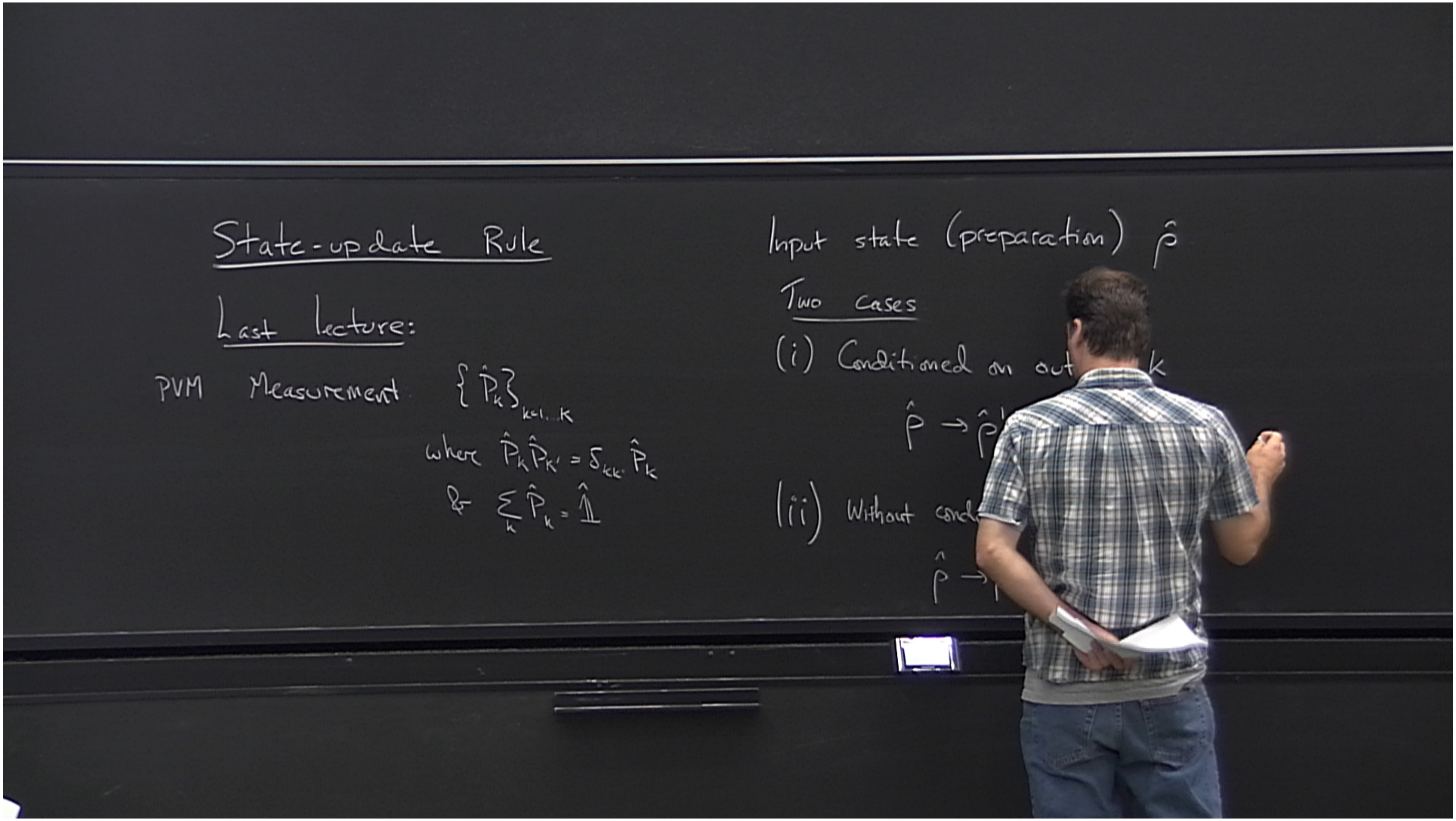
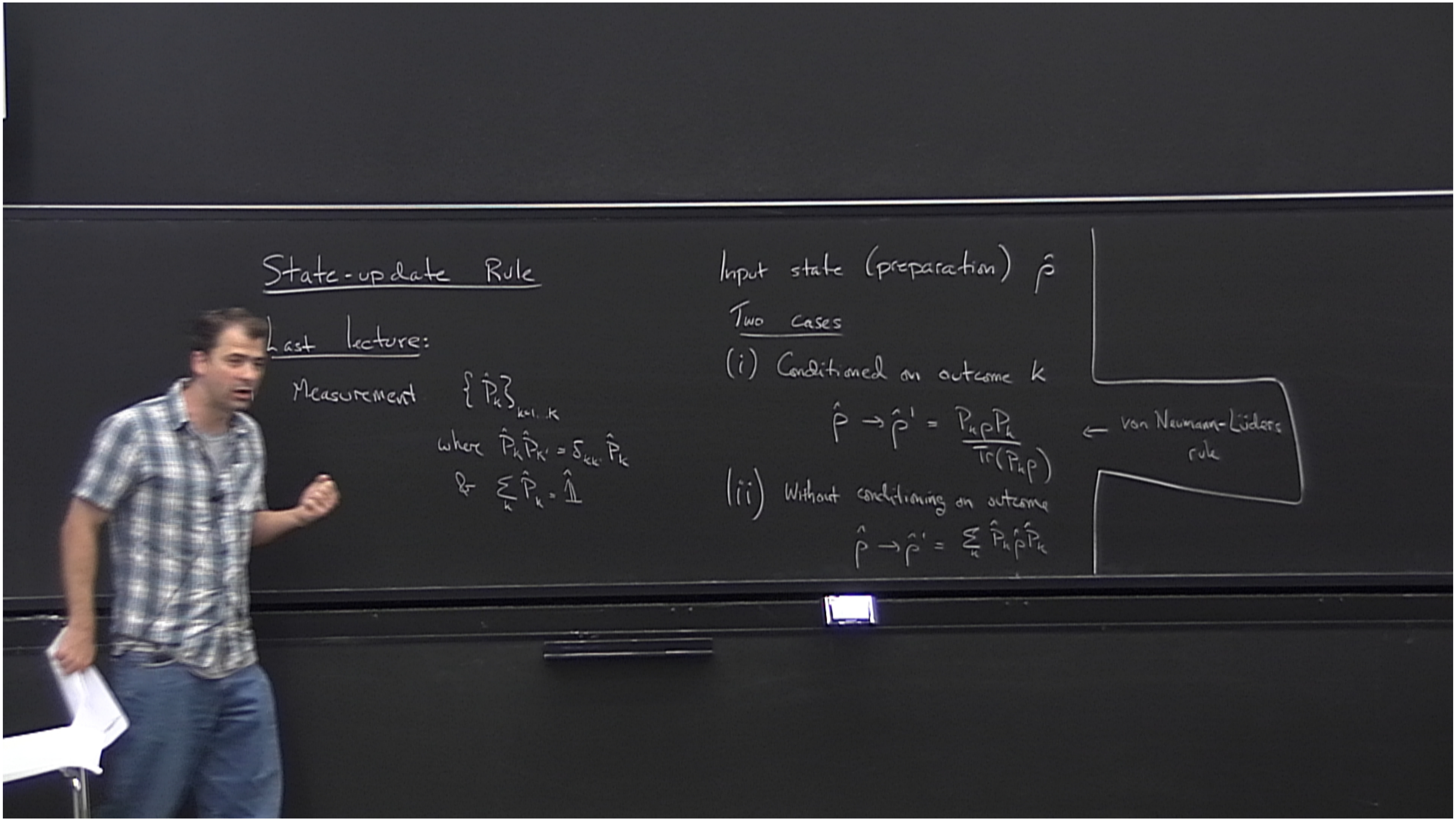
Title: Quantum Theory - Lecture 6

Date: Sep 17, 2012 09:00 AM

URL: http://pirsa.org/12090007

Abstract:


State-update Rule
Last lecture:
PVM Measurement $\{\hat{P}_k\}_{k=1..K}$
where $\hat{P}_k \hat{P}_{k'} = \delta_{kk'} \hat{P}_k$
& $\sum_k \hat{P}_k = \hat{\mathbb{1}}$
Input state (preparation) $\hat{\rho}$
Two cases
(i) Conditioned on out
(ii) Without cond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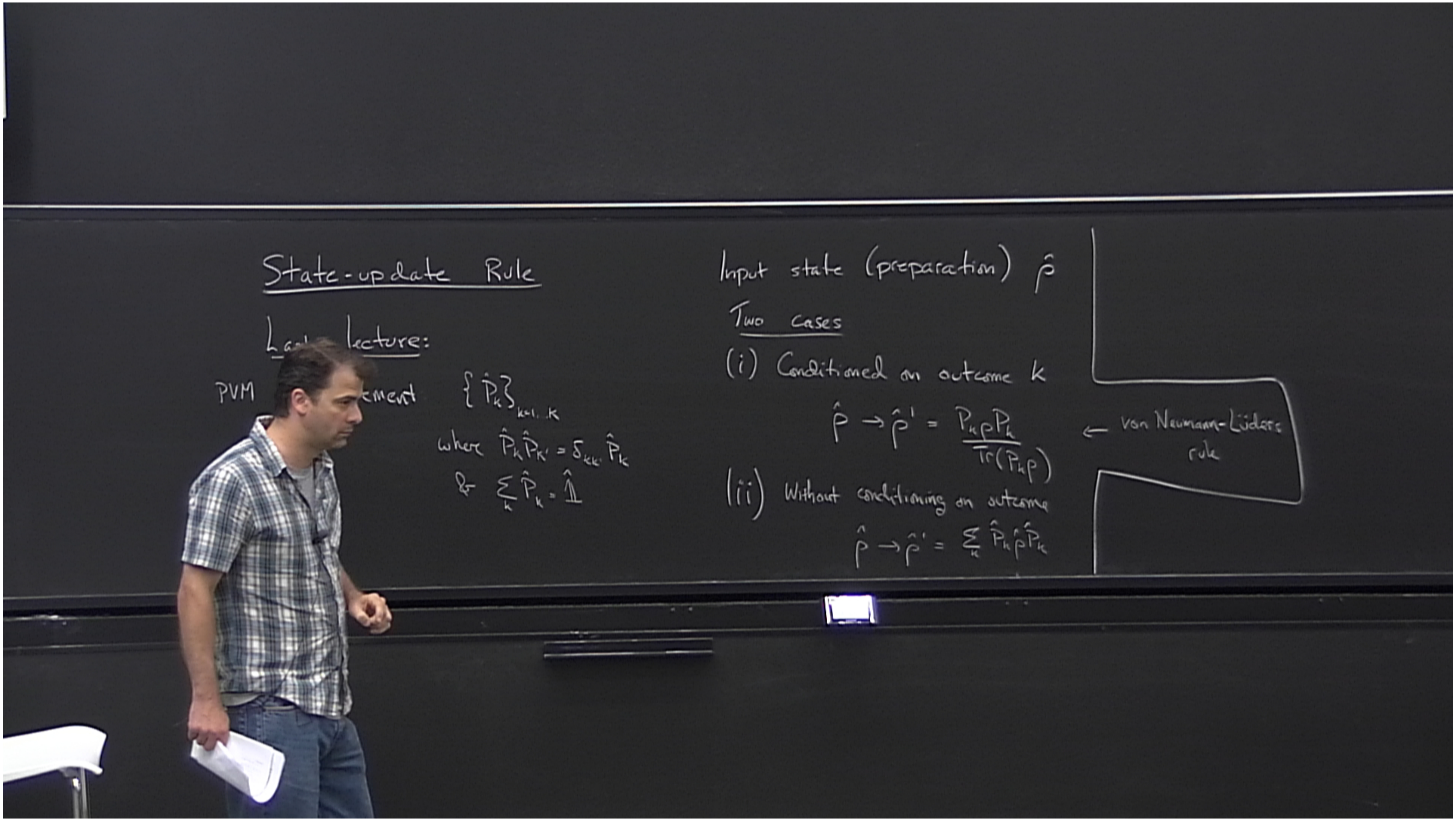
(i) Conditioned on outcome k
$\hat{\rho} \to \hat{\rho}' = \frac{P_k \rho P_k}{Tr(P_k \rho)}$
← von Neumann-Lüders rule
(ii) Without conditioning on outcome
$\hat{\rho} \to \hat{\rho}' = \sum_k \hat{P}_k \hat{\rho} \hat{P}_k$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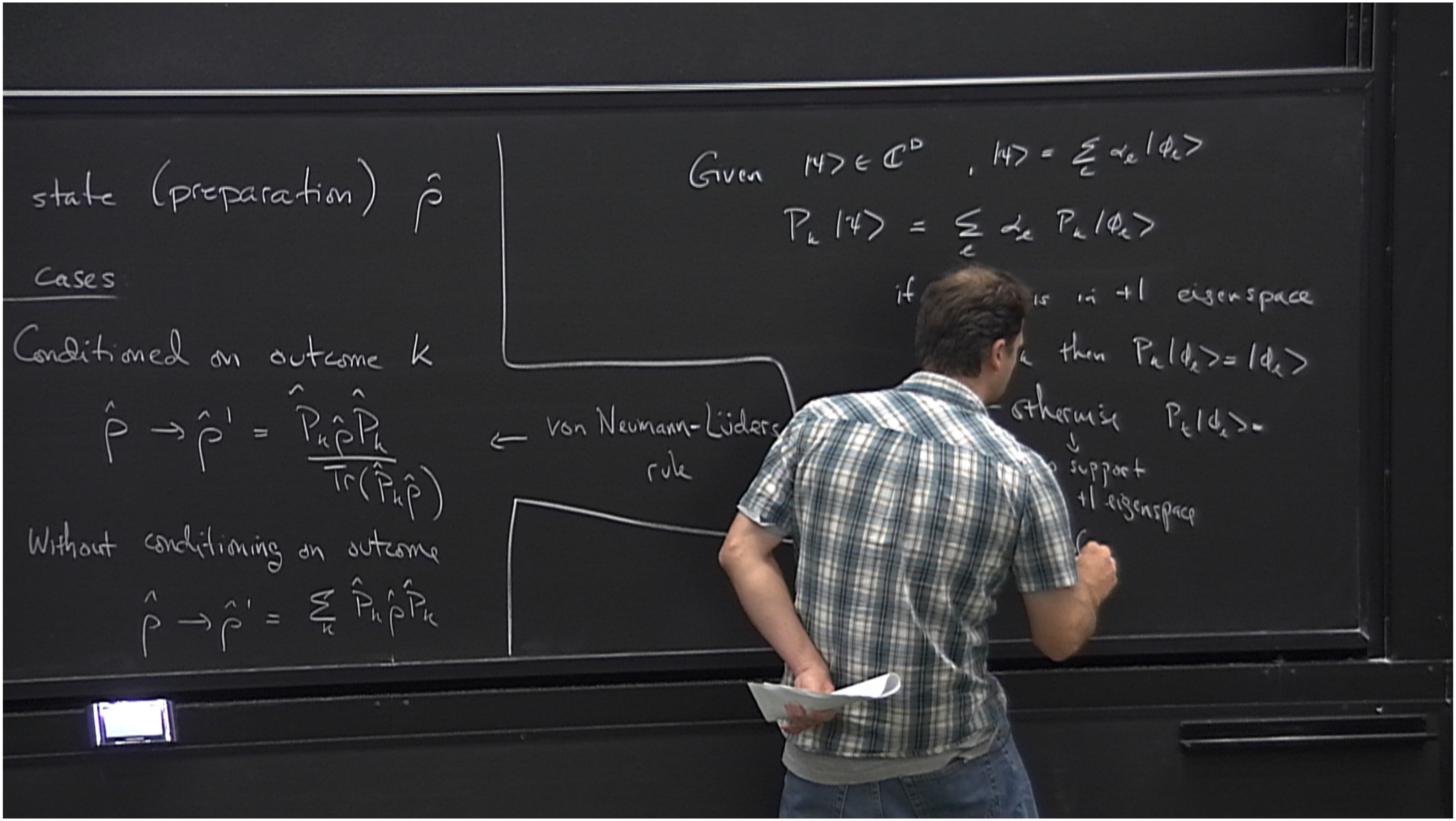

state (preparation) $\hat{\rho}$
Cases
Conditioned on outcome k
$\hat{\rho} \to \hat{\rho}' = \frac{\hat{P}_k \hat{\rho} \hat{P}_k}{\mathrm{Tr}(\hat{P}_k \hat{\rho})}$
$\leftarrow$ von Neumann-Lüders rule
Without conditioning on outcome
$\hat{\rho} \to \hat{\rho}' = \sum_k \hat{P}_k \hat{\rho} \hat{P}_k$
Given $|\psi\rangle \in \mathbb{C}^D$, $|\psi\rangle = \sum_\ell \alpha_\ell |\phi_\ell\rangle$
$P_k |\psi\rangle = \sum_\ell \alpha_\ell P_k |\phi_\ell\rangle$
if
is in +1 eigenspace
then $P_k|\phi_\ell\rangle = |\phi_\ell\rangle$
otherwise $P_k|\phi_\ell\rangle =$
support
+1 eigenspace

state (preparation) $\hat{\rho}$

Cases

Conditioned on outcome k

$$\hat{\rho} \rightarrow \hat{\rho}' = \frac{\hat{P}_k \hat{\rho} \hat{P}_k}{\mathrm{Tr}(\hat{P}_k \hat{\rho})}$$

$\leftarrow$ von Neumann-Lüders rule

Without conditioning on outcome

$$\hat{\rho} \rightarrow \hat{\rho}' = \sum_k \hat{P}_k \hat{\rho} \hat{P}_k$$

Given $|\psi\rangle \in \mathbb{C}^D$, $|\psi\rangle = \sum_\ell \alpha_\ell |\phi_\ell\rangle$

$$P_k |\psi\rangle = \sum_\ell \alpha_\ell \underline{P_k |\phi_\ell\rangle}$$

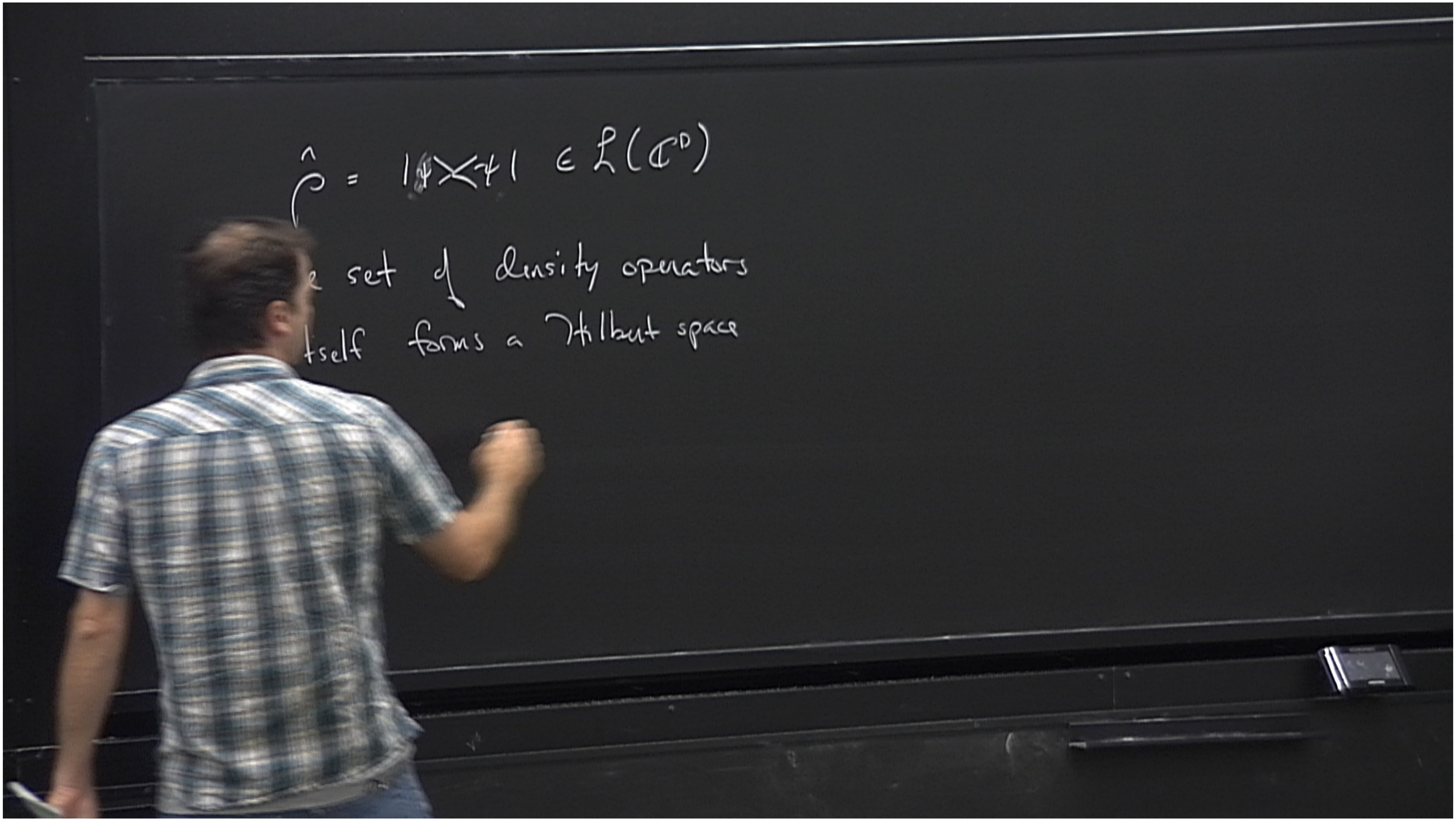
if $|\phi_\ell\rangle$ is in +1 eigenspace of P_k then $P_k |\phi_\ell\rangle = |\phi_\ell\rangle$

& otherwise $P_k |\phi_\ell\rangle = 0$

$\downarrow$

no support in +1 eigenspace of P_k

$$\hat{\rho} = |\psi\rangle\langle\psi| \in \mathcal{L}(\mathbb{C}^D)$$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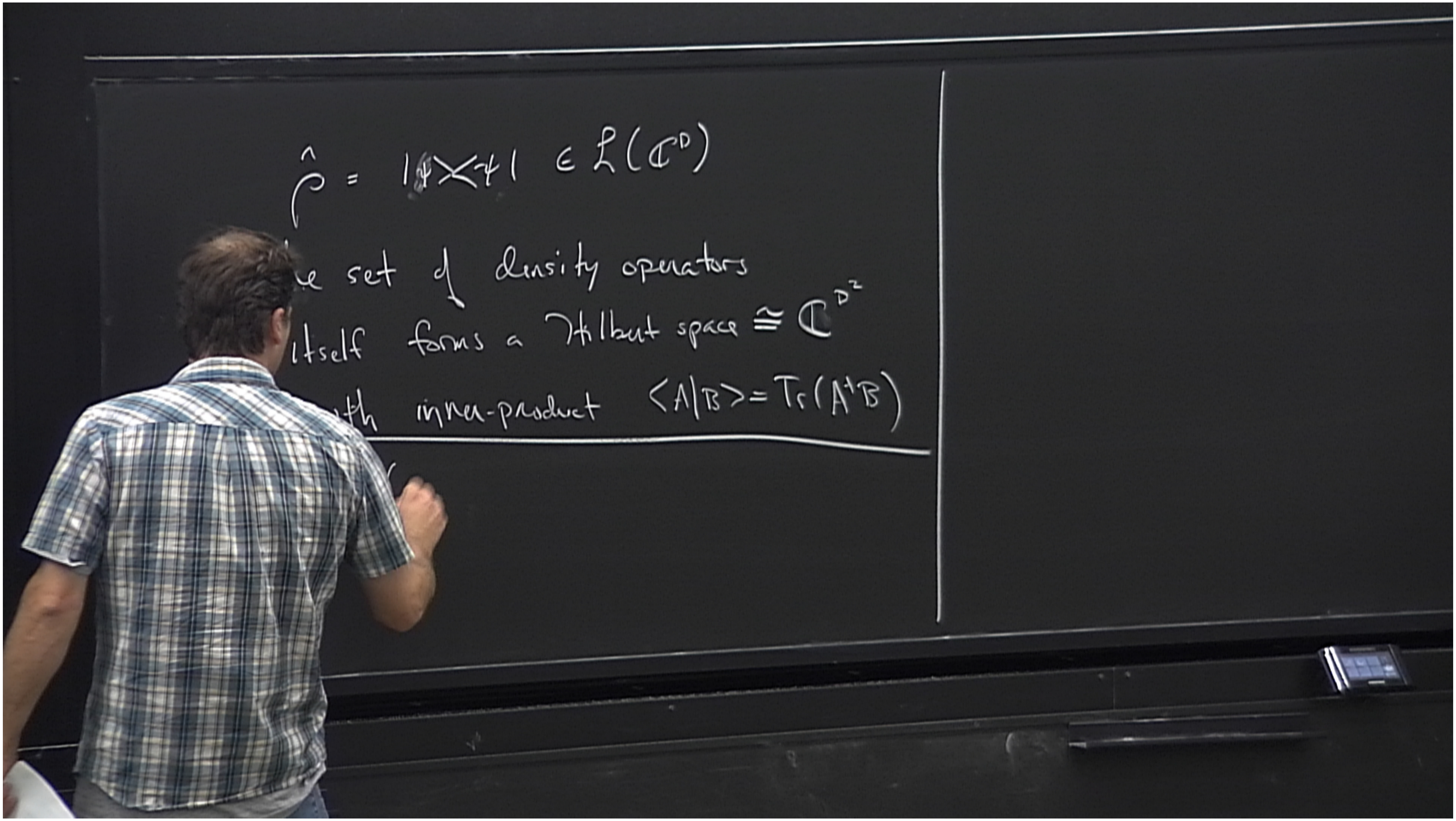
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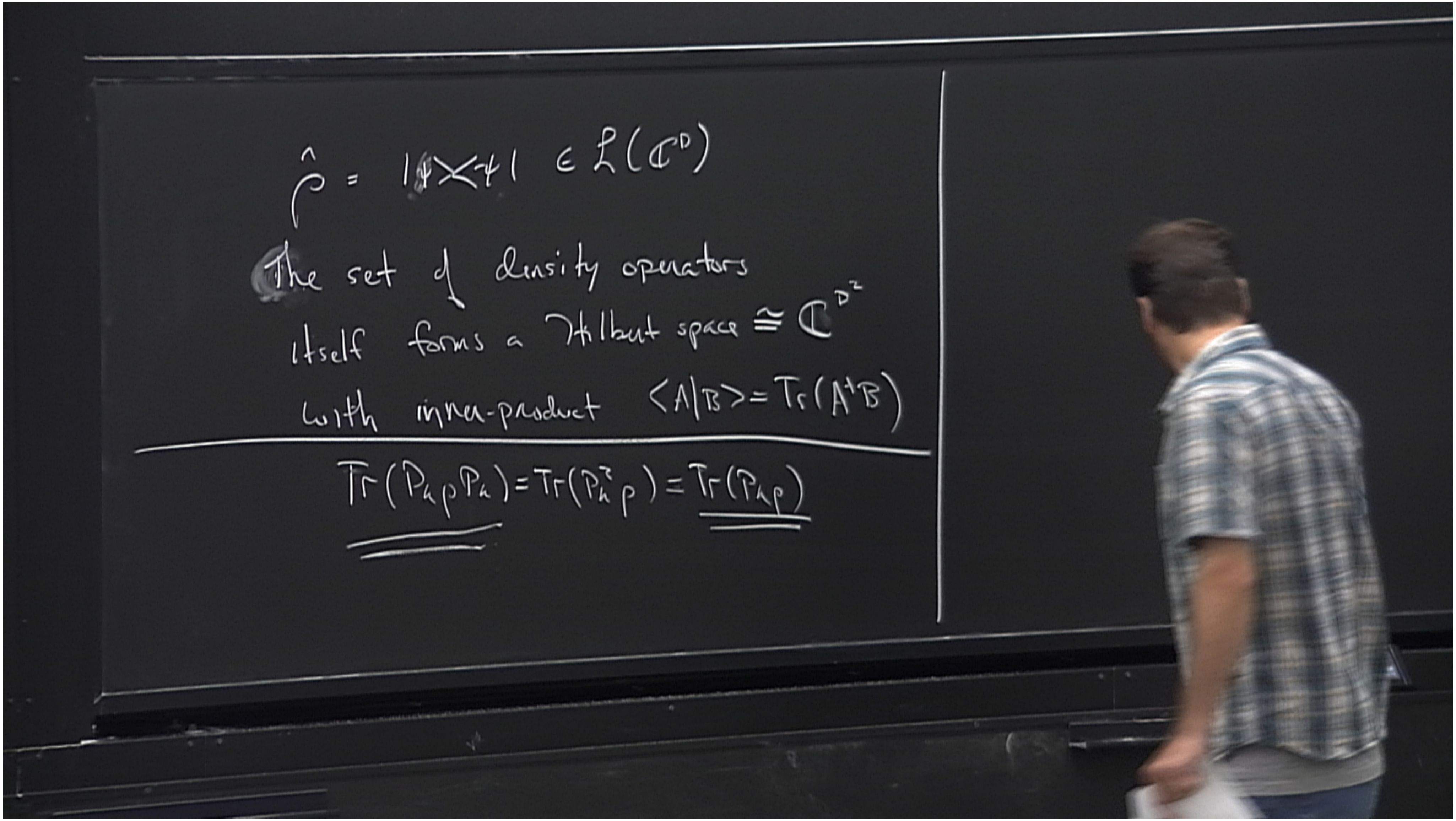
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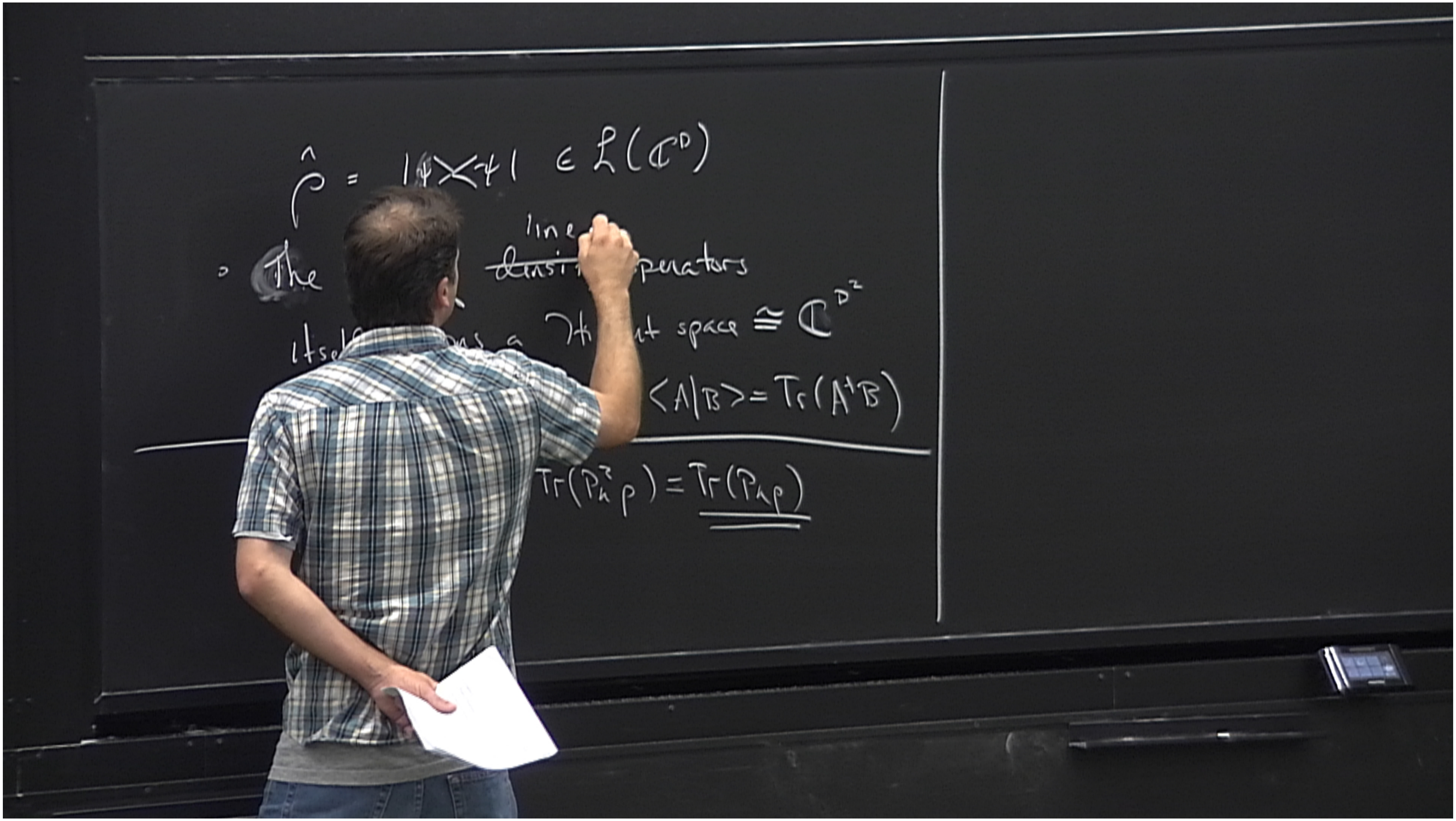
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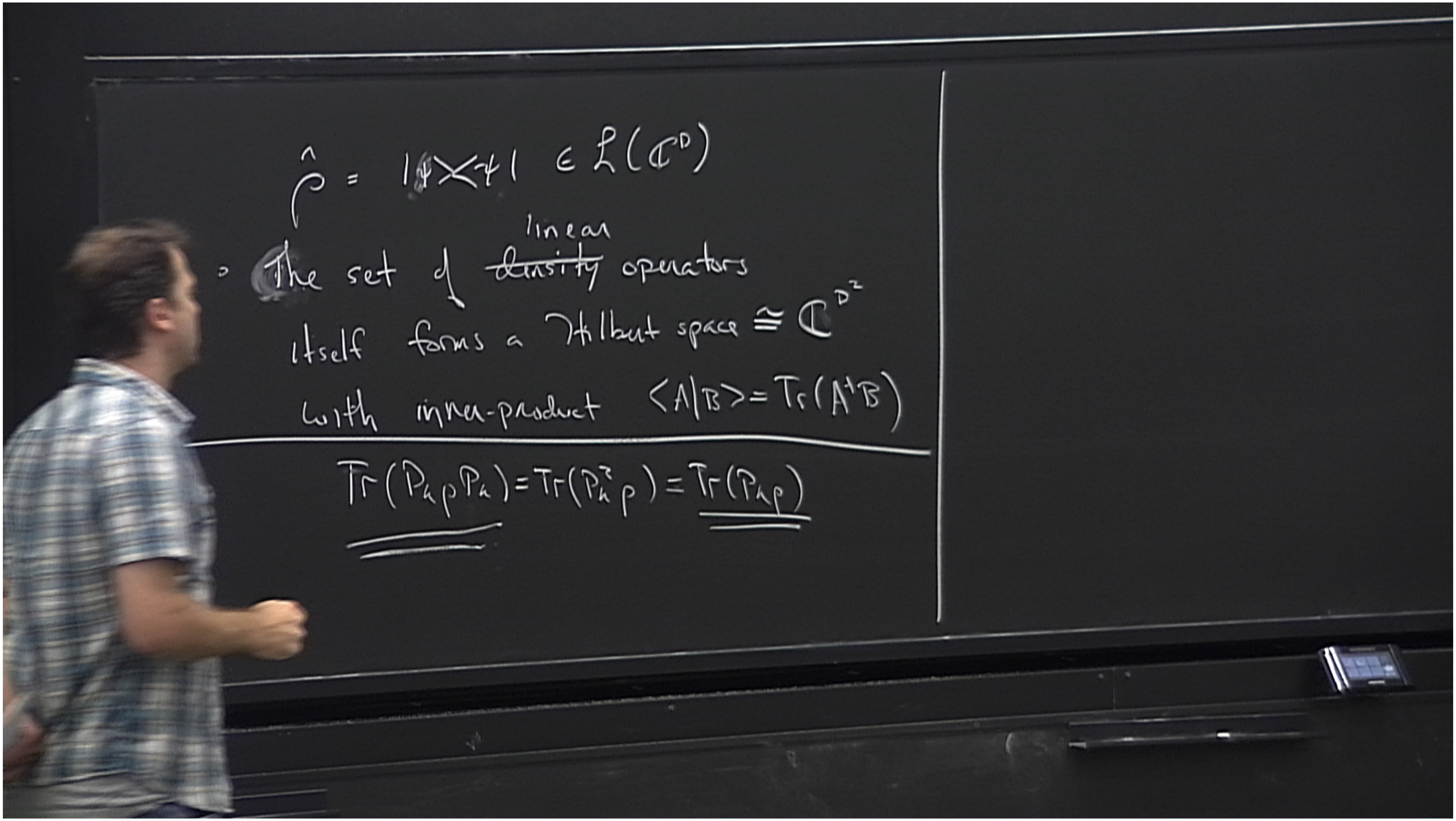

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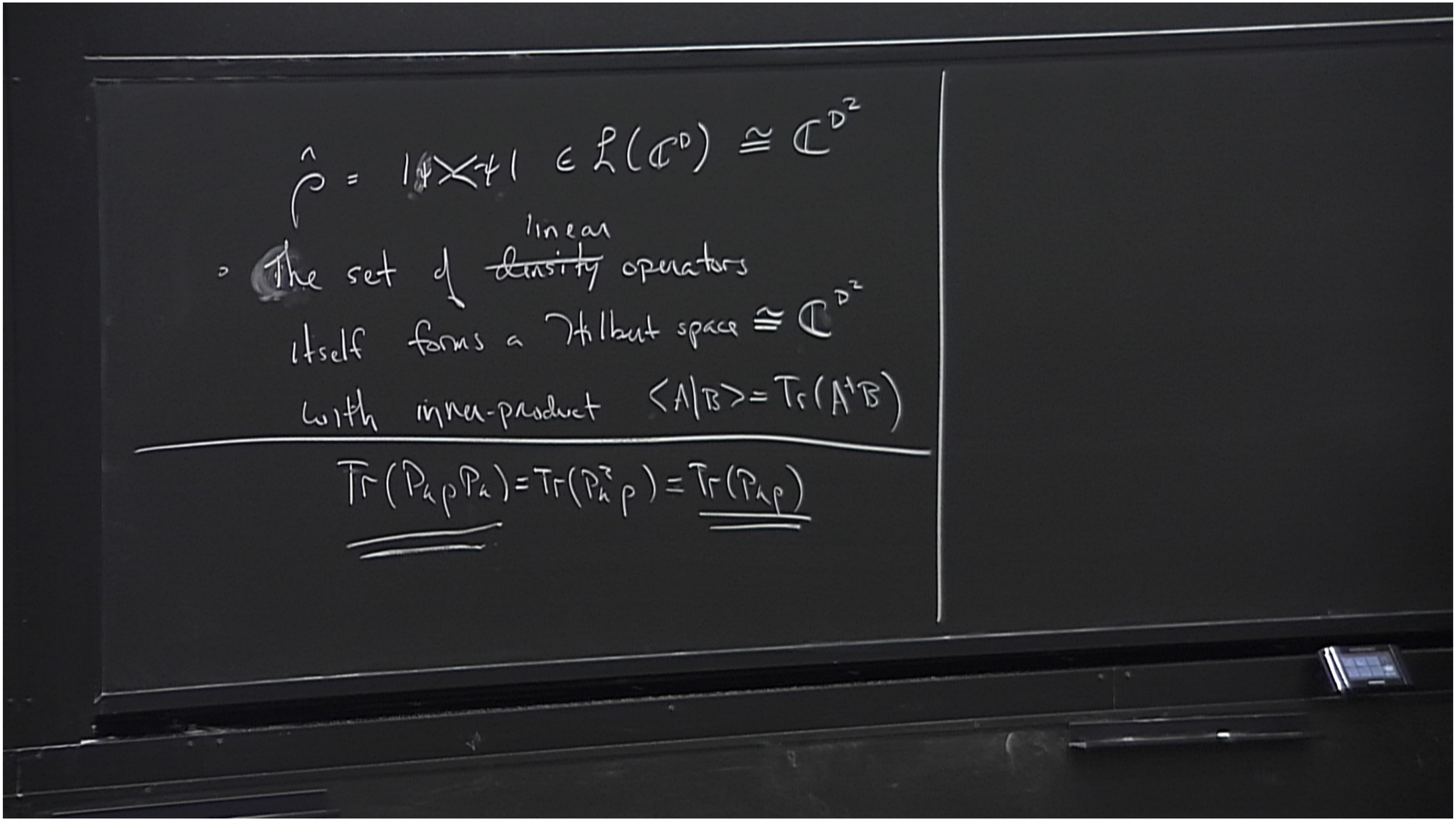
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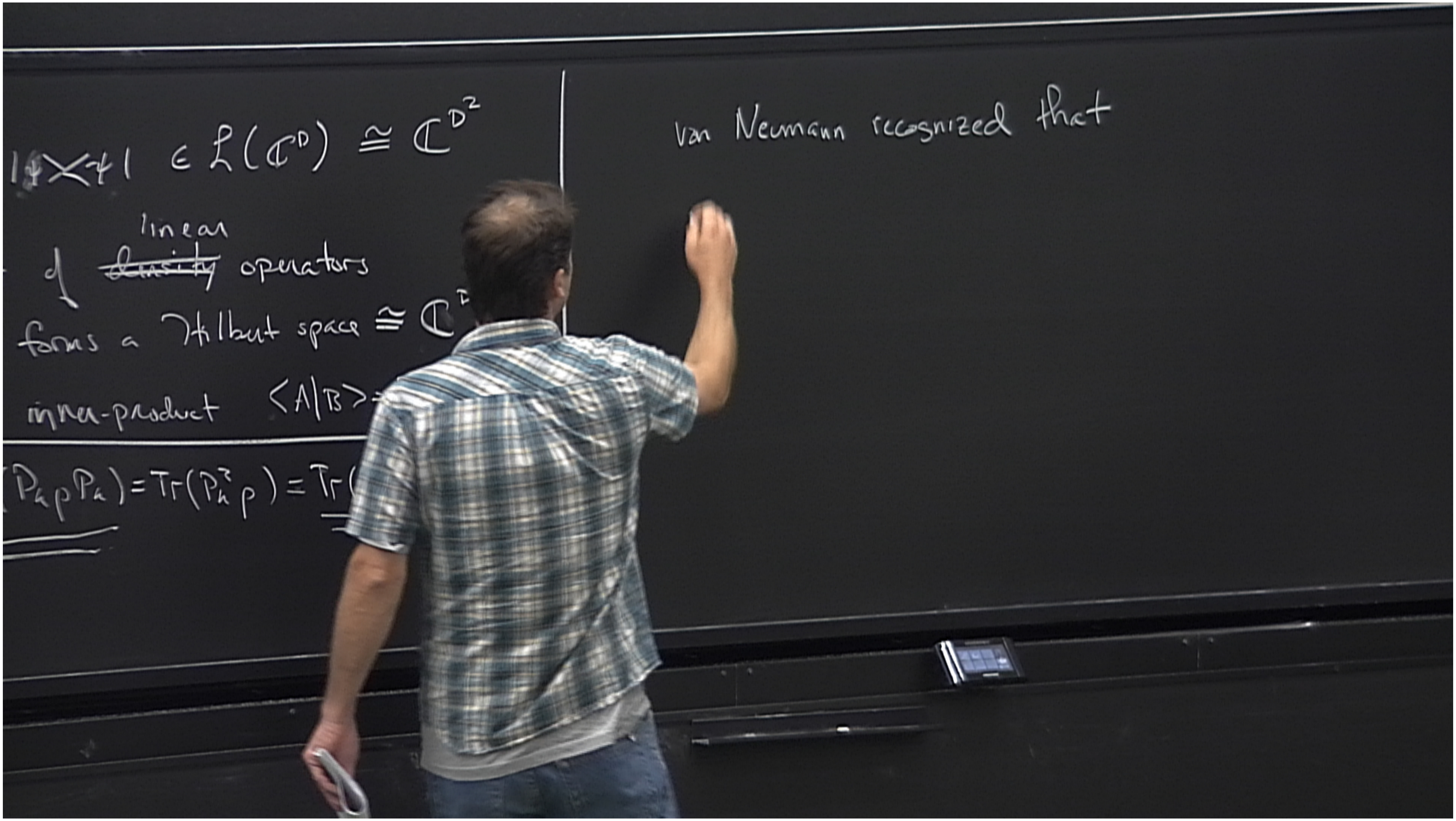
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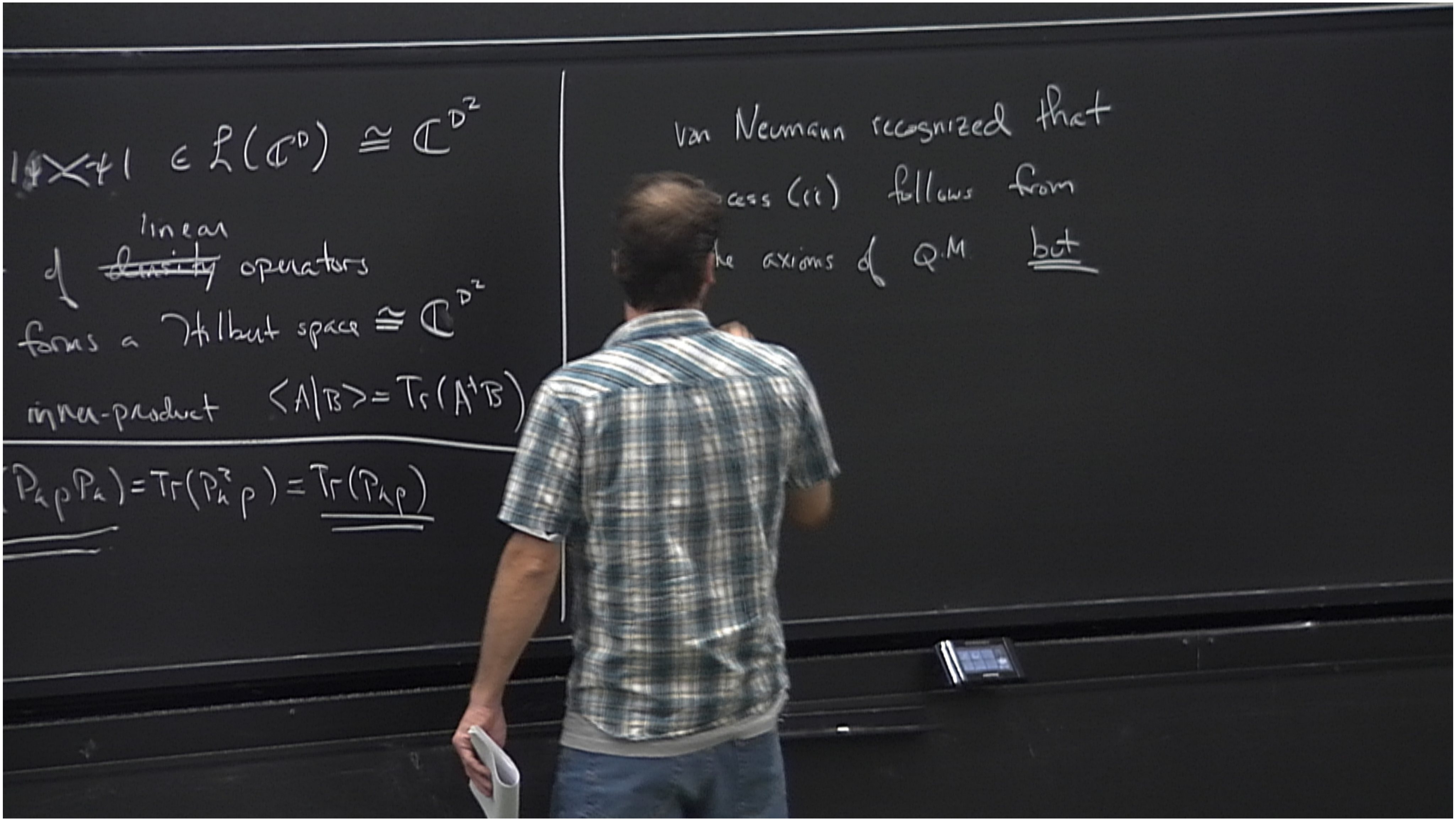
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van Neumann recognized that
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$|\psi\rangle\langle\psi| \in \mathcal{L}(\mathbb{C}^D) \cong \mathbb{C}^{D^2}$

of ~~density~~ linear operators

forms a Hilbert space $\cong \mathbb{C}^{D^2}$

inner-product $\langle A|B\rangle = \mathrm{Tr}(A^\dagger B)$

$(P_k \rho P_k) = \mathrm{Tr}(P_k^2 \rho) = \underline{\underline{\mathrm{Tr}(P_k \rho)}}$

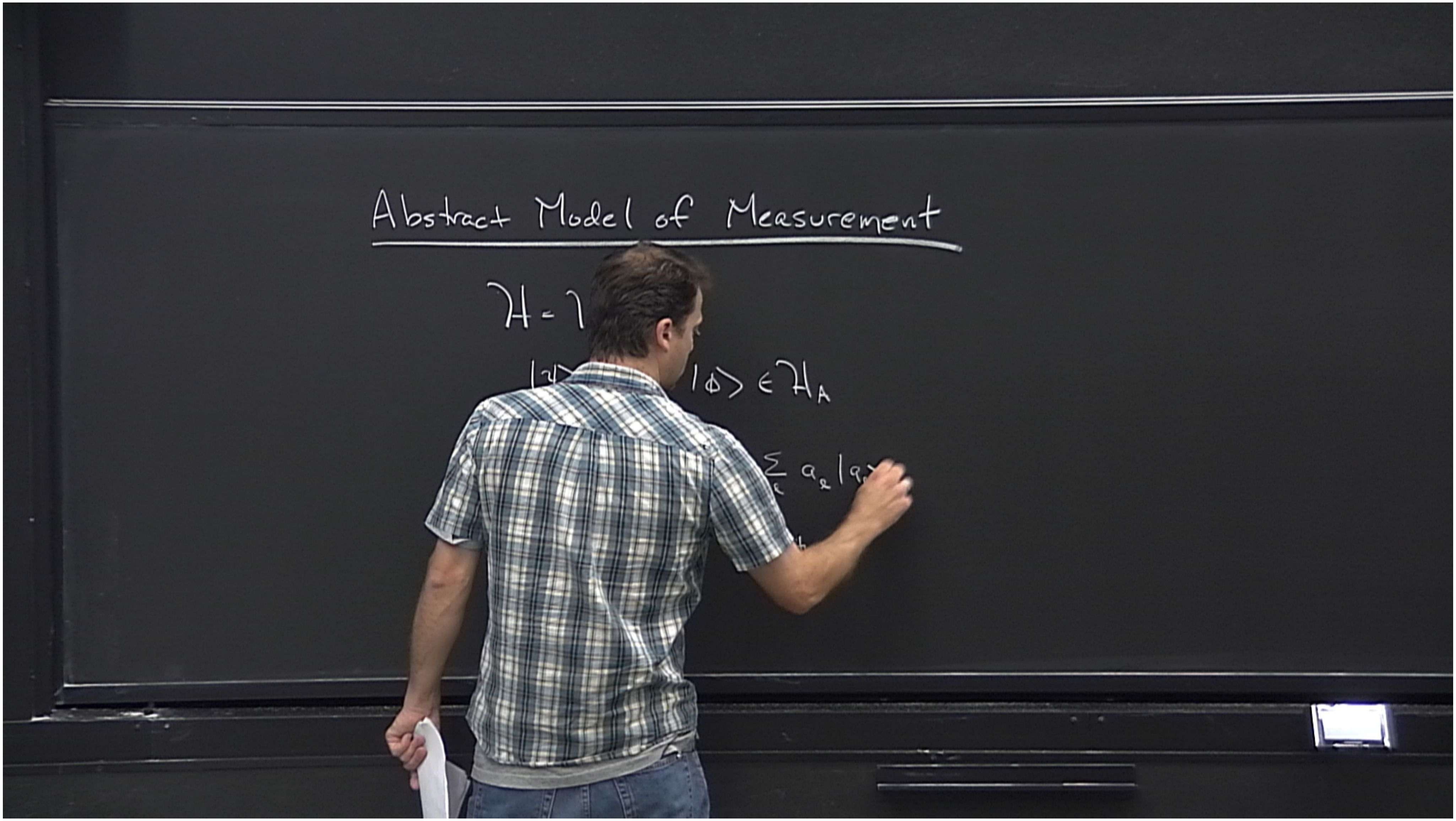
van Neumann recognized that

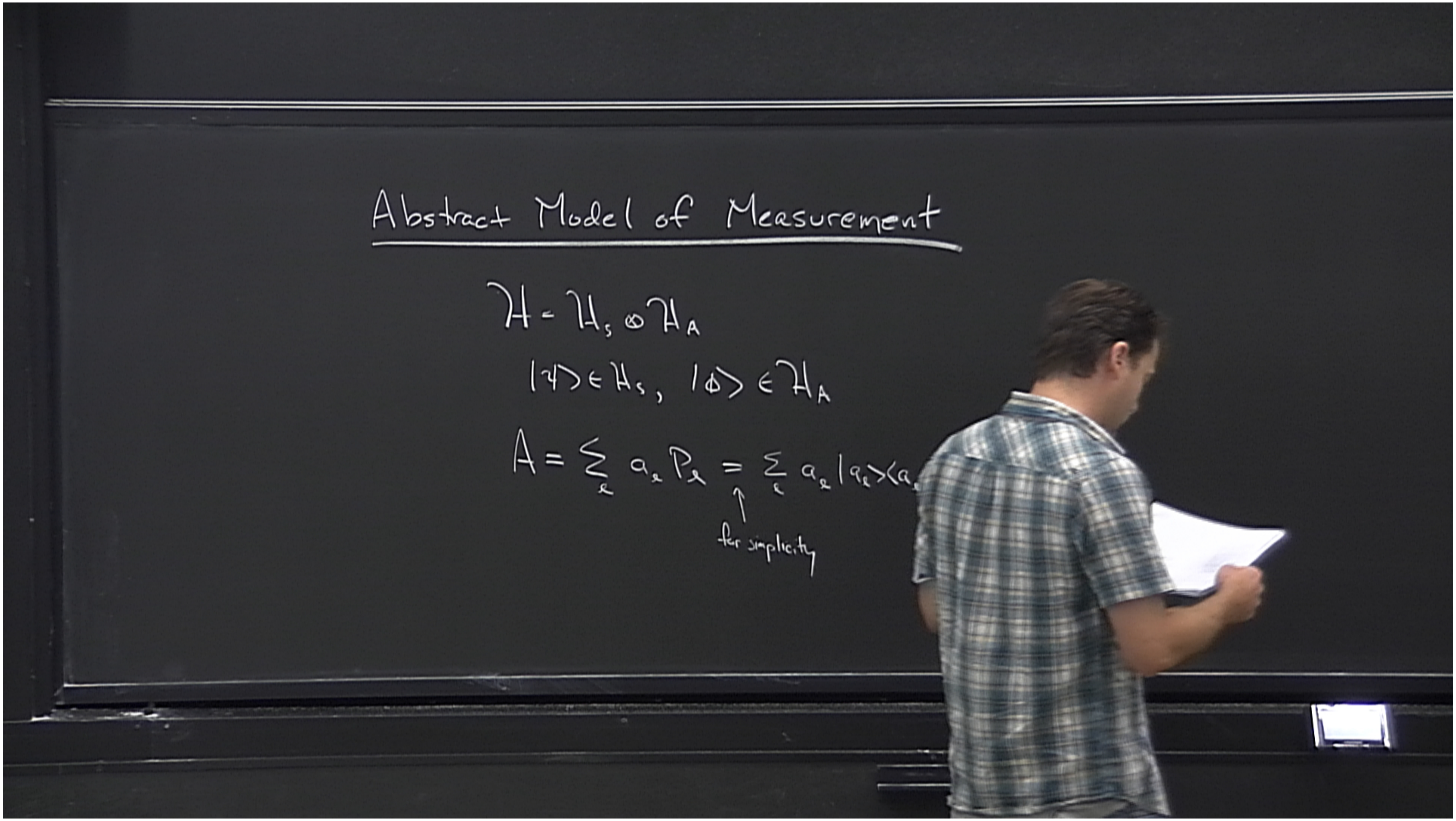
Process (ii) follows from

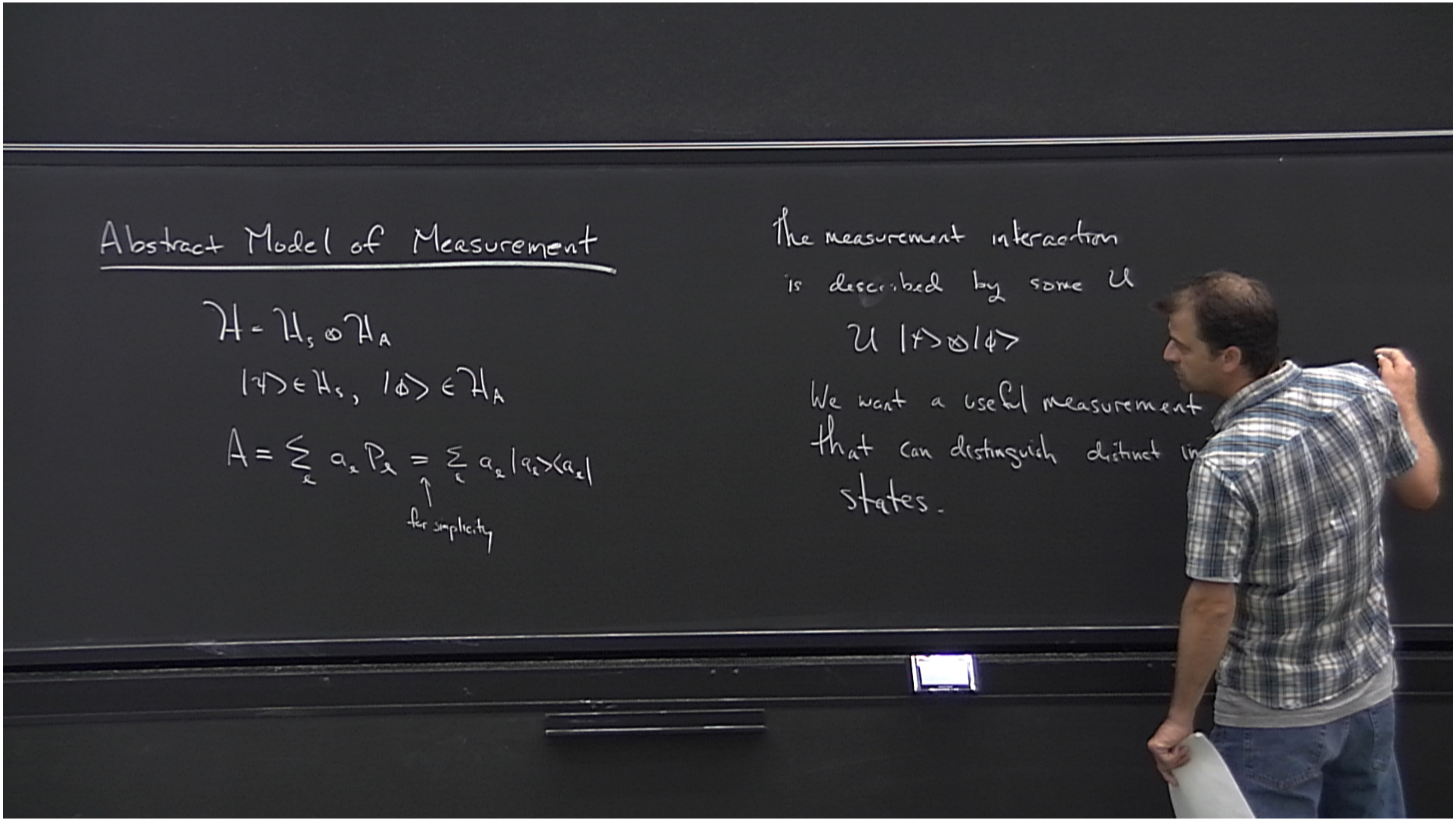
the axioms of Q.M $\underline{\underline{\text{but}}}$

Process (i) does not

↳ collapse, projection postulate etc.


Abstract Model of Measurement


Abstract Model of Measurement
$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A$
$|\psi\rangle \in \mathcal{H}_S, \; |\phi\rangle \in \mathcal{H}_A$
$A = \sum_l a_l P_l = \sum_l a_l |a_l\rangle\langle a_l$
for simplicity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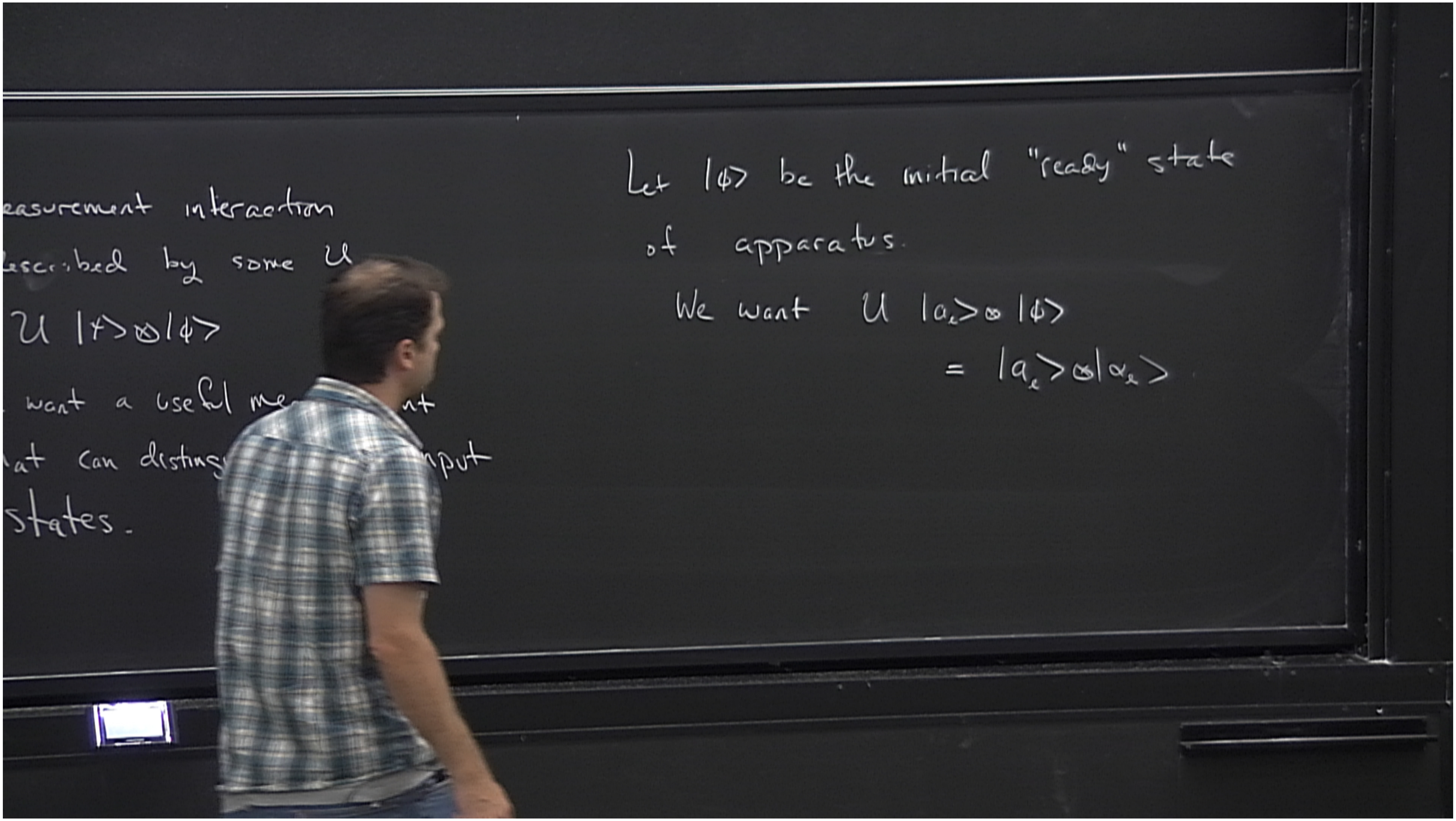
The measurement interaction
is described by some $\mathcal{U}$
$\mathcal{U}\, |\psi\rangle \otimes |\phi\rangle$
We want a useful measurement
that can distinguish distinct in
states.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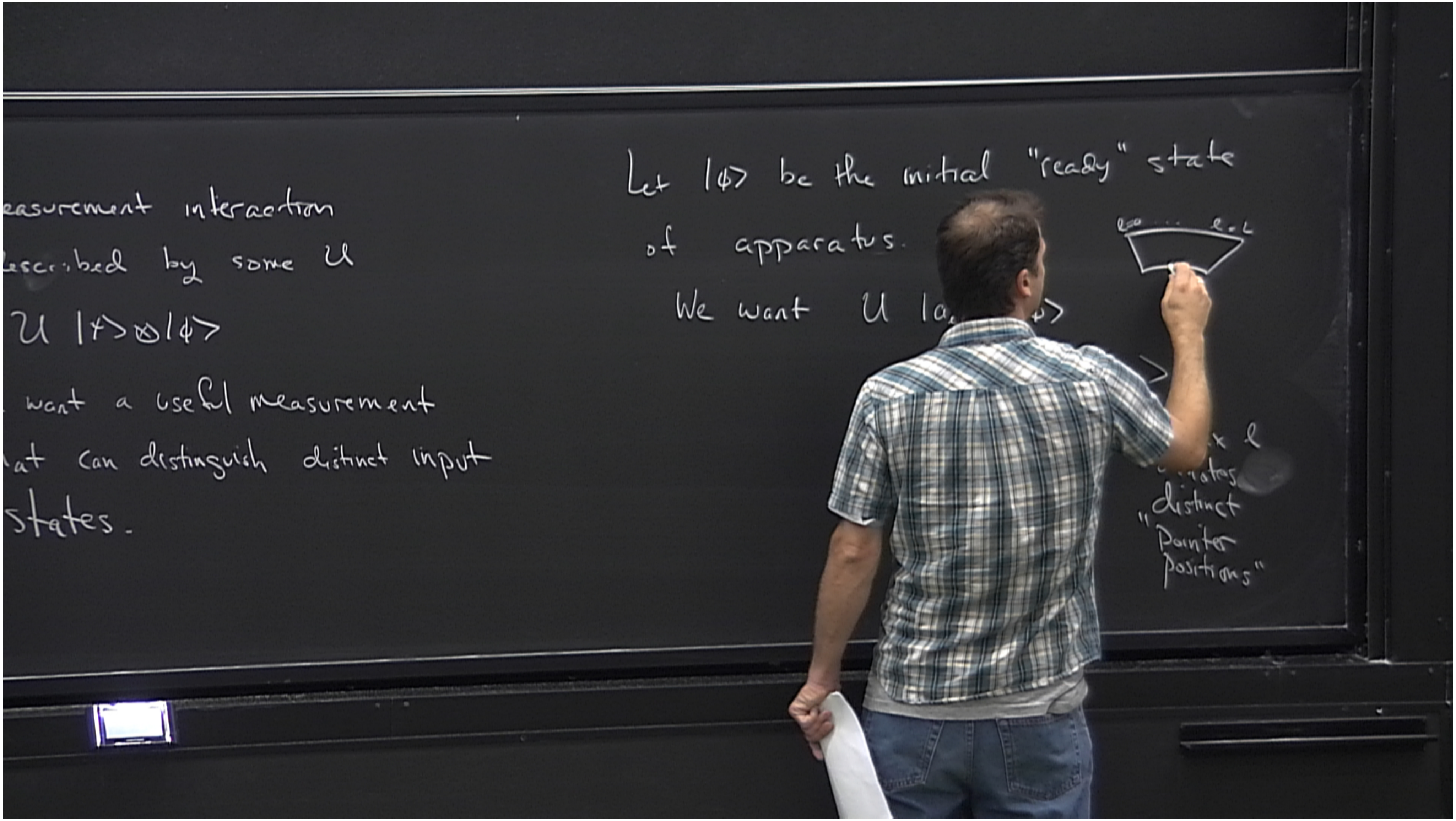
Let $|\phi\rangle$ be the initial "ready" state
of apparatus.
We want $\mathcal{U}\,|a_e\rangle \otimes |\phi\rangle$
$= |a_e\rangle \otimes |\alpha_e\rangle$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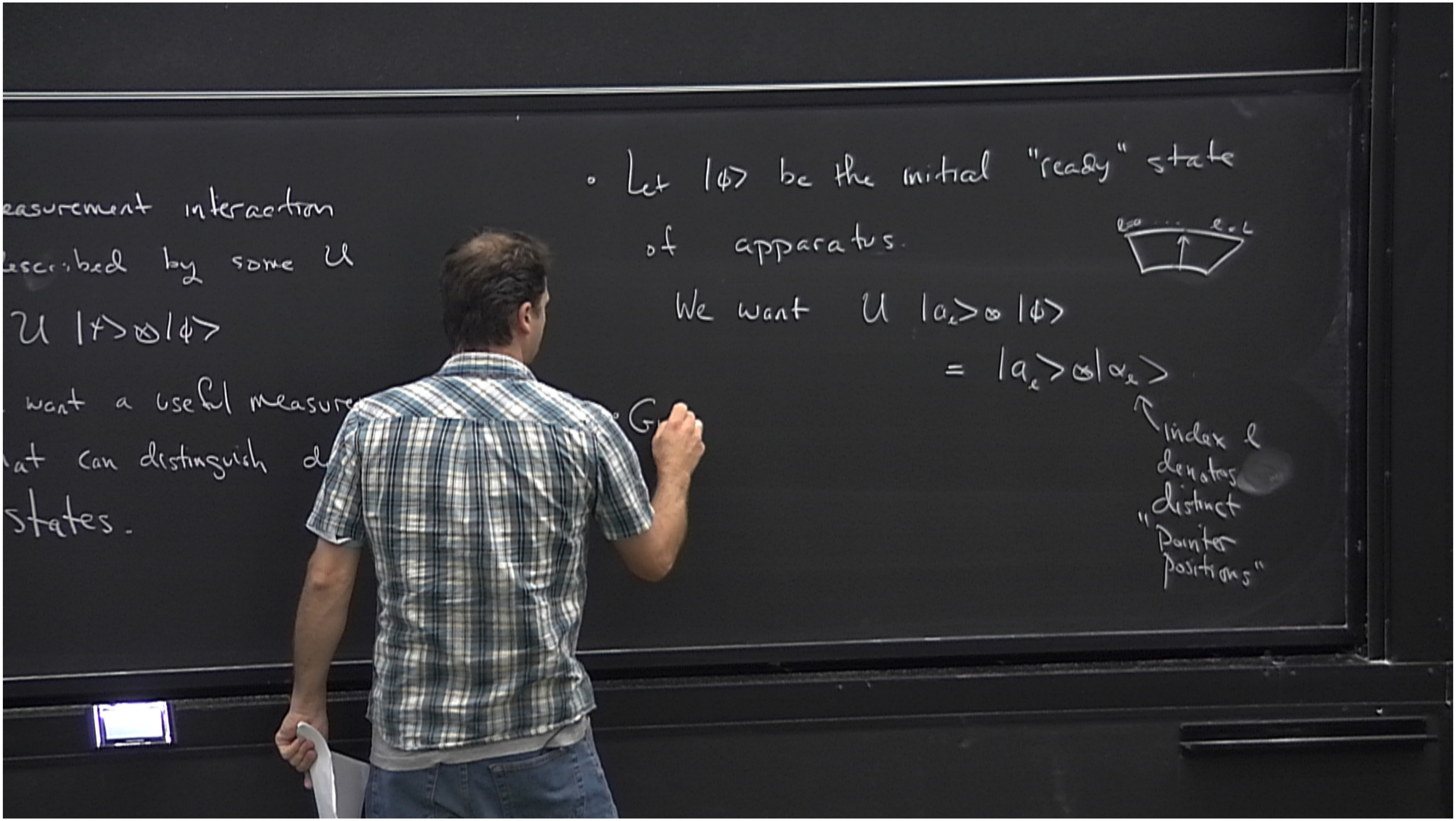
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= |a_l> ⊗ |α_l>
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$\mathcal{U}\, |\psi\rangle \otimes |\phi\rangle$

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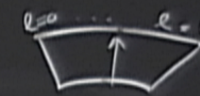

We want $\mathcal{U}\, |a_\ell\rangle \otimes |\phi\rangle$

$= |a_\ell\rangle \otimes |\alpha_\ell\rangle$

index ℓ denotes distinct "pointer positions"

• Given $|\psi\rangle = \sum_\ell c_\ell\, |a_\ell\rangle$

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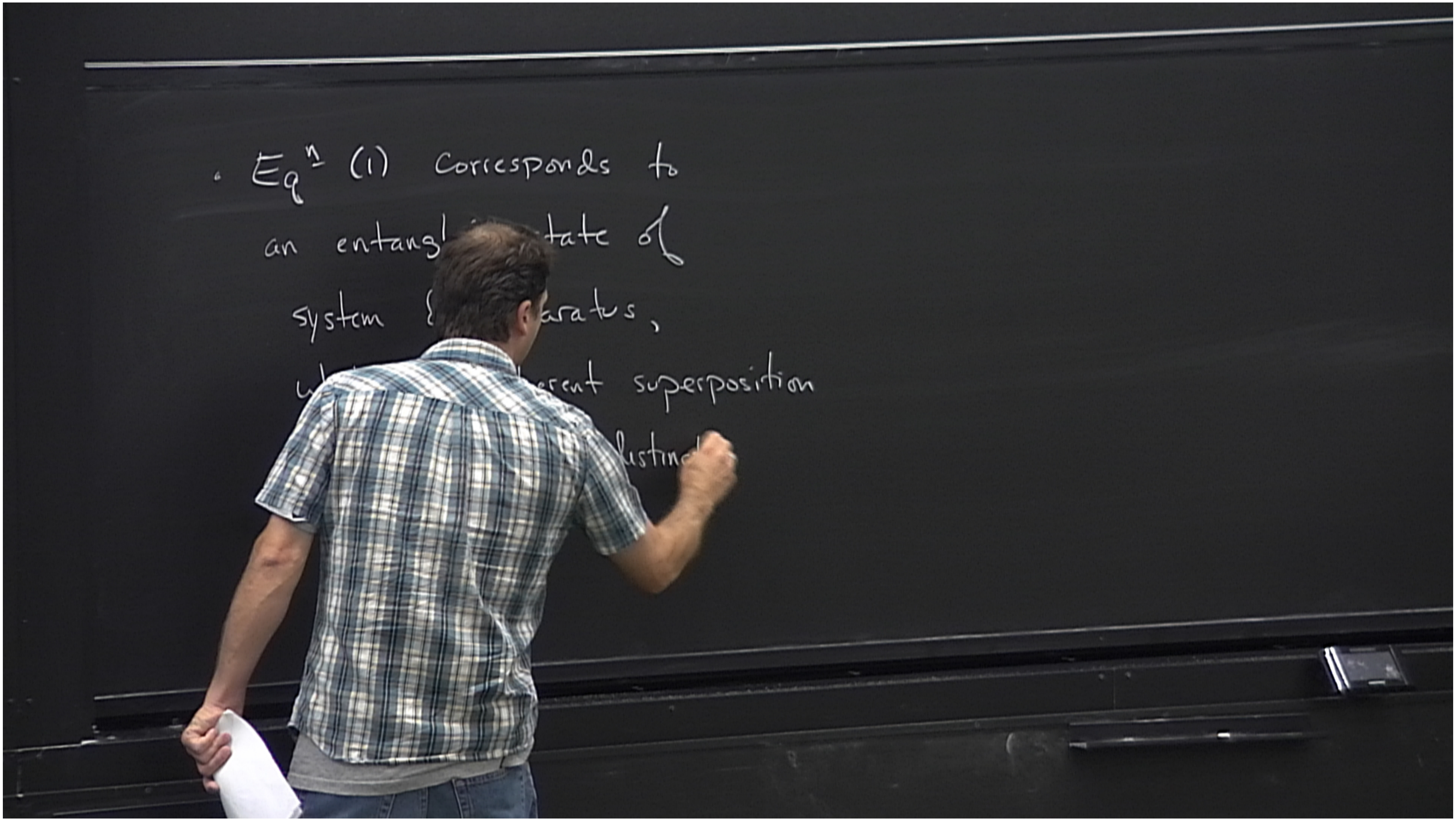
• Given $|\psi\rangle = \sum_\ell c_\ell\, |a_\ell\rangle$,

by linearity,

$\mathcal{U}\, |\psi\rangle \otimes |\phi\rangle = \sum_\ell c_\ell\, |a_\ell\rangle \otimes |\alpha_\ell\rangle$



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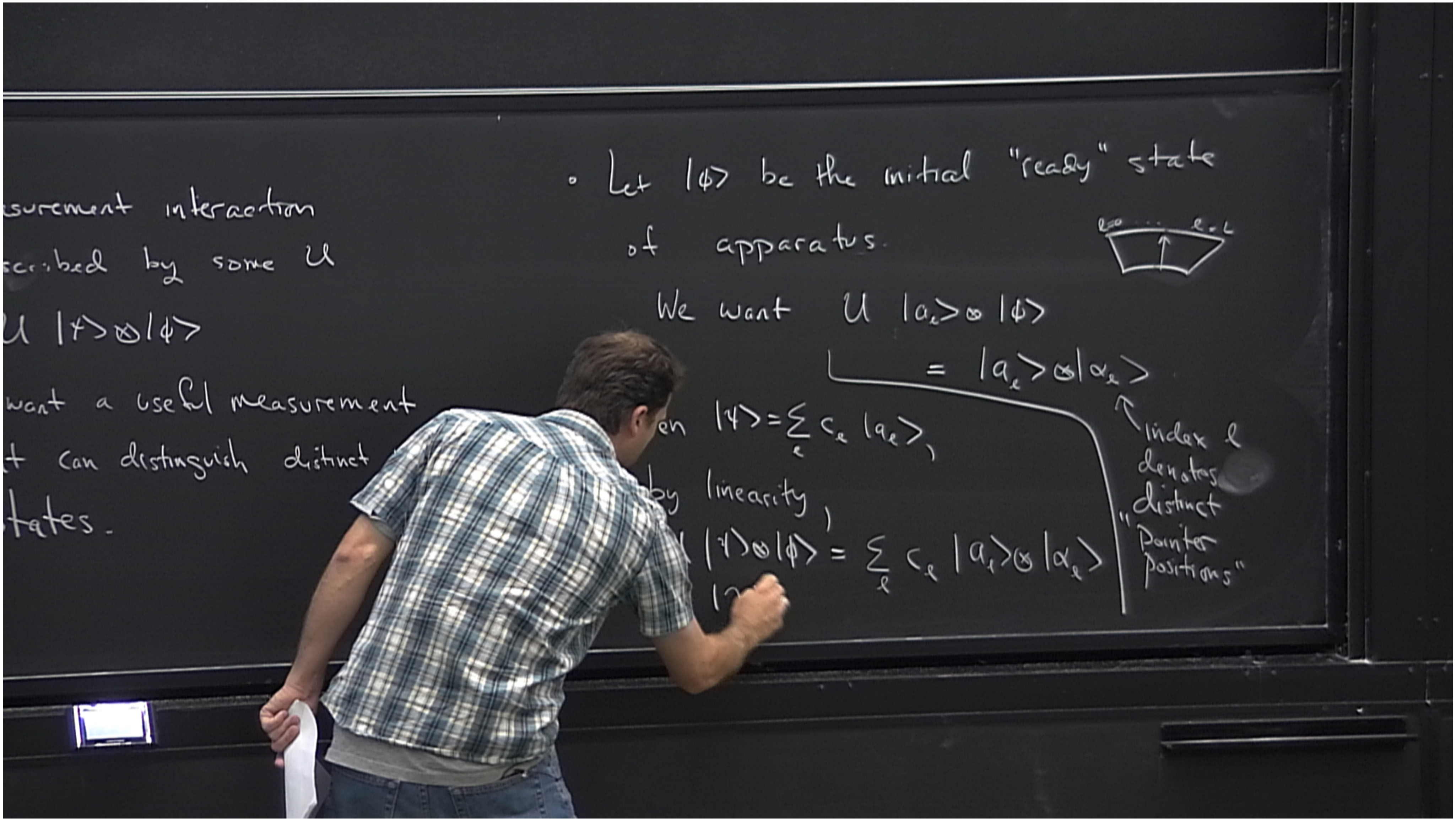
If we partial-trace over the apparatus then $\hat{\rho}_s = \mathrm{Tr}_A \left(|\chi\rangle\langle\chi| \right)$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• Eq^{n} (1) corresponds to an entangled state of system & apparatus, which is a coherent superposition of macroscopically distinct configurations.

If we partial-trace over the apparatus

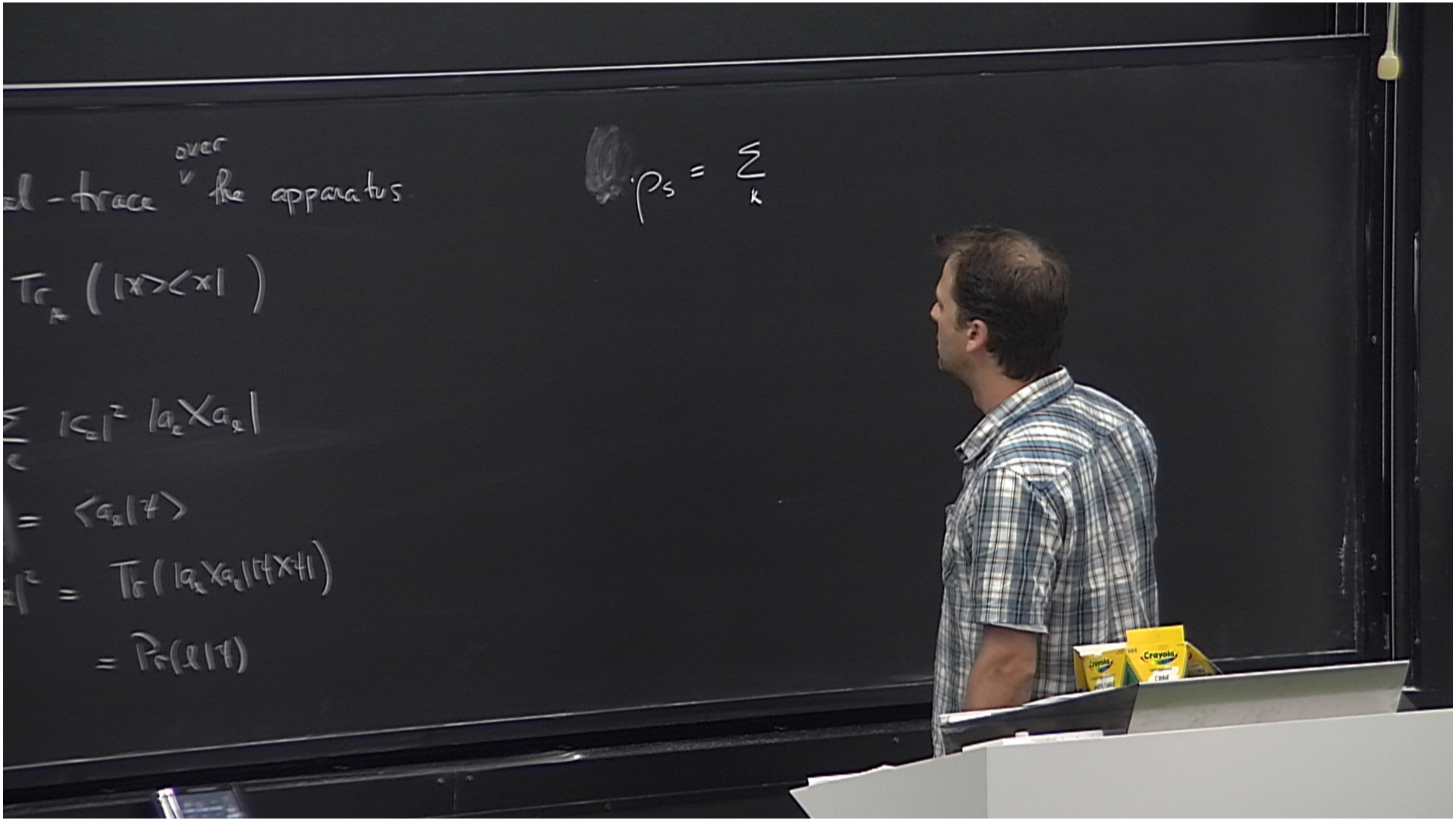
then $\hat{\rho}_s = Tr_A \left(|\chi\rangle\langle\chi| \right)$

$$\hat{\rho}_s = \sum_{\ell} |c_\ell|^2 \, |a_\ell\rangle\langle a_\ell|$$

$$c_\ell = \langle a_\ell | \psi \rangle$$

$$|c_\ell|^2 = Tr\left(|a_\ell\rangle\langle a_\ell| |\psi\rangle\langle\psi| \right)$$

$$= Pr(\ell|\psi)$$



over
-trace the apparatus
$\mathrm{Tr}_A\left(|x\rangle\langle x| \right)$
$\sum_l |c_l|^2 |a_l\rangle\langle a_l|$
$= \langle a_l|\psi\rangle$
$= \mathrm{Tr}\left(|a_l\rangle\langle a_l| |\psi\rangle\langle\psi| \right)$
$= \Pr(l|\psi)$
$\rho_s = \sum_k$
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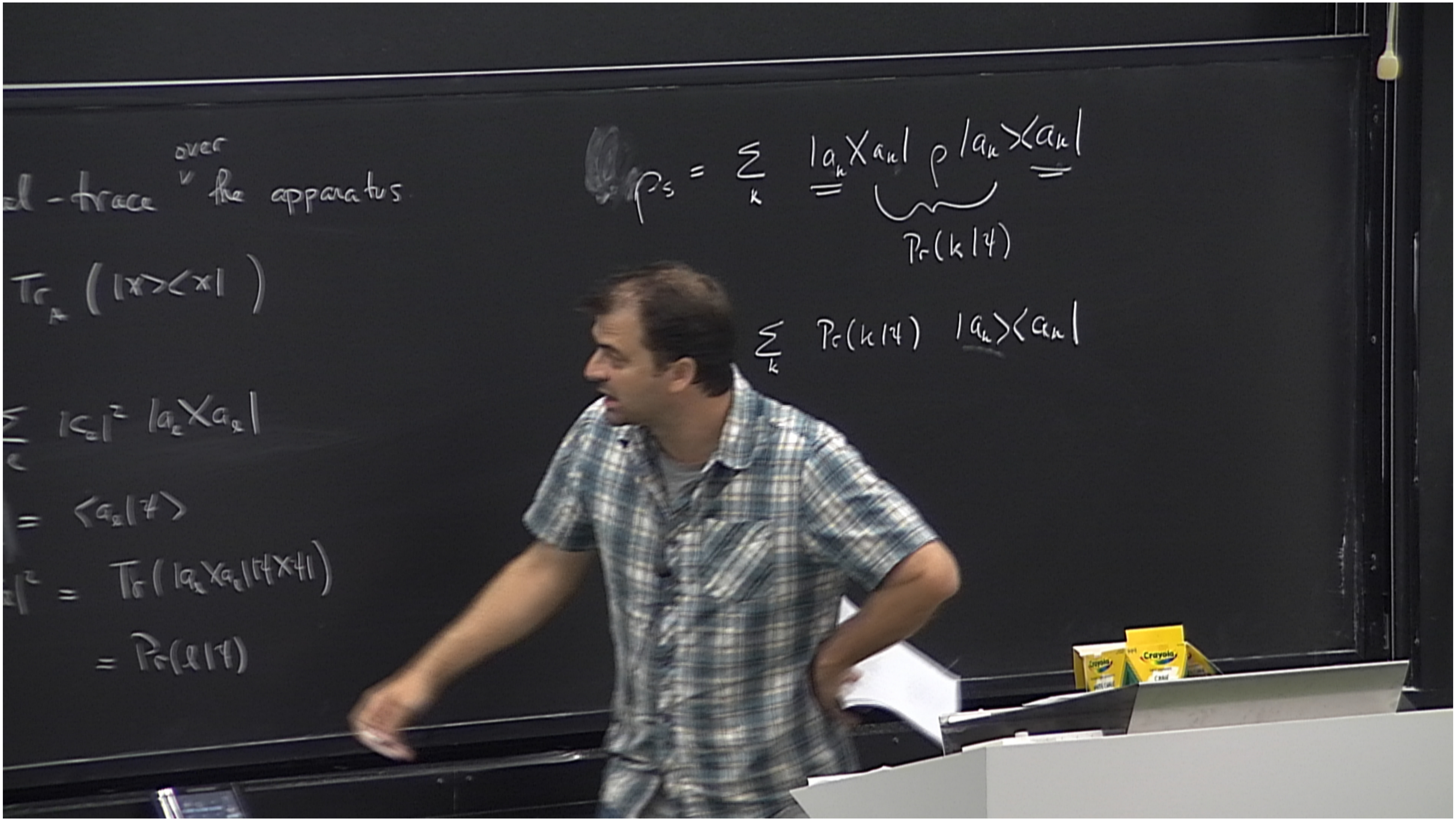
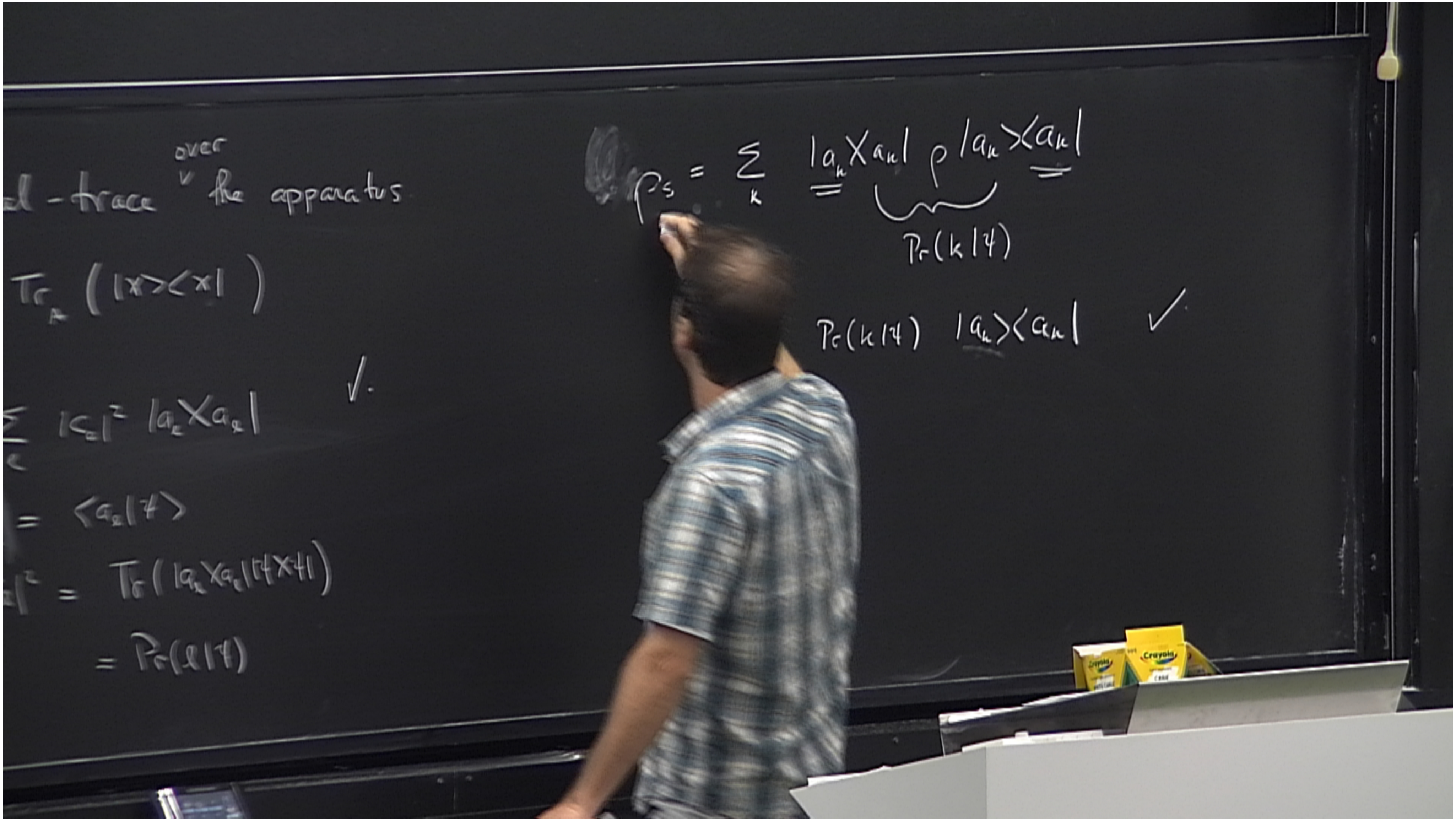
Crayola


over
-trace the apparatus
ρs = Σk |ak⟩⟨ak| ρ |ak⟩⟨ak|
Pr(k|ψ)
= Σk Pr(k|ψ) |ak⟩⟨ak|
= ⟨aℓ|ψ⟩
= Tr(|aℓ⟩⟨aℓ|ψ⟩⟨ψ|)
= Pr(ℓ|ψ)


-trace over the apparatus
$\mathrm{Tr}_A(|x\rangle\langle x|)$
$\sum |c_\ell|^2 |a_\ell\rangle\langle a_\ell|$
$= \langle a_\ell|\psi\rangle$
$= \mathrm{Tr}(|a_\ell\rangle\langle a_\ell|\psi\rangle\langle\psi|)$
$= \Pr(\ell|\psi)$
$\rho_s = \sum_k |a_k\rangle\langle a_k| \rho |a_k\rangle\langle a_k|$
$\Pr(k|\psi)$
$\Pr(k|\psi) |a_k\rangle\langle a_k|$
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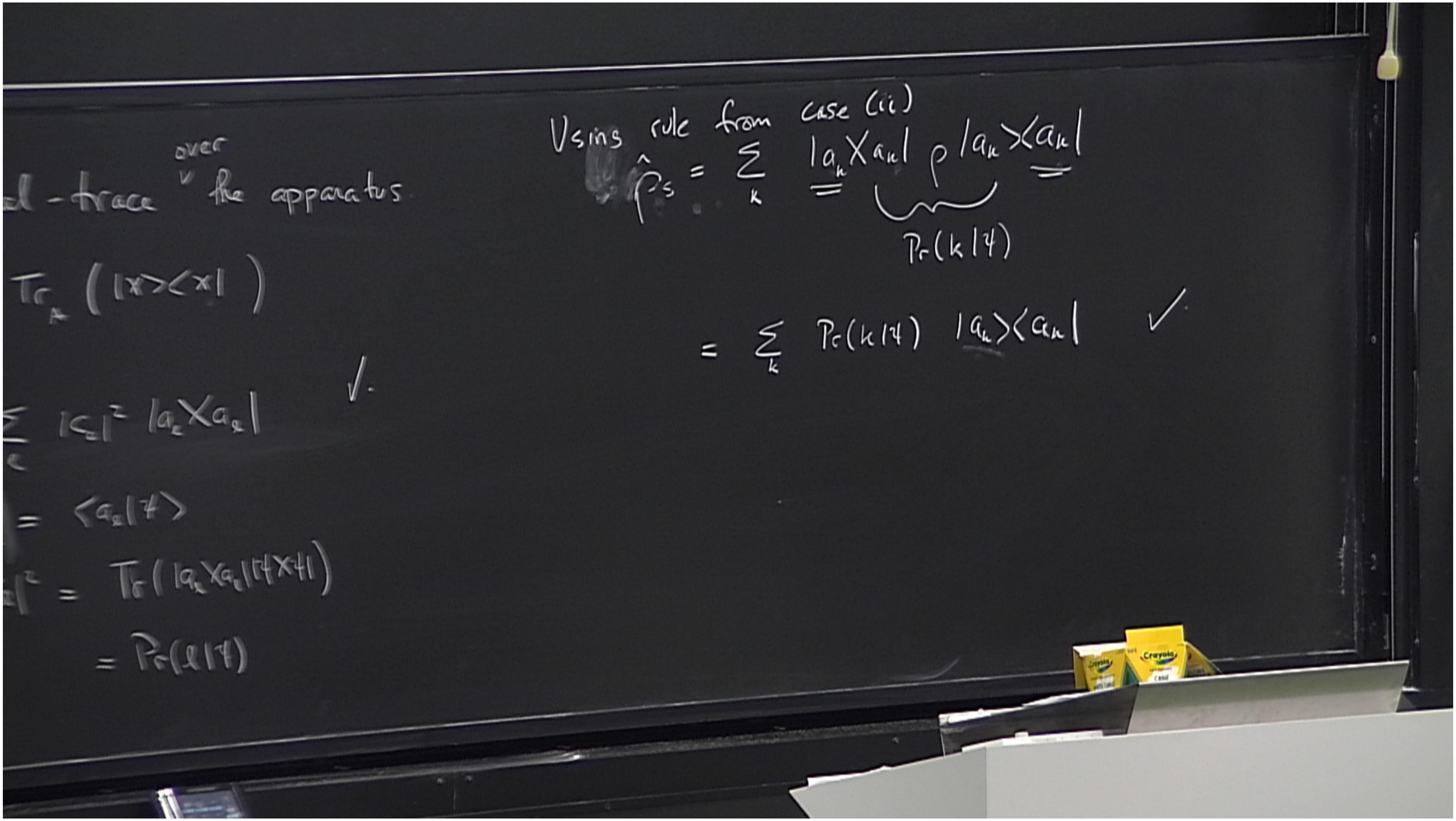
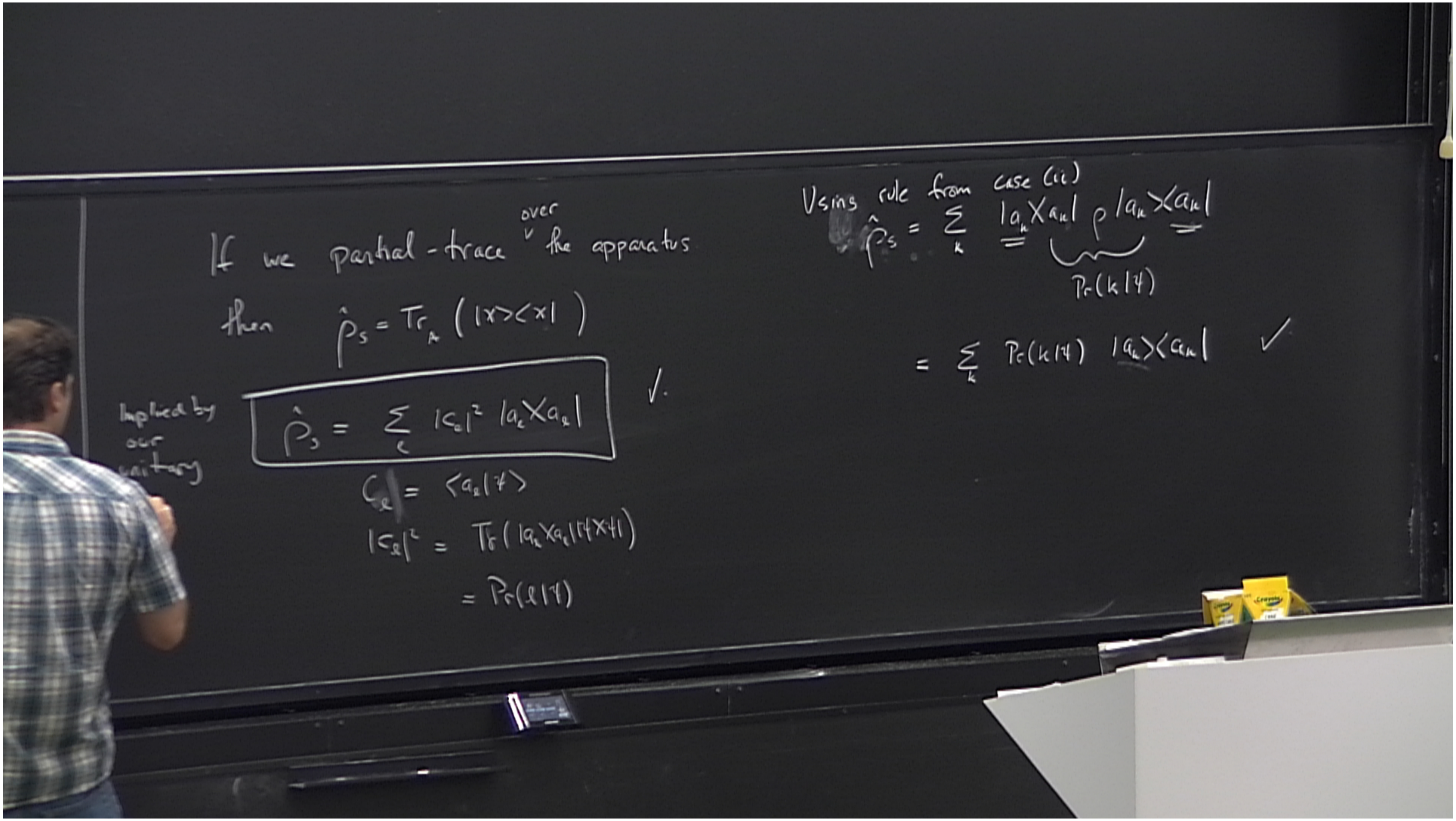
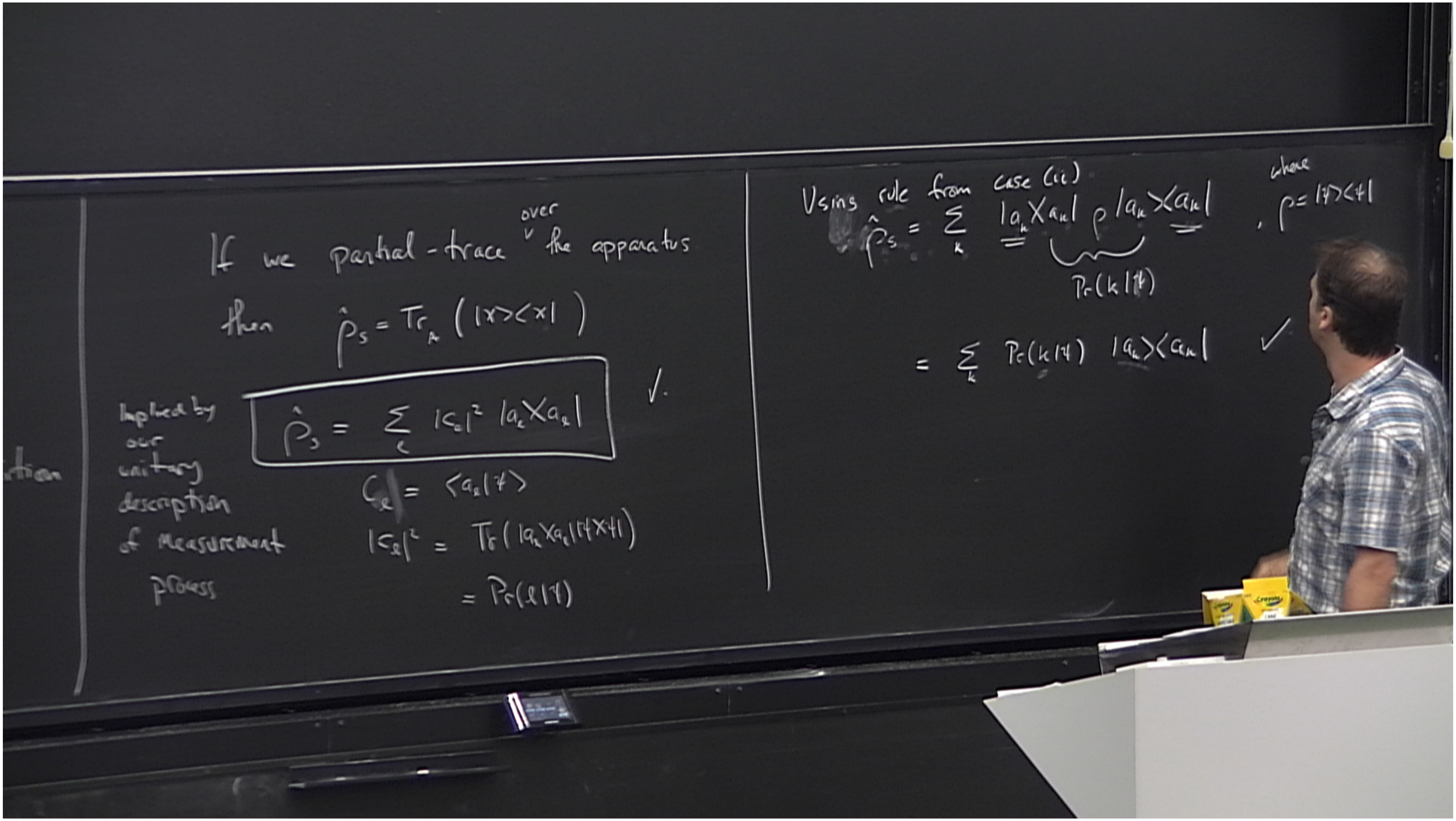


l - trace over the apparatus
$\mathrm{Tr}_A (|x\rangle\langle x|)$
$\sum_l |c_l|^2 |a_l\rangle\langle a_l|$ ✓.
$= \langle a_l|\psi\rangle$
$|^2 = \mathrm{Tr}(|a_l\rangle\langle a_l|\psi\rangle\langle\psi|)$
$= \Pr(l|\psi)$
Using rule from case (ii)
$\hat{\rho}_s = \sum_k |a_k\rangle\langle a_k| \underbrace{\rho}_{\Pr(k|\psi)} |a_k\rangle\langle a_k|$
$= \sum_k \Pr(k|\psi) |a_k\rangle\langle a_k|$ ✓


If we partial-trace over the apparatus
then $\hat{\rho}_s = \mathrm{Tr}_A (|X\rangle\langle X|)$
Implied by our unitary
$\boxed{\hat{\rho}_s = \sum_\ell |c_\ell|^2 |a_\ell\rangle\langle a_\ell|}$ ✓.
$c_\ell = \langle a_\ell | \psi \rangle$
$|c_\ell|^2 = \mathrm{Tr}(|a_\ell\rangle\langle a_\ell| \psi\rangle\langle\psi|)$
$= \mathrm{Pr}(\ell|\psi)$
Using rule from case (ii)
$\hat{\rho}_s = \sum_k |a_k\rangle\langle a_k| \underbrace{\rho\, |a_k\rangle\langle a_k|}_{\mathrm{Pr}(k|\psi)}$
$= \sum_k \mathrm{Pr}(k|\psi)\, |a_k\rangle\langle a_k|$ ✓



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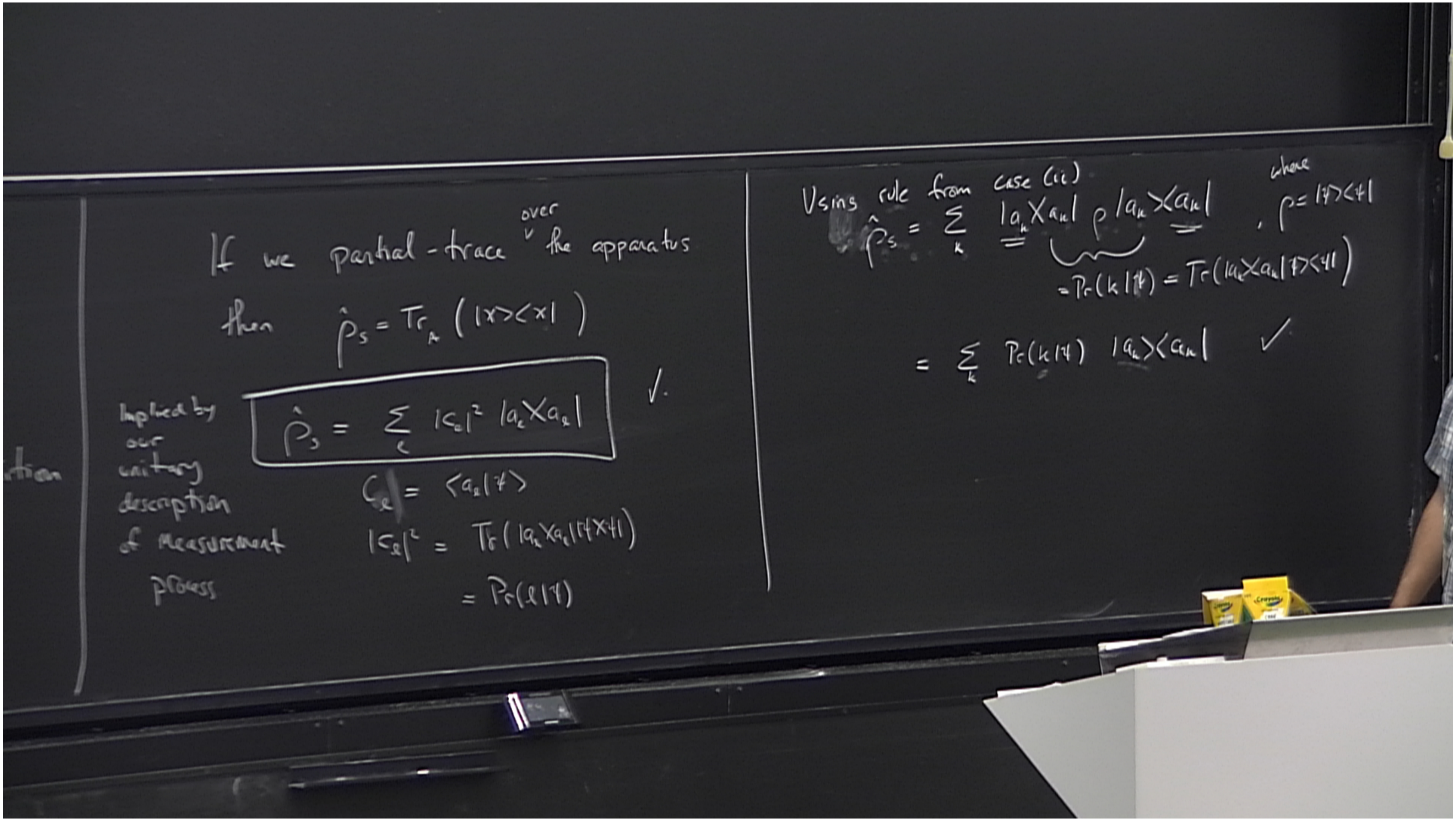
If we partial-trace over the apparatus
then $\hat{\rho}_s = \mathrm{Tr}_A\left(|\chi\rangle\langle\chi| \right)$
Implied by our unitary description of measurement process
$\boxed{\hat{\rho}_s = \sum_\ell |c_\ell|^2 \, |a_\ell\rangle\langle a_\ell|}$ ✓.
$c_\ell = \langle a_\ell|\psi\rangle$
$|c_\ell|^2 = \mathrm{Tr}\left(|a_\ell\rangle\langle a_\ell|\psi\rangle\langle\psi| \right)$
$= \Pr(\ell|\psi)$
Using rule from case (ii)
$\hat{\rho}_s = \sum_k |a_k\rangle\underbrace{\langle a_k| \rho |a_k\rangle}_{\Pr(k|\psi)}\langle a_k|$, where $\rho = |\psi\rangle\langle\psi|$
$= \sum_k \Pr(k|\psi) \, |a_k\rangle\langle a_k|$ ✓



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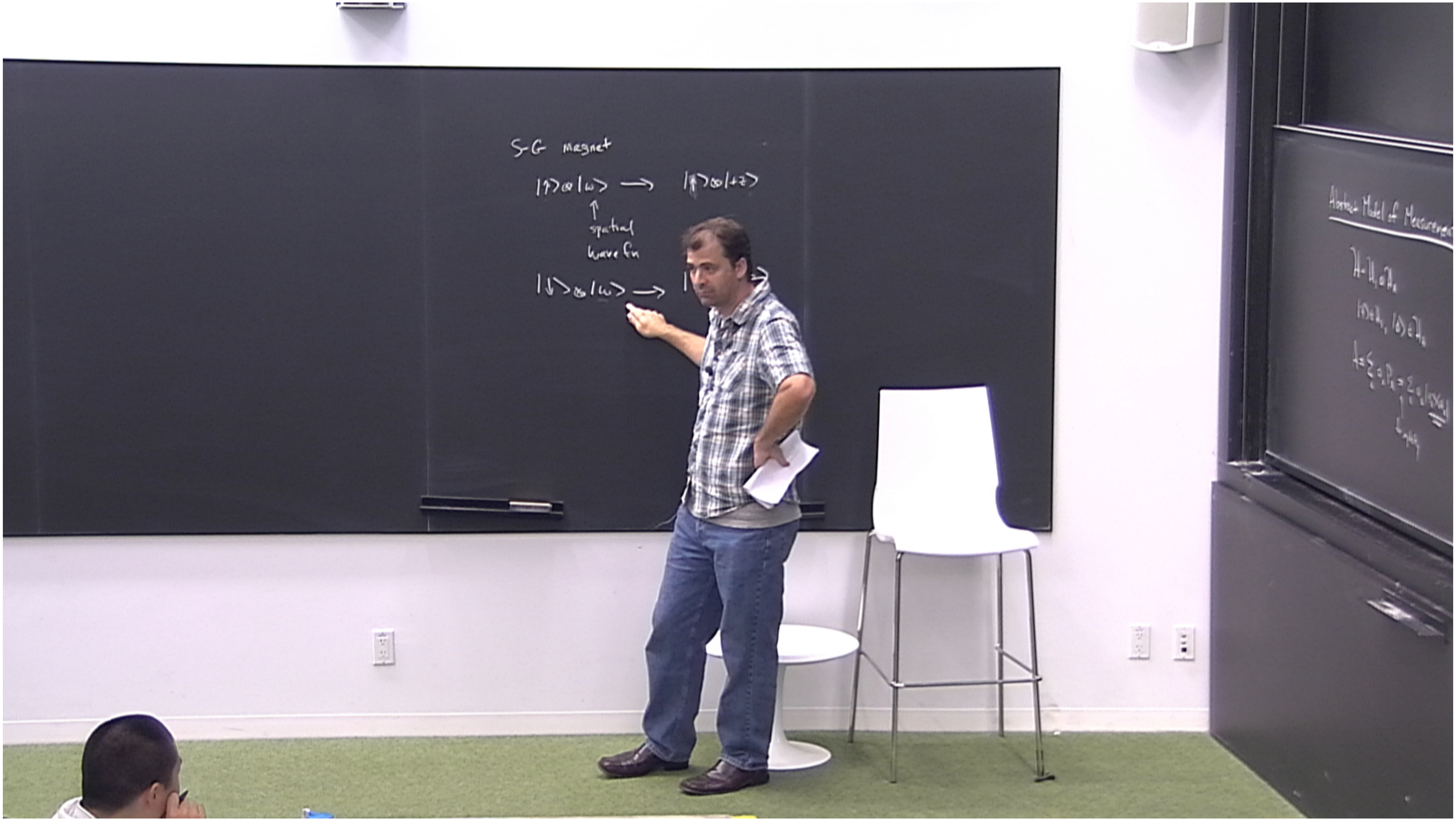
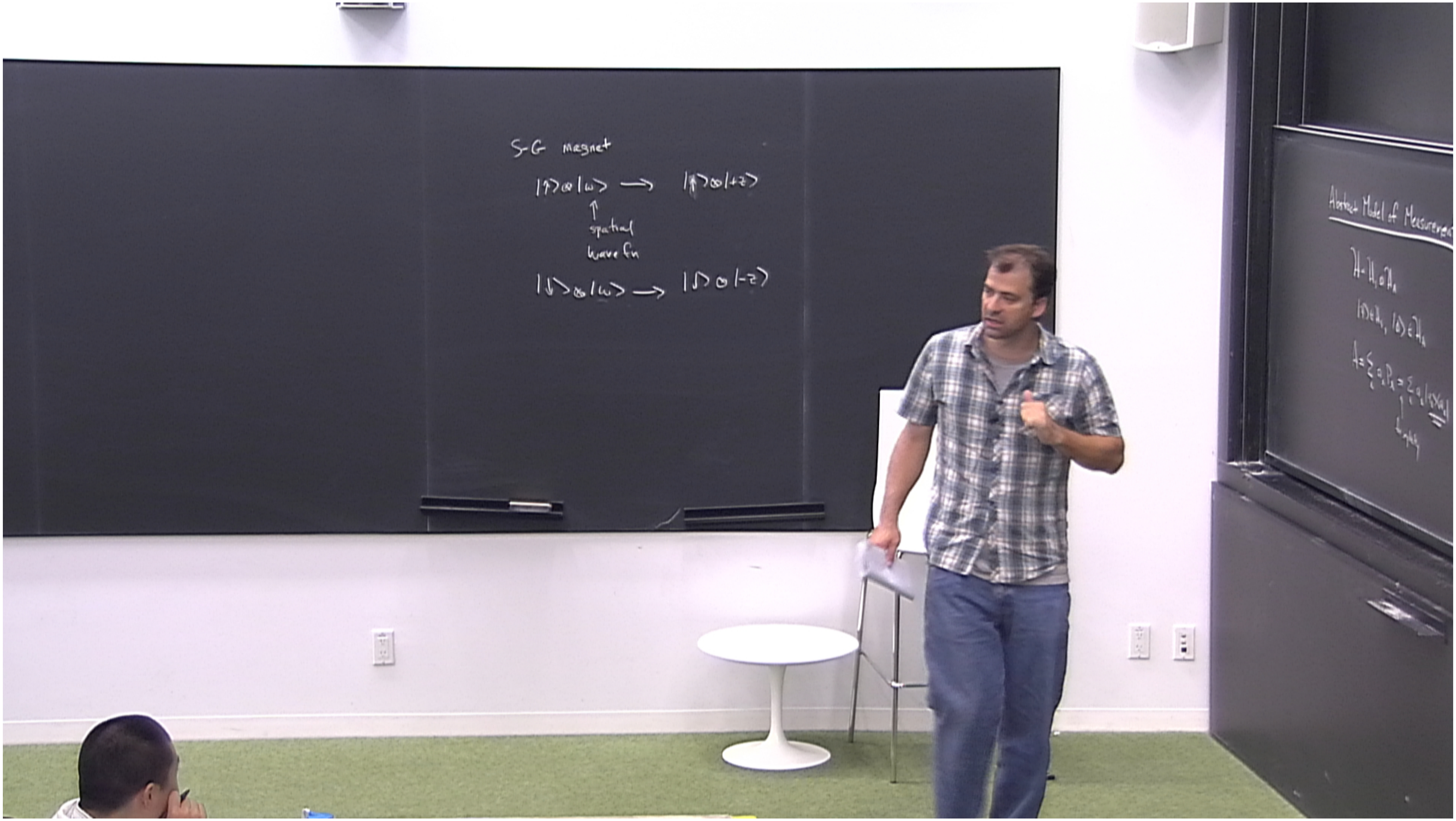
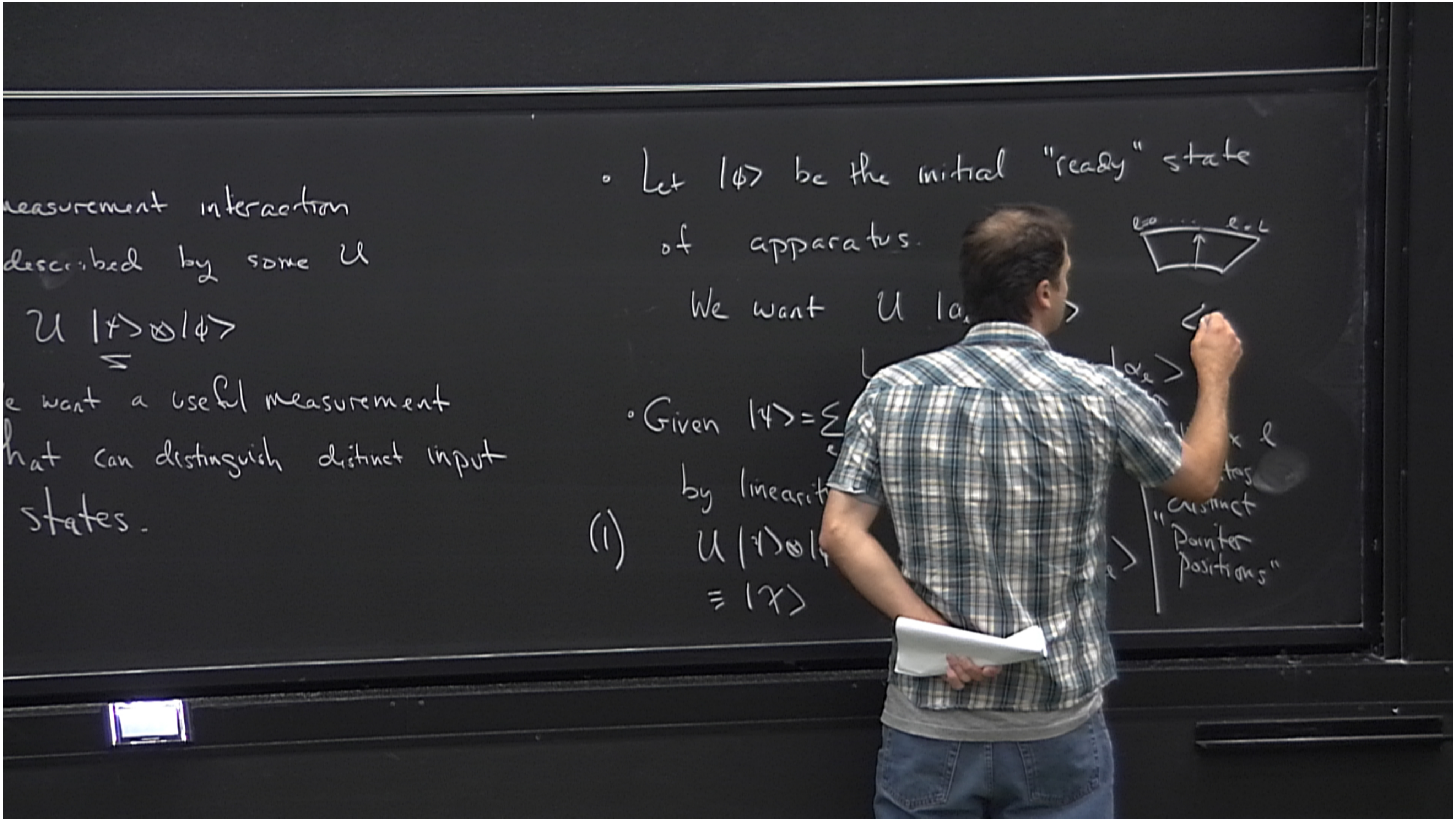
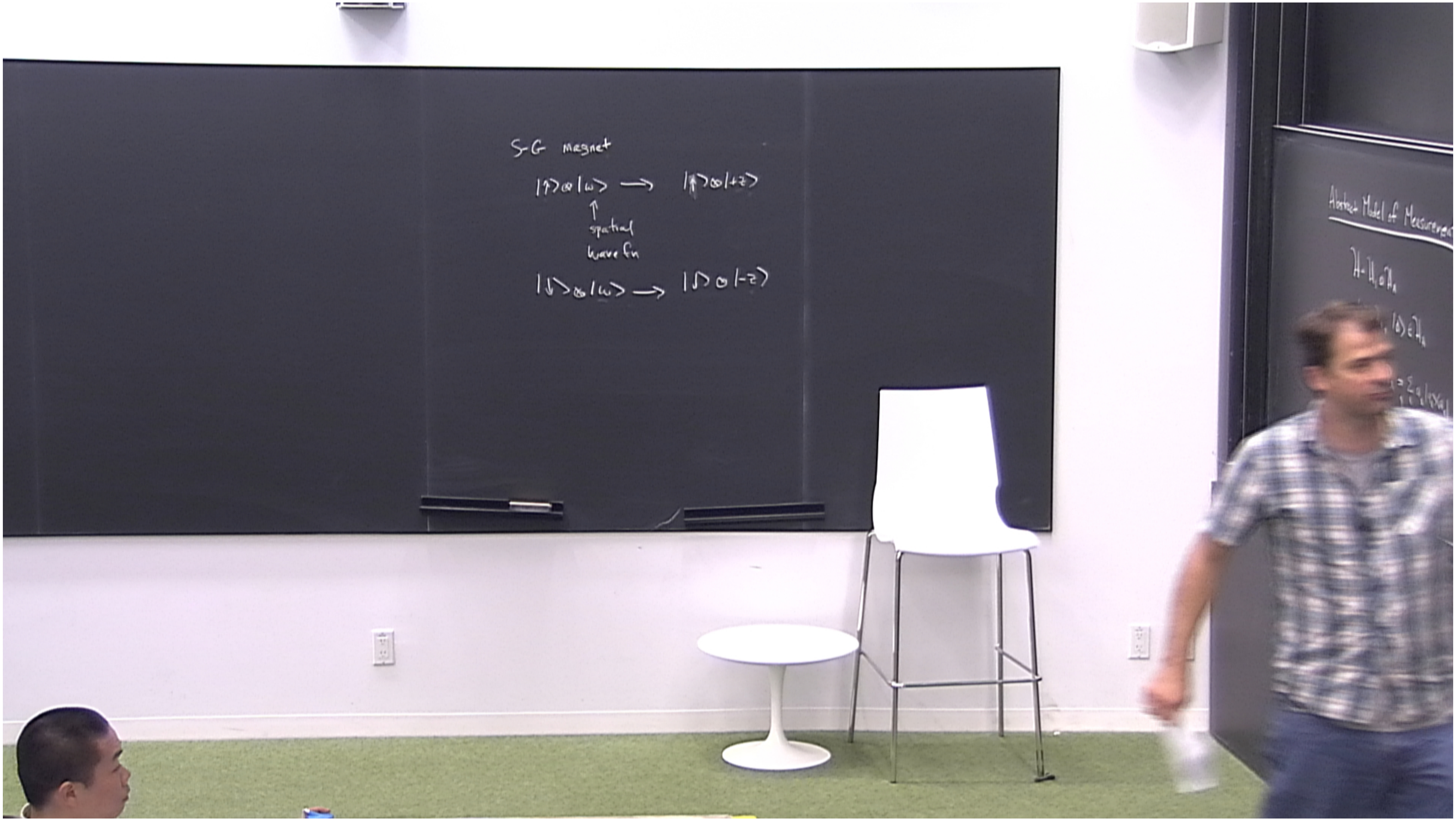
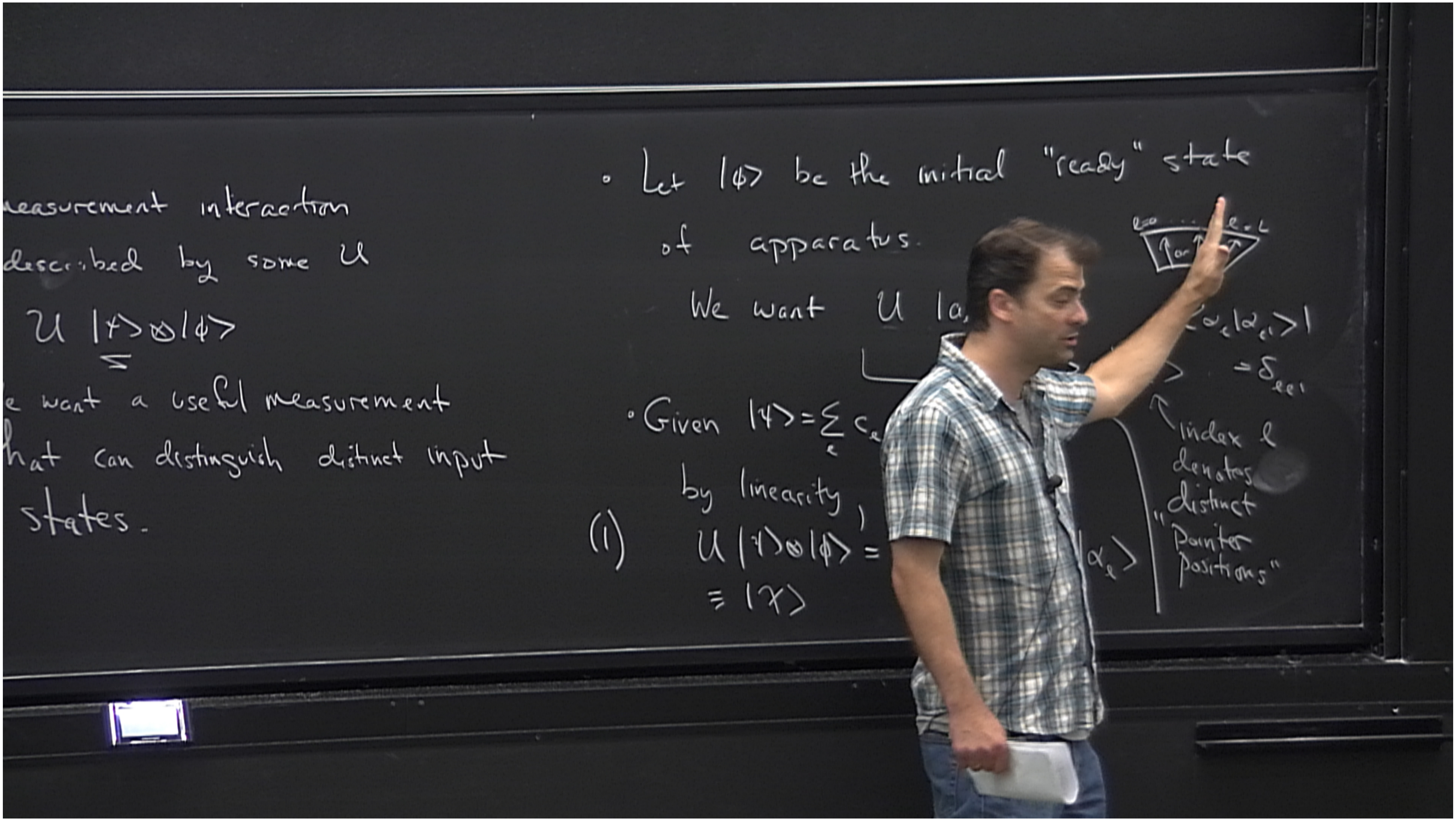
$c_\ell = \langle a_\ell|\psi\rangle$
$|c_\ell|^2 = Tr(|a_\ell\rangle\langle a_\ell|\psi\rangle\langle\psi|)$
$= Pr(\ell|\psi)$
Using rule from case (ii)
$\hat{\rho}_s = \sum_k |a_k\rangle\langle a_k| \underbrace{\rho |a_k\rangle}\langle a_k|$, where $\rho = |\psi\rangle\langle\psi|$
$= Pr(k|\psi) = Tr(|a_k\rangle\langle a_k|\psi\rangle\langle\psi|)$
$= \sum_k Pr(k|\psi) \; |a_k\rangle\langle a_k|$ ✓

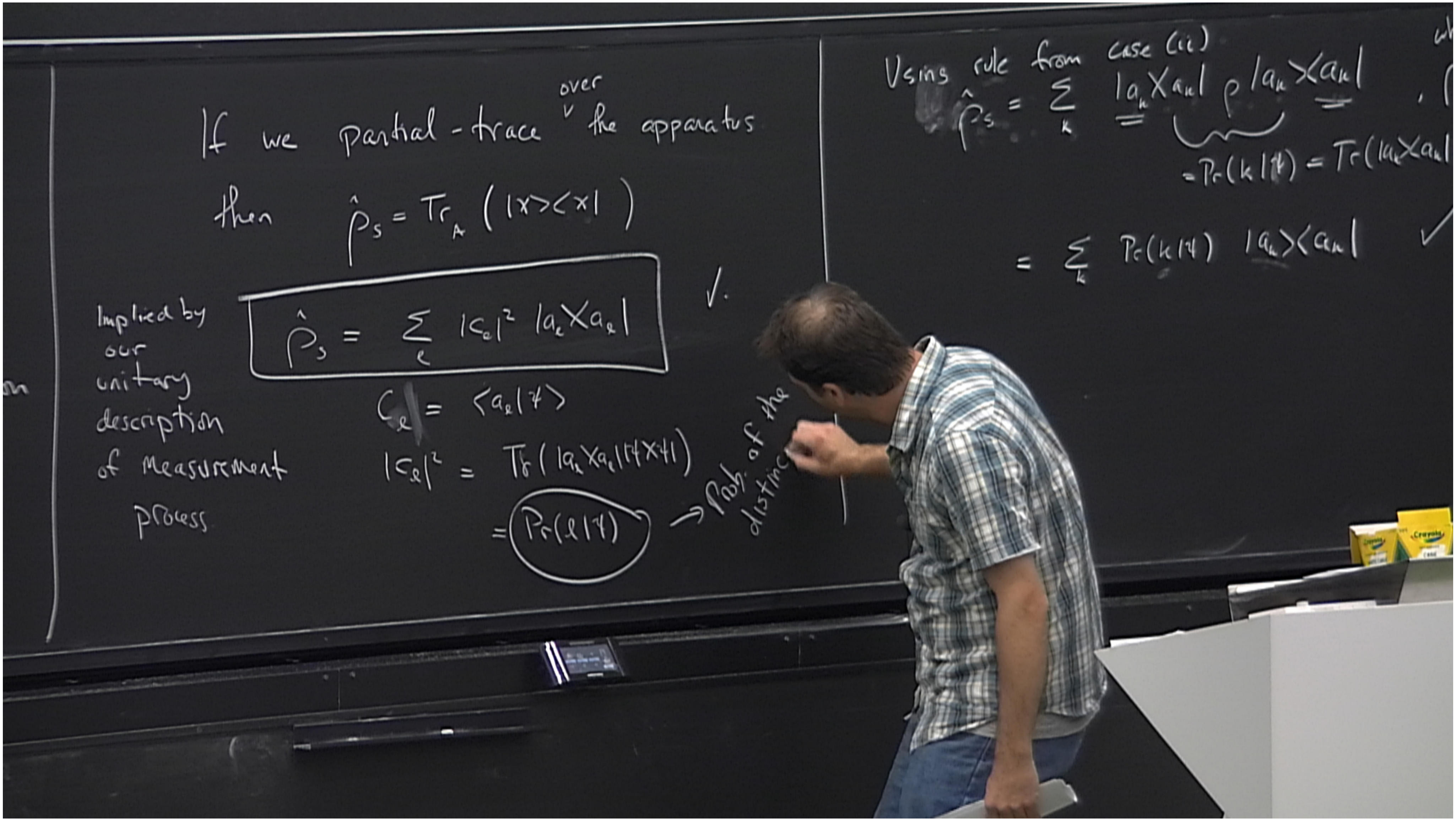

S-G magnet
|↑⟩⊗|ω⟩ ⟶ |↑⟩⊗|+z⟩
spatial
wave fn
|↓⟩⊗|ω⟩ ⟶

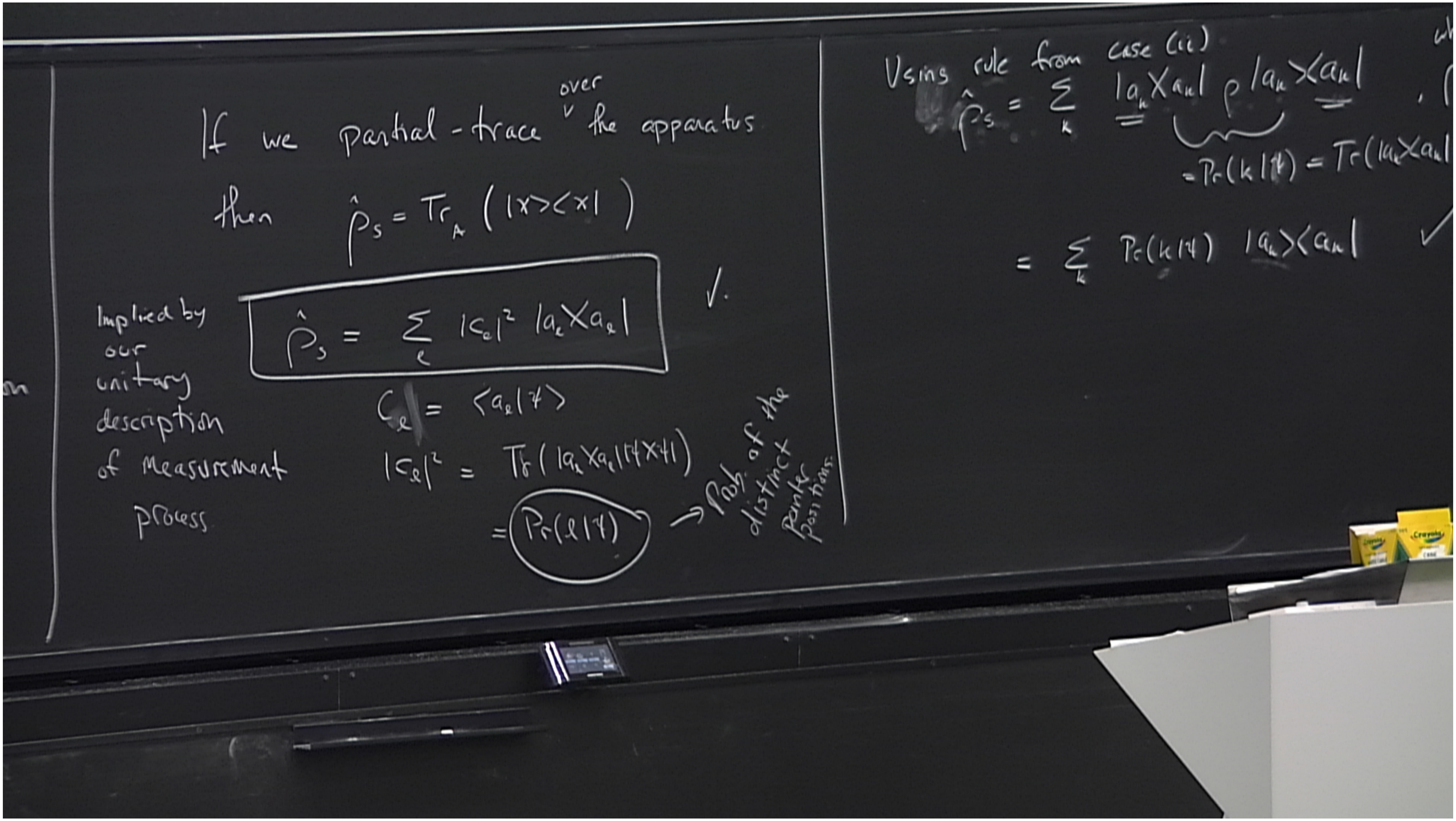

S-G magnet
$|\uparrow\rangle \otimes |\omega\rangle \longrightarrow |\uparrow\rangle \otimes |+z\rangle$
spatial
wave fn
$|\downarrow\rangle \otimes |\omega\rangle \longrightarrow |\downarrow\rangle \otimes |-z\rangle$

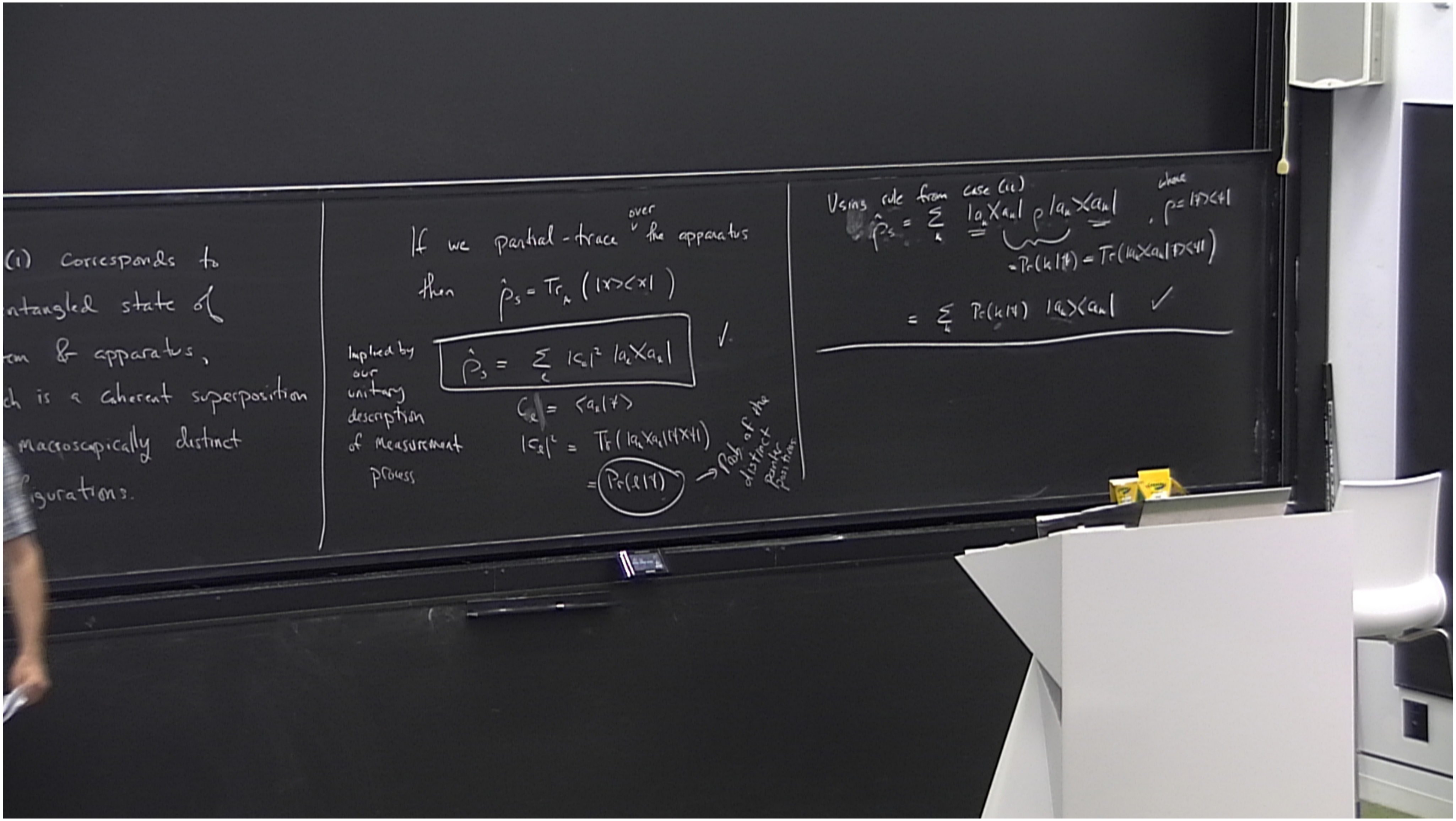

easurement interaction
described by some U
U |ψ⟩⊗|φ⟩
e want a useful measurement
hat can distinguish distinct input
states.
• Let |φ⟩ be the initial "ready" state
of apparatus.
We want U |α
• Given |ψ⟩=Σ
by linearit
(1) U|ψ⟩⊗|φ
≡ |χ⟩
distinct
"pointer
positions"

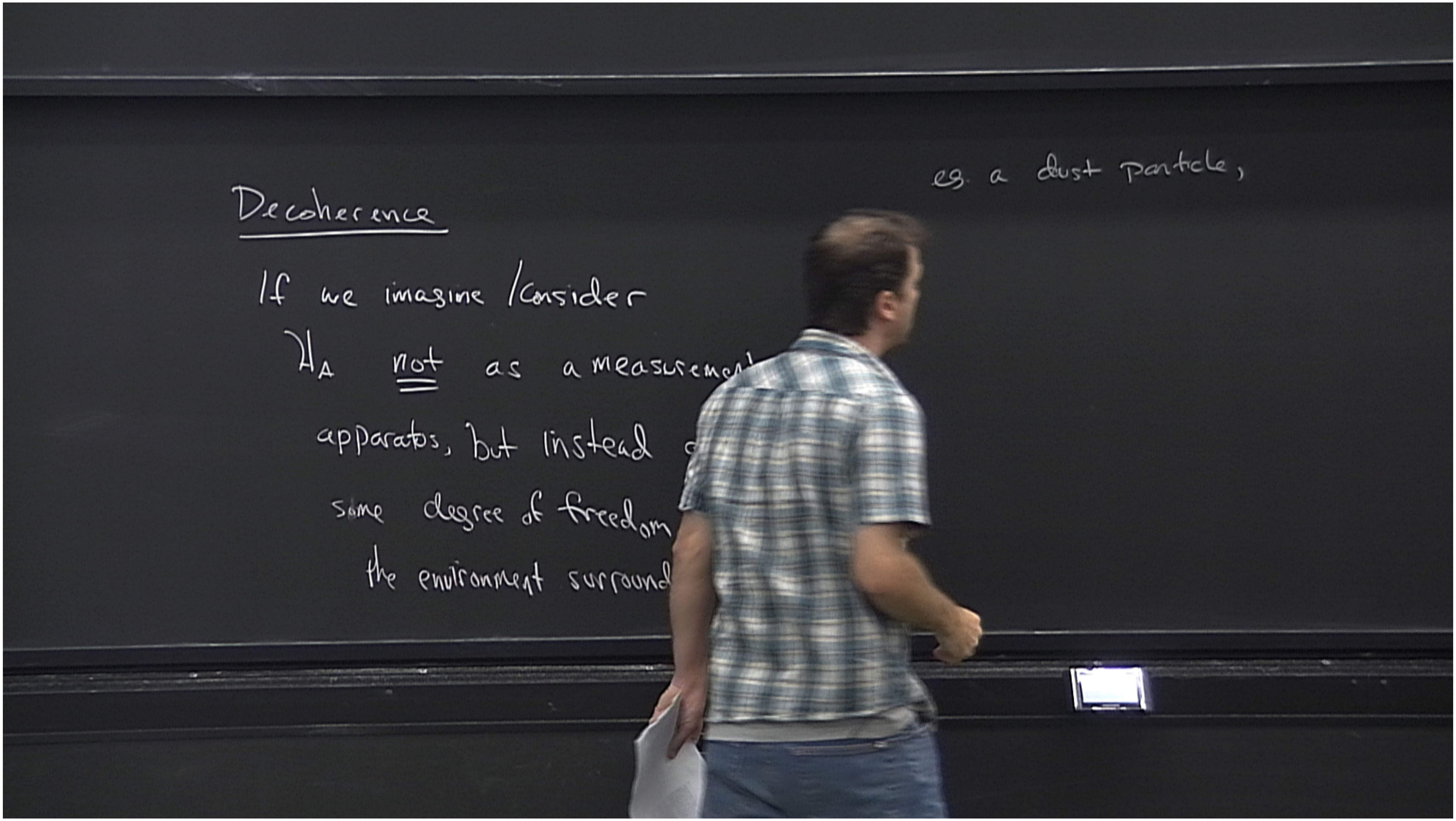

S-G magnet
|↑⟩⊗|ω⟩ ⟶ |↑⟩⊗|+z⟩
spatial
wave fn
|↓⟩⊗|ω⟩ ⟶ |↓⟩⊗|-z⟩
Abstract Model of

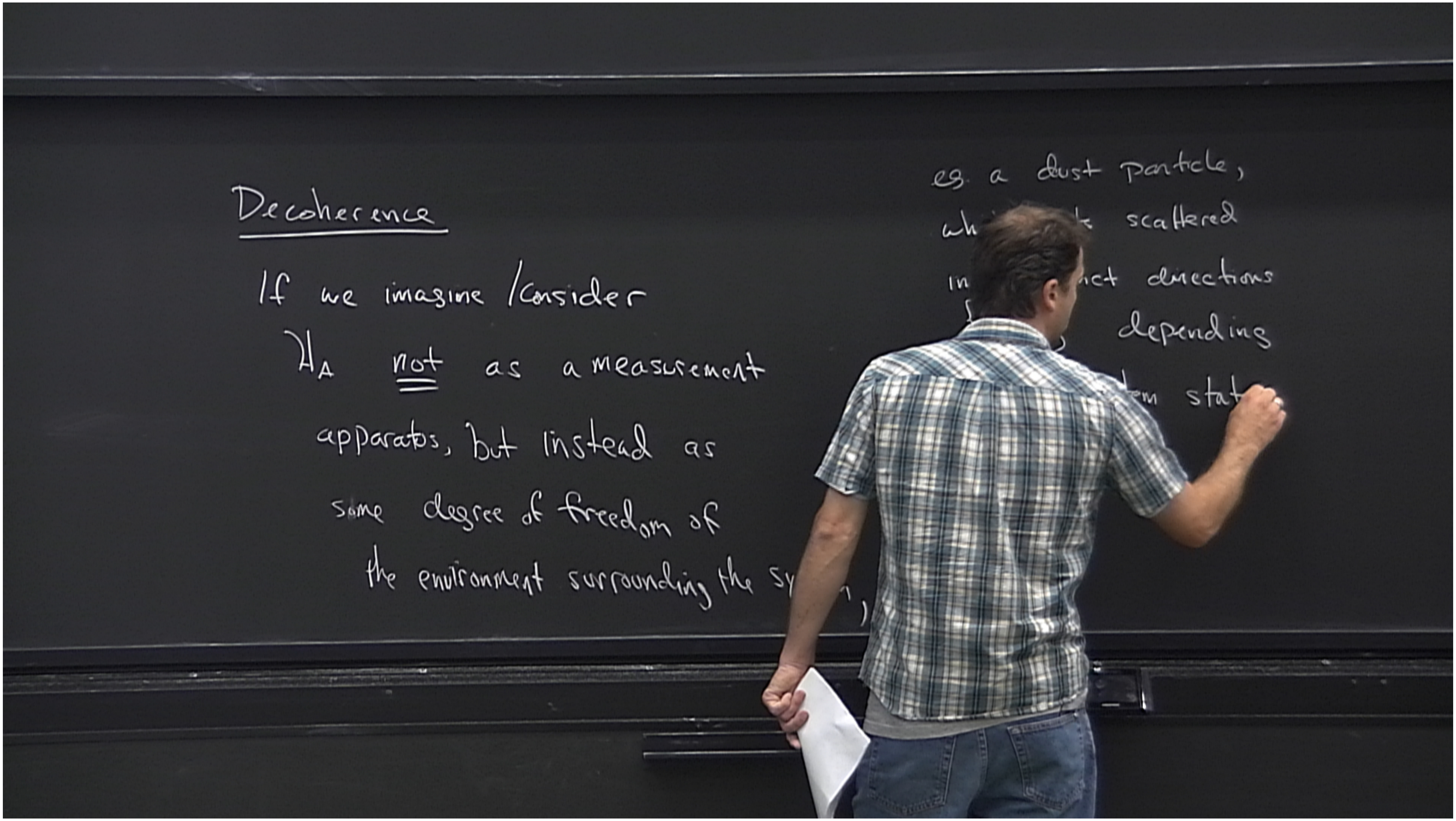

easurement interaction
described by some U
U |ψ> ⊗ |φ>
e want a useful measurement
hat can distinguish distinct input
states.
Let |φ> be the initial "ready" state
of apparatus.
We want U |α
<α_ℓ|α_ℓ'> = δ_ℓℓ'
Given |ψ> = Σ c
by linearity,
(1) U |ψ> ⊗ |φ> =
≡ |χ>
index ℓ
denotes
distinct
"pointer
positions"

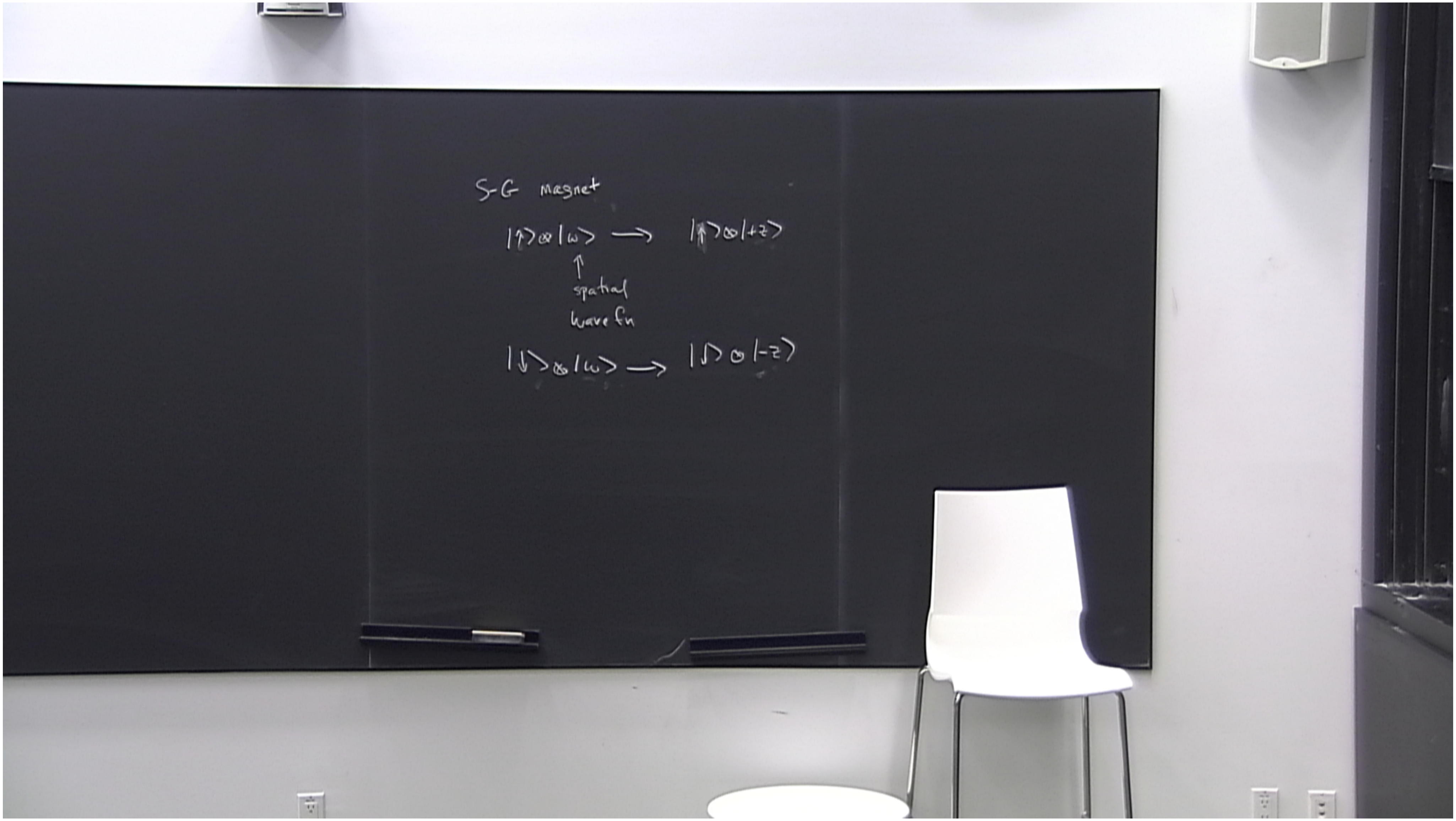

If we partial-trace over the apparatus
then $\hat{\rho}_s = Tr_A (|x\rangle\langle x|)$
Implied by our unitary description of measurement process
$\hat{\rho}_s = \sum_l |c_l|^2 |a_l\rangle\langle a_l|$
$c_l = \langle a_l|\psi\rangle$
$|c_l|^2 = Tr(|a_l\rangle\langle a_l||\psi\rangle\langle\psi|)$
$= Pr(l|\psi)$
Prob. of the distinc
Using rule from case (ii)
$\hat{\rho}_s = \sum_k |a_k\rangle\langle a_k| \rho |a_k\rangle\langle a_k|$
$= Pr(k|\psi) = Tr(|a_k\rangle\langle a_k|$
$= \sum_k Pr(k|\psi) |a_k\rangle\langle a_k|$

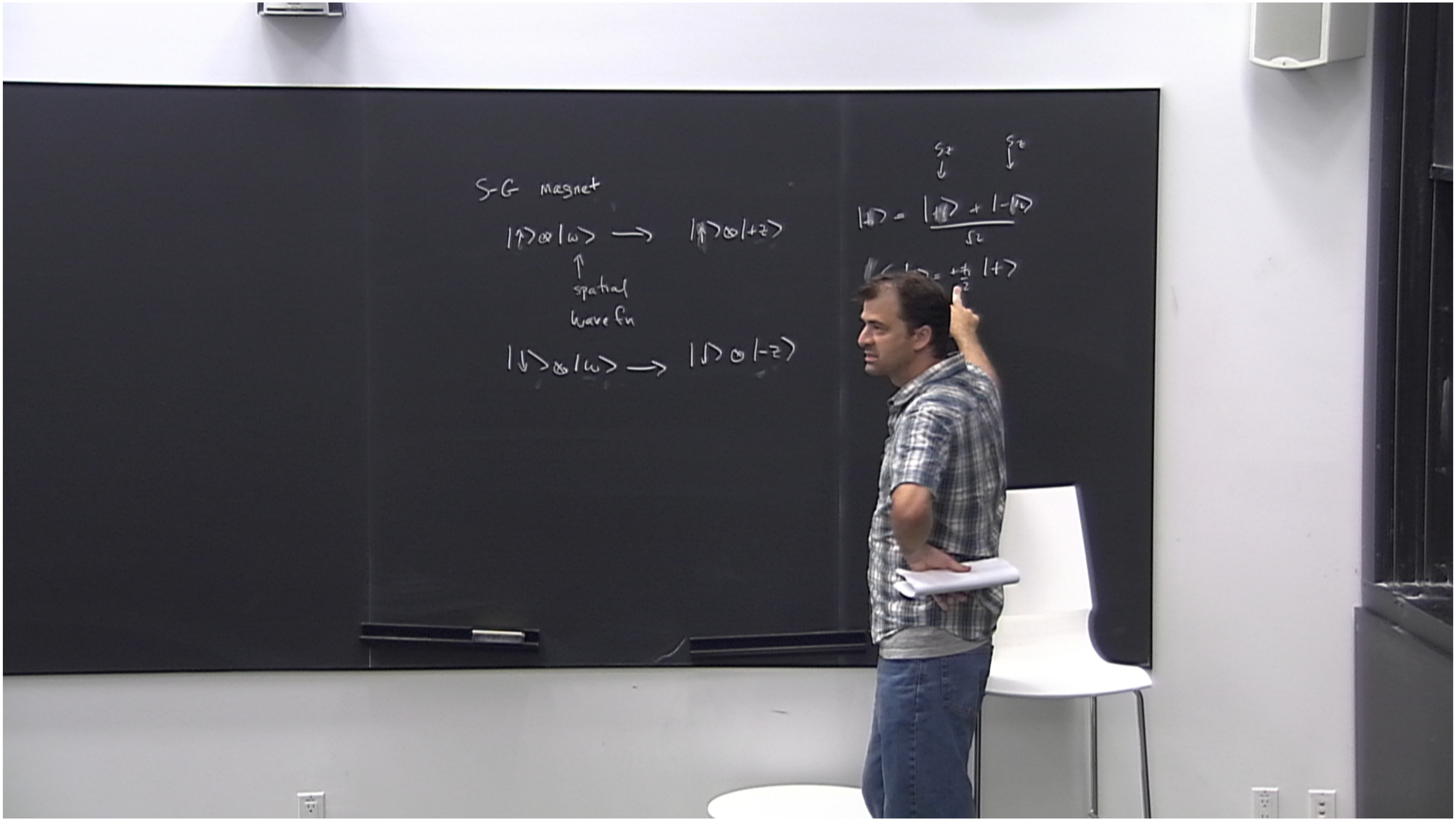

If we partial-trace over the apparatus
then $\hat{\rho}_S = Tr_A(|x\rangle\langle x|)$
$\boxed{\hat{\rho}_S = \sum_\ell |c_\ell|^2 |a_\ell\rangle\langle a_\ell|}$ ✓.
Implied by our unitary description of measurement process
$c_\ell = \langle a_\ell|\psi\rangle$
$|c_\ell|^2 = Tr(|a_\ell\rangle\langle a_\ell||\psi\rangle\langle\psi|)$
$= Pr(\ell|\psi)$ → Prob. of the distinct pointer positions.
Using rule from case (ii)
$\hat{\rho}_S = \sum_k |a_k\rangle\langle a_k| \underbrace{\rho\, |a_k\rangle\langle a_k|}_{= Pr(k|\psi) = Tr(|a_k\rangle\langle a_k|}$
$= \sum_k Pr(k|\psi)\, |a_k\rangle\langle a_k|$ ✓

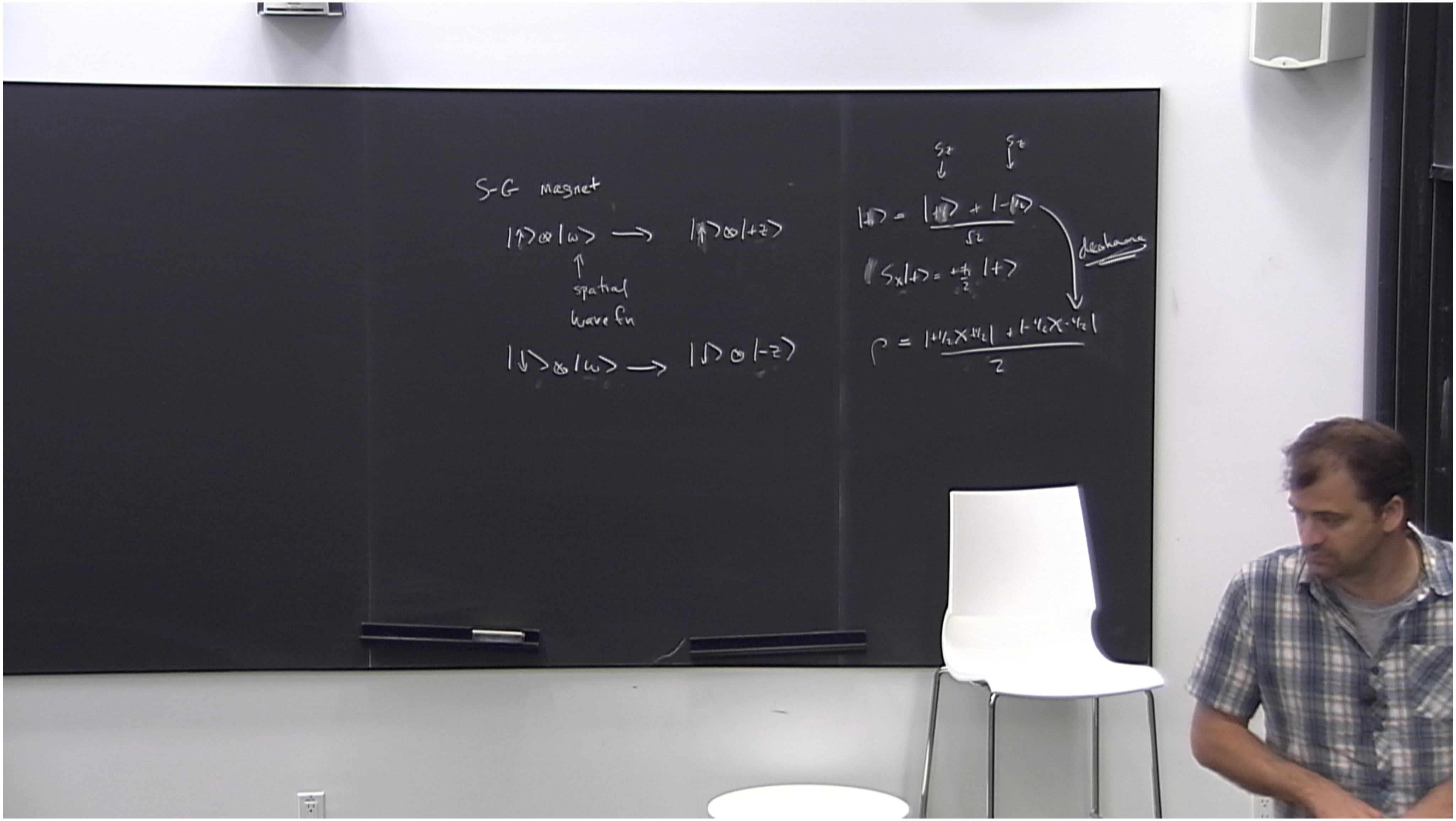

(i) corresponds to
ntangled state of
em & apparatus,
ch is a coherent superposition
macroscopically distinct
figurations.
If we partial-trace over the apparatus
then $\hat{\rho}_s = Tr_A (|\chi\rangle\langle\chi|)$
Implied by our unitary description of measurement process
$\boxed{\hat{\rho}_s = \sum_\ell |c_\ell|^2 |a_\ell\rangle\langle a_\ell|}$ ✓
$c_\ell = \langle a_\ell | \psi \rangle$
$|c_\ell|^2 = Tr(|a_\ell\rangle\langle a_\ell| \psi\rangle\langle\psi|)$
$= Pr(\ell|\psi) \rightarrow$ Prob. of the distinct pointer positions
Using rule from case (ii)
$\hat{\rho}_s = \sum_k |a_k\rangle\langle a_k| \rho |a_k\rangle\langle a_k|$, where $\rho = |\psi\rangle\langle\psi|$
$= Pr(k|\psi) = Tr(|a_k\rangle\langle a_k| \psi\rangle\langle\psi|)$
$= \sum_k Pr(k|\psi) |a_k\rangle\langle a_k|$ ✓

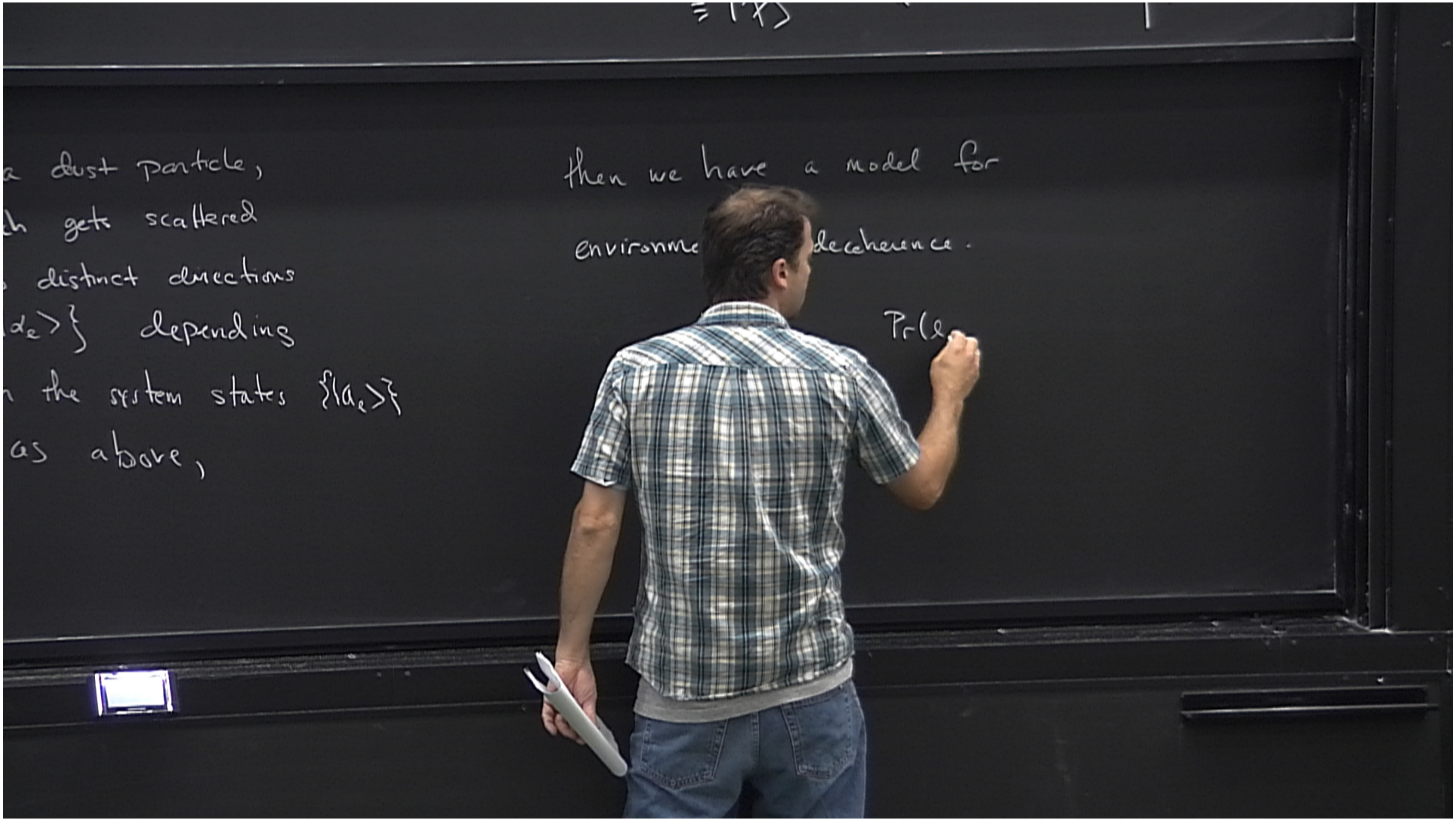

Decoherence
If we imagine /consider
$\mathcal{H}_A$ not as a measureme
apparatus, but instead
some degree of freedom
the environment surround
e.g. a dust particle,

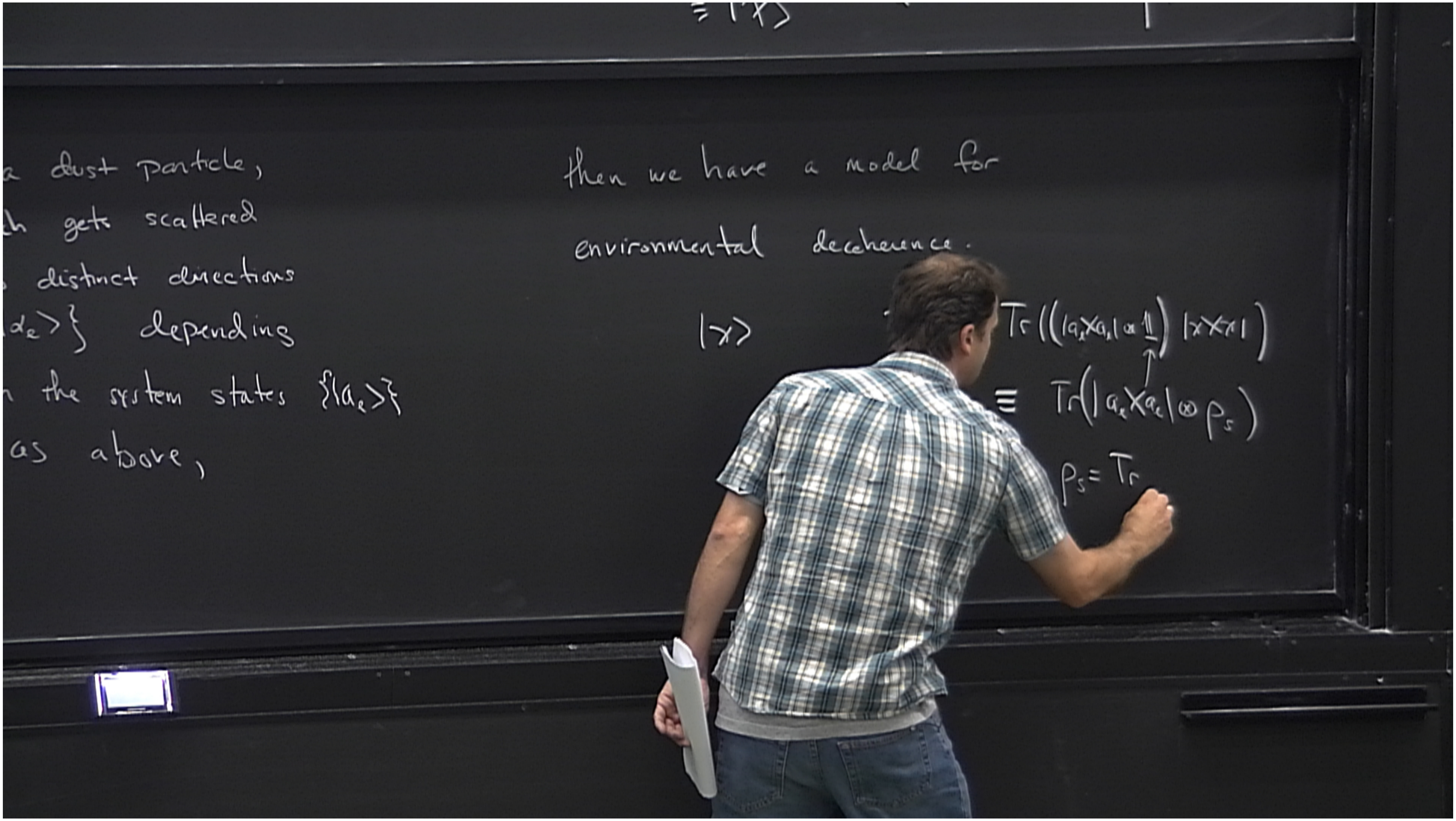

Decoherence
If we imagine /consider
$\mathcal{H}_A$ not as a measurement
apparatus, but instead as
some degree of freedom of
the environment surrounding the
eg. a dust particle,
scattered
directions
depending

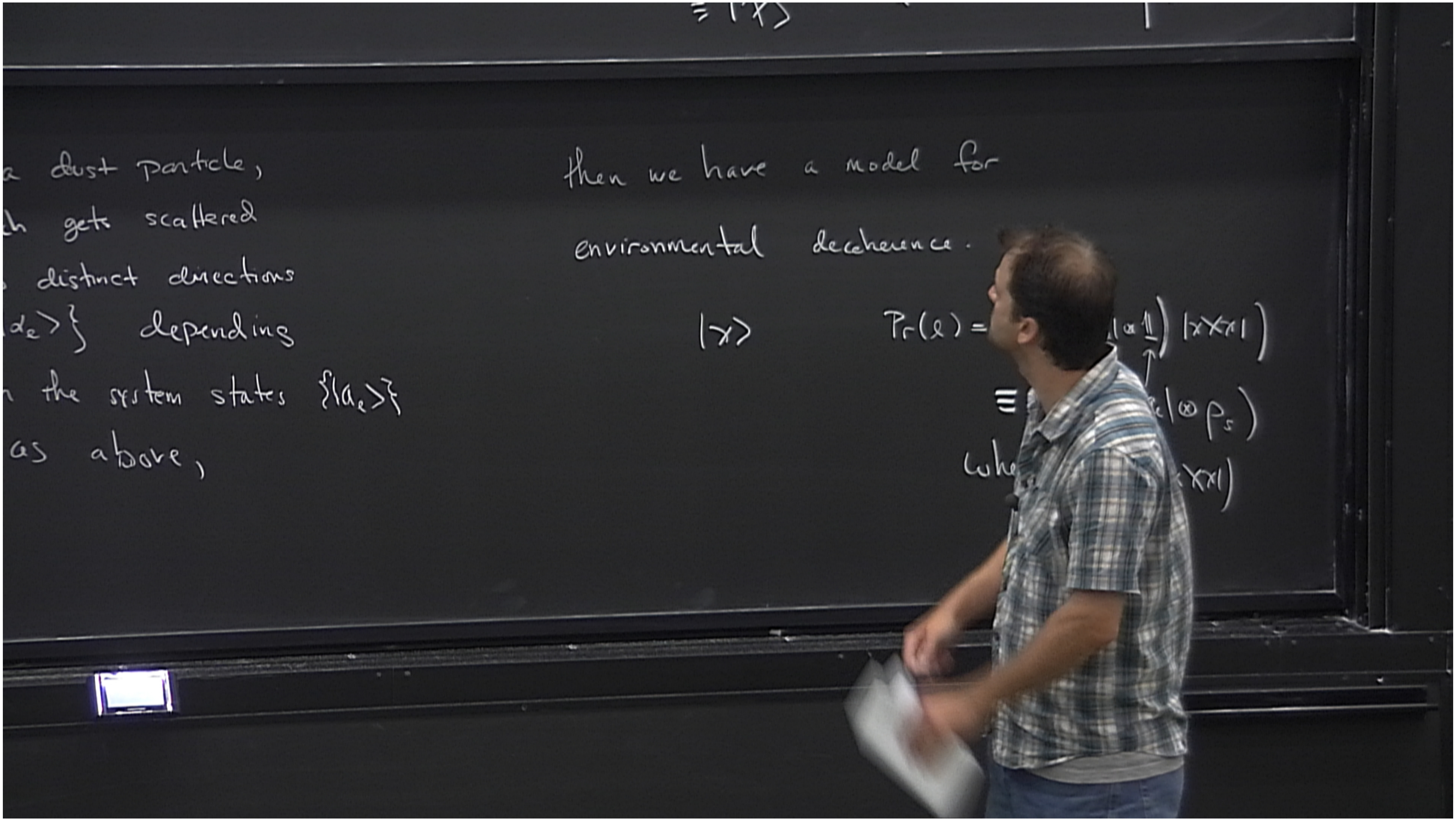

S-G magnet
$|\uparrow\rangle \otimes |\omega\rangle \longrightarrow |\uparrow\rangle \otimes |+z\rangle$
spatial
wave fn
$|\downarrow\rangle \otimes |\omega\rangle \longrightarrow |\downarrow\rangle \otimes |-z\rangle$

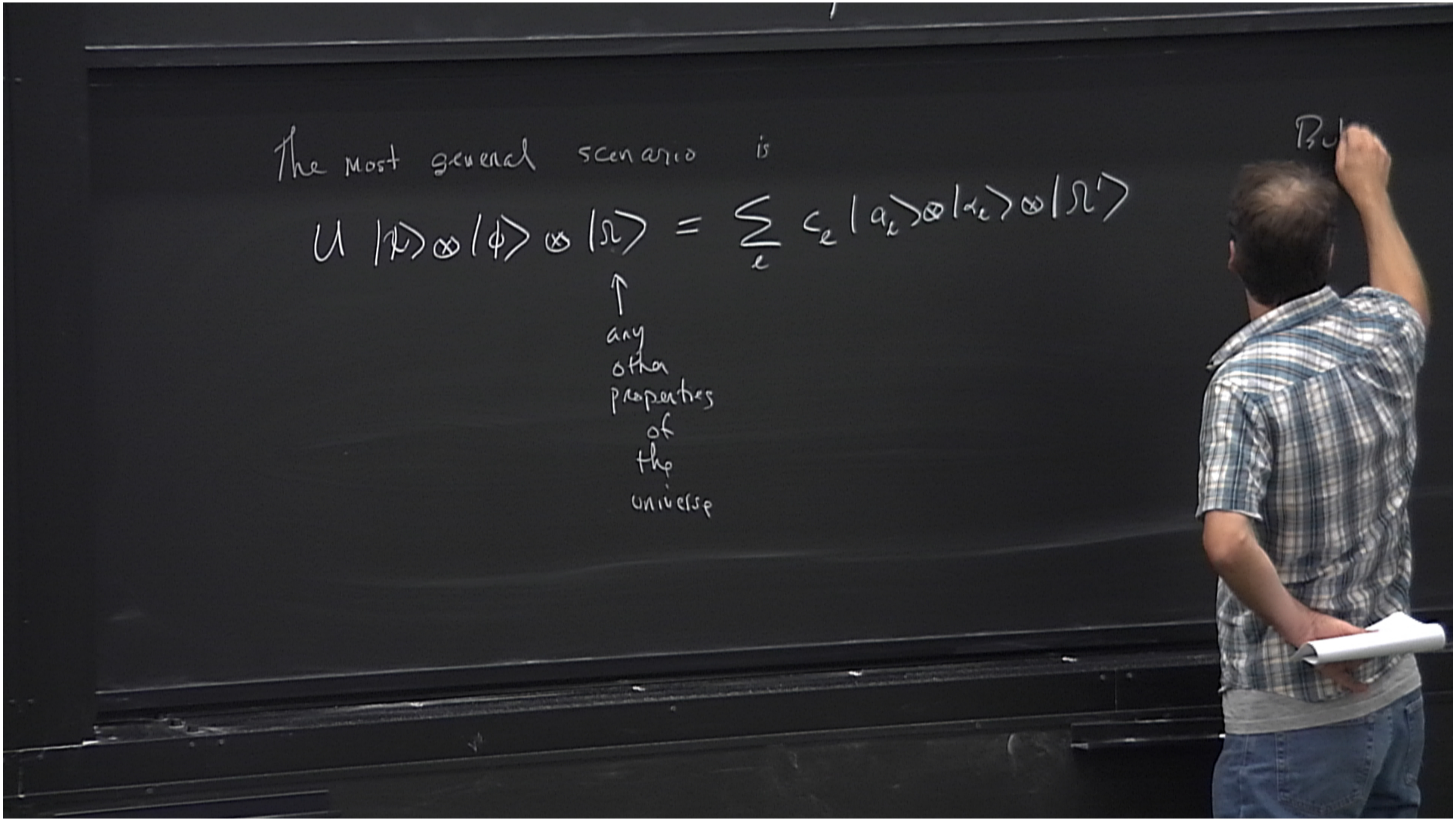

S-G magnet
|↑⟩⊗|ω⟩ ⟶ |↑⟩⊗|+z⟩
spatial
wave fn
|↓⟩⊗|ω⟩ ⟶ |↓⟩⊗|-z⟩

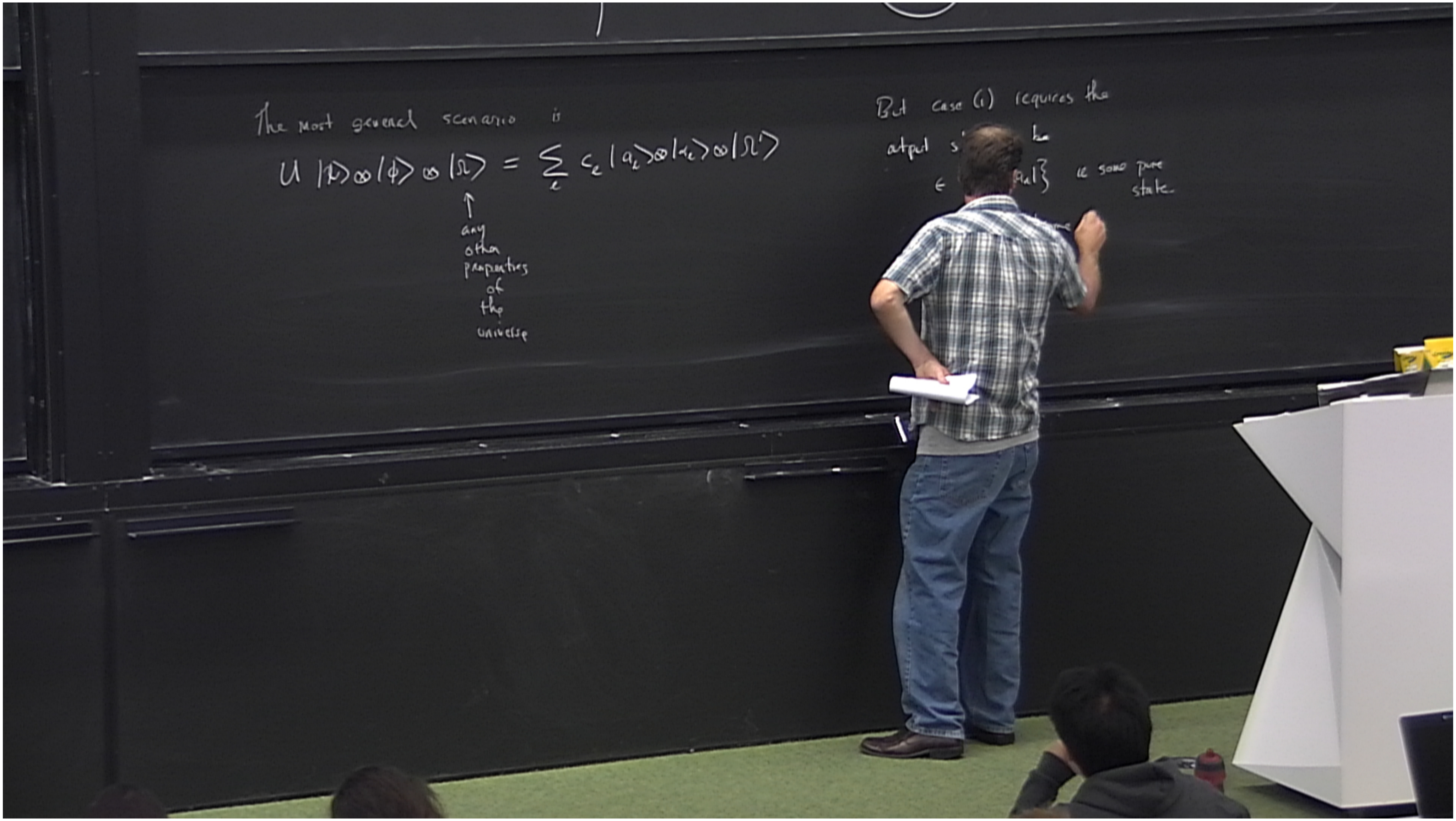

S-G magnet
$|\uparrow\rangle \otimes |\omega\rangle \longrightarrow |\uparrow\rangle \otimes |+z\rangle$
spatial wave fn
$|\downarrow\rangle \otimes |\omega\rangle \longrightarrow |\downarrow\rangle \otimes |-z\rangle$
S_z
$|+\rangle = \frac{|+\frac{1}{2}\rangle + |-\frac{1}{2}\rangle}{\sqrt{2}}$
$S_x|+\rangle = +\frac{\hbar}{2}|+\rangle$
decoherence
$\rho = \frac{|+\frac{1}{2}\rangle\langle+\frac{1}{2}| + |-\frac{1}{2}\rangle\langle-\frac{1}{2}|}{2}$

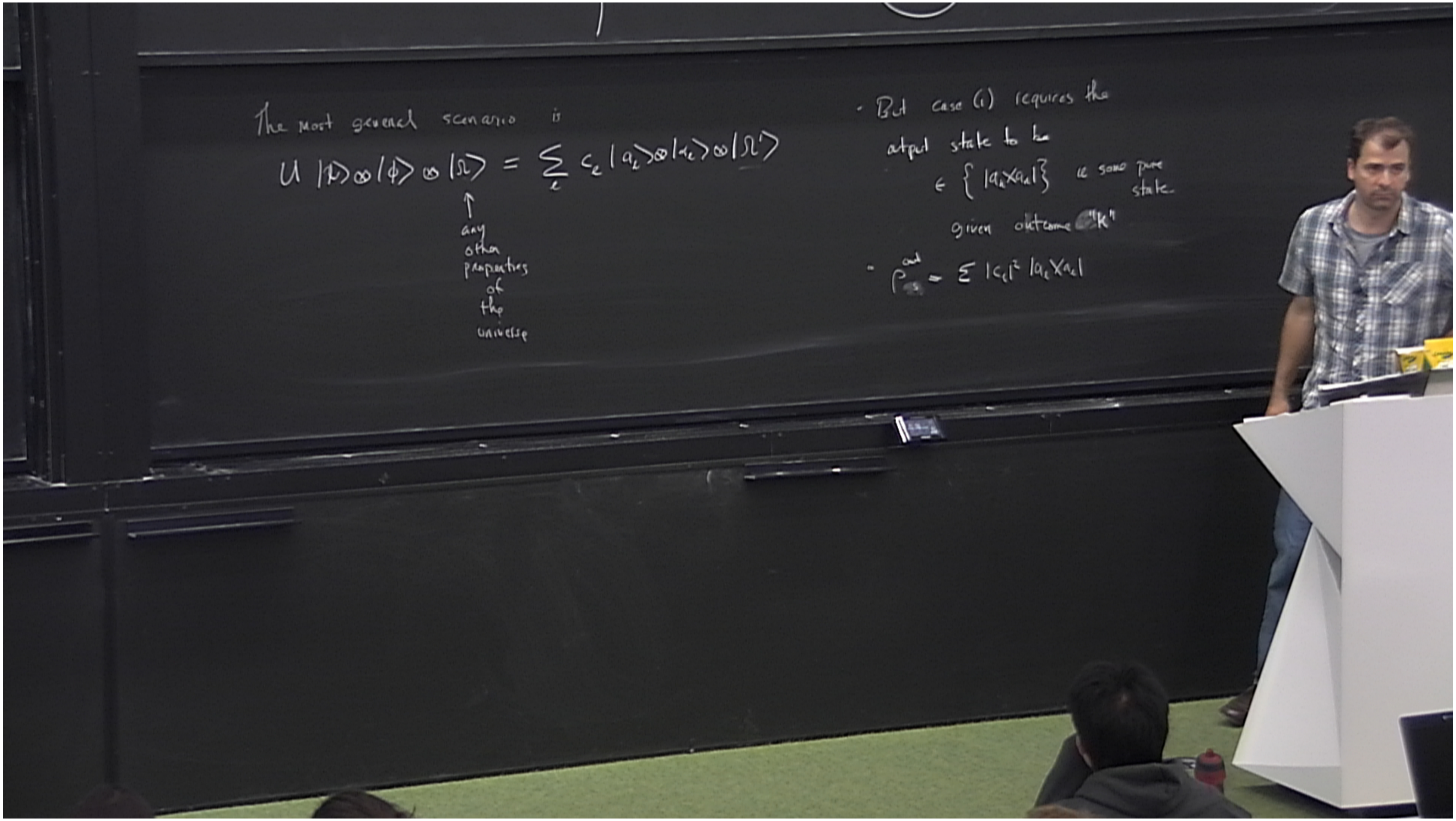

a dust particle,
ch gets scattered
distinct directions
$|d_e\rangle\}$ depending
n the system states $\{|a_e\rangle\}$
as above,
then we have a model for
environme decoherence.
Pr(e


dust particle,
gets scattered
distinct directions
depending
the system states $\{|a_e\rangle\}$
as above,
then we have a model for
environmental decoherence.
$|\chi\rangle$
$\mathrm{Tr}\left(\left(|a_e\rangle\langle a_e| \otimes \mathbb{1}\right) |\chi\rangle\langle\chi|\right)$
$\equiv \mathrm{Tr}\left(|a_e\rangle\langle a_e| \otimes \rho_s\right)$
$\rho_s = \mathrm{Tr}$


a dust particle,
h gets scattered
distinct directions
α_e>} depending
the system states {|a_e>}
as above,
then we have a model for
environmental decoherence.
|x>
Pr(l) =


The most general scenario is
$U |\psi\rangle \otimes |\phi\rangle \otimes |\Omega\rangle = \sum_i c_i |a_i\rangle \otimes |\alpha_i\rangle \otimes |\Omega'\rangle$
any other properties of the universe


The most general scenario is
$U\,|R\rangle \otimes |\phi\rangle \otimes |\Omega\rangle = \sum_i c_i\,|a_i\rangle \otimes |r_i\rangle \otimes |\Omega'\rangle$
any other properties of the universe
But case (i) requires the
output
ie some pure state.


The most general scenario is
$U |\psi\rangle \otimes |\phi\rangle \otimes |\Omega\rangle = \sum_i c_i |a_i\rangle \otimes |\alpha_i\rangle \otimes |\Omega'\rangle$
any other properties of the universe
· But case (i) requires the output state to be
$\in \{ |a_i\rangle\langle a_i| \}$ ie some pure state
given outcome "k"
· $\rho^{out}_{s} = \sum |c_i|^2 |a_i\rangle\langle a_i|$

The most general scenario is

$$U\,|\psi\rangle\otimes|\phi\rangle\otimes|\Omega\rangle = \sum_{\ell} c_\ell\,|a_\ell\rangle\otimes|\alpha_\ell\rangle\otimes|\Omega'\rangle$$

any other properties of the universe

- But case (i) requires the output state to be $\in \{\,|a_k\rangle\langle a_k|\,\}$ i.e. some pure state given outcome "k"
- $\rho_S^{out} = \sum |c_\ell|^2\,|a_\ell\rangle\langle a_\ell|$