Title: Quantum Theory - Lecture 1

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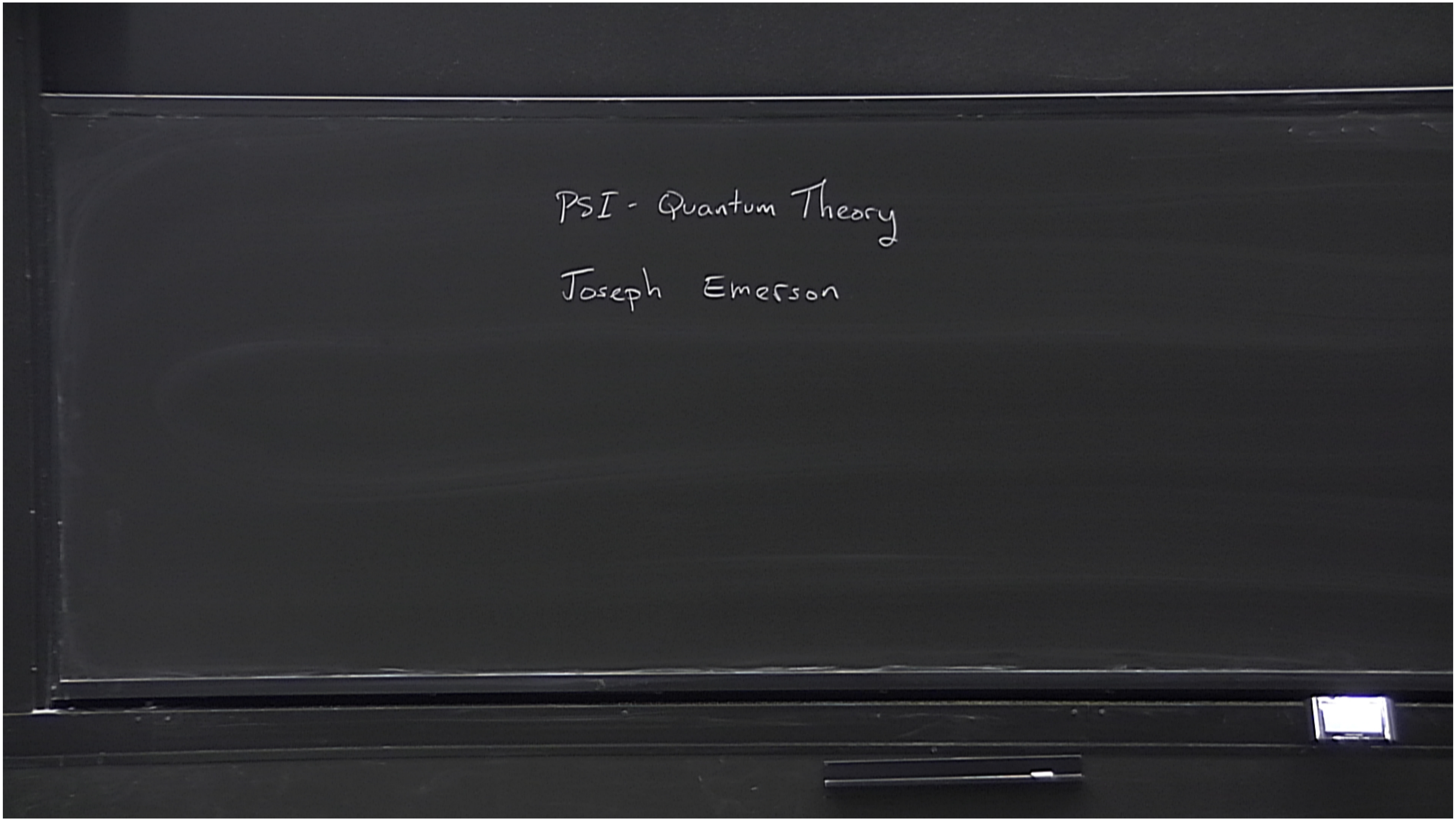
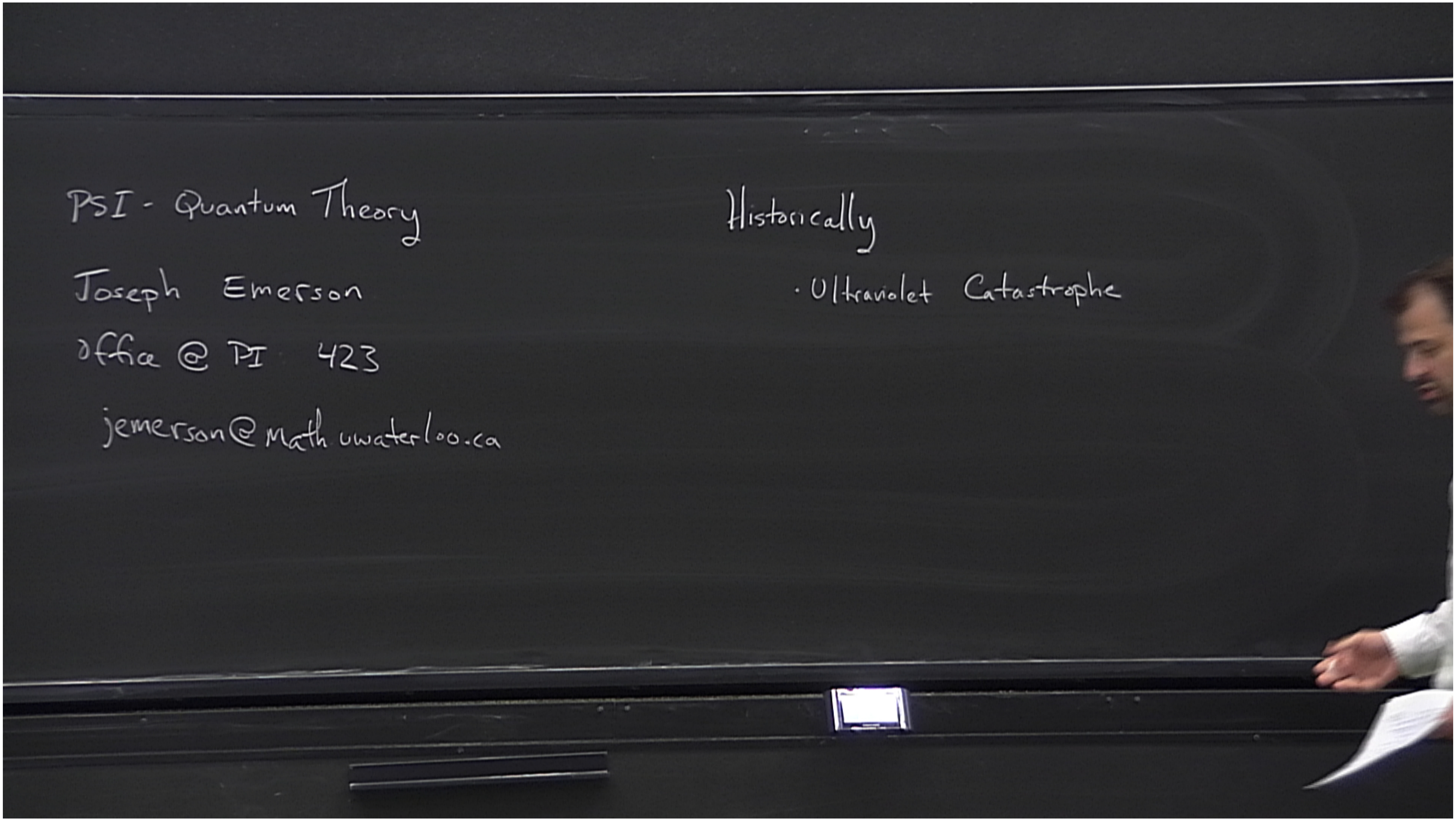
URL: http://pirsa.org/12090000

Abstract:



PSI - Quantum Theory

Joseph Emerson



PSI - Quantum Theory

Joseph Emerson

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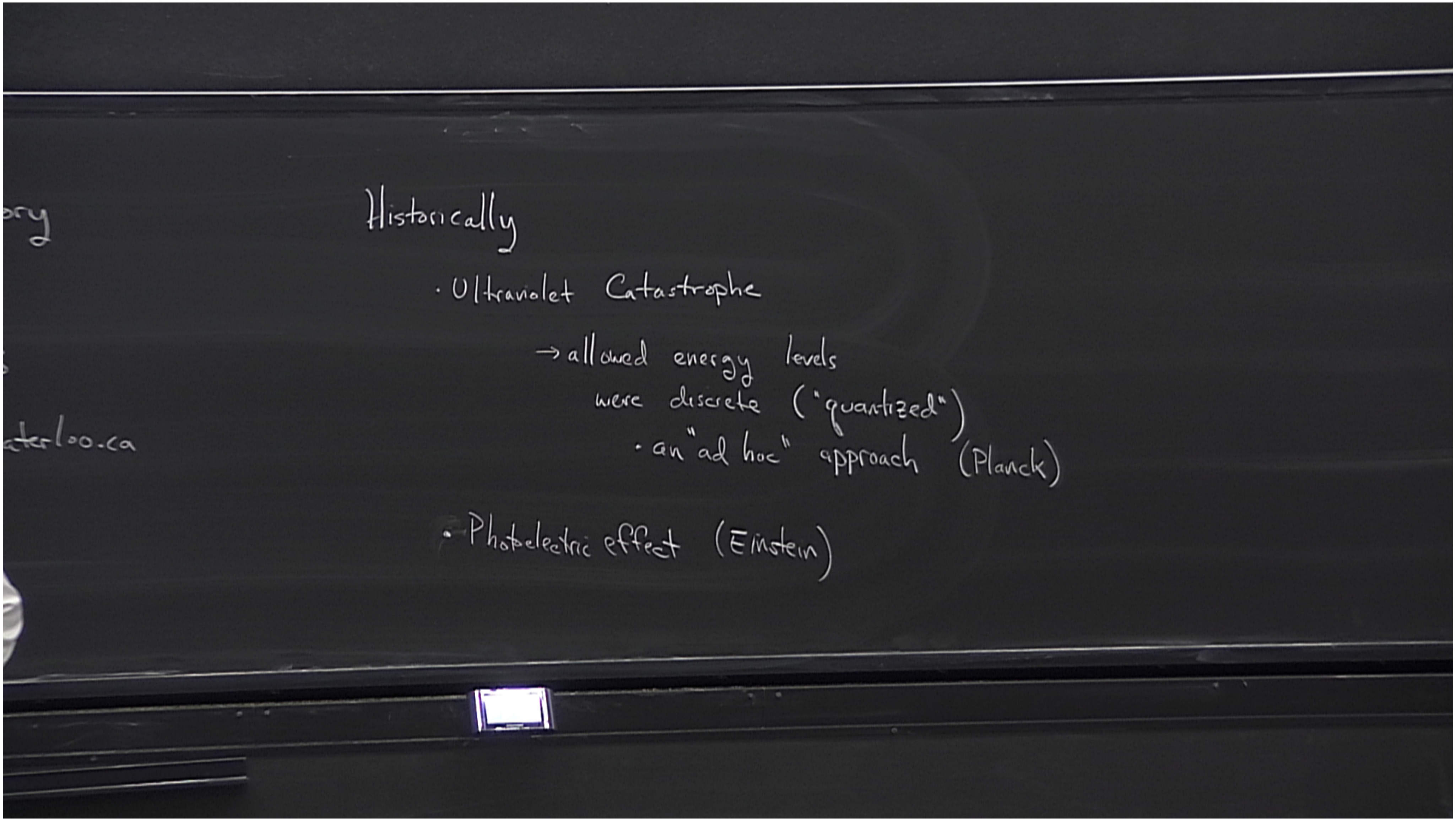
Historically

- Ultraviolet Catastrophe



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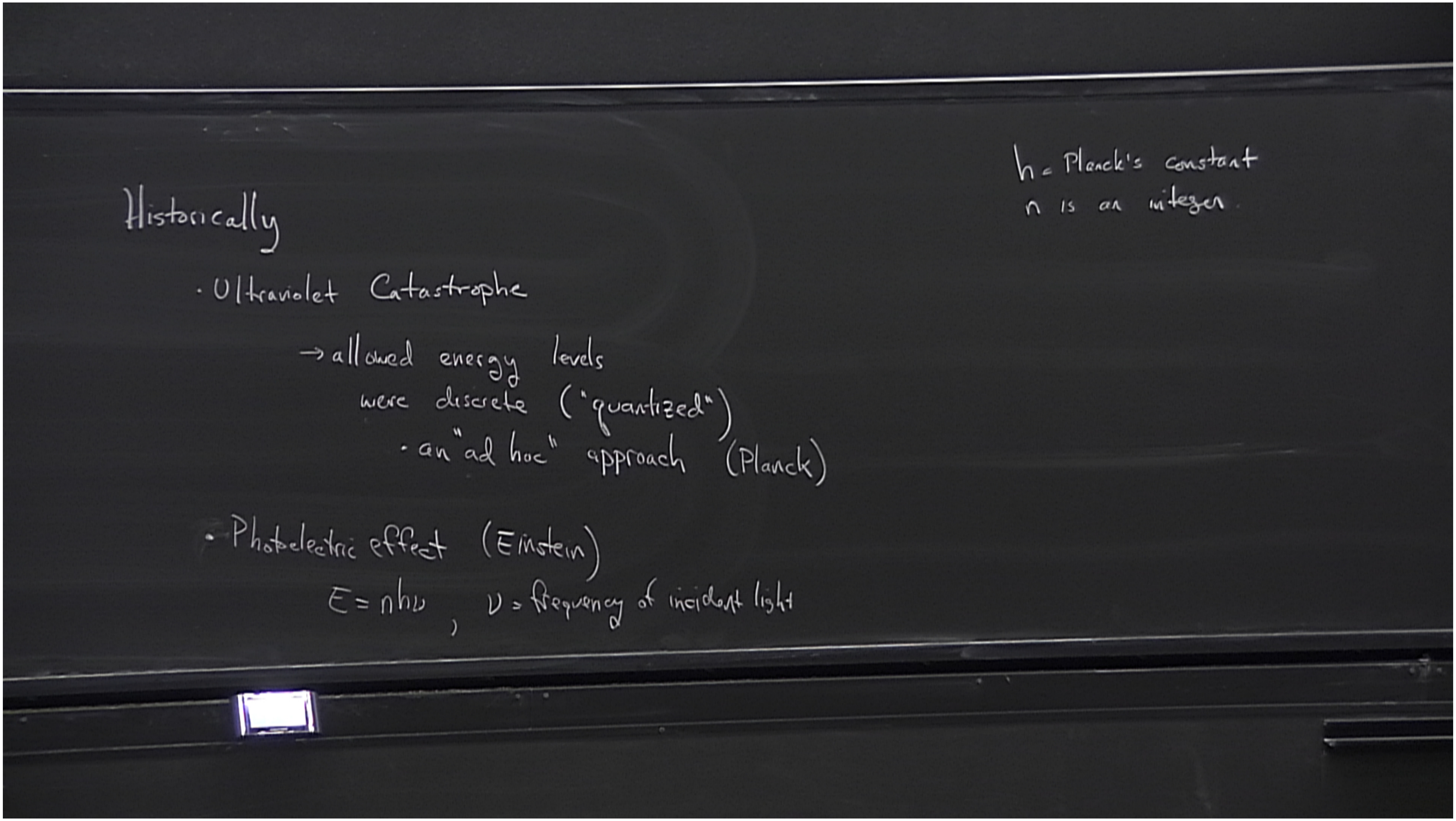
Historically

- Ultraviolet Catastrophe
  - → allowed energy levels were discrete ("quantized")
    - an "ad hoc" approach (Planck)
- Photoelectric effect (Einstein)



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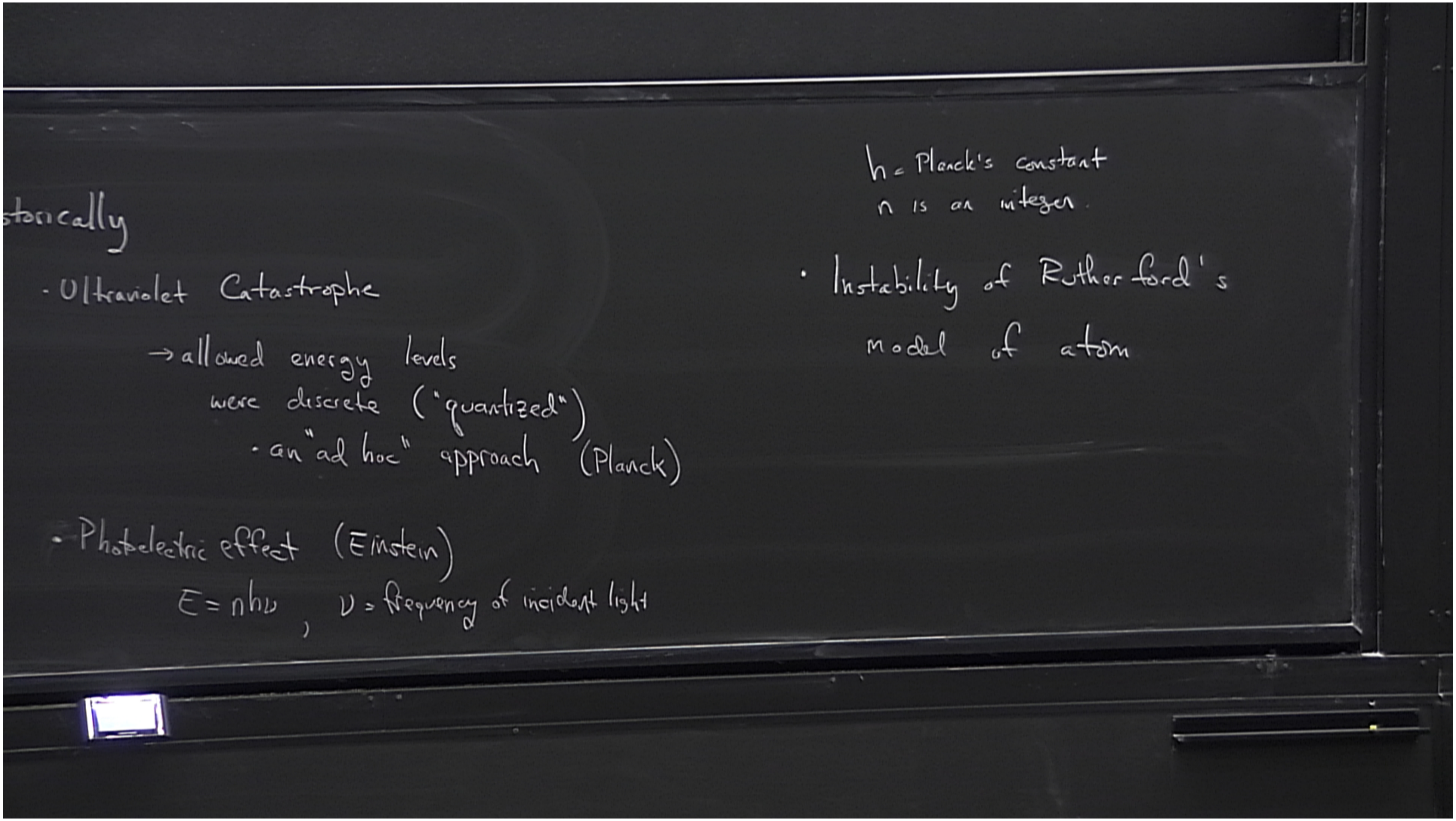
  - $E = nh\nu$, ν = frequency of incident light

h = Planck's constant

n is an integer.



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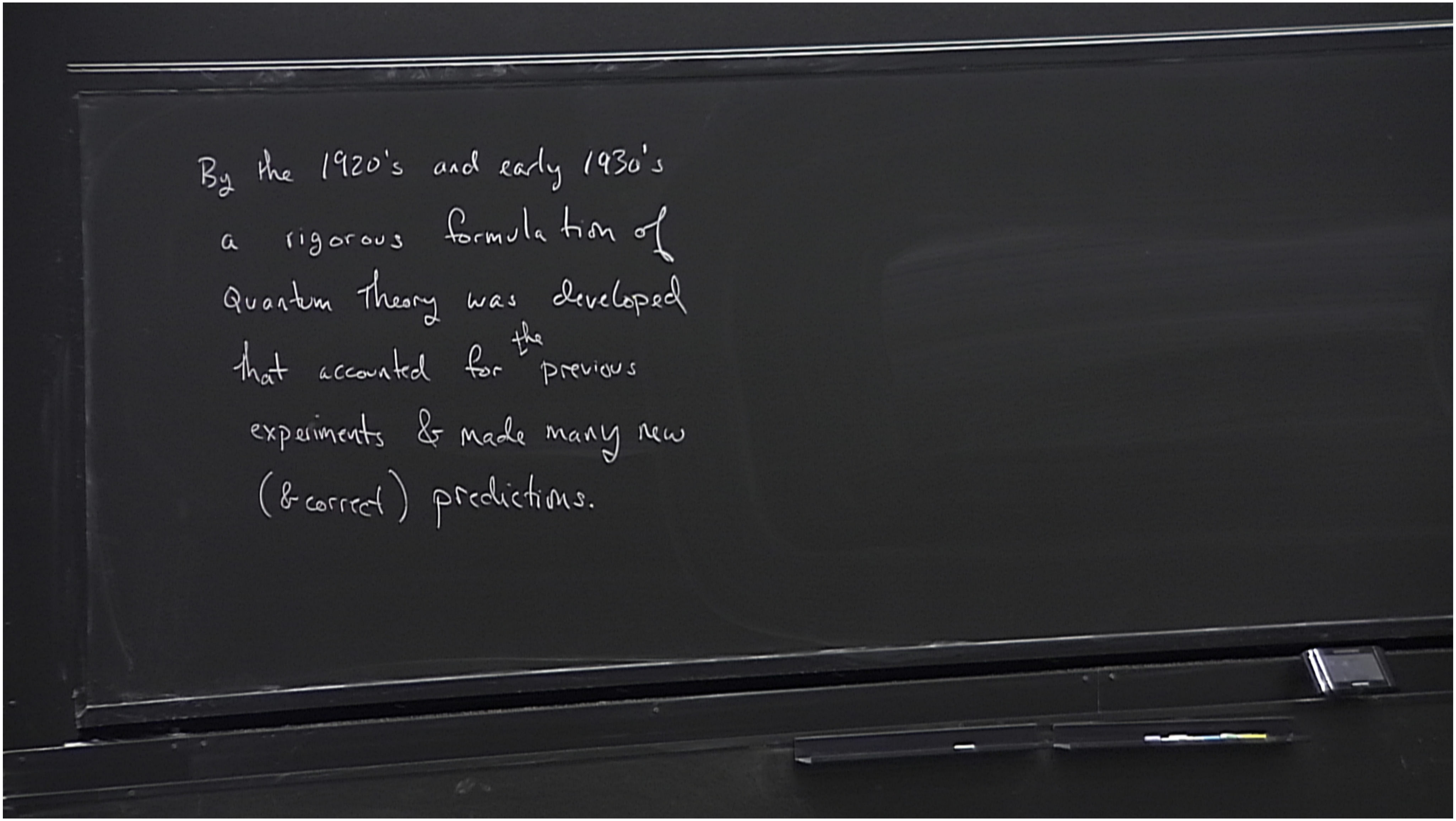
- Instability of Rutherford's model of atom



By the 1920's and early 1930's
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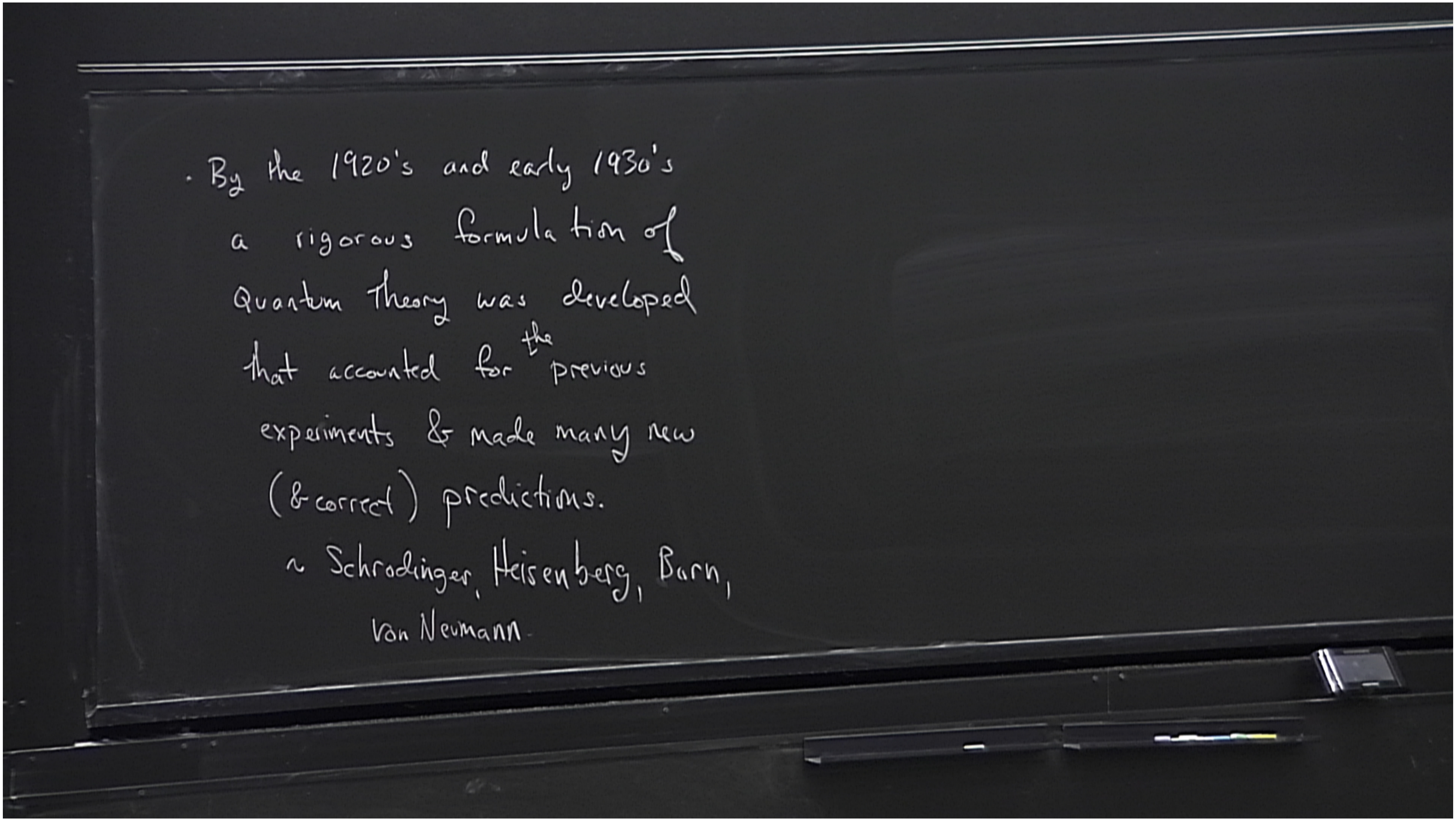

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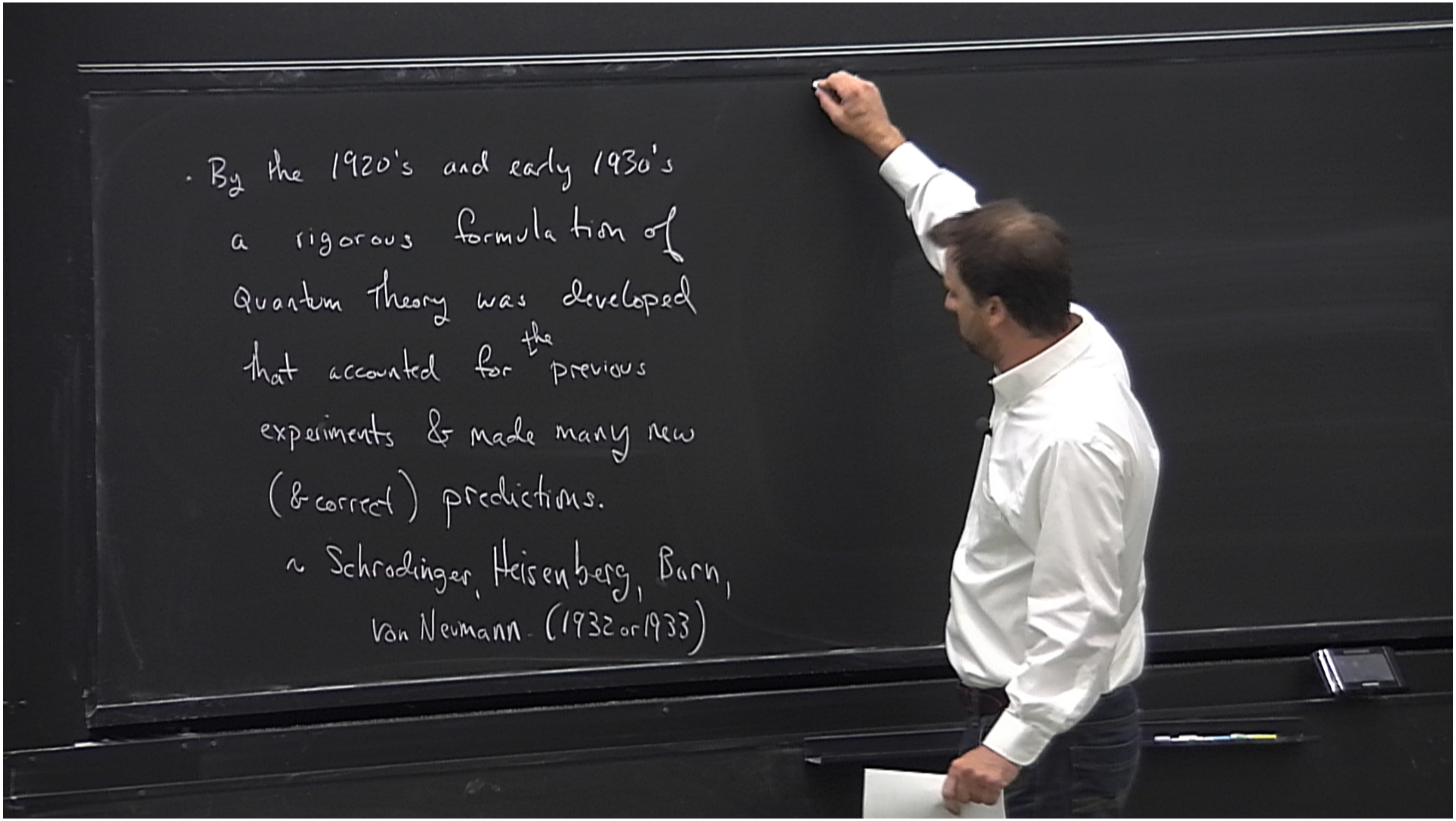

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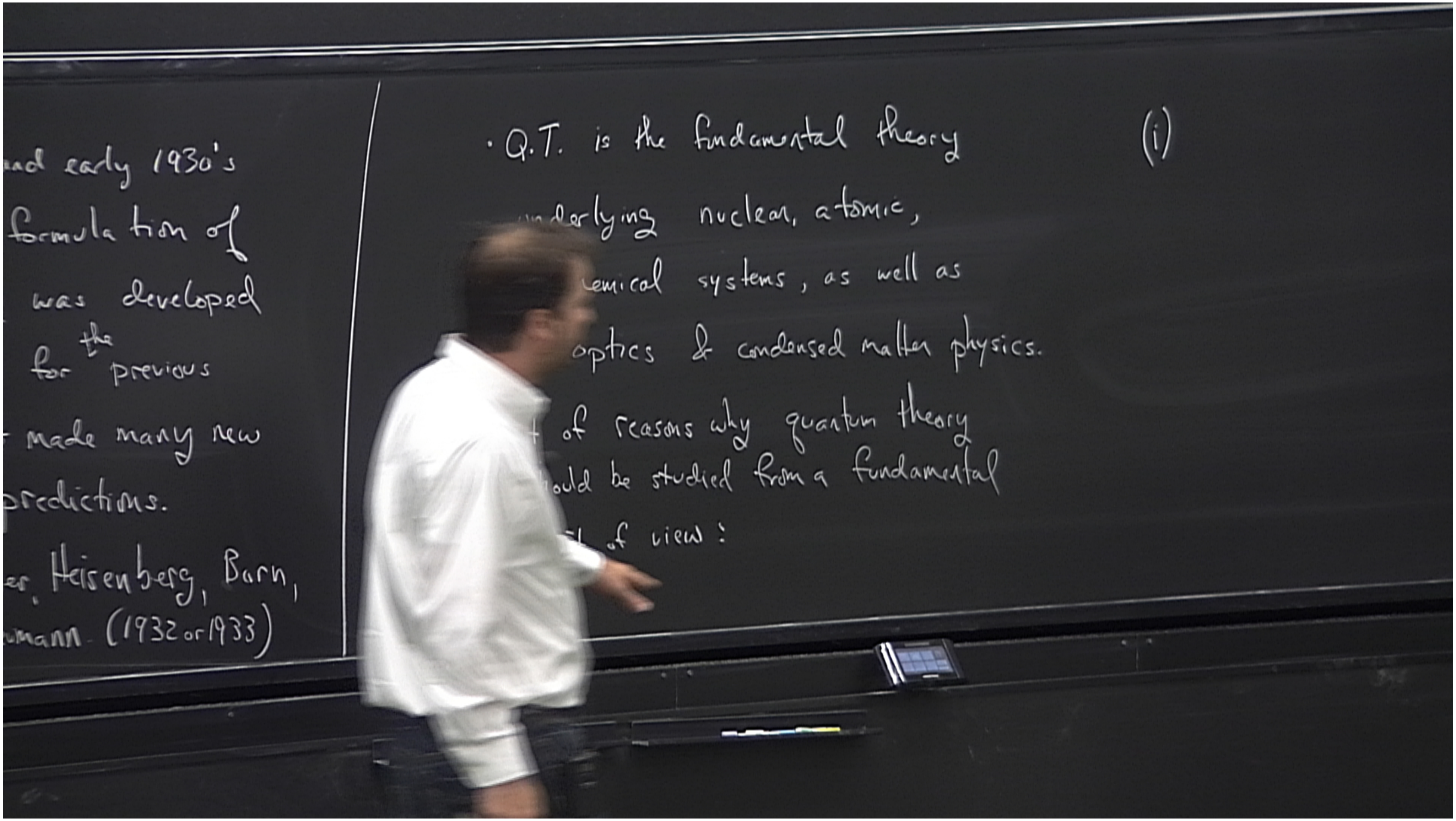

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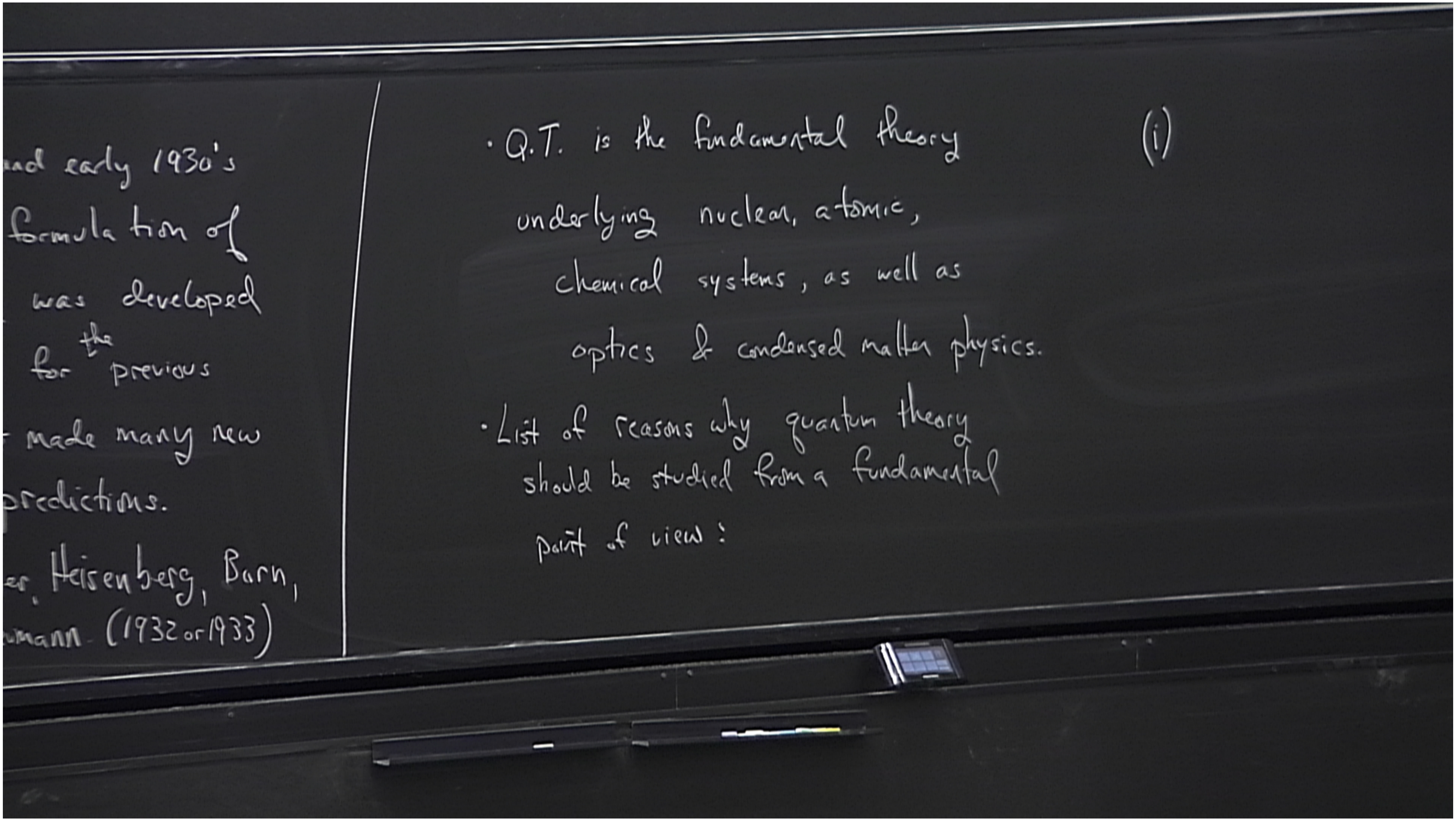

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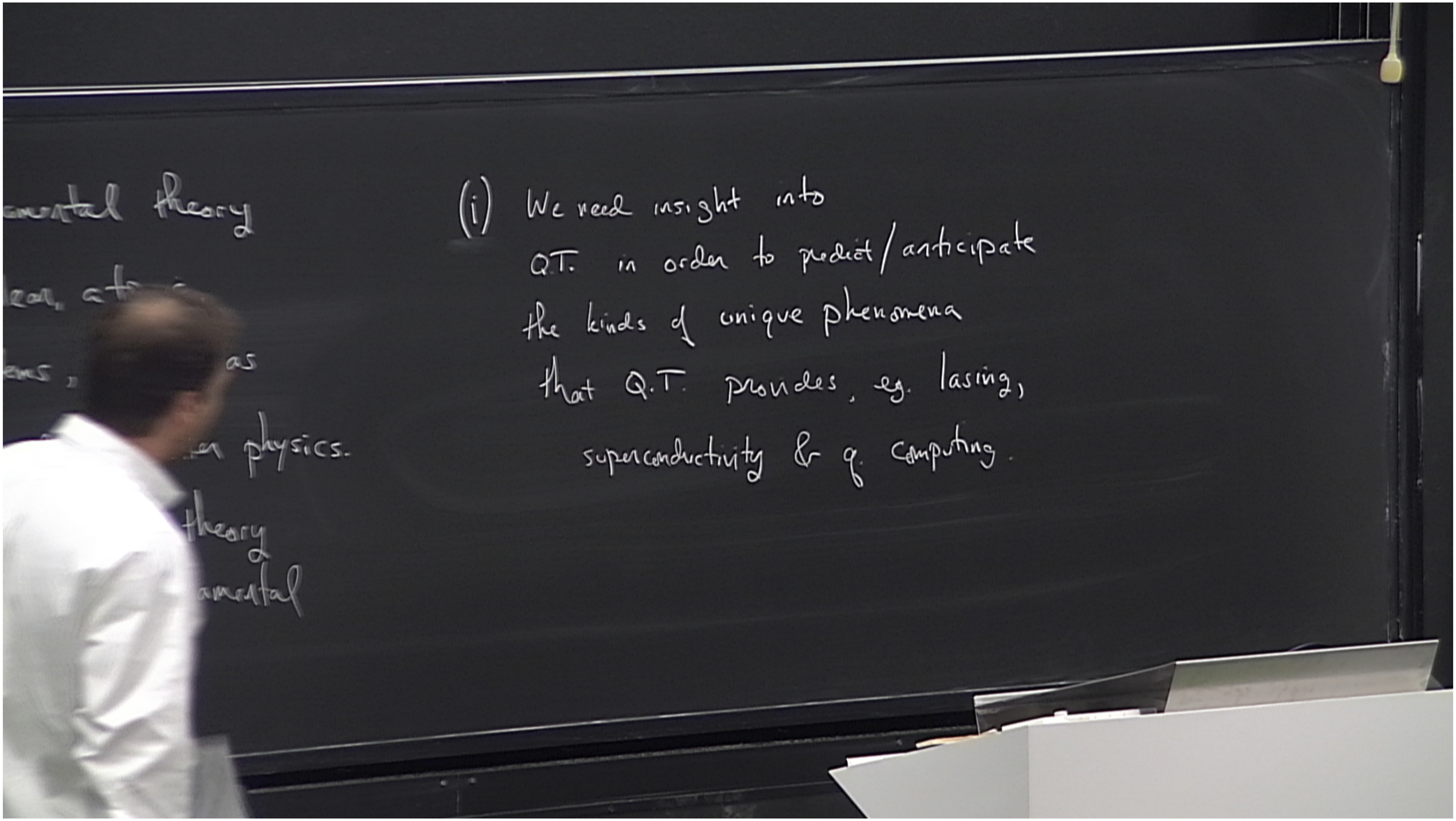
- Q.T. is the fundamental theory underlying nuclear, atomic, chemical systems, as well as optics & condensed matter physics.
- List of reasons why quantum theory should be studied from a fundamental point of view:



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(i) We need insight into
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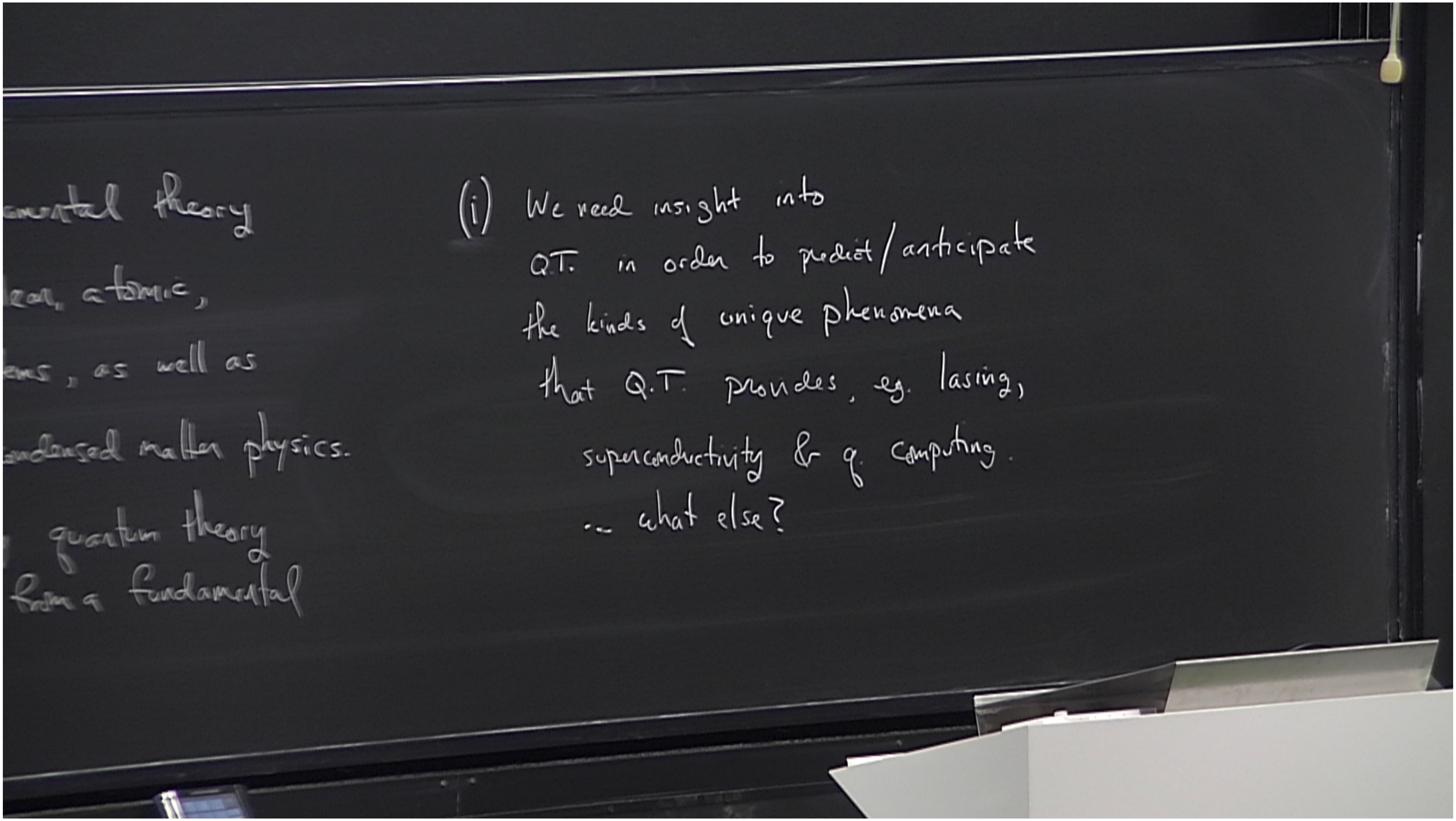

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(i) We need insight into Q.T. in order to predict/anticipate the kinds of unique phenomena that Q.T. provides, e.g. lasing, superconductivity & q. computing.
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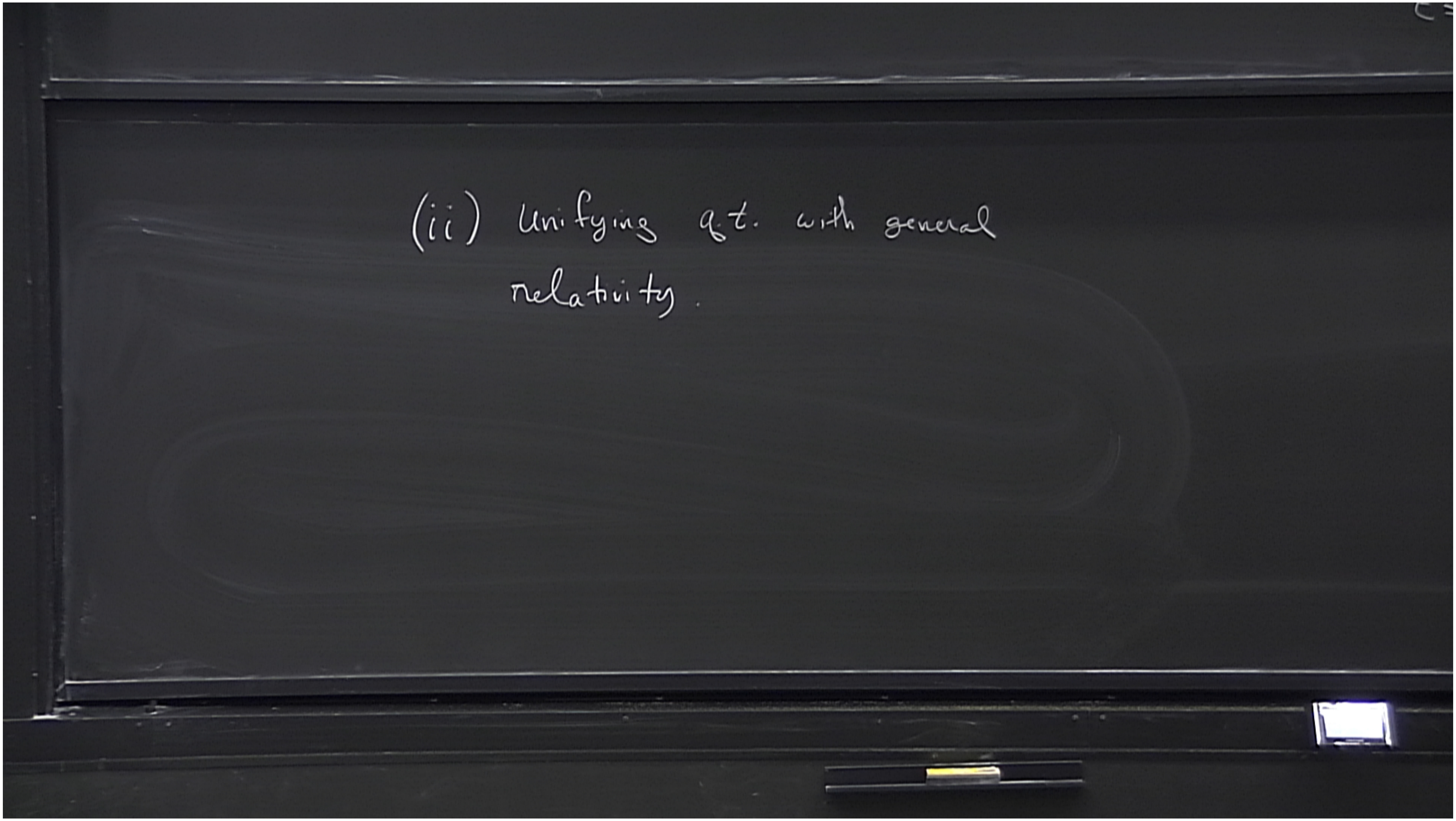
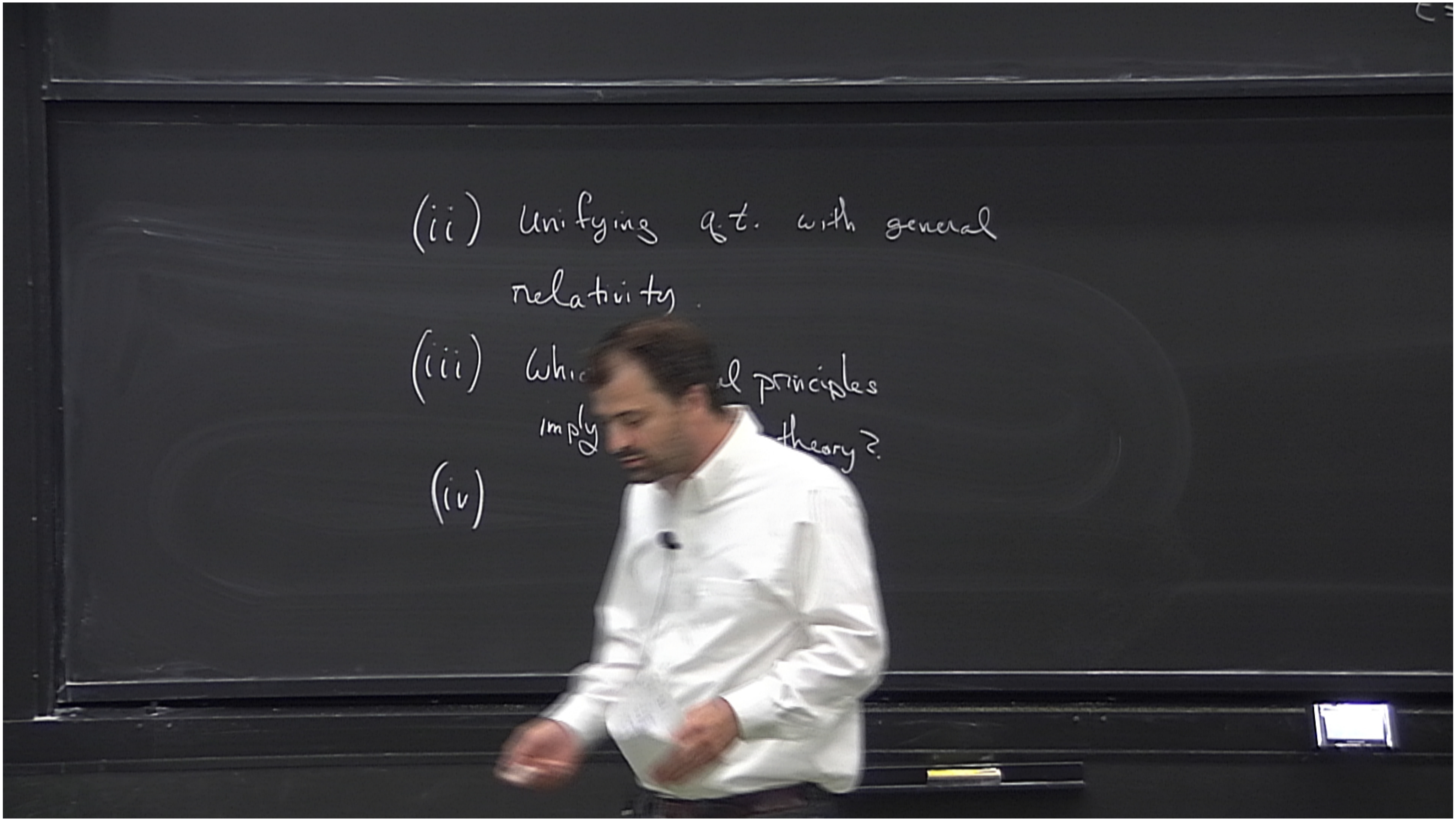


(ii) Unifying q.t. with general relativity.



(ii) Unifying q.t. with general relativity.

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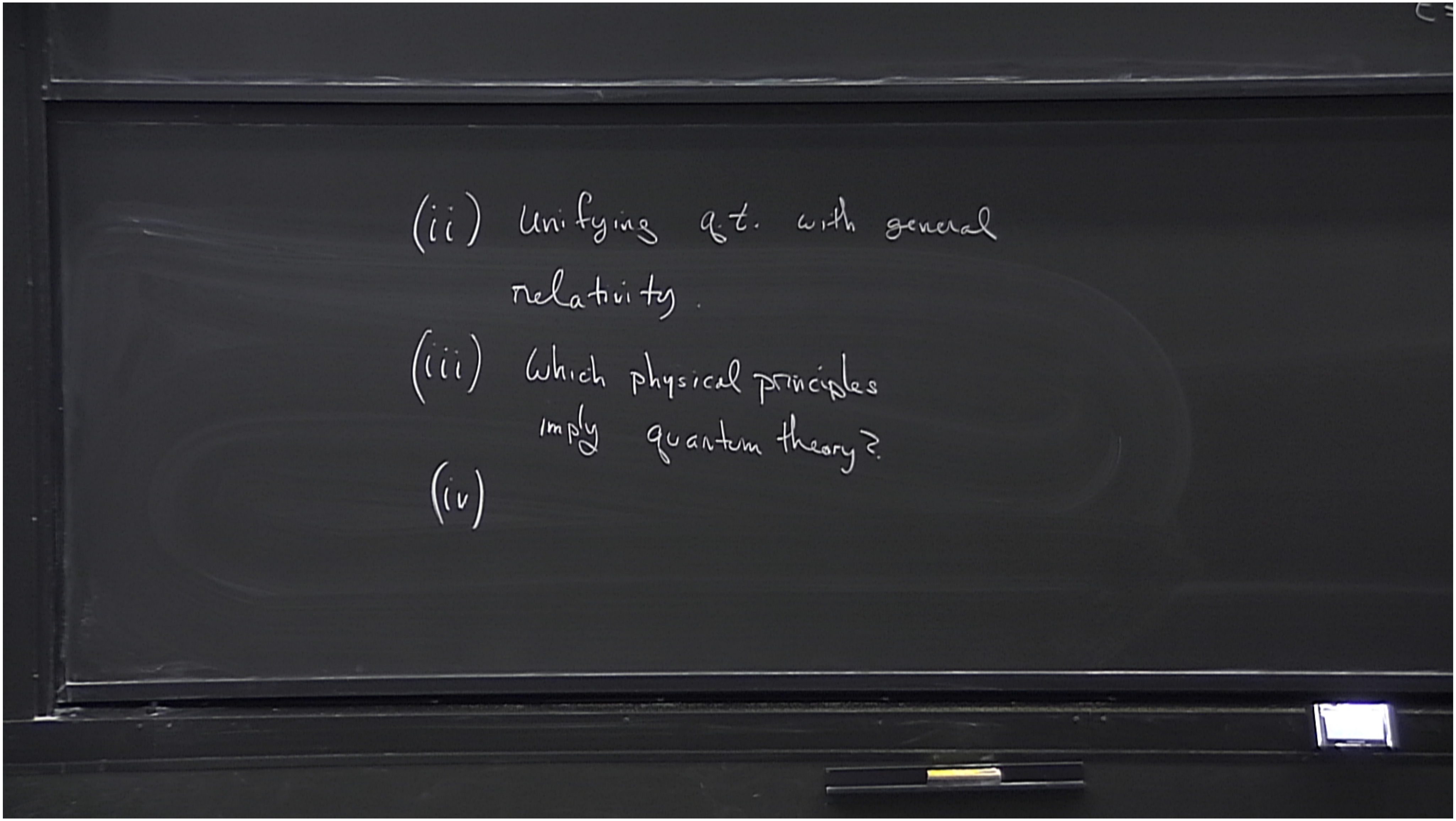

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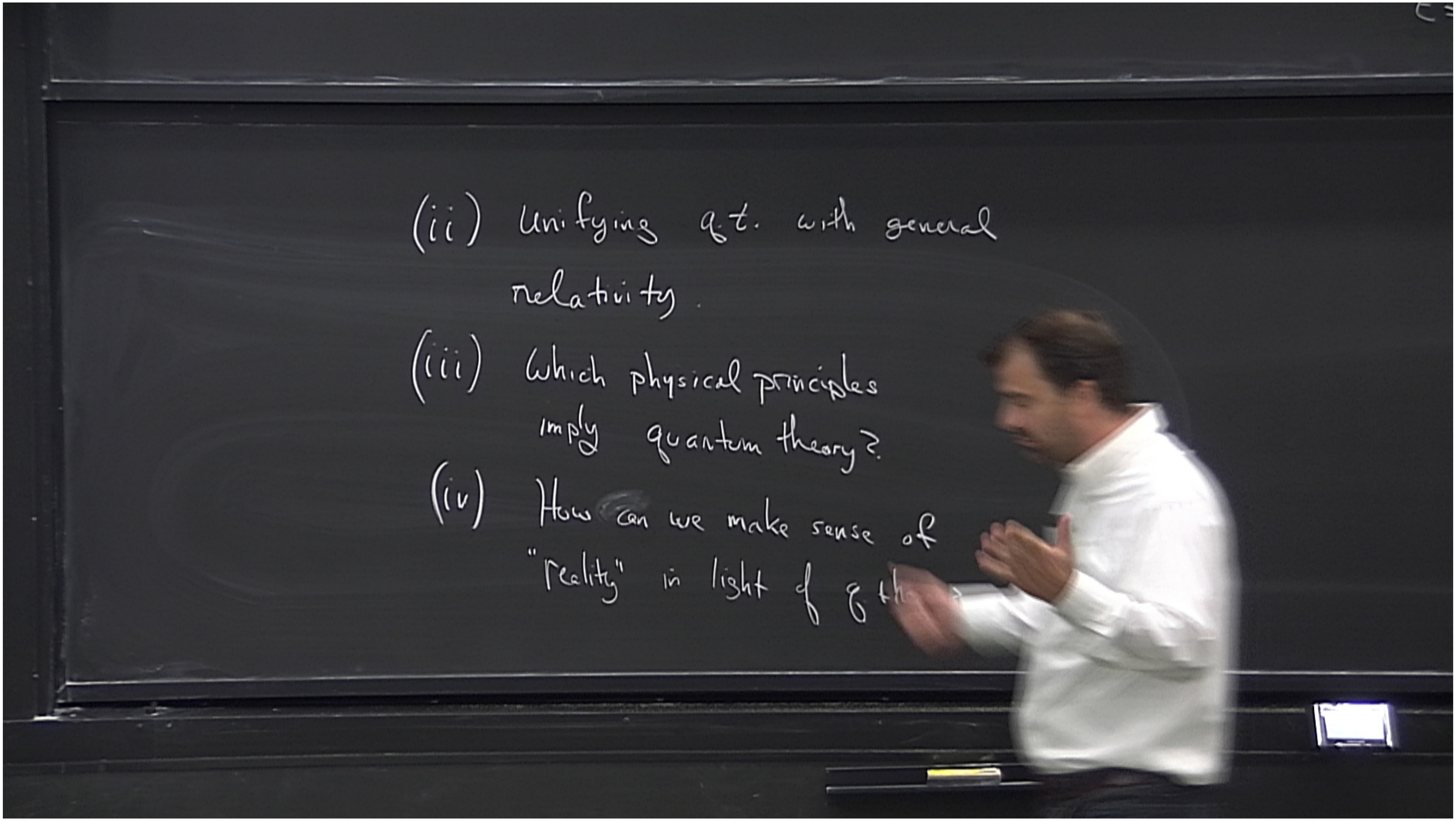

(ii) Unifying q.t. with general relativity.

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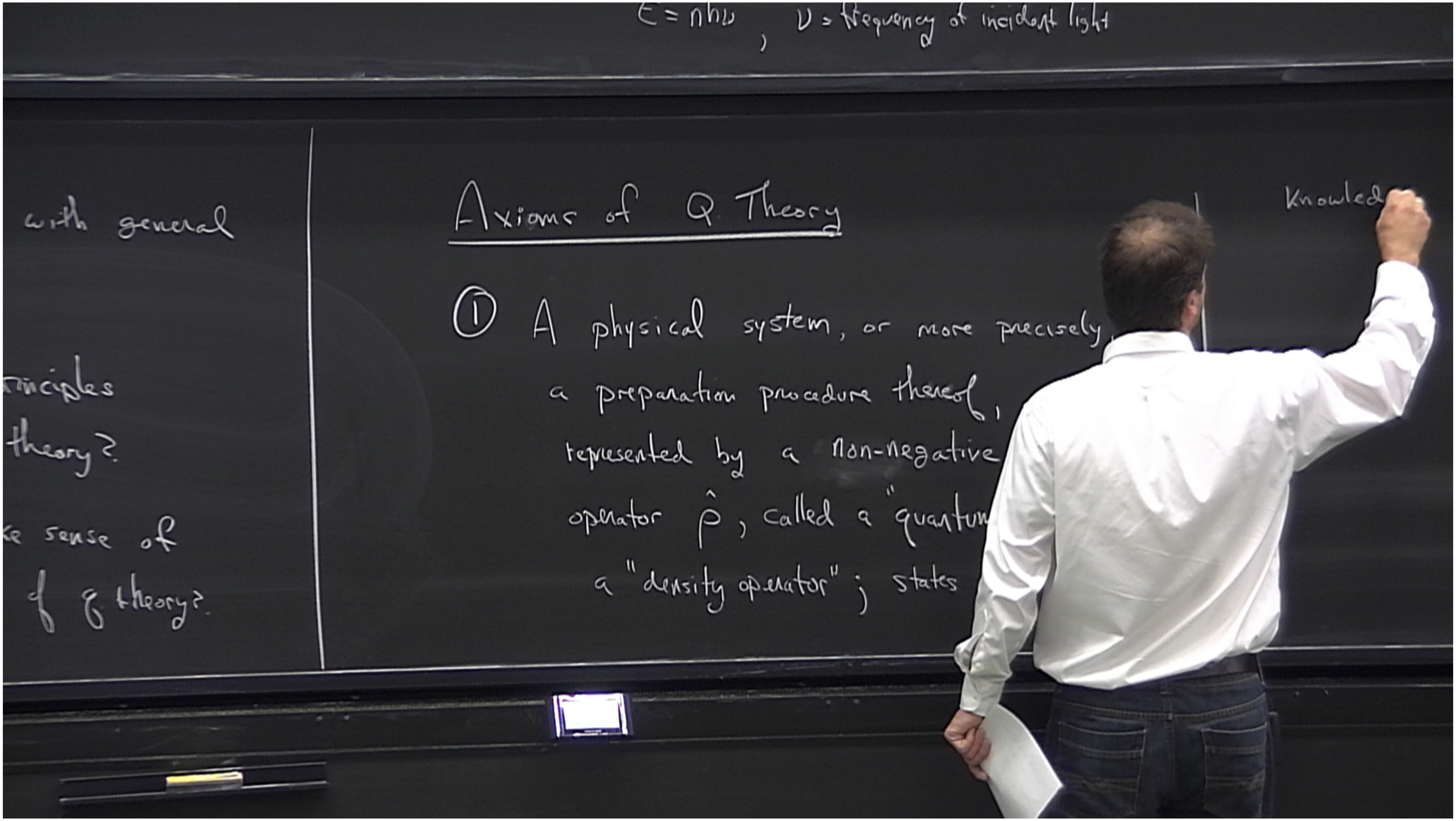
(iv) How can we make sense of "reality" in light of q th



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Axioms of Q Theory

① A physical system, or more precisely,
a preparation procedure thereof,
represented by a non-negative
operator $\hat{\rho}$, called a "quantum
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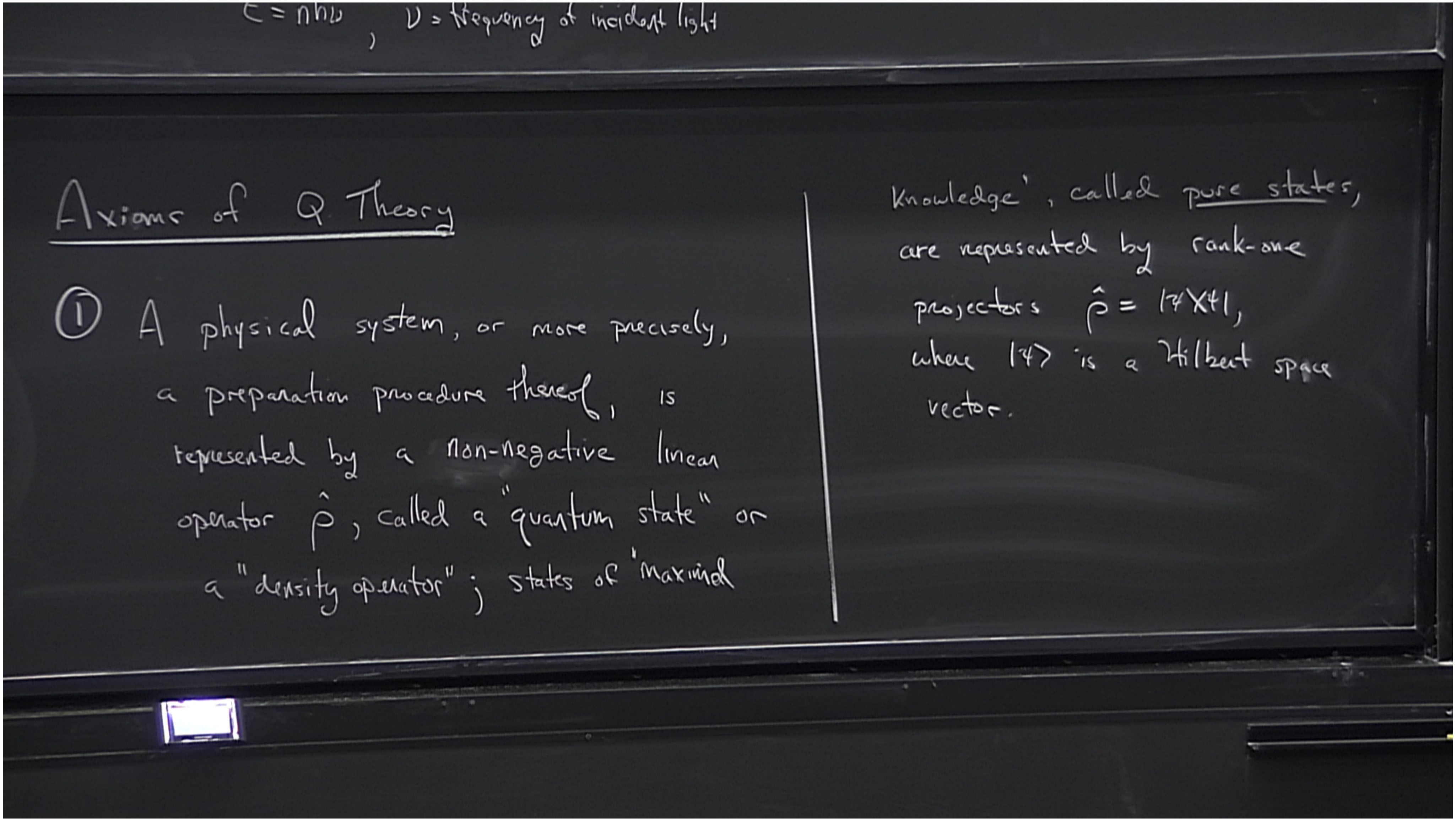
Knowled



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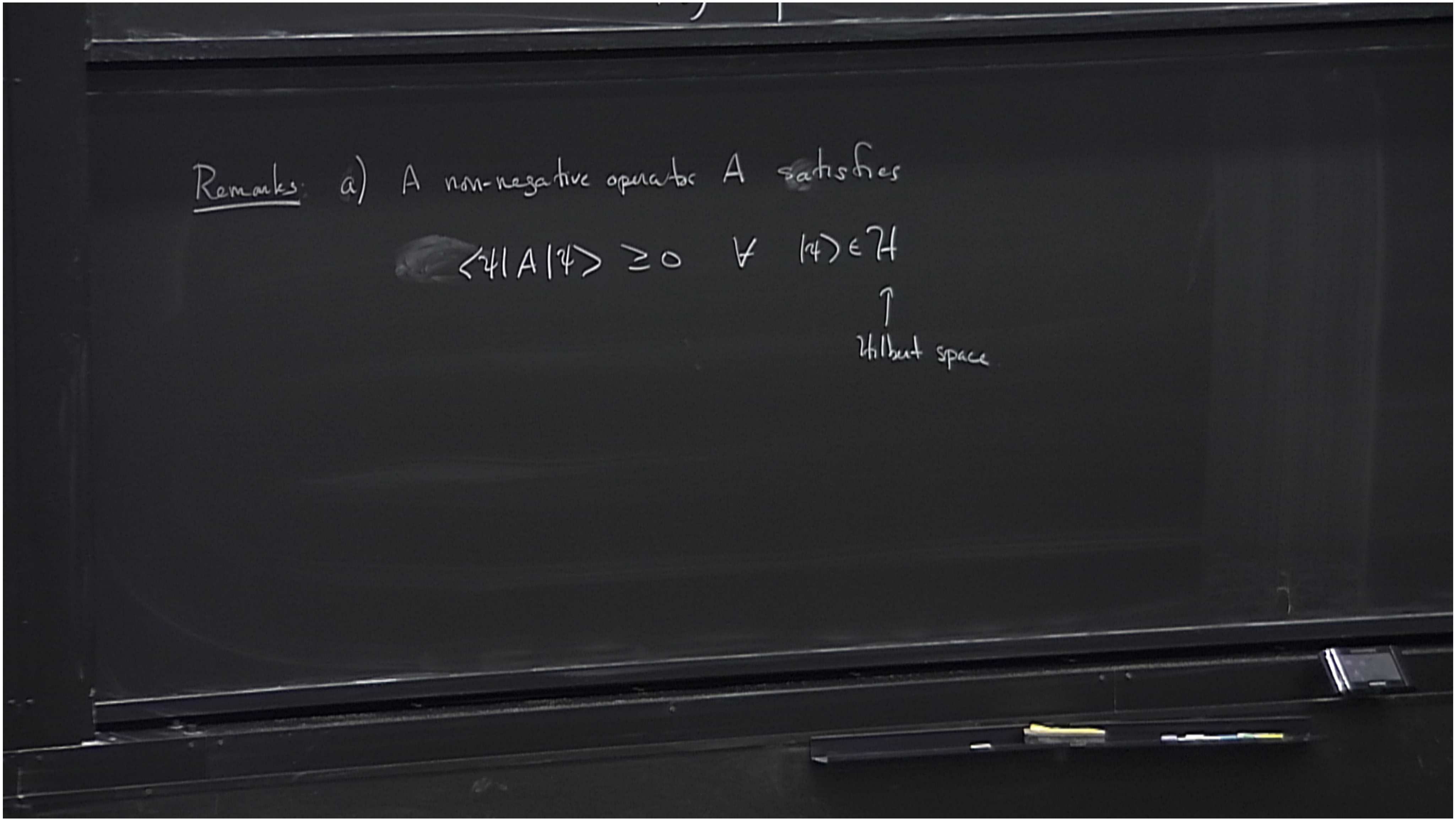
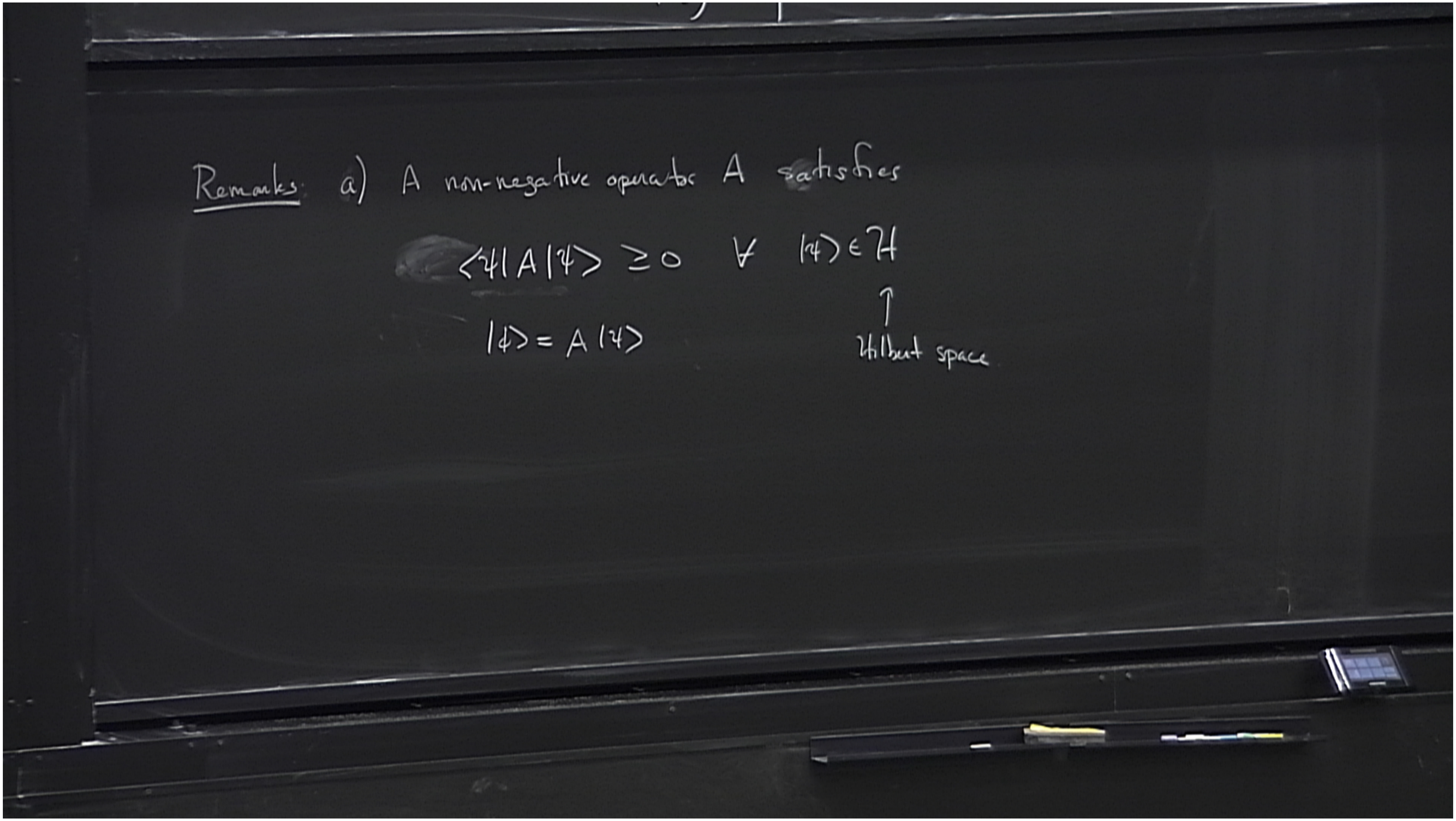
Axioms of Q Theory

① A physical system, or more precisely, a preparation procedure thereof, is represented by a non-negative linear operator $\hat{\rho}$, called a "quantum state" or a "density operator"; states of 'maximal knowledge', called pure states, are represented by rank-one projectors $\hat{\rho} = |\psi\rangle\langle\psi|$, where $|\psi\rangle$ is a Hilbert space vector.



Remarks: a) A non-negative operator A satisfies

$$\langle \psi | A | \psi \rangle \geq 0 \quad \forall \quad |\psi\rangle \in \underset{\substack{\uparrow \\ \text{Hilbert space}}}{\mathcal{H}}$$



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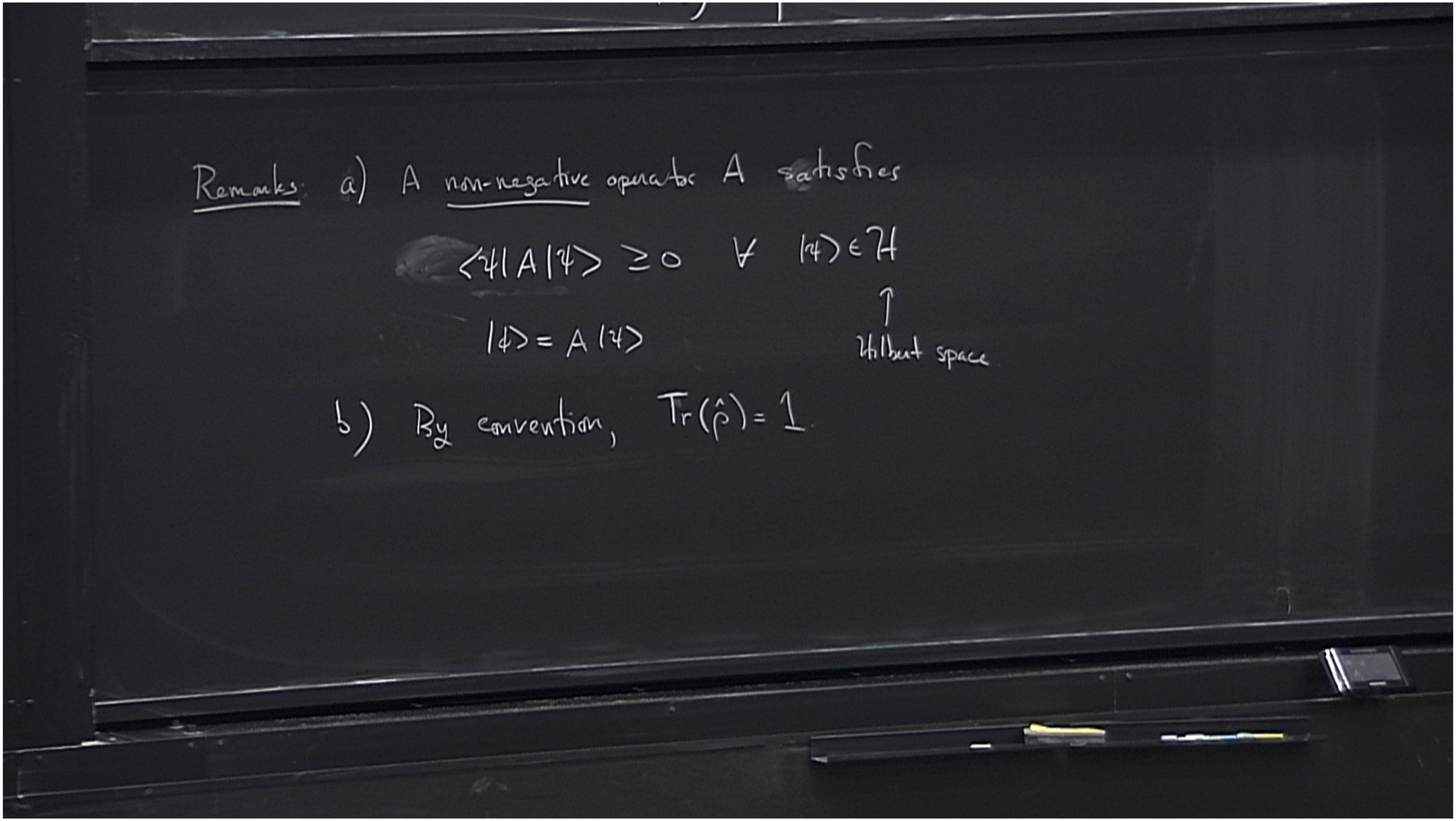
$$|\phi\rangle = A|\psi\rangle$$



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↑ Hilbert space

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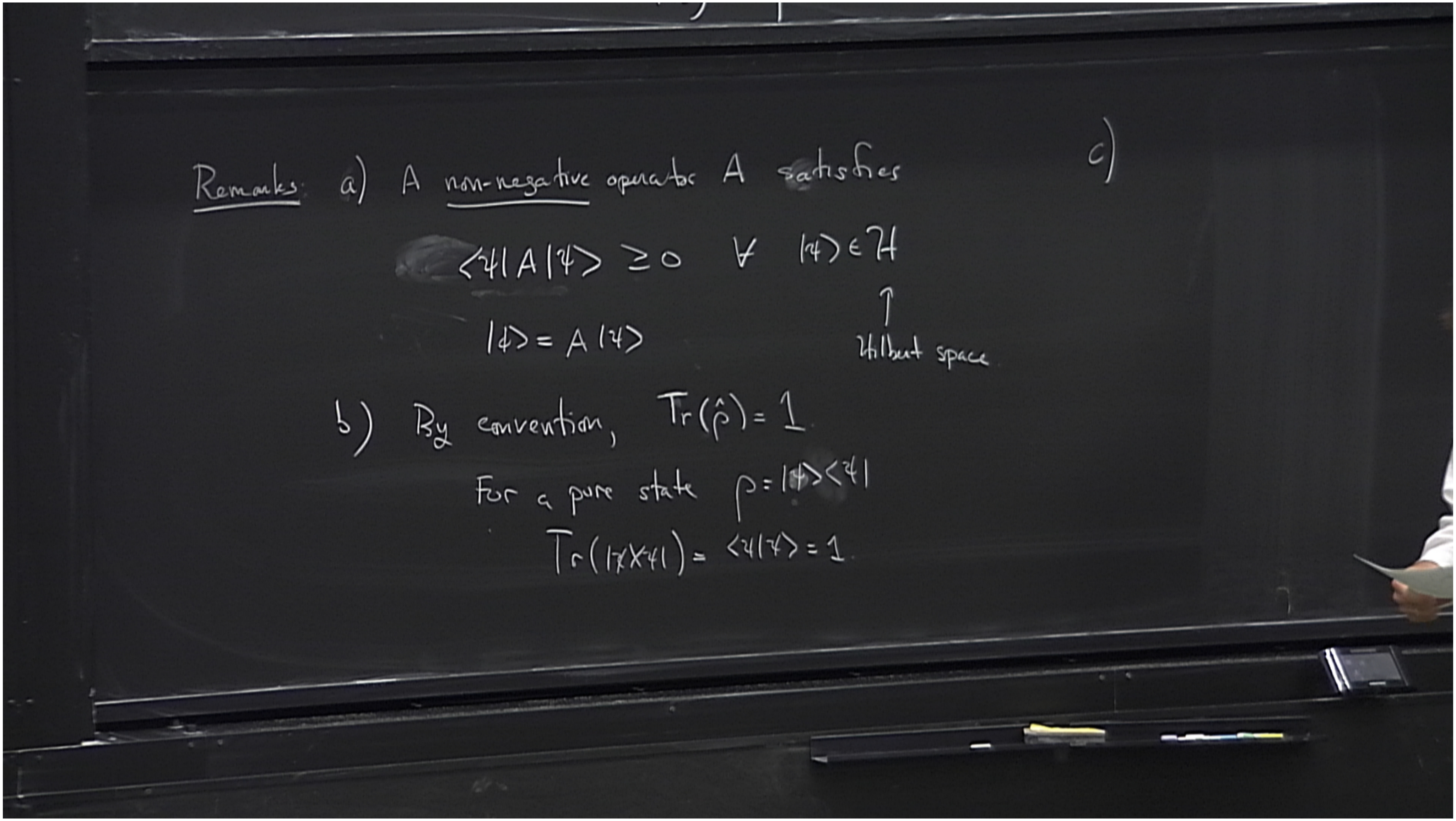
b) By convention, $\mathrm{Tr}(\hat{\rho}) = 1$.



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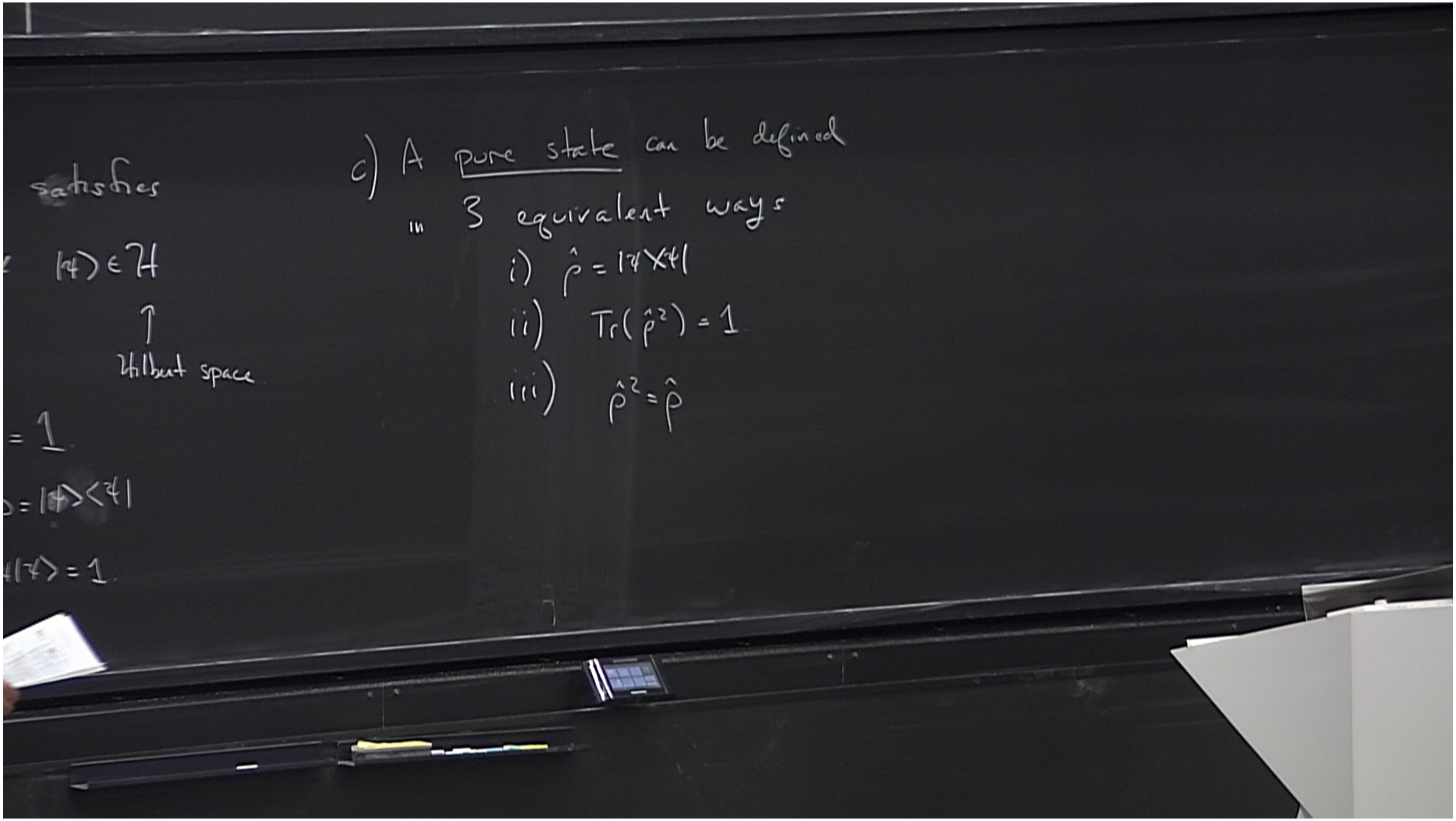
For a pure state $\rho = |\psi\rangle\langle\psi|$

$$\mathrm{Tr}(|\psi\rangle\langle\psi|) = \langle\psi|\psi\rangle = 1.$$

c)



satisfies

$|\psi\rangle \in \mathcal{H}$

$\uparrow$

Hilbert space

$= \underline{1}$

$= |\psi\rangle\langle\psi|$

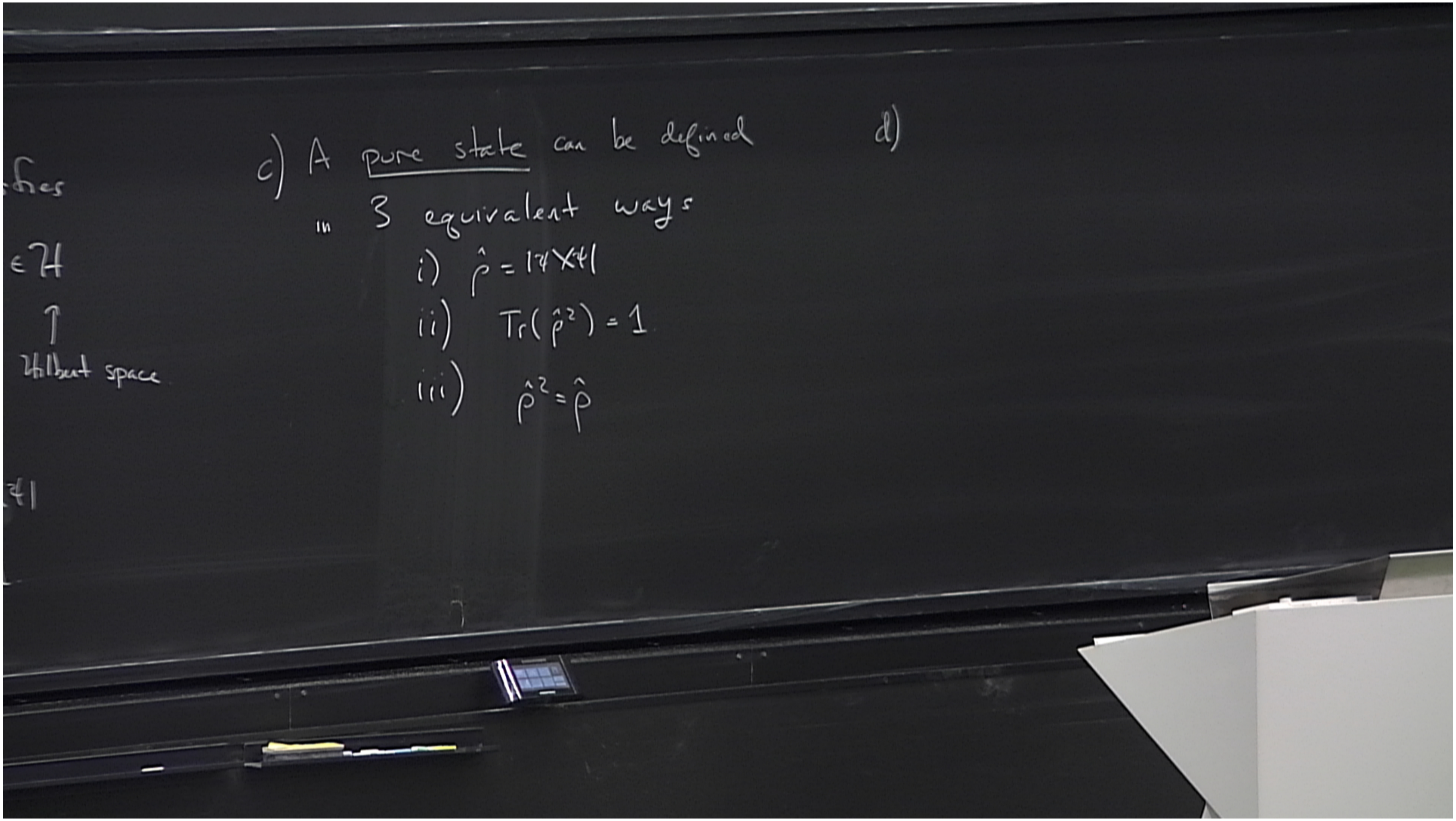
$\psi|\psi\rangle = 1$

c) A <u>pure state</u> can be defined in 3 equivalent ways

i) $\hat{\rho} = |\psi\rangle\langle\psi|$

ii) $\mathrm{Tr}(\hat{\rho}^2) = 1$

iii) $\hat{\rho}^2 = \hat{\rho}$



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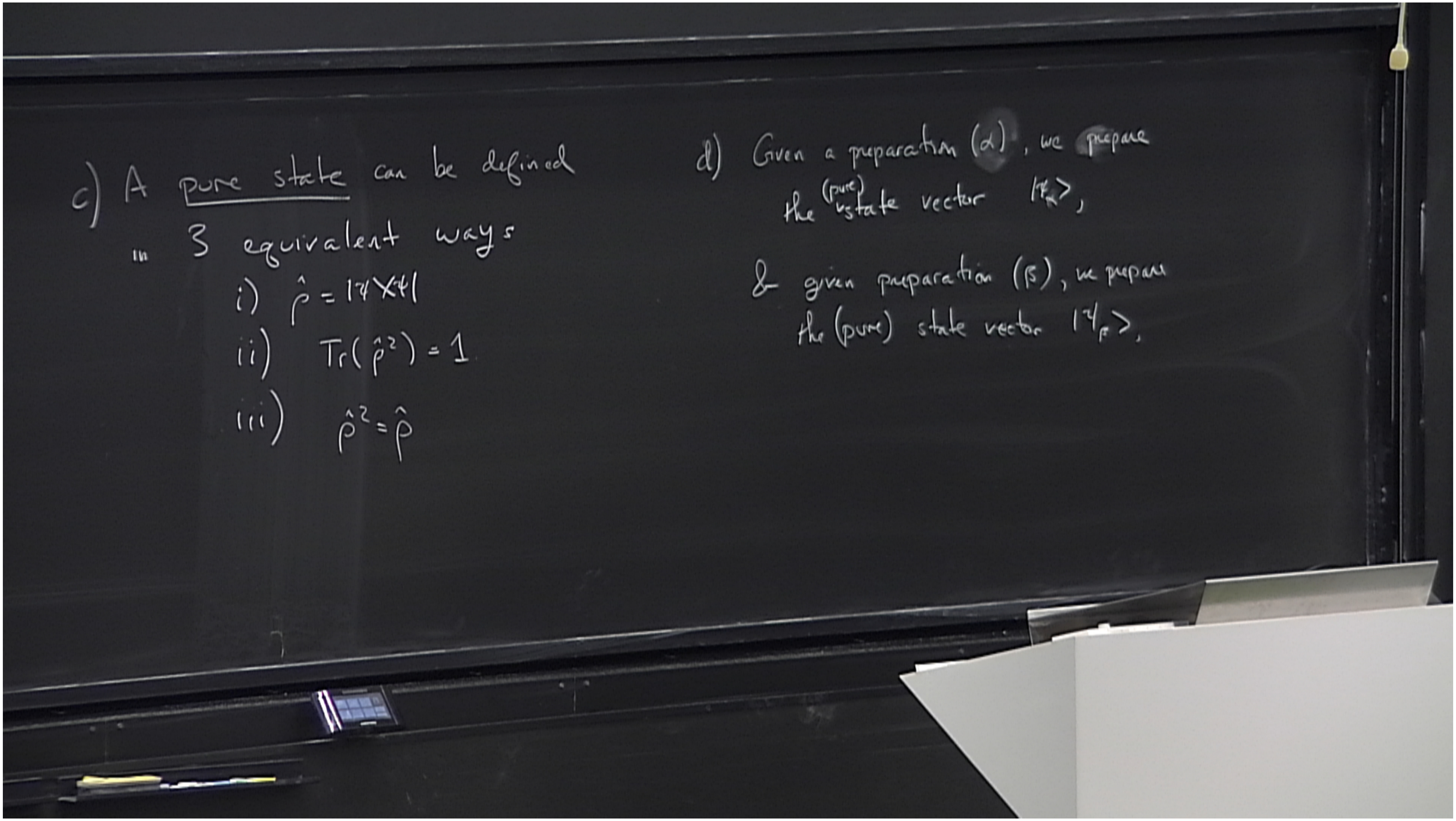

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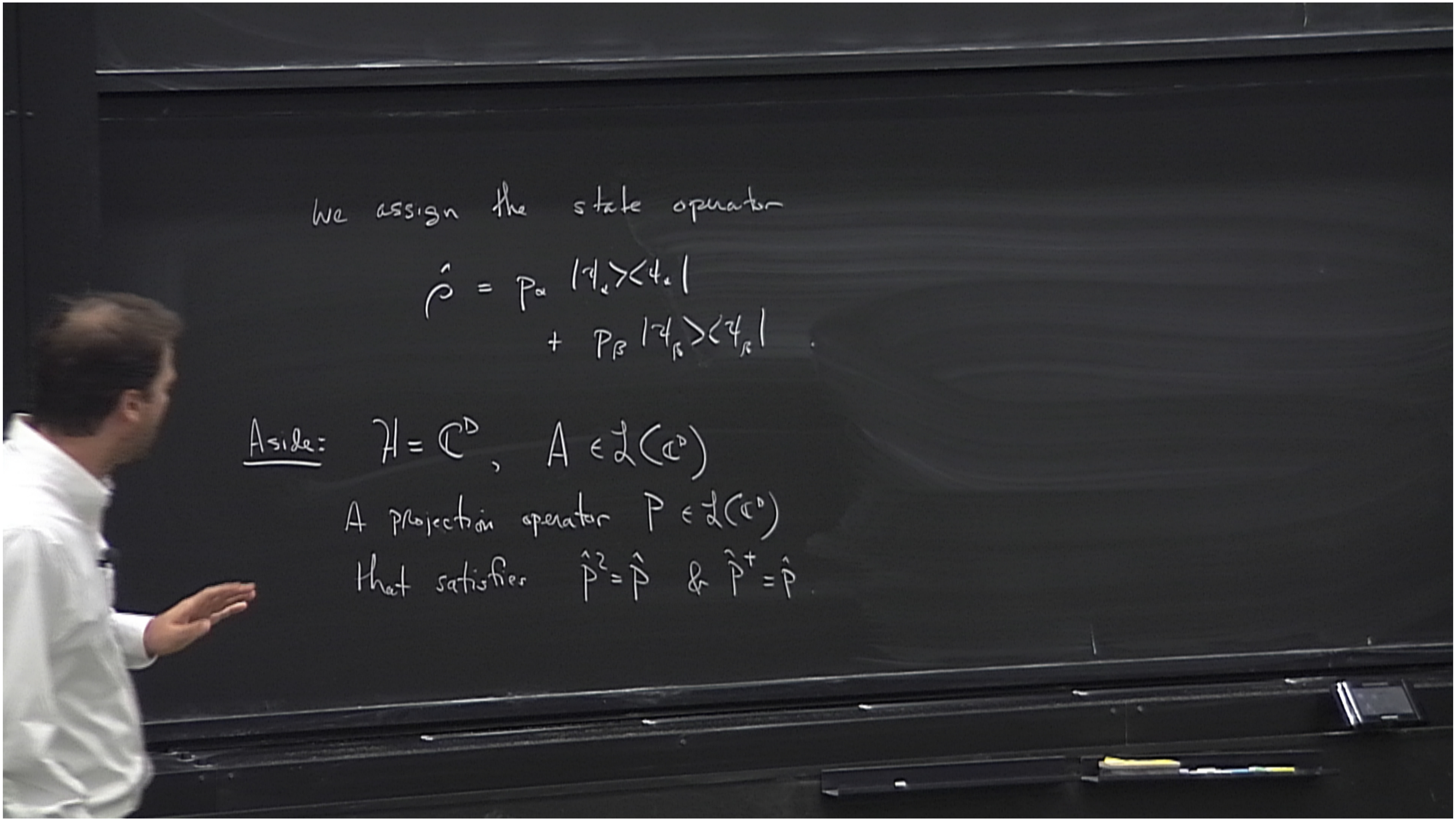
d) Given a preparation (α), we prepare the (pure) state vector $|\psi_\alpha\rangle$,

& given preparation (β), we prepare the (pure) state vector $|\psi_\beta\rangle$,



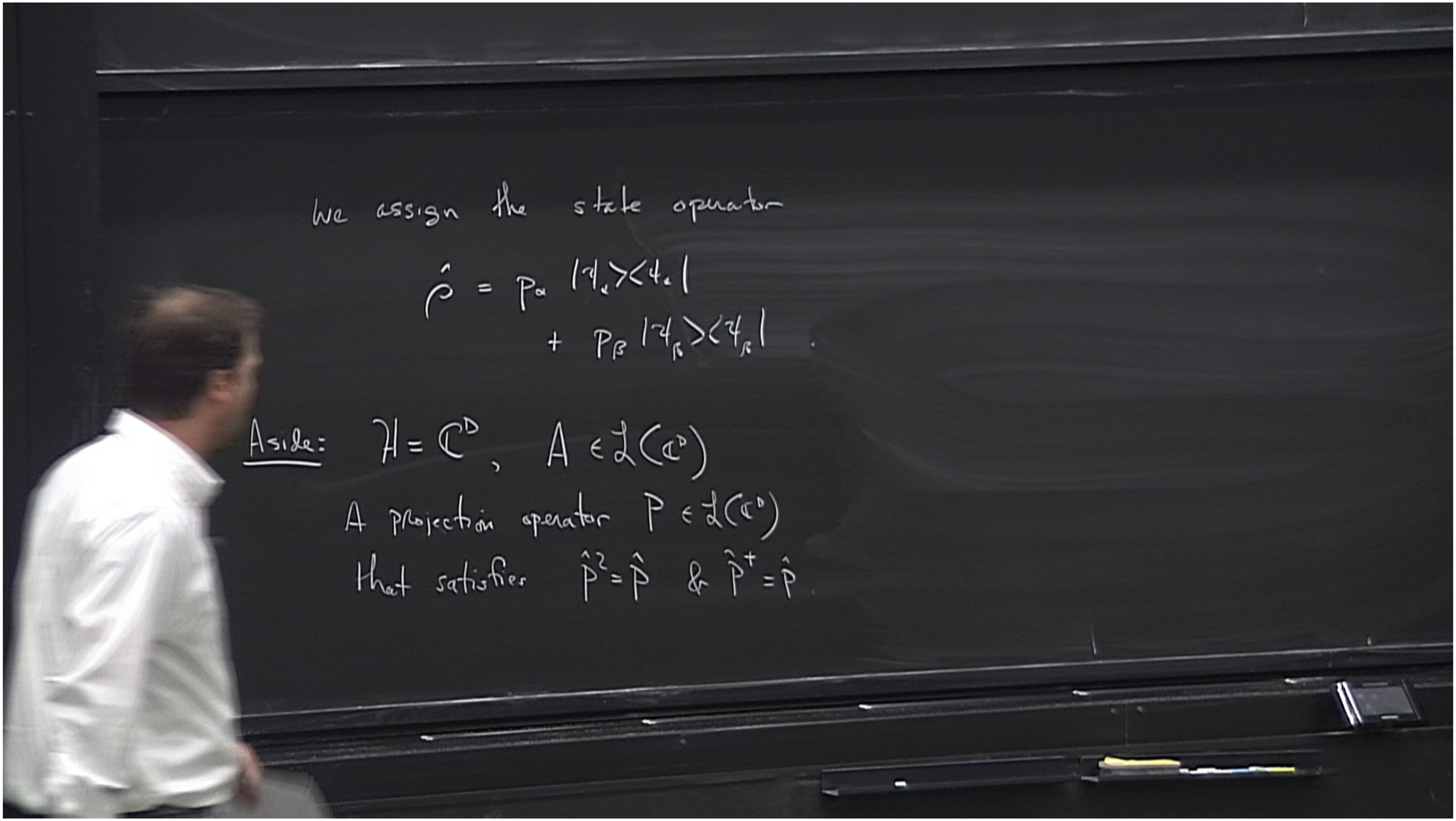
We assign the state operator

$$\hat{\rho} = p_\alpha |\psi_\alpha\rangle\langle\psi_\alpha| + p_\beta |\psi_\beta\rangle\langle\psi_\beta| .$$

<u>Aside:</u> $\mathcal{H} = \mathbb{C}^D$, $A \in \mathcal{L}(\mathbb{C}^D)$

A projection operator $P \in \mathcal{L}(\mathbb{C}^D)$

that satisfies $\hat{P}^2 = \hat{P}$ & $\hat{P}^\dagger = \hat{P}$.



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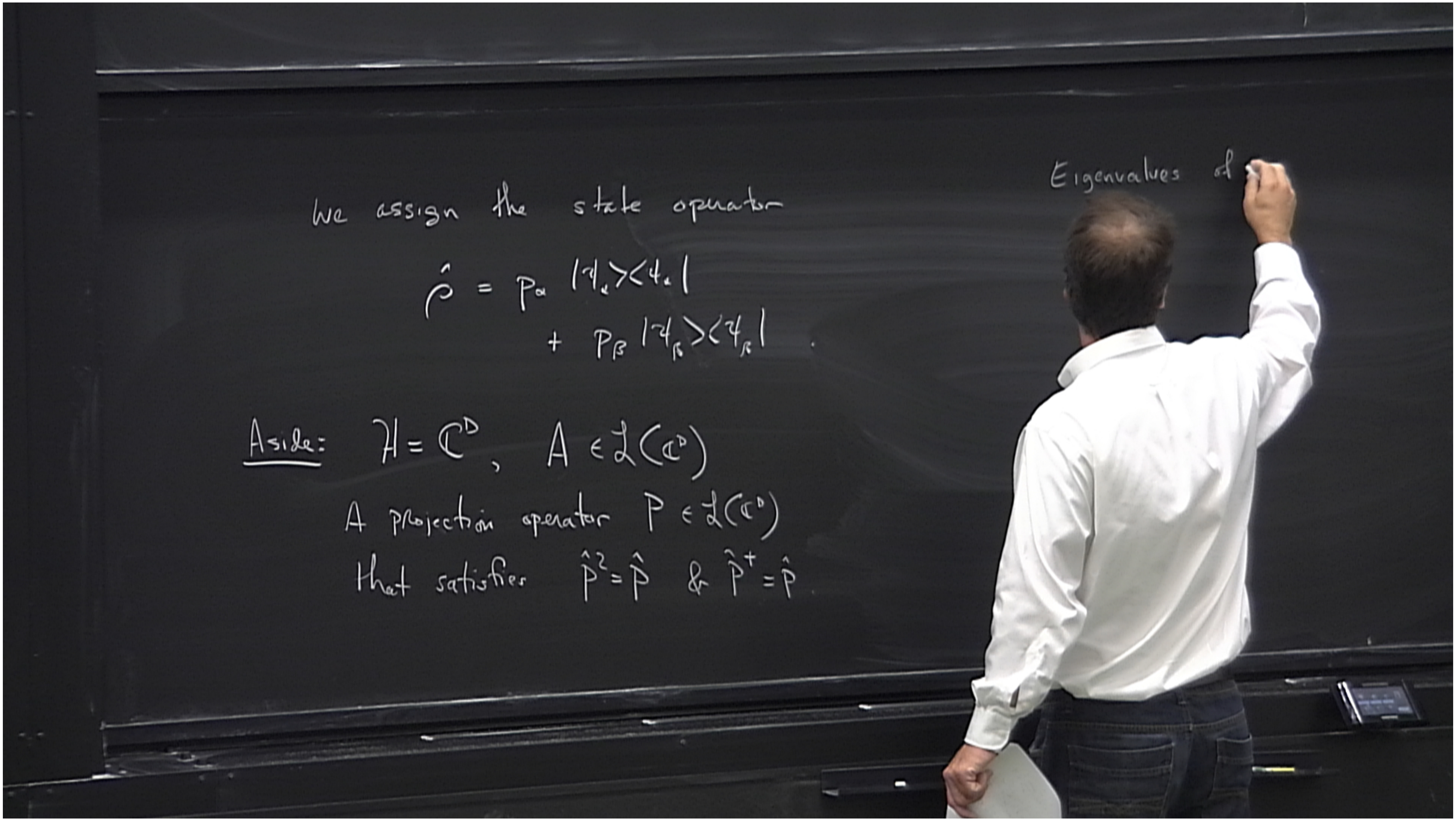

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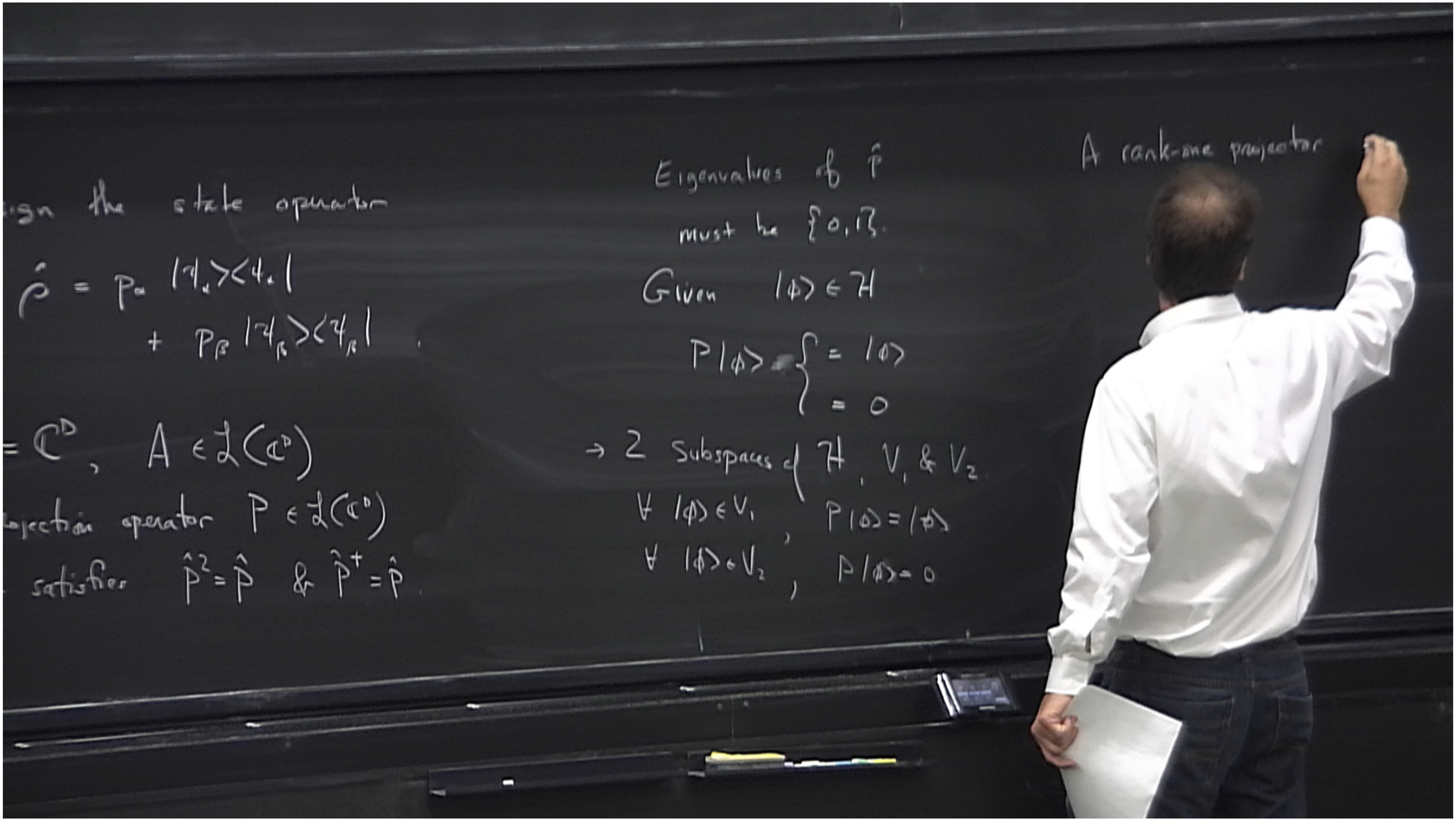
Eigenvalues of



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$= \mathbb{C}^D , \quad A \in \mathcal{L}(\mathbb{C}^D)$

ojection operator $P \in \mathcal{L}(\mathbb{C}^D)$

satisfies $\hat{P}^2 = \hat{P}$ & $\hat{P}^\dagger = \hat{P}$.

Eigenvalues of $\hat{P}$ must be $\{0, 1\}$.

Given $|\phi\rangle \in \mathcal{H}$

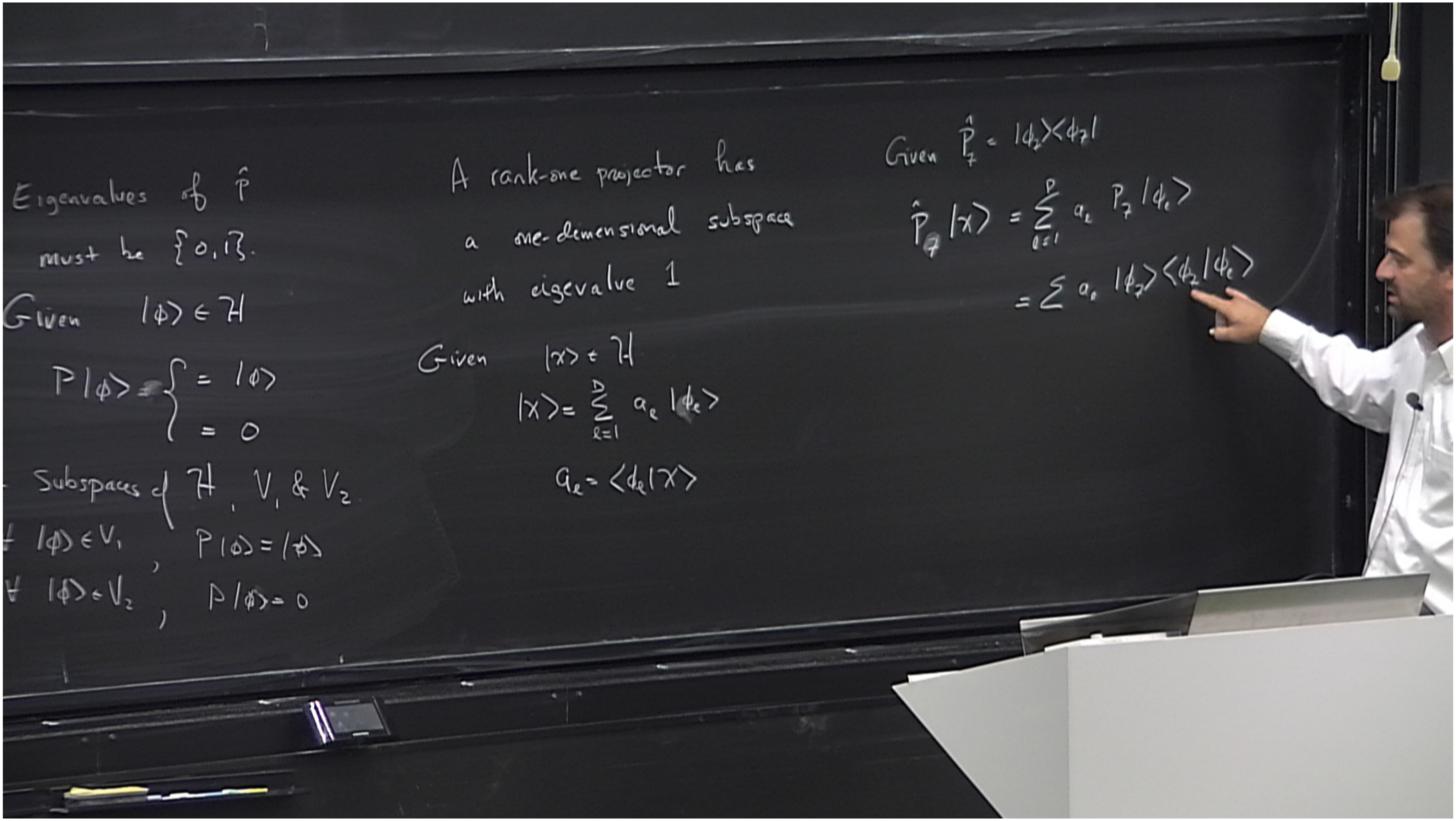
$$P|\phi\rangle \begin{cases} = |\phi\rangle \\ = 0 \end{cases}$$

→ 2 Subspaces of $\mathcal{H}$, V_1 & V_2.

$\forall\ |\phi\rangle \in V_1$, $P|\phi\rangle = |\phi\rangle$

$\forall\ |\phi\rangle \in V_2$, $P|\phi\rangle = 0$

A rank-one projector



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A rank-one projector has a one-dimensional subspace with eigevalue 1

Given $|\chi\rangle \in \mathcal{H}$

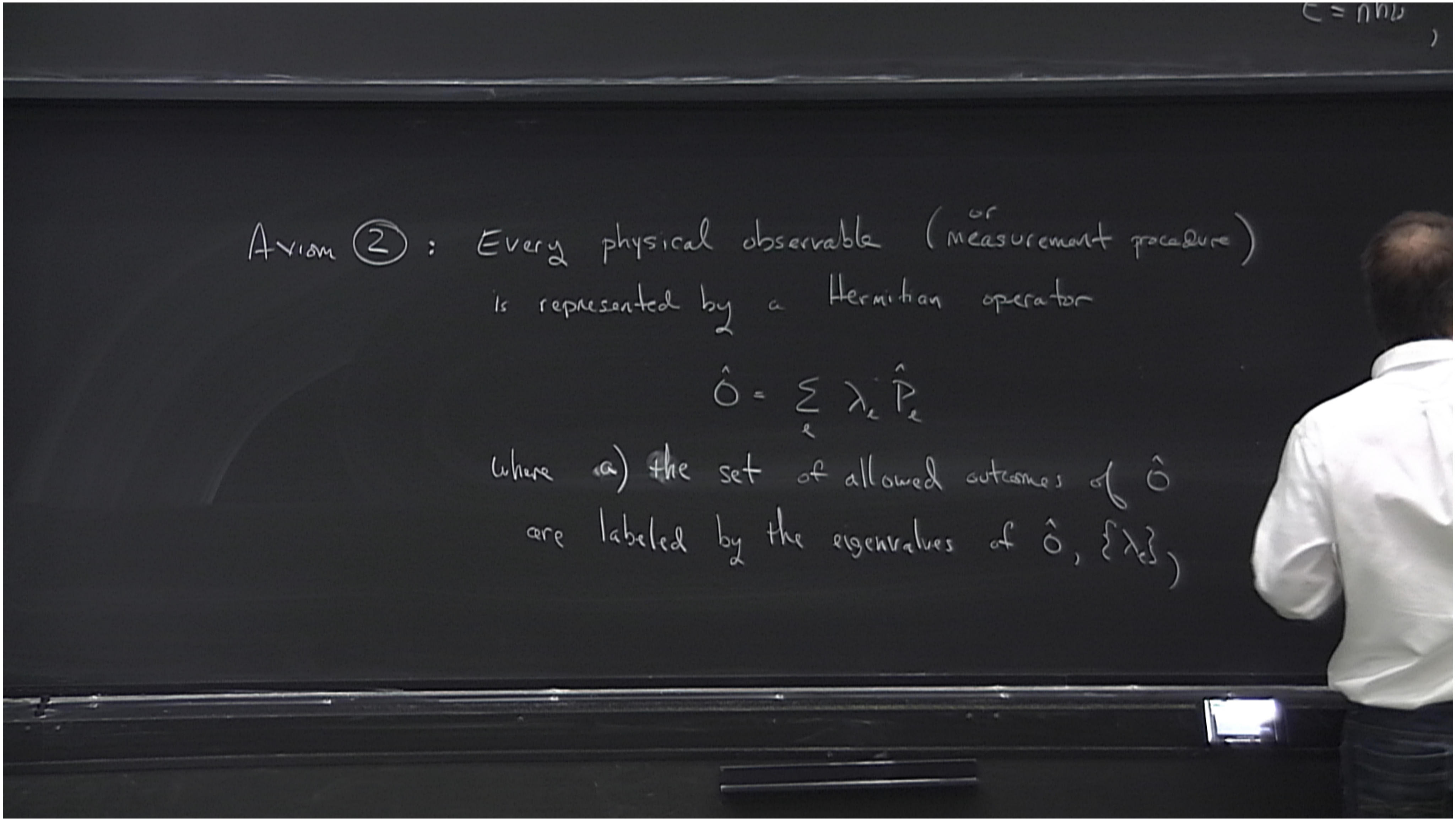
$$|\chi\rangle = \sum_{\ell=1}^{D} a_\ell |\phi_\ell\rangle$$

$$a_\ell = \langle\phi_\ell|\chi\rangle$$

Given $\hat{P}_7 = |\phi_7\rangle\langle\phi_7|$

$$\hat{P}_7|\chi\rangle = \sum_{\ell=1}^{D} a_\ell P_7|\phi_\ell\rangle$$

$$= \sum a_\ell |\phi_7\rangle\langle\phi_7|\phi_\ell\rangle$$



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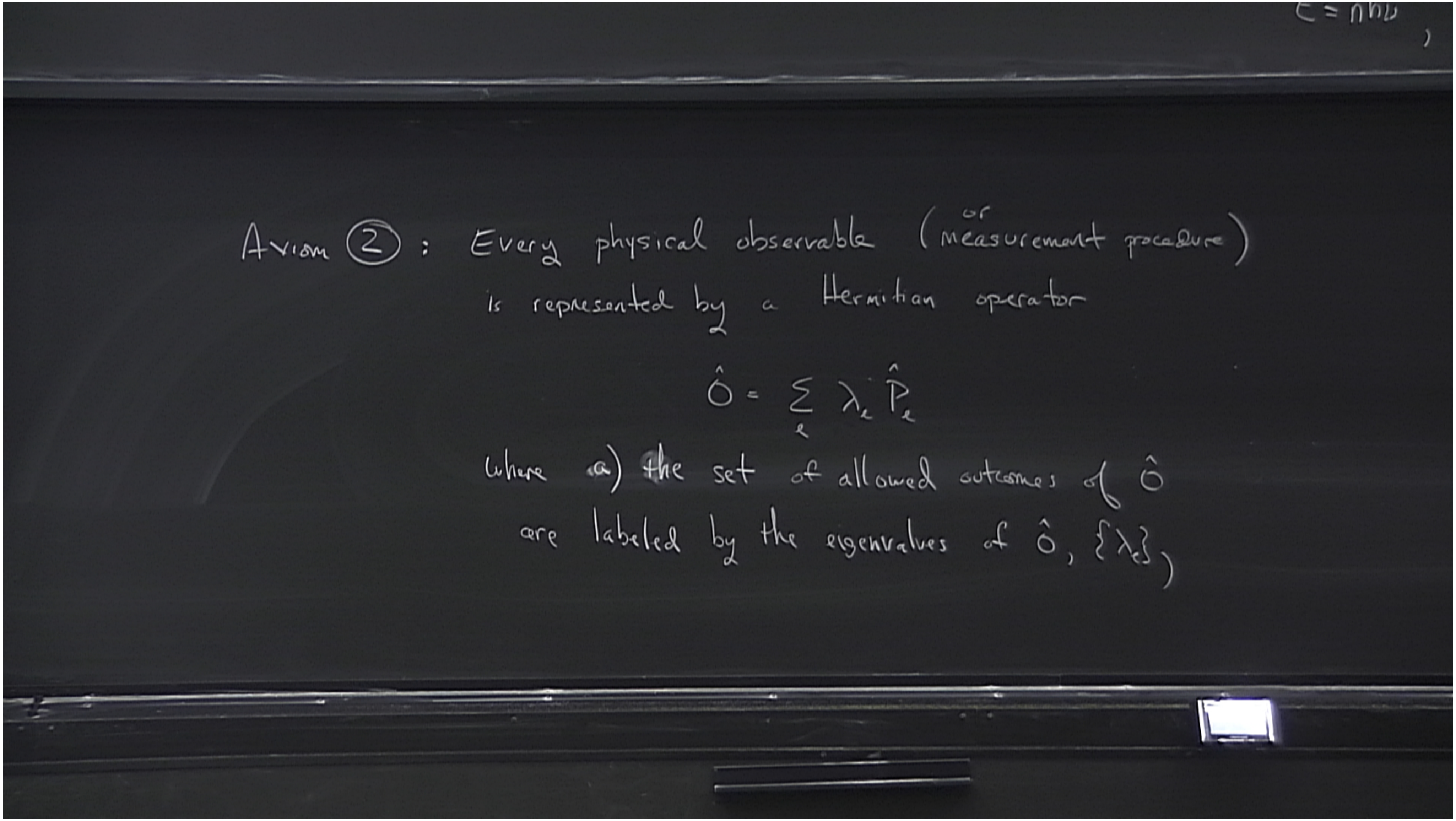
Axiom ② : Every physical observable (or measurement procedure) is represented by a Hermitian operator

$$\hat{O} = \sum_e \lambda_e \hat{P}_e$$

where a) the set of allowed outcomes of $\hat{O}$ are labeled by the eigenvalues of $\hat{O}$, $\{\lambda_e\}$,



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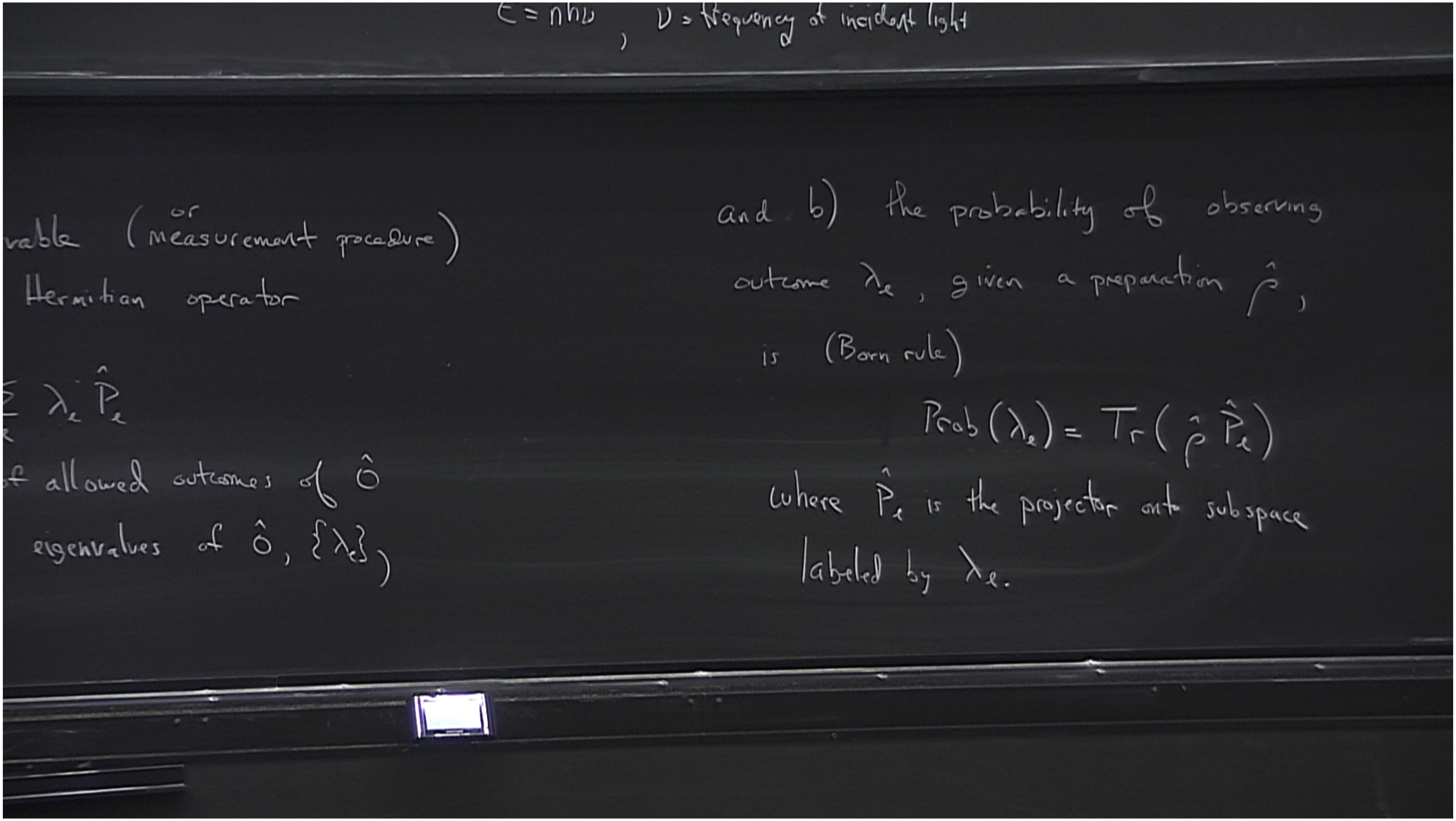

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Hermitian operator

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f allowed outcomes of $\hat{O}$

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and b) the probability of observing outcome λ_e, given a preparation $\hat{\rho}$, is (Born rule)

$$\mathrm{Prob}(\lambda_e) = \mathrm{Tr}(\hat{\rho}\,\hat{P}_e)$$

where $\hat{P}_e$ is the projector onto subspace labeled by λ_e.