Title: Fundamental quantum effects in the laboratory?

Date: Aug 30, 2012 02:30 PM

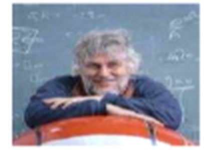
URL: http://pirsa.org/12080045

Abstract: There are several
fundamental predictions of quantum field theory, such as Hawking radiation (i.e., black hole evaporation) or the Sauter-Schwinger effect (i.e., electron-positron pair creation out of the quantum vacuum by a strong electric field), which have so far eluded direct experimental verification.
However, it should be possible to gain some experimental access to these effects via suitable condensed matter analogues. In this talk, some possibilities for reproducing such fundamental
quantum effects in the laboratory are discussed.<strongr>

</strongr>

Which Effects?





Relativistic quantum fields ($\hbar$, c) in vacuum state $|0\rangle$

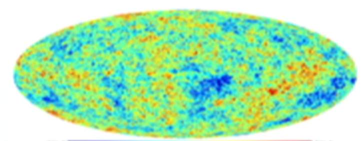
$\longrightarrow$ creation of particle pairs due to extreme conditions

- Sauter-Schwinger effect — electric field
- Hawking radiation — gravitational field
- Unruh effect — acceleration
- Dynamical Casimir effect — mirror motion
- Cosmological particle creation — expansion
- And many others — ...

Vacuum fluctuations $\longrightarrow$ **squeezing** $\longrightarrow$ entangled pairs

Experiments/observations?

WMAP:



See, e.g., C.M. Wilson *et al*, Nature **479**, 376 (2011).

Dirac Sea

Schrödinger equation
(non-relativistic)

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \quad \rightsquigarrow \quad \mathcal{E} = \frac{p^2}{2m} + V$$

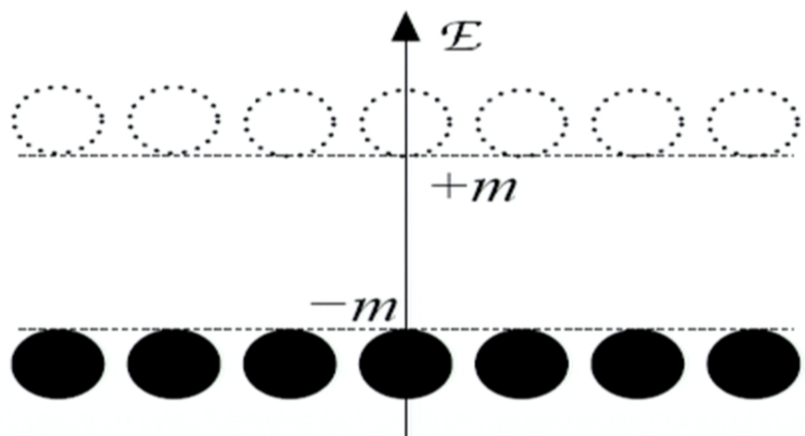
Dirac equation (relativistic)

$$\gamma^\mu\left(i\hbar\partial_\mu + qA_\mu\right)\Psi = mc\Psi \;\rightsquigarrow\; \mathcal{E} = V \pm \sqrt{c^2p^2 + m^2c^4}$$

$\longrightarrow$ positive and negative energy levels!

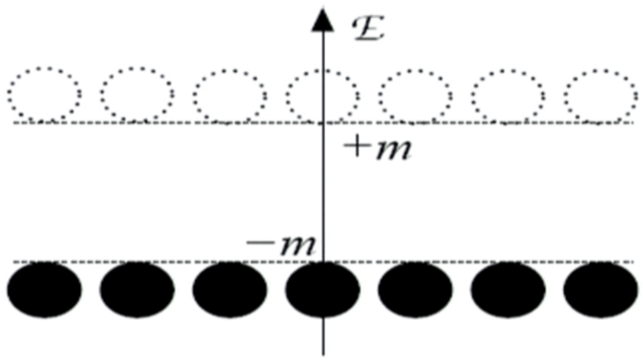
$\longrightarrow$ filled up in vacuum (Pauli principle)

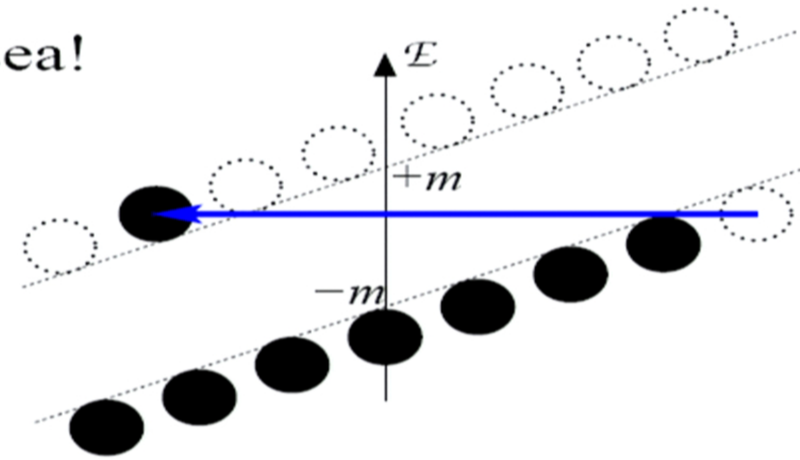
$\longrightarrow$ holes: positrons (prediction!)



Sauter-Schwinger Effect

Constant electric field E
$\longrightarrow$ potential $V(x) = qEx$
$\longrightarrow$ tilt of level spectrum
$\longrightarrow$ tunnelling from Dirac sea!

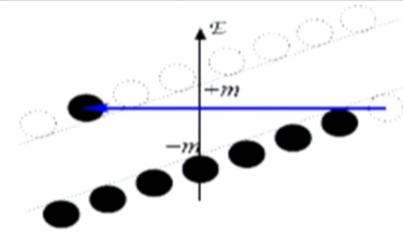




Creation of e^+e^- pairs out of the quantum vacuum

$$P_{e^+e^-} \propto \exp\left\{-2L\,\frac{\sqrt{2m\Delta V}}{\hbar}\right\} \propto \exp\left\{-2\,\frac{2mc^2}{qE}\,\frac{\mathcal{O}(mc)}{\hbar}\right\}$$

Critical Field Strength



Exact calculation yields exponent

$$P_{e^+e^-} \propto \exp\left\{-\pi \frac{c^3}{\hbar} \frac{m^2}{qE}\right\} = \exp\left\{-\pi \frac{E_S}{E}\right\}$$

Non-perturbative QED vacuum effect ($\longrightarrow$ difficult...)

Critical field strength (QED birefringence etc.)

$$E_S = \frac{c^3}{\hbar} \frac{m^2}{q} \approx 1.3 \times 10^{18}\,\mathrm{V/m}$$

Corresponds to intensity $I_S = \mathcal{O}(10^{29}\mathrm{W/cm^2})$

Planned ultra-strong lasers $I = \mathcal{O}(10^{26}\mathrm{W/cm^2})$

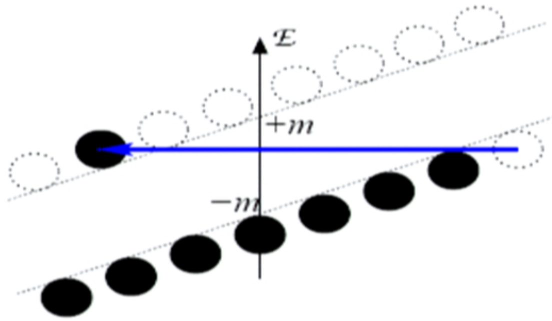
Exponential suppression $\exp\{-\pi E_S/E\} = \mathcal{O}(10^{-61})$

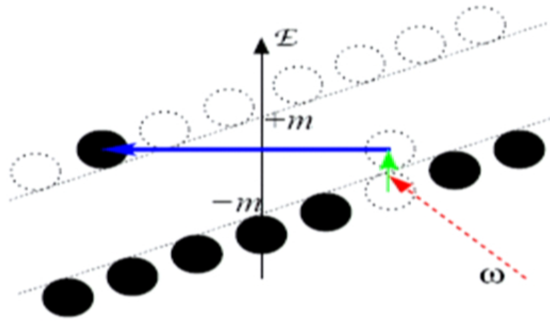
Four-volume factor $(\lambda_{\mathrm{laser}}/\lambda_{\mathrm{Compton}})^4 \approx 10^{22} \ldots$

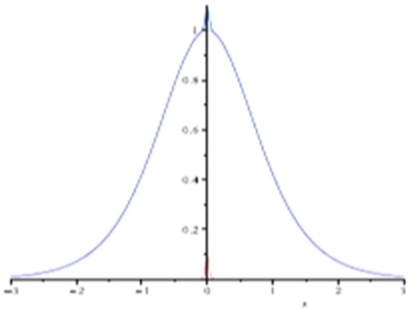
Further enhancement?

Assisted Tunnelling

Quantum-optics intuition: assisted tunnelling







Shorter way to tunnel $\longrightarrow$ reduction of exponent

Problem: $E \neq \text{const}$, e.g., $E = E(t)$ or $E = E(t, \boldsymbol{r})$

Tunnelling: non-perturbative $\exp\{-\pi E_S / E\}$

versus multi-photon: perturbative $\mathcal{O}([qE]^{2n})$

R. S., H. Gies, G. Dunne, Phys. Rev. Lett. **101**, 130404 (2008);

G. Dunne, H. Gies, R. S., Phys. Rev. D **80** (R), 111301 (2009);

C. Fey and R. S., Phys. Rev. D **85**, 025004 (2012).

Experiments?

Bose-Hubbard Model

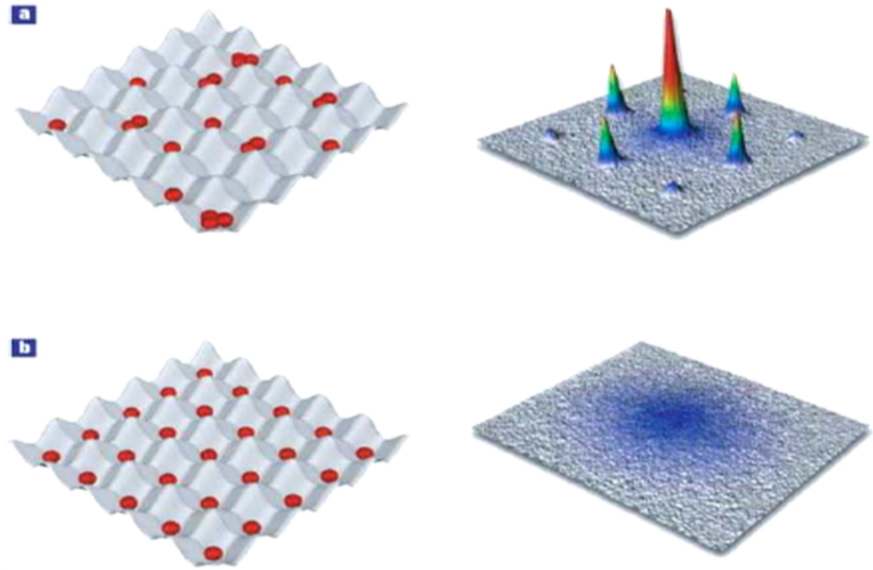
Atoms in optical lattice (lattice sites μ, ν) $\hbar = 1$

$$\hat{H} = -\frac{J}{Z}\sum_{\mu\nu} T_{\mu\nu}\hat{a}^\dagger_\mu \hat{a}_\nu + \frac{U}{2}\sum_{\mu}(\hat{a}^\dagger_\mu)^2\hat{a}^2_\mu$$

Hopping J dominates: superfluid phase

Tunnelling matrix $T_{\mu\nu}$ coordination number $Z = \sum_\nu T_{\mu\nu}$

Interaction U stronger: Mott insulator state $\rightarrow$ energy gap $\Delta\mathcal{E}$

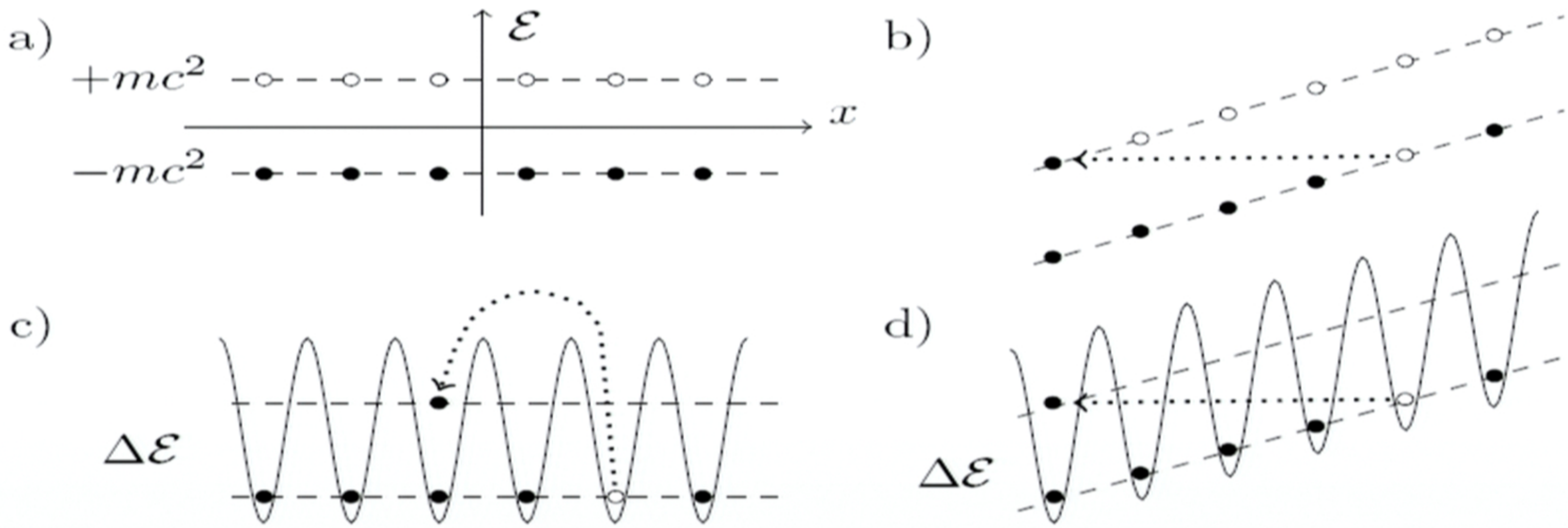


I. Bloch, Nature Physics **1**, 23 (2005).

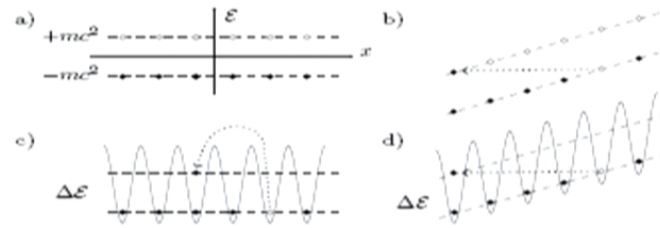
Lattice with Small Tilt $\longrightarrow V_\mu$

$$\hat{H} = -\frac{J}{Z}\sum_{\mu\nu} T_{\mu\nu}\hat{a}^\dagger_\mu \hat{a}_\nu + \frac{U}{2}\sum_\mu (\hat{a}^\dagger_\mu)^2 \hat{a}^2_\mu + \sum_\mu V_\mu \hat{n}_\mu$$

Particle-hole pair creation via tunnelling (Mott state)



Analogy to QED



Particle $\hat{p}_\mu = |1\rangle_\mu\langle 2|$ and

hole $\hat{h}_\mu = |0\rangle_\mu\langle 1|$ operators

$\longrightarrow$ effective scalar field $\hat{\phi}(t, \boldsymbol{r}_\mu) = \hat{h}_\mu(t) + \hat{p}_\mu(t)$

$$(i\partial_t - V)^2\,\hat{\phi} = \left[m_{\rm eff}^2 c_{\rm eff}^4 - c_{\rm eff}^2 \boldsymbol{\nabla}^2 + \mathcal{O}(\boldsymbol{\nabla}^4)\right]\hat{\phi}$$

Energy gap $\Delta\mathcal{E} = \sqrt{J^2 - 6JU + U^2} = 2m_{\rm eff}c_{\rm eff}^2$

Propagation speed $c_{\rm eff}^2 \propto (3JU - J^2)/2$

Klein-Fock-Gordon equation in continuum limit!

Quantitative analogy – R. Feynman:

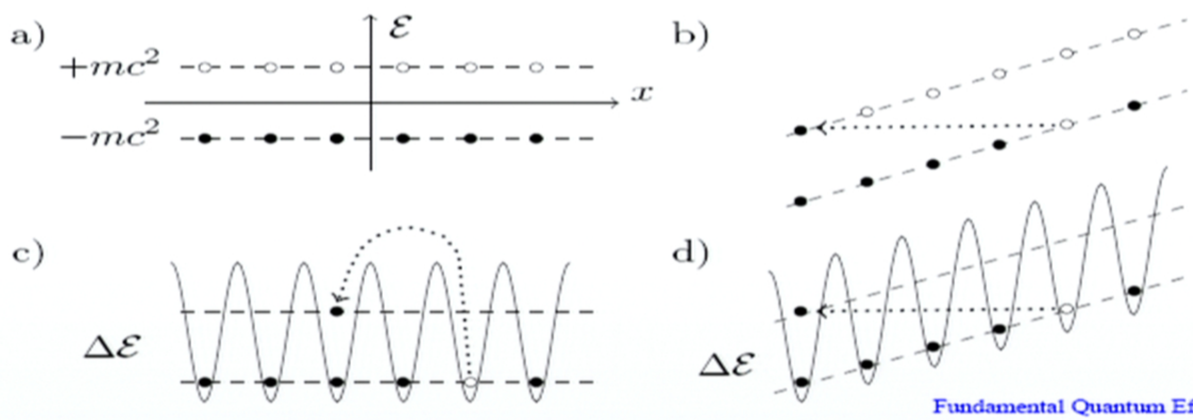
"*The same equations have the same solutions.*"

F. Queisser, P. Navez, R. S., Phys. Rev. A **85**, 033625 (2012).

Dictionary

F. Queisser, P. Navez, R. S., Phys. Rev. A **85**, 033625 (2012).

Sauter-Schwinger effect	Bose-Hubbard model
electrons & positrons	particles & holes
Dirac sea	Mott state
mass of electron/positron	energy gap $\Delta\mathcal{E}$
electric field $\boldsymbol{E}(t)$	lattice tilt $V_\mu(t)$
speed of light c	velocity c_{eff}



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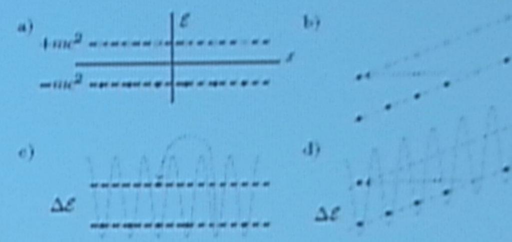

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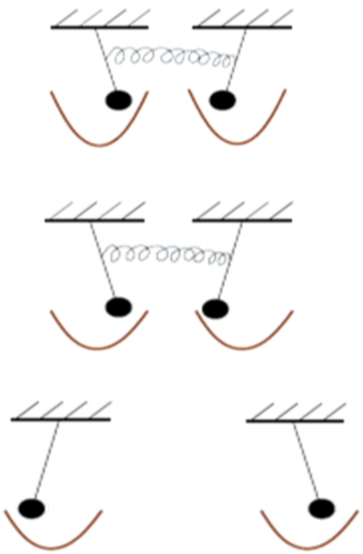
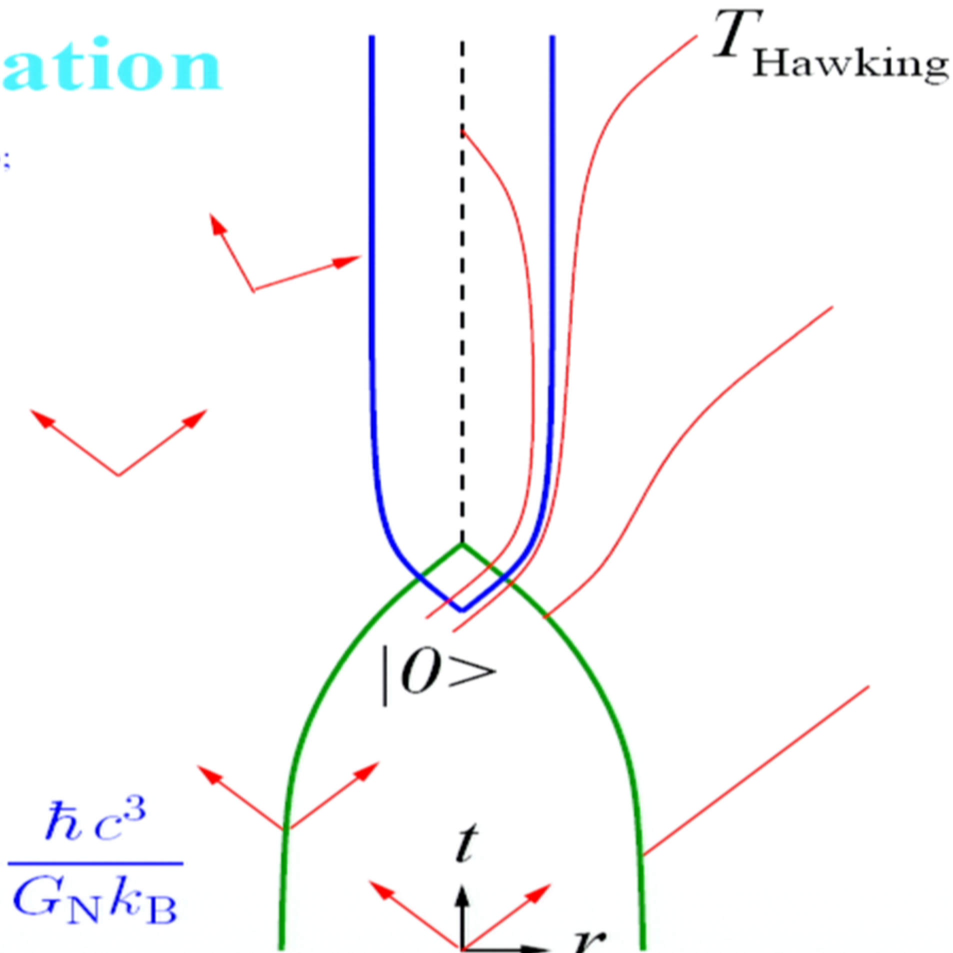
Hawking Radiation

S. W. Hawking, Nature **248**, 30 (1974);

Comm. Math. Phys. **43**, 199 (1975).

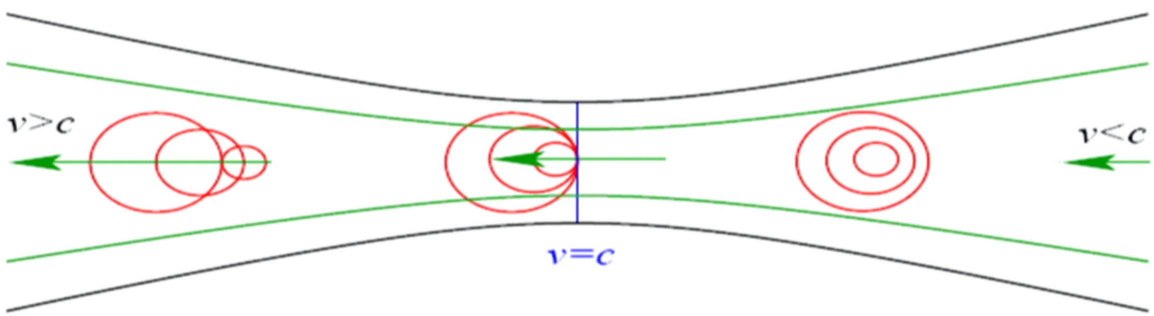


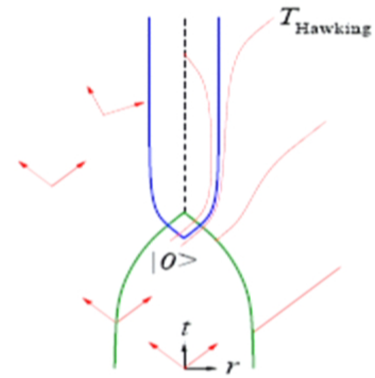


$$T_{\rm Hawking} = \frac{1}{8\pi M}\frac{\hbar\, c^3}{G_{\rm N} k_{\rm B}}$$

But: trans-Planckian problem

Quantitative Analogy





W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981).

"The same equations have the same solutions."

$$T_{\text{Hawking}} = \frac{\hbar}{2\pi\, k_{\text{B}}} \left| \frac{\partial}{\partial r} (v - c) \right|$$

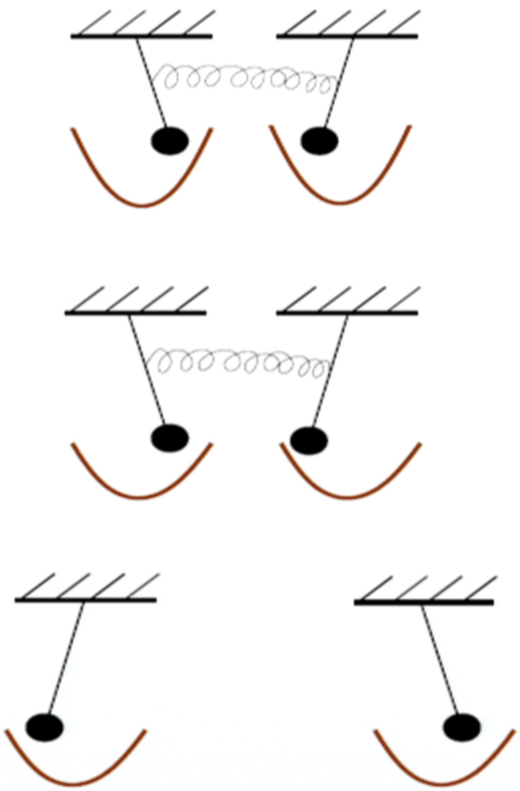
Theory: toy model for quantum gravity

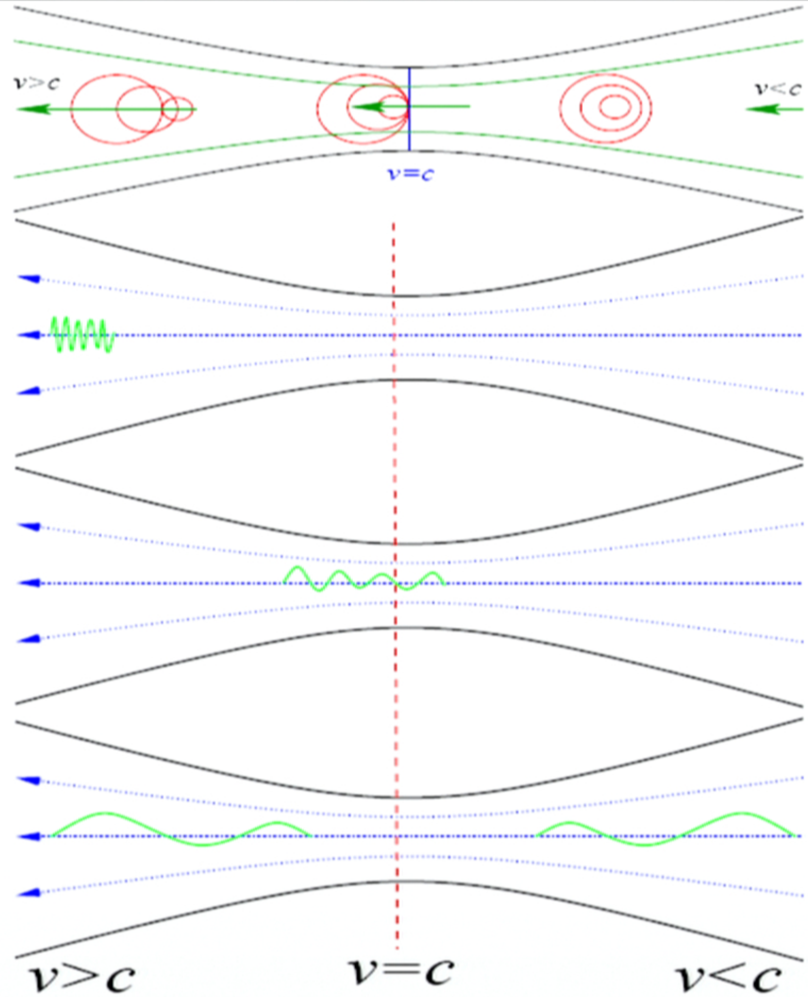
E.g., trans-Planckian problem

E.g.: R.S., W.G. Unruh, Phys. Rev. D **81**, 124033 (2010).

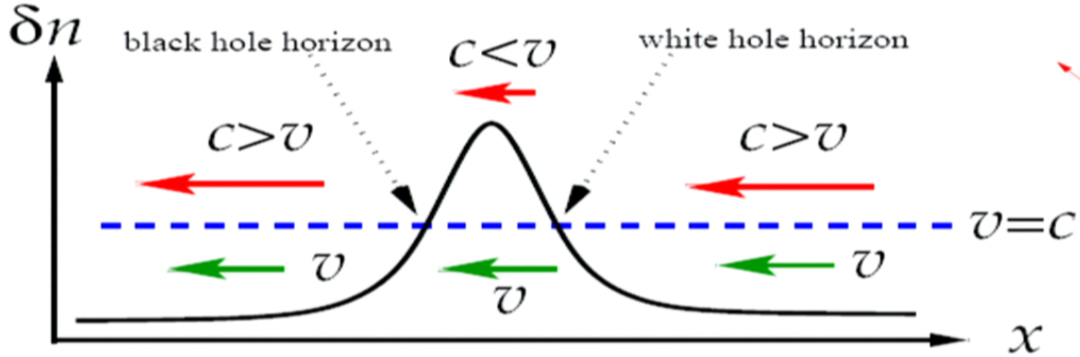
Experiments?

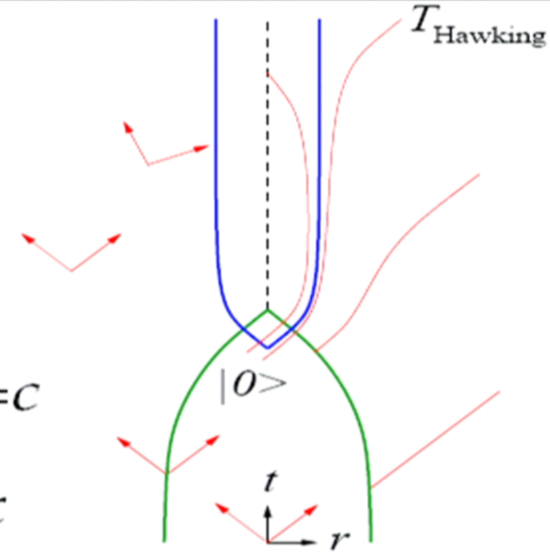
Pair Creation





Moving Pulse





W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981). R. S., W. G. Unruh, ibid. **95**, 031301 (2005).

Moving pulse in non-linear dielectric medium

"The same equations have the same solutions."

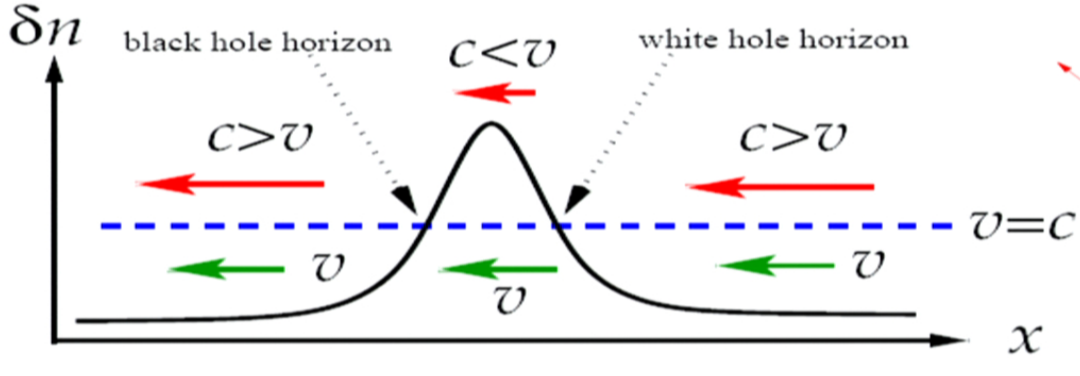
$$T_{\mathrm{Hawking}} = \frac{1}{8\pi M}\frac{\hbar c^3}{G_{\mathrm{N}} k_{\mathrm{B}}} \rightarrow \frac{\hbar}{2\pi\, k_{\mathrm{B}}}\frac{\partial c}{\partial r}$$

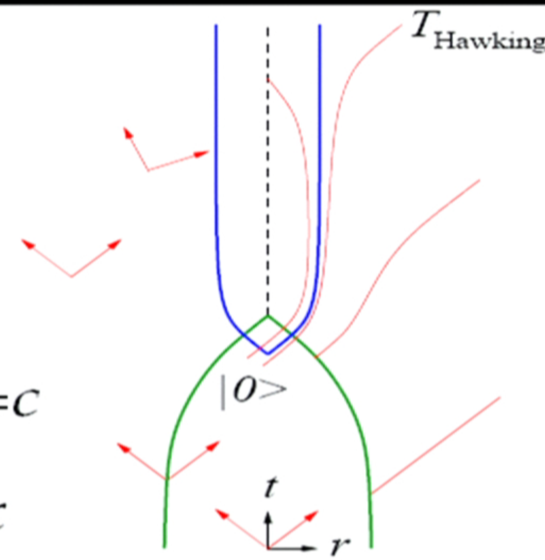
F. Belgiorno *et al*, Phys. Rev. Lett. **105**, 203901 (2010).

U. Leonhardt, *Strahlung hinterm Horizont*, Physik Journal 12/2010.

R.S., W.G. Unruh, Phys. Rev. Lett. **107**, 149401 (2011).

Moving Pulse





W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981). R. S., W. G. Unruh, ibid. **95**, 031301 (2005).

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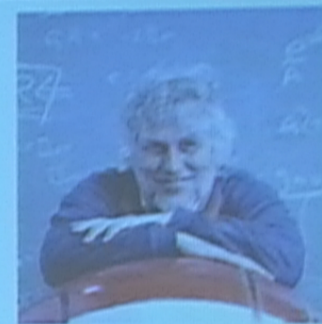
R.S., W.G. Unruh, Phys. Rev. Lett. **107**, 149401 (2011).

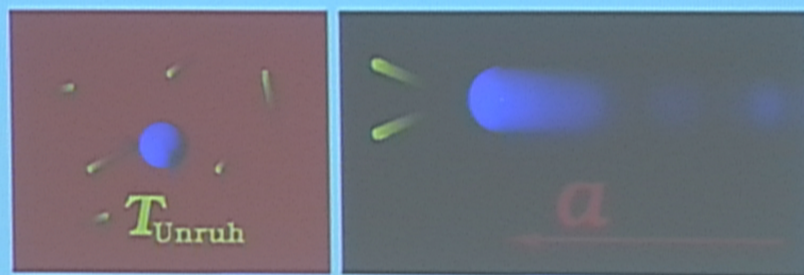
Unruh Effect

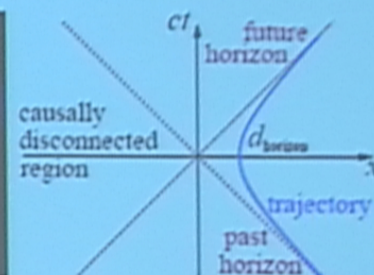


$$T_{\mathrm{Unruh}} = \frac{\hbar}{2\pi k_{\mathrm{B}} c}\, a$$

W. G. Unruh, Phys. Rev. D **14**, 870 (1976).

Scattering in accelerated frame (thermal bath)





Translation back into inertial (laboratory) frame

W. G. Unruh and R. M. Wald, Phys. Rev. D **29**, 1047 (1984).

Creation of entangled photon *pairs*

E.g., electron in strong laser field

R. S., G. Schaller, and D. Habs, Phys. Rev. Lett. **97**, 121302 (2006); ibid. **100**, 091301 (2008).

P. Chen and T. Tajima, Phys. Rev. Lett. **83**, 256 (1999).

Unruh Effect

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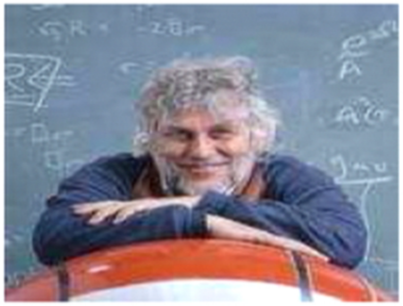

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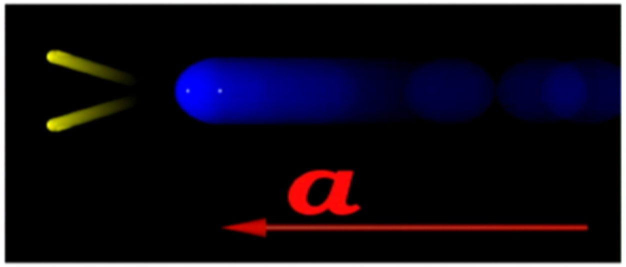

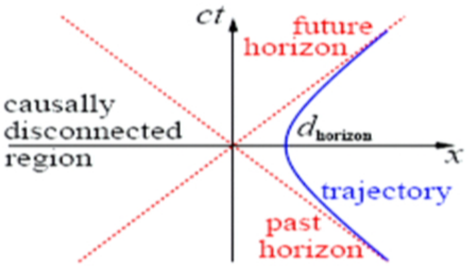

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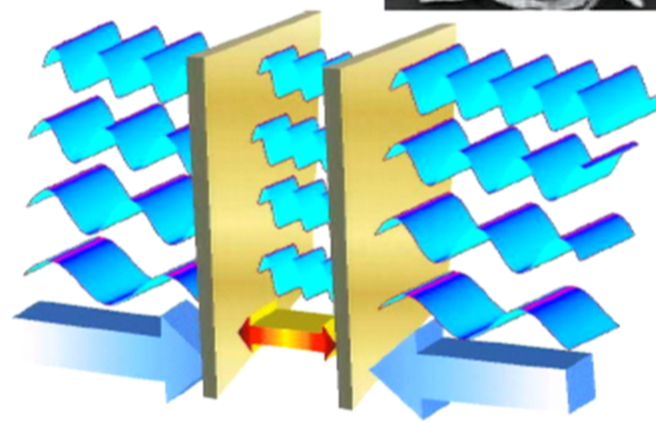
P. Chen and T. Tajima, Phys. Rev. Lett. **83**, 256 (1999).

Dynamical Casimir Effect

Static Casimir effect:
Attraction/repulsion in vacuum

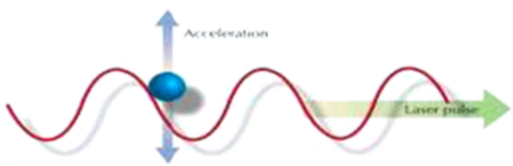
Dynamical Casimir effect:
Creation of photon pairs out of quantum vacuum due to non-inertial mirror motion

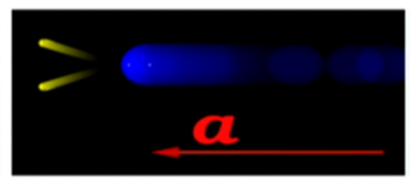


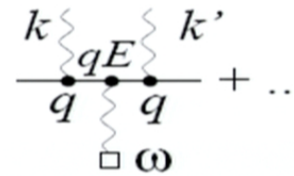
a) single mirror

b) cavity (resonance)

See, e.g., C.M. Wilson *et al*, Nature **479**, 376 (2011).

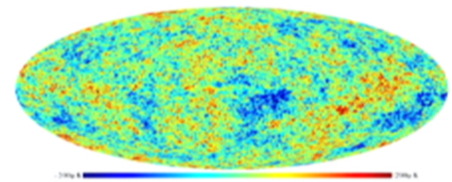






Relation to Unruh effect. . .

Cosmological Particle Creation



Quantum field in expanding Universe

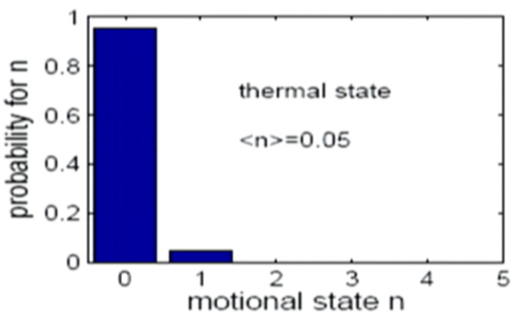
$\longleftrightarrow$ quantised phonons in ion chain

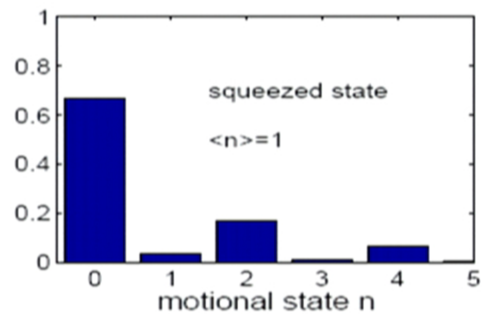
R. S. *et al.*, Phys. Rev. Lett. **99**, 201301 (2007).

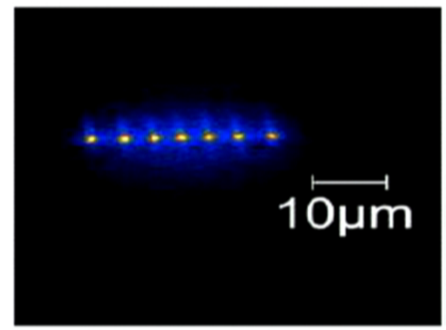
P.M. Alsing, J.P. Dowling, and G.J. Milburn, ibid. **94**, 220401 (2005).

"The same equations have the same solutions."

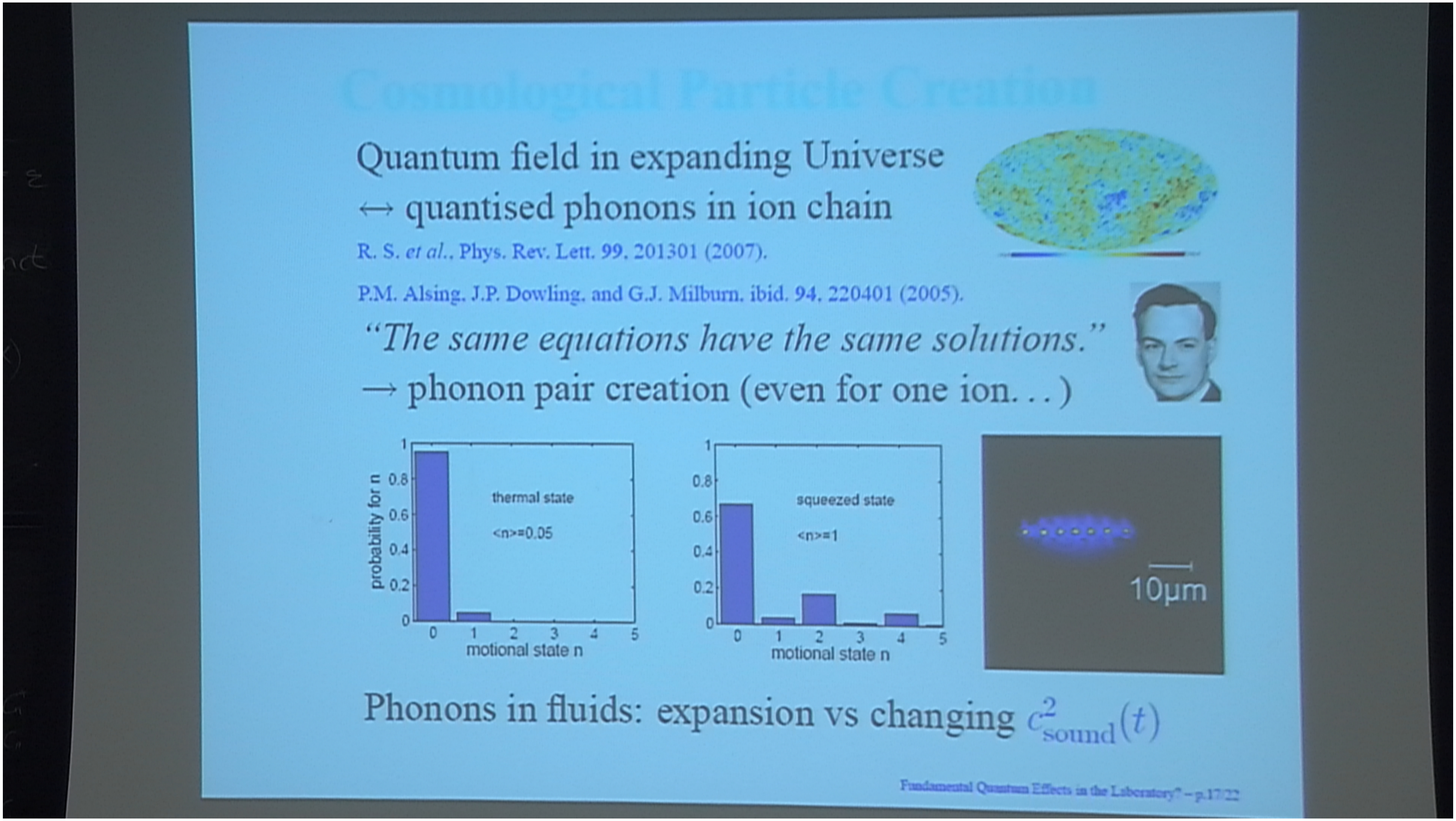
$\longrightarrow$ phonon pair creation (even for one ion. . .)







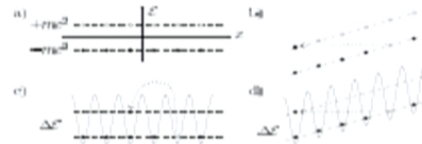
Phonons in fluids: expansion vs changing $c^2_{\text{sound}}(t)$


Cosmological Particle Creation
Quantum field in expanding Universe
↔ quantised phonons in ion chain
R. S. et al., Phys. Rev. Lett. 99, 201301 (2007).
P.M. Alsing, J.P. Dowling, and G.J. Milburn, ibid. 94, 220401 (2005).
"The same equations have the same solutions."
→ phonon pair creation (even for one ion…)
probability for n
thermal state
<n>=0.05
motional state n
squeezed state
<n>=1
motional state n
10µm
Phonons in fluids: expansion vs changing $c^2_{\mathrm{sound}}(t)$
Fundamental Quantum Effects in the Laboratory? – p.17/22

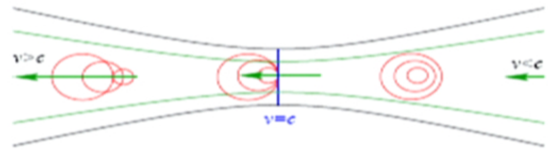
Summary

Effects of quantum fields under extreme conditions:

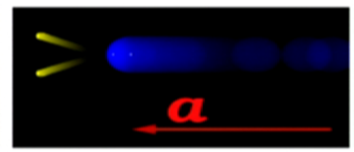
- Sauter-Schwinger effect
  $\longrightarrow$ atoms in optical lattices



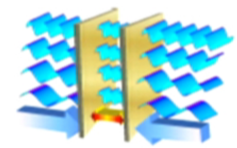

- Hawking radiation
  $\longrightarrow$ trans-sonic fluids



- Unruh effect
  $\longrightarrow$ electrons in lasers



- Dynamical Casimir effect
  $\longrightarrow$ non-inertial scatterers




- cosmological particle creation
  $\longrightarrow$ ion traps, condensates