Title: Effect of thermal fluctuations in topological p-wave superconductors

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Abstract:

Microsoft Research

Effect of thermal fluctuations in topological p-wave superconductors

Bela Bauer
Roman Lutchyn, Matt Hastings, Matthias Troyer

arXiv:1206.1326

Topological quantum computation

Quantum computers need ***low-decoherence qubit*** and ***reliable operations*** on it.

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Topological quantum computation

Quantum computers need ***low-decoherence qubit*** and ***reliable operations*** on it.

Many proposals:
- Quantum dots
- Trapped ions
- Superconducting circuits
- Cold atoms

Need error correction!

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Topological quantum computation

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Quantum computing with topological phases

No error correction!

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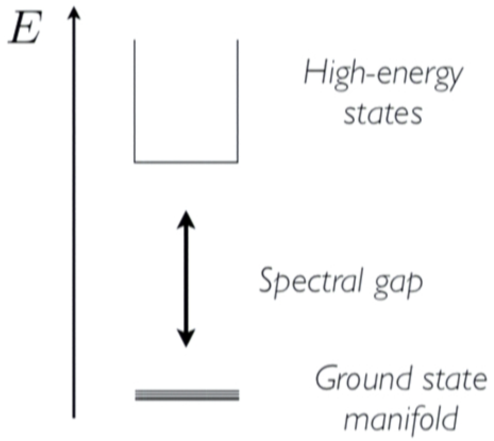
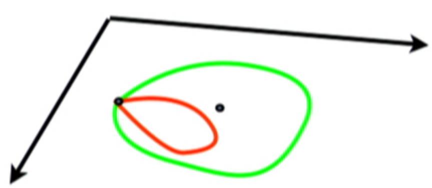
Topological phases



- Ground-state degeneracy depending on the topology with exponentially small splitting
- ***Ground states cannot be mixed by local perturbations***

$$\langle \Psi_1 | O | \Psi_2 \rangle \to 0$$



- Non-Abelian phase: Moving the quasiparticles changes the ground state, but ***operations are independent of details of the path***

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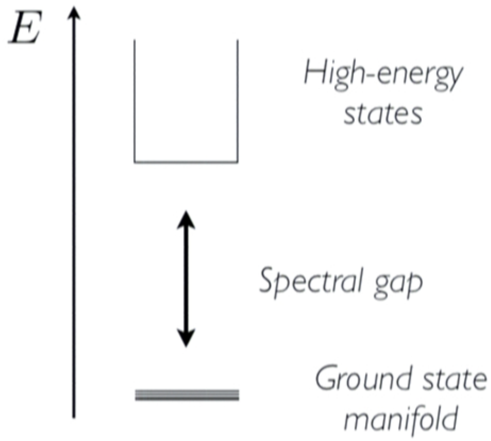
Topological phases



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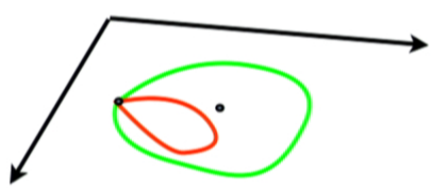

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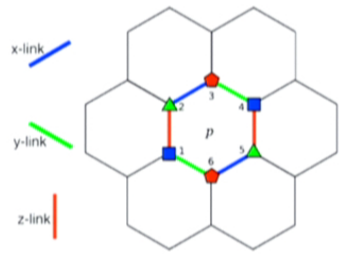
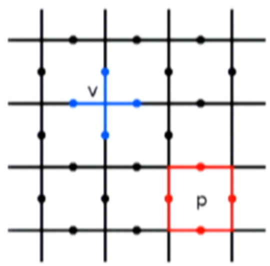
Examples of topological phases

Toy models

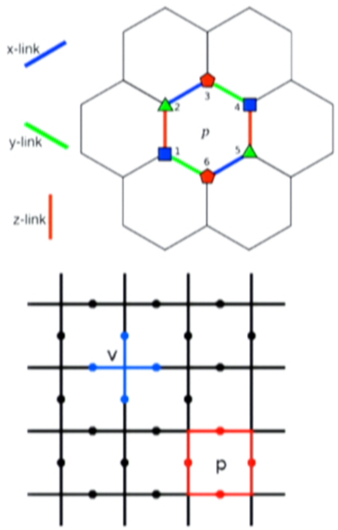




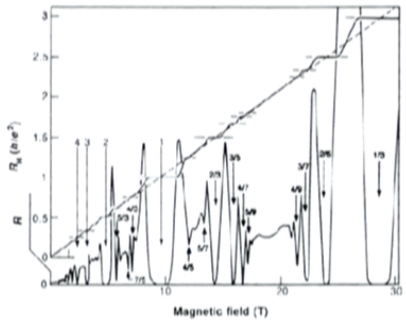
Research

Examples of topological phases

Toy models



Fractional Quantum Hall effect



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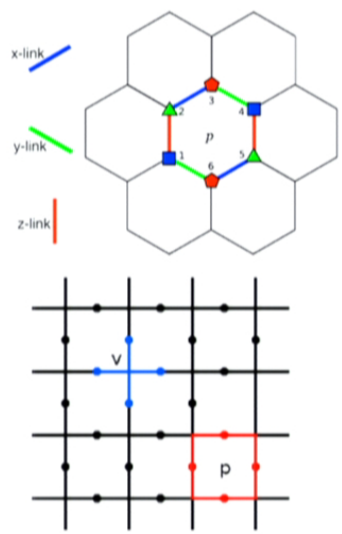
Examples of topological phases

Toy models



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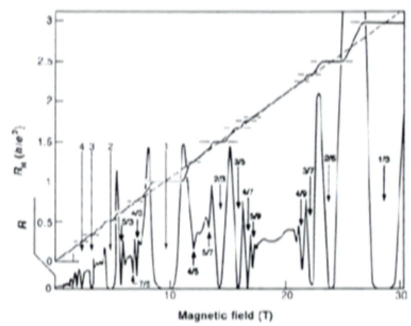
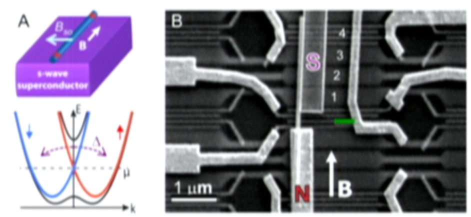


Topological superconductors



Topological superconductors

Seminal paper I: *Read & Green, PRB 2001*

- *p*-wave superconductor has a "weak-pairing phase" where the BCS wave function shares many properties with the Moore-Read state for $\nu=5/2$ FQH
  - Ground state degeneracy
  - Edge states
  - Non-Abelian excitations
  - Hall conductivity

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Seminal paper II: *Kitaev, 2000*

- One-dimensional *p*-wave superconducting wire has zero-energy *"edge"* modes
  - Non-local storage of quantum information
  - Alicea 2010: non-Abelian statistics

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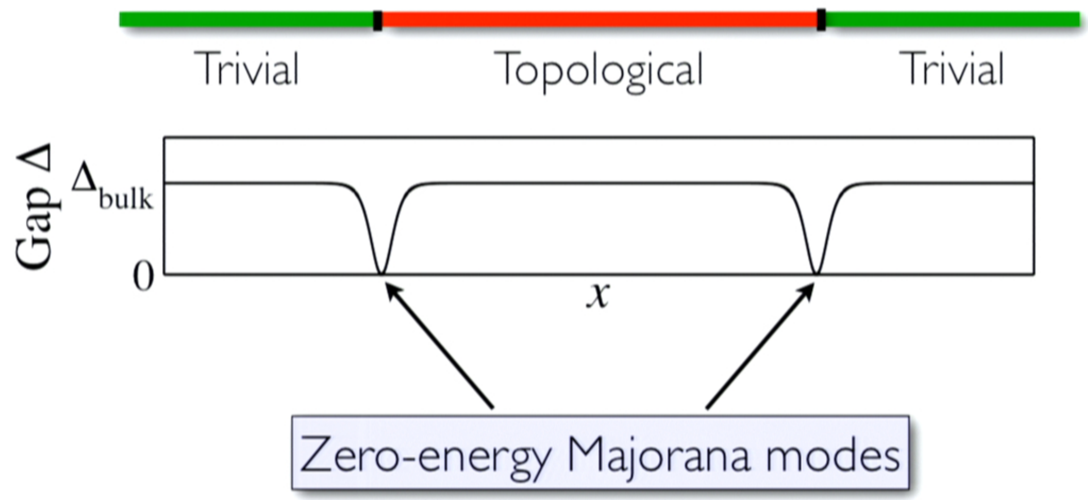
1d edge states



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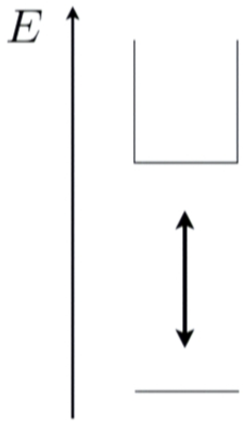
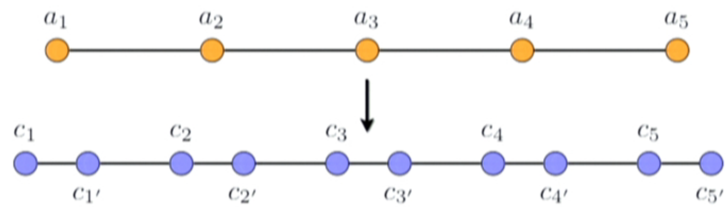
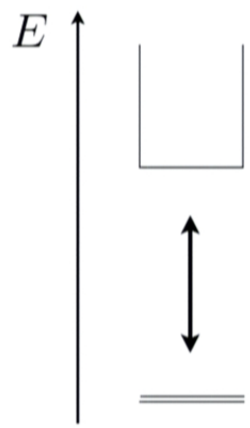
Kitaev's 1d model



$$H = -t\sum_j a_j^\dagger a_{j+1} - \mu \sum_j a_j^\dagger a_j + \sum_j \Delta a_j a_{j+1} + \text{h.c.}$$



Kitaev's *1d* model



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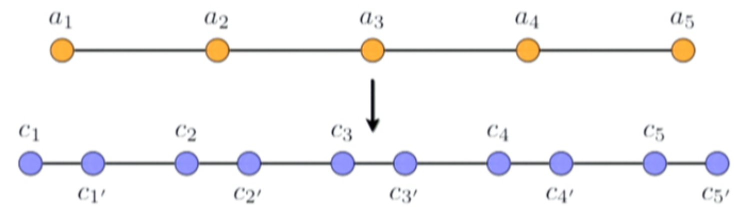
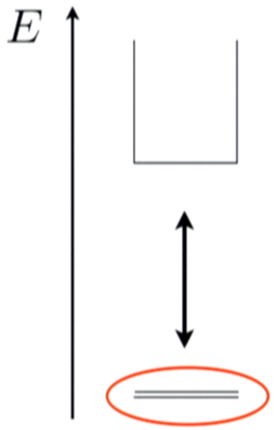


H doesn't contain $c_l, c_{L'}$

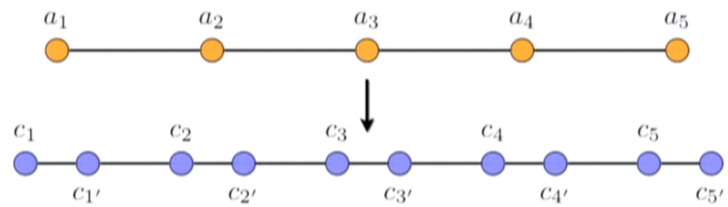
$$-ic_1 c_{L'} |\psi_0\rangle = |\psi_0\rangle$$
$$-ic_1 c_{L'} |\psi_1\rangle = -|\psi_1\rangle$$

Kitaev's 1d model



Two-level system

$$H = -t\sum_j a_j^\dagger a_{j+1} - \mu \sum_j a_j^\dagger a_j + \sum_j \Delta a_j a_{j+1} + \text{h.c.}$$



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$$-ic_1 c_{L'}|\psi_0\rangle = |\psi_0\rangle$$
$$-ic_1 c_{L'}|\psi_1\rangle = -|\psi_1\rangle$$

- Phase stable for a range of couplings up to exponentially small splitting

$$H_{\text{eff}} = -i\epsilon c_1 c_{L'} \qquad \epsilon \sim \exp(-L/l_0)$$

- "Symmetry-protected topological phase": protected by fermionic parity

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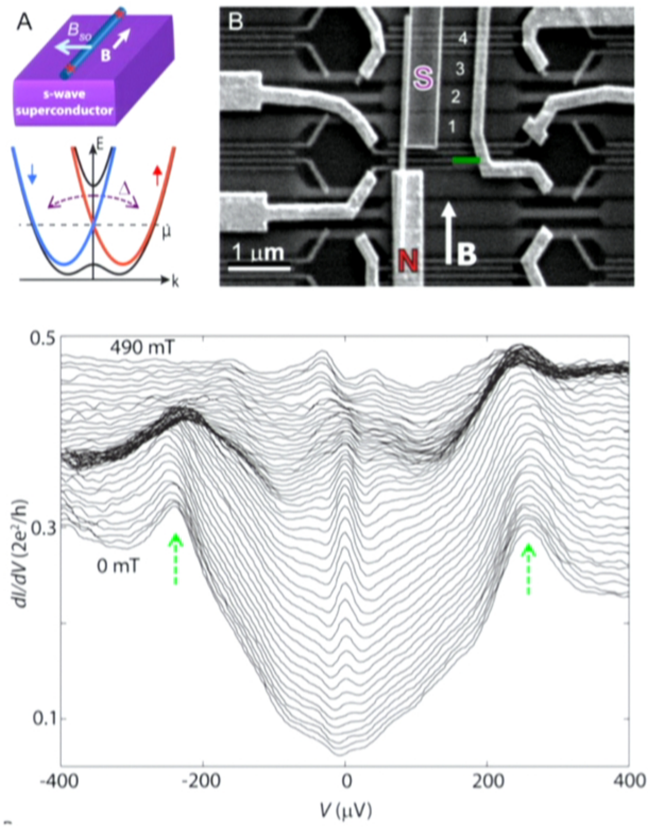
Engineered Majorana wires

Theoretical proposals:

- *Kitaev, 2000*
- *Fu & Kane, PRL 2008*
- *Lutchyn et al, PRL 2010*
- *Oreg et al, PRL 2010*

Experimental realization:

- *Kouwenhoven group, Science 2012*
- Lund, Weizmann, Purdue, ...



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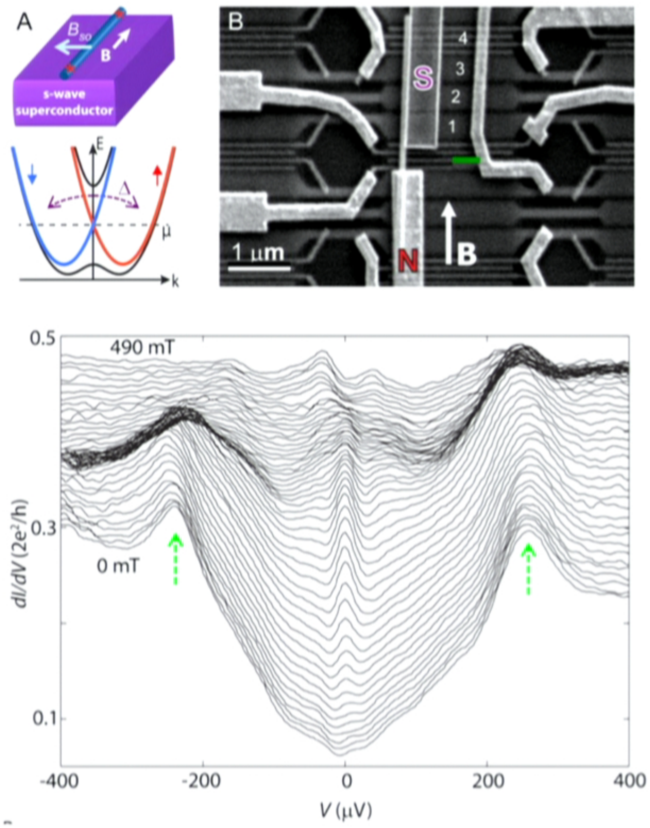

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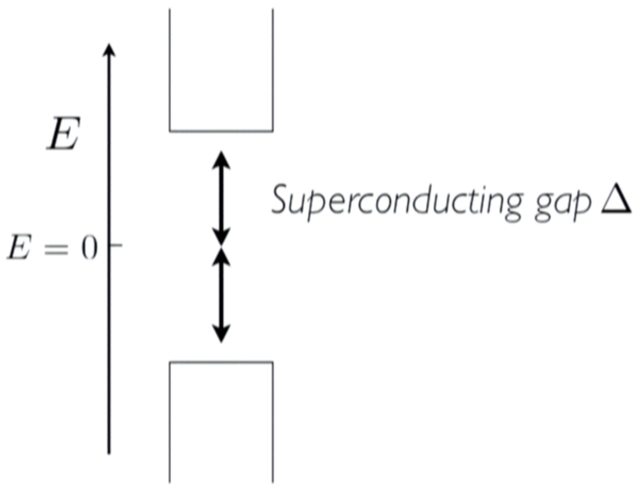
2d *p+ip* superconductor

$$H = \sum_{\mathbf{p}} \left[(\epsilon_{\mathbf{p}} - \mu) c^\dagger_{\mathbf{p}} c_{\mathbf{p}} + \frac{\Delta}{2} \left((p_x + i p_y) c_{-\mathbf{p}} c_{\mathbf{p}} + (p_x - i p_y) c^\dagger_{\mathbf{p}} c^\dagger_{-\mathbf{p}} \right) \right]$$



- Properties (*Read & Green, 2001*):
  - Non-zero Chern number
  - Edge states
  - ***Thermal*** Hall conductance
  - ***Non-Abelian quasi-particles...***

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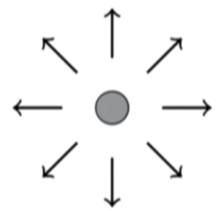
Majorana zero mode

Read & Green (2001)
Ivanov (2001)

Vortex:



$$\Delta \to \Delta_0 f(r) e^{i\phi}$$
$$f(r < R) = 0$$
$$f(r \geq R) > 0$$

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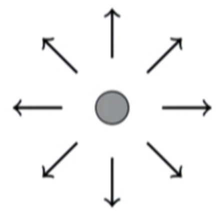

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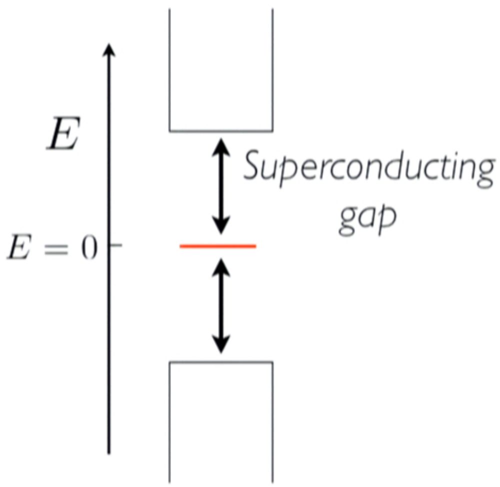

Solution of the BdG eqn. with $E_0 = 0$ and $\gamma = \gamma^\dagger$

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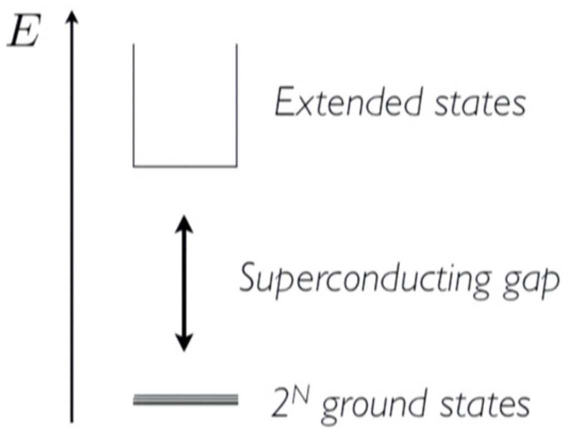
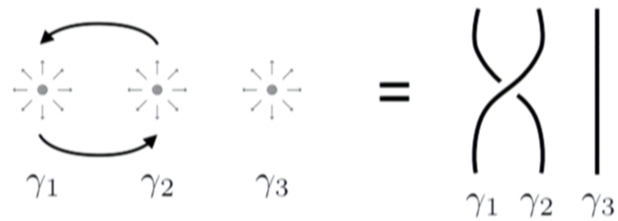
Majorana zero modes

- **Topological degeneracy:**
  - $2N$ vortices = N complex fermions
  - N fermions = 2^N ground states
  - $2N$ vortices = N-1 topologically protected qubits



Research

Braiding



"Braid"

D.A. Ivanov, PRL 2001

$$T_i: \begin{array}{ccc} \gamma_i & \rightarrow & \gamma_{i+1} \\ \gamma_{i+1} & \rightarrow & -\gamma_i \\ \gamma_j & \rightarrow & \gamma_j (j \neq i, i+1) \end{array}$$

- Braiding of non-Abelian anyons performs non-trivial operations in ground-state manifold
- In some cases, braid matrices are dense in unitary matrices, i.e. all operations can be decomposed into braiding operations
- Not the case for Majorana fermions: need to augment with "$\pi/8$ gate"

September 21, 2012

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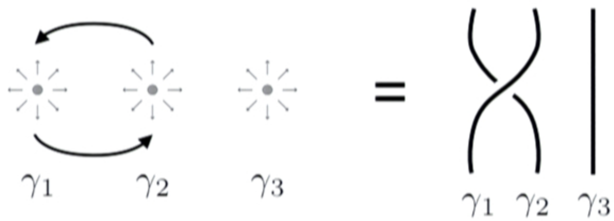
Braiding



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Thermal fluctuations

Robustness for TQC

- Interactions between Majoranas
- Interaction effects in the superconductor
- Disorder
- Finite temperature

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Thermal fluctuations

Robustness for TQC

Chiral pseudo-gap phase

- **Interactions between Majoranas**
- Interaction effects in the superconductor
- **Disorder**
- **Finite temperature**

- Pseudo-gap phase in high-T_c superconductor: no superconductivity, but local superconducting correlations
- *Nandkishore, PRB 2012*: A chiral pseudogap phase exists and has the same topological properties as the *T=0* phase

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Robustness

How stable are the topological properties of Majorana fermions?

- Interactions between vortices: exponential splitting (*Cheng et al, 2010*)

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- Interactions between vortices: exponential splitting (*Cheng et al, 2010*)
- Thermal fluctuations, *Cheng et al, 2011:* suppressed as $\exp(-\Delta/T)$, but which Δ?
  - Bulk gap: gap to (extended) bulk states
  - Minigap: gap to other localized vortex states, $\ll$ bulk gap

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Robustness

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  - Bulk gap: gap to (extended) bulk states
  - Minigap: gap to other localized vortex states, ≪ bulk gap
- Chiral pseudo-gap, Nandkishore 2012: Chern number not affected by finite temperature

Both calculations ignore thermally created vortices - what happens if we include them?

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Thermal phase fluctuations

$$H = \sum_{\langle i,j \rangle} \left(-t c_i^\dagger c_j + \Delta_0 \chi_{ij} e^{i\theta_{ij}} c_i^\dagger c_j^\dagger + \text{h.c.} \right) - \mu \sum_i c_i^\dagger c_i$$

$$\chi_{ij} = \begin{cases} \pm 1 & j = i \pm \hat{x} \\ \pm i & j = i \pm \hat{y} \end{cases}$$

- Consider thermal fluctuations of the phase of the order parameter θ_{ij}
- Magnitude of order parameter remains fixed
- Phase fluctuations are governed by classical **XY model**:

$$H = -J \sum \cos(\theta_{ij} - \theta_{jk})$$

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XY model

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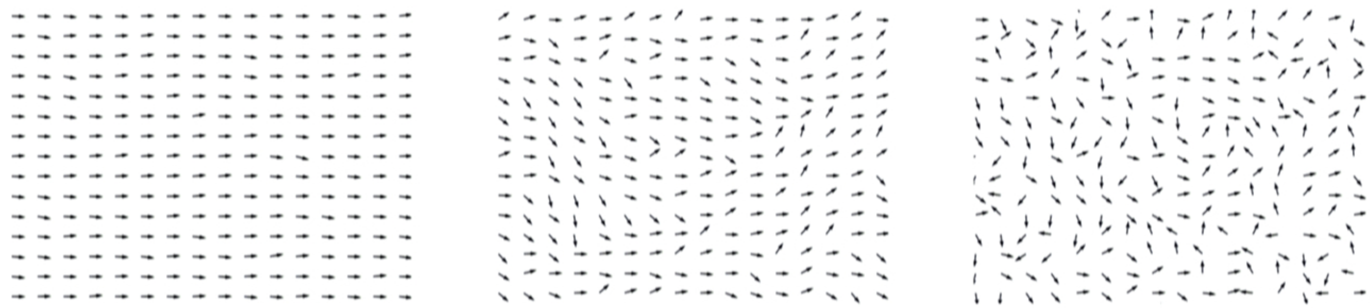


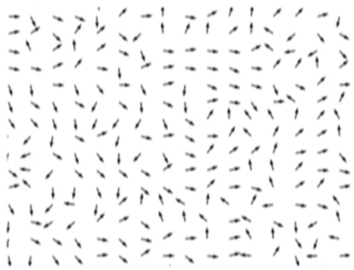
$T/J=0.01$ $T/J=0.4$ $T/J=1.5$

$T_{KT}=0.89$ T

Thermally excited vortices are bound to pairs by logarithmic interaction

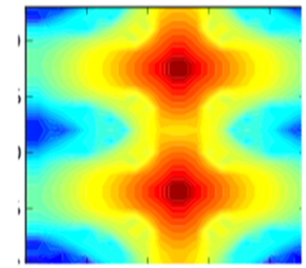
Thermally excited vortices are unbound and proliferate

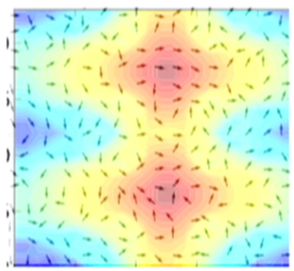
Method



Phase configuration from XY model at finite T

Vortex-antivortex pair at fixed location *R*=*L*/2





→ BdG equation

↓

Thermal average

Adiabatic approximation: vortex dynamics much slower than fermion dynamics

↓

Solve fermions with static order parameter configurations

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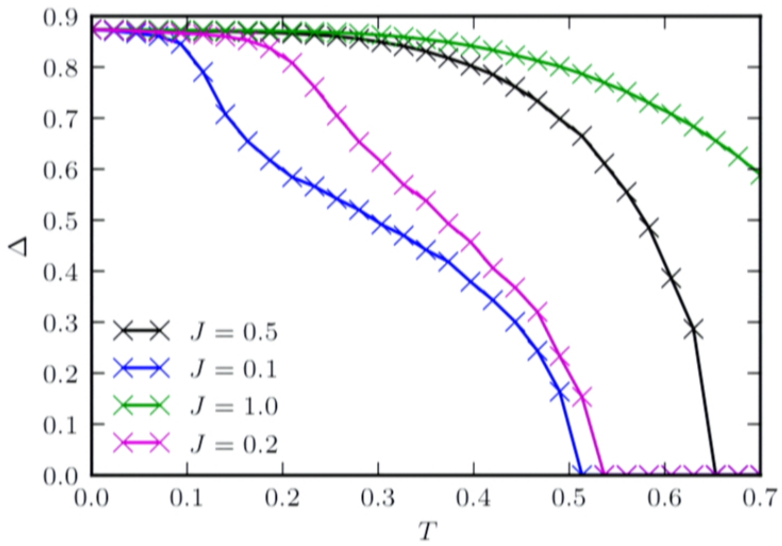
Self-consistency

Phase disordered but finite magnitude - is that physical?



$$H = -J \sum \cos(\theta_{ij} - \theta_{jk})$$

$$\Delta_0 = \langle |U \langle c_i c_j \rangle| \rangle_{ij}$$

U=5

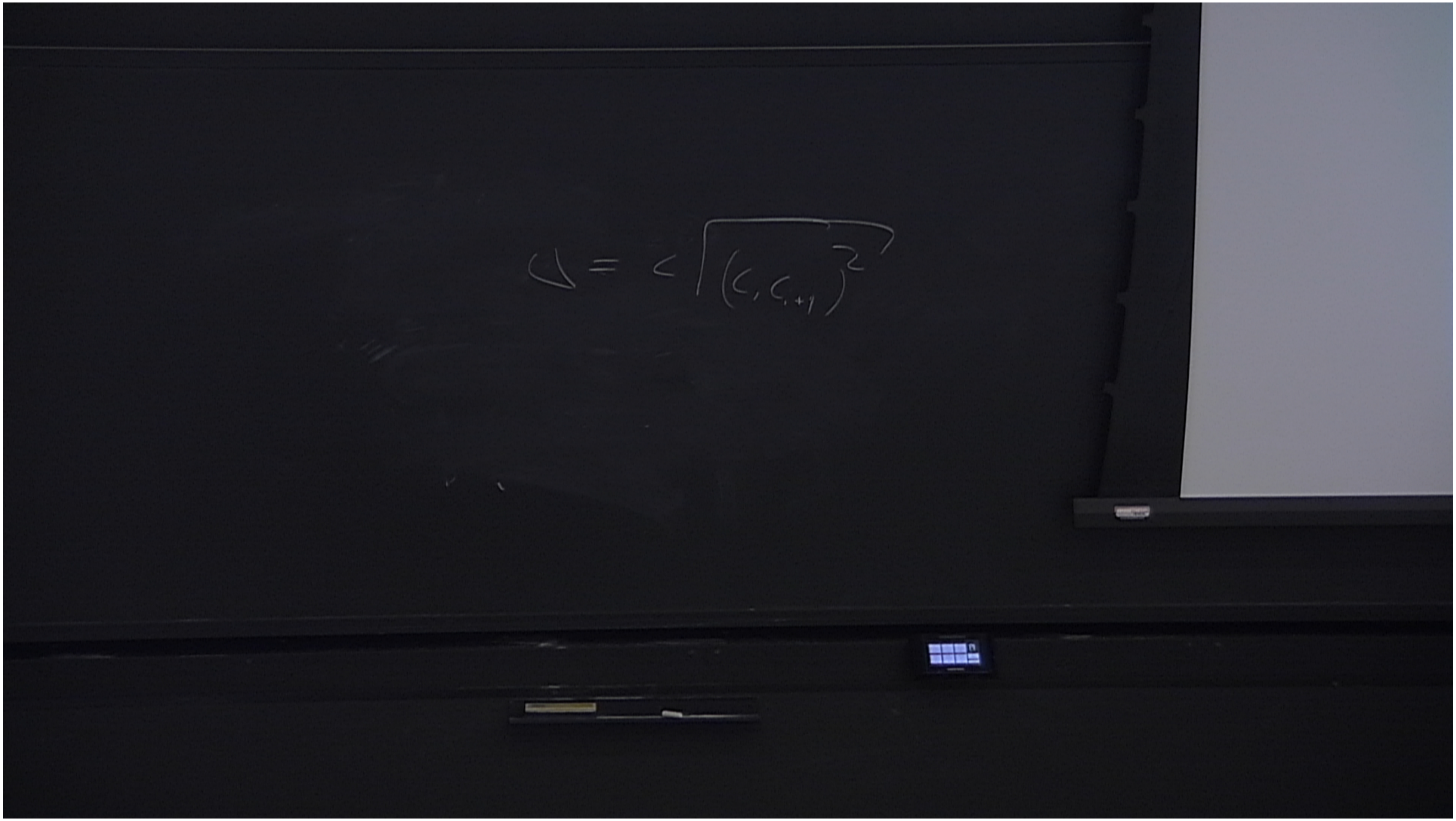
Vortices at finite distance

Cheng & Lutchyn, PRL 2010

$$\epsilon \sim (kR)^{-1/2} Y(kR) \exp\left(-R/\xi\right)$$

$$Y(kR) = \cos(kR+\alpha) - \frac{2}{\lambda}\sin(kR+\alpha) + \frac{2(1+\lambda^2)^{1/4}}{\lambda}$$

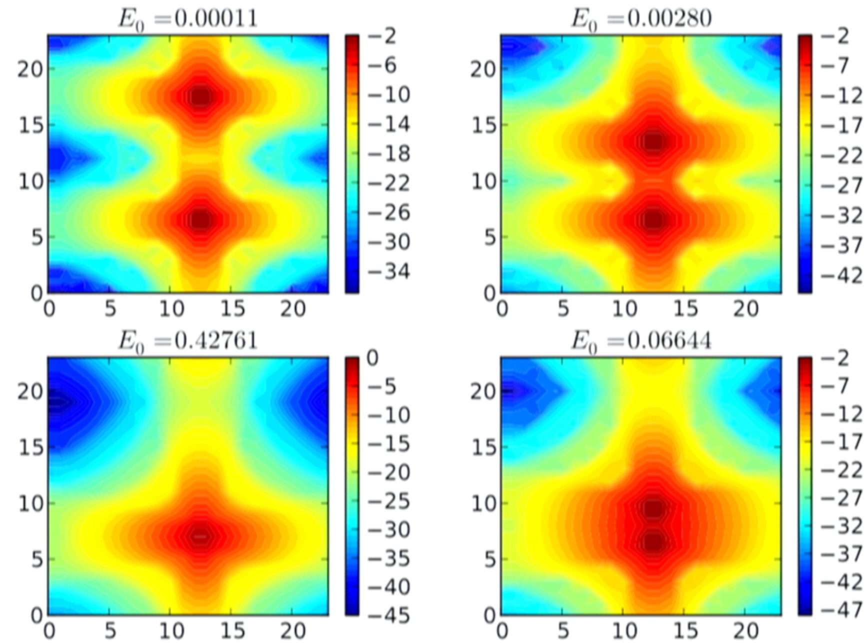
- Ground state in a finite system is split by an exponentially small amount
- Oscillatory terms at short scales



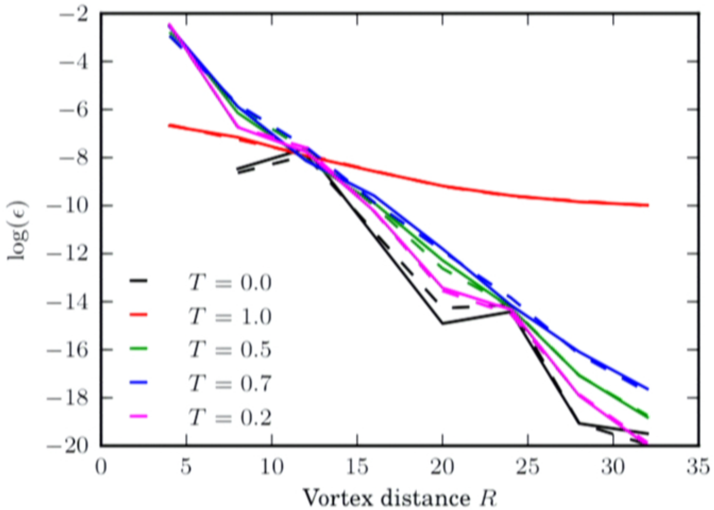
Vortices at finite distance

Torus
24 x 24

Logarithm
of local DoS
at *T=0*

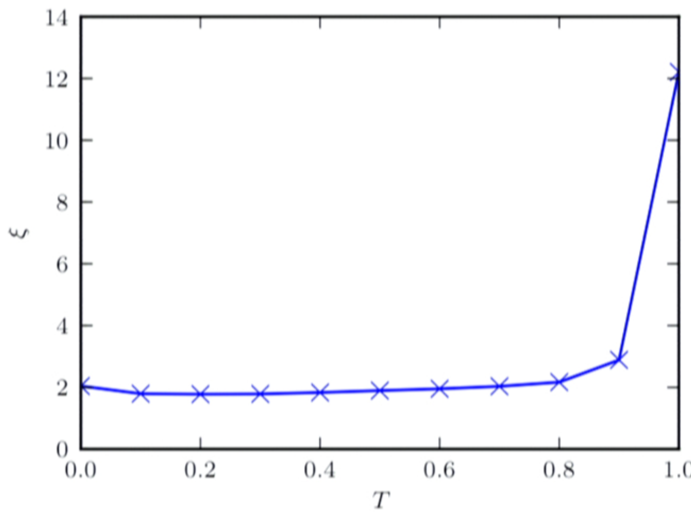


Low *T*: splitting

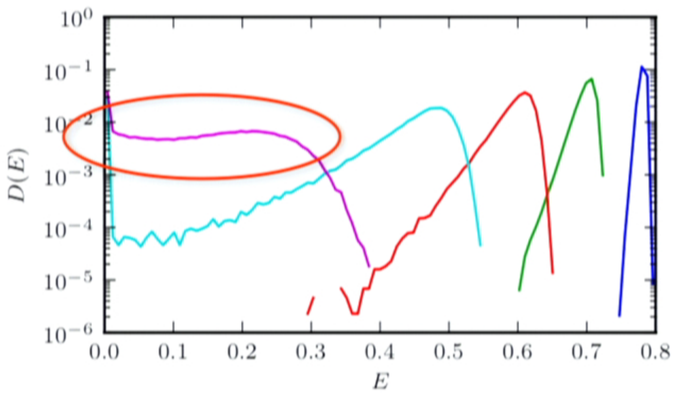


$$\varepsilon = \frac{c_1}{\sqrt{R}} \exp\left(-\frac{x}{\xi}\right)(1 + c_2 \cos(c_3 x + c_4))$$

Topological properties persist for $T < T_{KT}$



Low T: DoS



$T = 0.1, 0.3, 0.5, 0.7, 0.9$

$$D(E,T) = \frac{1}{N}\left\langle \sum_n \delta(E - E_n) \right\rangle_T$$

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Disorder phase diagram

- Symmetry classification of single-particle Hamiltonians (*Dyson 1962, Altland and Zirnbauer, PRB 1996*):

Cartan label	T	C	S
A (unitary)	0	0	0
AI (orthogonal)	+1	0	0
AII (symplectic)	−1	0	0
AIII (ch. unit.)	0	0	1
BDI (ch. orth.)	+1	+1	1
CII (ch. sympl.)	−1	−1	1
D (BdG)	0	+1	0
C (BdG)	0	−1	0
DIII (BdG)	−1	+1	1
CI (BdG)	+1	−1	1

Ryu, NJP 2010

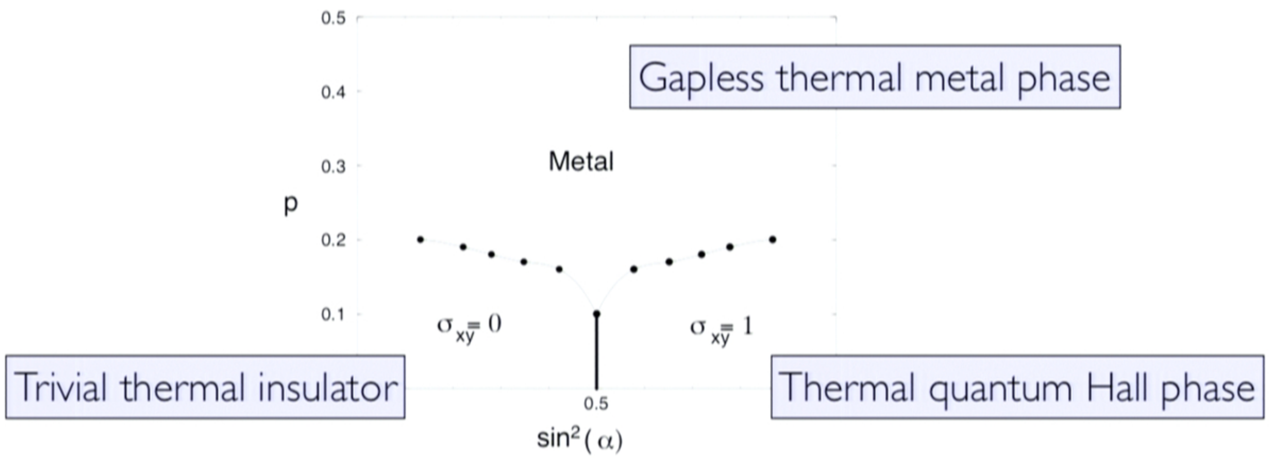
Class	time-rev.	spin-rot.
D	no	no
C	no	yes
DIII	yes	no
CI	yes	yes

Altland & Zirnbauer, PRB 1996

- Predictions for disordered systems:
  - Generic phase diagram for disordered superconductors in class *D*
  - Low-energy level statistics for gapless systems from random matrix theory

Disordered superconductors

- Class *D* superconductor: generic phase diagram expected from field theoretical considerations and numerical simulations (network models)
  - Chalker 2001, Mildenberger 2007:



  - Laumann et al, 2010: interacting Majorana anyons, triangular lattice, sign disorder

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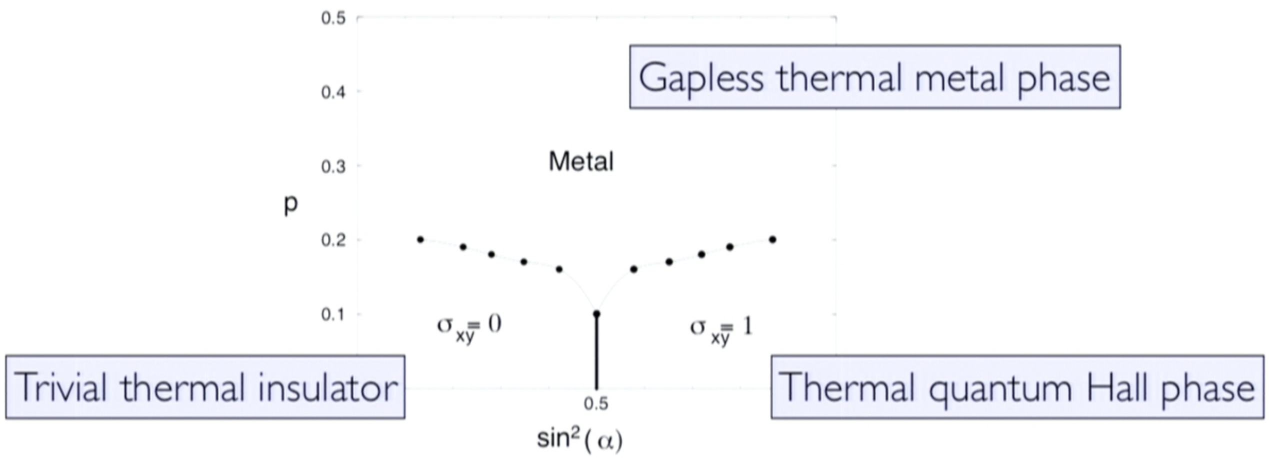

  - Laumann et al, 2010: interacting Majorana anyons, triangular lattice, sign disorder

Disordered phases

Thermal metal

- Quasi-particles form a band of extended states
- Thermal conductivity becomes finite even at *T=0*
- Two smoking gun characterizations:
  - Logarithmic divergence of the DoS

$$D(E) \sim \log \frac{1}{E}$$

  - Level statistics at lowest energies described by random matrix theory

$$D(E) \sim 1 + \frac{\sin(2\pi\gamma E L^2)}{2\pi E L^2}$$

Insulator

- Quasi-particles form localized states and the thermal conductivity vanishes
- Not necessarily a spectral gap: DoS at *E=0* can be finite, but must be independent of system size, and states at *E=0* must be localized

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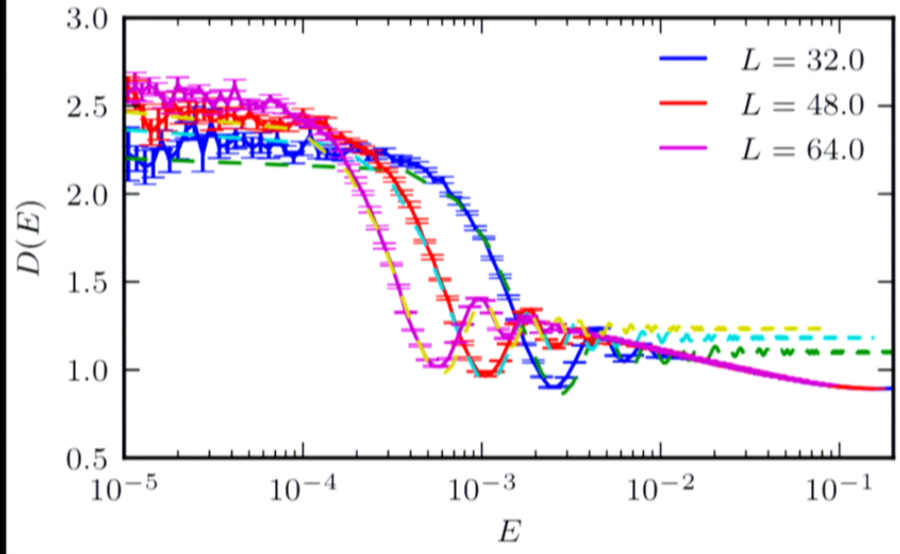
Thermal metal



$T = 1.5 > T_{KT}$

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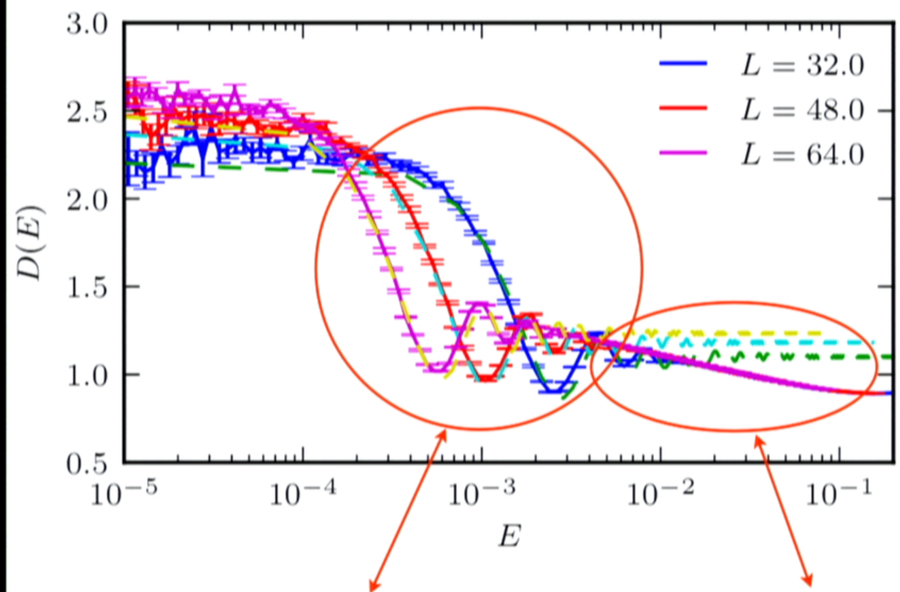
Thermal metal



Random matrix theory oscillations

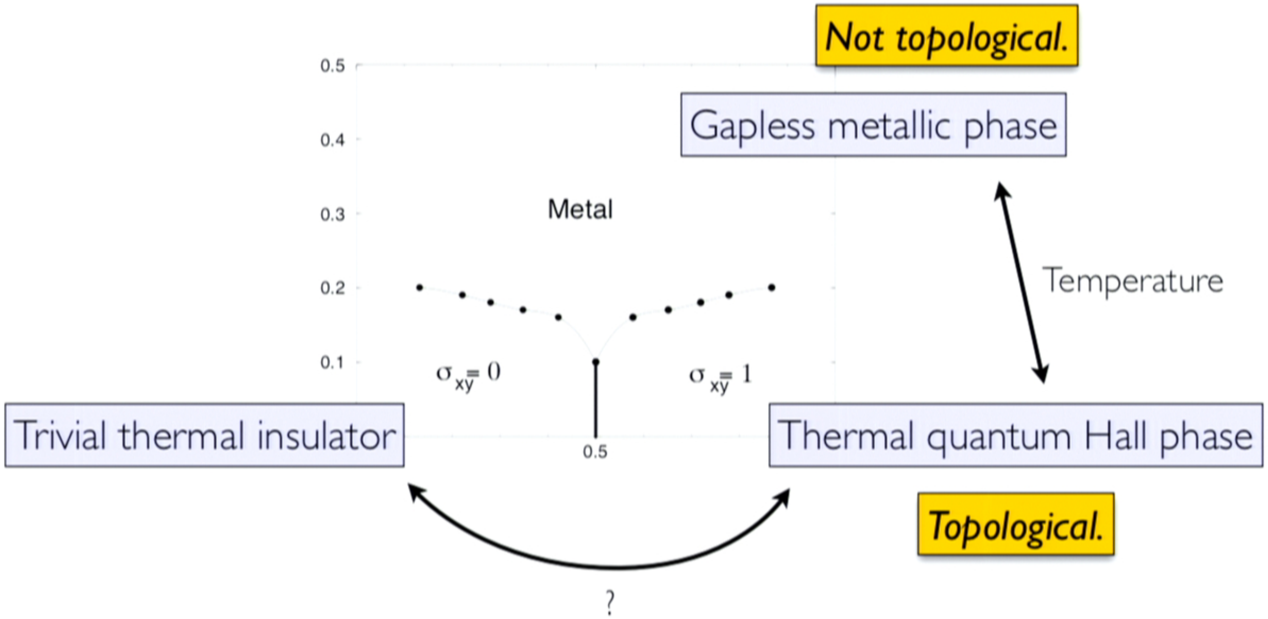
Logarithmic divergence

$T = 1.5 > T_{KT}$

RMT fit (dashed lines):

$$D(E) \sim \gamma + \frac{\sin(2\pi\gamma E L^2)}{2\pi E L^2}$$

Other phases?



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Full and half quantum vortices

$$H = -\sum_{\langle i,j\rangle,\sigma} t\left(c^\dagger_{i\sigma}c_{j\sigma} + c^\dagger_{j\sigma}c_{i\sigma}\right) - \mu\sum_{i\sigma} c^\dagger_{i\sigma}c_{i\sigma}$$
$$+ \sum_{\langle i,j\rangle,\sigma,\sigma'} \left(\Delta^{\sigma\sigma'}_{ij} c^\dagger_{i\sigma}c^\dagger_{j\sigma'} + (\Delta^{\sigma\sigma'}_{ij})^* c_{j\sigma'}c_{i\sigma}\right)$$

Full quantum vortices

$$\Delta^{\sigma\sigma'}_{ij} = \delta_{\sigma\sigma'}\Delta_{ij}$$

Not topological!

In-plane field: $H' = H + \alpha\sum\sigma_y$

Vortices at finite energy: $E_0 \approx \alpha$

Half quantum vortices

$$\Delta^{\uparrow\uparrow}_{ij} = \Delta_{ij} \qquad \Delta^{\downarrow\downarrow}_{ij} = 1$$

Topological!

In-plane field: $H' = H + \alpha\sum\sigma_y$

Vortices at zero energy: $E_0 \approx 0$

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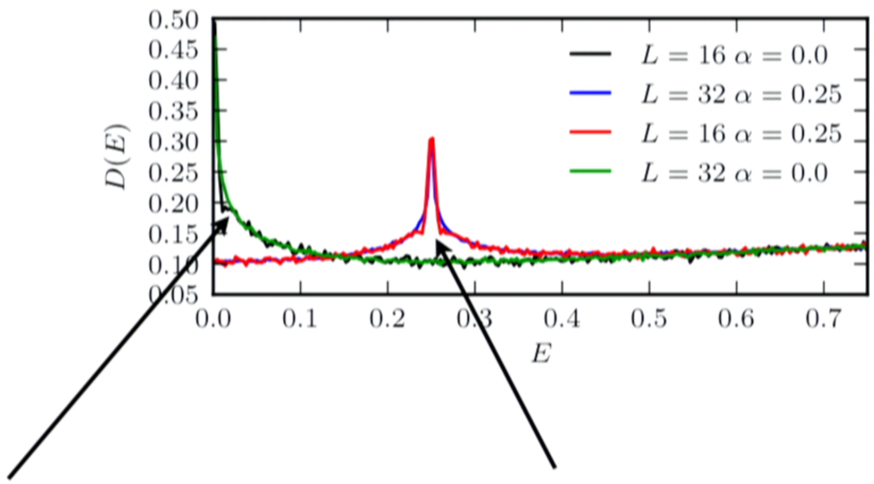
Trivial thermal insulator



Thermal metal at *E=0*

Band at $E \sim \alpha$?

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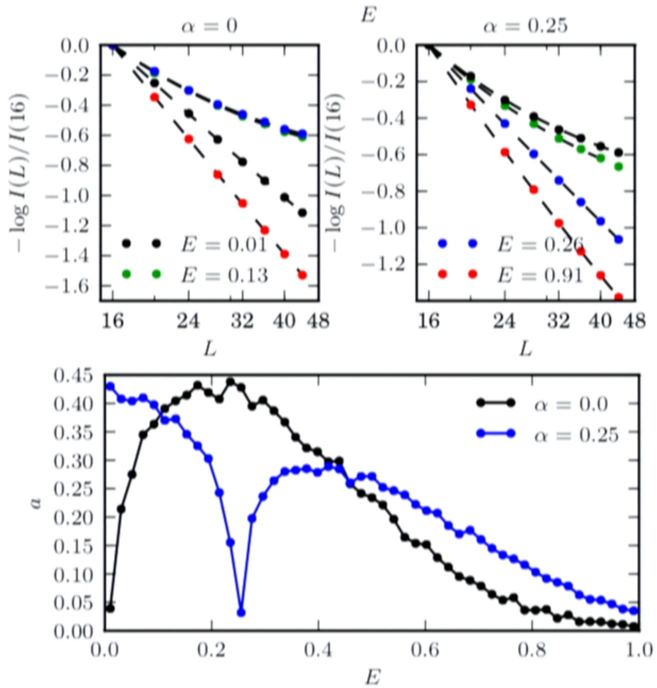
Inverse participation ratio

$$I(E) = \left\langle \sum_n \frac{\langle u_n \rangle^4 + \langle v_n \rangle^4}{(\langle u_n \rangle^2 + \langle v_n \rangle^2)^2} \delta(E - E_n) \right\rangle_T$$



Extended states

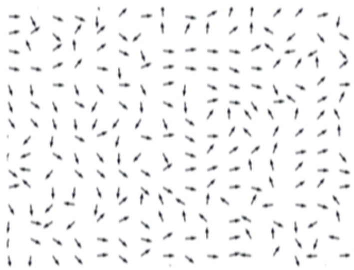
$I(E) \sim L^{2-\nu}$

Localized states

$\lim_{L \to \infty} I(E) \neq 0$

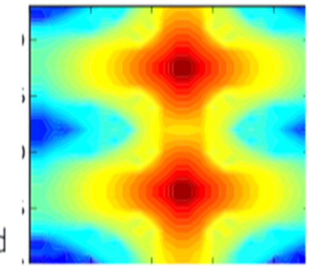
Fit: $I(E) \sim a + L^b$

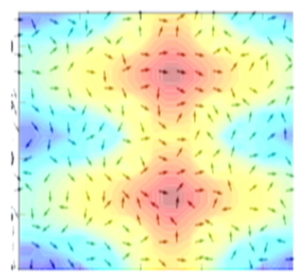
HQVs + FQVs



Phase configuration from XY model at finite T: ***Full quantum vortices***

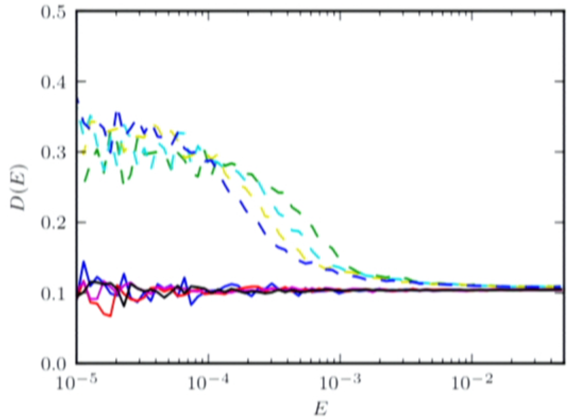
Half-quantum-vortex-antivortex pair at fixed location *R=L/2*



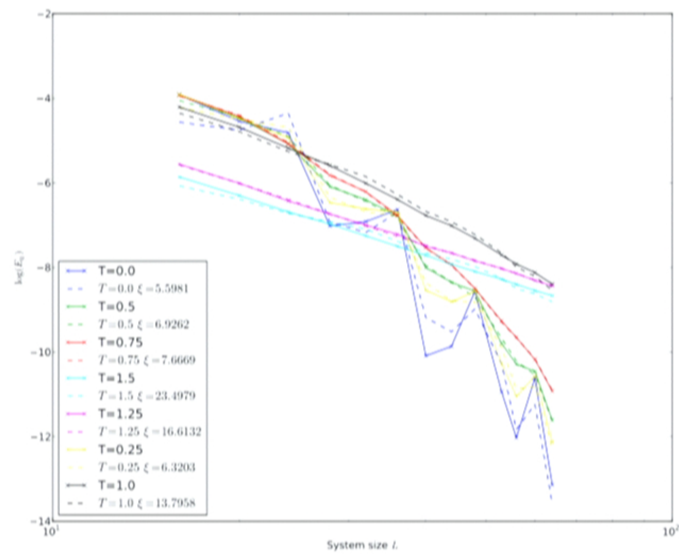


Research

Topological degeneracy?



- HQVs give discernible contribution to low-energy DoS
- Splitting is power-law in the high-temperature phase



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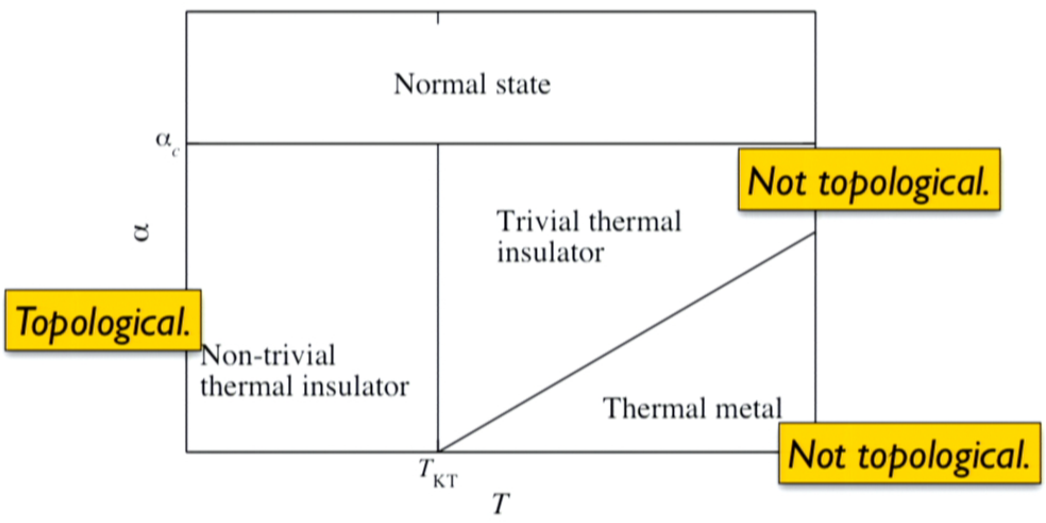
Conclusion



- KT temperature may be a relevant temperature scale for quantum computation
- Thermal fluctuations may generate the disorder phase diagram of a class *D* superconductor

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