Title: Large-N Collective Fields and de Sitter / CFT correspondence

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Abstract: We derive the collective theory of singlet fields of large-N Sp(N) spin models and display its unconventional features. Using the known relationship between O(N) collective fields and higher spin fields of Vasiliev theory in AdS space, we argue that the unique features of the Sp(N) collective theory suggest an interpretation in terms of a higher spin theory in de Sitter space, as proposed by Anninos, Hartman and Strominger. This is reminiscent of the way the Liouville mode is naturally interpreted as time in supercritical worldsheet string theory.

Dimensions from Large-N

- Since the early 1980's we have had many examples where the large-N degrees of freedom organize themselves in the form of extra dimensions – usually **space-like**

1. Eguchi-Kawai Models
2. "Old" Matrix Models
3. Matrix Theory
4. AdS/CFT

In some of these cases, this emergence of additional dimensions can be in some sense "deduced" – e.g. c = 1 Matrix Model

(S.R.D. and A. Jevicki; Polchinski – 1990's,…….).

There are also interesting suggestions for AdS/CFT

(*R. Gopakumar*, *S.S. Lee,……..*). However no solid understanding.

- All these models involve large-N matrices : they give rise to String theories – and in lucky circumstances (e.g. strong coupling limit of a large-N theory) the String Theory can be truncated to low energy supergravity.
- Another class of models which are known to display AdS/CFT dualities are vector models. The dual theories are not String theories – rather they are theories of an infinite number of higher spin gauge fields considered by *Vasiliev.*
- The classic example of this type is Klebanov-Polyakov duality of the singlet sector of O(N) vector model in 2+1 dimensions to Vasiliev theory of even spin gauge fields in AdS_4
- The higher spin currents are dual to the higher spin fields in the bulk.

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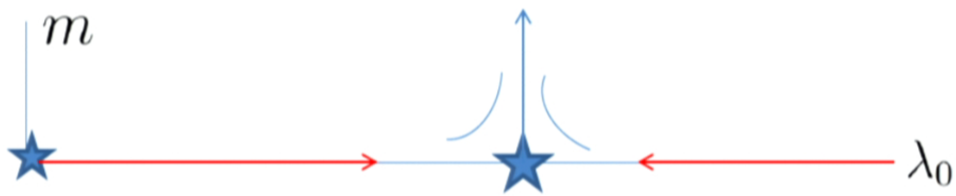
- The lagrangian for the O(N) linear sigma model in 2+1 dim is

$$L = \frac{1}{2}\left[(\partial\vec{\phi})^2 + m^2\vec{\phi}^2 + (\lambda_0\Lambda)(\vec{\phi}^2)^2\right]$$

- To leading order in 1/N this has a RG flow diagram



- The Vasiliev theory has a conformally coupled scalar - the UV fixed point corresponds to the "alternative" quantization, while the IR Wilson-Fisher fixed point corresponds to the "standard" quantization.

- There has been a fair amount of evidence for this conjecture recently *(Giombi and Yin, ...)*
- Related dualities which relate CFT2 with AdS3 have uncovered an interesting story *(Gabardiel and Gopakumar)* and dualities which involve fixed lines rather than fixed points have been discovered.
- It has also been realized that these dualities could be special cases of dualities in String Theory. *(Aharony et.al, Minwalla et.al.)*

Bilocal Collective Fields

- Quite some time ago we argued that there is a direct way to construct the bulk higher spin fields from those of the O(N) theory – *(S.R.D. and A. Jevicki, 2003)*
- In the hamiltonian formalism, singlet states are described by wave functionals of all the O(N) invariants.

- This is like looking at the zero angular momentum sector of a D dimensional quantum mechanical problem with a spherically symmetric potential – i.e. wave functions which are functions of the radial variable alone $\psi(r,t)$
- The hamiltonian acting on $\chi(r,t) = J(r)^{1/2}\psi(r,t)$ with the jacobian $J(r) \sim r^{D-1}$ is given by

$$H_r = -\frac{1}{2}\frac{d^2}{dr^2} + \frac{(D-1)(D-3)}{2r^2} + V(r)$$

- We can do the same for a euclidean theory – the hamiltonian is then the log of the transfer matrix.

- For the O(N) field theory the analog of the radial variable is a bilocal field $\sigma(\vec{x},\vec{y}) = \phi^i(\vec{x}) \cdot \phi^i(\vec{y})$
- We thus want states which are functionals of this bilocal field alone, not the "angles",
- Starting with some general O(N) model with a hamiltonian

$$H = \frac{1}{2}\int d^{d-1}x \left[-\frac{\delta^2}{\delta\phi^i(\vec{x})\delta\phi^i(\vec{x})} + \nabla\phi^i(\vec{x})\nabla\phi^i(\vec{x}) + U[\phi^i(\vec{x})\phi^i(\vec{x})] \right]$$

The singlet sector hamiltonian can then be derived using a key technique developed by *Sakita and Jevicki (1980).* The result is

$$H_{coll}^{O(N)} = 2\mathrm{Tr}\left[(\Pi_\sigma \sigma \Pi_\sigma) + \frac{N^2}{16}\sigma^{-1} \right] - \frac{1}{2}\int d\vec{x} \nabla_x^2 \sigma(\vec{x},\vec{y})|_{\vec{y}=\vec{x}} + U(\sigma(\vec{x},\vec{x}))$$

where $\sigma(\vec{x},\vec{y})$ is now considered as a matrix, and Π_σ is the momentum canonically conjugate to σ

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Collective Fields and AdS

- The conjecture is that the bilocal field is a collection of an infinite number of fields – one for each even spin – in one more dimension. These are identified with Vasiliev's fields
- At conformal points this higher dimensional bulk is a AdS space.
- It is easy to see that this has the right number of fields. Consider for example the 2+1 dimensional model. Define

$$\vec{u} = \frac{1}{2}(\vec{x} + \vec{y}) \qquad \vec{v} = \vec{x} - \vec{y} = (r, \theta)$$

- Fourier series $$\sigma(\vec{x}, \vec{y}) = \sum_{m=0}^{\infty} \sigma_{2m}(\vec{u}, r) e^{2im\theta}$$
- The fields $\sigma_{2m}(\vec{u}, r)$ represent 2 helicities for each even spin in 3+1 dimensions – exactly as in Vasiliev theory

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Sp(N) and dS/CFT

- Recently *Anninos, Hartmann and Strominger* conjectured that the Sp(2N) vector model in 2+1 dimensions is dual to Vasiliev theory in 3+1 dimensional de Sitter space
- This is a proposal for a microscopic realization of dS/CFT – the emergent direction is time-like.
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Sp(2N) Model

- The Sp(2N) model is defined by

$$S = i\int dtd^{d-1}x\ \left[\{\partial_t\phi_1^i\partial_t\phi_2^i - \nabla\phi_1^i\nabla\phi_2^i\} - V(i\phi_1^i\phi_2^i)\right]$$

- The ϕ_1^i, ϕ_2^i with $i = 1\cdots N$ are N pairs of grassmann variables.
- The hamiltonian is given by

$$H = i\int d^{d-1}x\left[P_2^iP_1^i + \nabla\phi_1^i\nabla\phi_2^i + V(i\phi_1^i\phi_2^i)\right]$$

- The only nontrivial anticommutators are

$$\begin{aligned}\{\phi_i^a(\vec{x}), P_j^b(\vec{y})\} &= -i\delta_{ij}\delta^{ab}\delta^{d-1}(\vec{x}-\vec{x}') \\ \{\phi_a^i(\vec{x}), \phi_b^j(\vec{y})\} &= \{P_a^i(\vec{x}), P_b^j(\vec{y})\} = 0\end{aligned}$$

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- The $Sp(2N)$ invariants are now

$$\rho(\vec{x},\vec{y}) \equiv \frac{i}{2}\epsilon^{ab}\phi_a^i(\vec{x})\phi_b^i(\vec{y})$$

- Note that the same action has a $U(N)$ symmetry as well. This is made manifest by writing it in terms of complex fields

$$\phi^i = \frac{1}{\sqrt{2}}(\phi_a^i + i\phi_2^i) \qquad \bar{\phi}^i = \frac{1}{\sqrt{2}}(\phi_a^i - i\phi_2^i)$$

- So that the action is

$$S = -\int d^dx[(\partial\bar{\phi}^i)(\partial\phi^i) + V(\bar{\phi}\phi)]$$

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- The conjecture of duality to de Sitter comes from the observation that

$$< \rho(\vec{x}_1, \vec{y}_2, t_1) \cdots \quad \rho(\vec{x}_n, \vec{x}_n, t_n) >^{conn}_{Sp(2N)} = \\ - < \sigma(\vec{x}_1, \vec{y}_2, t_1) \cdots \sigma(\vec{x}_n, \vec{x}_n, t_n) >^{conn}_{O(2N))}$$

- Since to leading order each side is proportional to N , this is like making the replacement $N \to -N$
- However, in the dual gravity $R^2_{AdS} = N l_p^2$ and under

$$N \to -N \qquad z \to i\tau \qquad R_{Ads} = iR_{dS}$$

- The *euclidean* AdS metric

$$ds^2 = \frac{R^2_{AdS}}{z^2}[dz^2 + d\vec{x}^2]$$

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- In the following we will try to argue why the collective theory of the fields ρ lead to a natural *interpretation* of the emergent direction as a time coordinate.

Quantization- Free theory

- We first quantize this model by treating $\phi_i^a(\vec{x})$ as hermitian. The conjugate momenta are then anti-hermitian. The mode expansion is standard

$$\phi_a^i(\vec{x},t) = \int \frac{d^{d-1}k}{(2\pi)^{d-1}\sqrt{2|k|}} \left[\alpha_a^i(\vec{k})e^{-i(|k|t-\vec{k}\cdot\vec{x})} + \alpha_a^{i\dagger}(\vec{k})e^{i(|k|t-\vec{k}\cdot\vec{x})}\right]$$

$$\{\alpha_1^i(\vec{k}), \alpha_2^{\dagger j}(\vec{k}')\} = i\delta^{ij}\delta(\vec{k}-\vec{k}') \quad \{\alpha_1^{\dagger i}(\vec{k}), \alpha_2^j(\vec{k}')\} = -i\delta^{ij}\delta(\vec{k}-\vec{k}')$$

- The hamiltonian is

$$H = i\int [d\vec{k}][\alpha_1^{\dagger\ i}(\vec{k})\alpha_2^i(\vec{k}) - \alpha_2^{\dagger\ i}(\vec{k})\alpha_1^i(\vec{k})]$$

$$[H, \alpha_a^i(k)] = -k\alpha_a^i(k) \qquad [H, \alpha_a^{i\dagger}] = k\alpha_a^{i\dagger}(k)$$

- *Henneaux and Teitelboim (1982); Finkelstein and Villasante (1986)*

The Sp(2N) Oscillator

- To read off the spectrum consider the Sp(2N) oscillator – this is a momentum mode of the field.
- Since the variables are *fermionic* there is a highest energy state as well as a lowest energy state

$$\alpha_a^i|0>=0 \qquad \alpha_a^{i\dagger}|N>=0$$

- In a "coordinate" representation the wavefunctions are

$$\Psi_0 = \exp[-ik\phi_1^i\phi_2^i] \qquad \Psi_N = \exp[ik\phi_1^i\phi_2^i]$$

- Where ϕ_a^i are now grassmann variables. The energy spectrum is

$$E_m = k[n-N] \qquad n = 0, 1, \cdots 2N$$

- Of course most of these are non-singlets. We need to look at the singlet states, and their collective theory.

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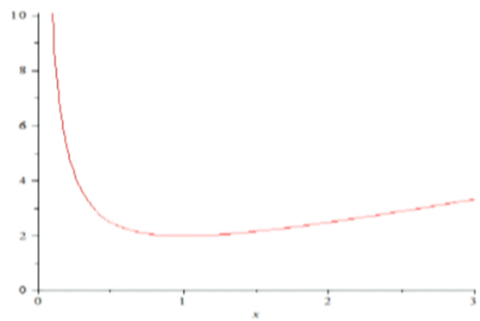
Collective theory of the O(N) Oscillator

- Before doing that let us review the O(N) oscillator.
- An efficient way of looking at the singlet sector is to write the theory in terms of the singlet variables $\sigma = X^i X^i$
- The jacobian for the change is

$$J(\sigma) = \frac{1}{2}\sigma^{(N-2)/2}\Omega_{N-1}$$

- And the s-wave hamiltonian is

$$H_{coll} = 2\Pi_\sigma \sigma \Pi_\sigma + \frac{(N-2)^2}{8\sigma} + \frac{1}{2}k^2\sigma$$



- In the large-N expansion, we need to expand around the saddle point of the potential. The saddle point equation gives

$$\sigma_0^2 = \frac{N^2}{4k^2}$$

- The quantity σ is however a positive real quantity, so it is quite clear that we need to consider the saddle

$$\sigma_0 = \frac{N}{2k}$$

- This gives the correct ground state energy $E_0 = \frac{N}{2}k$

and consistent with the equal time two point function $< 0|X^i(t)X^i(t)|0 >$

The leading corrections are computed by expanding around the saddle point solution

$$\sigma = \sigma_0 + \sqrt{\frac{2N}{k}}\eta \qquad \Pi_\sigma = \sqrt{\frac{k}{2N}}\pi_\eta$$

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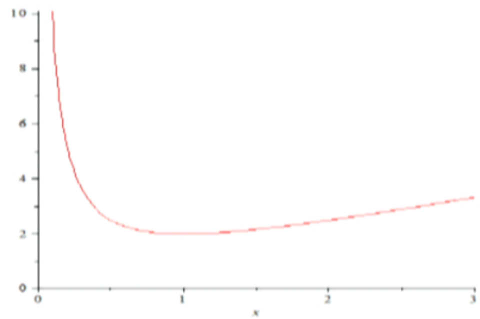

- In what follows we need to know about the singlet wavefunctions.
- The ground state wavefunction is of course the gaussian

$$\Psi_0(X^i) = \exp[-\frac{k}{2}X^iX^i] = \exp[-\frac{k}{2}\sigma] \sim \exp[-\sqrt{\frac{kN}{2}}\eta]$$

- From the collective theory, the wavefunction is – to O(1)

$$\Psi_0'(\eta) = [J(\sigma)]^{-\frac{1}{2}} \exp[-k\eta^2]$$

- Expanding this around the saddle point leads to an expression which contains all powers of η - however we can only trust the terms which are *upto second order*, since we did not consider higher orders in writing down the wave function.
- One can easily check that the quadratic term in η cancels and the linear term agrees. *(Jackiw and Strominger,1981)*

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- For the interacting O(N) field theory, this propagator, together with the nonlinear terms in the fluctuations which come from the expansion of $\frac{N^2}{8}\sigma^{-1}$ provide the set of Feynman rules to compute correlators of singlet operators.
- With the identification of the Poincare coordinate z mentioned above, these provide the bulk correlators of the higher spin theory in AdS.
- For example the quadratic action for the bulk scalar is

$$S = \frac{1}{2}\int dtdzdxdy \left[\frac{1}{z^2}\left((\partial_t\varphi)^2 - (\partial_y\varphi)^2 - (\partial_x\varphi)^2 - (\partial_z\varphi)^2\right) + \frac{2}{z^4}\varphi^2\right]$$

- We did this long detour to O(N) theory since we are going to follow the same steps for Sp(2N) and see where we end up.

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Collective theory of Sp(2N) oscillator

- For the Sp(2N) oscillator the collective variables are
$$\rho(\vec{x},\vec{y}) = i\epsilon^{ab}\phi_a^i(\vec{x})\phi_b^i(\vec{y})$$
- It is important to note that while this is grassmann even, it is not a usual bosonic field. In fact **some power of** ρ **vanishes**. This fact will play an important role later.
- To probe the singlet sector, let us first examine the Sp(2N) oscillator, which has the collective variable $\rho = i\epsilon^{ab}\phi_a^i\phi_b^i$
- The direct quantization of the model shows an important relationship of the correlators with those of the O(N) oscillator

$$\langle\rho(t_1)\rho(t_2)\cdots\rho(t_n)\rangle^{conn}_{Sp(2N)} = -\langle\sigma(t_1)\sigma(t_2)\cdots\sigma(t_n)\rangle^{conn}_{SO(2N)}$$

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- It is important to note that while this is grassmann even, it is not a usual bosonic field. In fact **some power of** ρ **vanishes**. This fact will play an important role later.
- To probe the singlet sector, let us first examine the Sp(2N) oscillator, which has the collective variable $\rho = i\epsilon^{ab}\phi_a^i\phi_b^i$
- The direct quantization of the model shows an important relationship of the correlators with those of the O(N) oscillator

$$\langle\rho(t_1)\rho(t_2)\cdots\rho(t_n)\rangle_{Sp(2N)}^{conn} = -\langle\sigma(t_1)\sigma(t_2)\cdots\sigma(t_n)\rangle_{SO(2N)}^{conn}$$

- Using the coordinate representation

$$\phi_a^i \to \phi_a^i \qquad P_a^i \to -i\frac{\partial}{\partial \phi_a^i}$$

- And making a change of variables to ρ we get the expected jacobian for the transformation

$$J'(\rho) = A' \; \rho^{-(N+1)}$$

- The inverse power reflects the fact that we are dealing with grassmann variables.
- Surprisingly the final form of the collective hamiltonian is the **same** as that for the O(2N) oscillator

$$H_{coll}^{Sp(2N)} = -2\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + \frac{N^2}{2\rho} + \frac{1}{2}k^2\rho$$

- These expressions are formal, since ρ is not a true bosonic variable. Nevertheless they make sense in the 1/N expansion

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- So we need to find the saddle point, which satisfies the equation

$$\rho_0^2 = \frac{N^2}{k^2}$$

- Now we have a choice of sign, since ρ is not restricted to be positive.
- The sign which we need to choose is in fact dictated by the fact that the saddle point value should agree with the coincident time two point function of the basic fields – and this requires us to choose

$$\rho_0 = -\frac{N}{k}$$

- This leads, as expected, to a negative ground state energy

$$E_{gs} = -Nk$$

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- Interestingly, the O(N) and Sp(2N) models are **two different solutions of the same collective hamiltonian.**

- As in the usual harmonic oscillator we need to expand around the saddle point

$$\rho = \rho_0 + \sqrt{\frac{4N}{k}}\xi \qquad \Pi_\rho = \sqrt{\frac{k}{4N}}\pi_\xi$$

- This leads to a quadratic hamiltonian

$$H_\xi^{(2)} = -\frac{1}{2}\left[\pi_\xi^2 + 4k^2\xi^2\right]$$

- Under normal circumstances this would have a spectrum which is unbounded from below. Defining usual oscillators

$$\xi = \frac{1}{\sqrt{4k}}[a_\xi + a_\xi^\dagger] \qquad \pi_\xi = i\sqrt{k}[a_\xi - a_\xi^\dagger]$$

- The algebra has a negative sign $[a_\xi, a_\xi^\dagger] = -1$ but still

$$[H, a_\xi] = -2ka_\xi$$

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- The highest energy state would be annihilated by $a_{\xi}^{\dagger}$ and have a "normalizable" wavefunction $e^{-k\xi^2}$
- The a_{ξ} then create states of lower and lower energy.
- This does not appear to resemble the Sp(2N) oscillator spectrum which we derived earlier, which has a lower bound,

$$E_m = k[n - N] \qquad n = 0, 1, \cdots 2N$$

- The point is that ξ is not a true bosonic variable and we have other possibiltiies.

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- The correct state which has to be identified with the ground state of the Sp(2N) oscillator is in fact the state which is annihilated by the operator a_ξ .
- However, because of the wrong sign in the commutation relation, this has a wavefunction

$$\exp[k\xi^2]$$

- If ξ was a usual bosonic variable, this wavefunction would be of course inadmissible.
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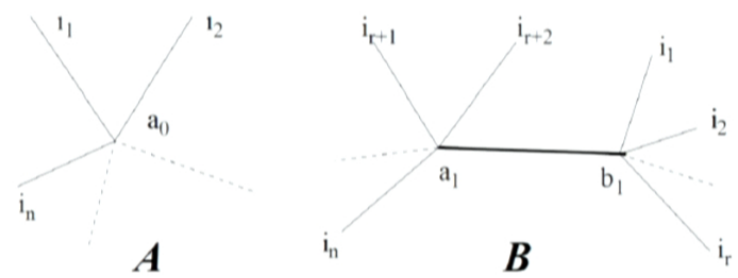
- However, the true normalization of the wavefunction has to be computed in terms of the grassmann partons of ξ - and for the grassmann integration this is just fine.

- In fact with this choice of the "ground state" ,the wavefunction agrees with what we found in the direct quantization,

$$\Psi'_{0\xi}[\xi] = [J'(\rho)]^{-1/2}\exp[k\xi^2] \quad \sim \exp[-\sqrt{Nk}\xi]$$

- The O(1) corrections to the spectrum are positive, just as in the exact answer.
- However, unlike the exact answer we cannot see the upper bound of the energy. This is because we are working around the limit $N=\infty$ where this upper bound is at infinity.
- And the two point correlation function of the collective field is the negative of that of the O(N) theory – as required.

- It is easy to see from the form of the hamiltonian that terms which have even powers of the fluctuation have a negative sign compared to the O(N) theory, while terms with odd powers of the fluctuation have the same sign.
- This ensures that **all tree level correlation functions have a negative sign compared to the O(N) result.**
- **Every time we split a vertex we get an even power of (-1)**



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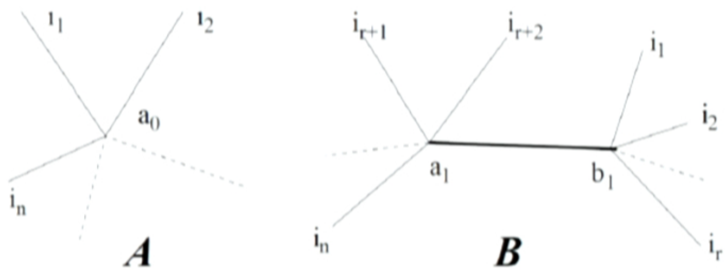

- The collective theory of ρ gives the right result in a perturbative expansion in 1/N – but at various stages we have to remember about its grassmann origin.
- The most important aspect of the grassmann origin, viz. the finite dimensionality of the Hilbert space is however not visible in the 1/N expansion.

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- The most important aspect of the grassmann origin, viz. the finite dimensionality of the Hilbert space is however not visible in the 1/N expansion.

Sp(2N) Field Theory

- These results can be easily generalized to the Sp(2N) field theory by working in momentum space. The bilocal collective field is

$$\rho(\vec{x},\vec{y}) \equiv \frac{i}{2}\epsilon^{ab}\phi_a^i(\vec{x})\phi_b^i(\vec{y})$$

- Which now has a saddle point solution

$$\rho_0(\vec{k},\vec{k}',t) = -\frac{N}{2|\vec{k}|}\delta(\vec{k}-\vec{k}')$$

- And the correlators satisfy the required relationship

$$< \rho(\vec{k}_1,\vec{k}_1',t_1)\rho(\vec{k}_2,\vec{k}_2',t_2)\cdots\rho(\vec{k}_n,\vec{k}_n',t_n) >^{conn}_{Sp(2N)} =$$
$$-\langle\sigma(\vec{k}_1,\vec{k}_1',t_1)\sigma(\vec{k}_2,\vec{k}_2',t_2)\cdots\sigma(\vec{k}_n,\vec{k}_n',t_n)\rangle^{conn}_{SO(2N)}$$

Why de Sitter

- Using the known relation between the bilocal field of the O(N) field theory and higher spin fields in AdS_4 we conclude that the bilocal theory in Sp(2N) gives the same kinetic terms, but with an overall negative sign.
- For example the action for the scalar which would follow from our Sp(2N) theory is

$$S = -\frac{1}{2}\int dt dz dx dy \left[\frac{1}{z^2}\left((\partial_t\varphi)^2 - (\partial_y\varphi)^2 - (\partial_x\varphi)^2 - (\partial_z\varphi)^2\right) + \frac{2}{z^4}\varphi^2\right]$$

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- This cries out for a double analytic continuation

$$z = i\tau \qquad t = -iw$$

- And we get an action

$$S' = \frac{1}{2}\int d\tau dw dx dy \left[\frac{1}{\tau^2}\left((\partial_\tau\varphi)^2 - (\partial_y\varphi)^2 - (\partial_x\varphi)^2 - (\partial_w\varphi)^2\right) - \frac{2}{\tau^4}\varphi^2\right]$$

- Note that the mass term did not change sign – but this is precisely what is required to get the action of a conformally coupled scalar in de Sitter space.
- The double analytic continuation makes the original Sp(2N) model euclidean – so this is a euclidean Sp(2N) – de Sitter correspondence.
- The holographic direction becomes time-like, pretty much like the way the Liouville mode appears as target space time for supercritical string theory *(S.R.D., S. Naik and S.R. Wadia, 1988)*

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- We started out with a model which contained ghosts – and we lived with them for a while.
- The singlet sector of the theory had kinetic terms which are negative – this motivates a double analytic continuation, leading to actions in de Sitter space. This renders the original model euclidean.
- The grassmann origin of the the "bulk" fields plays a role at various steps – in particular non-normalizable wavefunctionals.
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Dimensionality of Hilbert Space

- We have emphasized the fact that the dimensionality of the Hilbert space for each momentum mode is finite – this follows from the grassman origin.
- The expansion around $N = \infty$ makes this invisible, but this must have important consequences at any finite N .
- There cannot be states in this theory which have indefinitely large number of particles of any given momentum.
- The lesson is that any duality of the Sp(2N) theory to higher spin theory in de Sitter space has to be taken with this important **grain of salt** – the higher spin description can at best be a perturbative description in $G = R_{dS}^2/N$

- The bilocal collective field we used looks even worse.
- Suppose that there are K points in space. Then there are K^2 collective oscillators, and each oscillator can give rise to an infinite number of multiparticle states.
- On the other hand, the K fermionic operators can give rise to at most 2^{NK} states, and one would think that most of these states are in fact not singlets.
- We will now describe an algebraic treatment of the bilocal collective theory which automatically incorporates the grassmann origin and correctly counts the number of singlet states in the theory.

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- We will now describe an algebraic treatment of the bilocal collective theory which automatically incorporates the grassmann origin and correctly counts the number of singlet states in the theory.

- Consider the Sp(2N) singlet operators

$$
\begin{aligned}
S(p_1,p_2) &= \frac{-i}{2\sqrt{N}} a^T(p_1)\epsilon_N a(p_2) = \frac{i}{2\sqrt{N}} \sum_{i=1}^{N} (a^i_{p_1+} a^i_{p_2-} + a^i_{p_2+} a^i_{p_1-}) \\
S^\dagger(p_1,p_2) &= \frac{-i}{2\sqrt{N}} \tilde{a}^T(p_1)\epsilon_N \tilde{a}(p_2) = \frac{i}{2\sqrt{N}} \sum_{i=1}^{N} (a^{i\dagger}_{p_1+} a^{i\dagger}_{p_2-} + a^{i\dagger}_{p_2+} a^{i\dagger}_{p_1-}) \\
B(p_1,p_2) &= \tilde{a}^T(p_1)\epsilon_N a(p_2) = \sum_{i=1}^{N} a^{i\dagger}_{p_1+} a^i_{p_2+} + a^{i\dagger}_{p_1-} a^i_{p_2-}
\end{aligned}
$$

- Their algebra is

$$
\begin{aligned}
\left[S(\vec{p}_1,\vec{p}_2), S^\dagger(\vec{p}_3,\vec{p}_4)\right] &= \frac{1}{2}\left(\delta_{\vec{p}_2,\vec{p}_3}\delta_{\vec{p}_4,\vec{p}_1} + \delta_{\vec{p}_2,\vec{p}_4}\delta_{\vec{p}_3,\vec{p}_1}\right) - \frac{1}{4N} \\
&\quad + \delta_{\vec{p}_1,\vec{p}_3} B(\vec{p}_4,\vec{p}_2) + \delta_{\vec{p}_1,\vec{p}_4} B(\vec{p}_3,\vec{p}_2)] \\
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- If there are K points in space, i.e. K possible values of momenta, these are K X K matrix-valued operators, and are generalizations of SU(2)
- The operators $S^\dagger$ create all the $Sp(2N)$ singlet states.
- The Casimir can be calculated by the action on e.g. the vacuum and leads to the condition

$$\frac{4}{N}S^\dagger \star S + (1 - \frac{1}{N}B) \star (1 - \frac{1}{N}B) = \mathbb{I}$$

- The dimensionality of the Hilbert space can be now calculated using a method due to *Berezin*, and the result is

$$\text{Dim } \mathcal{H}_B = \prod_{j=0}^{K-1} \frac{\Gamma(j+1)\Gamma(N+K+j+1)}{\Gamma(K+j+1)\Gamma(N+j+1)}$$

- This is the total number of singlet states – its finiteness is a consequence of the grassmann origin.

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- One can solve this in terms of a canonically conjugate pair Ψ , Π_ψ

$$
\begin{aligned}
S &= \frac{\sqrt{-\gamma}}{2}\left[-\frac{2}{k}\Pi_\psi\Psi\Pi_\psi - \frac{1}{2\gamma^2 k\Psi} + \frac{k}{2}\Psi - i(\Psi\Pi_\psi + \Pi_\psi\Psi)\right] \\
S^\dagger &= \frac{\sqrt{-\gamma}}{2}\left[-\frac{2}{k}\Pi_\psi\Psi\Pi_\psi - \frac{1}{2\gamma^2 k\Psi} + \frac{k}{2}\Psi + i(\Psi\Pi_\psi + \Pi_\psi\Psi)\right] \\
B &= \frac{1}{\gamma} + \left[\frac{2}{k}\Pi_\psi\Psi\Pi_\psi + \frac{1}{2\gamma^2 k\Psi} + \frac{1}{2}k\Psi\right]
\end{aligned}
$$

- This is the representation in terms of canonical collective fields, and B is basically the collective hamiltonian – which is the same in either case.

$$
\begin{aligned}
\gamma &= 1/N \qquad O(N) \\
\gamma &= -1/N \qquad Sp(N)
\end{aligned}
$$

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- For small values of K the dimensionality behaves as N^{K^2}
- However this steeply rises with increasing K and one gets an asymptotic behavior

$$\ln(\mathrm{Dim}\ \mathcal{H}_B) \sim 2NK\ \ln 2$$

- Which is exactly the dimensionality without any singlet constraint.
- Perturbative higher spin theory would of course give an infinite result.

EAdS or dS ?

- We have a conformal field theory on *euclidean* three dimensional space – symmetry is SO(4,1).
- This is the symmetry group of both $EAdS_4$ and dS_4 .
- How do we figure out which would be the correct dual ?
- One hint comes from holographic RG equations.
- These equations are different for $EAdS_4$ and dS_4.
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(Diptarka Das, S.R. Das and Gautam Mandal, to appear)

EAdS or dS ?

- We have a conformal field theory on *euclidean* three dimensional space – symmetry is SO(4,1).
- This is the symmetry group of both $EAdS_4$ and dS_4 .
- How do we figure out which would be the correct dual ?
- One hint comes from holographic RG equations.
- These equations are different for $EAdS_4$ and dS_4.
- It turns out that these equations in dS_4 point to a specific kind of non-unitary CFT's

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- Consider a lagrangian of a *euclidean* large-N CFT

$$L = L_{CFT} + \frac{g}{N}\mathcal{O}^2$$

- The beta function for the coupling g is, to leading order in large-N *(Witten, Vechhi)*

$$\beta(g) = (2\Delta - d)g + vg^2$$

- Here $$< \mathcal{O}(\vec{x})\mathcal{O}(\vec{y}) >_{CFT} = \frac{v}{|\vec{x} - \vec{y}|^{2\Delta}}$$

  For usual theories $v > 0$, while for e.g. $Sp(2N)$ we have $v < 0$

- The result is the following : The beta functions for $v > 0$ can be obtained by holographic RG in $EAdS_4$. Holographic RG in dS_4 gives rise to the beta functions for $v < 0$
- This may provide a different perspective into the kind of CFT's which could be dual to quantum gravity in de Sitter space.

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Epilouge

- dS/CFT is rather poorly understood.
- Our hope is that a direct connection of the higher spin theory to the collective field may be a good tool in addressing some of the deep questions in dS/CFT
- Of course the most tantalizing question is the finite entropy of de Sitter space !