Title: Quantum Theory - Lecture 7

Date: Sep 18, 2012 09:00 AM

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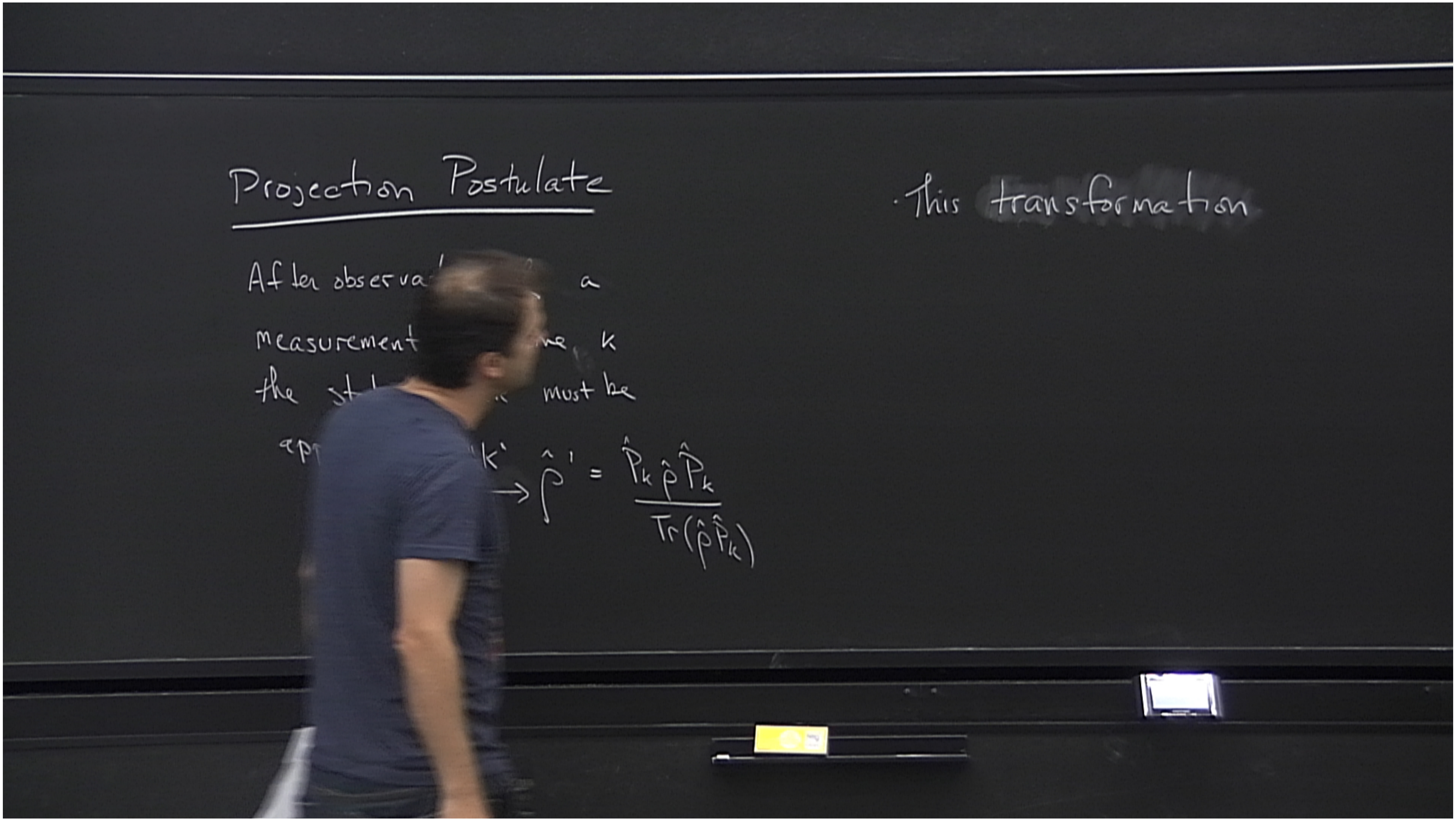
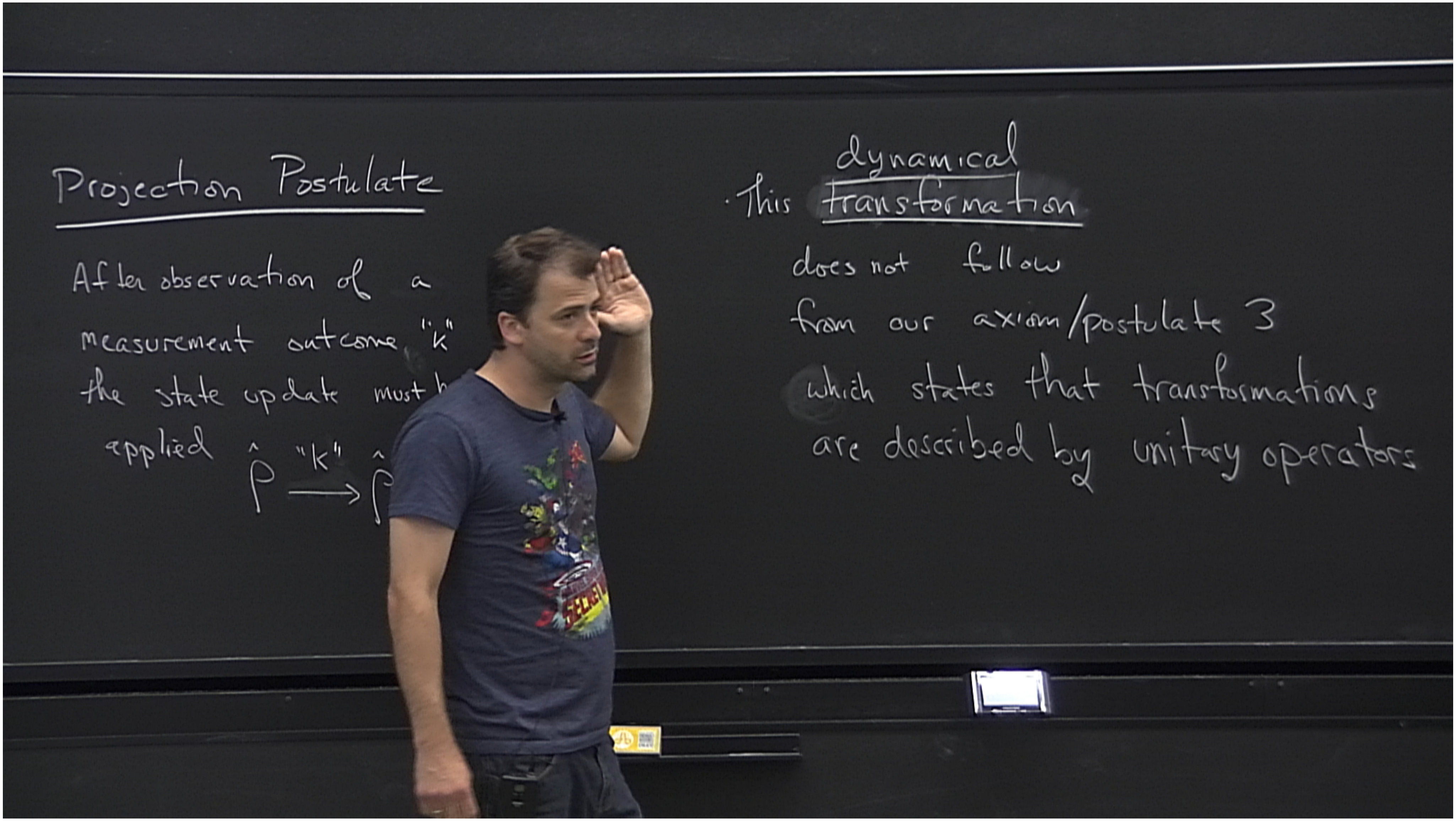
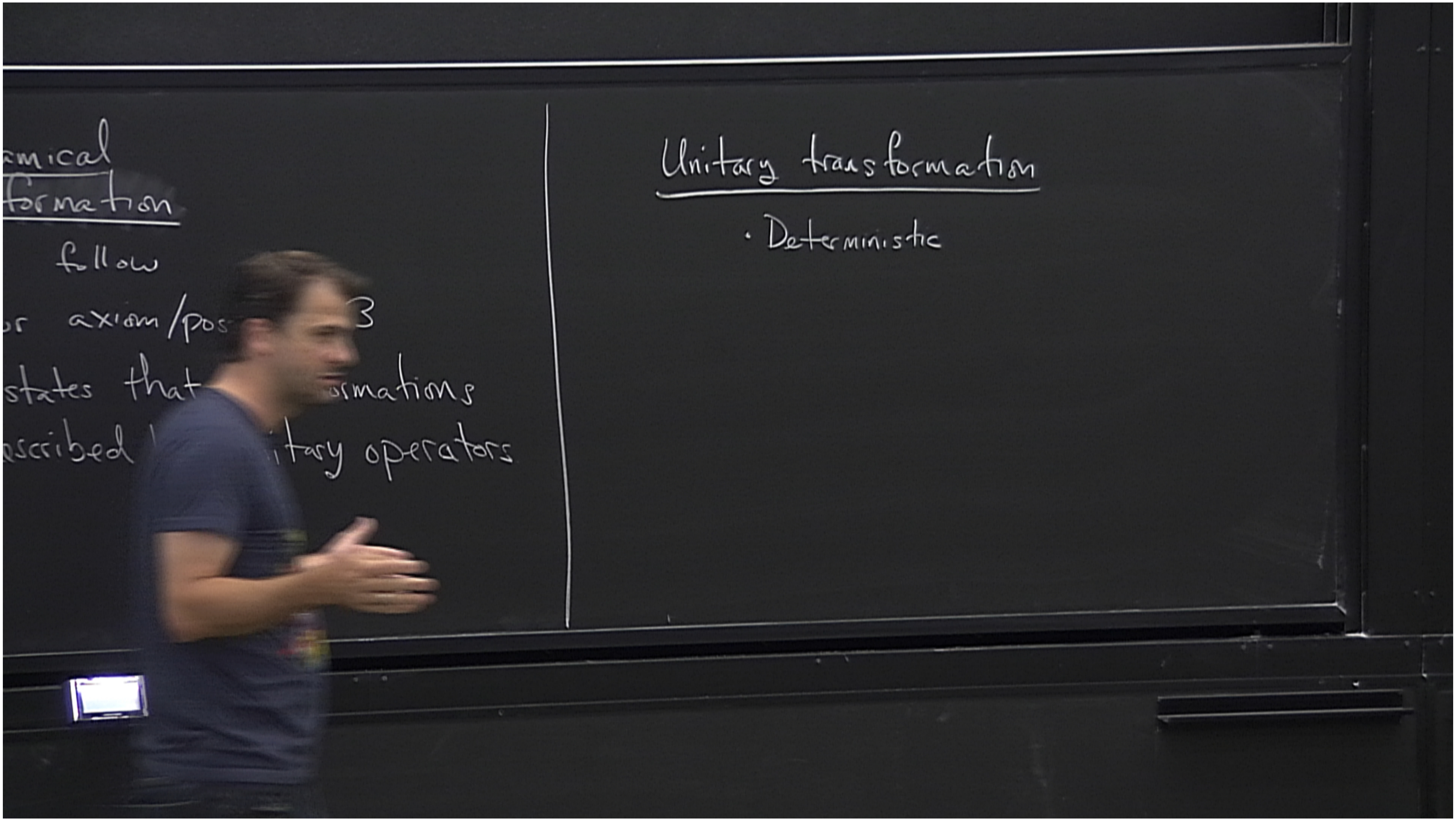
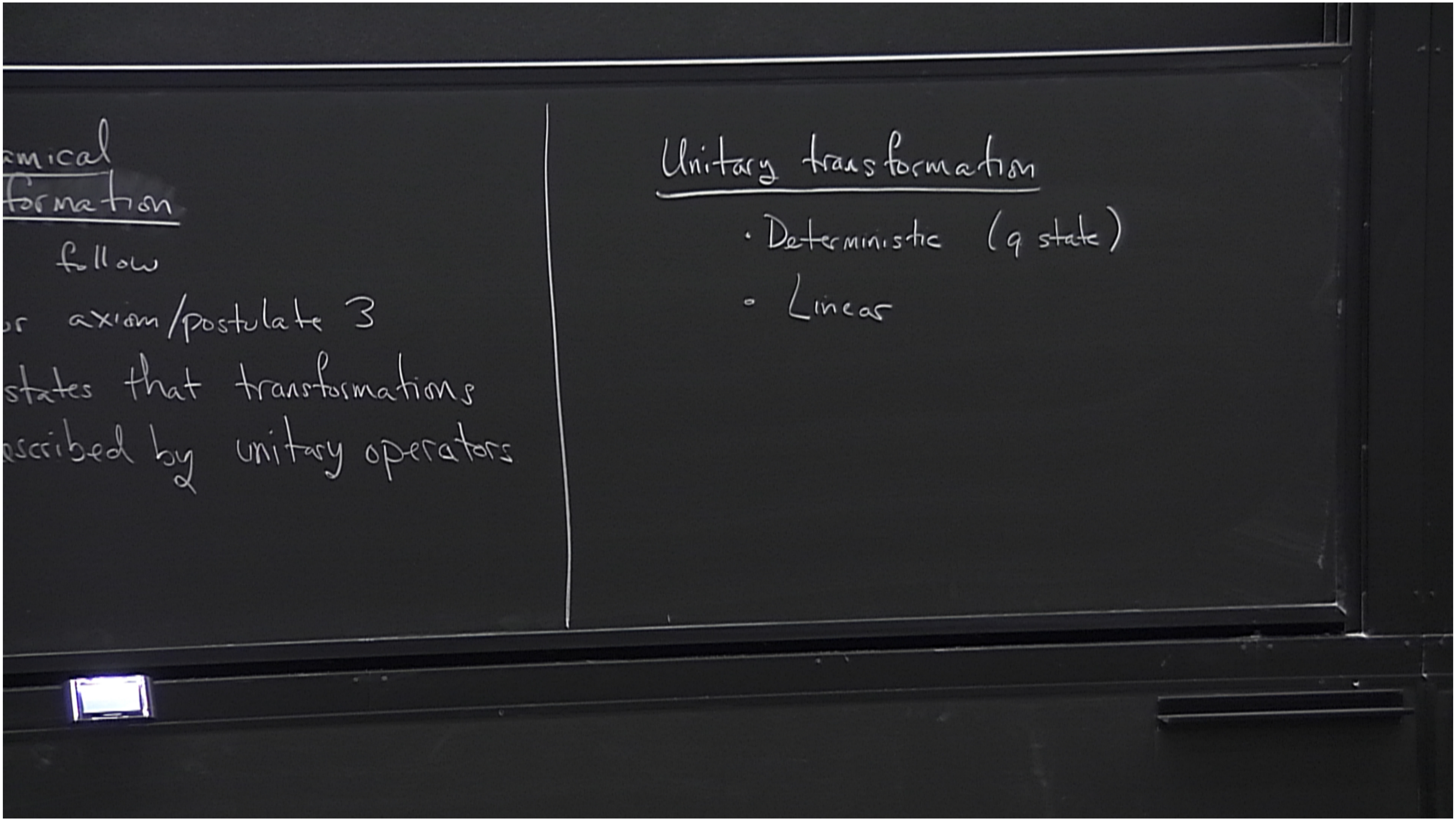
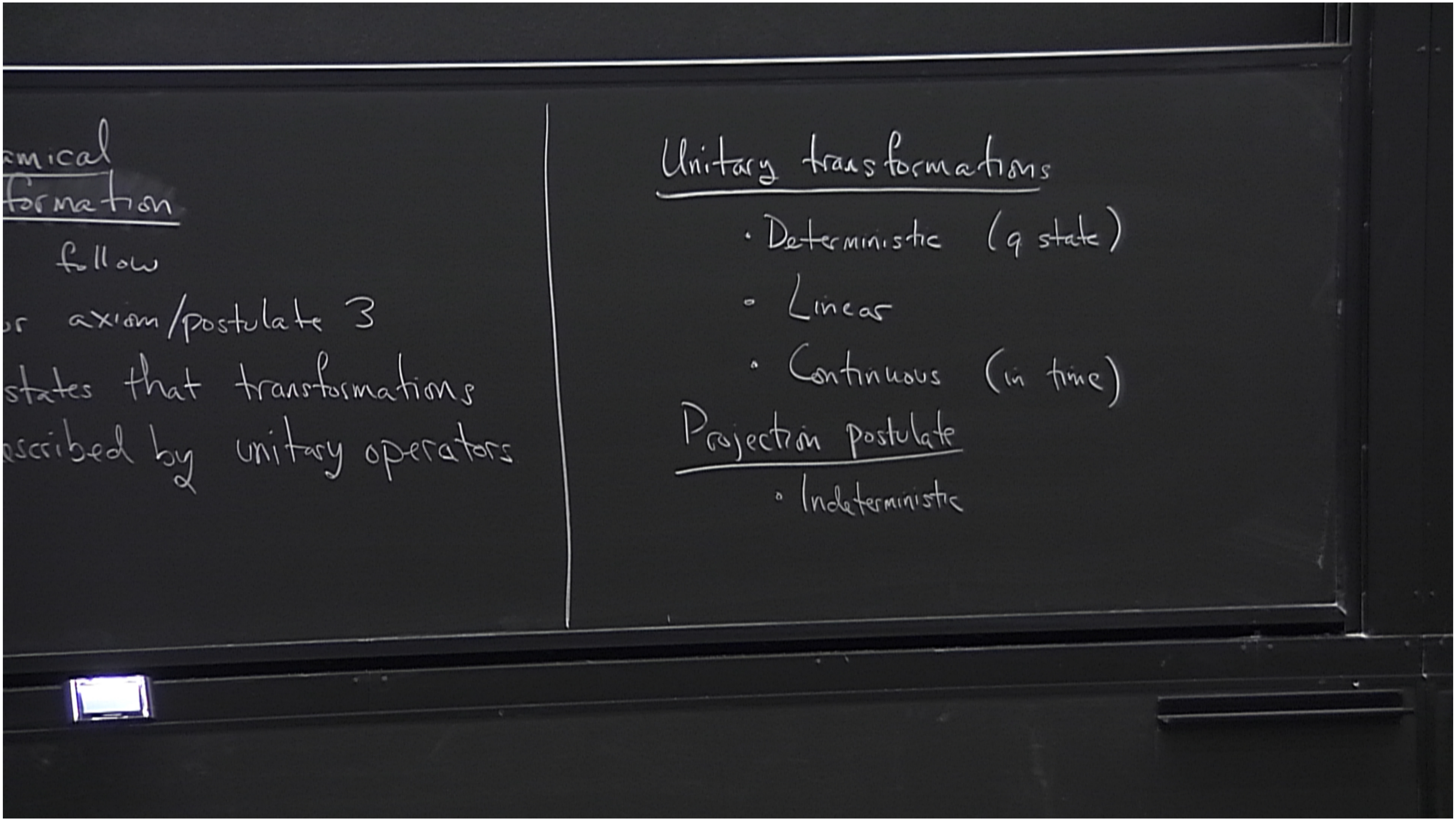
Abstract:

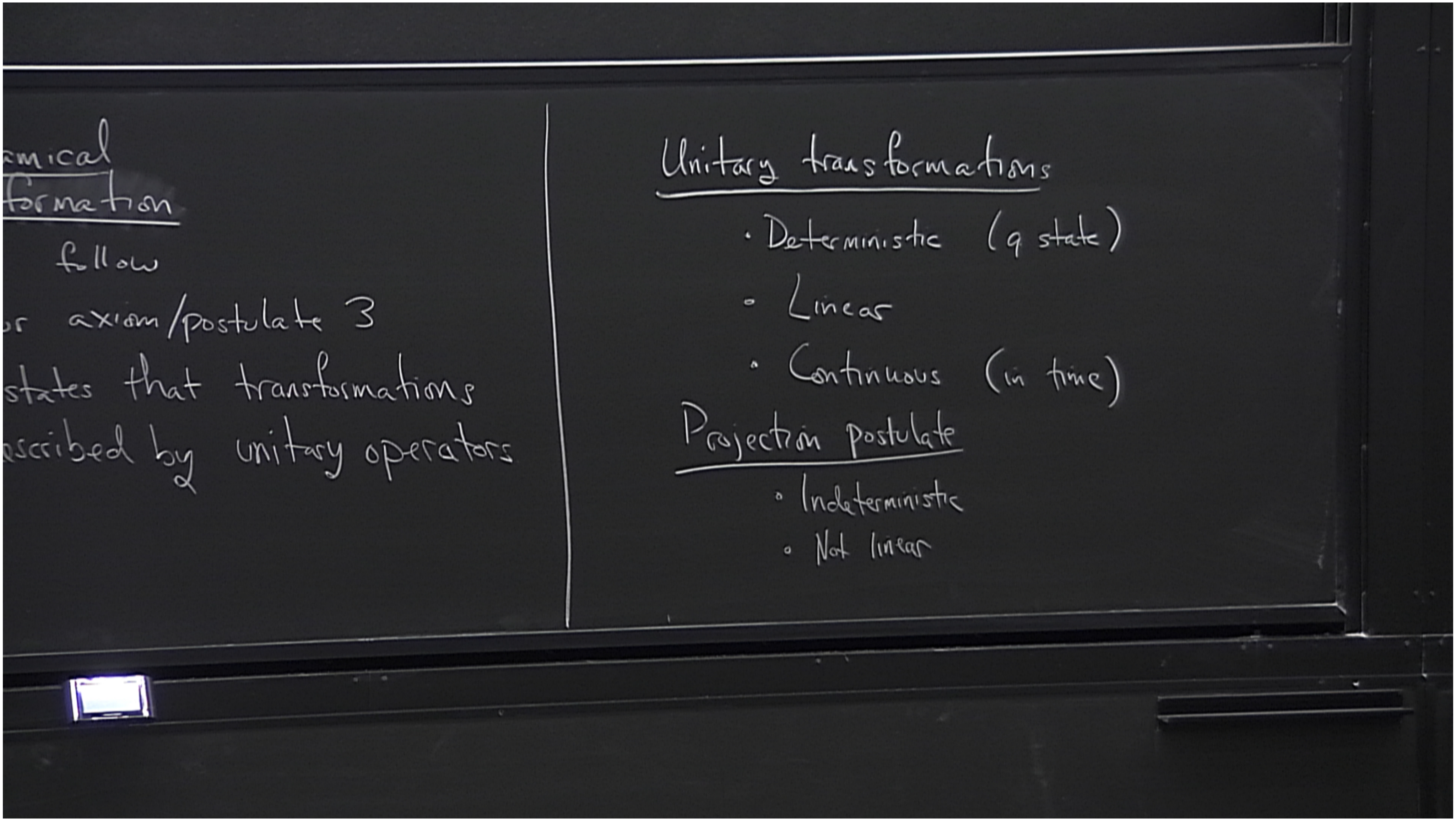

Projection Postulate
After observa a
measurement k
the must be
$\hat{\rho}' = \frac{\hat{P}_k \hat{\rho} \hat{P}_k}{Tr(\hat{\rho}\hat{P}_k)}$
· This transformation


Projection Postulate
After observation of a
measurement outcome "k"
the state update must
applied
"k"
dynamical
· This transformation
does not follow
from our axiom/postulate 3
which states that transformations
are described by unitary operators


Unitary transformation
· Deterministic
follow
states that
operators


follow
axiom/postulate 3
states that transformations
by unitary operators
Unitary transformation
· Deterministic (q state)
· Linear


mical
formation
follow
r axiom/postulate 3
states that transformations
scribed by unitary operators
Unitary transformations
· Deterministic (q state)
· Linear
· Continuous (in time)
Projection postulate
· Indeterministic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Unitary transformations
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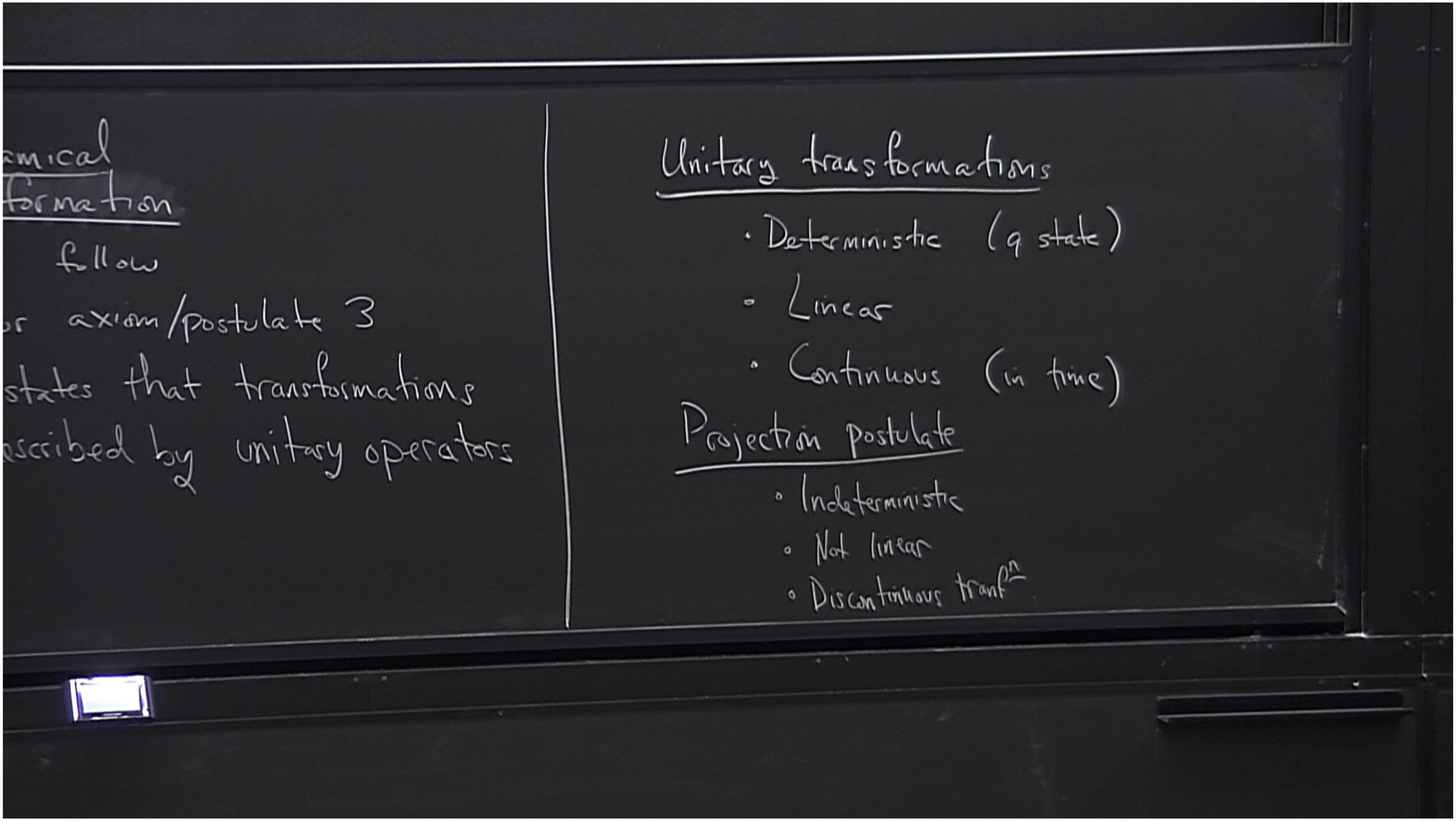
· Continuous (in time)
Projection postulate
· Indeterministic
· Not linear



mical
formation
follow
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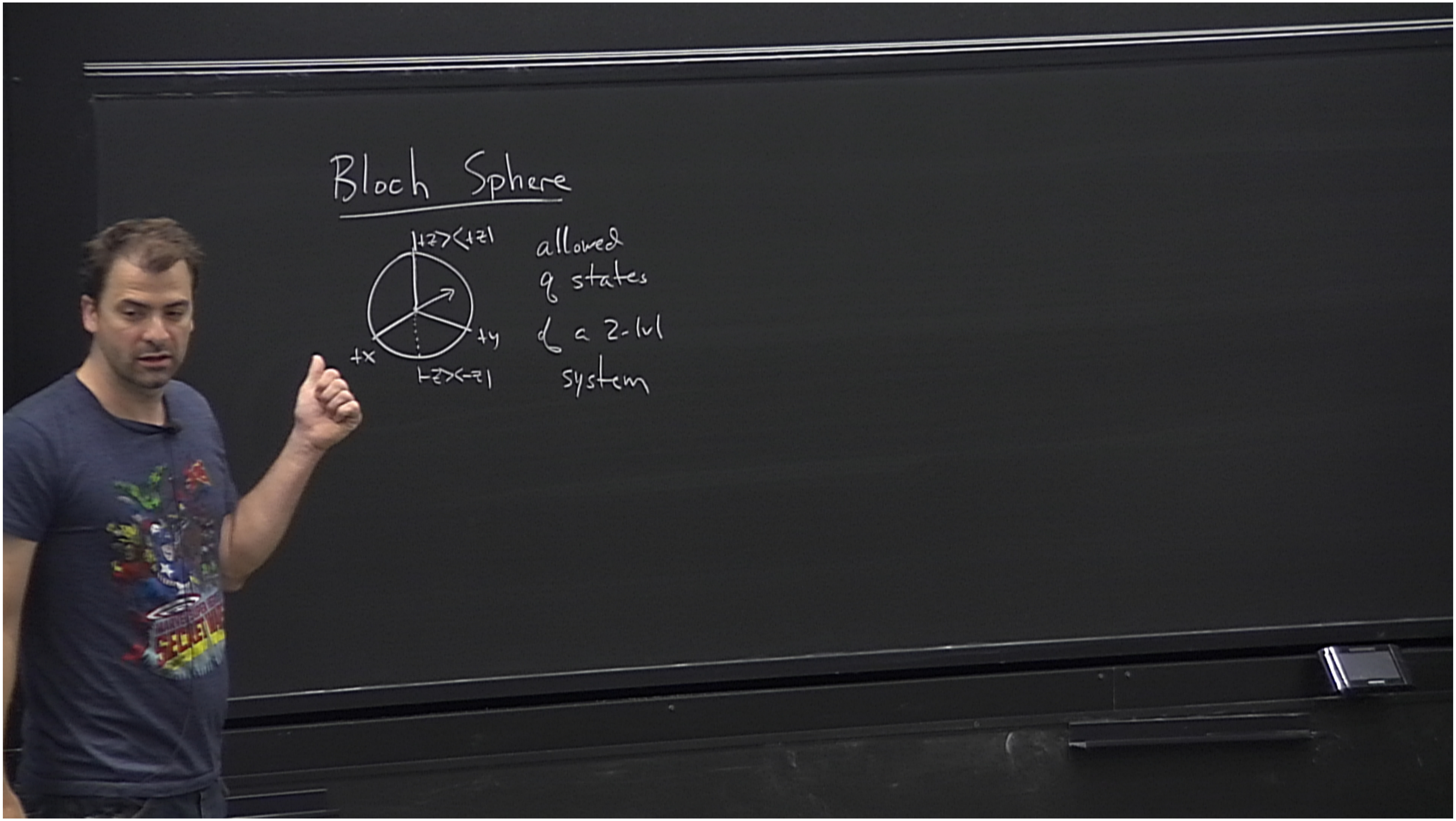
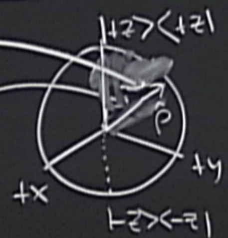
states that transformations
scribed by unitary operators
Unitary transformations
· Deterministic (q state)
· Linear
· Continuous (in time)
Projection postulate
· Indeterministic
· Not linear
· Discontinuous tranf^{n}


Bloch Sphere
|+z><+z|
allowed
q states
+y
of a 2-lvl
+x
system
|-z><-z|

Bloch Sphere

$\langle \hat{\vec{\sigma}} \rangle$

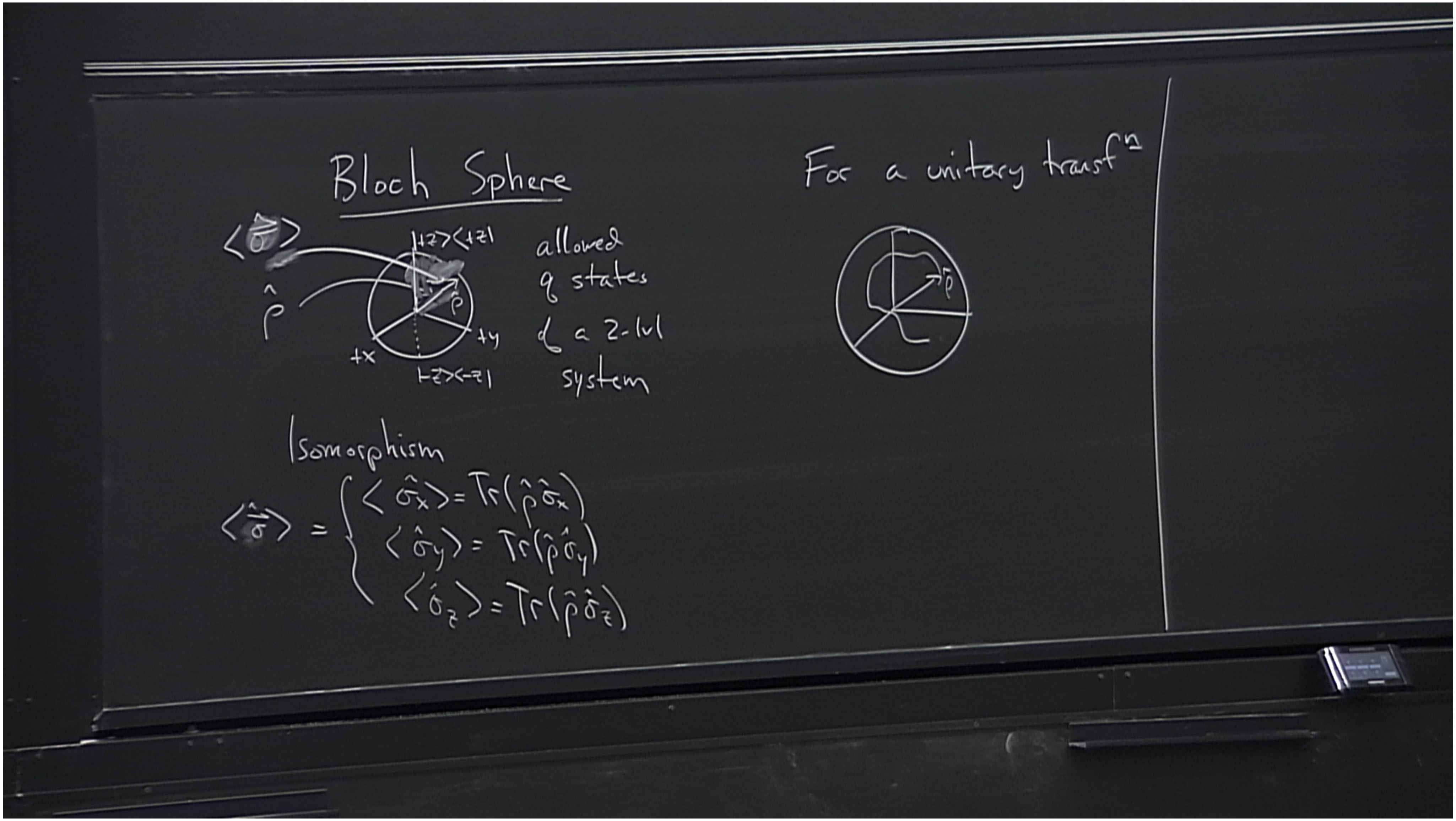
$\hat{\rho}$



allowed q states of a 2-lvl system

Isomorphism

$$\langle \hat{\vec{\sigma}} \rangle = \begin{cases} \langle \hat{\sigma}_x \rangle = \mathrm{Tr}(\hat{\rho}\hat{\sigma}_x) \\ \langle \hat{\sigma}_y \rangle = \mathrm{Tr}(\hat{\rho}\hat{\sigma}_y) \\ \langle \hat{\sigma}_z \rangle = \mathrm{Tr}(\hat{\rho}\hat{\sigma}_z) \end{cases}$$



Bloch Sphere
$\langle \vec{\hat{\sigma}} \rangle$
$\hat{\rho}$
$|+z\rangle\langle +z|$
$\vec{\rho}$
+x
+y
$|-z\rangle\langle -z|$
allowed
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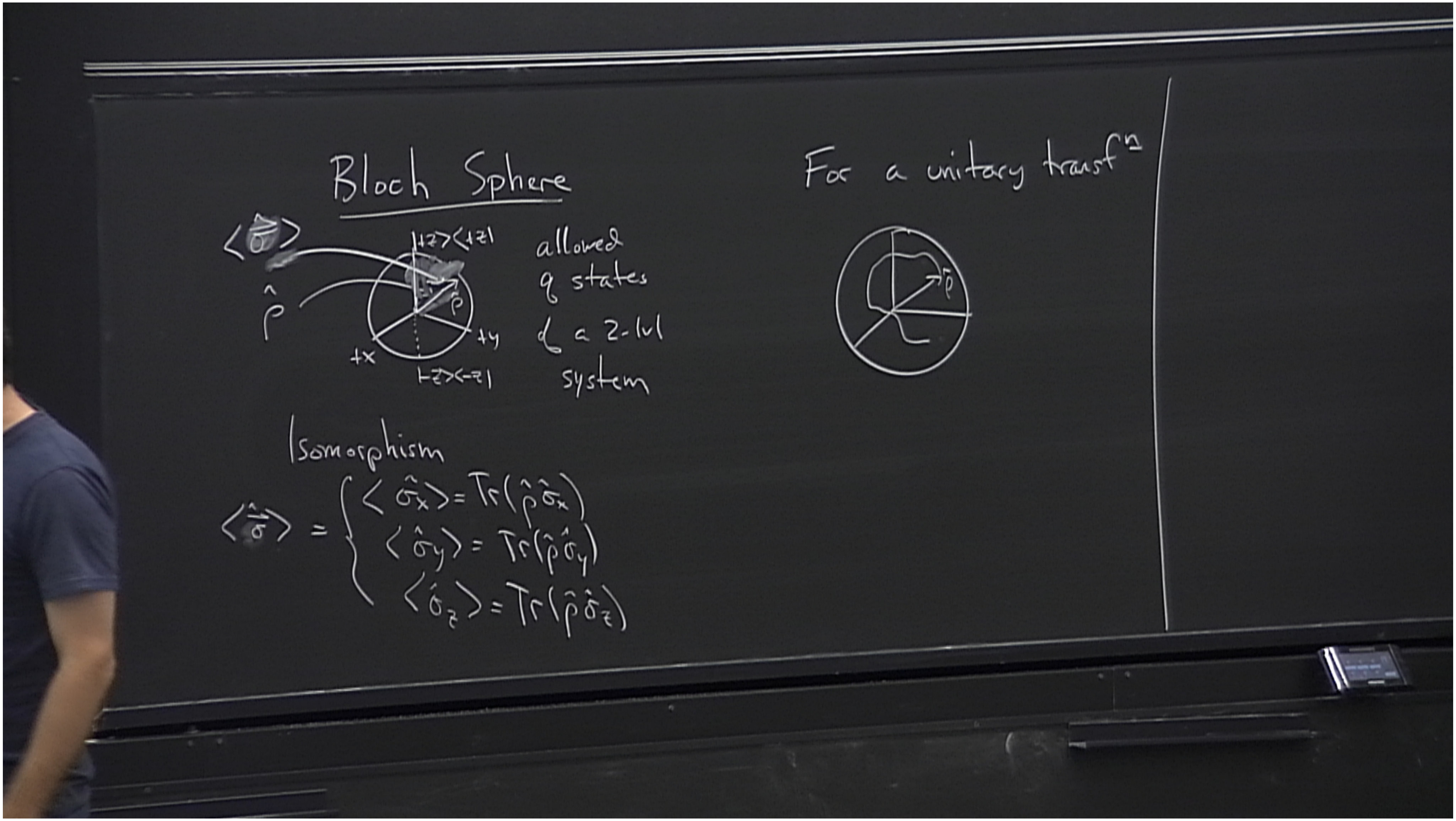
For a unitary transf^{n}
$\vec{\rho}$



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For a unitary transf$^{\underline{n}}$

von Neumann struggles to make sense of measurement

von Neumann felt the resulting situation was "unexplained":

> "We have then answered the question as to what happens in the measurement of [an observable]. To be sure, the "how" remains unexplained for the present."

von Neumann (1932/1955)

- von Neumann goes through a long analysis to show that, within his interpretation, the application of the projection postulate can be applied in a consistent way either to the system directly or to the system + apparatus.
- He insists that ultimately the postulate must be applied whenever an "interaction" takes place between the "measuring portion" and the "measured portion" of the world.

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von Neumann struggles to make sense of measurement

> Now quantum mechanics describes the events which occur in the observed portions of the world, so long as they do not interact with the observing portion, with the aid of the process 2 [Schrodinger evolution], but as soon as such an interaction occurs, i.e., a measurement, it requires the application of the process 1 [projection postulate]. The dual form is therefore justified."

von Neumann (1932/1955)

- This arbitrary boundary between the 'measurer' and the 'measuree' is sometimes called the "von Neumann cut." Clearly von Neumann took great pains to justify this boundary and its arbitrariness.

von Neumann struggles to make sense of measurement

> "First, it is inherently entirely correct that the measurement of the related process of the subjective perception is a new entity relative to the physical environment and is not reducible to the latter. Indeed, subjective perception leads us into the intellectual inner life of the individual, which is extra-observational by its every nature (since it must be take for granted by any conceivable observation or experiment). Nevertheless, it is a fundamental requirement of the scientific viewpoint - the so-called principle of the psycho-physical parallelism (!) - that it must be possible so to describe the extra-physical process of the subjective perception as if it were in reality in the physical world - i.e., to assign to its parts equivalent physical processes in the objective environment, in ordinary space. "

von Neumann (1955)

For Dirac, a consequence of literal realism is that the projection postulate must represent a physical process, an actual 'jump':

"When we measure a real dynamical variable, the disturbance involved in the act of measurement causes a jump in the state of the dynamical system. From physical continuity, if we make second measurement of the same dynamical variable immediately after the first, the result of the second measurement must be the same as that of the first. Thus after the first measurement has been made, there is no indeterminacy in the result of the second. Hence, after the first measurement has been made, the system is in an eigenstate of the dynamical variable, the eigenvalue it belongs to being equal to the result of the first measurement. This conclusion must still hold if the second measurement is not actually made. In this way we see that a measurement always causes the system to jump into an eigenstate of the dynamical variable that is being measured, the eigenvalue this eigenstate belongs to being equal to the result of the measurement."

Dirac (1958)

Literal realism implies the absence of causality

> "This concept of quantum mechanics, which accepts its statistical expression as the actual form of the laws of nature, and which abandons the principle of causality, is the so-called statistical interpretation."

von Neumann (1932/1955)

- Note that von Neumann is oddly using the label "statistical interpretation" to refer to his "literal realist" view that quantum states specify the "complete ontology".
- However, nowadays the label "statistical interpretation" refers to the exact opposite point of view, in particular that of Einstein and Ballentine, which posits that quantum states do not give a complete description of the properties of individual systems.
- In any case the key point here is that it is the **unnecessary interpretational assumption of completeness** that implies the loss of causality.

Literal Realism and Schrodinger's cat

Schrodinger's cat paradox is an expression of the *under-determined reality* resulting from *literal realism*:

> "A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny amount of radioactive substance, so small, that perhaps in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if mean while no atom has decayed. The first atomic decay would have poisoned it. The ψ-function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts."

Schrodinger (1935)

Literal Realism and Schrodinger's cat

- Many commentators take Schrodinger's argument as a literal claim about the *ambiguous ontology* that results from coherent superposition.
- As such they fail to appreciate that Schrodinger's cat argument was a *reduction ad absurdum* intended to ridicule the literal realism of the orthodox interpretation and the anti-realism of the Copenhagen interpretation.
- Consider how Schrodinger introduced the above passage:

"One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device ..."

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According to von Neumann and Dirac, the question of whether the cat is finally either alive or dead (as opposed to a coherent superposition) depends on when the "dynamical process" for collapse is supposed to have taken place.

- Is the cat's status in an undefined state until it is observed? Or is the cat's own observation enough to collapse the wavefunction?
- The ambiguity of this point has been stressed in a more comical way by John Bell (1990):

"What exactly qualifies some physical systems to play the role of *measurer*? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system ... with a PhD?"

The Measurement Problem

All of the above considerations are different aspects of what is now called "the measurement problem".

- The measurement problem is usually identified as the failure of the *unitary evolution* to account for the *unique outcomes* that are observed in practice.
- As we've seen, the projection postulate is designed to solve this problem, but it is creates new problems for the orthodox interpretation:
  - it implies that the fundamental ontology is governed by two different dynamical laws.
  - the question of when a measurement takes place (and which dynamical law should apply) is left unspecified.

The Measurement Problem

But wait! Is the measurement problem really a "problem" that needs to be solved?

- As I hope to convince you, the measurement problem is not a *problem of quantum mechanics*, but a problem created by the orthodox interpretation.
- Hideo Mabuchi, a contemporary quantum information scientist from Cal Tech, has put this best:

"The measurement problem is a set of people."

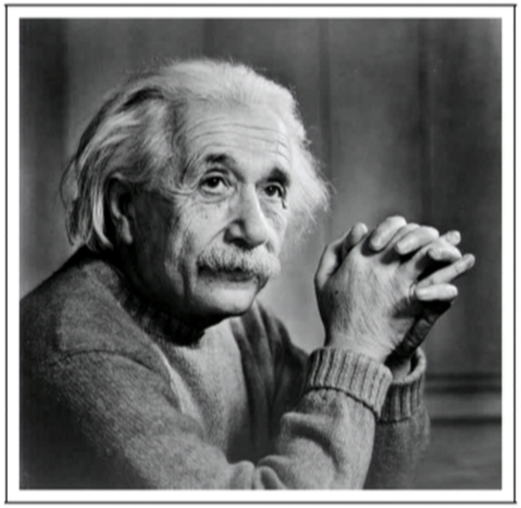
- So then, for whom is the measurement problem not a problem?

Einstein's Perspective

Einstein believed that quantum theory gave an incomplete description of reality. He advocated this view at least as early as 1927 and maintained it throughout his life.

> "The attempt to conceive the quantum-theoretical description as the complete description of the individual systems leads to unnatural theoretical interpretations, which become immediately unnecessary if one accepts the interpretation that the description refers to ensembles of systems and not to individual systems."

A. Einstein (1949)



Einstein's "reductio ad absurdum" argument

"The system is a substance in chemically unstable equilibrium, perhaps a charge of gunpowder that, by means of intrinsic forces, can spontaneously combust, and where the average life span of the whole setup is a year. In principle this can quite easily be represented quantum-mechanically. In the beginning the psi-function characterizes a reasonably well-defined macroscopic state. But, according to your equation (!), after the course of a year this is no longer the case. Rather, the psi-function then describes a sort of blend of not-yet and already-exploded systems. Through no art of interpretation can this psi-function be turned into an adequate description of a real state of affairs; in reality there is just no intermediary between exploded and not-exploded."

Einstein 1935 (letter to Schrodinger)

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Einstein 1935 (letter to Schrodinger)

"They somehow believe that the quantum theory provides a description of reality, and even a *complete* description; this interpretation is, however, refuted most elegantly by your system of radioactive atom + Geiger counter + amplifier + charge of gun powder (!) + cat in a box, in which the ψ-function of the system contains the cat both alive and blown to bits. Is the state of the cat to be created only when a physicist investigates the situation at some definite time? Nobody really doubts that the presence or absence of the cat is something independent of the act of observation. But then the description by means of the ψ-function is certainly incomplete, and there must be a more complete description. If one wants to consider the quantum theory as final (in principle), then one must believe that a more complete description would be useless because there would be no laws for it. If that were so then physics could only claim the interest of shopkeepers and engineers, the whole thing would be a wretched bungle."

Einstein 1950 (letter to Schrodinger)

Bohr didn't get it

Earlier we've opened the door to a favorable, "operational" reading of Bohr.

- We stressed that Bohr's view could be read as a strictly operational view if we replaced the awkward idea of "classical devices" with the modern notion of "input/ouput information".

But this may be too favorable. Bohr was not just an operationalist, but a staunch anti-realist (like Fuchs!).

- Bohr steadfastly refused to acknowledge even the logical possibility of Einstein's perspective being valid. For example, as late as 1949 he insisted that the more complete analysis Einstein seeks "is *in principle* excluded."

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Orthodox Interpretation and Non-locality

The most interesting criticism of the orthodox view was devised by Einstein, Podolsky, and Rosen in the celebrated "EPR paper" (1935).

- The goal of the paper was to show that the completeness assumption and a notion of locality were incompatible assumptions.
- For EPR, locality obviously held, and hence the assumption of completeness should be abandoned.

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Copyright 1935 by Science Service.

PRINCETON, N. J., May 3.—Professor Albert Einstein will attack science's important theory of quantum mechanics, a theory of which

New York Times, 1935.

The EPR Argument against Completeness

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

- The EPR argument is awkwardly convoluted. Einstein later acknowledged dissatisfaction with the way the argument was composed - the paper was written mainly by Podolsky.
- The basic dilemma for EPR is:

"...either (1) the quantum-mechanical description of reality given by the wave function is not complete, or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality."

The EPR Argument against Completeness

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The EPR Argument against Completeness

EPR considered a system of two particles initially produced in a joint eigenstate of their relative position and total linear momentum.
We will consider a simpler system involving a two spin-1/2 particles (proposed by Bohm (1951)) which illustrates the same features (now known as EPRB).

- Consider two particles prepared in the singlet-state,

$$\psi = \frac{1}{\sqrt{2}}\left(|+\rangle_1 \otimes |-\rangle_2 \; - \; |-\rangle_1 \otimes |+\rangle_2\right).$$

- This state has zero total angular momentum, so the spin of the first particle (system S_1) is anti-correlated with the spin of the second particle (system S_2).

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The EPR Argument against Completeness

- Assume that after the state preparation the particles are separated spatially in such a way that *the two particles can no longer interact.*
- Observe that if measurement of particle 1, along, say, the z-axis, yields $+\hbar/2$ then measurement of particle 2 (along the same z-axis) must yield $-\hbar/2$, and vice versa. Similarly, if we measure instead S_x for particle 1, then we can predict with certainty the outcome of an S_x measurement for particle 2.
- Hence we can predict with certainty the outcomes of measurements of either S_x or S_z of the second particle "without in any way disturbing the second system" - note the assumption of locality is invoked to guarantee that there can be no such disturbance.

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The EPR Argument against Completeness

- In accordance with the EPR criterion of reality (ii), there must therefore be elements of reality corresponding to both S_x and S_z for the second particle.
- Because quantum mechanical states do not assign definite properties simultaneously for these non-commuting observables (following the eigenvalue-eigenstate link) EPR deduced that the quantum-mechanical description of physical reality given by wave functions must not be complete.

Bad news for Einstein

In Einstein's view the gedanken experiments with entangled particles display a conflict between the two assertions:

- (1) the description given by the wavefunction is complete.
- (2) the real states (i.e. ontic states) of spatially separate objects are independent of each other.

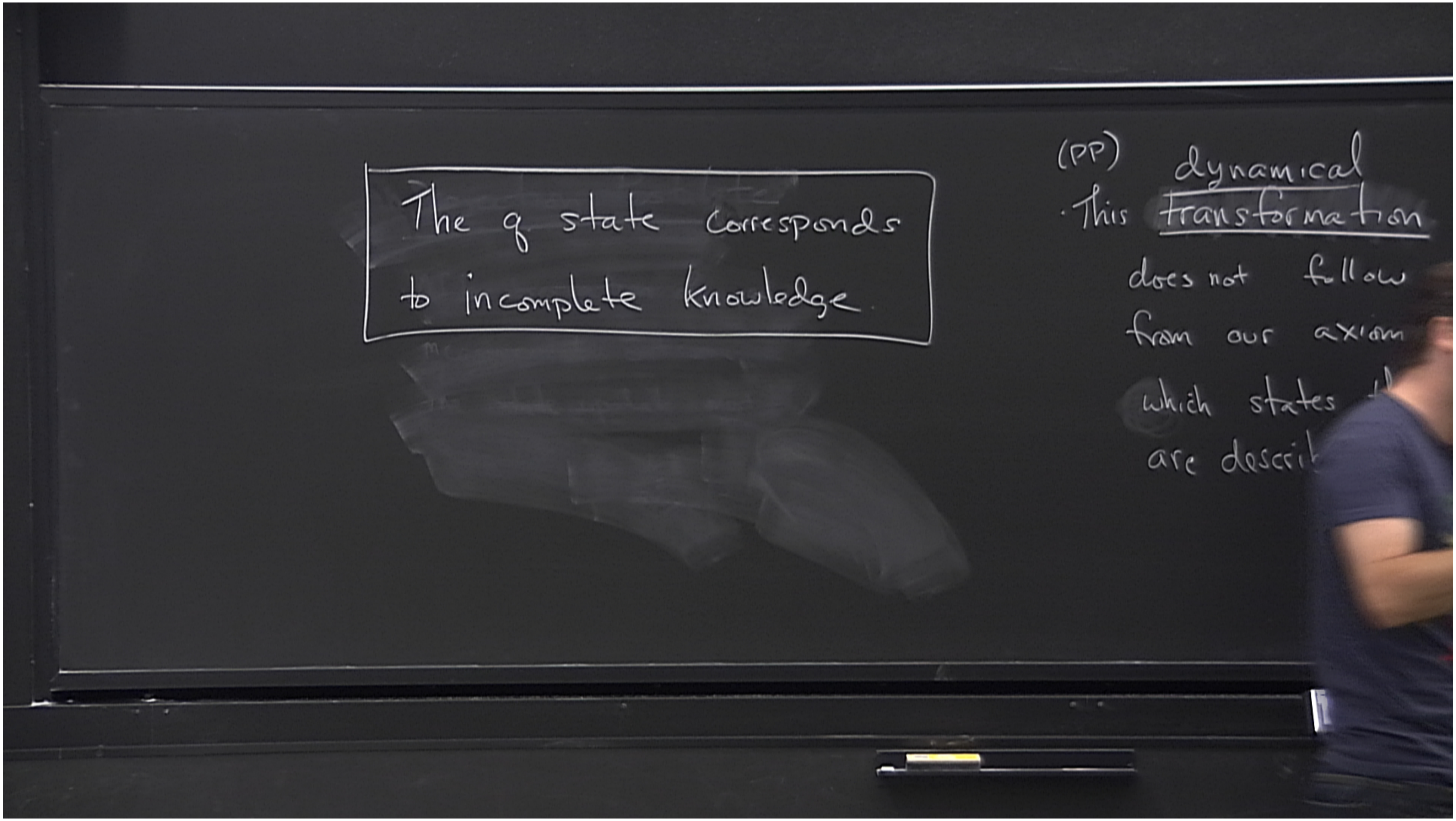
As we will see, John Bell later showed that any more complete theory, ie hidden variable interpretation, which reproduces the predictions of quantum mechanics must in fact be non-local (the real states of spatially separated objects are not independent of each other).

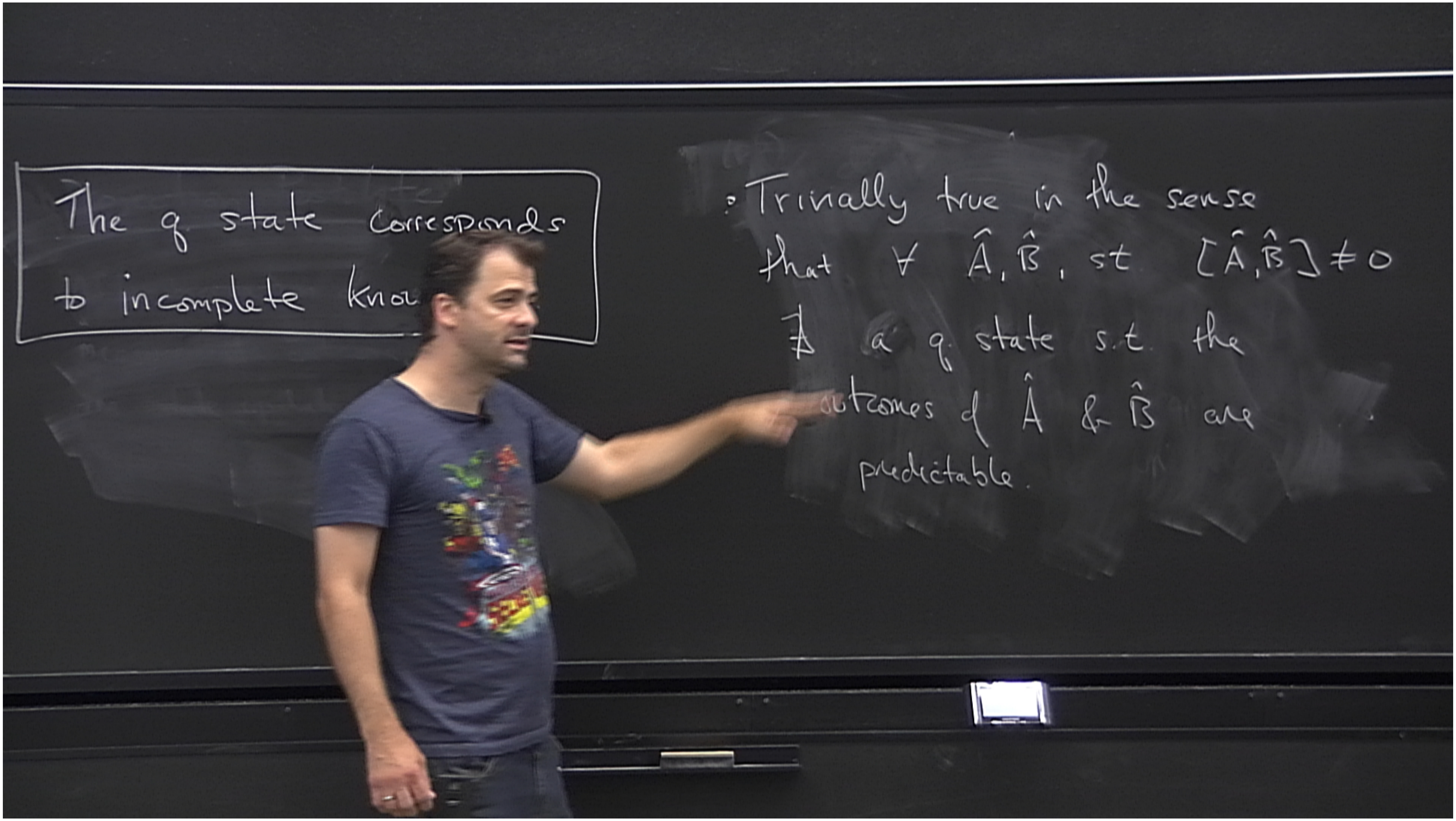
- As a result one is forced to reject the notion of locality (2) whether one accepts or rejects the assumption of completeness (1)!

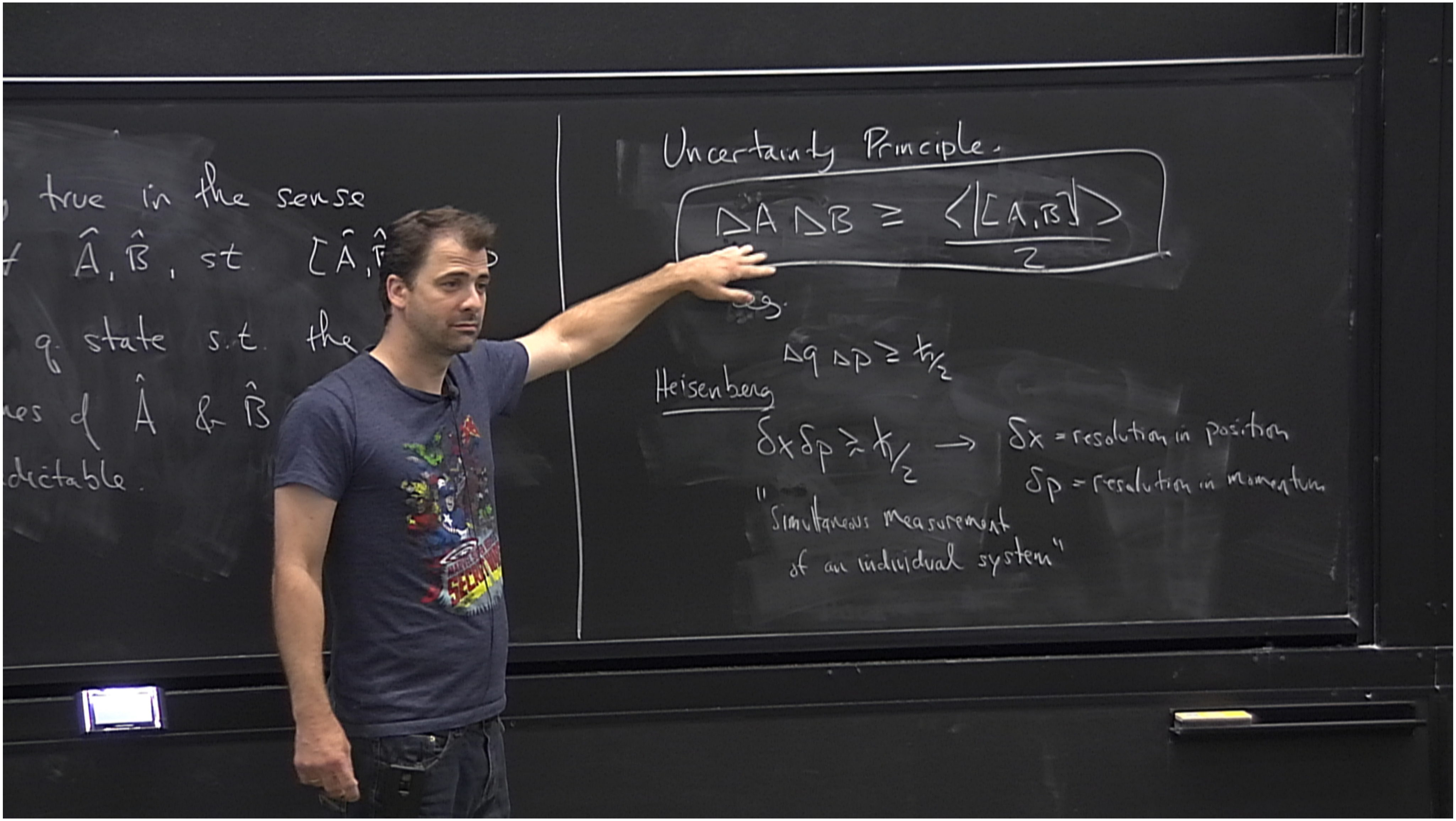
So where do we go from here?

"There exists, however, a simple psychological reason for that fact that this most nearly obvious [ensemble] interpretation is being shunned. For if the statistical quantum theory does not pretend to describe the individual system (and its development in time) completely, it appears unavoidable to look elsewhere for a complete description of the individual system . . . Assuming the success of efforts to accomplish a complete physical description, the statistical quantum theory would, within the framework of future physics, take an approximately analogous position to the statistical mechanics with the framework of classical mechanics. I am rather firmly convinced that the development of theoretical physics will be of that type; but the path will be lengthy and difficult."

A. Einstein (1949)


The q state corresponds
to incomplete knowledge
(PP) dynamical
· This transformation
does not follow
from our axiom
which states
are descri


The q state corresponds
to incomplete kno
Trivially true in the sense
that ∀ Â, B̂, st [Â, B̂] ≠ 0
∄ a q. state s.t. the
outcomes of Â & B̂ are
predictable.


true in the sense
Â, B̂, st [Â, B̂
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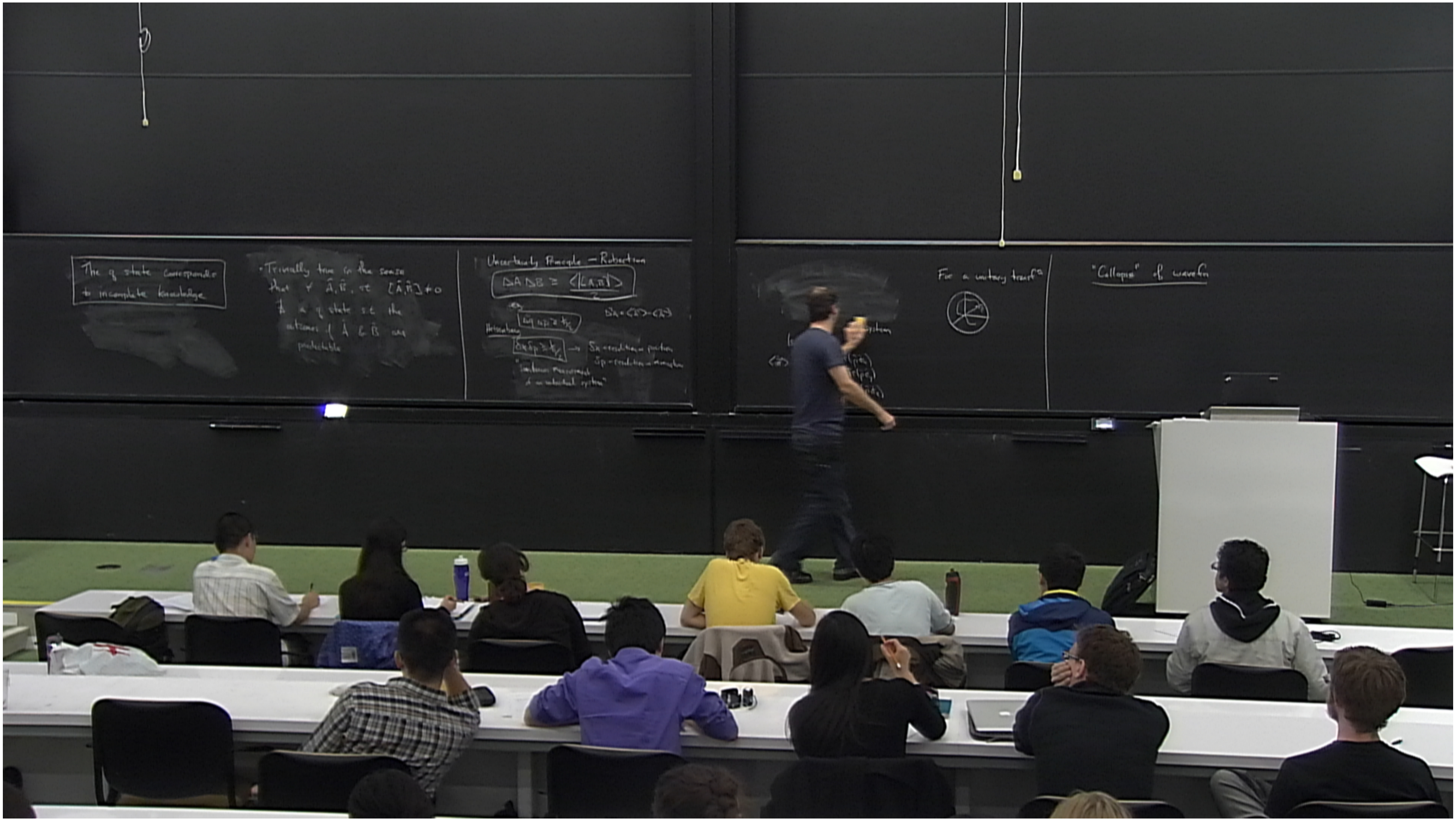
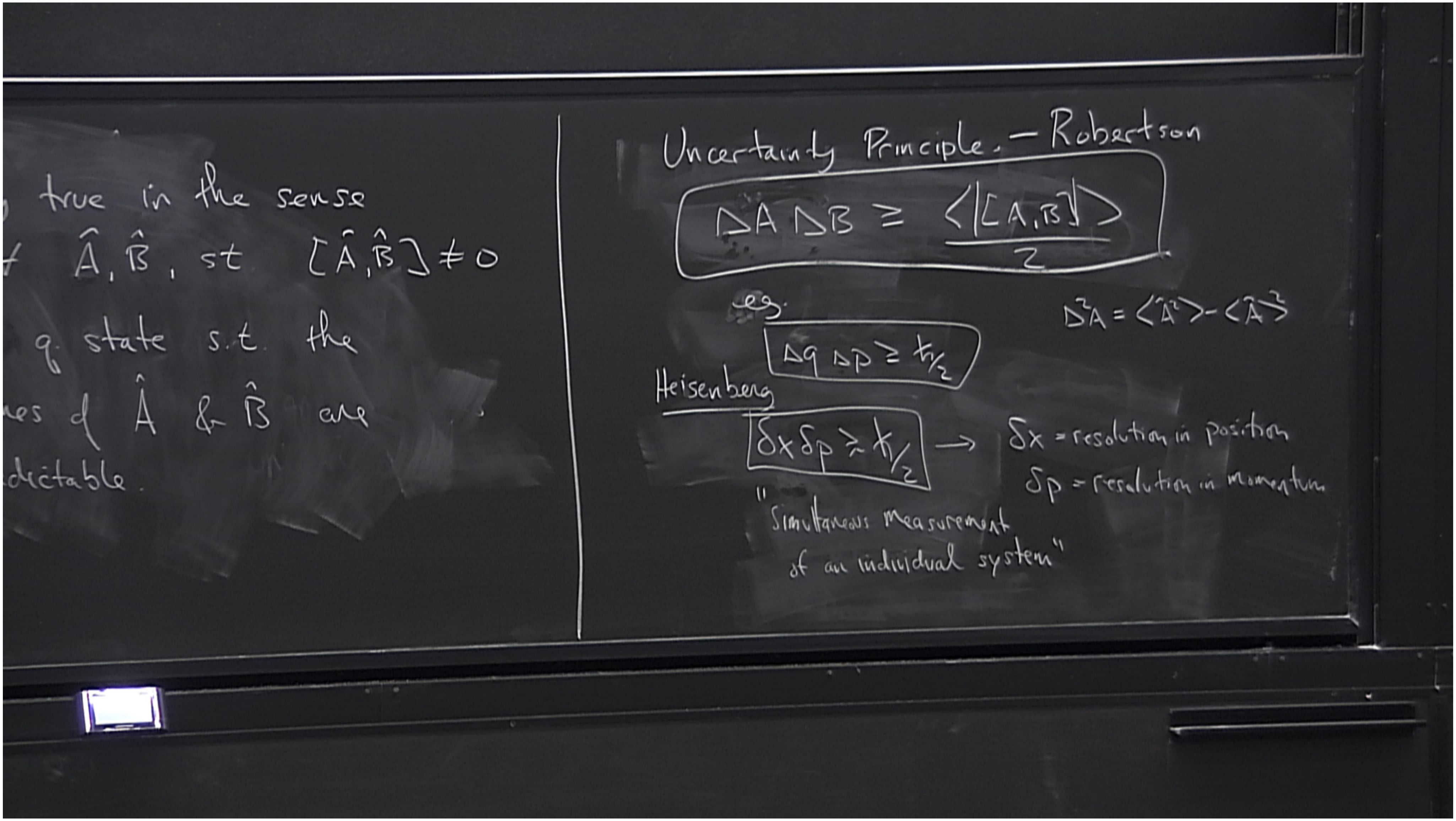
dictable.
Uncertainty Principle.
ΔA ΔB ≥ ⟨|[A,B]|⟩ / 2
eg.
Δq Δp ≥ ħ/2
Heisenberg
δx δp ≳ ħ/2 → δx = resolution in position
δp = resolution in momentum
"Simultaneous measurement
of an individual system"


The q state corresponds to incomplete knowledge
Uncertainty Principle — Robertson
For a unitary transf^n
"Collapse" of wavefn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ΔA ΔB ≥ ⟨|[A, B]|⟩ / 2
Δ²A = ⟨Â²⟩ – ⟨Â⟩²
eg.
Δq Δp ≥ ħ/2
Heisenberg
δx δp ≳ ħ/2
δx = resolution in position
δp = resolution in momentum
"Simultaneous measurement
of an individual system"

… true in the sense

… $\hat{A}, \hat{B}$, st $[\hat{A}, \hat{B}] \neq 0$

… q. state s.t. the

…es of $\hat{A}$ & $\hat{B}$ are

…dictable.

Uncertainty Principle. — Robertson

$$\Delta A \, \Delta B \geq \left| \frac{\langle [A, B] \rangle}{2i} \right|$$

"mutually exclusive / incompatible measurements" on a preparation

$$\Delta^2 A = \langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2$$

e.g.

$$\Delta q \, \Delta p \geq \hbar/2$$

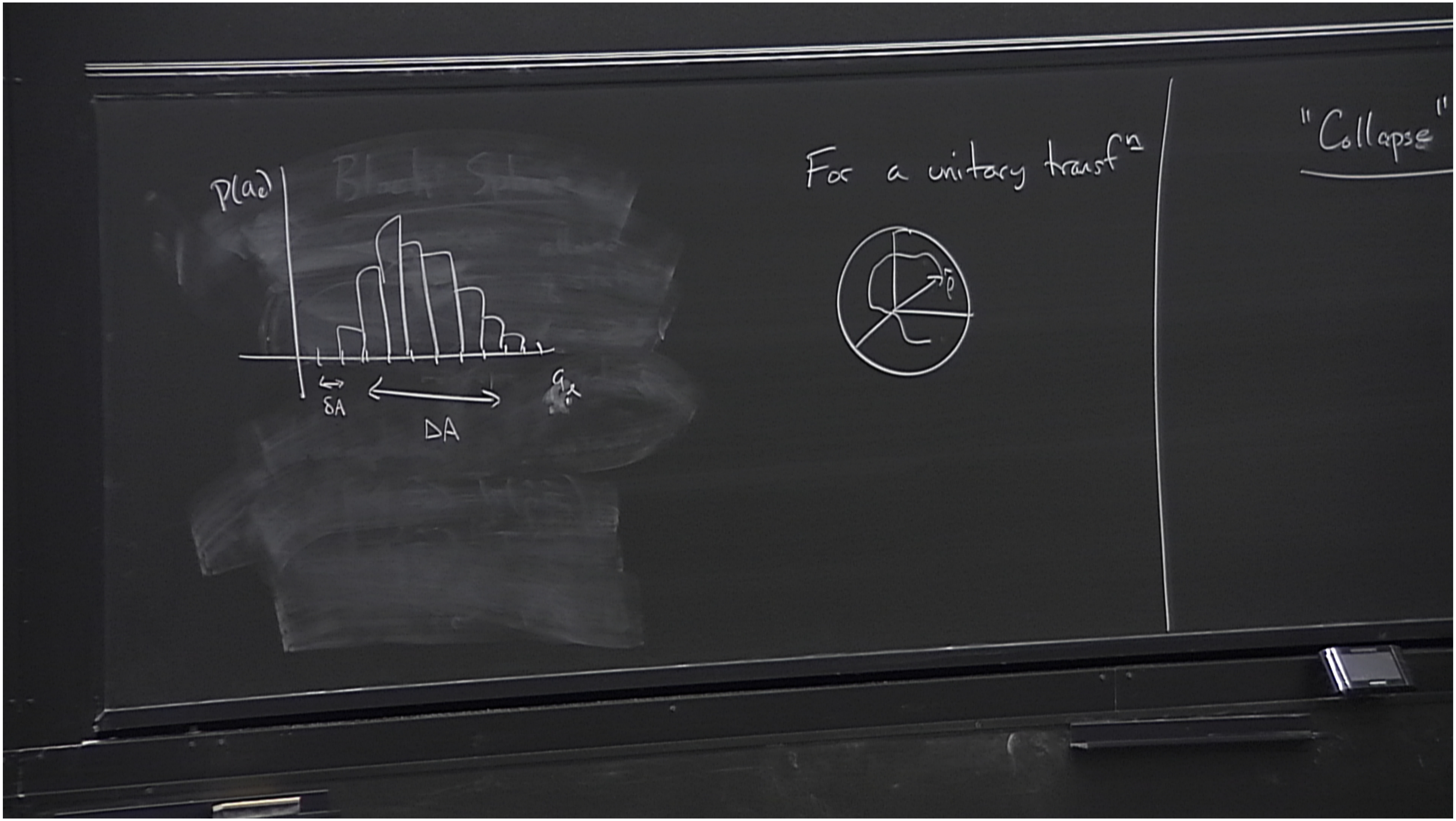
Heisenberg

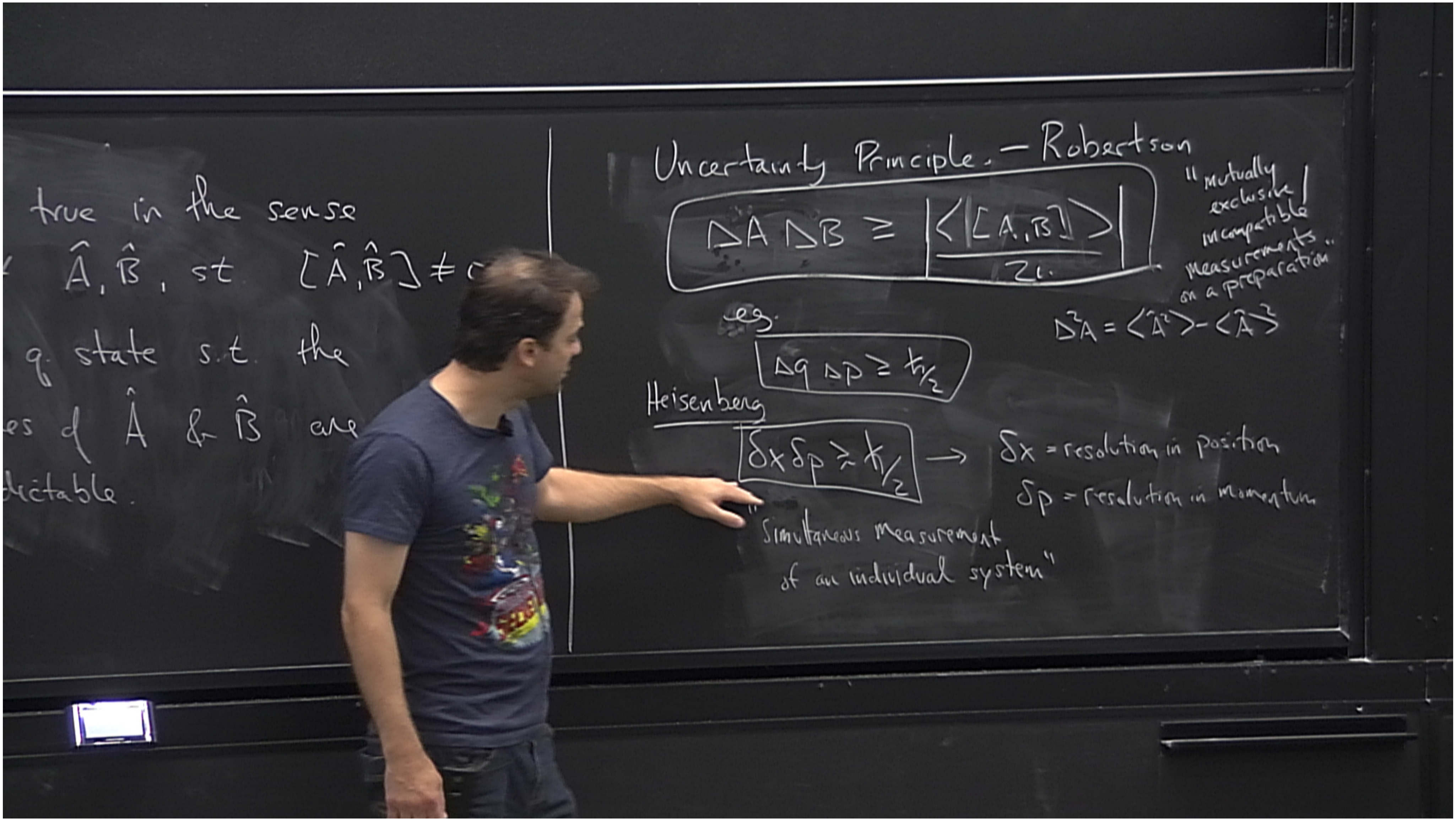
$$\delta x \, \delta p \gtrsim \hbar/2$$

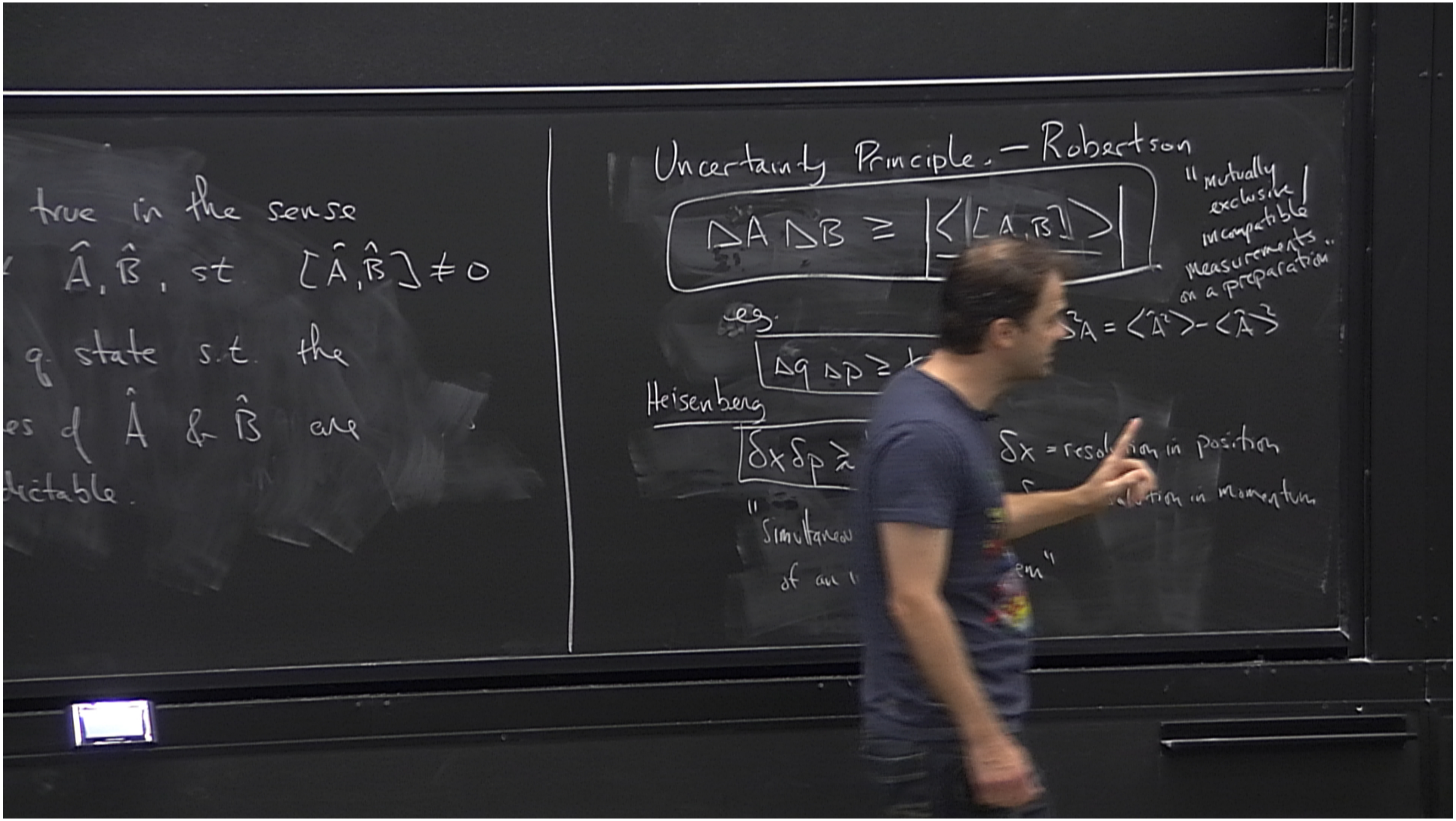
→ δx = resolution in position

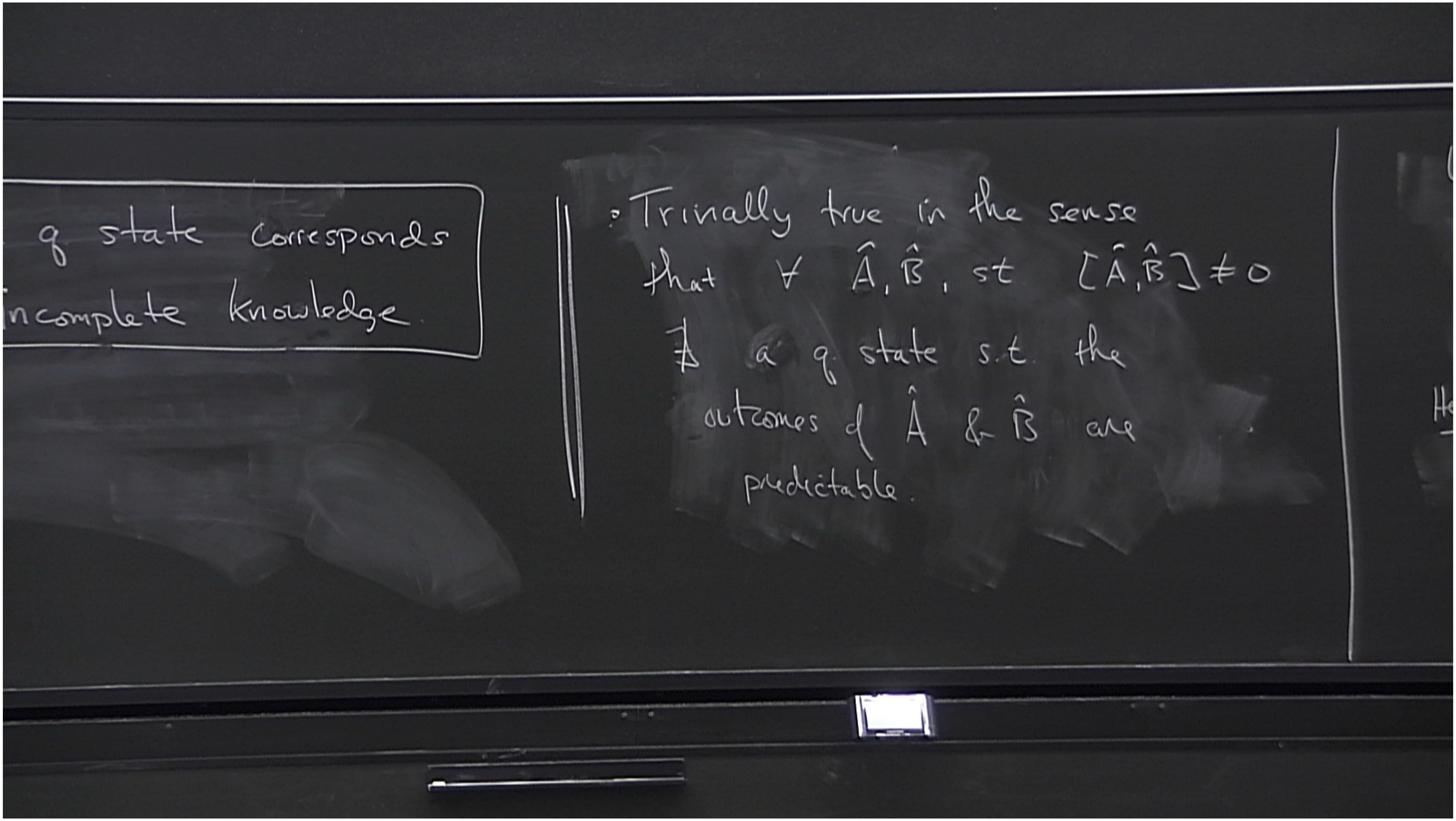
δp = resolution in momentum

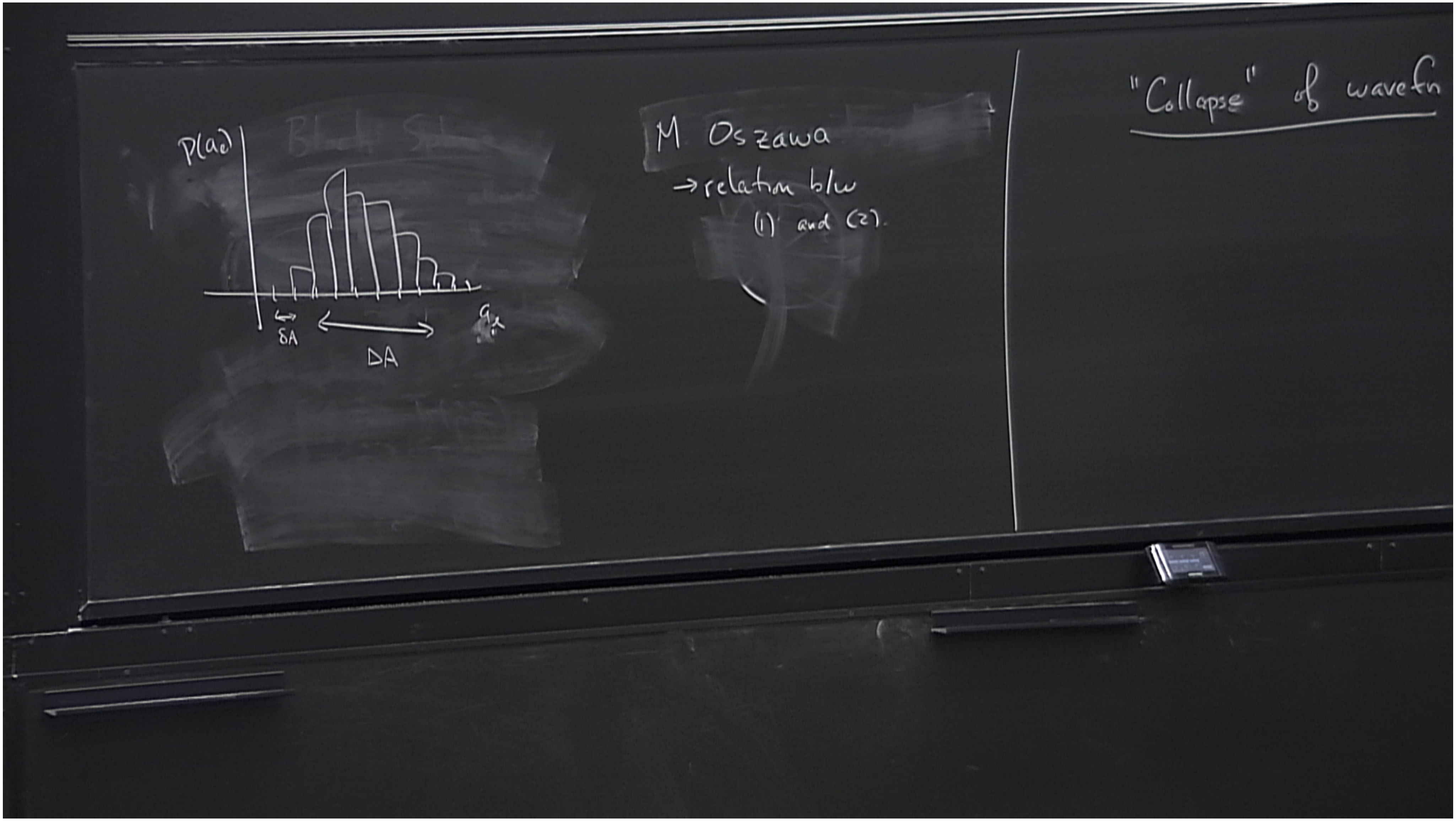
"Simultaneous measurement of an individual system"


p(a_e)
δA
ΔA
For a unitary transf^n
ρ̄
"Collapse"


true in the sense
Uncertainty Principle. — Robertson
$\Delta A \Delta B \geq \left| \frac{\langle [A,B] \rangle}{2i} \right|$
"mutually exclusive / incompatible measurements on a preparation"
$\Delta^2 A = \langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2$
e.g.
$\Delta q \Delta p \geq \hbar/2$
Heisenberg
$\delta x \delta p \gtrsim \hbar/2$
δx = resolution in position
δp = resolution in momentum
"Simultaneous measurement of an individual system"


true in the sense
Â, B̂, st [Â, B̂] ≠ 0
q. state s.t. the
es of Â & B̂ are
dictable.
Uncertainty Principle. — Robertson
ΔA ΔB ≥ |⟨[A, B]⟩|
"mutually exclusive/incompatible measurements.. on a preparation"
e.g.
Δq Δp ≥
A = ⟨Â²⟩ − ⟨Â⟩²
Heisenberg
δx δp ≳
δx = resol in position
in momentum
"Simultaneou
of an
em"


q state corresponds
incomplete knowledge
Trivially true in the sense
that ∀ Â, B̂, st [Â, B̂] ≠ 0
∄ a q state s.t. the
outcomes of Â & B̂ are
predictable.


M Oszawa
→ relation b/w
(1) and (2).
δA
ΔA
"Collapse" of wavefn