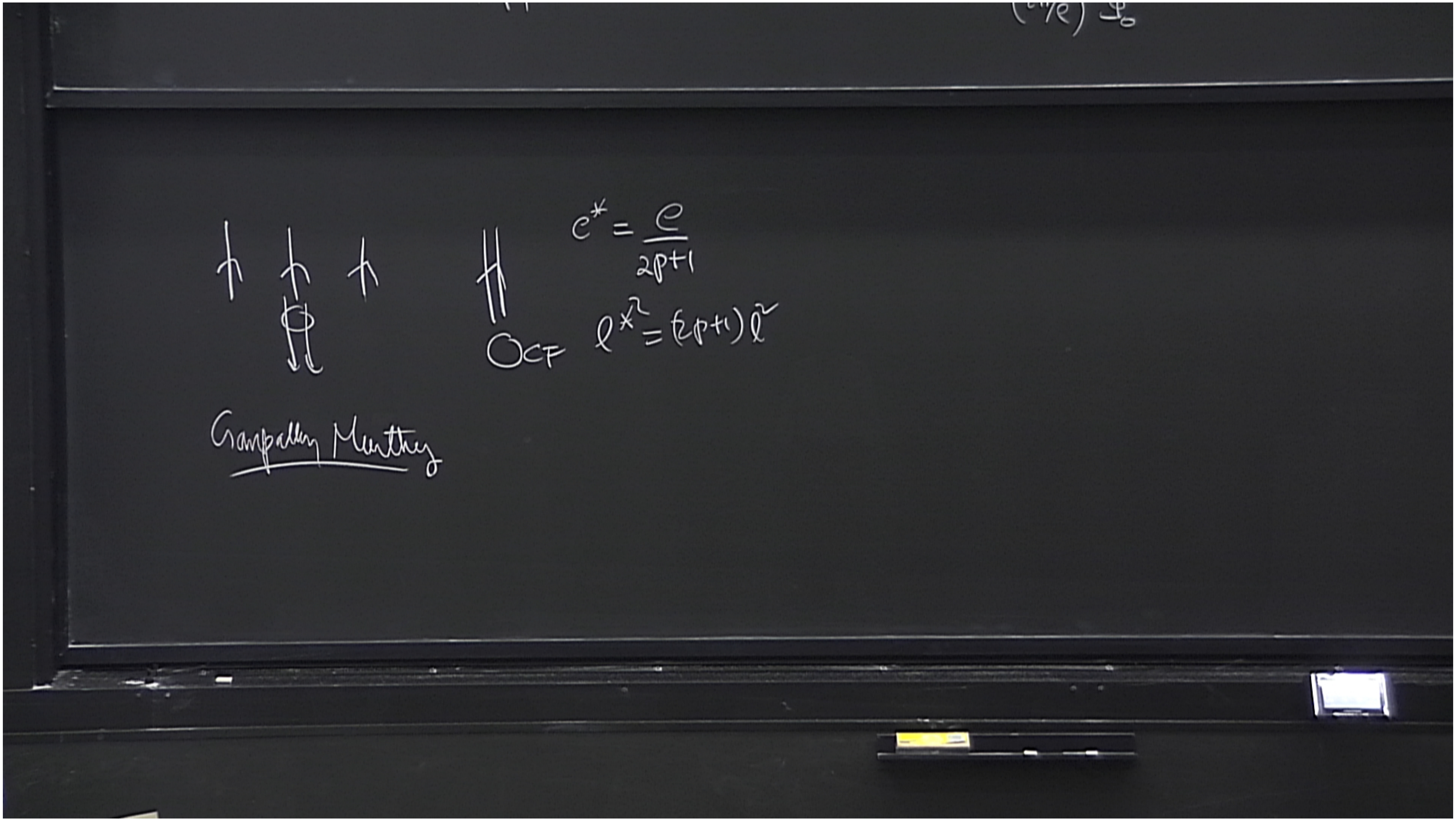
$H = 0 + \frac{1}{2}\sum \bar\rho(q)\, v(q)\, \bar\rho(-q) = \sum_q e^{i\bar q\cdot(\bar R_i - \bar R_j)}\, v$
$\rho(q) = \sum_j e^{i\bar q\cdot \bar r_j} = \sum_j \langle e^{i\bar q\cdot(\bar R_j + \bar\eta_j)}\rangle = e^{-\frac{q^2 \ell^2}{4}} \sum_j e^{i\bar q\cdot \bar R_j}$
GMP $[\bar\rho(\bar q_1), \bar\rho(\bar q_2)] = 2i \sin\left(\frac{\bar q \times \bar q' \ell^2}{2}\right) \bar\rho(\bar q + \bar q')$
$A = i\, u^* \nabla_r u$
$A_x = \frac{P_y}{2\pi}$
$A_y = 0$





$$e^* = \frac{e}{2p+1}$$

OCF $\ell^{*2} = (2p+1)\ell^2$


$e^* = \frac{e}{2p+1}$
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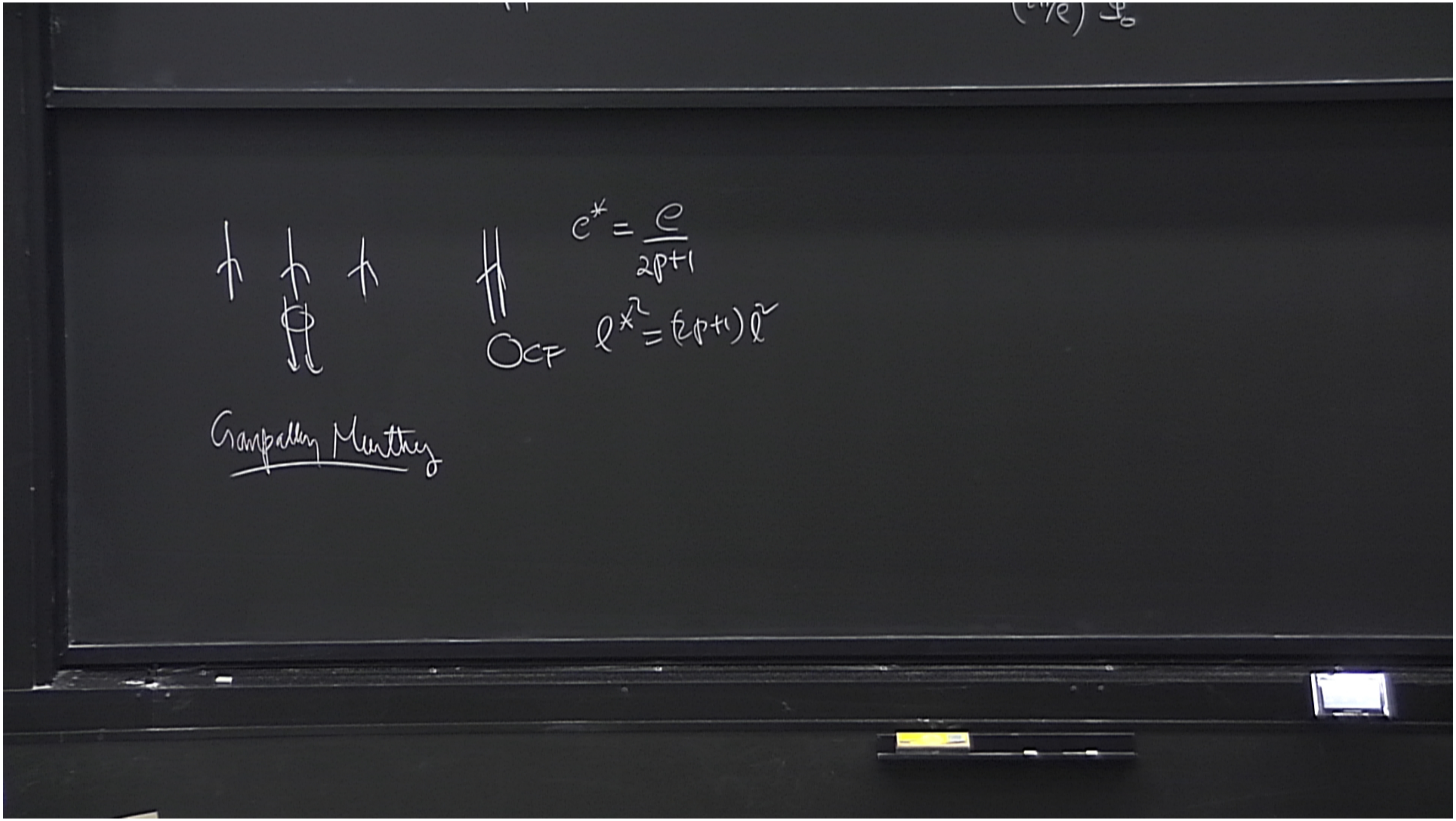
Ganpathy Murthy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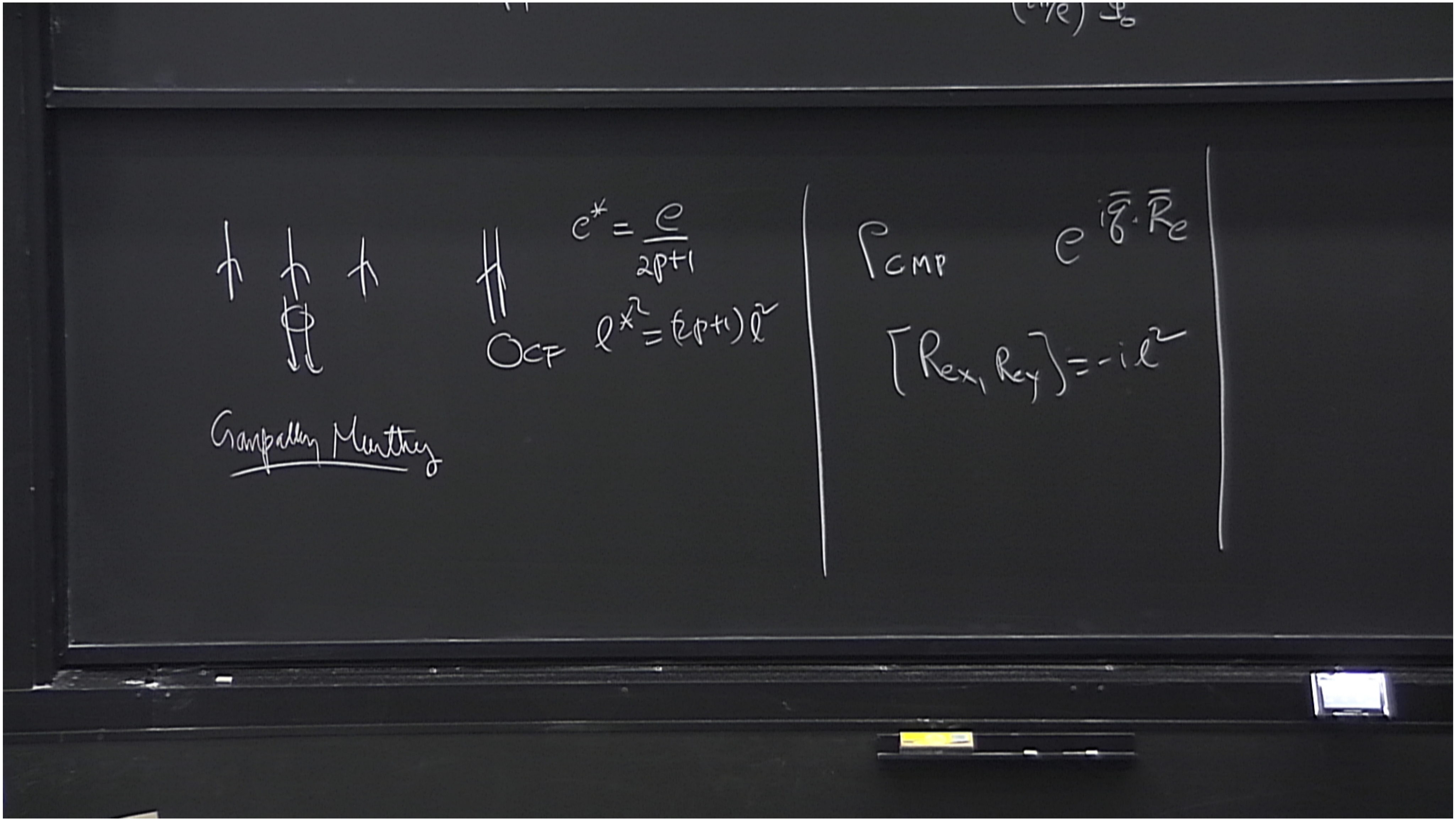
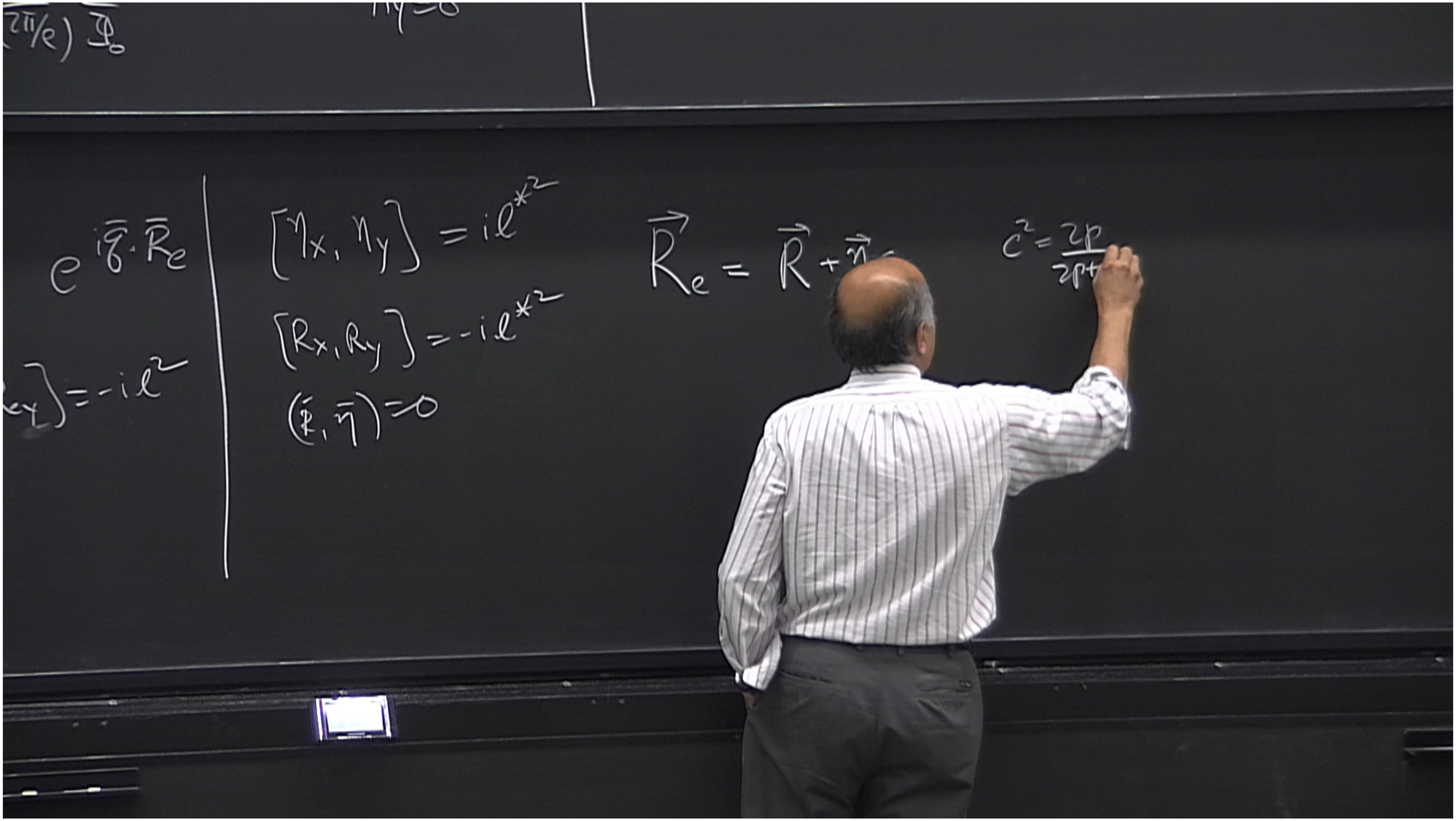
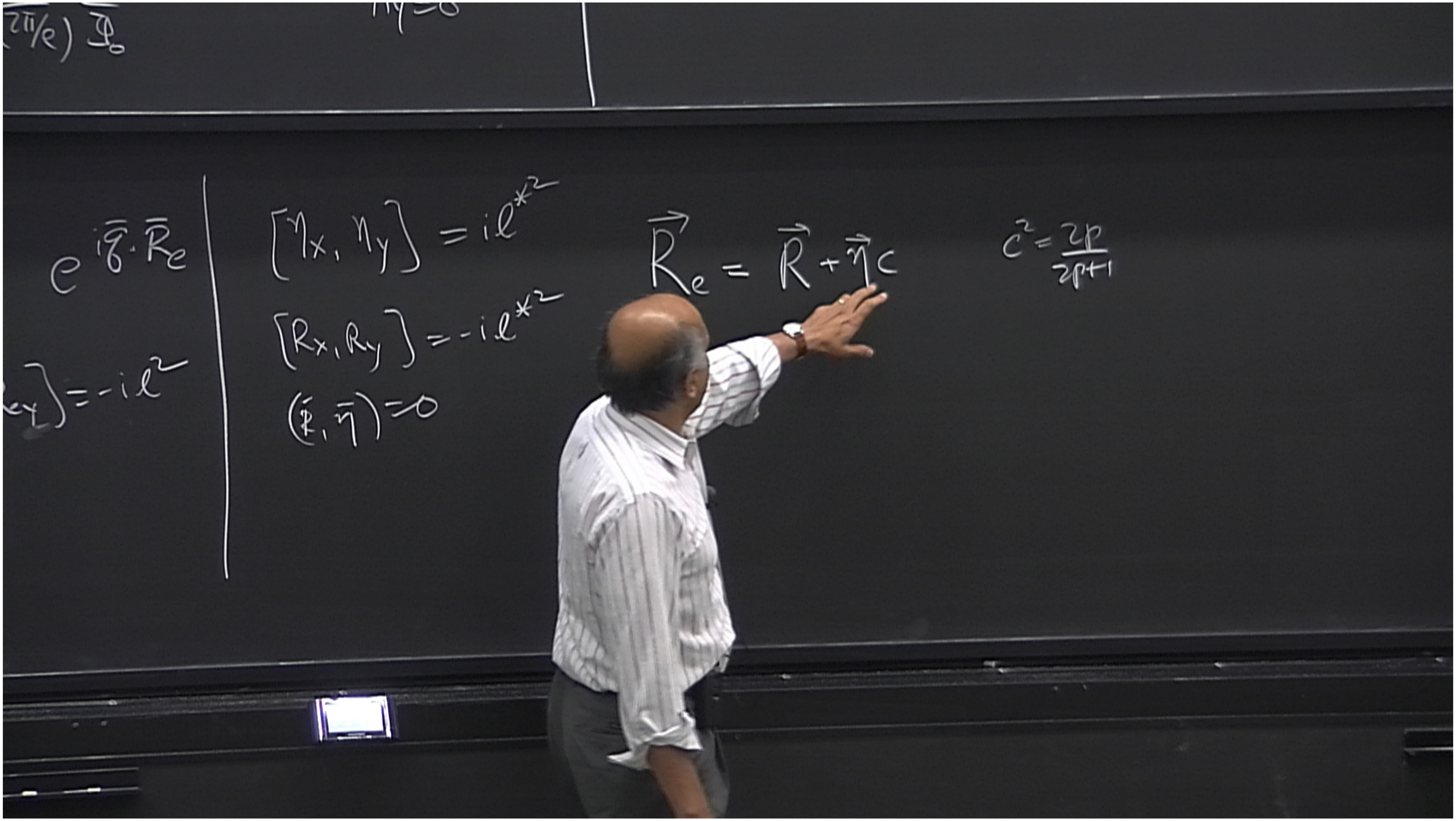
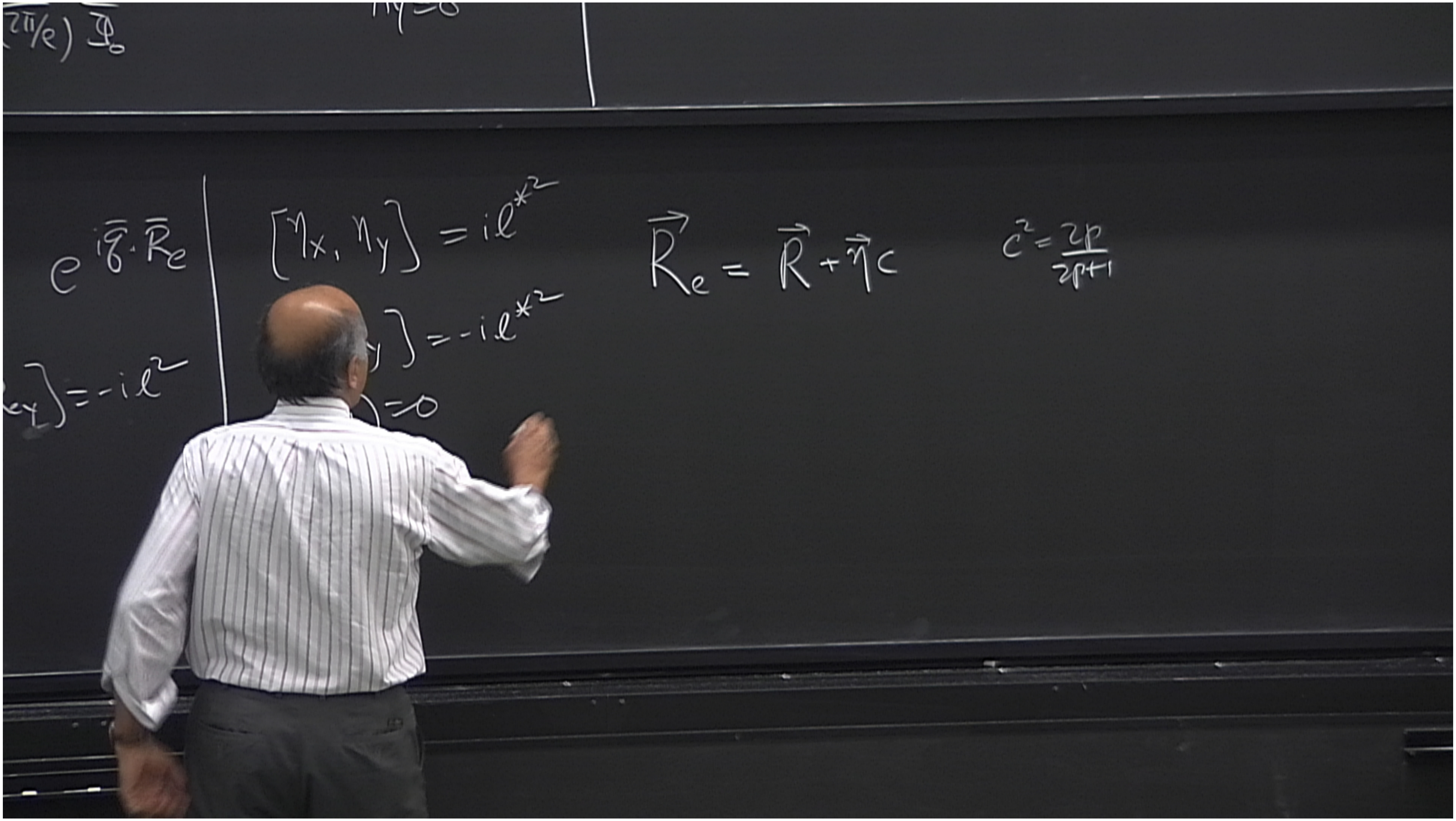
Ganpathy Murthy

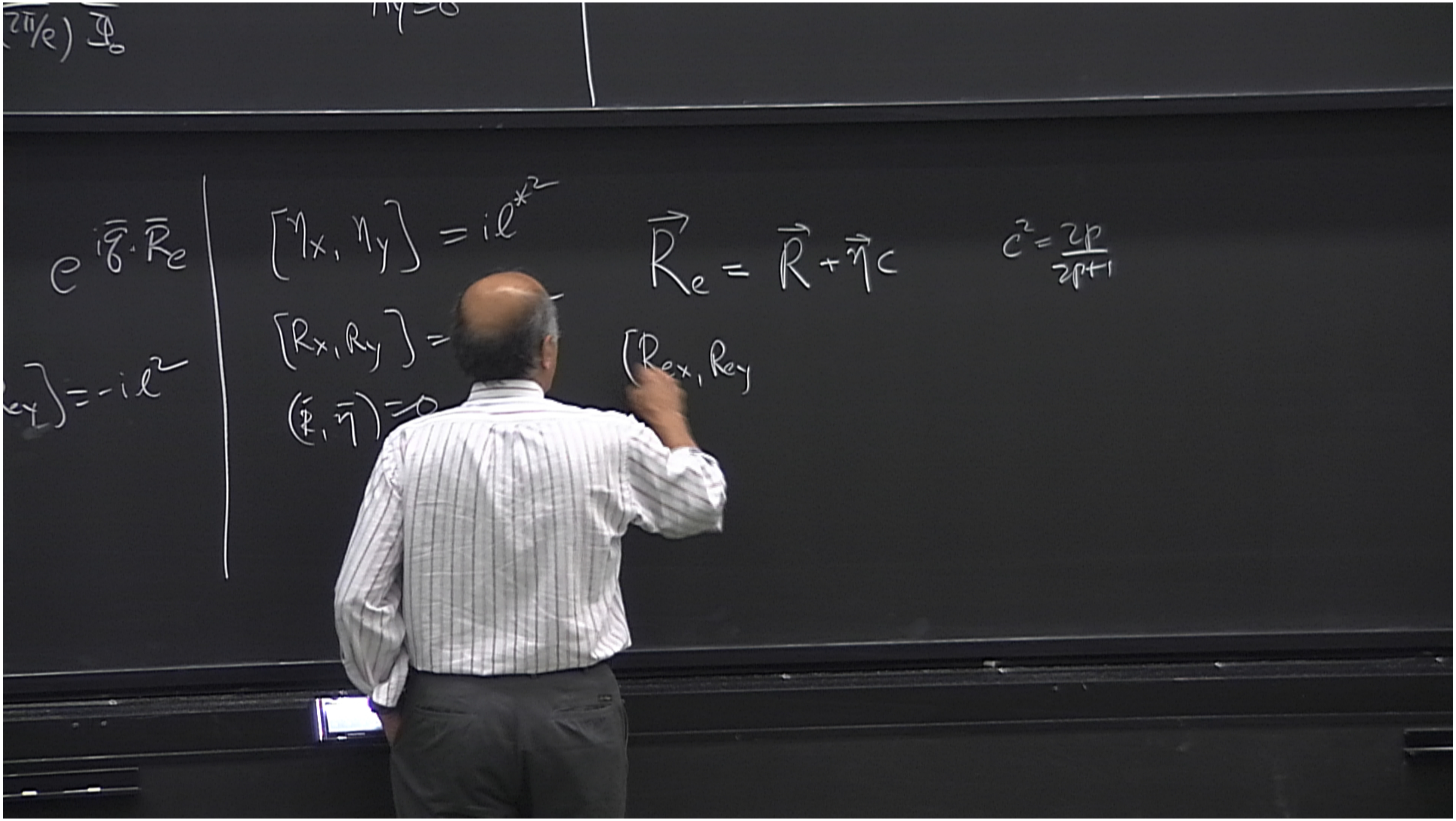


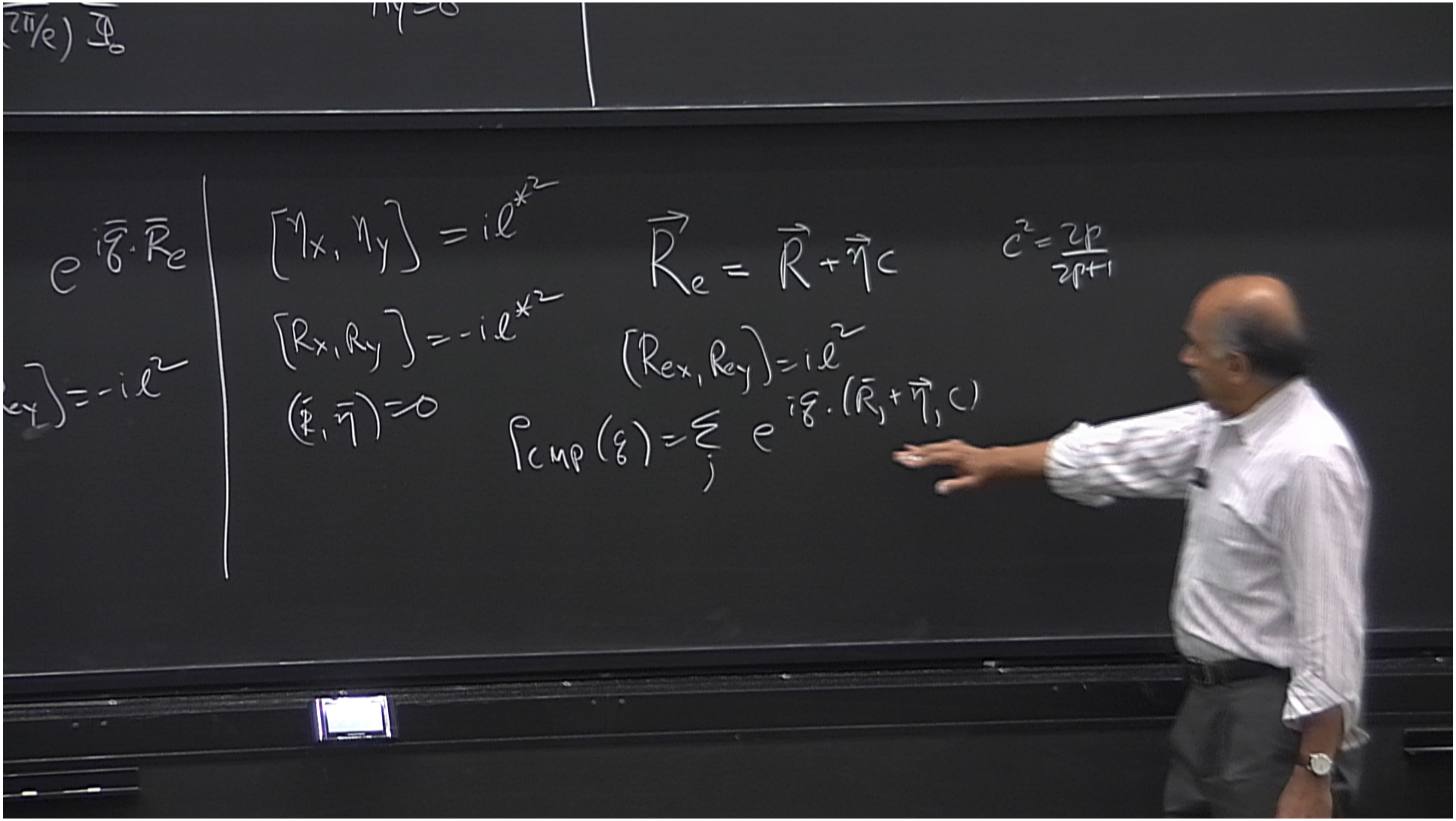
$e^* = \frac{e}{2p+1}$
OCF $\ell^{*2} = (2p+1)\ell^2$
Ganpathy Murthy
ρ_{CMP} $e^{i\bar{q}\cdot\bar{R}_e}$
$[R_{ex}, R_{ey}] = -i\ell^2$


[ηx, ηy] = iℓ*²
[Rx, Ry] = -iℓ*²
(R, η) = 0
Re = R + η


$e^{i\bar{q}\cdot\bar{R}_e}$
$[\eta_x, \eta_y] = i\ell^{*2}$
$[R_x, R_y] = -i\ell^{*2}$
$(\bar{R}, \bar{\eta}) = 0$
$\vec{R}_e = \vec{R} + \vec{\eta}\, c$
$c^2 = \frac{2p}{2p+1}$
$= -i\ell^2$


$(2\pi/e)\,\Phi_0$
$e^{i\bar{q}\cdot\bar{R}_e}$
$[\eta_x, \eta_y] = i\ell^{*2}$
$\vec{R}_e = \vec{R} + \vec{\eta} c$
$c^2 = \frac{2p}{2p+1}$
$] = -i\ell^{*2}$
$] = -i\ell^2$
$) = 0$


[ηx, ηy] = iℓ*²
Re = R + ηc
c² = 2p/(2p+1)
[Rx, Ry] =
[Rex, Rey
]= -iℓ²


$[\eta_x, \eta_y] = i\ell^{*2}$
$[R_x, R_y] = -i\ell^{*2}$
$(\bar{R}, \bar{\eta}) = 0$
$\vec{R}_e = \vec{R} + \vec{\eta}c$
$c^2 = \frac{2p}{2p+1}$
$[R_{ex}, R_{ey}] = i\ell^2$
$\rho_{CMP}(q) = \sum_j e^{iq\cdot(\bar{R}_j + \vec{\eta}_j c)}$
$e^{i\bar{q}\cdot\bar{R}_e}$

$(2\pi/e)\,\Phi_0$

$e^{i\bar{q}\cdot\bar{R}_e}$

$[\,R_{ex}, R_{ey}] = -i\ell^2$

$[\eta_x, \eta_y] = i\ell^{*2}$

$[R_x, R_y] = -i\ell^{*2}$

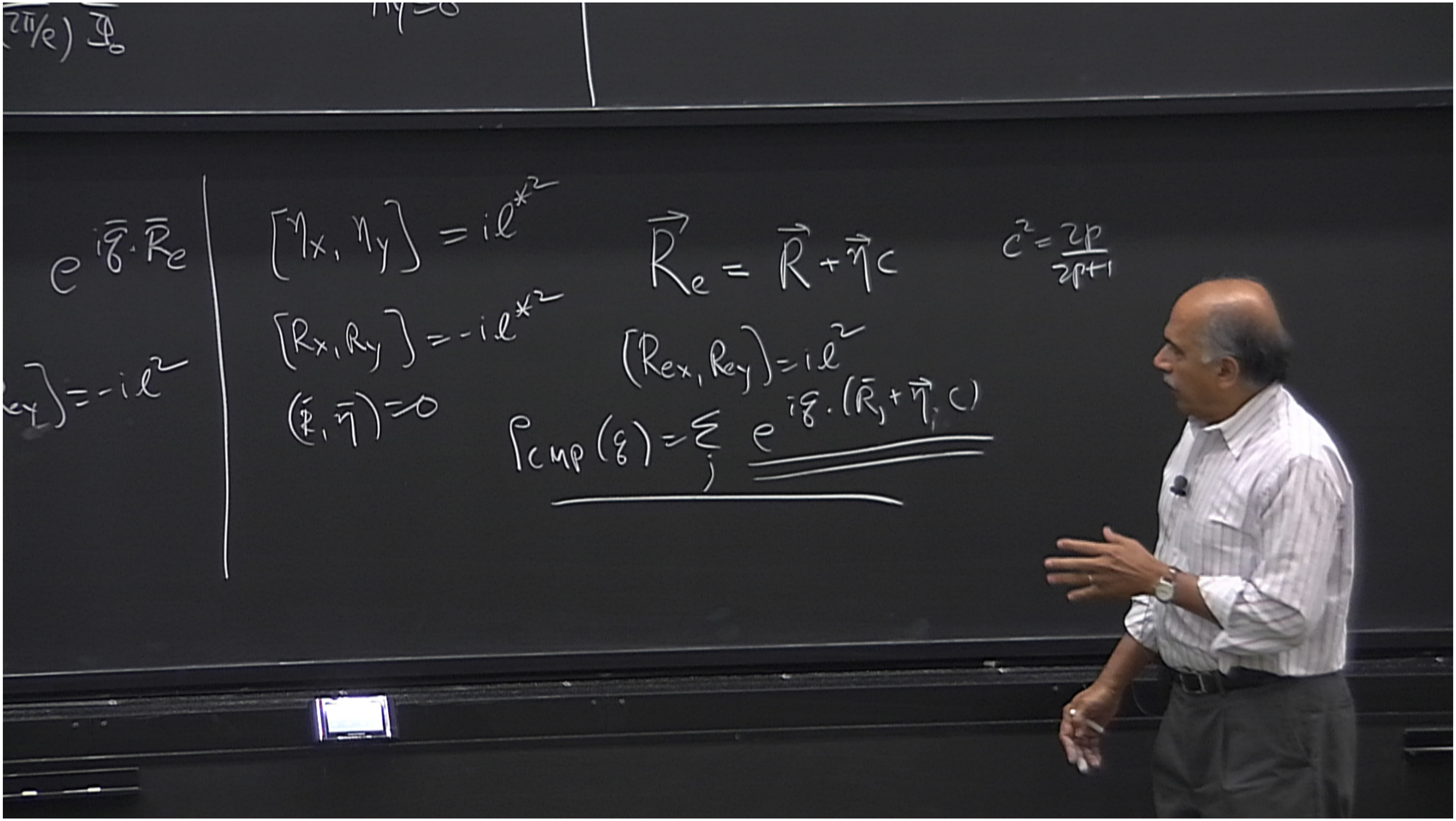
$(\bar{R}, \bar{\eta}) = 0$

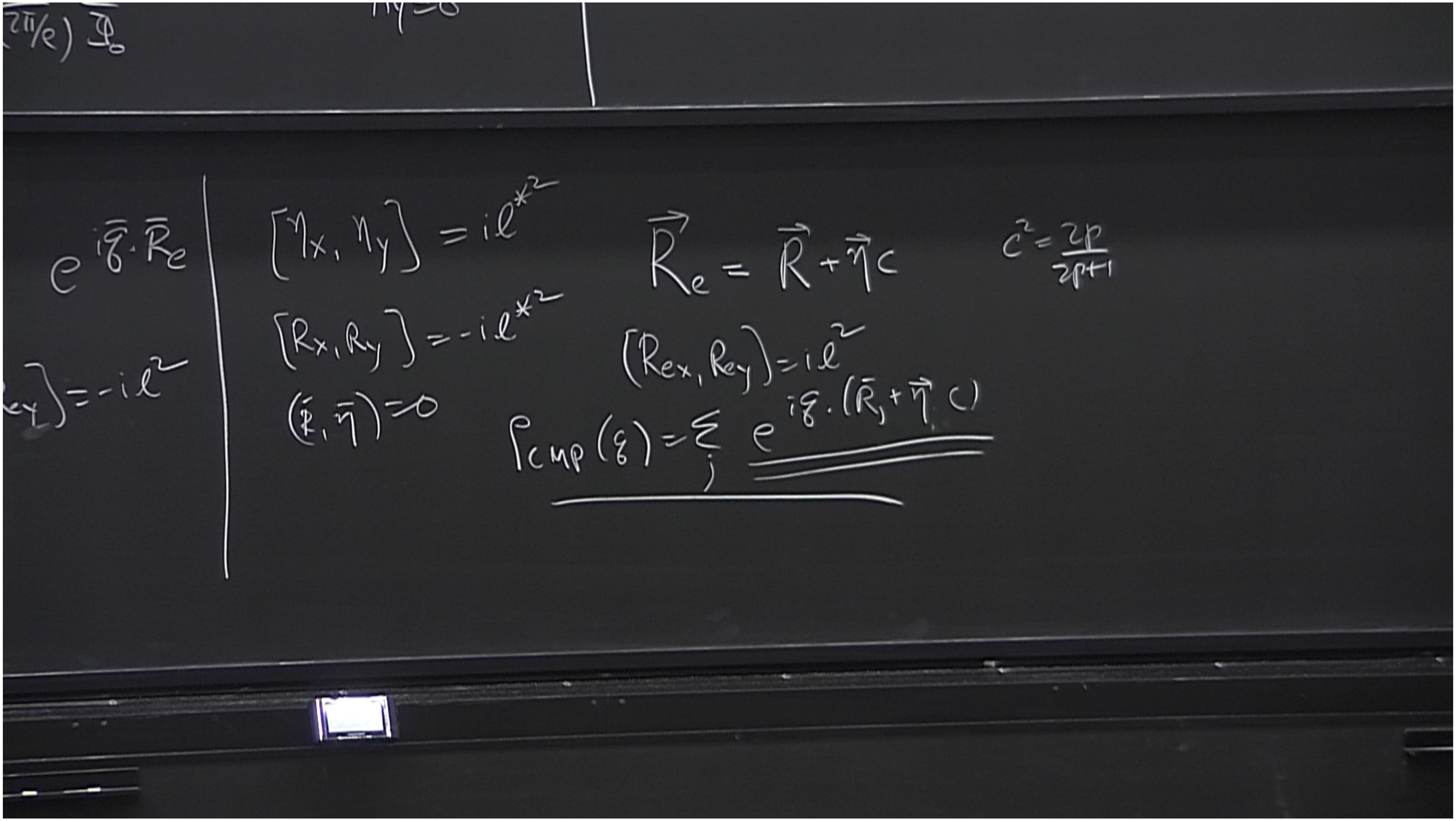
$\vec{R}_e = \vec{R} + \vec{\eta}c$

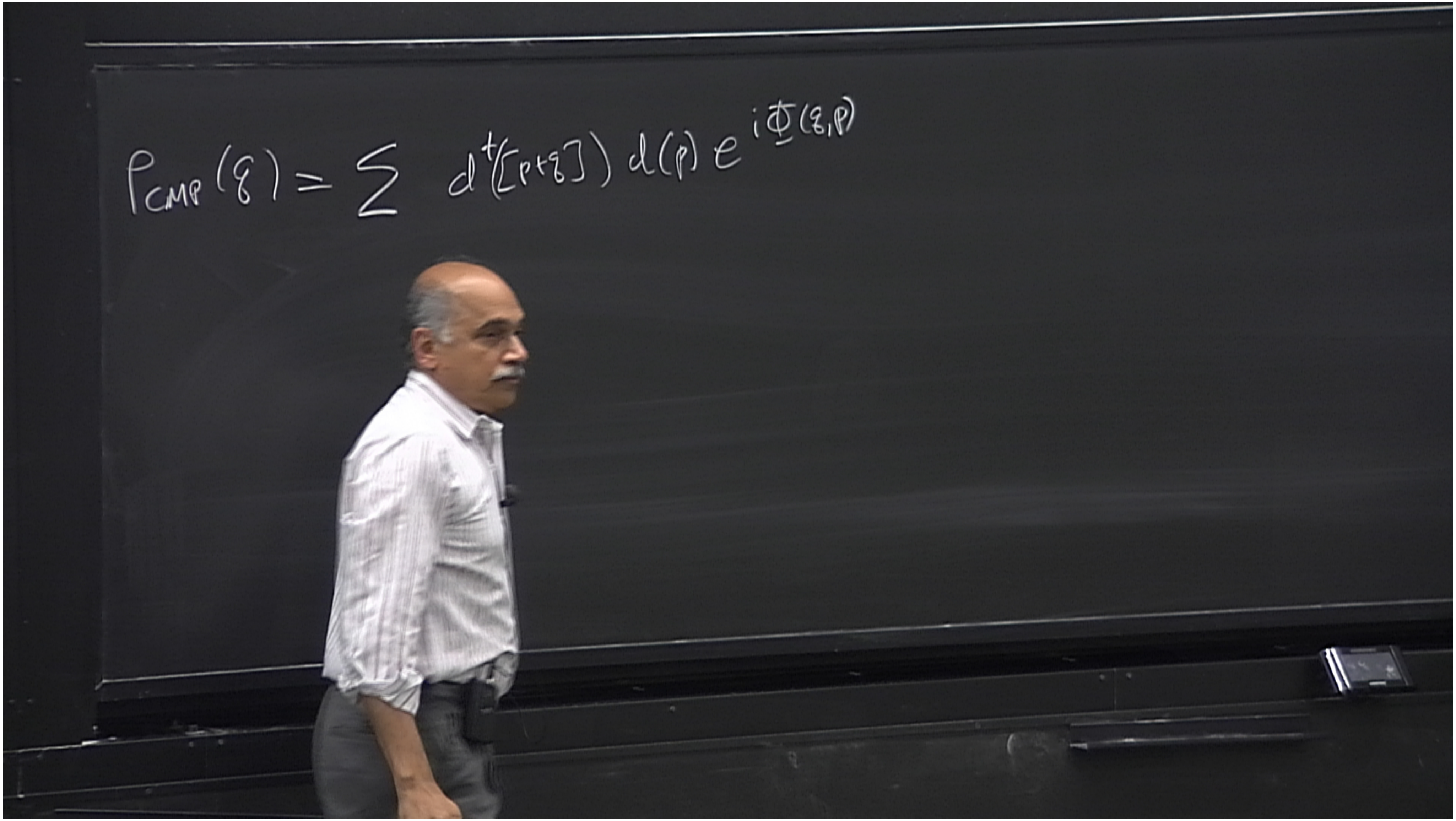
$c^2 = \frac{2p}{2p+1}$

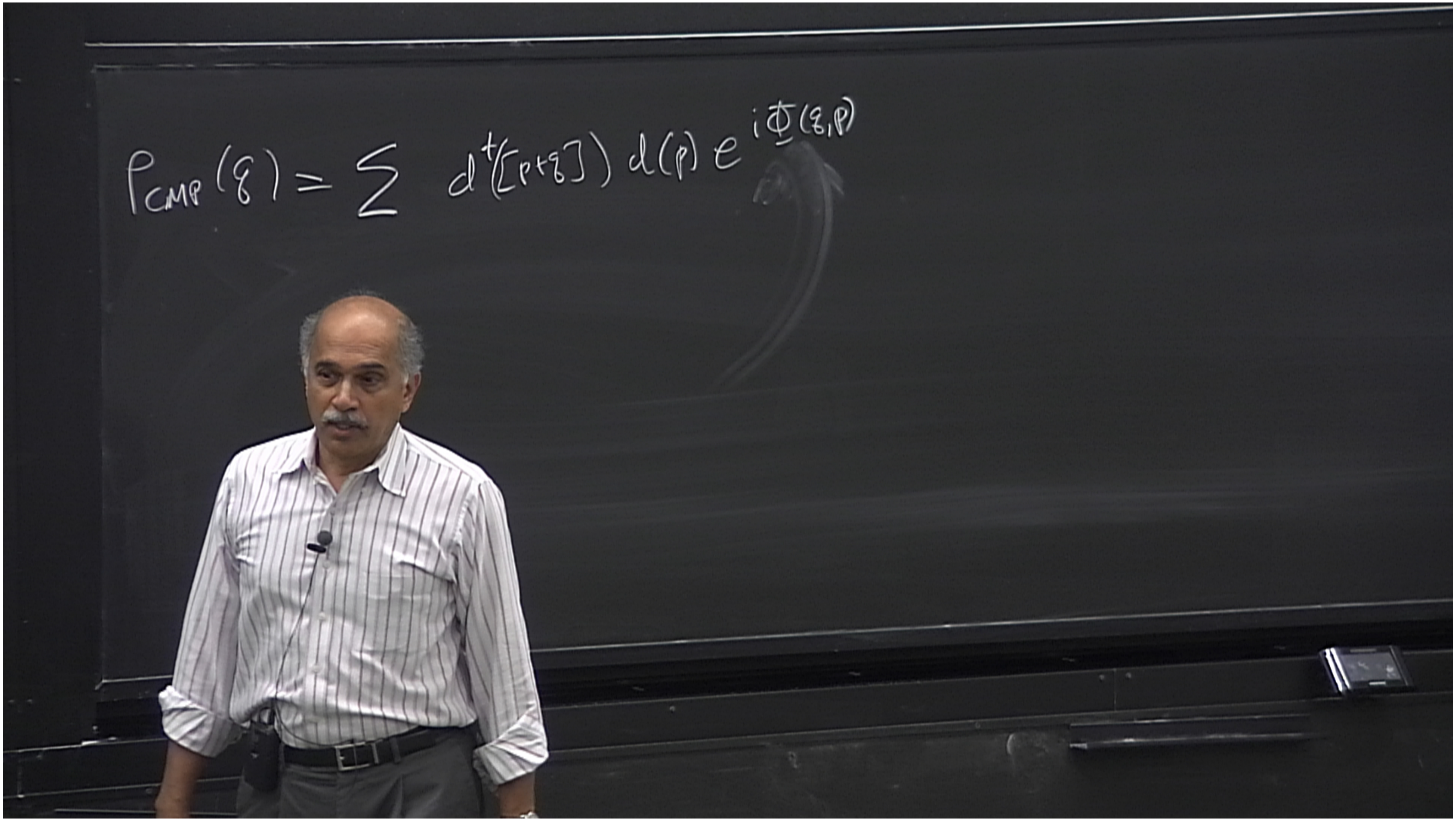
$[R_{ex}, R_{ey}] = i\ell^2$

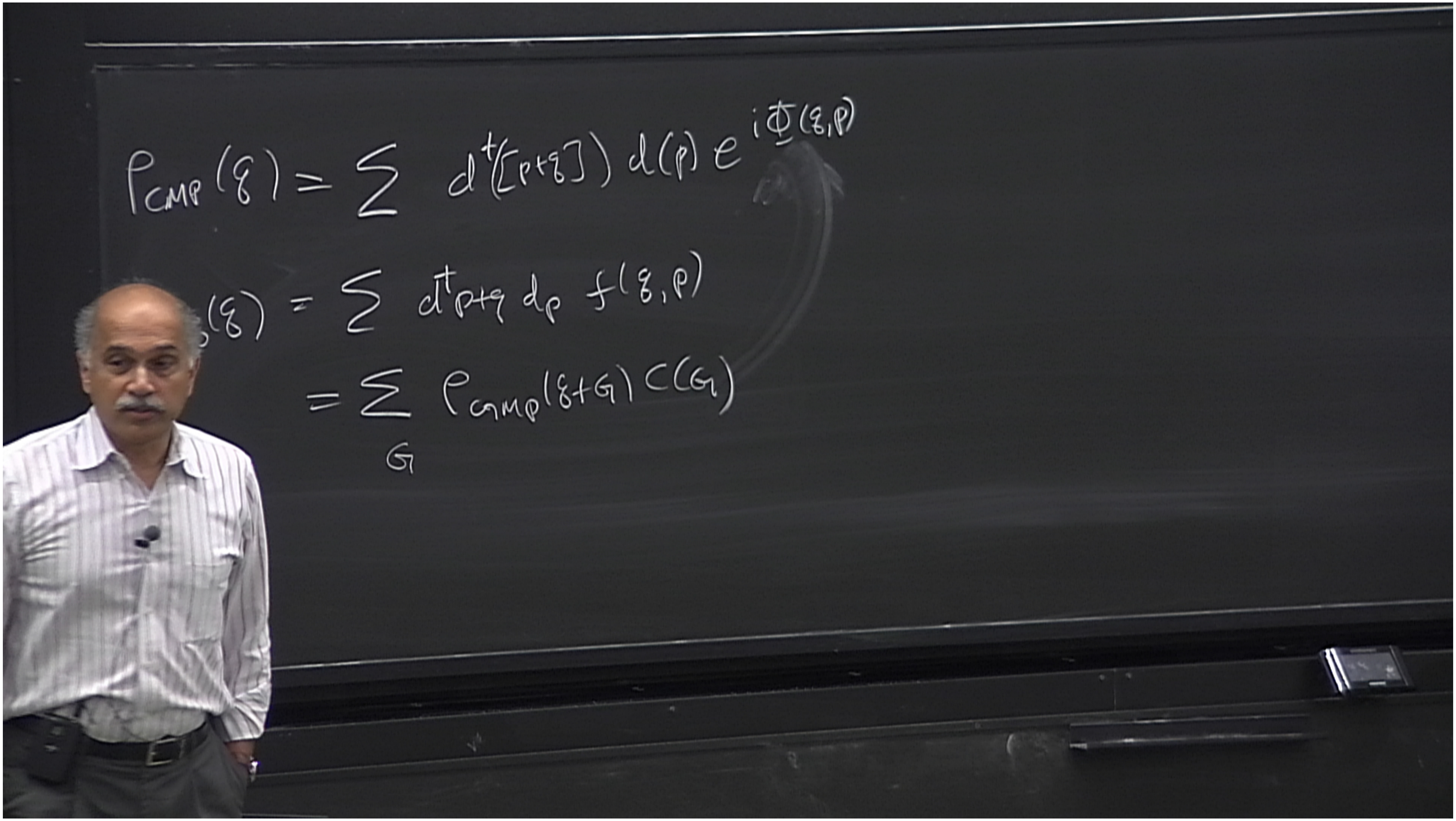
$\rho_{cmp}(q) = \sum_j e^{i\bar{q}\cdot(\bar{R}_j + \vec{\eta}_j c)}$

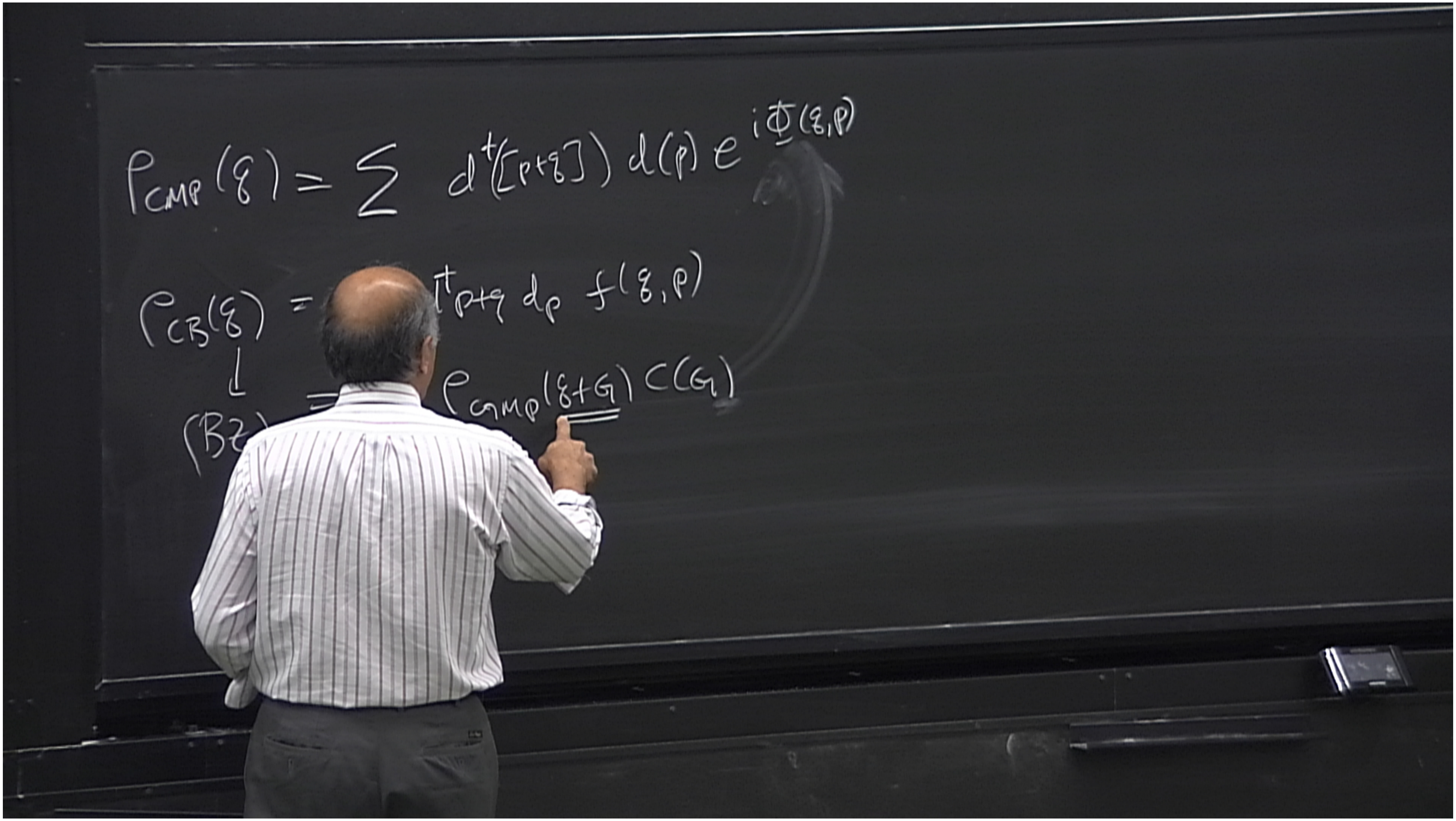

e^{iq·R_e}
Rey] = -i l^2
[ηx, ηy] = i l*^2
[Rx, Ry] = -i l*^2
(R, η) = 0
R_e = R + η c
c^2 = 2p/(2p+1)
[Rex, Rey] = i l^2
ρcmp(q) = Σ_j e^{iq·(R_j + η_j c)}

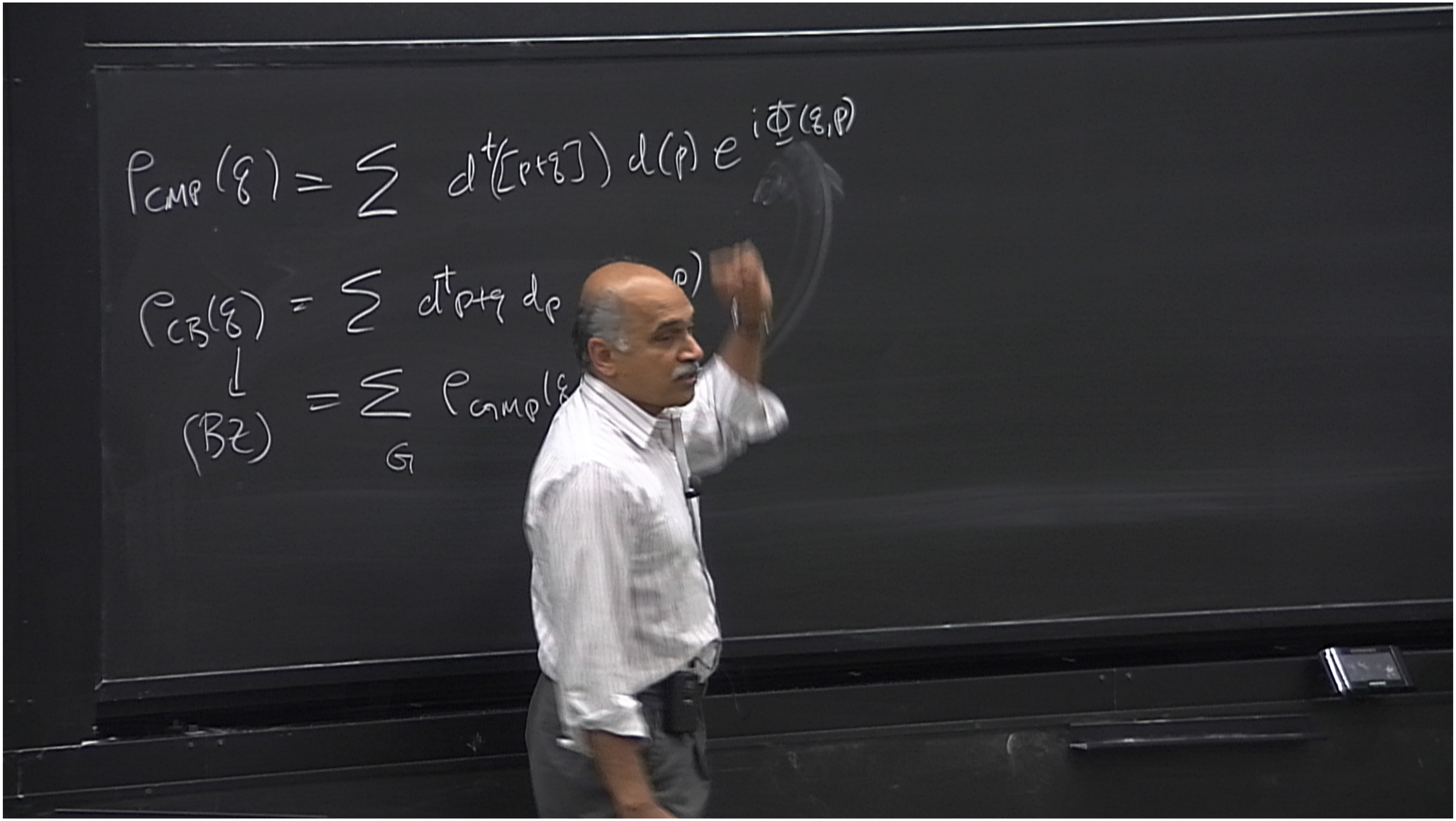

$(2\pi/e)\,\Phi_0$
$e^{i\bar{q}\cdot\bar{R}_e}$
$[\eta_x, \eta_y] = i\ell^{*2}$
$[R_x, R_y] = -i\ell^{*2}$
$(\bar{R}, \bar{\eta}) = 0$
$\vec{R}_e = \vec{R} + \vec{\eta}\, c$
$c^2 = \frac{2p}{2p+1}$
$[R_{ex}, R_{ey}] = i\ell^2$
$\rho_{CMP}(q) = \sum_j e^{i q\cdot(\bar{R}_j + \vec{\eta}\, c)}$
$e_y] = -i\ell^2$


$\rho_{CMP}(q) = \sum d^{\dagger}([p+q])\, d(p)\, e^{i\Phi(q,p)}$


$\rho_{CMP}(q) = \sum d^{\dagger}([p+q])\, d(p)\, e^{i\Phi(q,p)}$


$\rho_{CMP}(q) = \sum d^\dagger([p+q])\, d(p)\, e^{i\Phi(q,p)}$
$(q) = \sum d^\dagger_{p+q}\, d_p\, f(q,p)$
$= \sum_G \rho_{GMP}(q+G)\, C(G)$


ρCMP(q) = Σ d†([p+q]) d(p) e^{iΦ(q,p)}
ρCB(q) = d†p+q dp f(q,p)
(BZ)
ρGMP(q+G) C(G)


$\rho_{CMP}(q) = \sum d^{\dagger}([p+q])\, d(p)\, e^{i\Phi(q,p)}$
$\rho_{CB}(q) = \sum d^{\dagger}_{p+q}\, d_p$
(BZ)
$= \sum_{G} \rho_{GMP}$

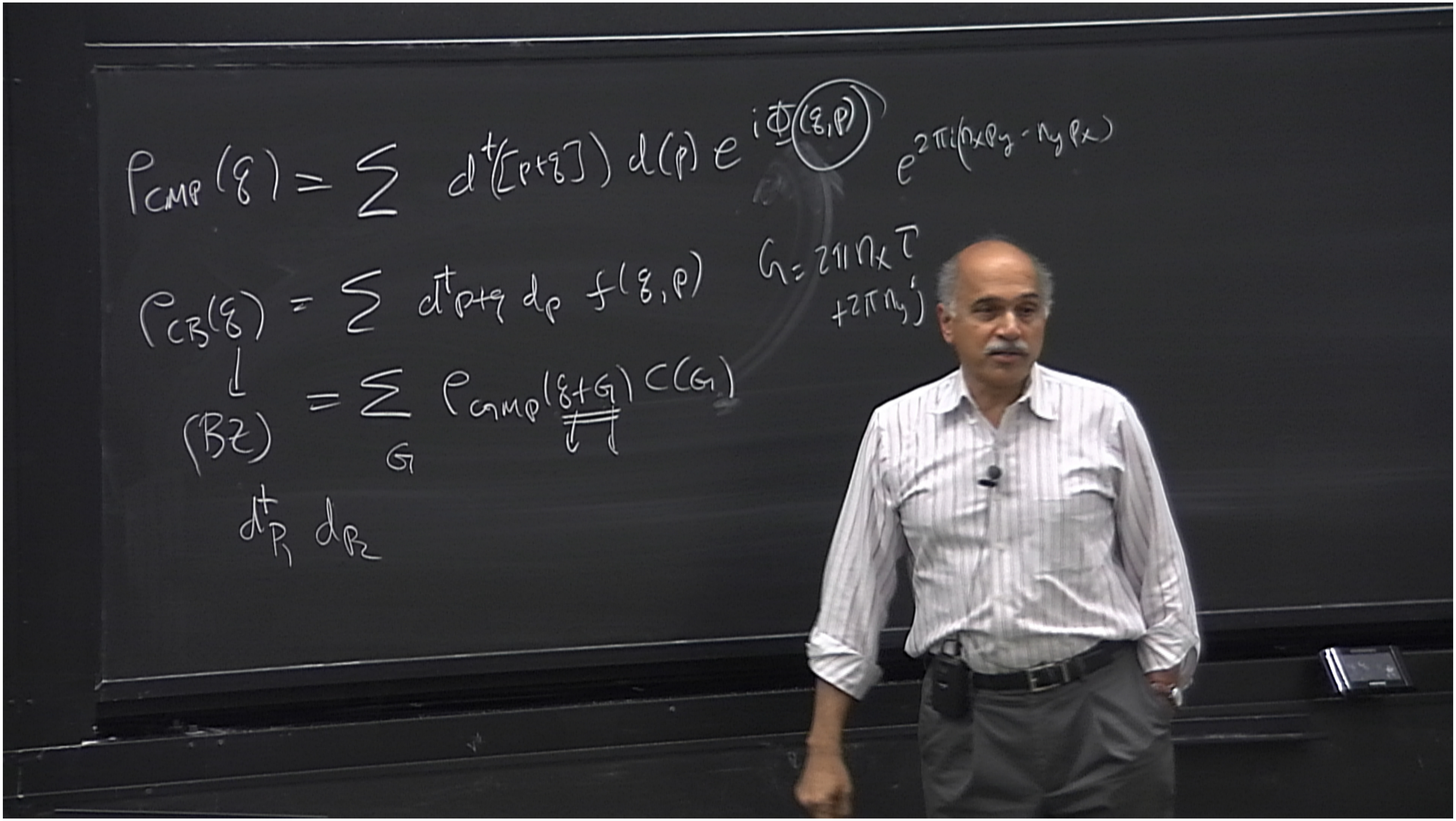
$$\rho_{CMP}(q) = \sum d^\dagger([p+q])\, d(p)\, e^{i\Phi(q,p)}$$

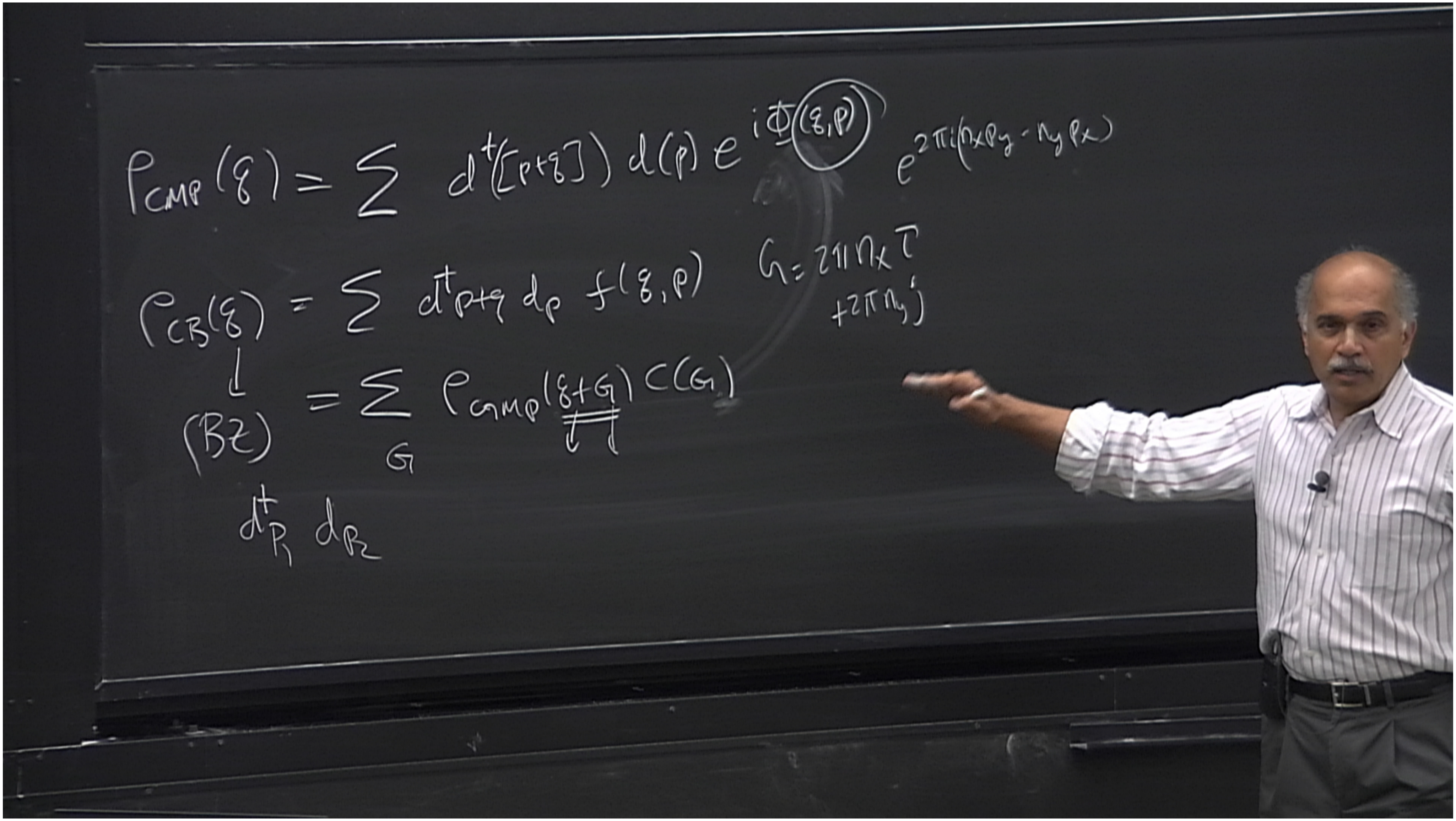
$$\rho_{CB}(q) = \sum d^\dagger_{p+q}\, d_p\, f(q,p)$$

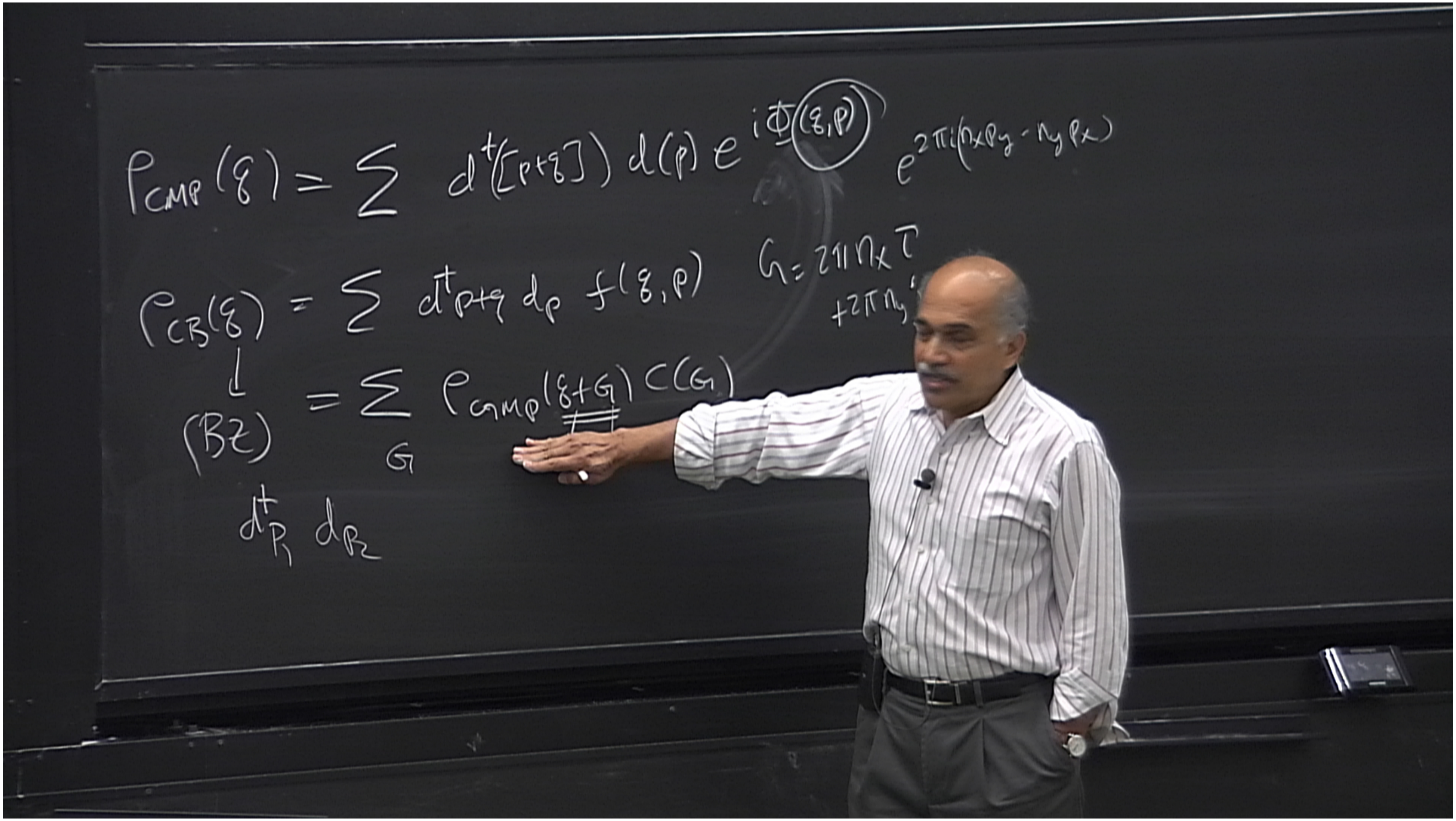
$$\downarrow$$

$$(BZ) \quad = \sum_G \rho_{GMP}(q+G)\, C(G)$$

$$d^\dagger_{p_1}\, d_{p_2}$$


$\rho_{CMP}(q) = \sum d^\dagger([p+q])\, d(p)\, e^{i\Phi(q,p)}$
$e^{2\pi i(n_x p_y - n_y p_x)}$
$\rho_{CB}(q) = \sum d^\dagger_{p+q}\, d_p\, f(q,p)$
$G = 2\pi n_x \tau + 2\pi n_y j$
$= \sum_G \rho_{CMP}(q+G)\, C(G)$
(BZ)
$d^\dagger_{p_1}\, d_{p_2}$


ρ_CMP(q) = Σ d†([p+q]) d(p) e^{iΦ(q,p)}
e^{2πi(n_x p_y − n_y p_x)}
ρ_CB(q) = Σ d†_{p+q} d_p f(q,p)
G = 2π n_x î + 2π n_y ĵ
(BZ)
= Σ_G ρ_CMP(q+G) C(G)
d†_{P_1} d_{P_2}


$\rho_{CMP}(q) = \sum d^\dagger([p+q])\, d(p)\, e^{i\Phi(q,p)}$
$e^{2\pi i(n_x p_y - n_y p_x)}$
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$+ 2\pi n_y$
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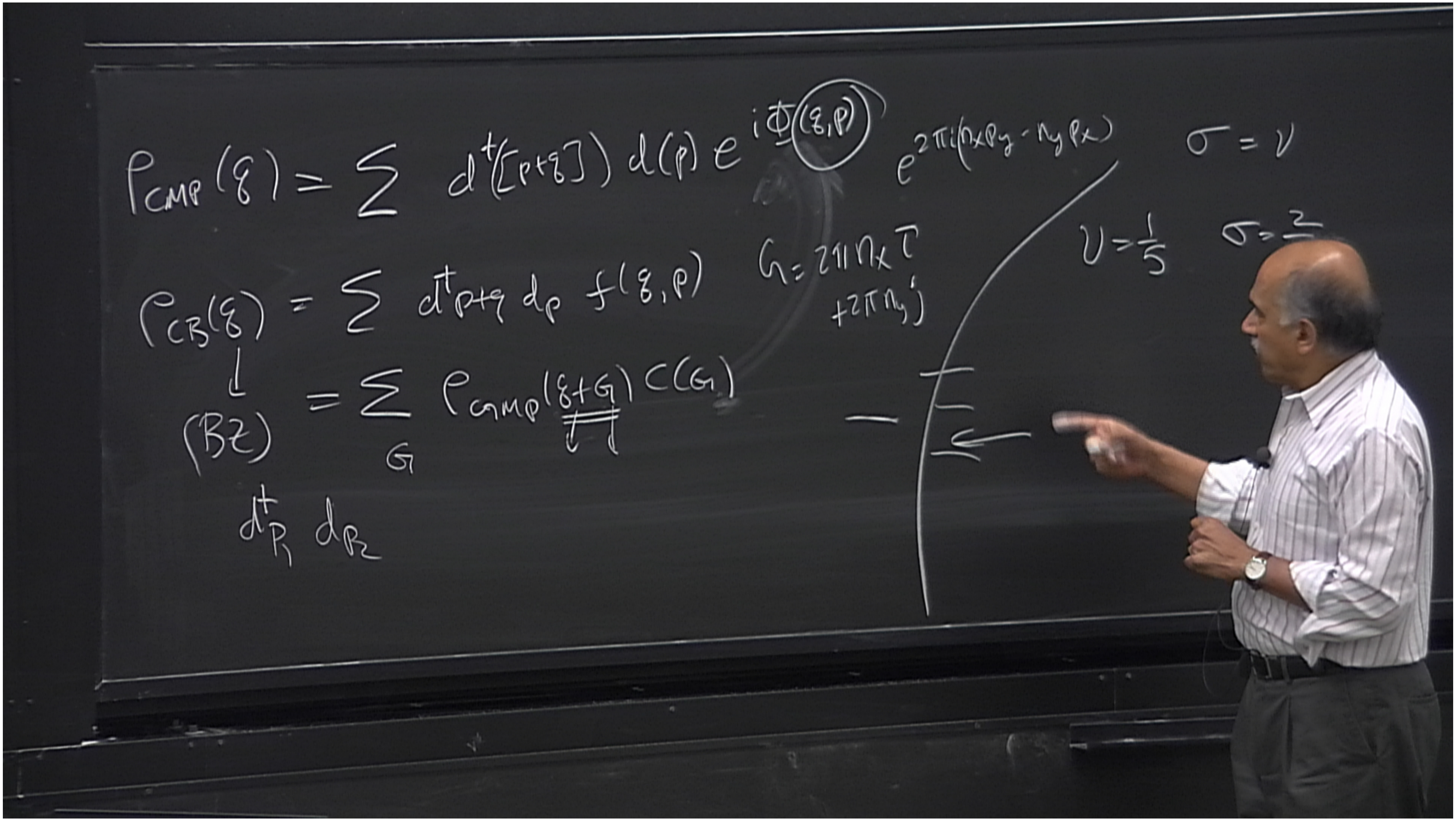
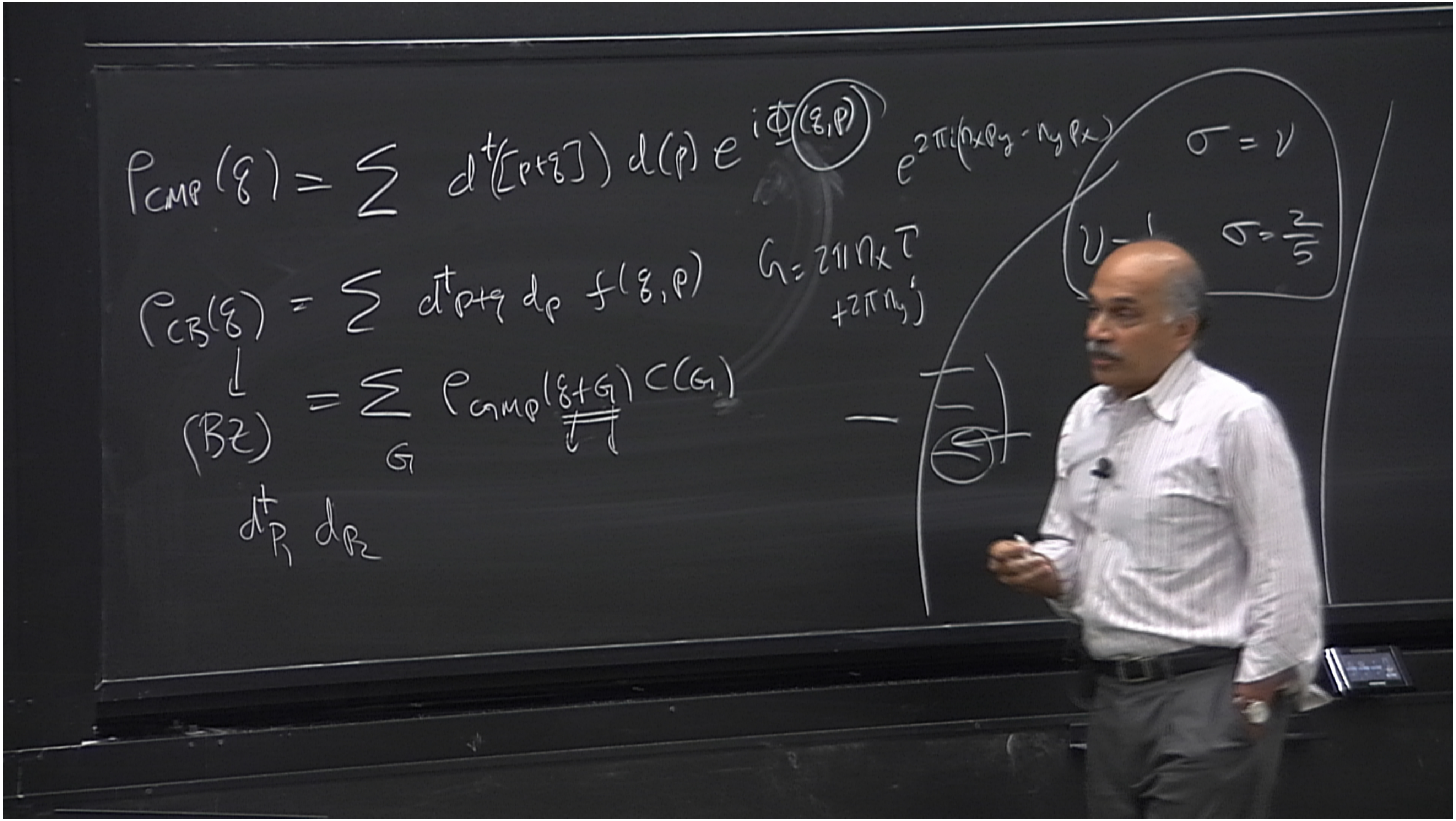
(BZ)
$d^\dagger_{P_1}\, d_{P_2}$

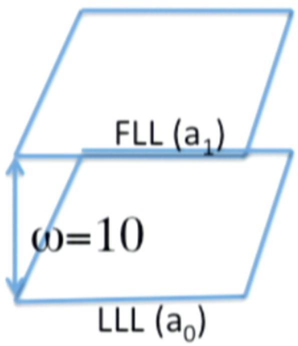

$\rho_{CMP}(q) = \sum d^\dagger([p+q])\, d(p)\, e^{i\Phi(q,p)}$
$e^{2\pi i(n_x p_y - n_y p_x)}$
$\sigma = \nu$
$\nu = \frac{1}{5}$
$\rho_{CB}(q) = \sum d^\dagger_{p+q}\, d_p\, f(q,p)$
$G = 2\pi n_x \hat{i} + 2\pi n_y \hat{j}$
$(BZ) = \sum_G \rho_{GMP}(q+G)\, C(G)$
$d^\dagger_{P_1}\, d_{P_2}$


$\rho_{CMP}(q) = \sum d^{\dagger}([p+q])\, d(p)\, e^{i\Phi(q,p)}$
$e^{2\pi i(n_x p_y - n_y p_x)}$
$\sigma = \nu$
$\sigma = \frac{2}{5}$
$\rho_{CB}(q) = \sum d^{\dagger}_{p+q}\, d_p\, f(q,p)$
$G = 2\pi n_x \tau + 2\pi n_y j$
(BZ)
$= \sum_G \rho_{GMP}(q+G)\, c(G)$
$d^{\dagger}_{P_1}\, d_{P_2}$

Hamiltonian Theory of Fractionally Filled Chern Bands

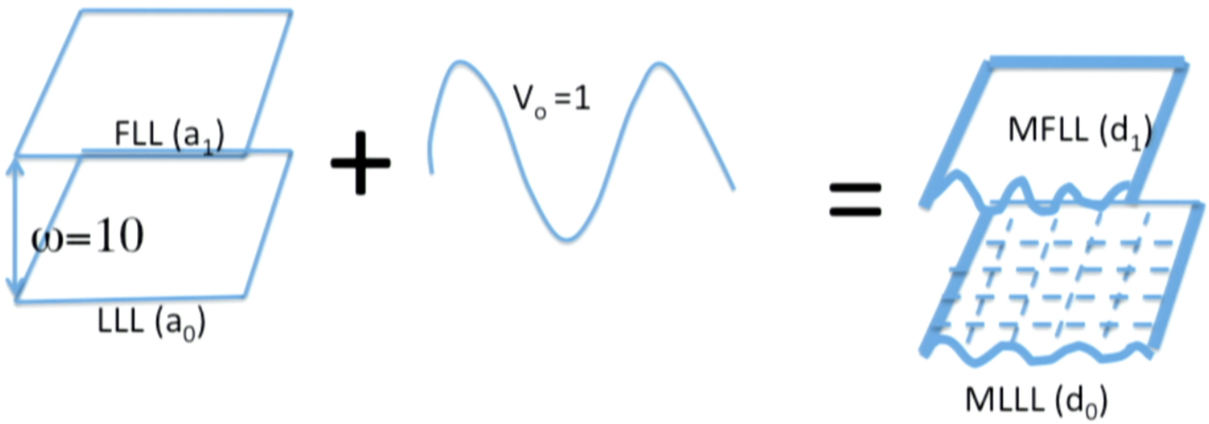
The embedding: Part II

- Start with 2 LL's, Bloch states $a^+(n,k)|0\rangle = |n,k\rangle$, $n=0,1$
- Both bands have uniform $B(k)=B$ and flat $E(k)=E$



The embedding: Part II

- Start with 2 LL's, Bloch states $a^{+}(n,k)|0\rangle = |n,k\rangle$, n=0,1
- Both bands have uniform B (k)=B and flat E(k)=E
- V_0 mixes LL's and turns B and E into B(k) and E(k)



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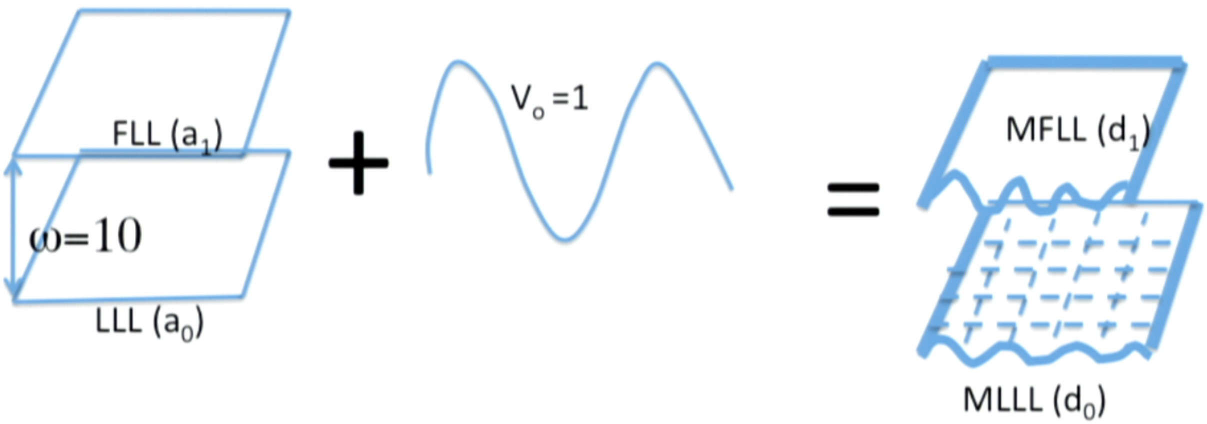

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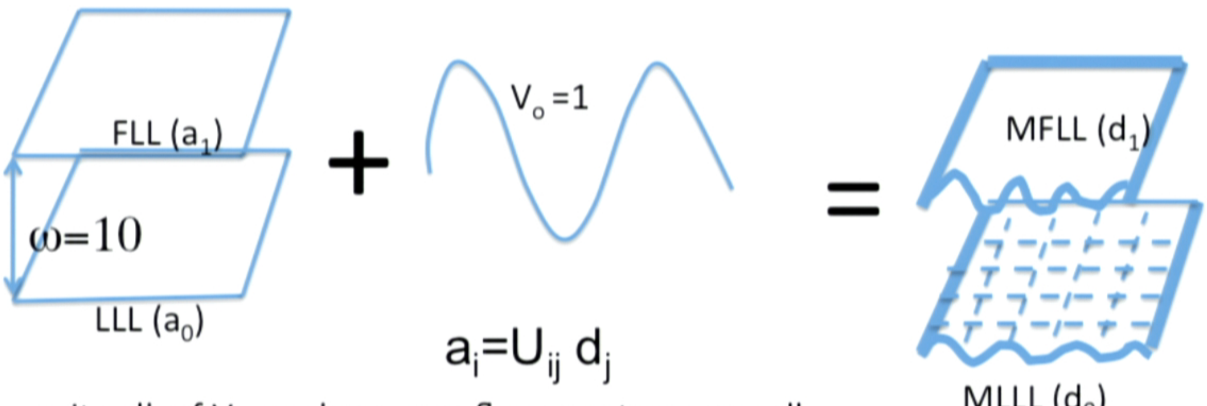
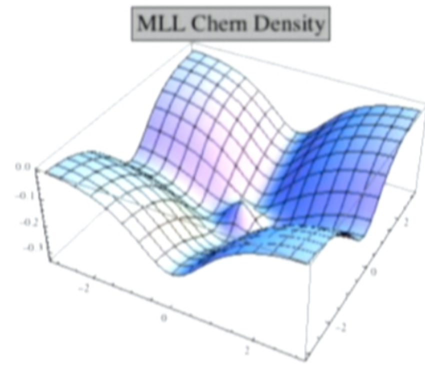


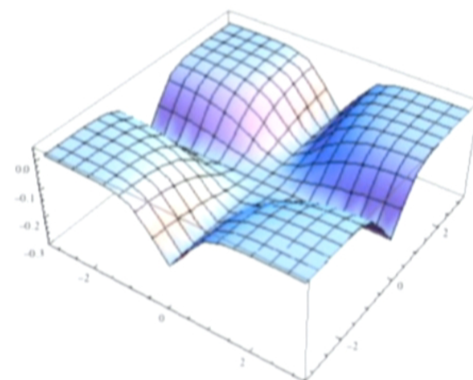
The unit cell of V_0 encloses one flux quantum per cell

MLL is our target band

$$h(k)=h_0(k)\,I+[-(\omega/2)+(.5\ \pi^{1/2})\,V_0\,(\mathrm{Cos}\,k_x+\mathrm{Cos}\,k_y)]\,\sigma_3\ -V_0\,(\mathrm{Sin}\,k_x)\,\sigma_2\ -V_0\,(\mathrm{Sin}\,k_y)\,\sigma_1$$

Comparison of Berry MLLL vs LDM





Next Step in the Program

- We have the band with the non-constant B (k)
- We need the electron density in this band
- We need to express that in terms of densities that obey GMP
- We can do the CF substitution in ρ_{GMP}, namely

$$\rho_{GMP} = \exp\left[-i\,\mathbf{q}\,(\mathbf{R} + \boldsymbol{\eta}\, c)\right]$$

in first quantization where **R** and $\boldsymbol{\eta}$ are CF guiding center and cyclotron coordinates.

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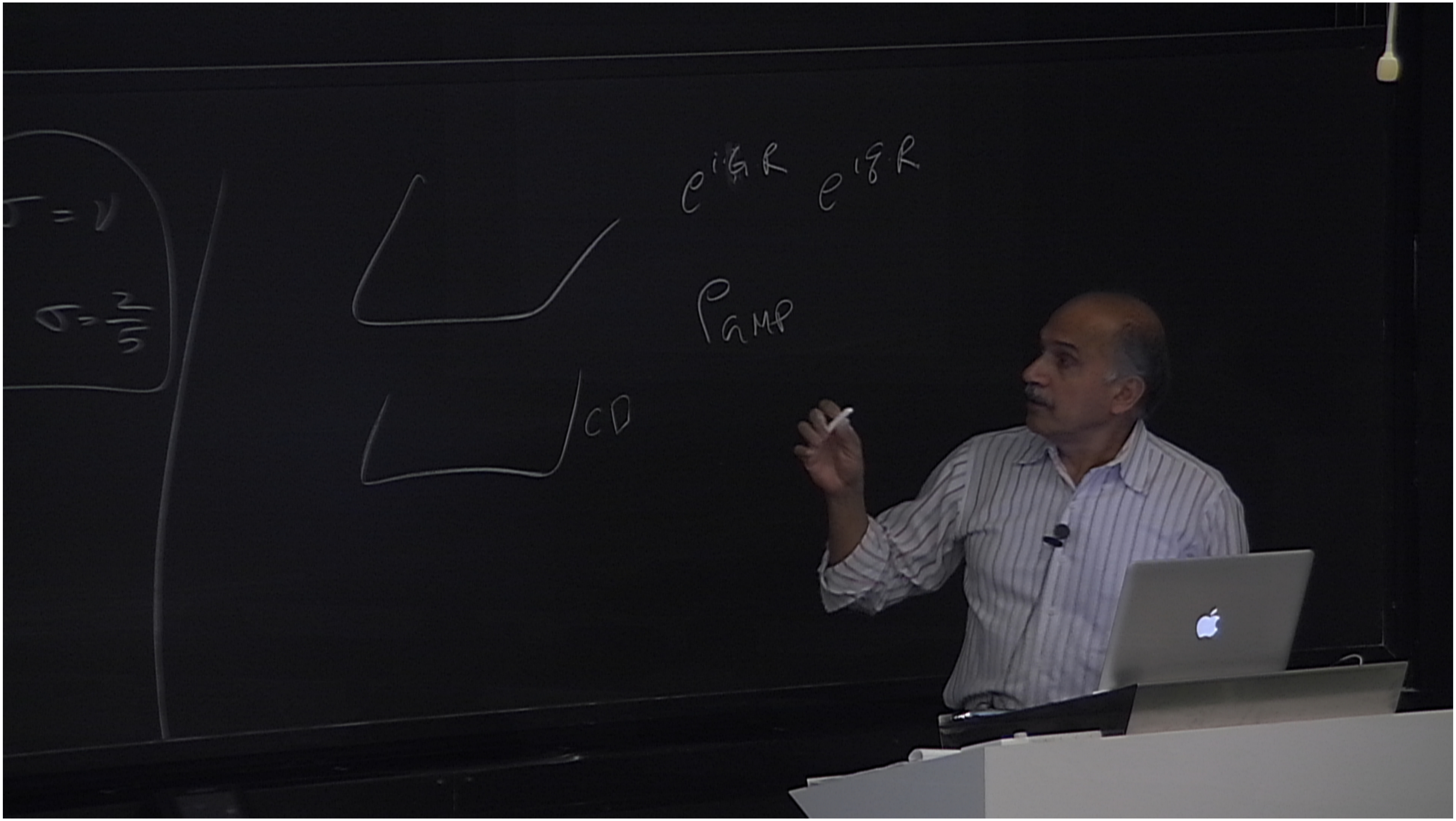
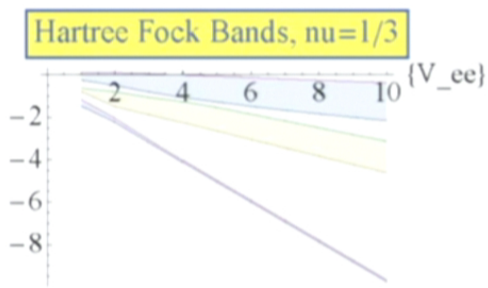
Step 4: Express $\rho_e(q)$ (= $\rho_{GMP}(q)$ at $V_0=0$) in terms of $\rho_{GMP}(q+G)$

- $\rho_e(q) = \Sigma_G \; c(G,q) \; \rho_{GMP}(q+G)$
- We have a closed expression for c(G,q). We can do the CF substitution in $\rho_{GMP}(q+G)$.

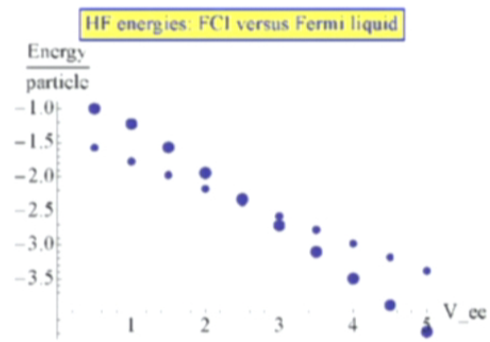


σ = 1
σ = 2/5
CD
e^{iq·R}
e^{iq·R}
ρ_{AMP}

HF for 1/3

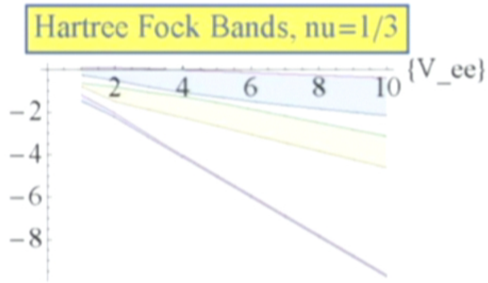


Hartree Fock Bands, nu=1/3
{V_ee}
2
4
6
8
10
−2
−4
−6
−8



HF energies: FCI versus Fermi liquid
Energy
particle
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
1
2
3
4
5
V_ee

HF for 1/3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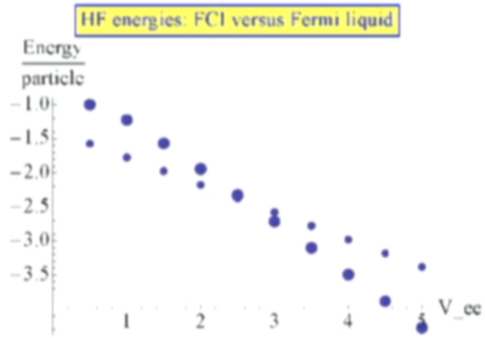
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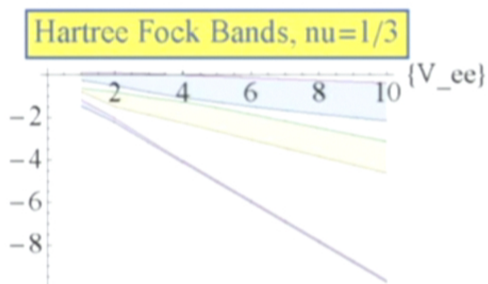

Hartree Fock Bands, nu=1/3
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8
10
−2
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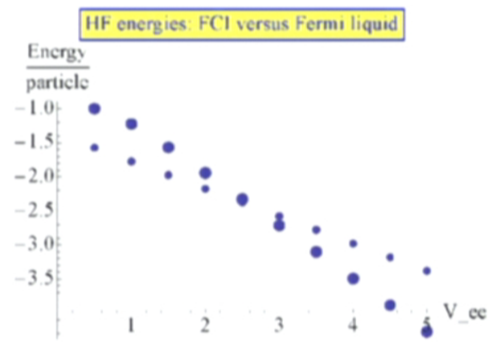
−6
−8



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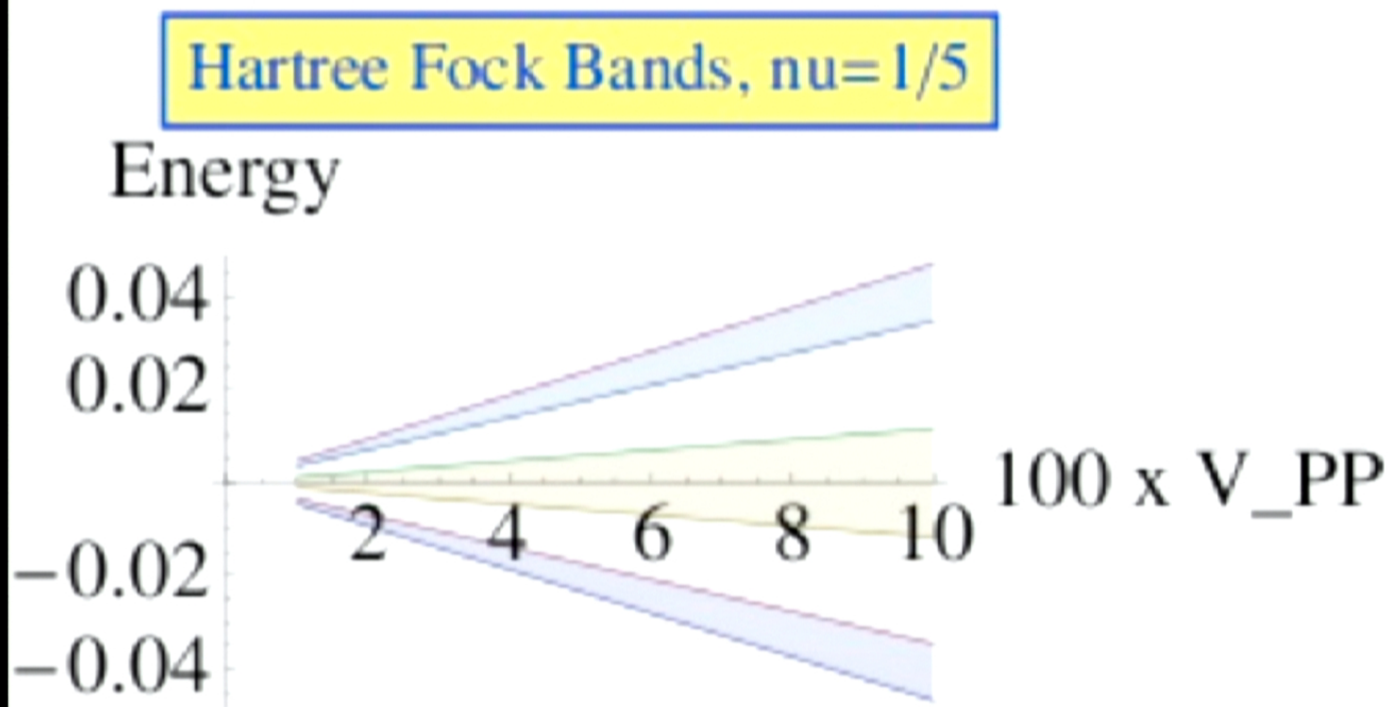
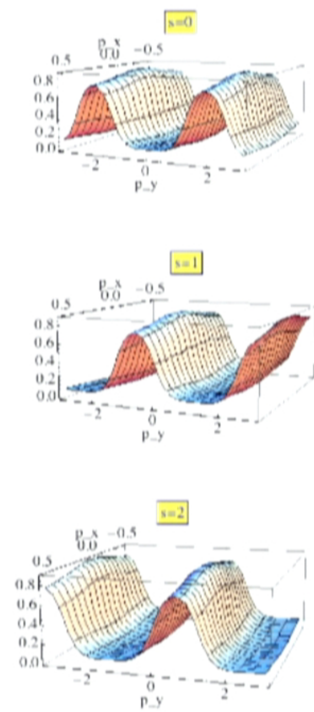
Energy / particle
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
1
2
3
4
5
V_ee

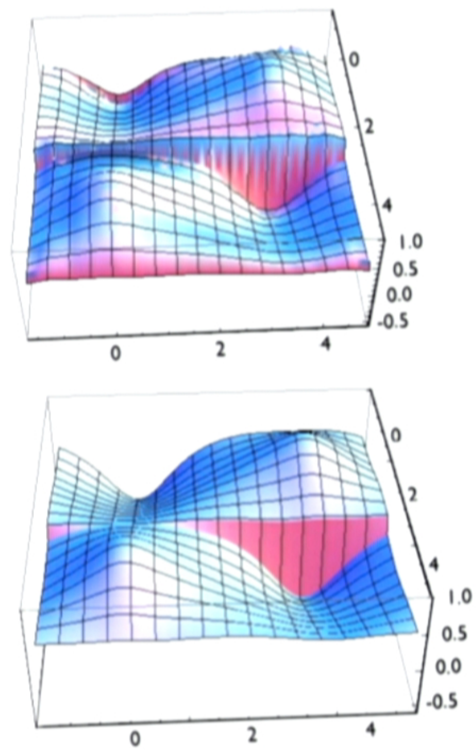
A fraction with $\nu \neq \sigma$



n= 1/5, V_ee=10,V_PP=.02



Comparison of f(q,p) and fit



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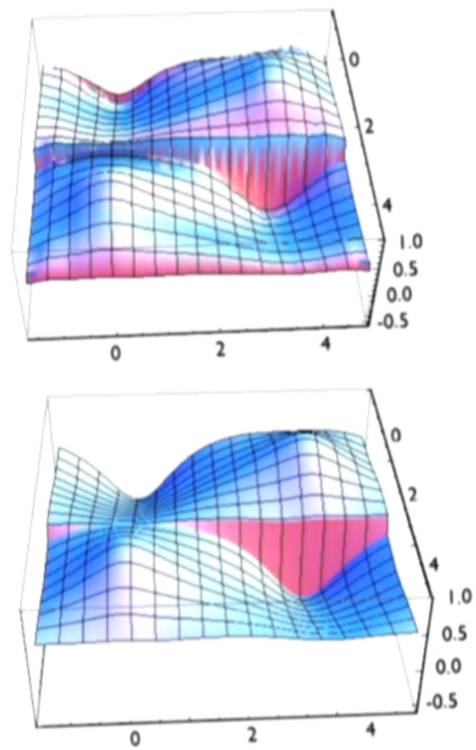


CAN A PARTIALLY FILLED C=0 BAND
SHOW
FQHE

CAN A PARTIALLY FILLED C=0 BAND SHOW FQHE

CAN A PARTIALLY FILLED C=0 BAND SHOW FQHE

Summary

FQHE STORY

$$\rho_{GMP}(q) = \exp i\, q\, R_e$$

$$R_e = R + c\,\eta \qquad c^2 = 2ps/(2ps+1)$$

FRACTIONAL CHERN BANDS

$\neq$

Summary

FQHE STORY

$$\rho_{GMP}(q) = \exp i\, q\, R_e$$

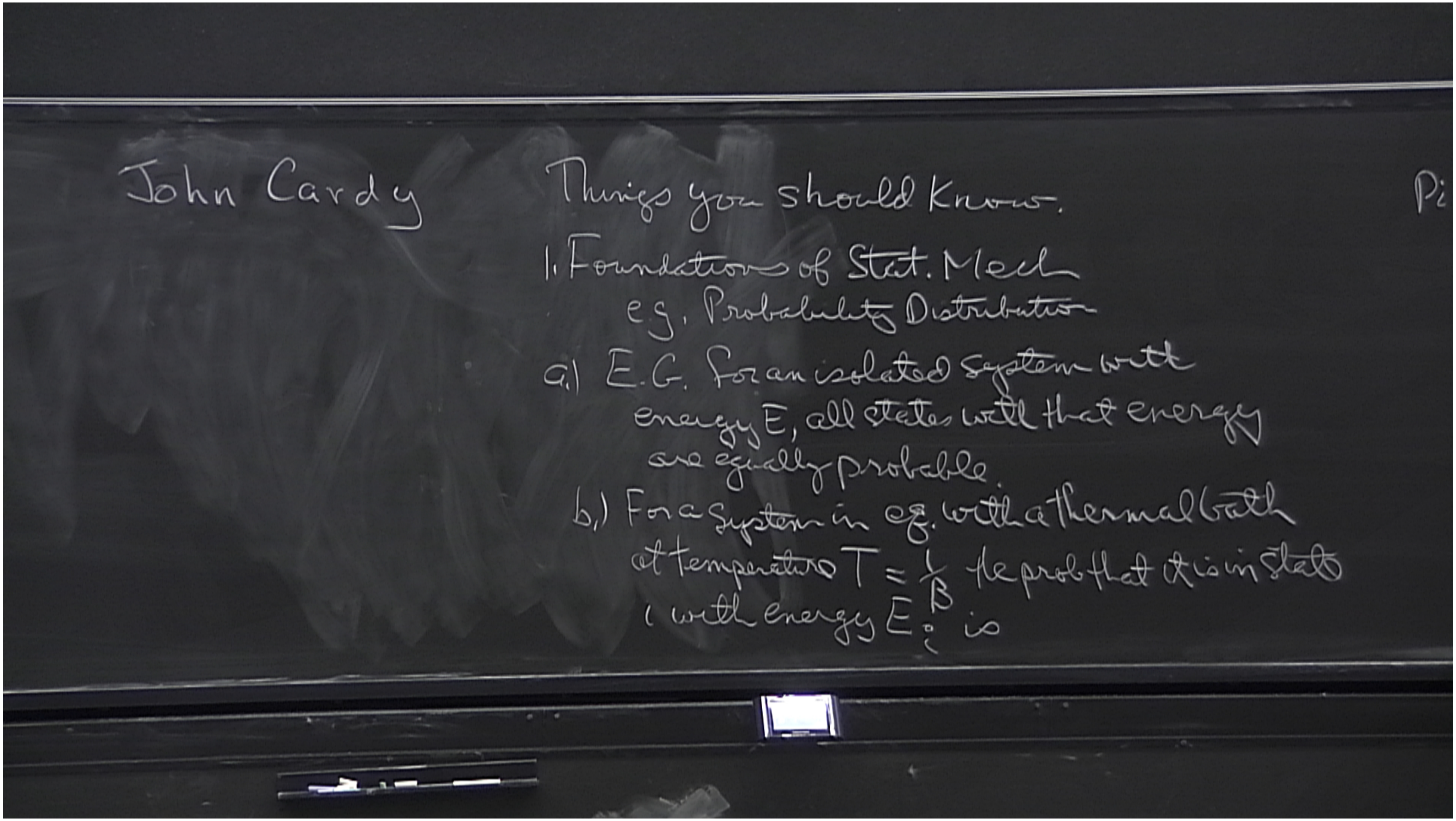
$$R_{e\,=}\ R + c\,\eta \qquad c^2 = 2ps/(2ps+1)$$

FRACTIONAL CHERN BANDS

$$\rho_{CB}\, q) = \Sigma\, d[p+q]d[p]\ f(q,p)$$

$$\rho_{CB}(q) = \Sigma\, c(q,G)\ \rho_{GMP}(q+G)$$

$\neq$


John Cardy
Things you should know.
1. Foundations of Stat. Mech
e.g. Probability Distribution
a.) E.G. for an isolated system with
energy E, all states with that energy
are equally probable.
b.) For a system in eq. with a thermal bath
at temperature T = 1/β the prob that it is in state
i with energy E_i is
Pi