

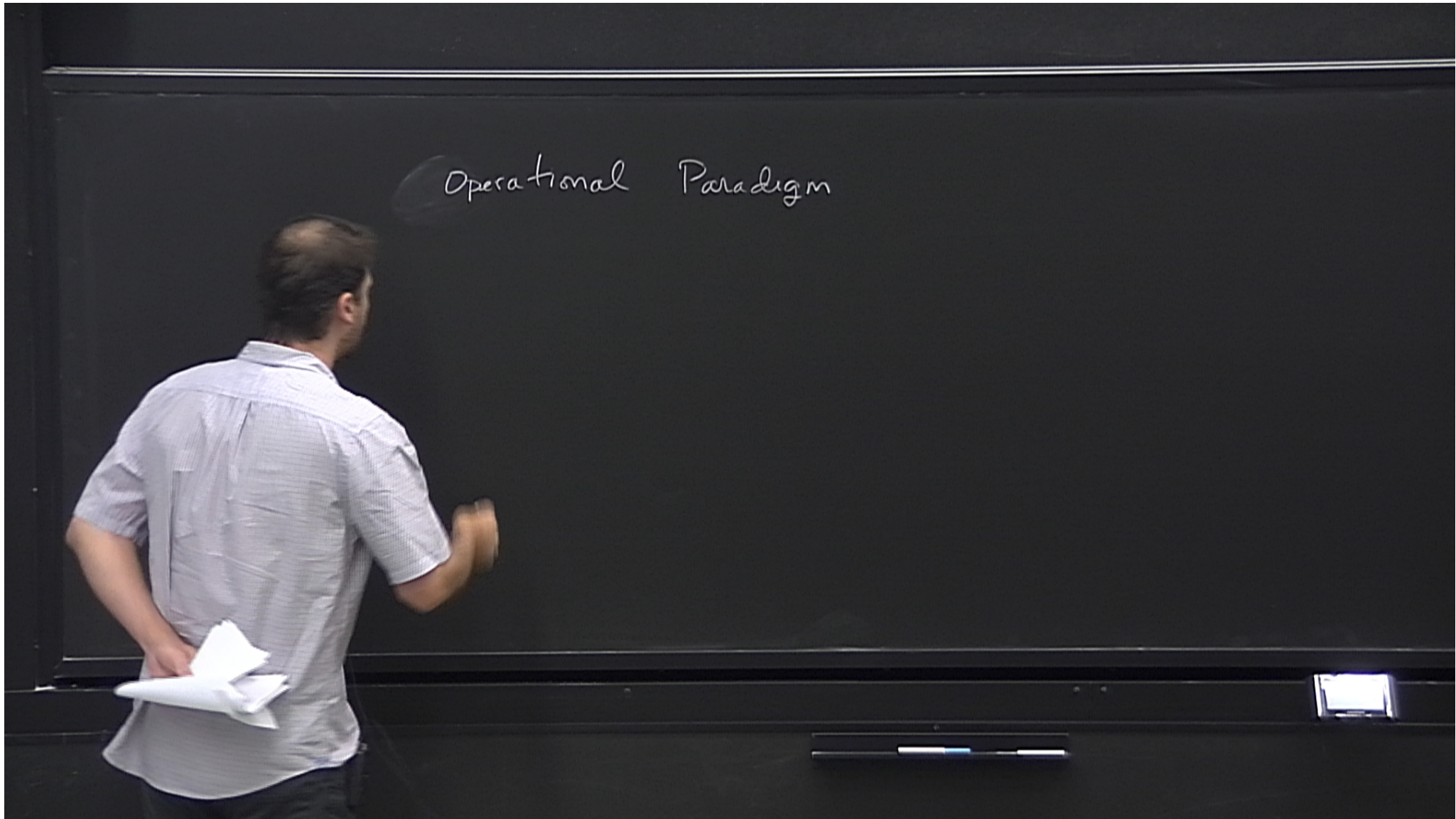
Title: Quantum Theory - Lecture 3

Date: Sep 12, 2012 09:00 AM

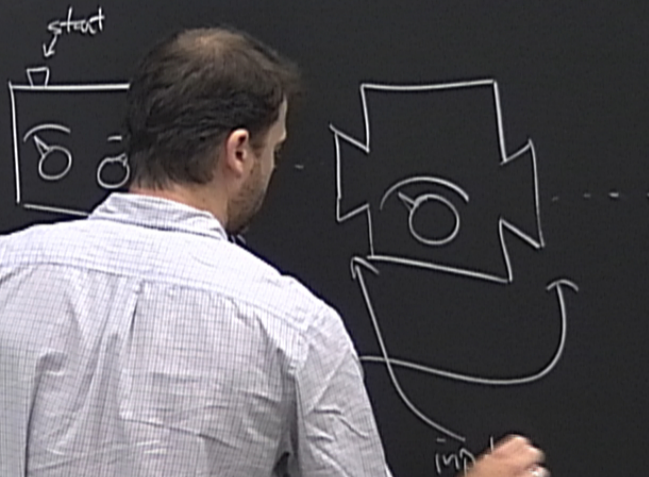
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Abstract:

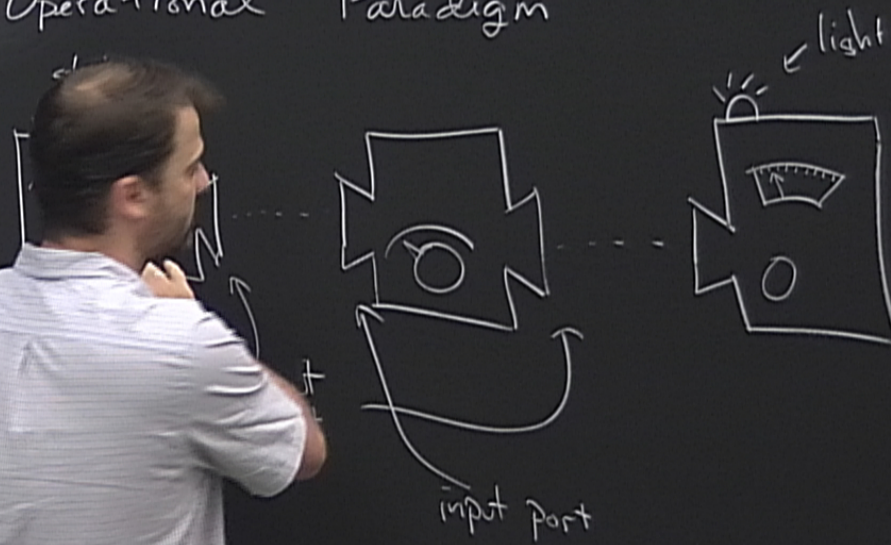


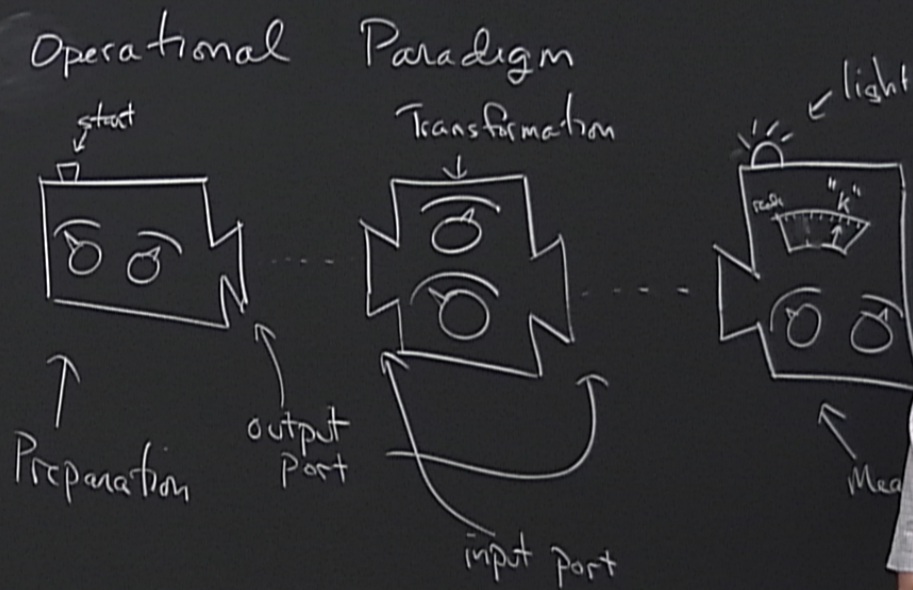


Operational Paradigm



Operational Paradigm





Unit

Unitary transformations
pr purity

• If $|\psi\rangle\langle\psi|$

$$\hat{U}^\dagger = \hat{U} |\psi\rangle\langle\psi| \hat{U}^\dagger$$

=

• Unitary transformations
preserve purity

• If $\hat{\rho} = |\psi\rangle\langle\psi|$

$$\begin{aligned}\rho(t) &= \hat{U} \hat{\rho} \hat{U}^\dagger = \hat{U} |\psi\rangle\langle\psi| \hat{U}^\dagger \\ &= |\phi\rangle\langle\phi|\end{aligned}$$

• Unitary transformations
preserve purity

• If $\hat{\rho} = |\psi\rangle\langle\psi|$

$$\rho(\psi) = \hat{U} \hat{\rho} \hat{U}^\dagger = \hat{U} |\psi\rangle\langle\psi| \hat{U}^\dagger \\ = |\phi\rangle\langle\phi|$$

Degree of purity

$$\leq \text{Tr}(\hat{\rho}^2) \leq 1$$

where $\hat{\rho} \in \mathcal{L}(\mathbb{C}^D)$

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$$\rho(\psi) = \hat{U} \hat{\rho} \hat{U}^\dagger = \hat{U} |\psi\rangle\langle\psi| \hat{U}^\dagger \\ = |\psi'\rangle\langle\psi'|$$

Degree of purity

$$\frac{1}{D} \leq \text{Tr}(\hat{\rho}^2) \leq 1$$

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Degree of purity
 $\frac{1}{\text{tr}(\hat{\rho}^2)} \leq 1$

$$\hat{\rho} \in \mathcal{L}(\mathbb{C}^D)$$

is pure then $\text{tr}(\hat{\rho}^2) = 1$

$\hat{\rho}$ is completely mixed, $\hat{\rho} = \frac{1}{D} \mathbb{I}$

- Unitary transformations
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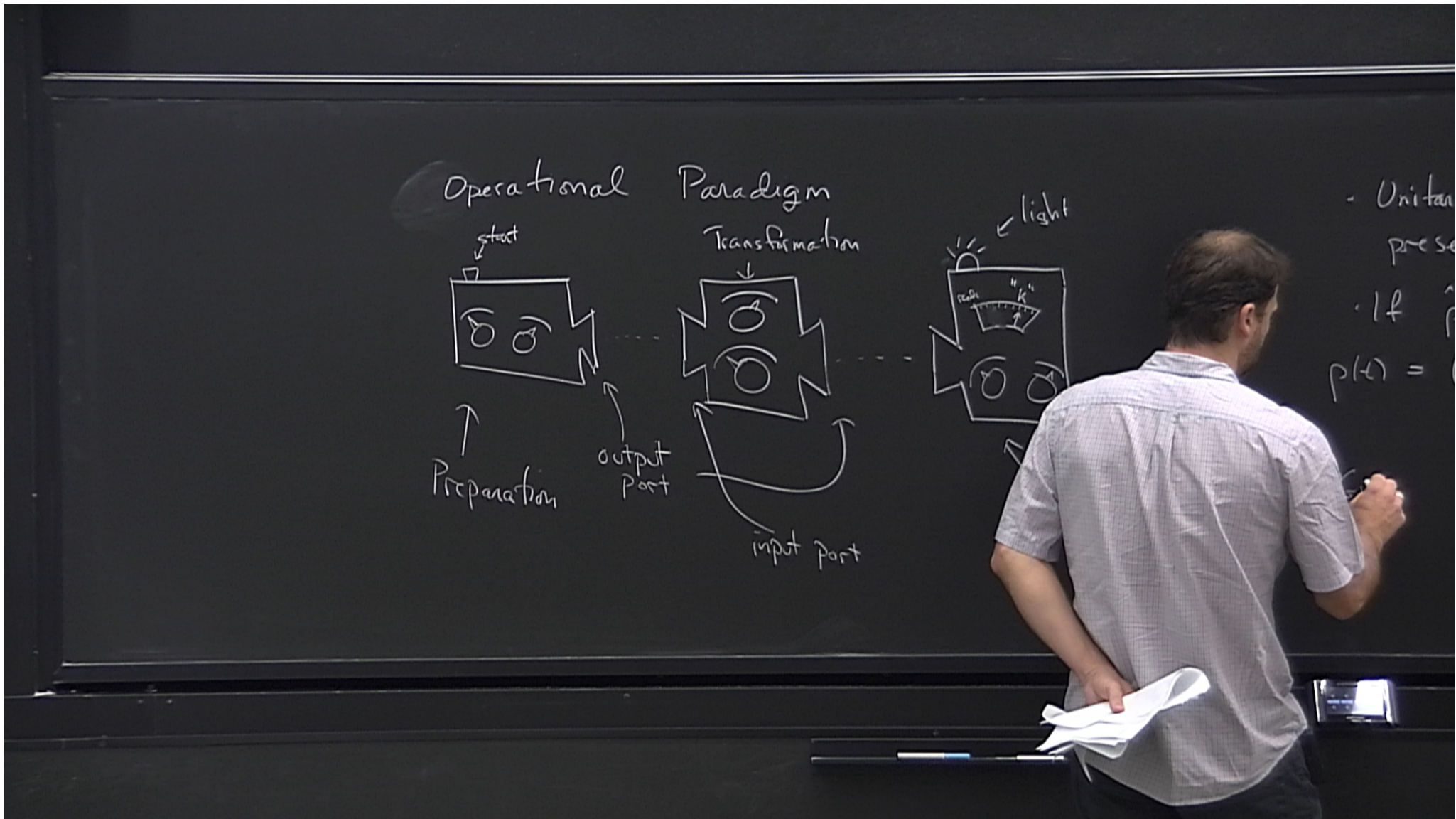
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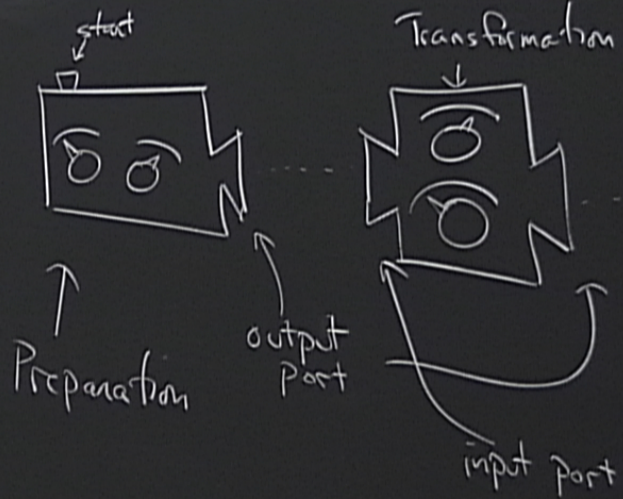
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Operational

Paradigm
Transformation

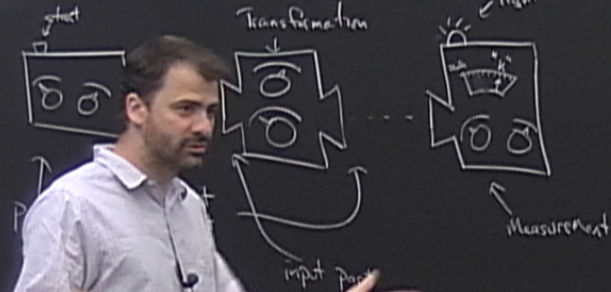


Unitary
preservation

$\cdot 1f$

$p(L) =$

Operational Paradigm



Unitary transformations preserve purity

$$\rho(t) = \hat{U} \rho \hat{U}^\dagger = \hat{U} |\psi\rangle\langle\psi| \hat{U}^\dagger = |\psi\rangle\langle\psi|$$

$$\text{Given observable } \hat{A} = \sum_k \lambda_k \hat{P}_k$$

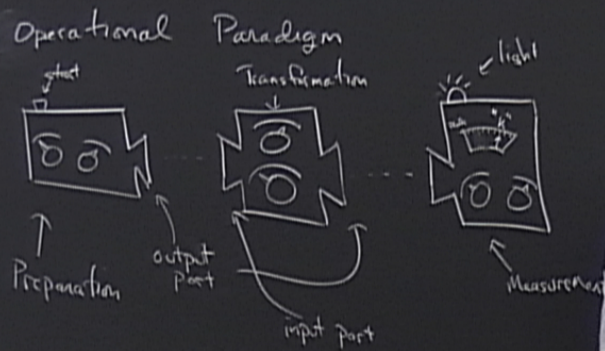
Degree of purity

$$\frac{1}{D} \leq \text{Tr}(\rho^2) \leq 1$$

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& if ρ is completely mixed, $\rho = \frac{\mathbb{I}}{D}$
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Unitary transformations preserve purity

$$\rho = |\psi\rangle\langle\psi|$$

$$\rho' = U \rho U^\dagger = U |\psi\rangle\langle\psi| U^\dagger = |U\psi\rangle\langle U\psi|$$

$$\text{an observable } \hat{A} = \sum_i \lambda_i \hat{P}_i$$

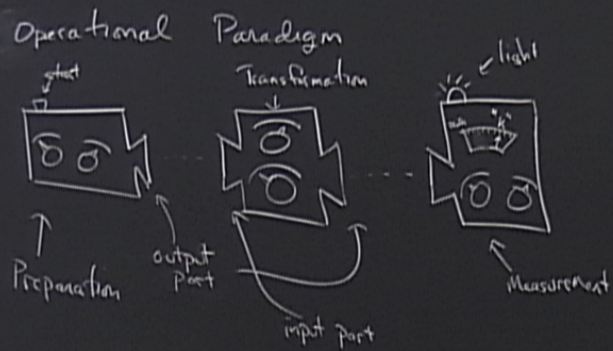
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- Unitary transformations preserve purity
- If $\hat{\rho} = |\psi\rangle\langle\psi|$

$$\rho(t) = \hat{U} \hat{\rho} \hat{U}^\dagger = \hat{U} |\psi\rangle\langle\psi| \hat{U}^\dagger = |\psi(t)\rangle\langle\psi(t)|$$
- Given observable $\hat{A} = \sum_k \lambda_k \hat{P}_k$
 $\lambda_k = \lambda_{k'}$
 $\lambda_k \neq \lambda_{k'}$ for any other values of k, k'

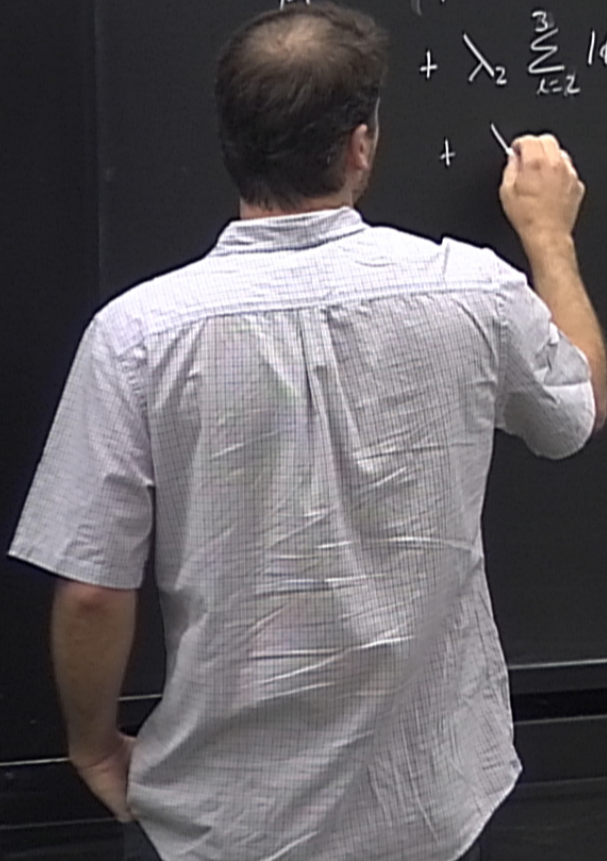
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A man with short dark hair, wearing a light blue short-sleeved button-down shirt, is seen from the back, writing on a blackboard. He is holding a piece of chalk in his right hand, positioned to write the next term in the equation. The blackboard is dark and has some white chalk markings. The equation written on the board is:

$$\hat{A} = \lambda_1 |\phi_1\rangle\langle\phi_1| + \lambda_2 \sum_{i=2}^3 |\phi_i\rangle\langle\phi_i| +$$

$$\hat{A} = \lambda_1 |\phi_1\rangle\langle\phi_1| + \lambda_2 \left(\sum_{i=2}^3 |\phi_i\rangle\langle\phi_i| \right) + \lambda_4 |\phi_4\rangle\langle\phi_4| + \dots + \lambda_n |\phi_n\rangle\langle\phi_n|$$

$$P_2 = |\phi_2\rangle\langle\phi_2| + |\phi_3\rangle\langle\phi_3|$$

$$\hat{A} = \lambda_1 |\phi_1 \times \phi_1| + \lambda_2 \left(\sum_{i=2}^3 |\phi_i \times \phi_i| \right) + \lambda_4 |\phi_4 \times \phi_4| + \dots + \lambda_D |\phi_D \times \phi_D|$$

$$P_2 = |\phi_2 \times \phi_2| \quad \text{then} \quad P_{2,3} = |\phi_2 \times \phi_2| + |\phi_3 \times \phi_3|$$

$$P_3 = |\phi_3 \times \phi_3|$$

$$\hat{A} = \lambda_1 |\phi_1 \times \phi_1| + \lambda_2 \left(\sum_{i=2}^3 |\phi_i \times \phi_i| \right) + \lambda_4 |\phi_4 \times \phi_4| + \dots + \lambda_D |\phi_D \times \phi_D|$$

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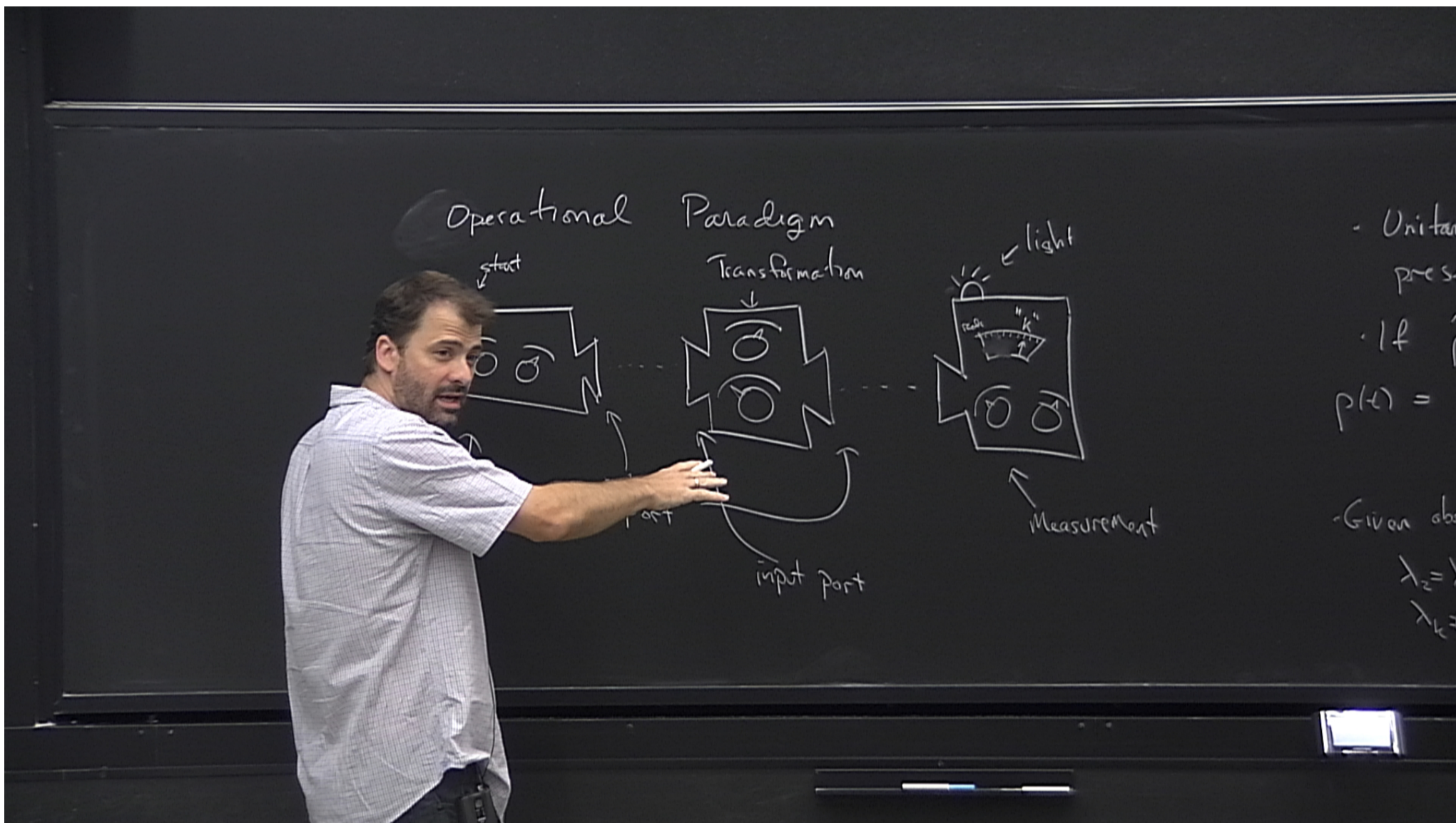
$$P_3 = |\phi_3 \times \phi_3|$$

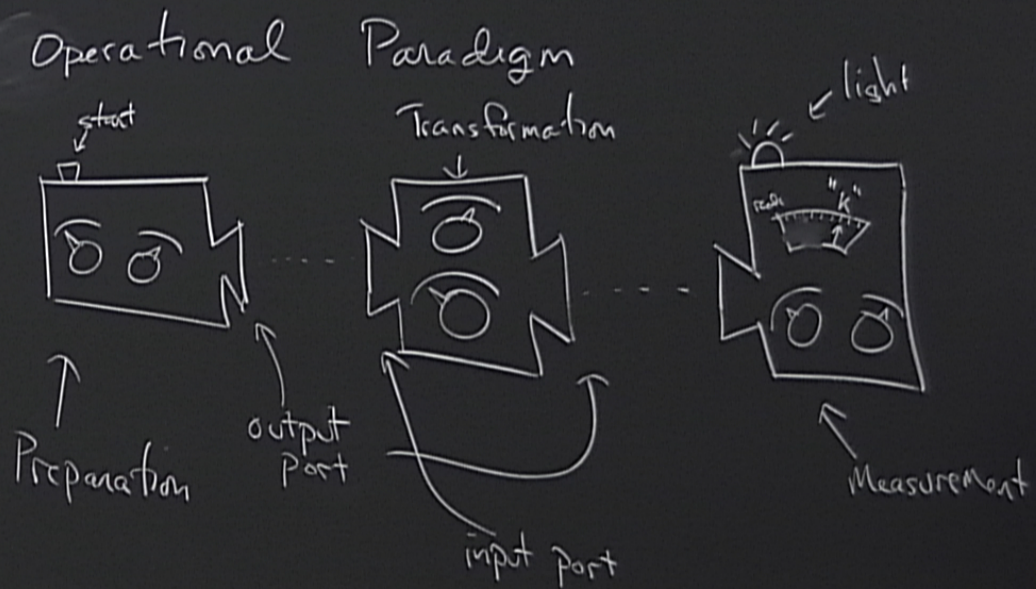
$$\hat{A} = \lambda_1 |\phi_1 \times \phi_1| + \lambda_2 \left(\sum_{i=2}^3 |\phi_i \times \phi_i| \right) + \lambda_4 |\phi_4 \times \phi_4| + \dots + \lambda_D |\phi_D \times \phi_D|$$

$$\hat{A} = \sum_{j=1}^{D-1} \lambda_j P_j$$

$$P_2 = |\phi_2 \times \phi_2| \quad \text{then} \quad P_{2,3} = |\phi_2 \times \phi_2| + |\phi_3 \times \phi_3|$$

$$P_3 = |\phi_3 \times \phi_3|$$





$$\hat{A} = \lambda_1 |\phi_1 \times \phi_1| + \lambda_2 \left(\sum_{i=2}^3 |\phi_i \times \phi_i| \right) + \lambda_4 |\phi_4 \times \phi_4| + \dots + \lambda_D |\phi_D \times \phi_D|$$

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$$\hat{A} = \sum_{j=1}^{D-1} \lambda_j P_j$$

The form of
characteristic of
"A" of the set
of vectors
compute

$$\hat{A} = \lambda_1 |\phi_1 \rangle \langle \phi_1| + \lambda_2 \left(\sum_{i=2}^3 |\phi_i \rangle \langle \phi_i| \right) + \lambda_4 |\phi_4 \rangle \langle \phi_4| + \dots + \lambda_D |\phi_D \rangle \langle \phi_D|$$

$$P_2 = |\phi_2 \rangle \langle \phi_2| \quad \text{then} \quad P_{2,3} = |\phi_2 \rangle \langle \phi_2| + |\phi_3 \rangle \langle \phi_3|$$

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$$\hat{A} = \sum_{j=1}^{D-1} \lambda_j P_j$$

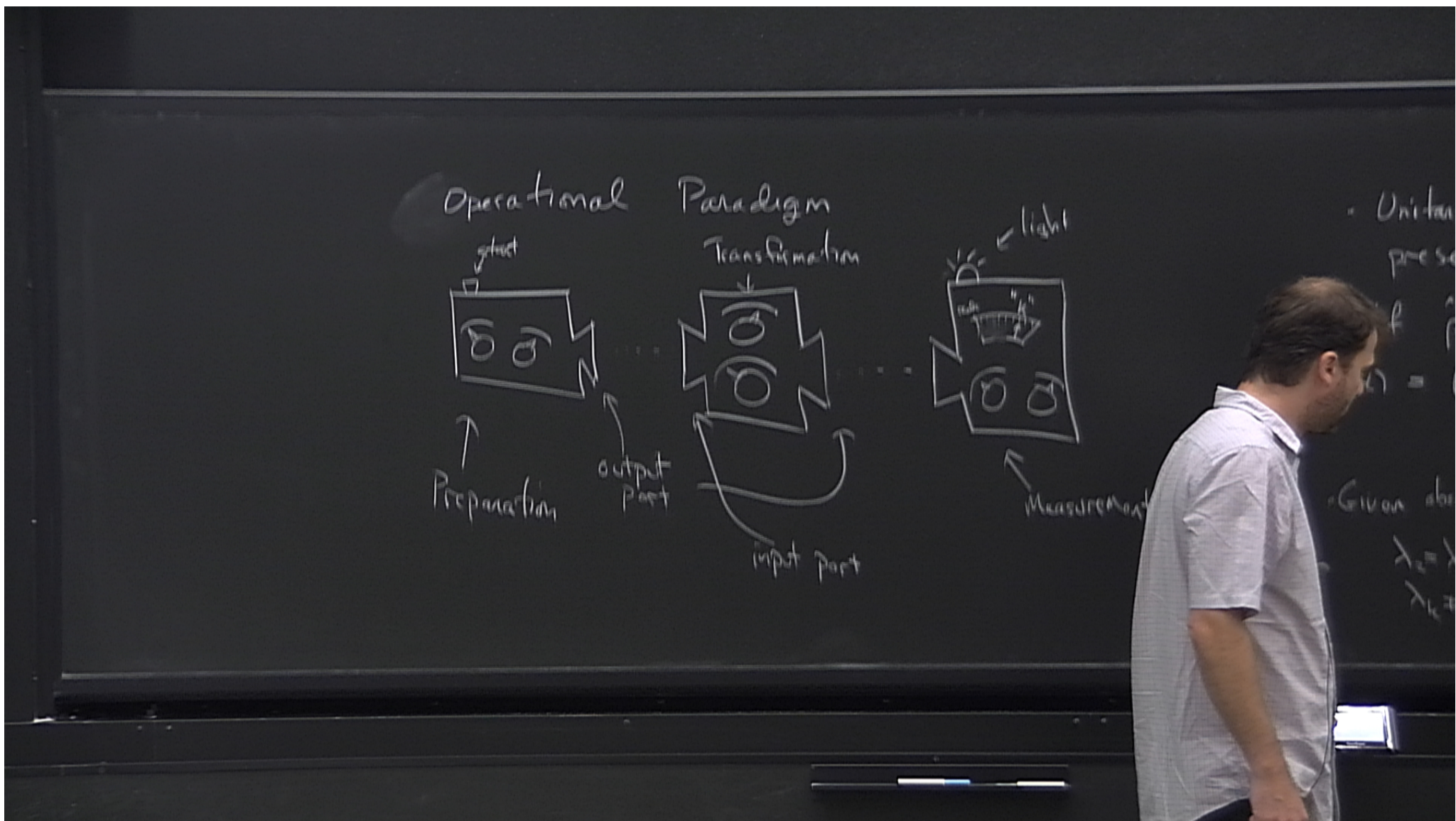
The fundamental characterization of " $\hat{A}$ " is the set of distinct projectors in the spectral decomposition of $\hat{A}$, namely $\{$

• The quantity $\{P_k\}$
is called PVM,
Projective Measure.
• the measurement

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Positive operator valued measure.



In Heisenberg picture one has

$$i\hbar \frac{\partial U(t, t_0)}{\partial t} = U(t, t_0) \hat{H}_H(t)$$

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$$i\hbar \frac{\partial U(t, t_0)}{\partial t} = U(t, t_0) \hat{H}_H(t)$$

$$\hat{U}^\dagger \hat{H} \hat{U}$$

$$[\hat{U}^\dagger \hat{H} \hat{U}, \hat{H}_H(t)] + (\hat{U}^\dagger \hat{H} \hat{U})$$

In Schrödinger picture,

Let $\rho = |\psi\rangle\langle\psi|$

$$\frac{d\hat{\rho}_s(t)}{dt} = \frac{1}{i\hbar} [\hat{H}_s(t), \hat{\rho}_s(t)]$$

↑ von Neumann eqⁿ

$$\rho_s(t) = U(t, t_0) \rho(t_0) U^\dagger(t, t_0)$$

In Schrodinger picture,

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Let $\rho = |\psi\rangle\langle\psi|$

$$|\psi\rangle = |\psi(t)\rangle$$

$$= \langle\psi(t)| U^\dagger$$

Evolution

$$\hat{H}|\psi\rangle$$

In Schrödinger picture,

$$\frac{d\hat{\rho}_s(t)}{dt} = \frac{1}{i\hbar} [\hat{H}_s(t), \hat{\rho}_s(t)]$$

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Let $\rho = |\psi\rangle\langle\psi|$

$$|\psi(t)\rangle = U|\psi(0)\rangle$$

$$\langle\psi(t)| = \langle\psi(0)|U^\dagger$$

By differentiation

$$i\hbar \frac{d|\psi\rangle}{dt} = \hat{H}|\psi\rangle$$

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$$U = \exp(-iHt).$$

for H time-independent.

$$U = \exp\left(-i \frac{H t}{\hbar}\right).$$

for H time-independent.

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for time-independent

$$U = \exp\left(-i \int dt\right)$$

Solu

$$U = \exp\left(-\frac{iHt}{\hbar}\right).$$

H time-independent

$$= T \exp\left(-\frac{i}{\hbar} \int_0^t d\tau H(\tau)\right)$$

Solution to eqⁿ in Axiom 3

• $U = \exp\left(-\frac{iHt}{\hbar}\right)$

for H time-independent

• $U = T \exp\left(-\frac{i}{\hbar} \int_0^t d\tau H(\tau)\right)$

in general.

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Recall that for any H
one has

Solution to eqⁿ in Axiom 3

• $U = \exp\left(\frac{-iHt}{\hbar}\right)$

for H time-independent

• $U = T \exp\left(-\frac{i}{\hbar} \int_0^t H d\tau\right)$

in general

Recall that for any $\mathcal{H}$

one has a dual space,

$\forall |\psi\rangle \in \mathcal{H} \exists \langle\psi| \in \mathcal{H}^+ \equiv \mathcal{H}$

$|\psi(t)\rangle \langle\psi(t)|$ $U |\psi(0)\rangle \langle\psi(0)| U^\dagger$
 $|\psi(t)\rangle$ $\langle\psi(t)|$

Solution to eqⁿ in Axiom 3

$$\bullet U = \exp\left(\frac{-iHt}{\hbar}\right)$$

for H time-independent

$$\bullet U = \mathcal{T} \exp\left(-\frac{i}{\hbar} \int_0^t d\tau H(\tau)\right)$$

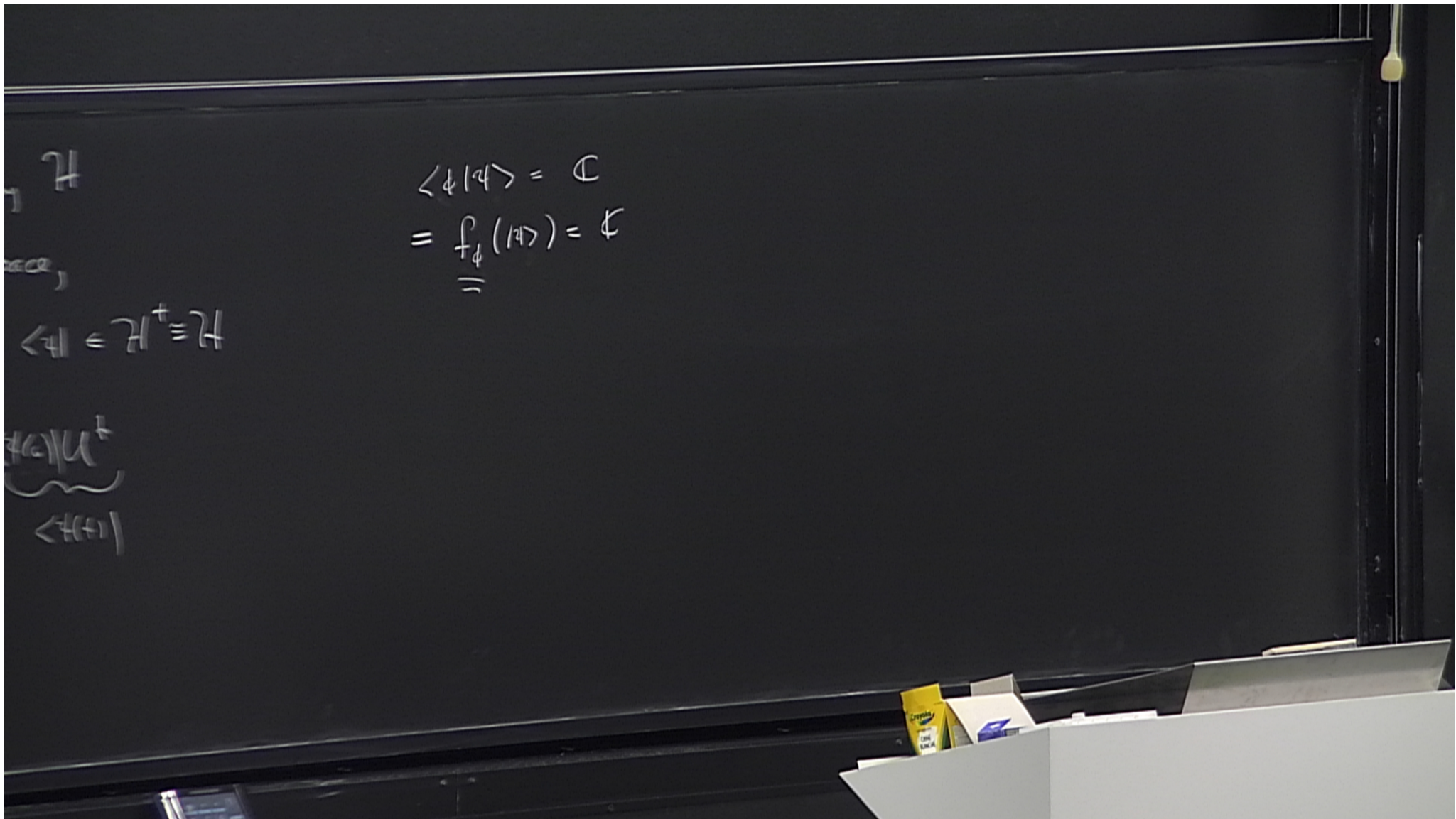
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$$|\psi(t)\rangle \langle\psi(t)| = \underbrace{U |\psi(0)\rangle \langle\psi(0)| U^\dagger}_{|\psi(t)\rangle \langle\psi(t)|}$$



Composite Systems & Entanglement

Consider two systems A & B

Composite Systems & Entanglement

- Consider two systems A & B
Hilbert spaces $\mathcal{H}_A$ & $\mathcal{H}_B$
Let $\mathcal{H}_A = \mathbb{C}^M$, $\mathcal{H}_B = \mathbb{C}^N$
↑
on basis
 $\{ |i\rangle \}$

Composite Systems & Entanglement

- Consider two systems A & B
with Hilbert spaces $\mathcal{H}_A$ & $\mathcal{H}_B$

$$\text{Let } \mathcal{H}_A = \mathbb{C}^M, \mathcal{H}_B = \mathbb{C}^N$$

↑
ON basis
 $\{|a_k\rangle\}_{k=1,\dots,M}$

↑
ON basis
 $\{|b_\ell\rangle\}_{\ell=1,\dots,N}$

The

Then the Hilbert space
joint/composite system

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$$

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$\mathcal{H}_{AB}$ is spanned by
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Then the Hilbert space
of joint/composite

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$\mathcal{H}_{AB}$ is spanned by
the O.N. basis

Any state $|\chi\rangle \in \mathcal{H}_{AB}$

$$|\chi\rangle = \sum_{l,k} c_{lk} |a_l\rangle \otimes |b_k\rangle$$

Then the Hilbert space
of joint/composite system

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$\mathcal{H}_{AB}$ is spanned by
the O.N. basis $\{|a_k\rangle \otimes |b_e\rangle\}$

Any state $|x\rangle \in \mathcal{H}_{AB}$

$$|x\rangle = \sum_{e,k} c_{e,k} |a_k\rangle \otimes |b_e\rangle$$

$$c_{e,k} = \underbrace{(\langle a_k| \otimes \langle b_e|)}_{\text{dual vector}} |x\rangle \in \mathbb{C}$$

Let $|\phi\rangle \in \mathcal{H}_A$, $|\psi\rangle \in \mathcal{H}_B$

$$|\phi\rangle = \sum_k \alpha_k |a_k\rangle$$

α

Let $|\phi\rangle \in \mathcal{H}_A$, $|\psi\rangle \in \mathcal{H}_B$

$$|\phi\rangle = \sum_k \alpha_k |a_k\rangle$$

$$\alpha_k = \langle a_k | \phi \rangle$$

$$|\psi\rangle = \sum_r \beta_r |b_r\rangle$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$$

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}$$

Let $|\omega\rangle = |\psi\rangle$

Let $|\phi\rangle \in \mathcal{H}_A$, $|\psi\rangle \in \mathcal{H}_B$

$$|\phi\rangle = \sum_k \alpha_k |a_k\rangle$$

$$\alpha_k = \langle a_k | \phi \rangle$$

$$|\psi\rangle = \sum_r \beta_r |b_r\rangle$$

$$\rightarrow \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}$$

$$\text{Let } |\omega\rangle = |\phi\rangle \otimes |\psi\rangle$$

$$|\omega\rangle \rightarrow \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Let $|\phi\rangle \in \mathcal{H}_A$, $|\psi\rangle \in \mathcal{H}_B$

$$|\phi\rangle = \sum_k \alpha_k |a_k\rangle$$

$$\alpha_k = \langle a_k | \phi \rangle$$

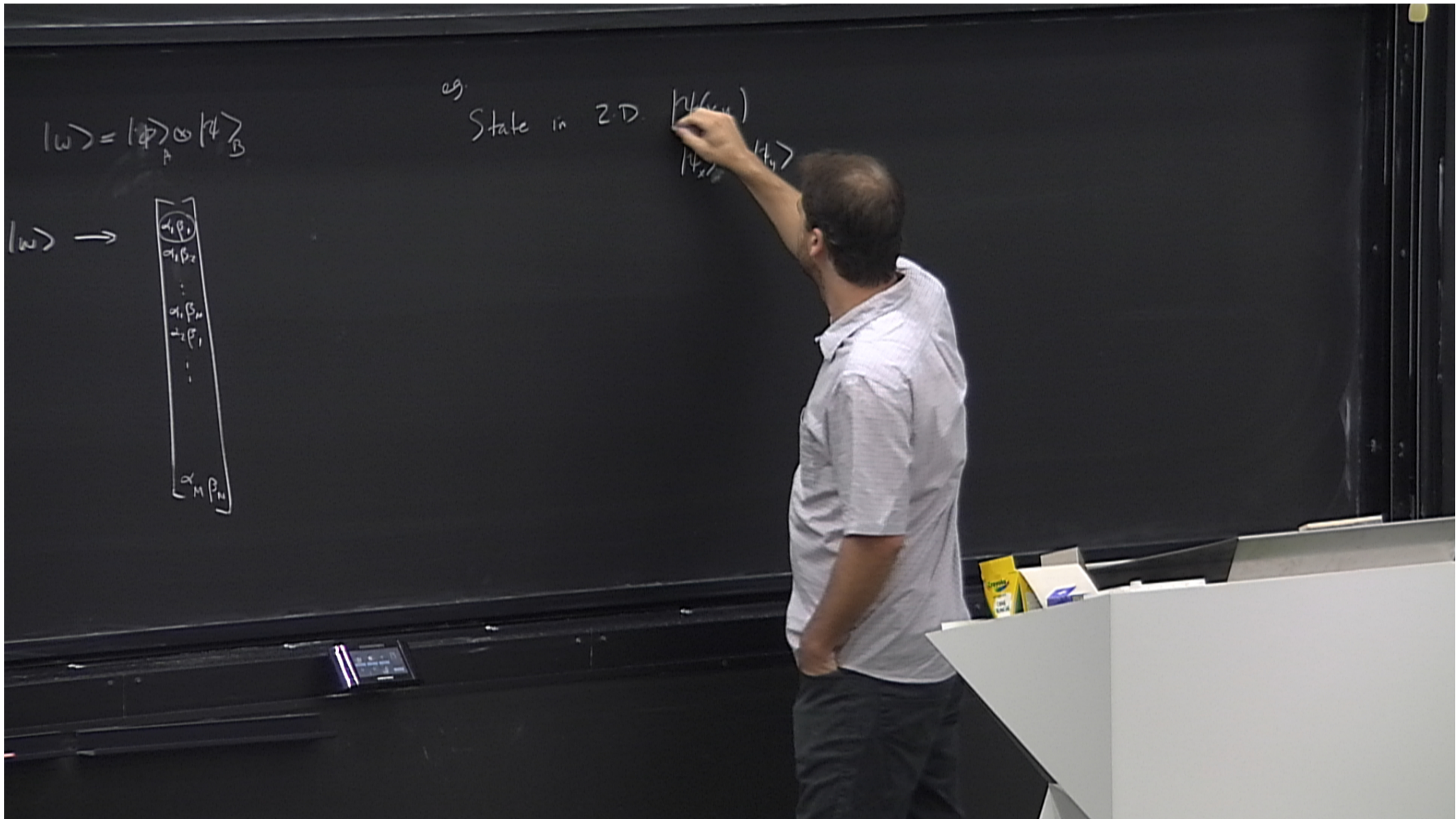
$$|\psi\rangle = \sum_r \beta_r |b_r\rangle$$

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$$\text{Let } |\omega\rangle = |\phi\rangle \otimes |\psi\rangle$$

$$\begin{bmatrix} \alpha_1 \\ \vdots \end{bmatrix}$$



$$|w\rangle = |\psi\rangle_A \otimes |\psi\rangle_B$$

eg
State in 2-D $|\psi(x,y)\rangle$
 $|\psi_x\rangle \otimes |\psi_y\rangle$

$$|w\rangle \rightarrow \begin{bmatrix} \alpha_1 \beta_1 \\ \alpha_1 \beta_2 \\ \vdots \\ \alpha_1 \beta_n \\ \vdots \\ \alpha_m \beta_1 \\ \vdots \\ \alpha_m \beta_n \end{bmatrix}$$

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State in 2D $|\psi(x,y)\rangle$
 $|\psi_x\rangle \otimes |\psi_y\rangle$

$$\text{eg } (U_A \otimes U_B) |x\rangle = \sum_{k,l} c_{k,l} |a_k\rangle \otimes \underline{\underline{(U_B |b_l\rangle)}}$$

$$|w\rangle = |\psi\rangle_A \otimes |\psi\rangle_B$$

$$|w\rangle \rightarrow \begin{bmatrix} \alpha_1 \beta_1 \\ \alpha_1 \beta_2 \\ \vdots \\ \alpha_1 \beta_n \\ \vdots \\ \alpha_m \beta_1 \\ \vdots \\ \alpha_m \beta_n \end{bmatrix}$$

eg
State in 2-D $|\psi(x,y)\rangle$
 $|\psi_x\rangle \otimes |\psi_y\rangle$

$$\text{eg } (U_A \otimes U_B) |x\rangle = \sum_{k,l} c_{k,l} (U_A |a_k\rangle \otimes U_B |b_l\rangle)$$

$$U_A \otimes U_B \rightarrow$$

$$U_A \rightarrow \begin{bmatrix} U_{1,1}^A & U_{1,2}^A & \dots & U_{1,n}^A \\ U_{2,1}^A & U_{2,2}^A & \dots & U_{2,n}^A \\ \vdots & \vdots & \ddots & \vdots \\ U_{m,1}^A & U_{m,2}^A & \dots & U_{m,n}^A \end{bmatrix}$$

$$|w\rangle = |\phi\rangle_A \otimes |\psi\rangle_B$$

$$|w\rangle \rightarrow \begin{bmatrix} \alpha_1 \beta_1 \\ \alpha_1 \beta_2 \\ \vdots \\ \alpha_1 \beta_n \\ \vdots \\ \alpha_m \beta_1 \\ \vdots \\ \alpha_m \beta_n \end{bmatrix}$$

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State in 2-D $|\psi(x,y)\rangle$
 $|\psi_x\rangle \otimes |\psi_y\rangle$

$$\text{eg } (U_A \otimes U_B) |x\rangle = \sum_{k,l} c_{k,l} |k\rangle \otimes (U_B |l\rangle)$$

$$U_A \otimes U_B \rightarrow \begin{bmatrix} U_{1,1}^A & U_{1,2}^A & \dots & U_{1,n}^A \\ U_{2,1}^A & U_{2,2}^A & \dots & U_{2,n}^A \\ \vdots & \vdots & \ddots & \vdots \\ U_{m,1}^A & U_{m,2}^A & \dots & U_{m,n}^A \end{bmatrix}$$

$$|w\rangle = |\psi\rangle_A \otimes |\phi\rangle_B$$

$$|w\rangle \rightarrow \begin{bmatrix} \alpha_1 \beta_1 \\ \alpha_1 \beta_2 \\ \vdots \\ \alpha_1 \beta_n \\ \vdots \\ \alpha_m \beta_1 \\ \vdots \\ \alpha_m \beta_n \end{bmatrix}$$

eg. State in 2-D $|\psi(x,y)\rangle$
 $|\psi_x\rangle \otimes |\psi_y\rangle$

$$\text{eg } (U_A \otimes U_B) |x\rangle = \sum_{k,l} c_{k,l} (\underline{U_A |a_k\rangle}) \otimes (\underline{U_B |b_l\rangle})$$

$$\begin{array}{c} U_A \otimes U_B \rightarrow \begin{bmatrix} U_{1,1}^A U_{1,1}^B & U_{1,2}^A U_{1,2}^B & \dots & U_{1,n}^A U_{1,n}^B \\ U_{2,1}^A U_{2,1}^B & U_{2,2}^A U_{2,2}^B & \dots & U_{2,n}^A U_{2,n}^B \\ \vdots & \vdots & \ddots & \vdots \\ U_{m,1}^A U_{m,1}^B & U_{m,2}^A U_{m,2}^B & \dots & U_{m,n}^A U_{m,n}^B \end{bmatrix} \end{array}$$

$$U_A \otimes U_B \rightarrow \begin{bmatrix} U_{1,1}^A & \dots \end{bmatrix}$$

Entanglement

Any $|x\rangle \in \mathcal{H}_{AB}$

$$|x\rangle = \sum c_{a,b} |a\rangle \otimes |b\rangle$$

Product basis

- Unitary
pres

- If $\hat{P}$

$$p(t) =$$

- Given obs

$$\lambda_2 = \lambda$$
$$\lambda_k \neq$$

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$$|x\rangle = \sum c_{a,b} \underbrace{|a\rangle \otimes |b\rangle}_{\text{Product basis}}$$

In special cases $\exists$ exist states in $\mathcal{H}_{AB}$

which have the form $|w\rangle \in \mathcal{H}_{AB}$

$$|w\rangle = |\xi\rangle \otimes |\eta\rangle$$

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Defⁿ

A pure state $|\chi\rangle \in \mathcal{H}_{AB}$
that can be expressed
as $|\chi\rangle = |\alpha\rangle \otimes |\beta\rangle$, for
some $|\alpha\rangle \in \mathcal{H}_A$, $|\beta\rangle \in \mathcal{H}_B$,
called a product state

Defⁿ

A pure state $|\chi\rangle \in \mathcal{H}_{AB}$
that can be expressed
as $|\chi\rangle = |\alpha\rangle \otimes |\beta\rangle$, for
some $|\alpha\rangle \in \mathcal{H}_A$, $|\beta\rangle \in \mathcal{H}_B$,
is called a product state,
or factorable state or an

unentangled state;
otherwise the state $|\chi\rangle$
is entangled.

Entanglement

Any $|x\rangle \in \mathcal{H}_{AB}$

$$|x\rangle = \sum c_{a,b} \underbrace{|a\rangle \otimes |b\rangle}_{\text{Product basis}}$$

in special cases $\exists$ exist states in $\mathcal{H}_{AB}$

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