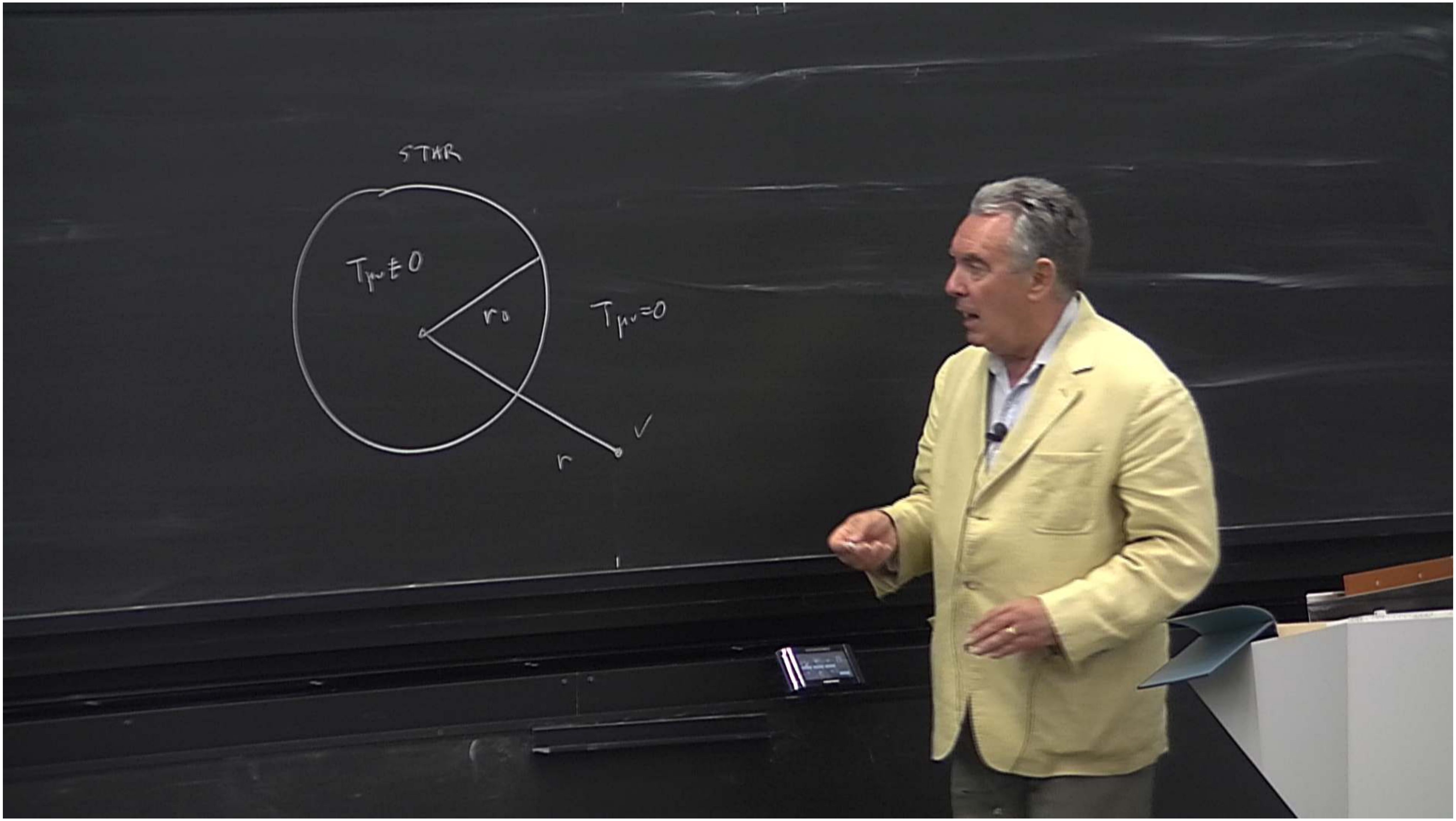
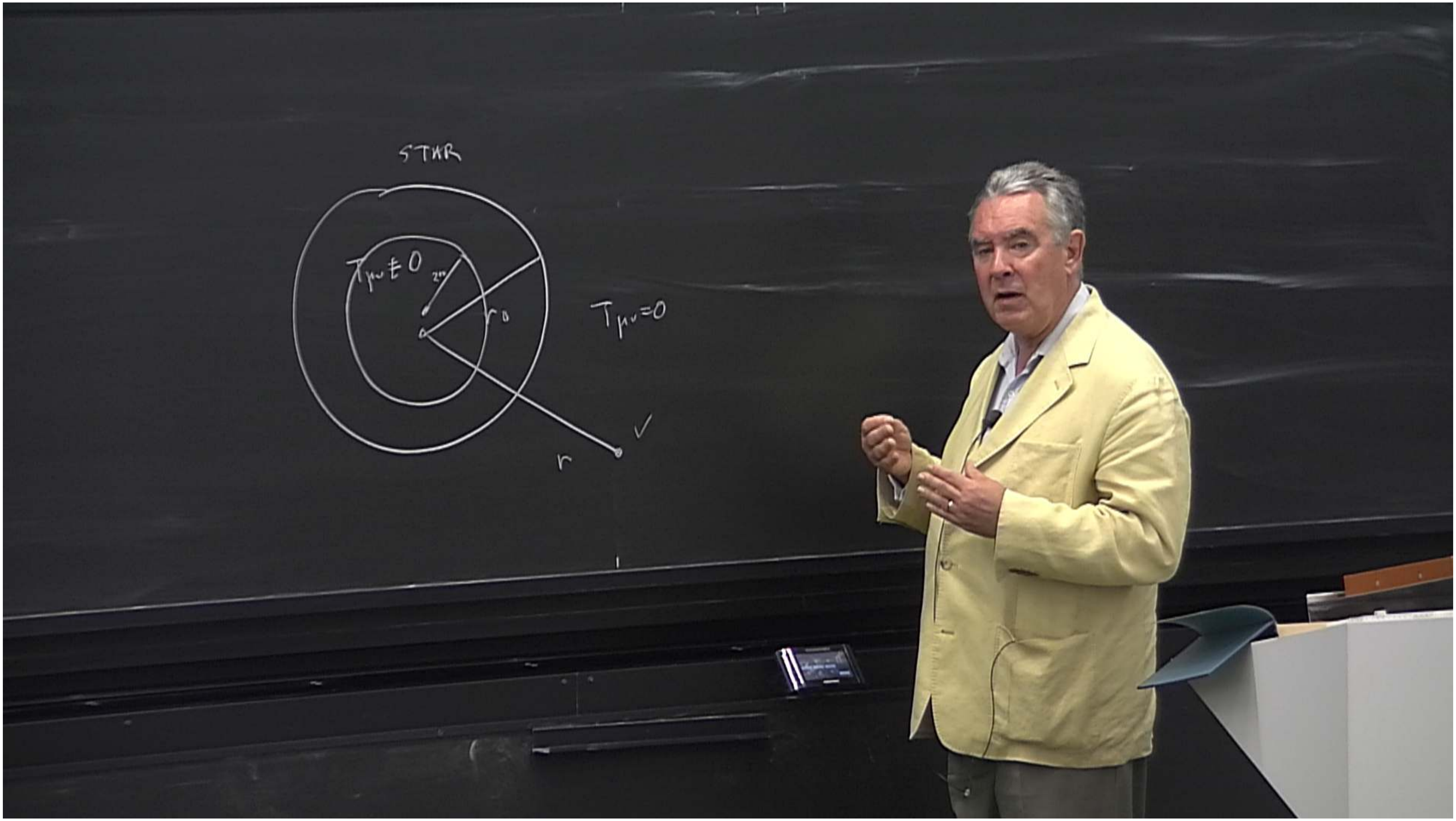
Title: Relativity - Lecture 8

Date: Sep 19, 2012 10:30 AM

URL: http://pirsa.org/12090028

Abstract:


STAR
$T_{\mu\nu} \neq 0$
r_0
$T_{\mu\nu} = 0$
r


STAR
Tμν ≠ 0
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Tμν=0
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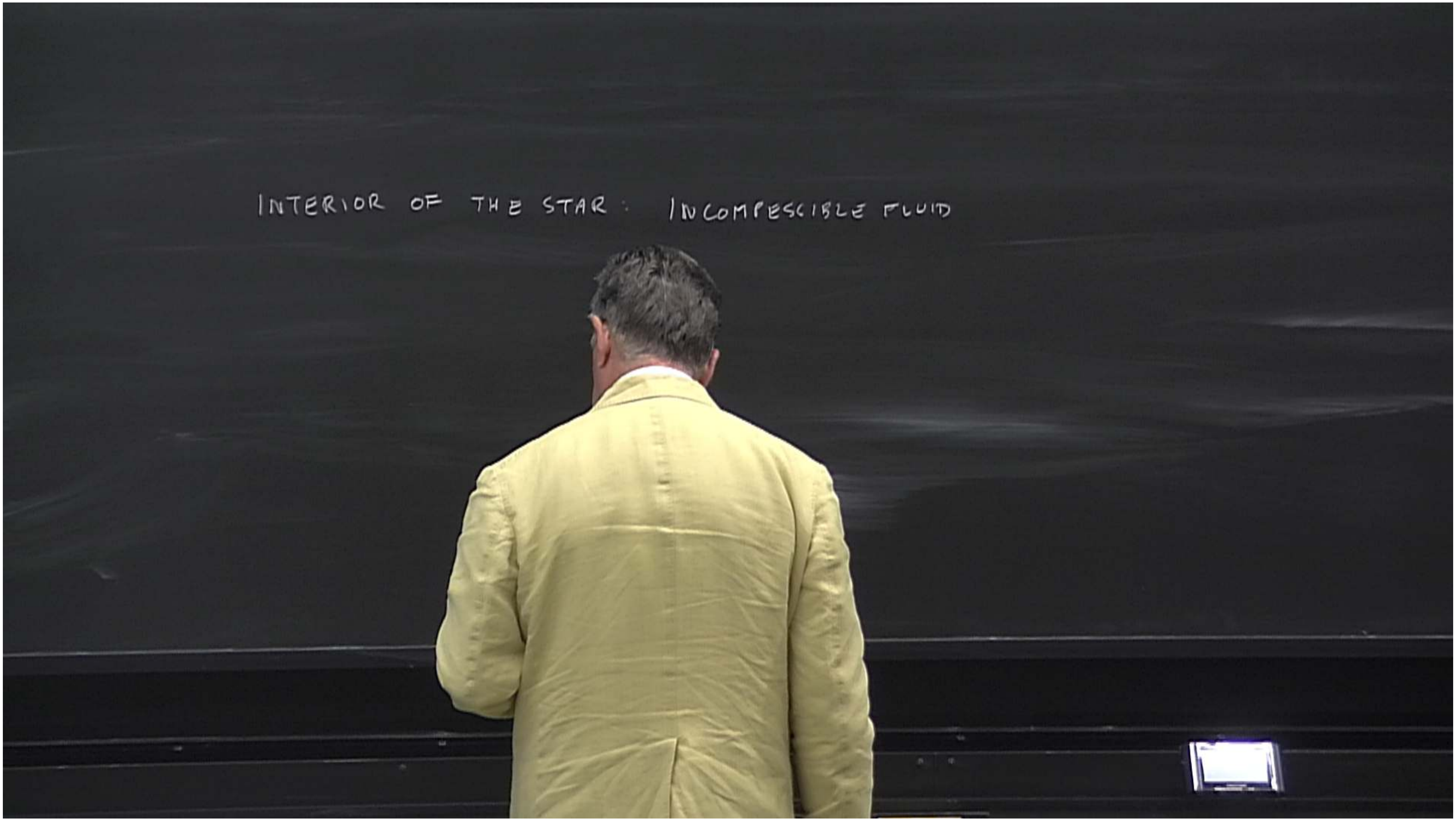
INTERIOR OF THE STAR: INCOMPESCIBLE FLUID

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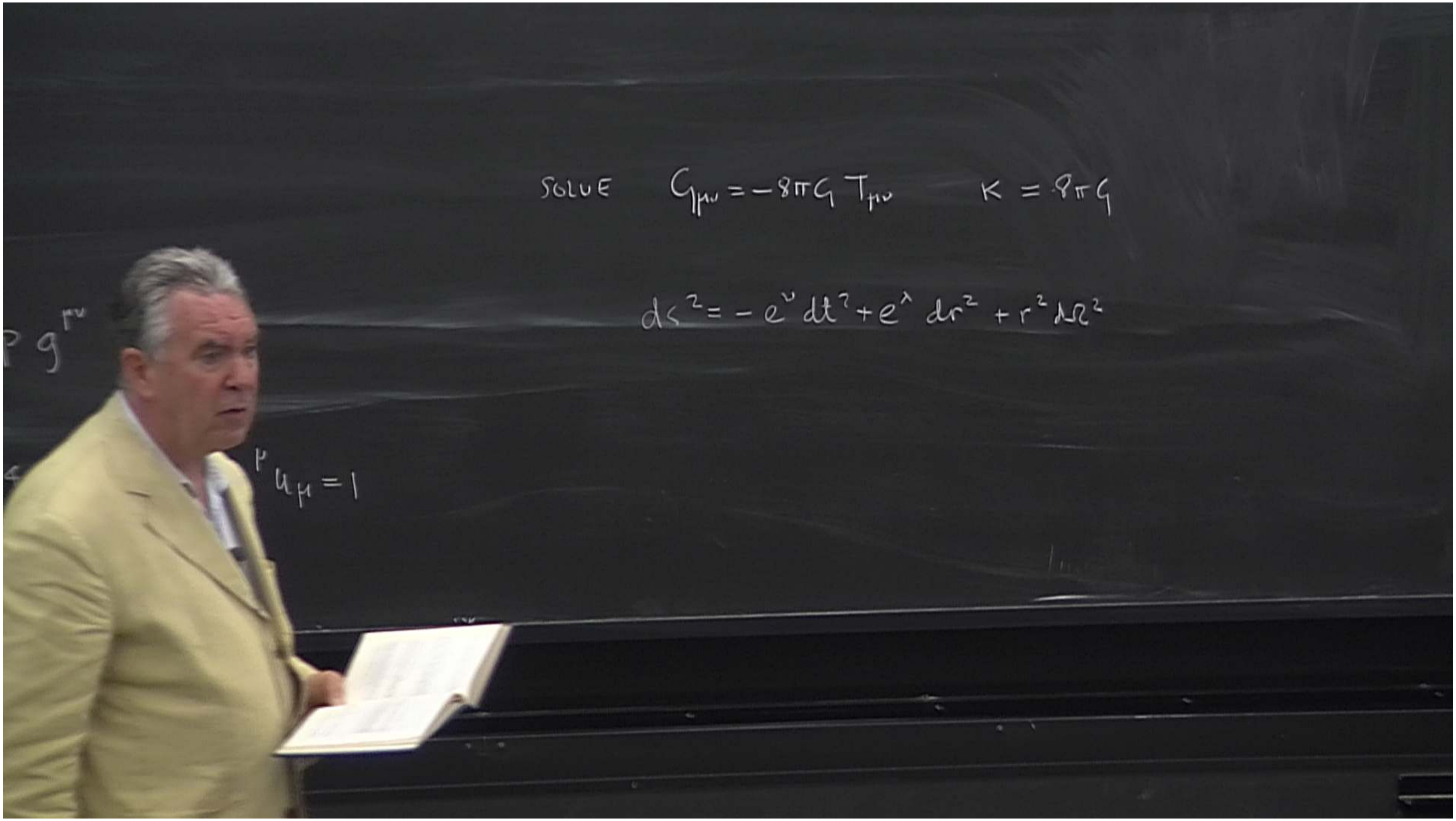
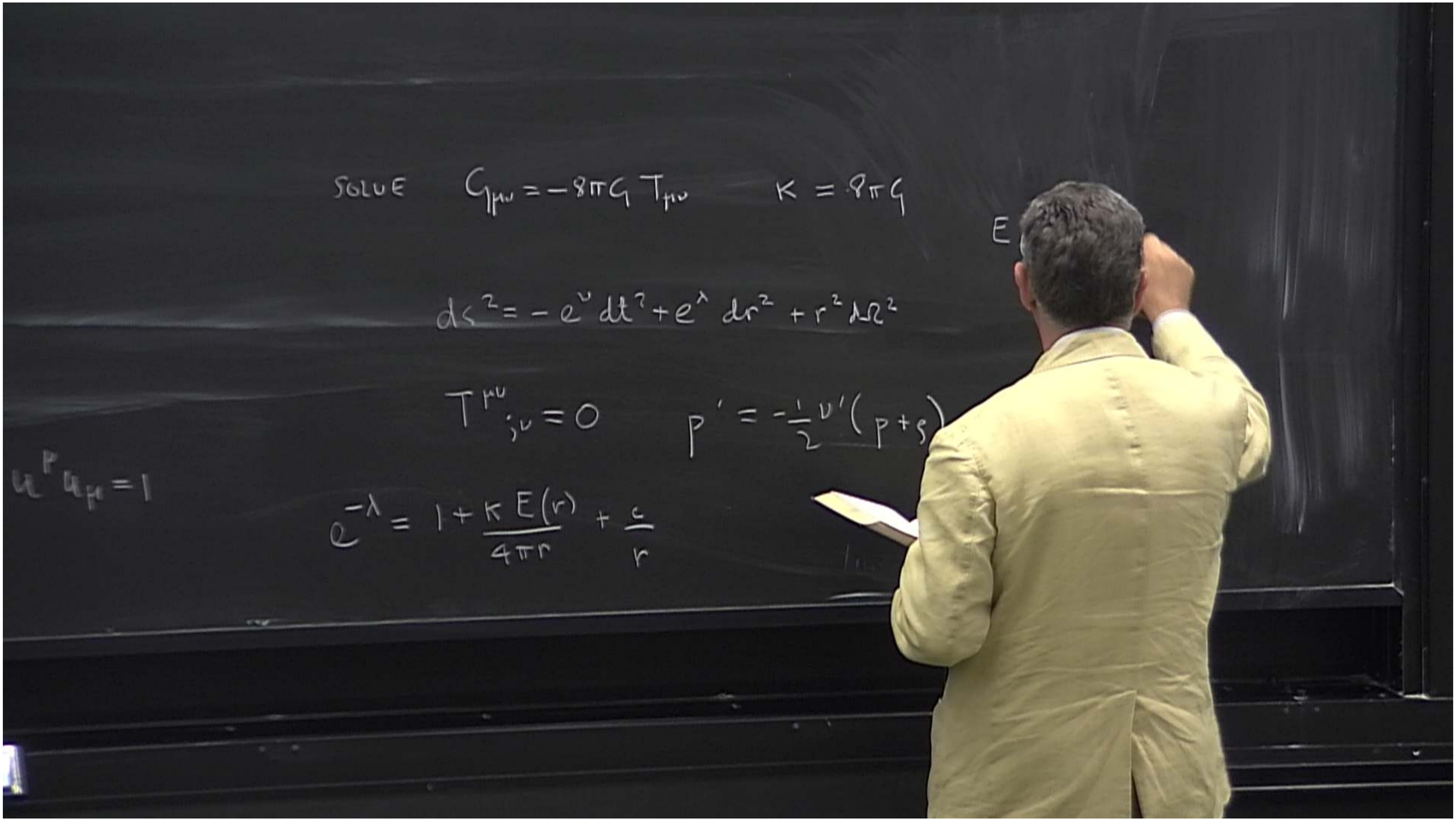
$$T^{\mu\nu} = (\rho + p)\, u^{\mu} u^{\nu} - p\, g^{\mu\nu}$$

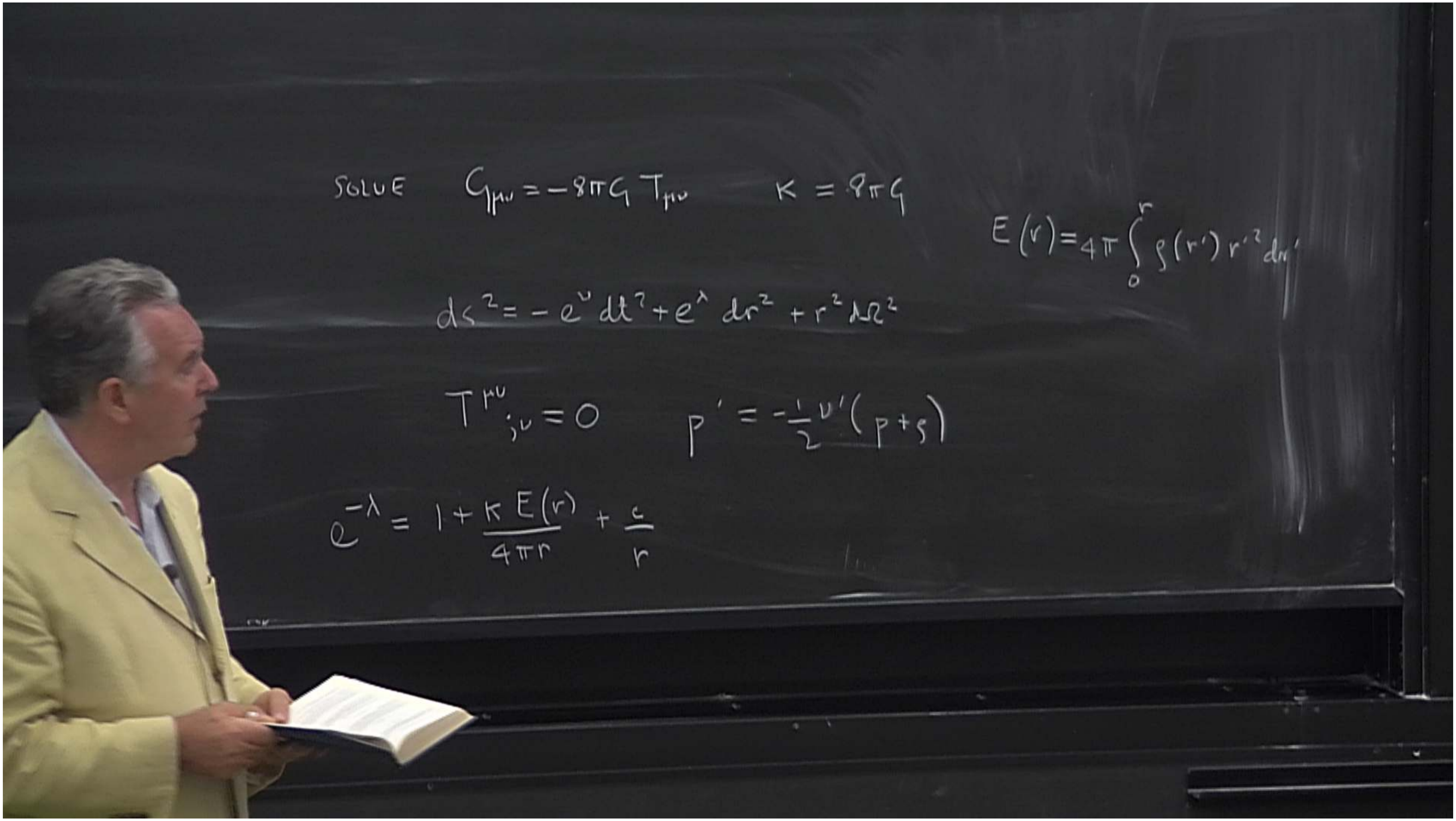
ρ = ENERGY DENISTY

u^{μ} 4 VELOCITY $u^{\mu} u_{\mu} = 1$

p = PRESSURE


SOLVE $G_{\mu\nu} = -8\pi G\, T_{\mu\nu}$ $\kappa = 8\pi G$
$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2 d\Omega^2$
$u_\mu = 1$


SOLVE $G_{\mu\nu} = -8\pi G \, T_{\mu\nu}$
$\kappa = 8\pi G$
E
$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2 d\Omega^2$
$T^{\mu\nu}{}_{;\nu} = 0$
$p' = -\frac{1}{2}\nu'(p+\rho)$
$u^{\mu} u_{\mu} = 1$
$e^{-\lambda} = 1 + \frac{\kappa E(r)}{4\pi r} + \frac{c}{r}$

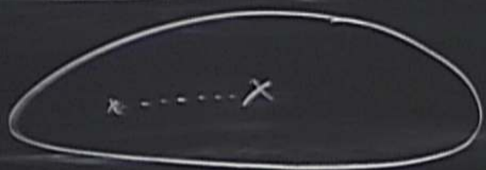

SOLVE $G_{\mu\nu} = -8\pi G\, T_{\mu\nu}$ $\kappa = 8\pi G$
$E(r) = 4\pi \int_0^r \rho(r') r'^2 dr'$
$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2 d\Omega^2$
$T^{\mu\nu}{}_{;\nu} = 0$ $p' = -\frac{1}{2}\nu'(p+\rho)$
$e^{-\lambda} = 1 + \frac{\kappa E(r)}{4\pi r} + \frac{c}{r}$

$$A = \frac{3}{2}\sqrt{1 - \frac{r_0^2}{R^2}} \qquad B = \frac{1}{2}$$

$$e^{\nu/2} = \frac{3}{2}\sqrt{1 - \frac{r_0^2}{R^2}} - \frac{1}{2}\sqrt{1 - \frac{r^2}{R^2}}$$

$$p = \rho\left[\frac{\sqrt{1 - r^2/R^2} - \sqrt{1 - r_0^2/R^2}}{3\sqrt{1 - r_0^2/R^2} - \sqrt{1 - r^2/R^2}}\right]$$

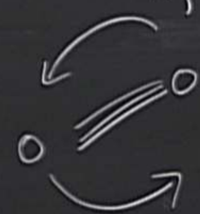
$$A = \frac{3}{2}\sqrt{1-\frac{r_0^2}{R^2}} \qquad B = \frac{1}{2}$$



$$e^{\nu/2} = \frac{3}{2}\sqrt{1-\frac{r_0^2}{R^2}} - \frac{1}{2}\sqrt{1-\frac{r^2}{R^2}}$$

$$p = \rho\left[\frac{\sqrt{1-r^2/R^2} - \sqrt{1-r_0^2/R^2}}{3\sqrt{1-r_0^2/R^2} - \sqrt{1-r^2/R^2}}\right]$$

$$e^{\lambda} = 1 - \frac{r^2}{R^2} \qquad R^2 = -\frac{3}{\kappa\rho}$$



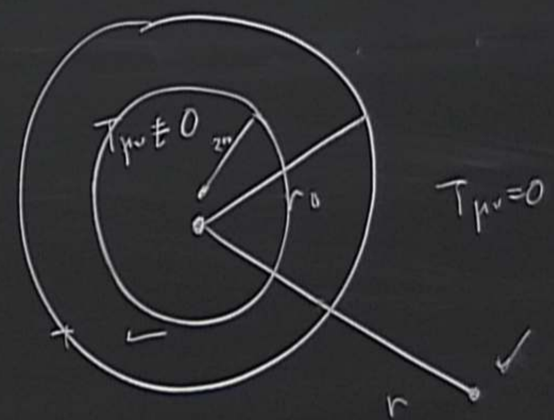
$$e^{\nu/2} = A - B\sqrt{1 - \frac{r^2}{R^2}}$$

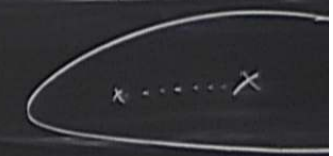
FINITE

PRESSURE

$$r_0^2 < \frac{8R^2}{9}$$

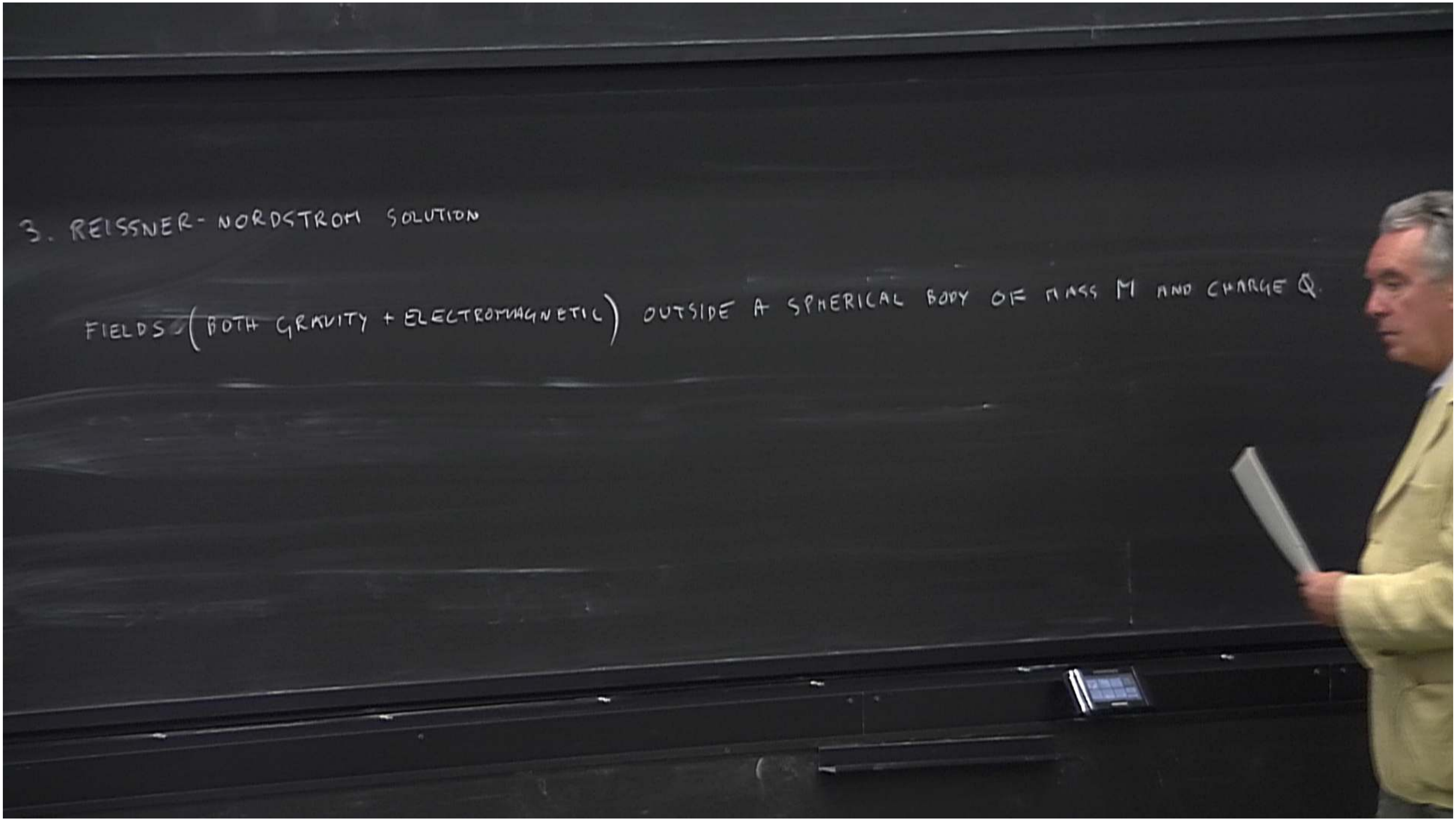
STAR





REAL:

$$r_0^2 < R^2$$


3. REISSNER-NORDSTROM SOLUTION
FIELDS (BOTH GRAVITY + ELECTROMAGNETIC) OUTSIDE A SPHERICAL BODY OF MASS M AND CHARGE Q.

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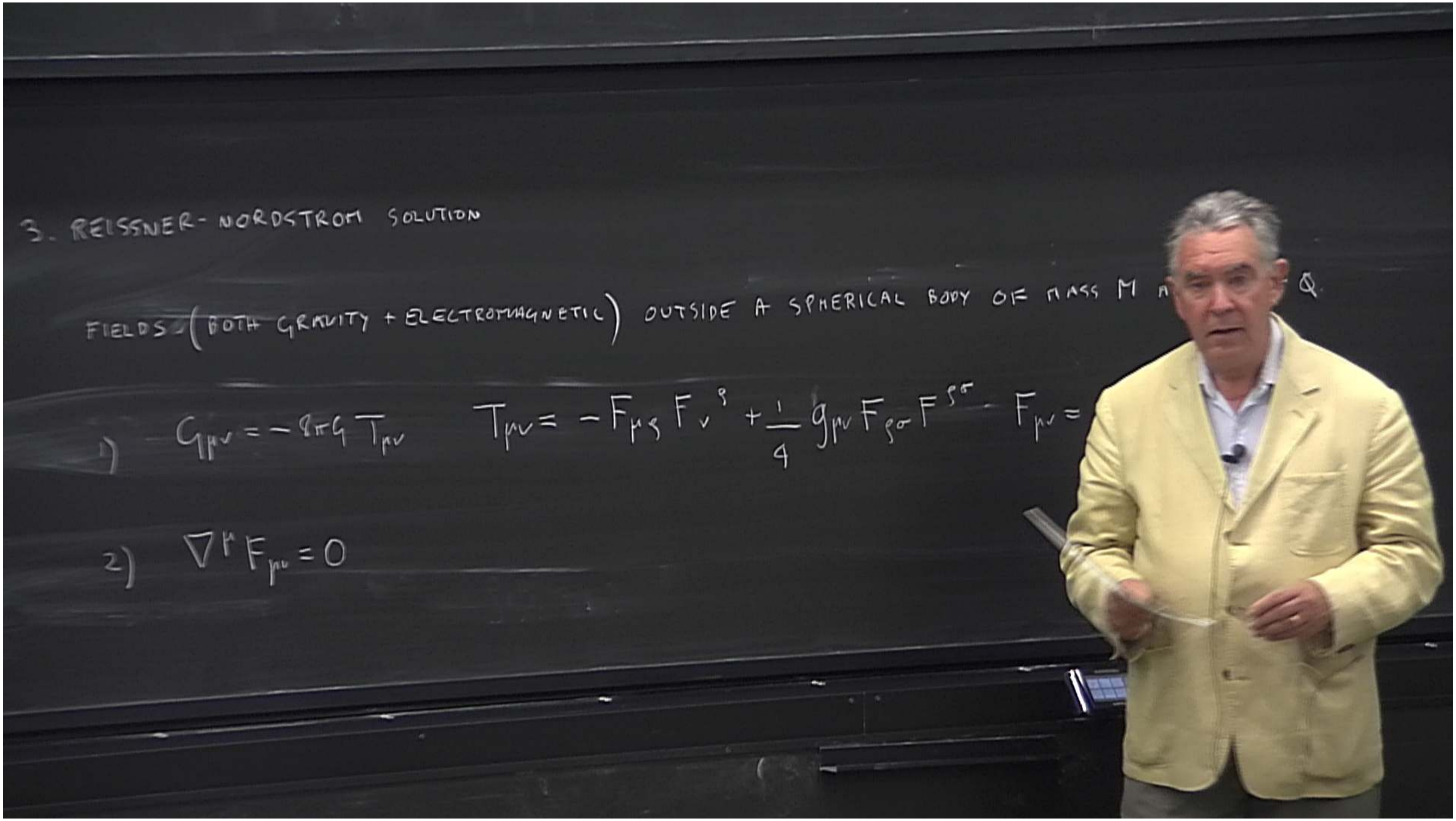
FIELDS (BOTH GRAVITY + ELECTROMAGNETIC) OUTSIDE A SPHERICAL BODY OF MASS M AND CHARGE Q.

$$G_{\mu\nu} = -8\pi G\, T_{\mu\nu} \qquad T_{\mu\nu} = -F_{\mu\rho} F_{\nu}{}^{\rho} + \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$


3. REISSNER-NORDSTROM SOLUTION
FIELDS (BOTH GRAVITY + ELECTROMAGNETIC) OUTSIDE A SPHERICAL BODY OF MASS M
Q.
1) $G_{\mu\nu} = -8\pi G\, T_{\mu\nu}$ $T_{\mu\nu} = -F_{\mu\rho} F_{\nu}{}^{\rho} + \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}$ $F_{\mu\nu} =$
2) $\nabla^{\mu} F_{\mu\nu} = 0$

3. REISSNER-NORDSTROM SOLUTION

FIELDS (BOTH GRAVITY + ELECTROMAGNETIC) OUTSIDE A SPHERICAL BODY OF MASS M AND CHARGE Q.

1) $G_{\mu\nu} = -8\pi G \, T_{\mu\nu}$ $\quad T_{\mu\nu} = -F_{\mu\rho} F_{\nu}{}^{\rho} + \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}$. $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ $\quad ($

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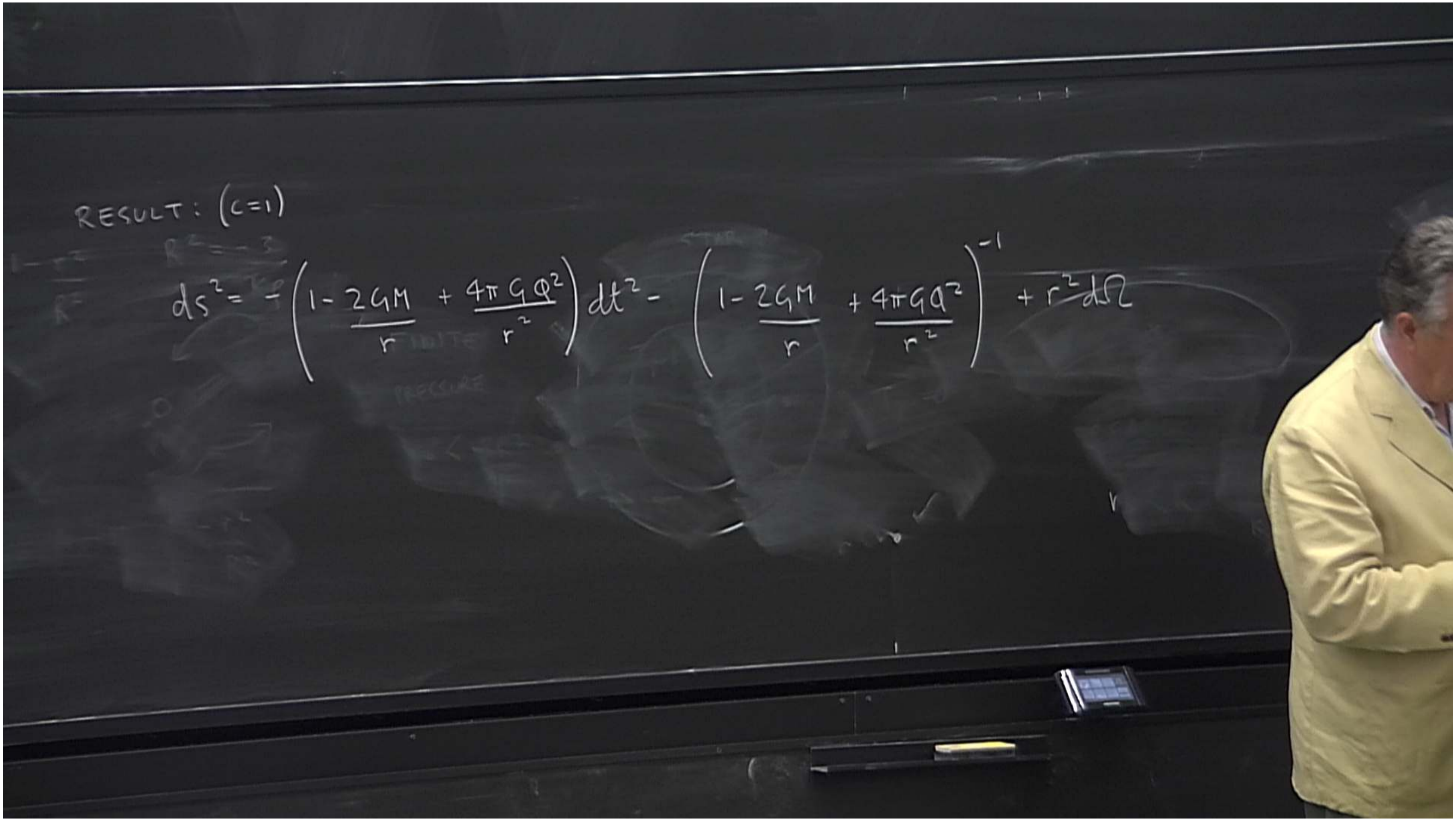
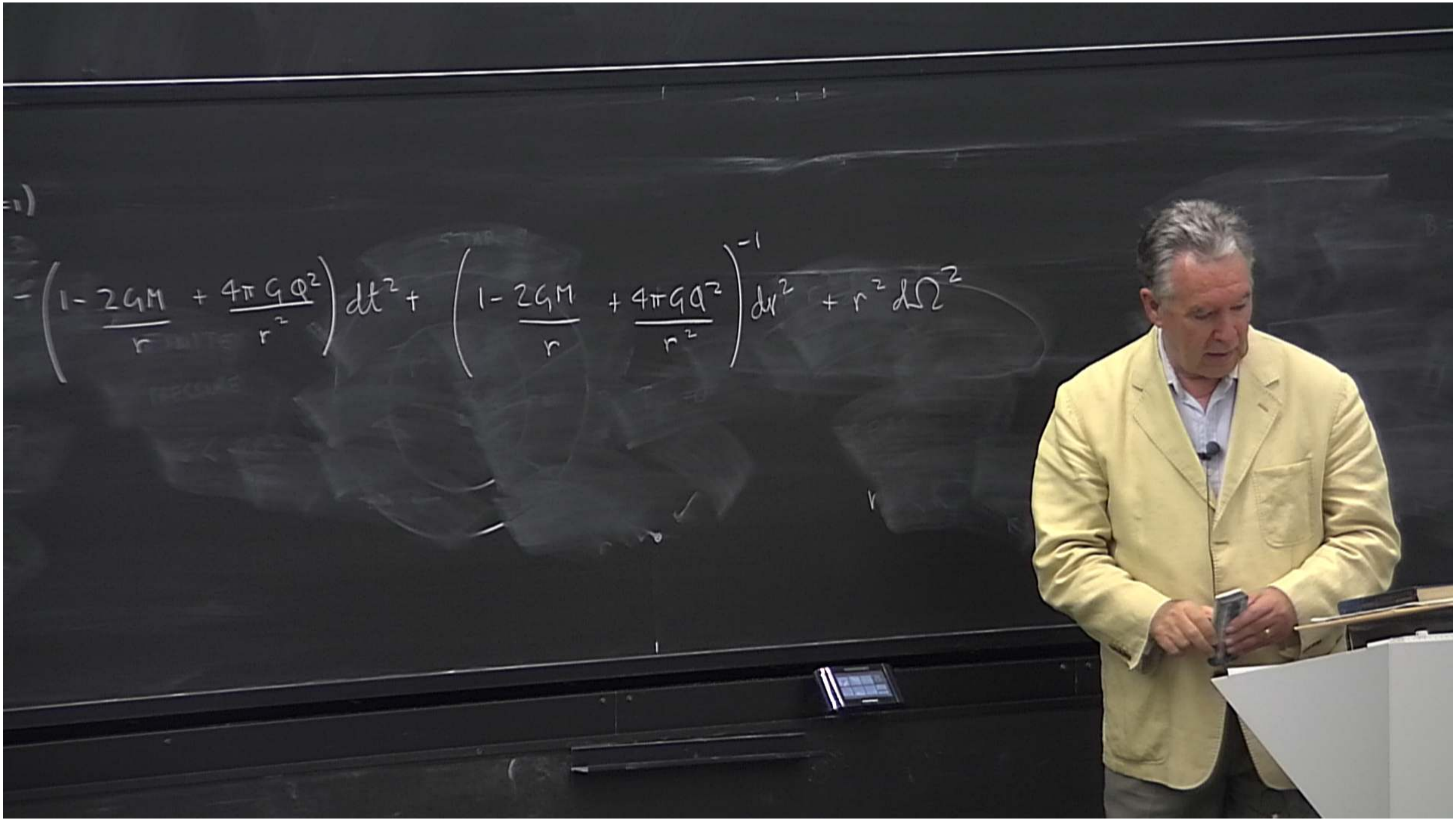
2) $\nabla^{\mu} F_{\mu\nu} = 0$

RESULT: $(c=1)$

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r}\right) dt^2 - \left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r}\right)^{-1} + r^2 d\Omega$$


RESULT: (c=1)
$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\right)^{-1} + r^2 d\Omega$

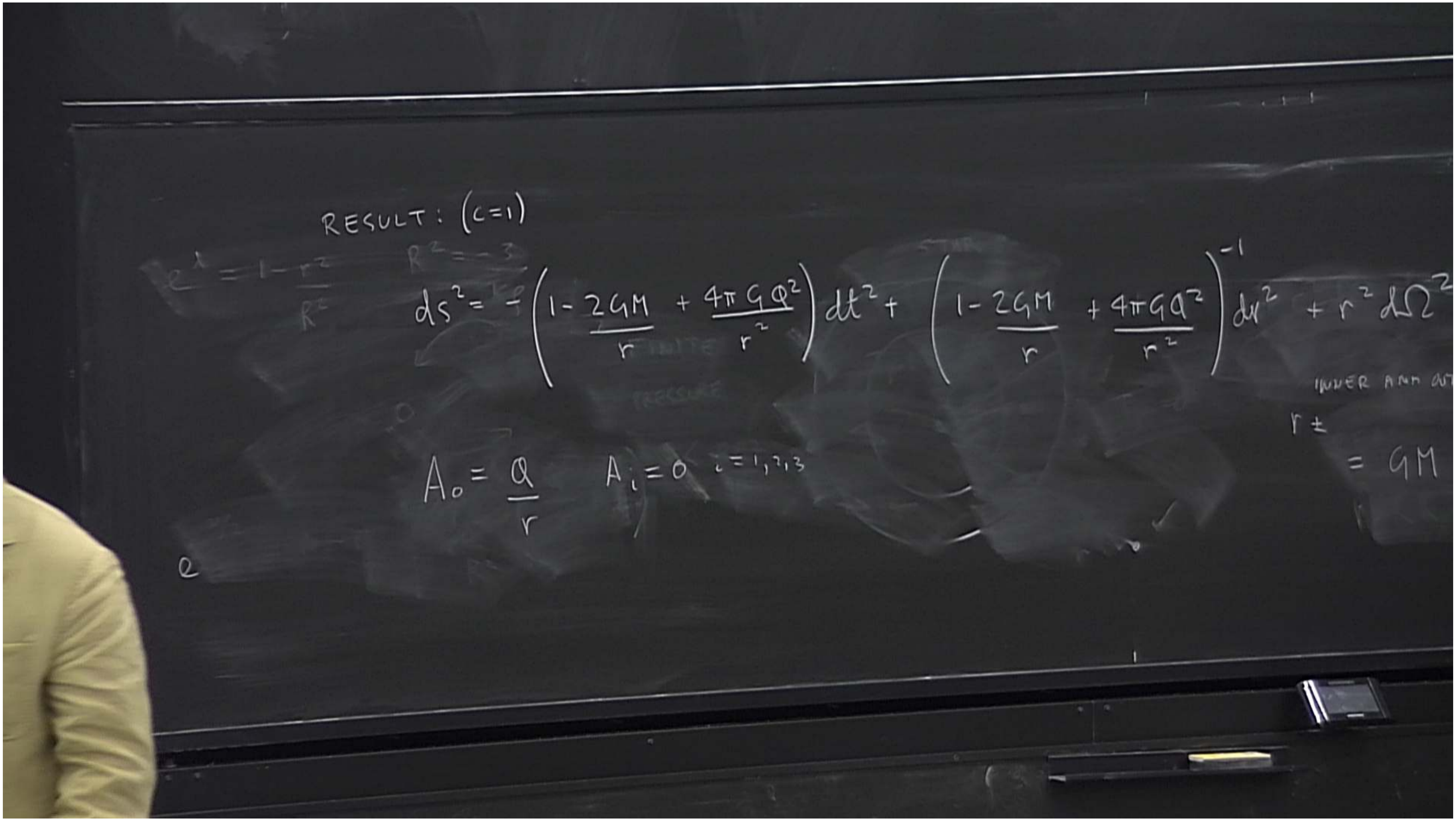

$-\left(1-\frac{2GM}{r}+\frac{4\pi G Q^2}{r^2}\right)dt^2+\left(1-\frac{2GM}{r}+\frac{4\pi G Q^2}{r^2}\right)^{-1}dr^2+r^2d\Omega^2$

$$\frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\Big)^{-1} dr^2 + r^2 d\Omega^2 = -\frac{(r-r_+)(r-r_-)}{r^2} dt^2 + \frac{r^2}{(r-r_+)(r-r_-)} dr^2 + r^2 d\Omega^2$$

INNER AND OUTER HORIZONS

$$r_\pm = GM \pm \sqrt{G^2M^2 - 4\pi G Q^2}$$

$$G^2M^2 > 4\pi G Q^2$$


RESULT: $(c=1)$
$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$
$A_0 = \frac{Q}{r} \quad A_i = 0 \quad i = 1,2,3$
INNER AND OUT
$r_\pm$
$= GM$

RESULT: $(c=1)$

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

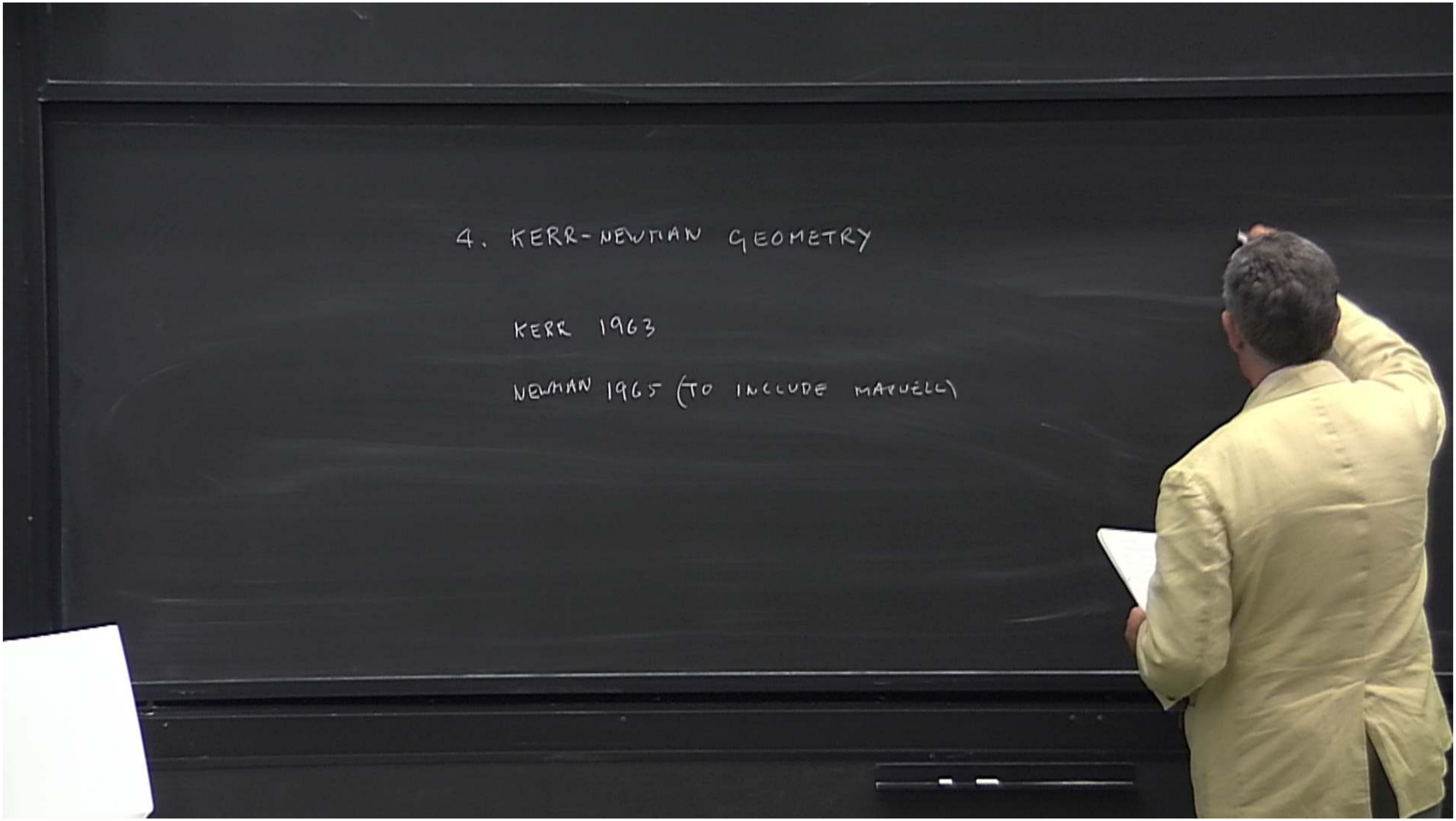
INNER AND OUT

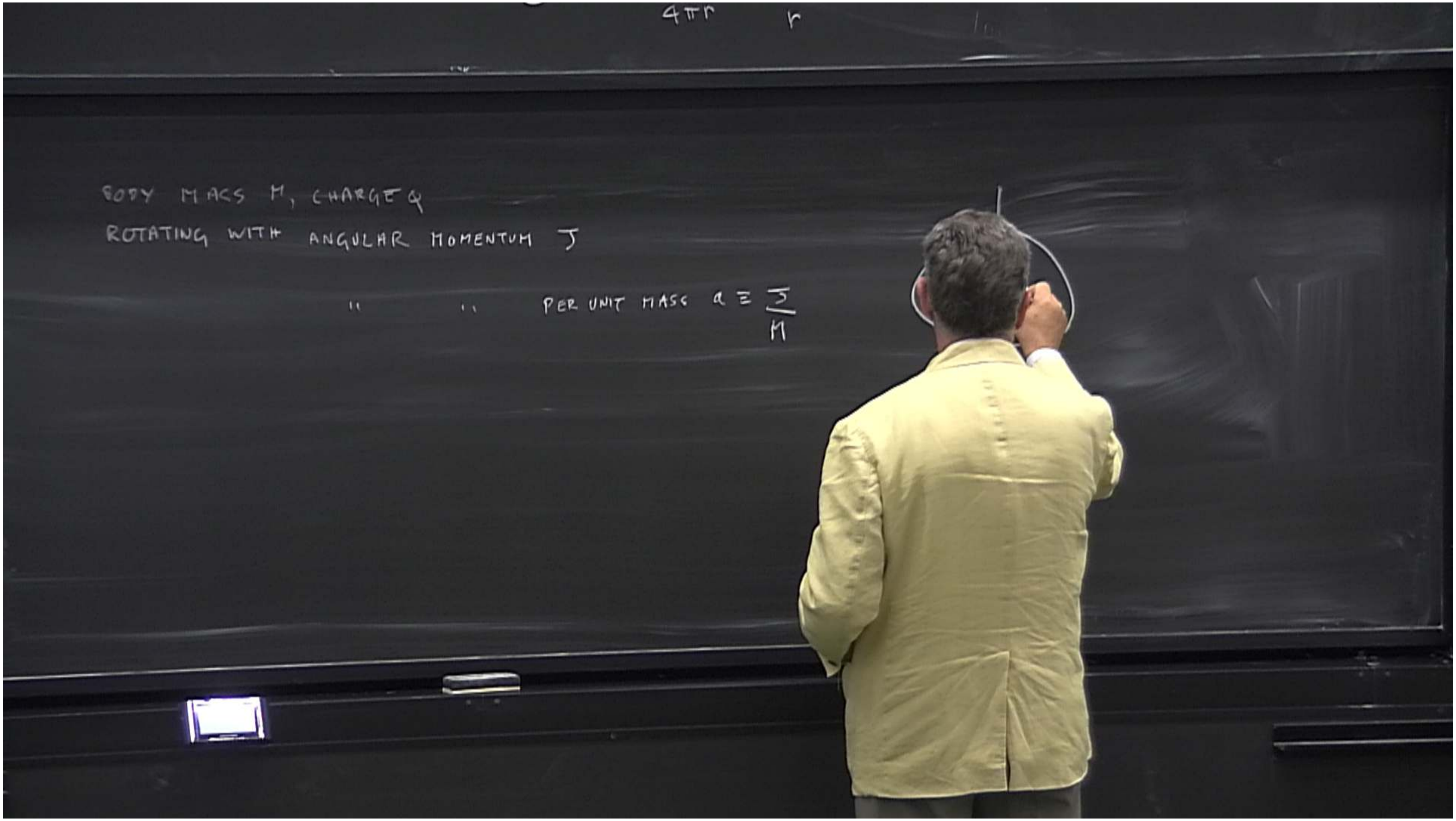
$r_\pm$

$= GM$

$$A_0 = \frac{Q}{r} \qquad A_i = 0 \quad i = 1,2,3$$

EXERCISE: INCLUDE MAGNETIC CHARGE


4. KERR-NEWMAN GEOMETRY
KERR 1963
NEWMAN 1965 (TO INCLUDE MAXWELL)

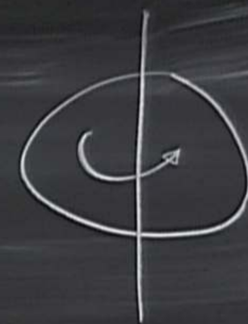

BODY MASS M, CHARGE Q
ROTATING WITH ANGULAR MOMENTUM J
" " PER UNIT MASS a ≡ J/M

$4\pi r \qquad r$

BODY MASS M, CHARGE Q

ROTATING WITH ANGULAR MOMENTUM J

" " PER UNIT MASS $a \equiv \frac{J}{M}$



$$ds^2 = -\frac{\Delta}{\rho}\left[dt^2 - a\sin^2\theta\, d\phi^2\right] + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)\,d\phi - a\,dt\right]^2$$

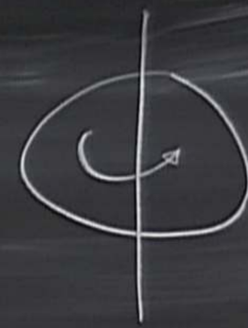
$$+ \frac{\rho^2}{\Delta}\,dr^2 + \rho^2\, d\theta^2 \qquad \Delta \equiv r^2 - 2Mr + a^2 + 4\pi Q^2$$

$4\pi r \quad r$

BODY MASS M, CHARGE Q

ROTATING WITH ANGULAR MOMENTUM J

" " PER UNIT MASS $a \equiv \frac{J}{M}$

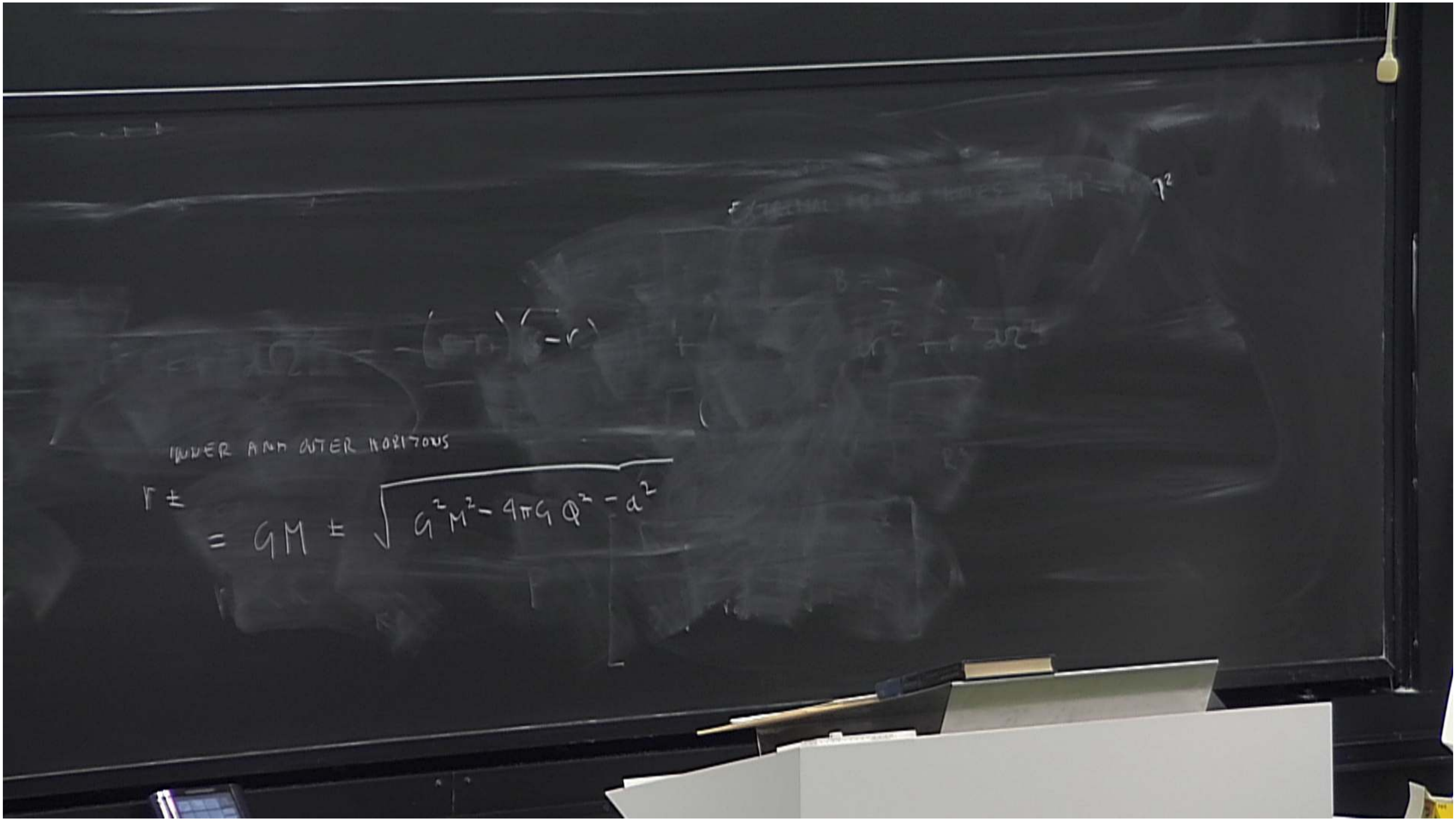


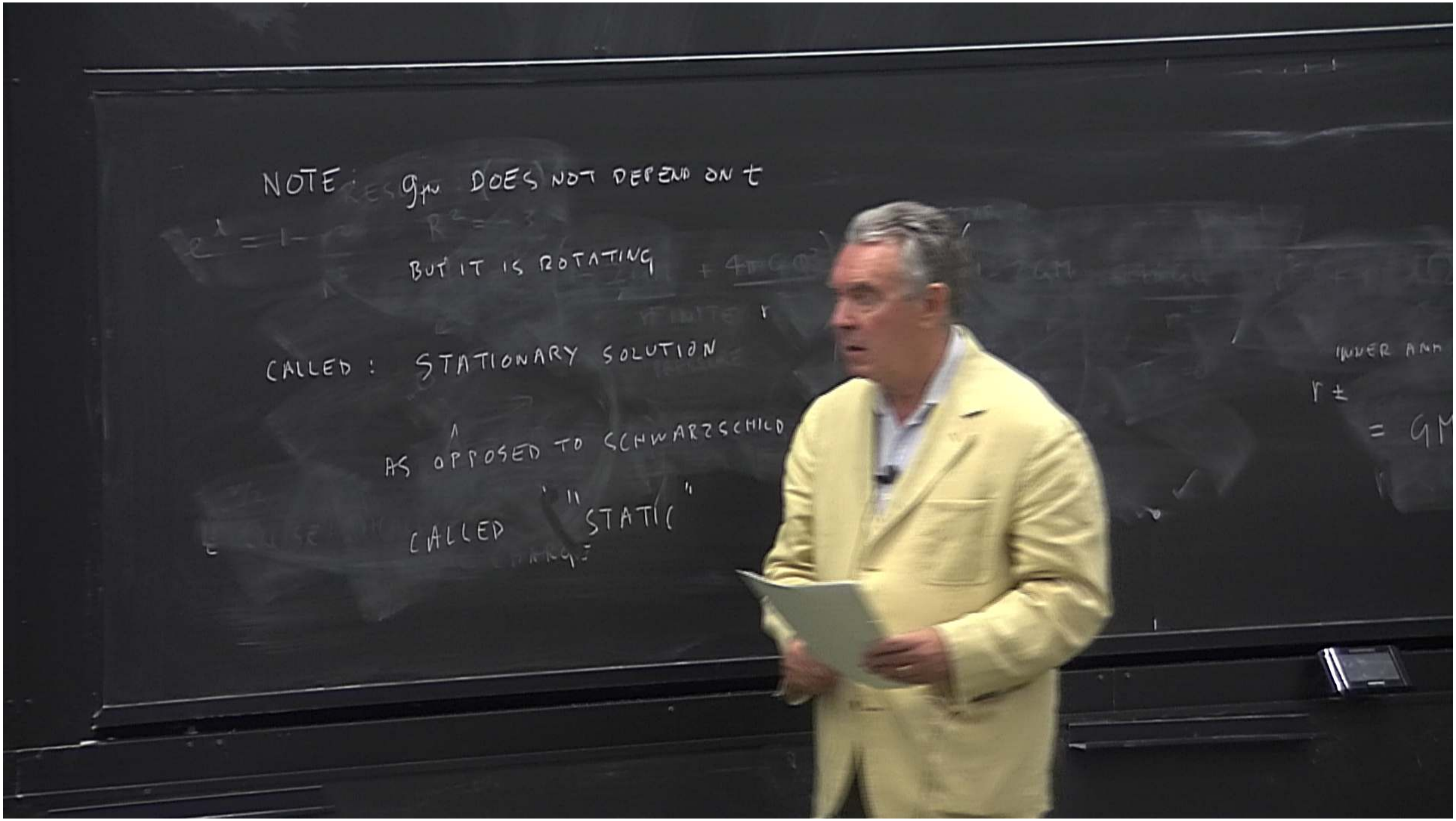
$$ds^2 = -\frac{\Delta}{\rho}\left[dt^2 - a\sin^2\theta\, d\phi^2\right] + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)\,d\phi - a\,dt\right]^2$$

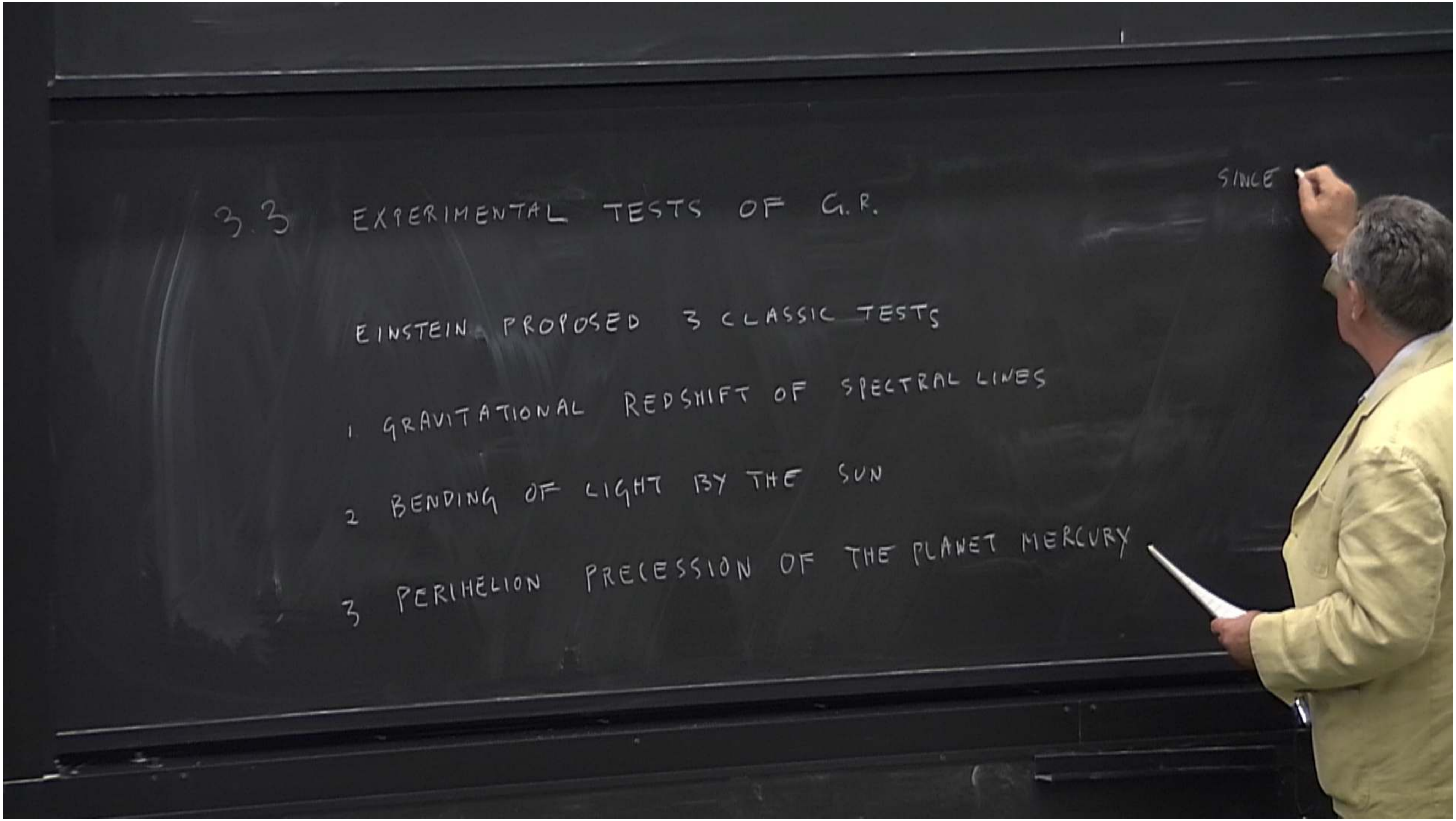
$$+ \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

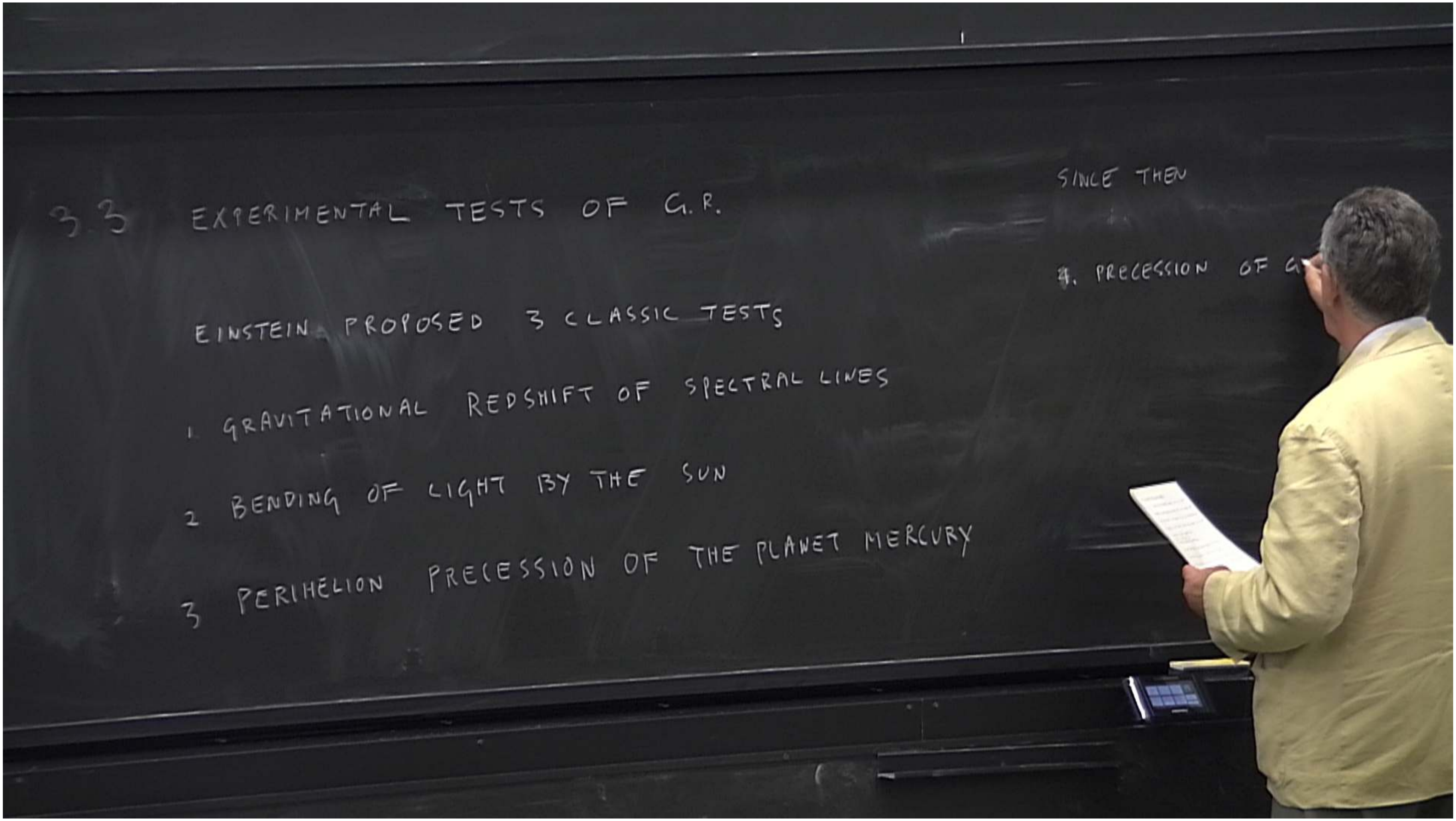
$$\Delta \equiv r^2 - 2Mr + a^2 + 4\pi Q^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$


INNER AND OUTER HORIZONS
$r_\pm = GM \pm \sqrt{G^2M^2 - 4\pi G Q^2 - a^2}$


NOTE: g DOES NOT DEPEND ON t
BUT IT IS ROTATING
CALLED: STATIONARY SOLUTION
AS OPPOSED TO SCHWARZSCHILD
CALLED "STATIC"
INNER AND
r ±
= GM


3.3 EXPERIMENTAL TESTS OF G.R.
SINCE
EINSTEIN PROPOSED 3 CLASSIC TESTS
1 GRAVITATIONAL REDSHIFT OF SPECTRAL LINES
2 BENDING OF LIGHT BY THE SUN
3 PERIHELION PRECESSION OF THE PLANET MERCURY


3.3 EXPERIMENTAL TESTS OF G.R.
EINSTEIN PROPOSED 3 CLASSIC TESTS
1 GRAVITATIONAL REDSHIFT OF SPECTRAL LINES
2 BENDING OF LIGHT BY THE SUN
3 PERIHELION PRECESSION OF THE PLANET MERCURY
SINCE THEN
4. PRECESSION OF G

SINCE THEN

4. PRECESSION OF

5. ENERGY LOSS IN

3.3 EXPERIMENTAL TESTS OF G.R.

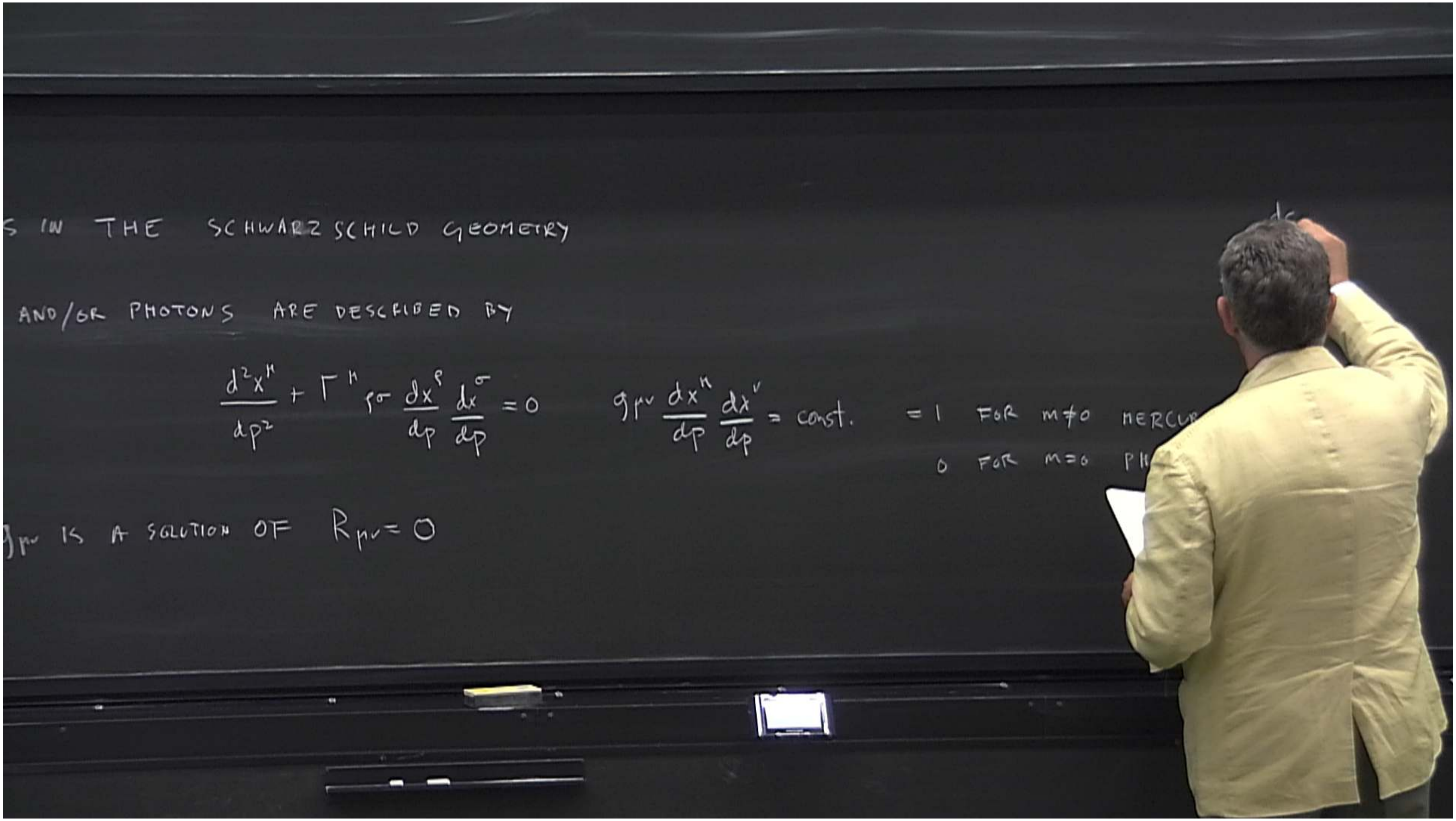
EINSTEIN PROPOSED 3 CLASSIC TESTS

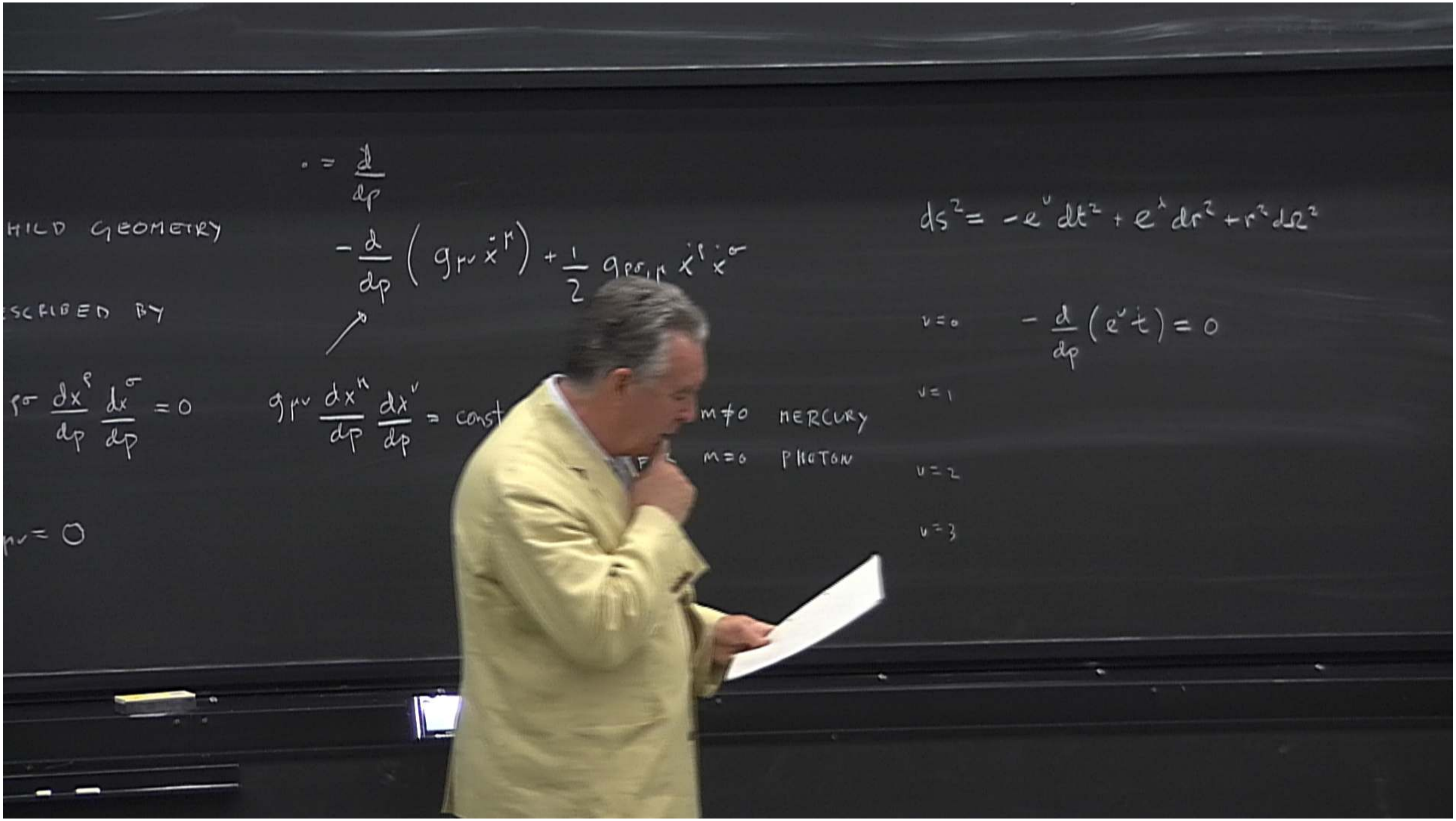
GEODESICS IN THE SCHWARZSCHILD GEOMETRY

1 GRAVITATIONAL REDSHIFT OF SPECTRAL LINES

2 BENDING OF LIGHT BY THE SUN

3 PERIHELION PRECESSION OF THE PLANET MERCURY


S IN THE SCHWARZSCHILD GEOMETRY
AND/OR PHOTONS ARE DESCRIBED BY
$\frac{d^2x^\mu}{dp^2} + \Gamma^\mu{}_{\rho\sigma} \frac{dx^\rho}{dp} \frac{dx^\sigma}{dp} = 0$
$g_{\mu\nu} \frac{dx^\mu}{dp} \frac{dx^\nu}{dp} = \text{const.}$
= 1 FOR $m \neq 0$ MERCU
0 FOR $m = 0$ PH
$g_{\mu\nu}$ IS A SOLUTION OF $R_{\mu\nu} = 0$


HILD GEOMETRY
SCRIBED BY
MERCURY
PHOTON
ds² = −e^ν dt² + e^λ dr² + r² dΩ²
ν = 0
ν = 1
ν = 2
ν = 3

$$\cdot = \frac{d}{dp}$$

$$-\frac{d}{dp}\left(g_{\mu\nu}\dot{x}^{\mu}\right) + \frac{1}{2} g_{\rho\sigma,\mu}\dot{x}^{\rho}\dot{x}^{\sigma}$$

$$g_{\mu\nu}\frac{dx^{\mu}}{dp}\frac{dx^{\nu}}{dp} = \text{const.} \quad = 1 \text{ FOR } m \neq 0 \text{ MERCURY}$$

$$0 \text{ FOR } m = 0 \text{ PHOTON}$$

$$ds^2 = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2 d\Omega^2$$

$\nu = 0$ $$-\frac{d}{dp}\left(e^{\nu}\dot{t}\right) = 0$$

$\nu = 1$ $$\frac{d}{dp}\left(e^{\lambda}\dot{r}\right) + \frac{1}{2}\nu' e^{\nu}\dot{t}^2 - \frac{1}{2}\lambda' e^{\nu}\dot{r}^2 - r\dot{\theta}^2 - r^2\sin^2\theta\,\dot{\phi}^2 = 0$$

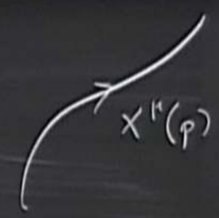
$\nu = 2$ $$\frac{d}{dp}\left(r^2\dot{\theta}\right) - r^2\sin^2\theta\cos\theta\,\dot{\phi}^2 = 0$$

$\nu = 3$ $$\frac{d}{dp}\left(r^2\sin^2\theta\,\dot{\phi}\right) = 0$$

$$0 = \frac{d}{dp}$$

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega^2$$

$$(g_{\mu\nu} \ddot{x}^\mu) + \frac{1}{2} g_{\rho\sigma,\mu} \dot{x}^\rho \dot{x}^\sigma$$



$= \text{const.}$ $= 1$ FOR $m \neq 0$ MERCURY

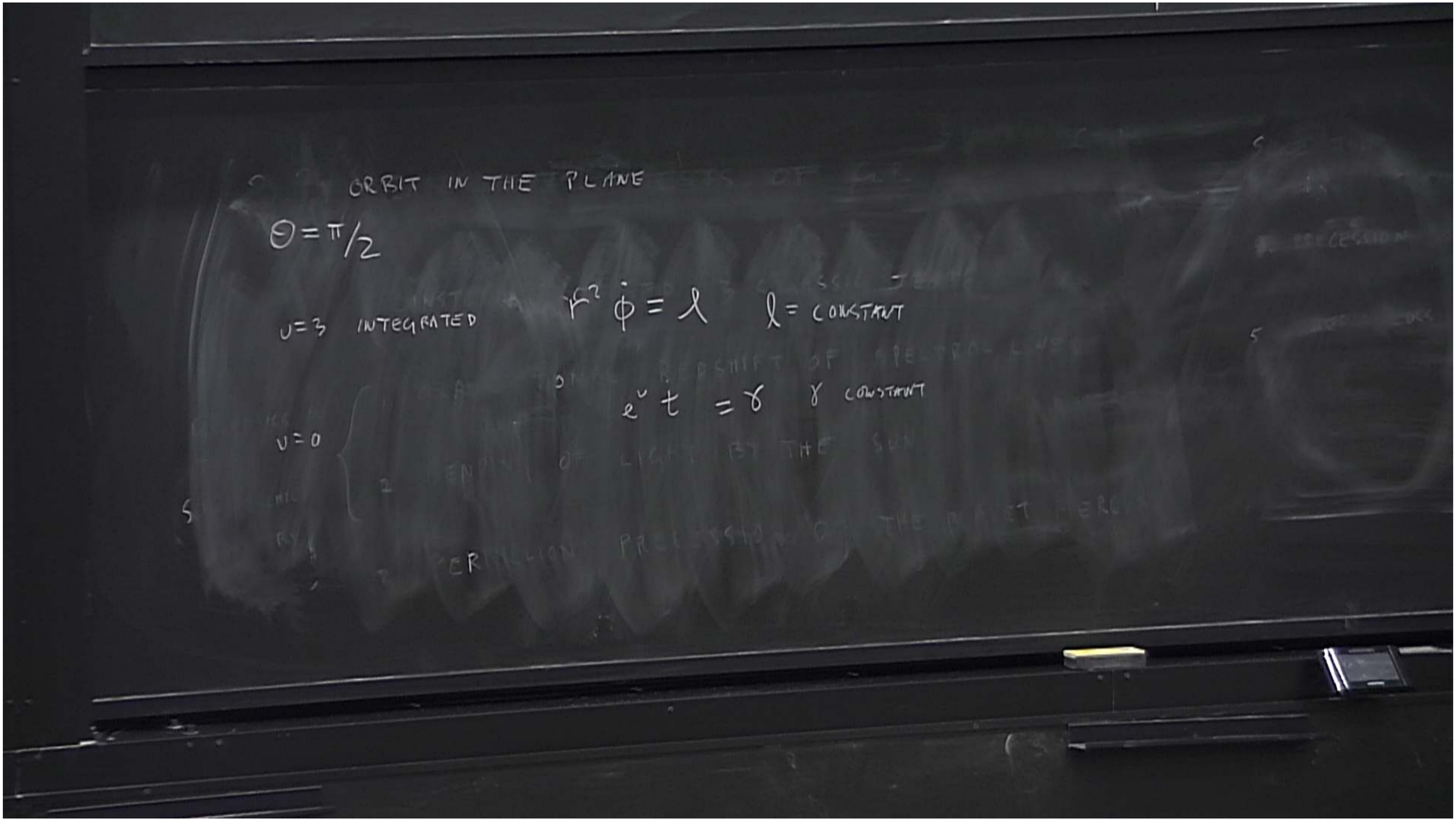
0 FOR $m = 0$ PHOTON

$\nu = 0$ $$-\frac{d}{dp}(e^\nu \dot{t}) = 0$$

$\nu = 1$ $$\frac{d}{dp}(e^\lambda \dot{r}) + \frac{1}{2}\nu' e^\nu \dot{t}^2 - \frac{1}{2}\lambda' e^\lambda \dot{r}^2 - r\dot{\theta}^2 - r^2 \sin^2\theta \, \dot{\phi}^2 = 0$$

$\nu = 2$ $$\frac{d}{dp}(r^2 \dot{\theta}) - r^2 \sin\theta \cos\theta \, \dot{\phi}^2 = 0$$

$\nu = 3$ $$\frac{d}{dp}(r^2 \sin^2\theta \, \dot{\phi}) = 0$$


ORBIT IN THE PLANE
θ = π/2
ν=3 INTEGRATED
r² φ̇ = λ
l = CONSTANT
e^ν ṫ = γ
γ CONSTANT
ν=0