Title: 12/13 PSI - Student Presentations 1A

Date: Aug 17, 2012 09:00 AM

URL: http://pirsa.org/12080038

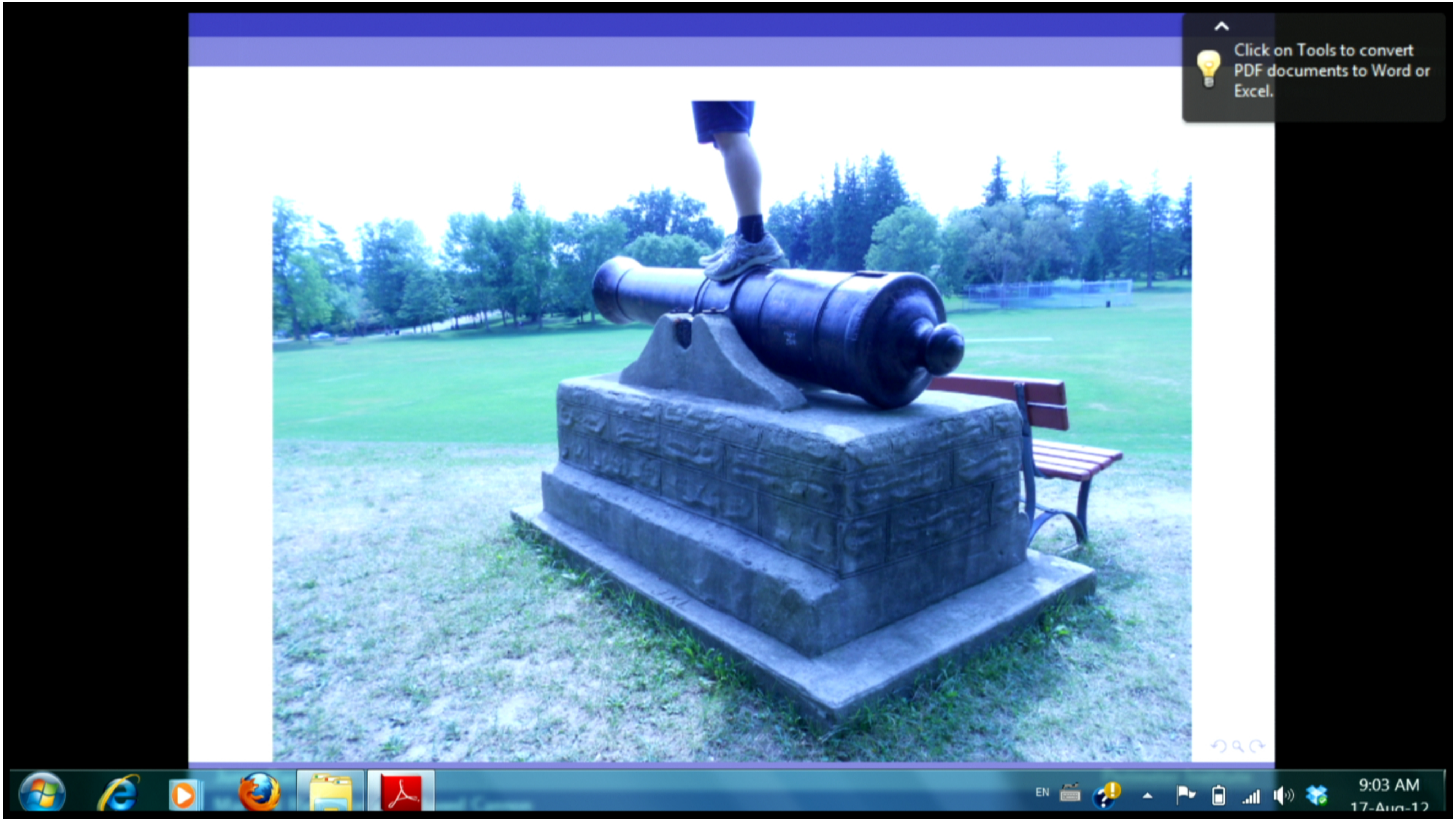
Abstract:

Maximal Range for a Hill-based Cannon

Junjie Rao

Perimeter Institute

perimeter scholars international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Motivation

- If you are leading an artillery ...

Motivation

- If you are leading an artillery ...
- For a fixed height of the hill, what is the maximal range of the cannon, and the corresponding angle? Regardless of the air resistance.
- Of course not $45°$!

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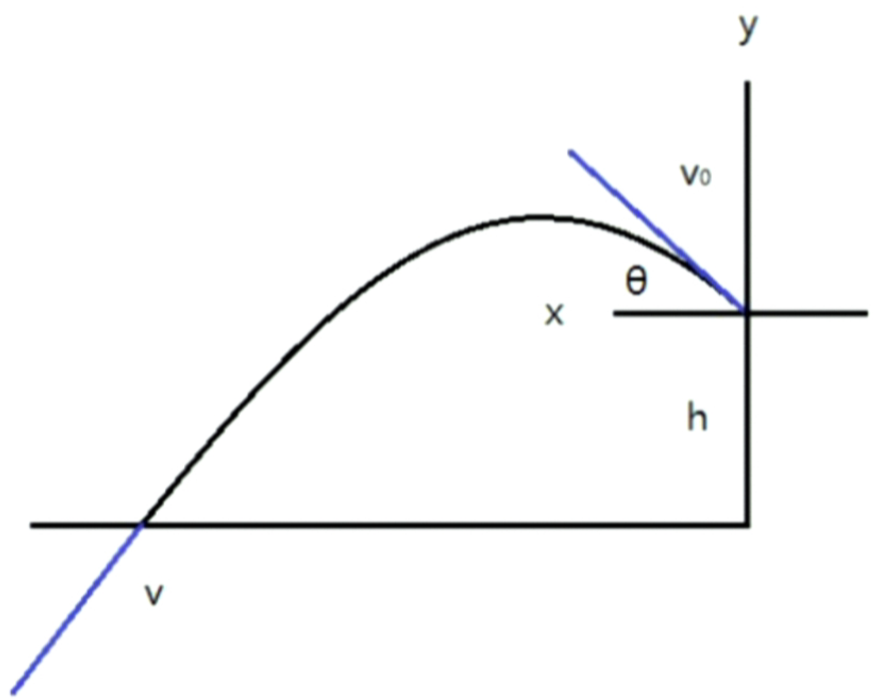
Outline

- Brute Force Solving
- Cleverer Approach - Vector Triangle
- Two Limits
- Conclusion

Brute Force Solving I



Brute Force Solving II

Motion Equations (due to Galileo?)

$$x = v_0 t \cos\theta \tag{1}$$

$$y = v_0 t \sin\theta - \frac{1}{2} g t^2 \tag{2}$$

Trajectory Equation

$$y = x \tan\theta - \frac{g x^2}{2 v_0^2 \cos^2\theta} \tag{3}$$

$$\Rightarrow x \tan\theta - \frac{g x^2}{2 v_0^2}(1 + \tan^2\theta) + h = 0 \tag{4}$$

- For $y = -h$, maximize x by varying θ.

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Brute Force Solving II

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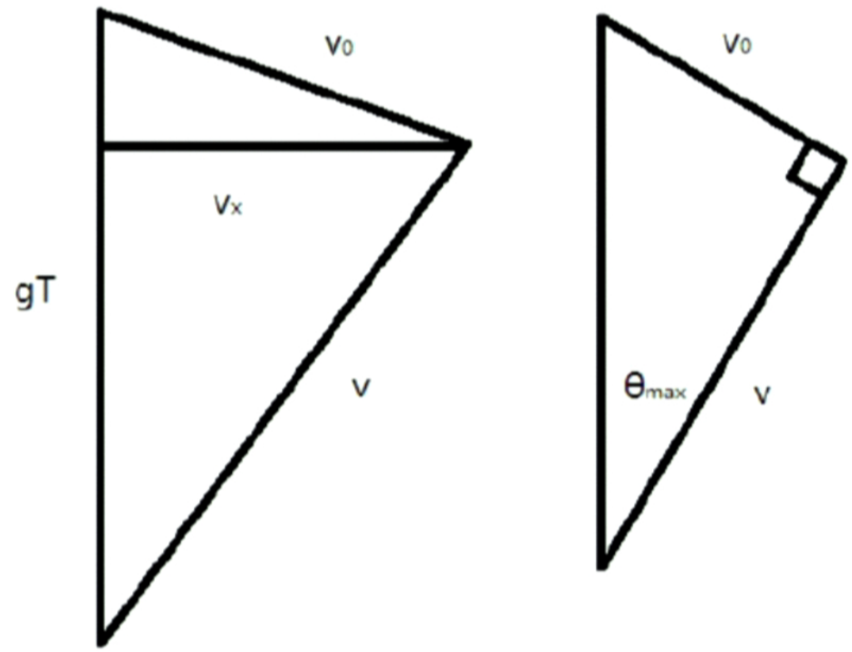
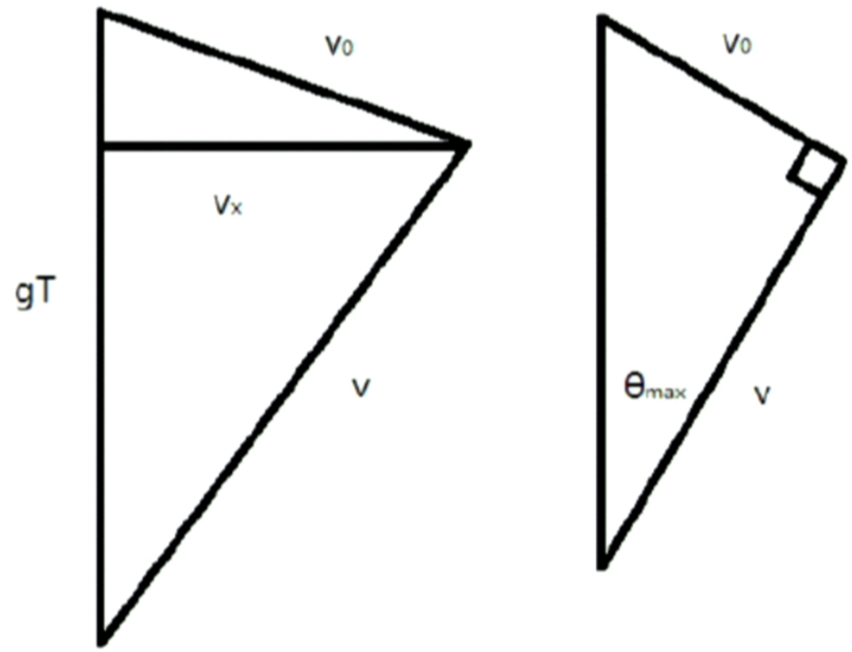
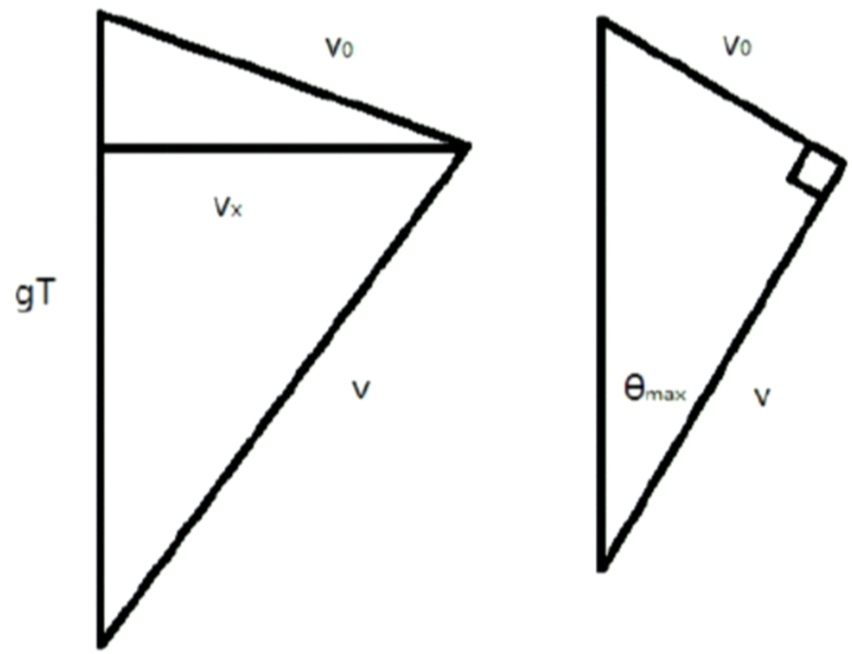
Cleverer Approach - Vector Triangle I



Cleverer Approach - Vector Triangle I



Cleverer Approach - Vector Triangle I



Two Limits

- $h \ll v_0^2/2g \Rightarrow$ Slant projectile (on the same level)

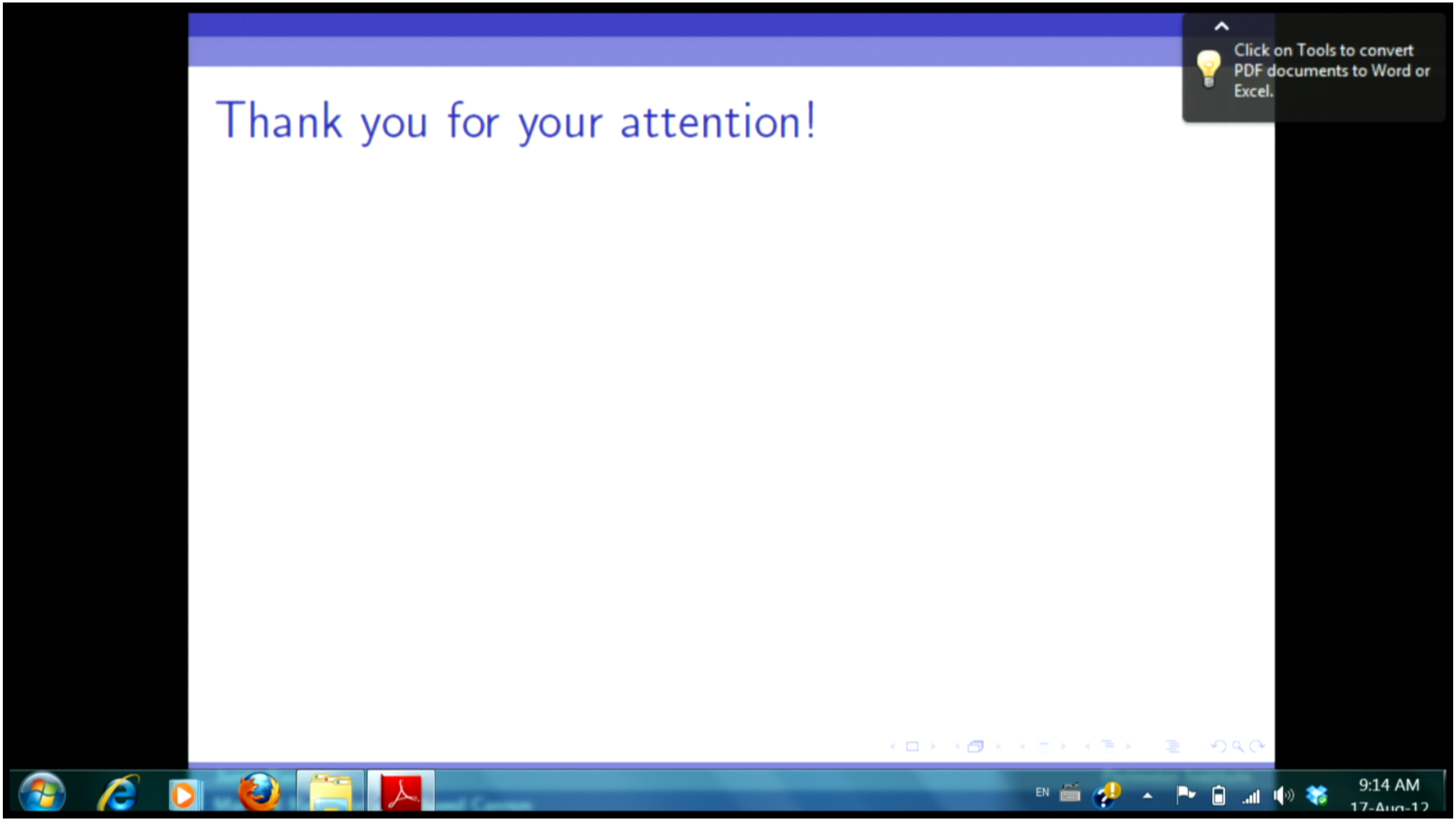
$$\tan\theta_{\max} = 1 \Rightarrow \theta_{\max} = 45^\circ \tag{9}$$

$$r_{\max} = \frac{v_0^2}{g} \tag{10}$$

- $h \gg v_0^2/2g \Rightarrow$ Horizontal projectile (on the cliff)

$$\tan\theta_{\max} = 0 \Rightarrow \theta_{\max} = 0^\circ \tag{11}$$

$$r_{\max} = v_0\sqrt{\frac{2h}{g}} \tag{12}$$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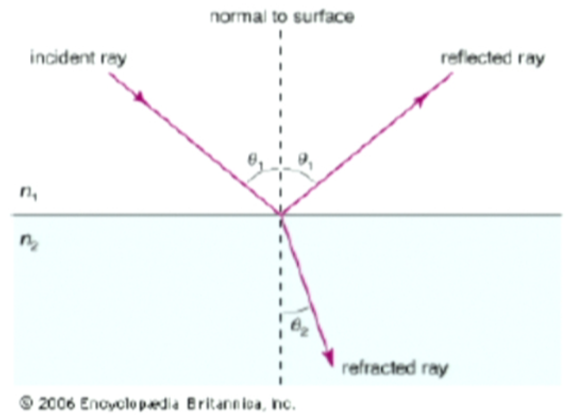
Water reflections

Reflection and refraction

- Familiar laws of reflection and refraction

Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



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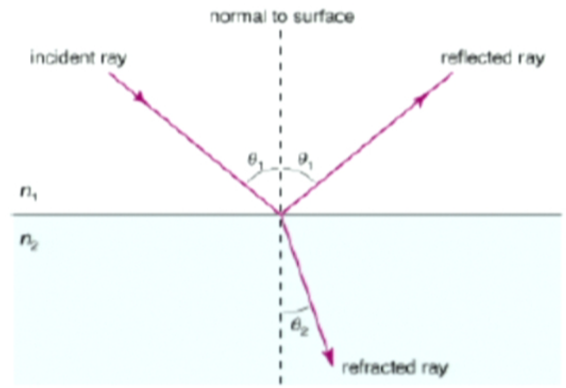

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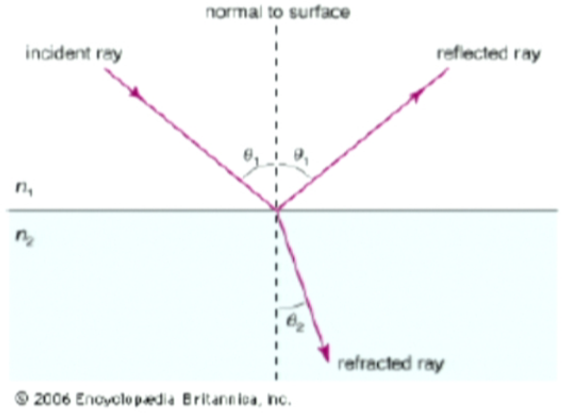

Fresnel equations

- Maxwell's equations on the boundary imply discontinuity conditions.

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

$$B_{1\perp} = B_{2\perp}$$

$$\frac{B_{1\parallel}}{\mu_1} = \frac{B_{2\parallel}}{\mu_2}$$

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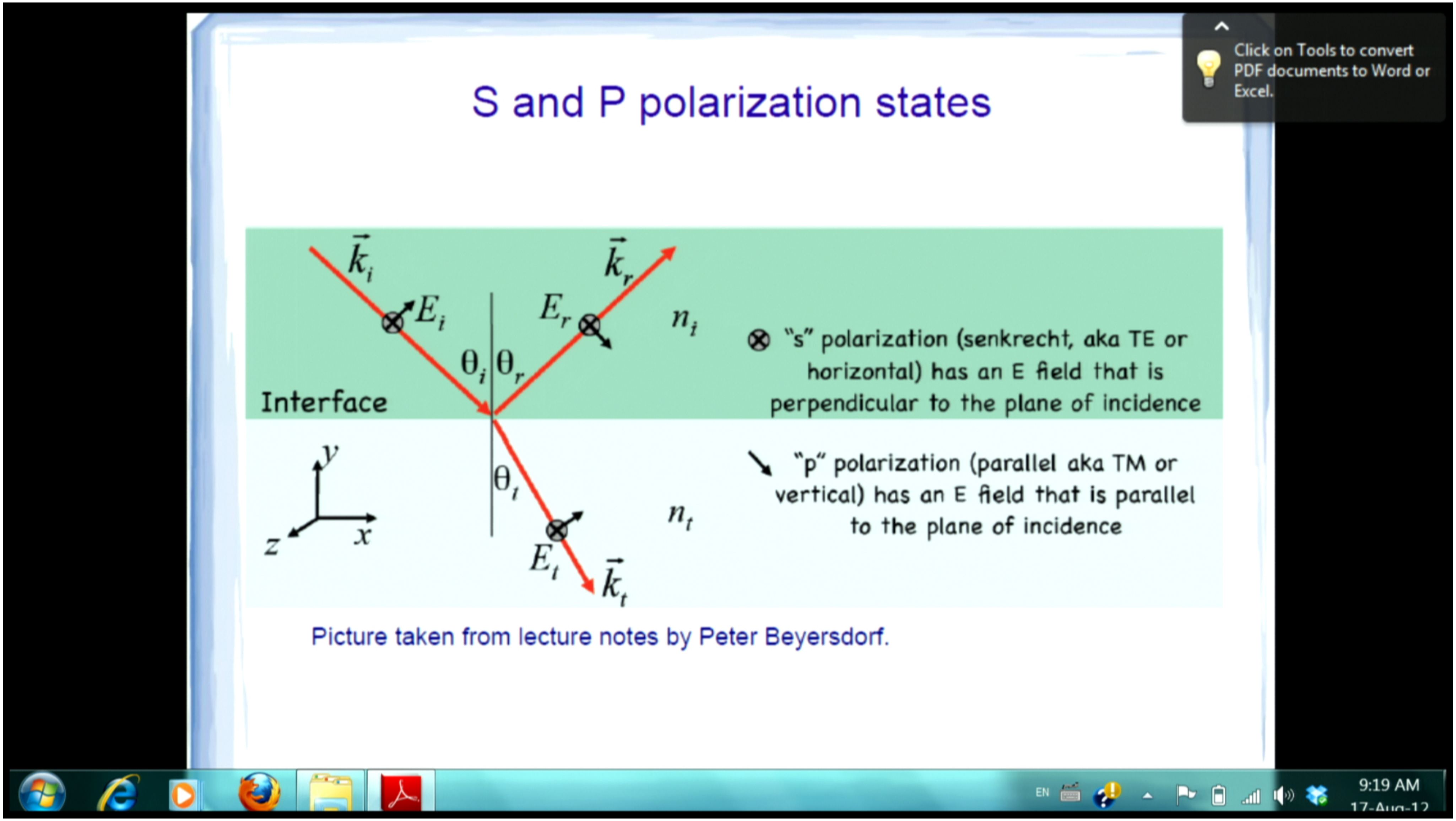
S and P polarization states
$\vec{k}_i$
E_i
$\vec{k}_r$
E_r
n_i
θ_i θ_r
Interface
θ_t
n_t
E_t
$\vec{k}_t$
y
x
z
"s" polarization (senkrecht, aka TE or horizontal) has an E field that is perpendicular to the plane of incidence
"p" polarization (parallel aka TM or vertical) has an E field that is parallel to the plane of incidence
Picture taken from lecture notes by Peter Beyersdorf.
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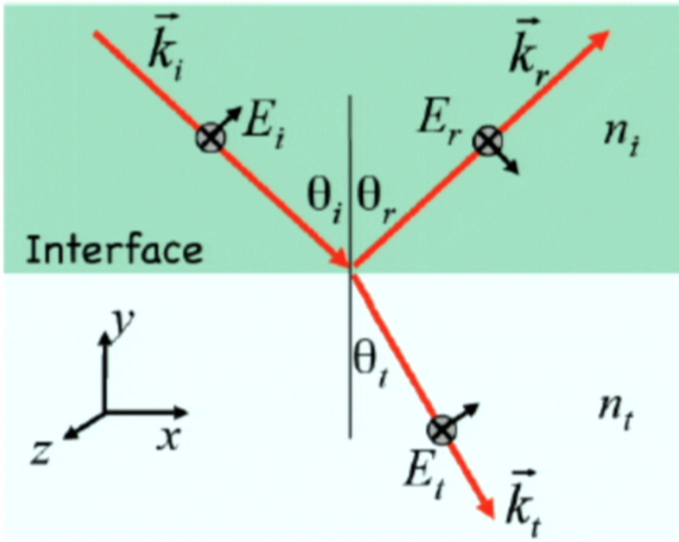
S and P polarization states



⊗ "s" polarization (senkrecht, aka TE or horizontal) has an E field that is perpendicular to the plane of incidence

↘ "p" polarization (parallel aka TM or vertical) has an E field that is parallel to the plane of incidence

Picture taken from lecture notes by Peter Beyersdorf.

Fresnel equations cont.

- Solving for the electric fields at the interface gives the reflectivity coefficients.

$$r_\perp = \frac{E_{0r}}{E_{0i}} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$r_\parallel = \frac{E_{0r}}{E_{0i}} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

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Fishing and 2D dynamical systems

Nick Jones
Perimeter Institute

17th August 2012

perimeter scholars international

9:29 AM
17-Aug-12

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Introduction

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INTRODUCTION

- I will discuss a model of fish population in the lake outside PI, and experiment with permitting fishing.
- I will show some numerical simulations, and then explain how to find behaviours without a computer.

MODELLING THE POPULATION OF FISH

- Let's use a predator/prey model (due to Lotka and Volterra):

MODELLING THE POPULATION OF FISH

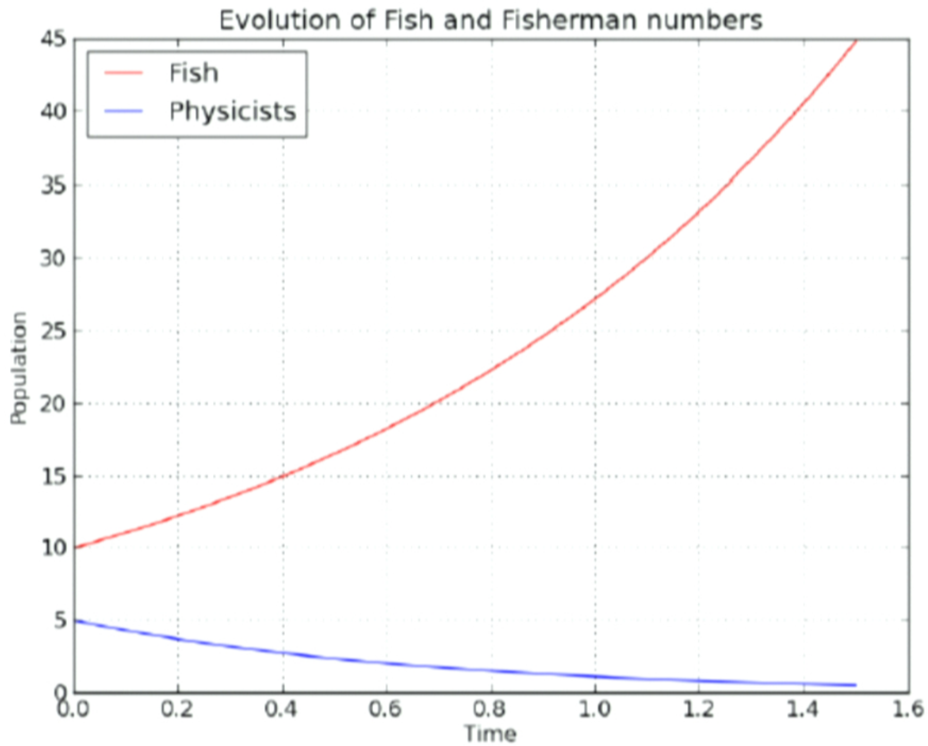
- Let's use a predator/prey model (due to Lotka and Volterra):

$$\dot{x} = \alpha x - \beta x p$$
$$\dot{p} = -\gamma p + \delta x p$$

- x is the number of fish in the pond, p is the number of fishing physicists.
- The greek letters are positive constants determining the rates of growth and strengths of coupling.

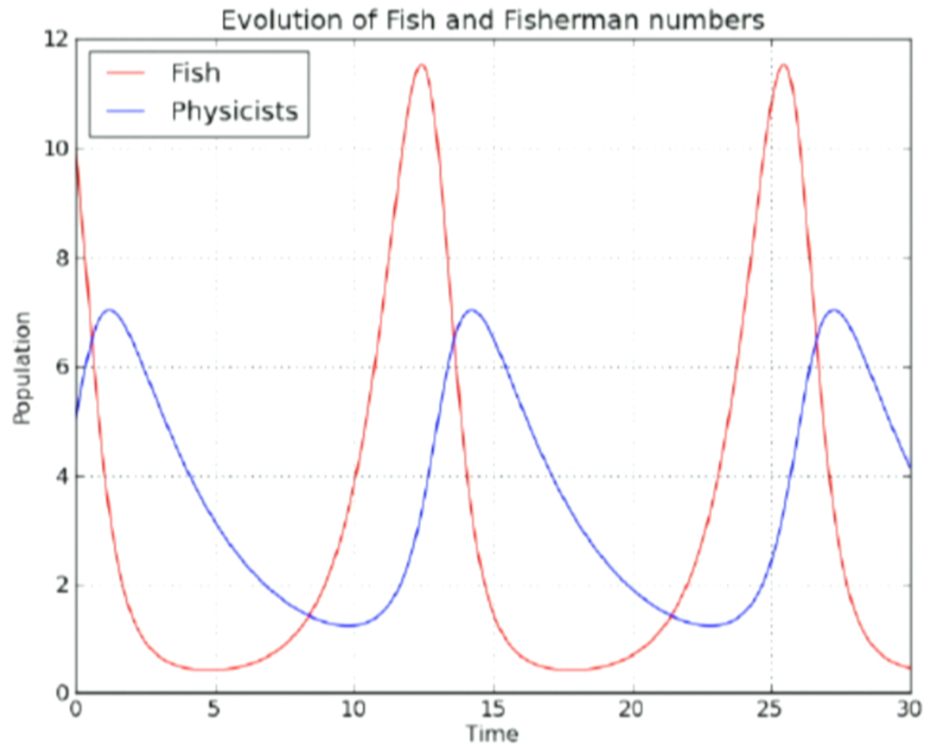
NO FISHING

$$\beta = 0$$



Relaxing the rules

$\beta = -0.3$



RELAXING THE RULES

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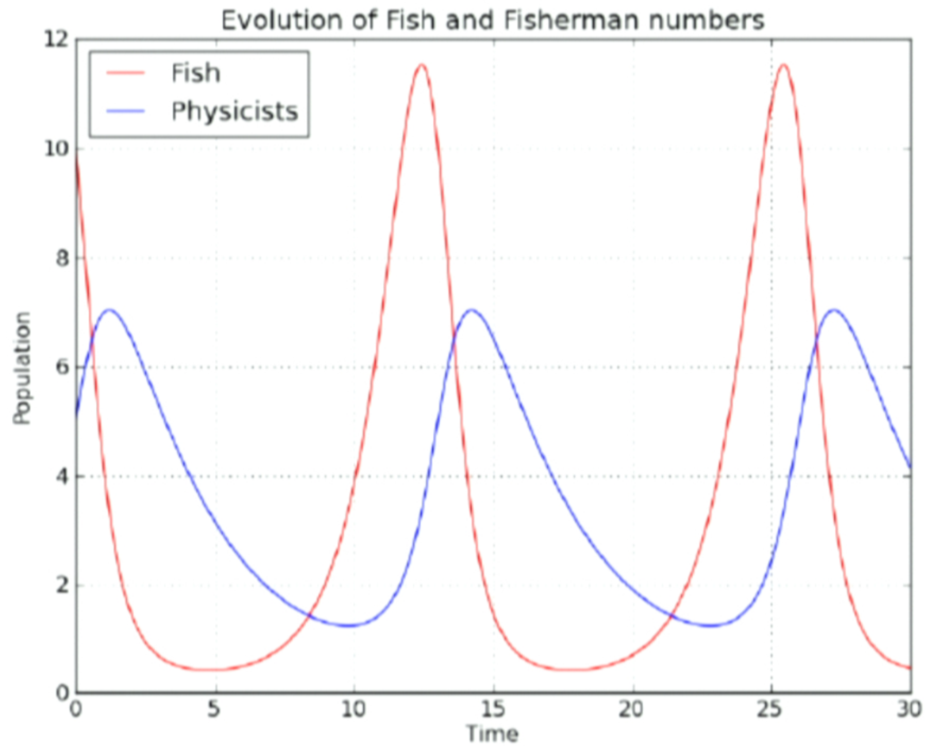

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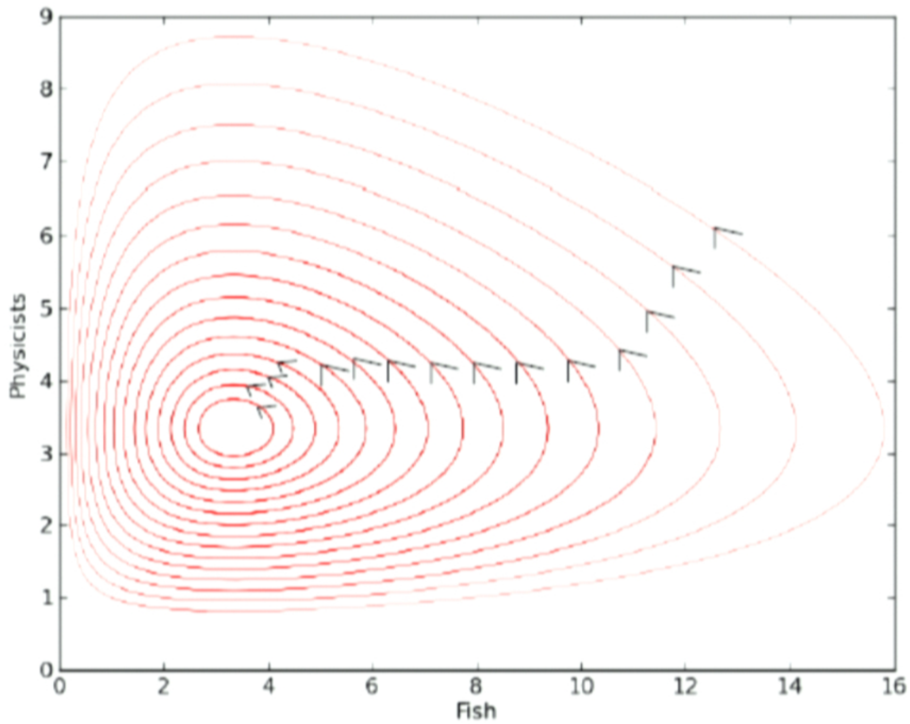
'Phase plane portrait'

$\beta = -0.3$



Behaviour without computation

- We can try to draw this picture without solving the system.

BEHAVIOUR WITHOUT COMPUTATION

- We can try to draw this picture without solving the system.

$$\dot{x} = \alpha x - \beta x p$$
$$\dot{p} = -\gamma p + \delta x p$$

- Linearise the system about fixed points and try to learn the behaviour this way.
- Fixed points are at $(0, 0)$ and $(\frac{\gamma}{\delta}, \frac{\alpha}{\beta})$

Linearising a nonlinear system

- We start with a system of ODEs:

$$\mathbf{x}' = \mathbf{F}(\mathbf{x})$$

- Then find a critical point and Taylor expand the right hand side of the system:

$$\mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{x}_0) \simeq D\mathbf{F}(\mathbf{x}_0)(\mathbf{x} - \mathbf{x}_0)$$

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- Then as $\mathbf{x_0}$ is fixed in time:

$$(\mathbf{x} - \mathbf{x_0})' = D\mathbf{F}(\mathbf{x_0})(\mathbf{x} - \mathbf{x_0})$$

and we have a linear system.

BEHAVIOUR NEAR FIXED POINTS

- There is a complete classification of behaviours for 2D linear systems.
- The behaviour is determined by the (complex) eigenvalues of the system, and the eigenvectors give an orientation.
- The trajectories behave as $\exp(\lambda t)$ and so we see oscillatory and exponential increasing and decreasing behaviour - as well as a combination of the two.

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LINEARISING A NONLINEAR SYSTEM

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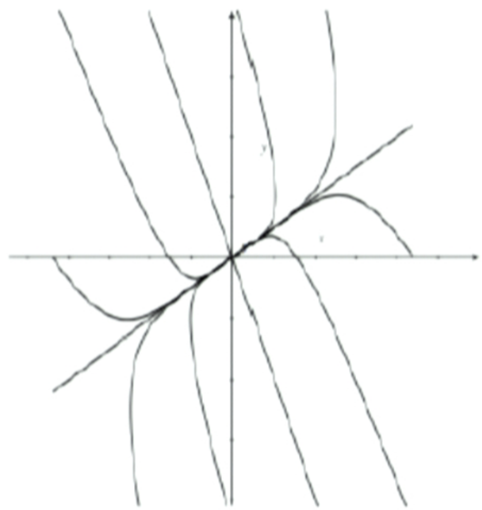
Behaviour near fixed points

Two different real eigenvalues of the same sign:



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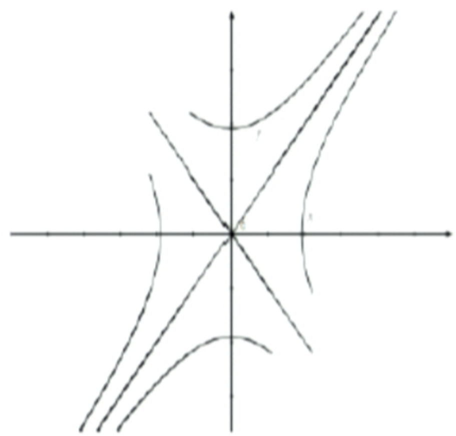
BEHAVIOUR NEAR FIXED POINTS

Two real eigenvalues of opposite sign:



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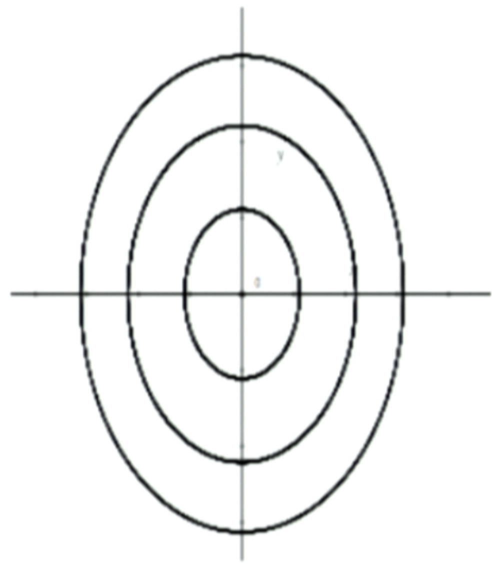
BEHAVIOUR NEAR FIXED POINTS

Two pure imaginary eigenvalues:



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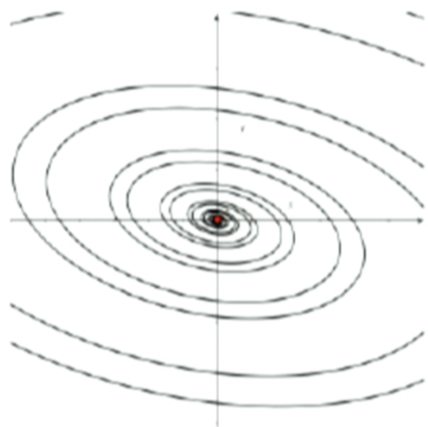
BEHAVIOUR NEAR FIXED POINTS

Two complex eigenvalues:



These pictures are due to Valeriy Slastikov.

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BEHAVIOUR WITHOUT COMPUTATION

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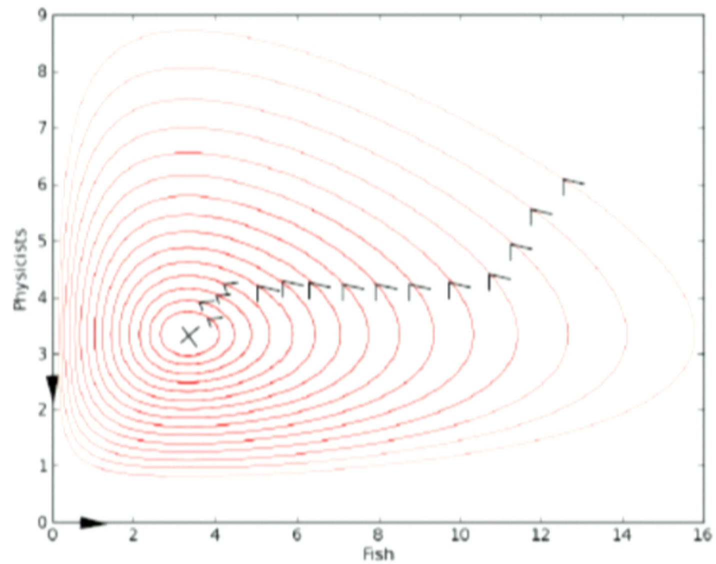
BEHAVIOUR NEAR FIXED POINTS

- Can see the stable and unstable directions here:

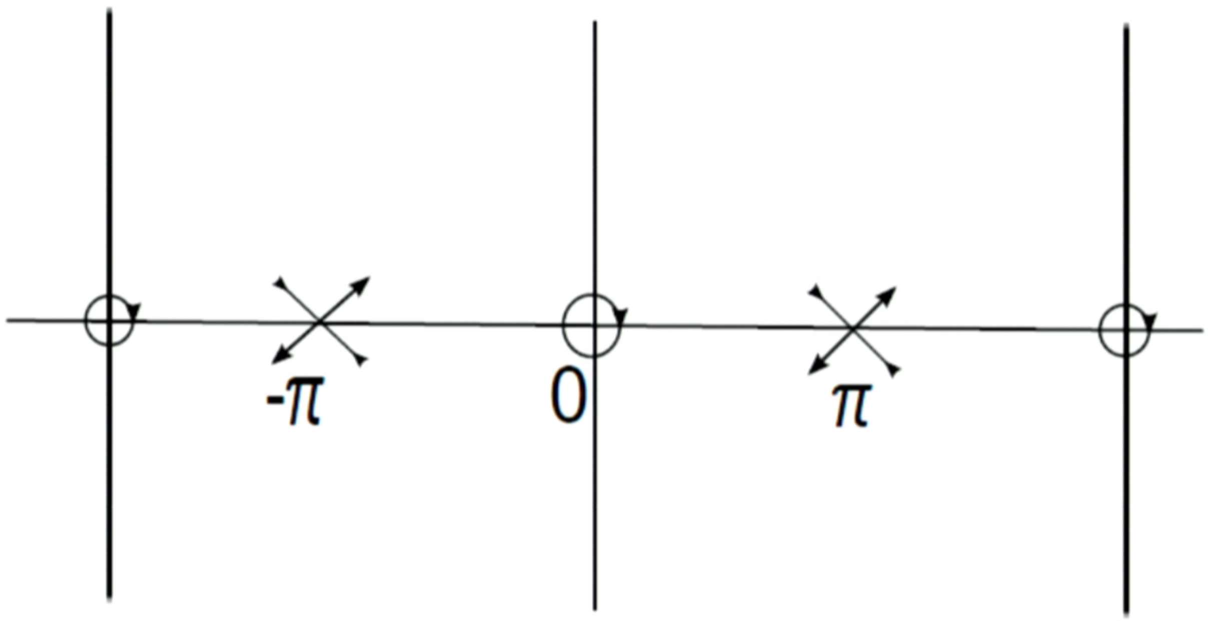


- also the other fixed point.

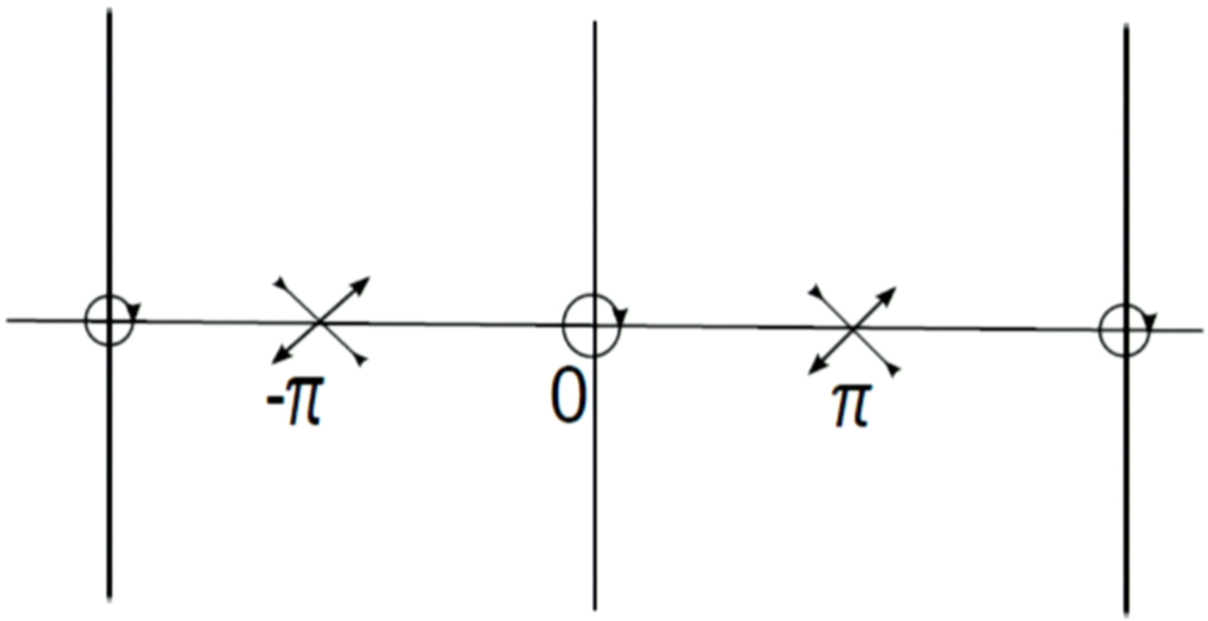
BEHAVIOUR NEAR FIXED POINTS

- The other fixed point has a Jacobian with imaginary eigenvalues.
- The behaviour of the local linearised system is a centre - and this is the same in the nonlinear system as we can see from the simulation - but need to do more work to transfer this stability. (Actually solve the system).

THE PENDULUM



THE PENDULUM



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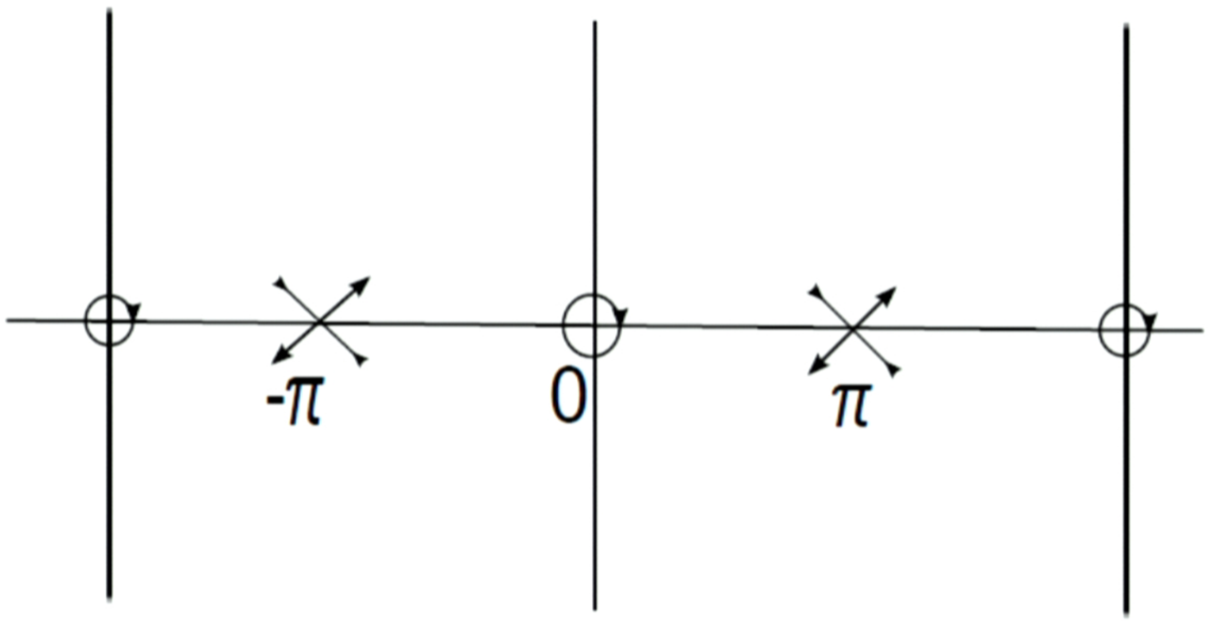
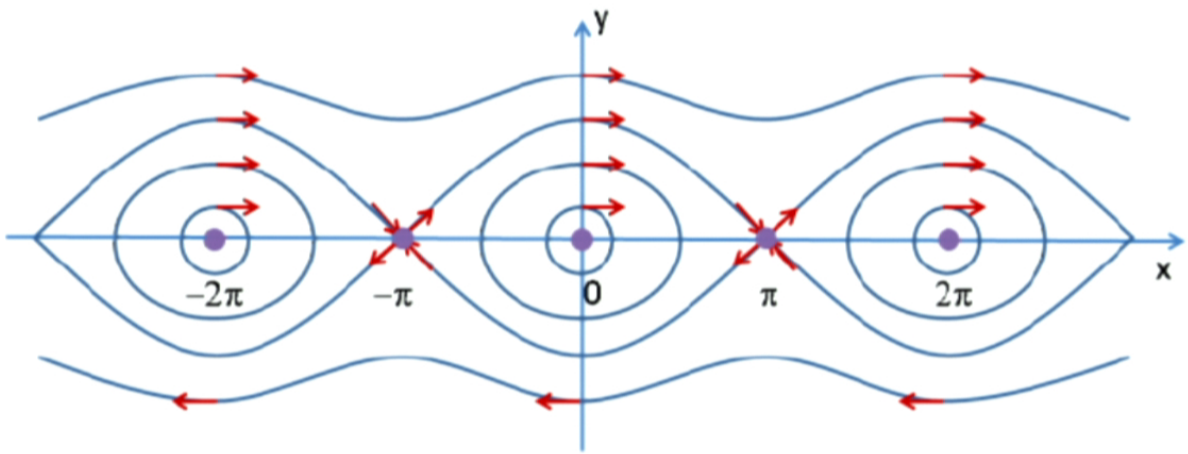


THE PENDULUM



Picture due to mathematicalgarden.wordpress.com.

CONCLUSION

- Have seen how linearisation helps us to find behaviour of 2D dynamical systems.

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- Have seen that the no fishing sign by the lake is not necessary to maintain the fish population (if the predator/prey model is a good one..)

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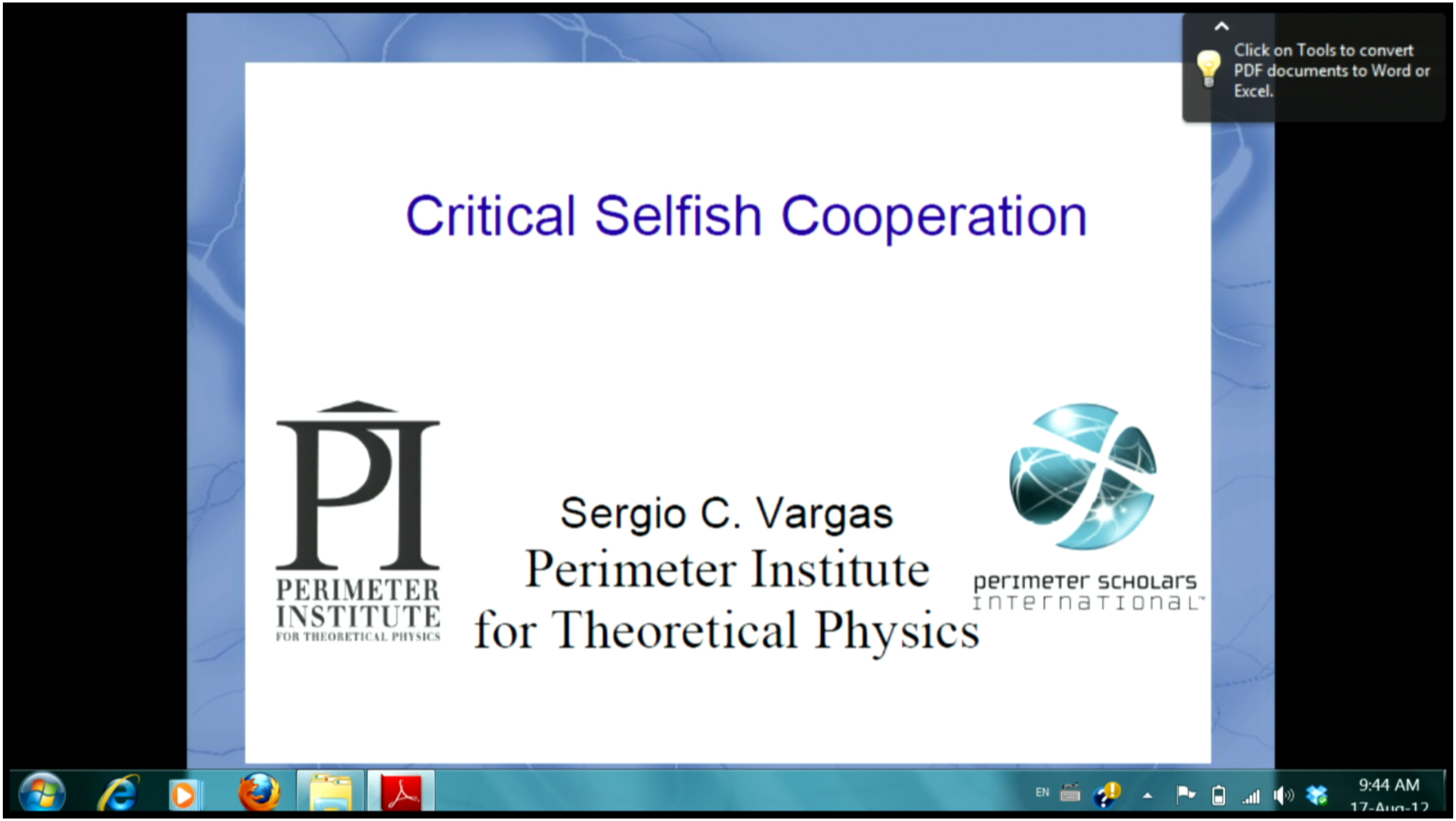
Conclusion

- Have seen how linearisation helps us to find behaviour of 2D dynamical systems.
- Have seen that the no fishing sign by the lake is not necessary to maintain the fish population (if the predator/prey model is a good one..)
- Thanks for listening.


Critical Selfish Cooperation
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS
Sergio C. Vargas
Perimeter Institute
for Theoretical Physics
perimeter scholars international
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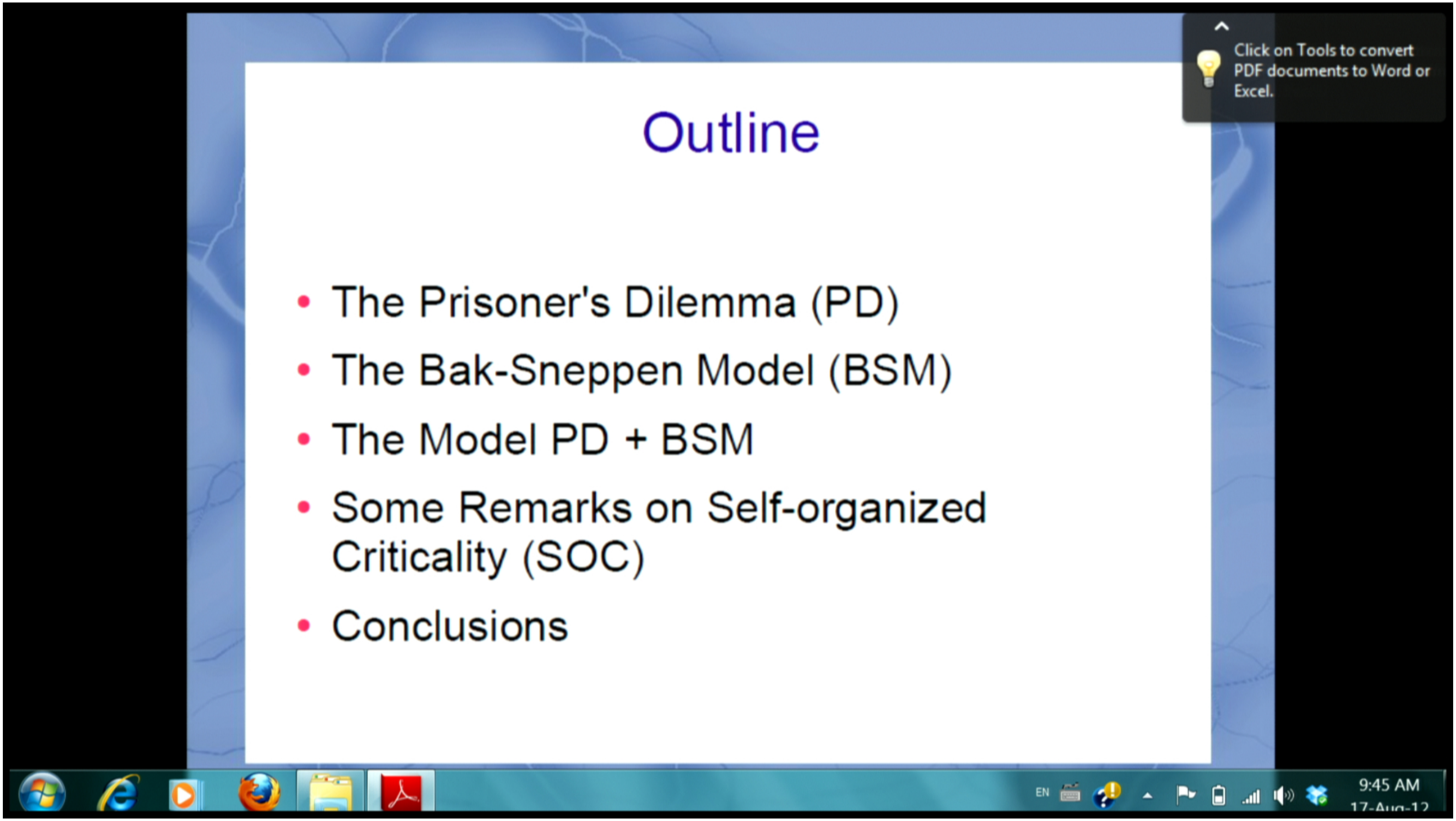
Motivations

How does Altruistic behavior emerge from mechanics that are essentially selfish (Natural Selection)?

Why does evolution take place in terms of intermittent bursts of activities?


Outline
The Prisoner's Dilemma (PD)
The Bak-Sneppen Model (BSM)
The Model PD + BSM
Some Remarks on Self-organized Criticality (SOC)
Conclusions
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The Prisoner's Dilemma I

Cooperate or Defect?

B GUY \ R GUY	COOPERATE	DEFECT
COOPERATE	A A	C B
DEFECT	B C	D D

B and R Pay-offs for each case.

$B > A > D > C$ $\qquad$ $2A > B + C$

The Bak-Sneppen Model I

A model of Co-evolution, which tries to explain key features of the Fossil Record:

- Extinction Events.
- Punctuated Equilibrium.

Critical Phenomena?

The Model PD + BSM

Jeong, Park, arXiv:1011.2013v4 [physics.data-an]

- Set *N* individuals in a region with periodic boundaries.
- Set a random Cooperation Probability (CP) for each one.
- Play PD a certain # of times between the individual and its closest neighbors accordingly.

The Model PD + BSM II

- Eliminate the individuals that hold the minimum fitness (the sum of its pay-offs). Also, there is a probability *w* of elimination of its closest neighbors.
- Replace those by individuals with random CP.
- Play PD again...

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Pay-offs

- Cost of cooperation: 1
- Benefit of cooperation: b (> 1)

B GUY \ R GUY	COOPERATE	DEFECT
COOPERATE	b b	0 b+1
DEFECT	b+1 0	1 1

Rescaled Pay-offs.

Fitness

- Consider a 1-D alignment of *N* individuals (i.e. 2 Neighbors) labeled with an integer index *i*.
- The fitness at each cycle is calculated with the use of the CPs (c_i):

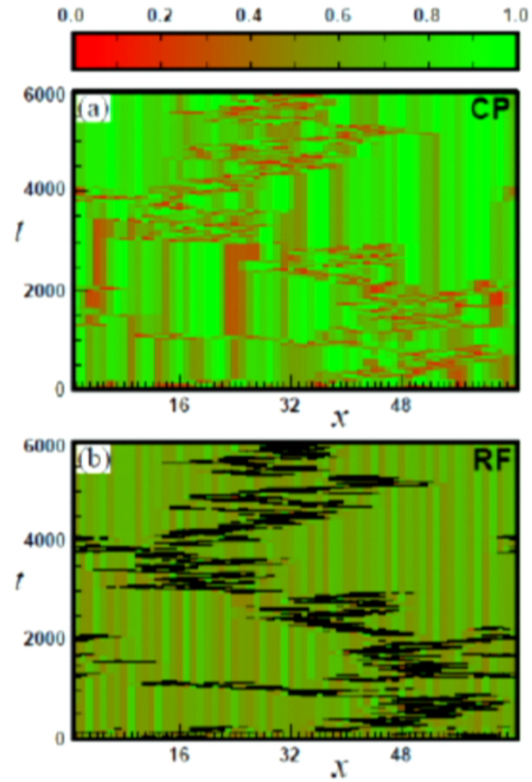
$$f_{(i)} = b[c_{(i-1)} - c_{(i+1)}] + 2[1 - c_{(i)}]$$

- It is defined the Reduced Fitness (RF) as

$$Reduf_{(i)} = f_{(i)} / (2b + 1)$$

Results I

Fig. 1. Real Space Configurations of the CP (a) and the RF (b) with $w = 1$, $b = 1.5$, and $N = 64$. The scale goes from 0 (red) to green (1). The black dots in (b) represent the least fit sites.



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Results II

- Consider a mean CP (MCP)

$$C_{(t)} = Average\left(\Sigma_i c_{(i)}\right) / N$$

, where the average is taken over several realizations of initial configurations.

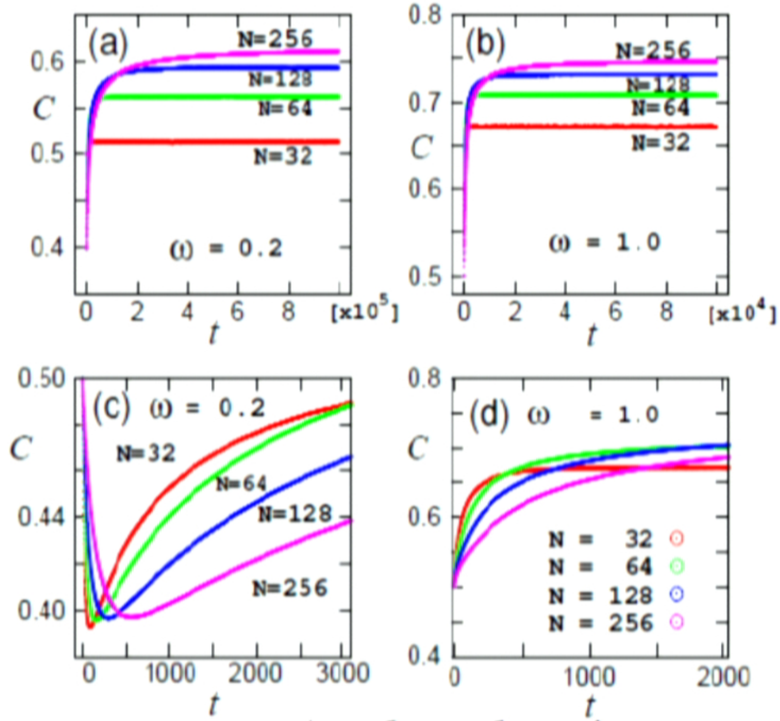
- Consider also the mean fitness

$$F_{(t)} = Average\left(\Sigma_i f_{(i)}\right) / N$$

- It can be shown that

$$F_{(t)} = 2 + 2(b-1) C_{(t)}$$

Results III



Avalanches !

Fig. 2. Time Dependance of the MCP for (a,c) w = 0.2 and (b,d) w = 1.0 for systems with N = 32, 64, 128 and 256. Overall (a,b) and transient (c,d) behaviors.

Remarks on Self-Organized Criticality (SOC)

- Emerging critical behaviors (failure of the statistical criteria of equilibrium... Thermodynamic fluctuations rule!).
- Power-law distributions of certain variables in the critical points (e.g. Thermodynamic potentials derivatives). Avalanche Size.
- This leads to a lack of a typical scale for the correlation functions. AVALANCHES!

Conclusions

- Cooperation can emerge as a SOC within the Natural Selection Process.

- Intermittent Bursts of activity are consistent with critical behaviors in this model.

Cooperation... It is Natural... It is the spirit of PSI!

Thank you!

The Physics of Trees

Xinyu Li

The total mass of air???

Can we estimate the value from this picture?

- Leaves on the top need water
- If waters are pumped by the atmospheric pressure, then

$$\rho_{\mathrm{H_2O}} g h = p_{\mathrm{atm}} \sim 10^5 \mathrm{Pa}$$

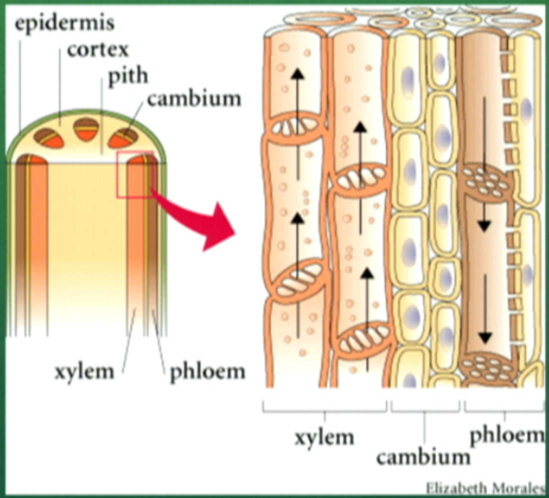
$$p_{\mathrm{atm}} = \frac{W_{\mathrm{atm}}}{S_{\mathrm{earth}}}$$

$$W_{\mathrm{atm}} = p_{\mathrm{atm}} \times 4\pi R_{\mathrm{earth}}^2 \sim 5.3 \times 10^{19} \mathrm{N}$$

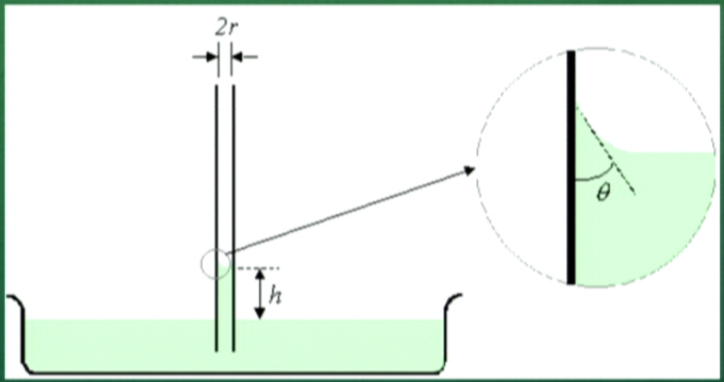
- Sealevel standard atmospheric pressure = 101,325 Pa
- It can only pump water to 10m high
- The tallest living tree known, named Hyperion is 115.5m
- How do the leaves on top get water?

Capillary action

- Narrow tube draw liquid into itself via the adhesive force between the liquid and the tube wall
- Xylem draw water in trees



- Model a tree as a single tube of height h with circular cross section of radius r
- Consider the balance between gravity and the adhesive force
- Let γ be the surface tension of water, θ be the contact angle



- The gravity

$$F_{\mathrm{g},1} = \rho \pi r^2 h g$$

- The adhesive force

$$F_{\mathrm{ad},1} = \gamma 2 \pi r \cos\theta$$

- The height derived from balance of two forces

$$h = \frac{2\gamma\cos\theta}{\rho g r} < 0.72\mathrm{m}\left(\frac{\gamma}{7.2\times10^{-2}\mathrm{N/m}}\right)\left(\frac{\mathrm{r}}{20\mu\mathrm{m}}\right)^{-1}$$

- The radius of xylem in trees is about 20 micro meter, thus it can support at most 0.72m height of water, much less than the height of the tallest tree

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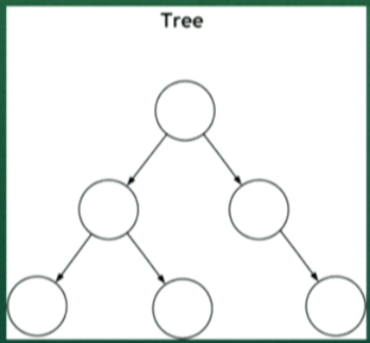
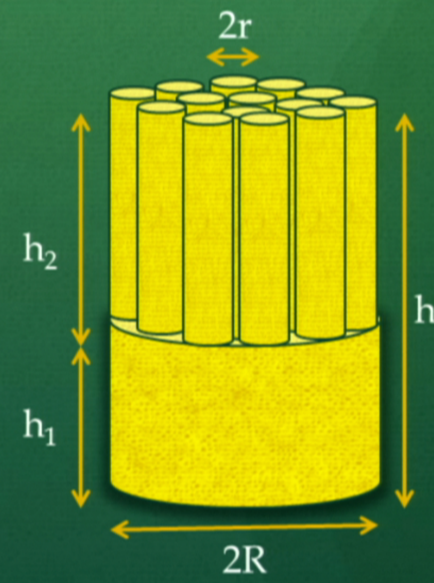
- The radius of xylem in trees is about 20 micro meter, thus it can support at most 0.72m height of water, much less than the height of the tallest tree

- However, a tree is not composed of parallel tubes of constant radius from bottom to top; trees will branch
- Graph theory: A tree is an undirected graph where any two vortices are connected by one simple path



- When the radius goes smaller, the height goes up
- Xylem in leaves can be as narrow as 5nm
- Model a tree of height h as a single tube of height h_1, radius R branching into N upper tubes of height h_2=H-h_1 and radius r



- The gravity

$$F_{\mathrm{g},1} = \rho \pi r^2 h g$$

- The adhesive force

$$F_{\mathrm{ad},1} = \gamma 2 \pi r \cos\theta$$

- The height derived from balance of two forces

$$h = \frac{2\gamma\cos\theta}{\rho g r} < 0.72\mathrm{m}\left(\frac{\gamma}{7.2\times 10^{-2}\mathrm{N/m}}\right)\left(\frac{\mathrm{r}}{20\mu\mathrm{m}}\right)^{-1}$$

- The radius of xylem in trees is about 20 micro meter, thus it can support at most 0.72m height of water, much less than the height of the tallest tree

- Define η by

$$\eta = \frac{N\pi r^2}{\pi R^2}$$

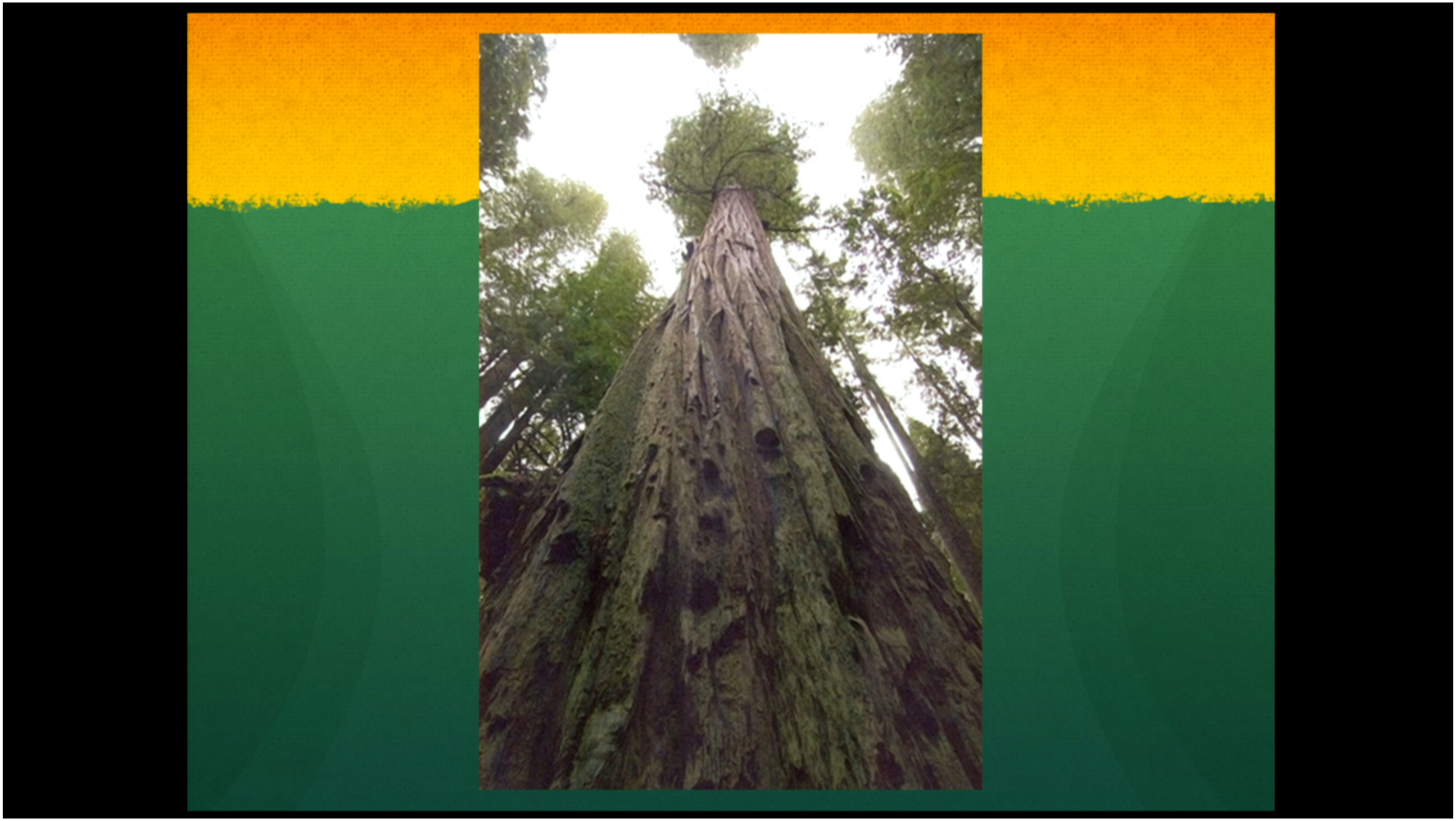
- For 2D hexagonal packed arrangements, η≈0.9
- The total upward force

$$F_{\rm ad,N} = N\gamma 2\pi r \cos\theta \approx \gamma 2\pi R\sqrt{\eta N} = \sqrt{\eta N} F_{\rm ad,1}$$

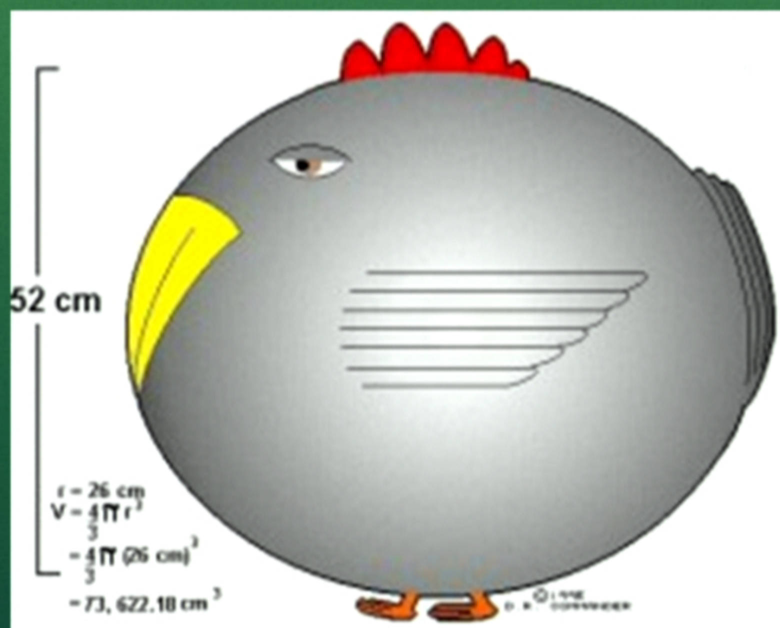
- The gravitational force

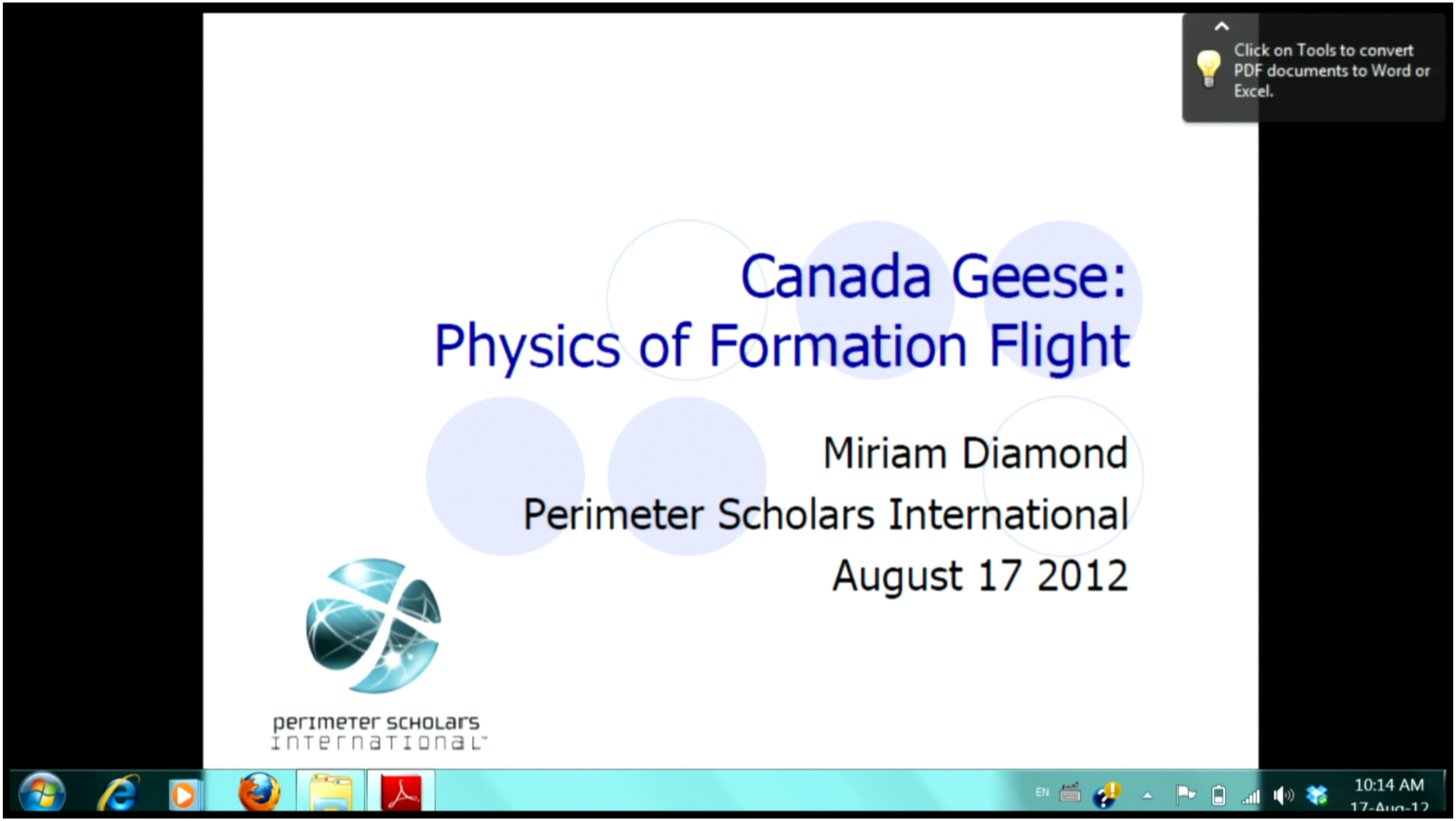
$$F_{\rm g,N} = \rho g(h_1 \pi R^2 + N h_2 \pi r^2) = F_{\rm g,1}\left(1 - (1-\eta)\frac{h_2}{h}\right)$$

- The upward force increases when N increases, independent of h_2
- The gravitational force increases when h_2 decreases
- To maximize the height h and the amount of water needs N to be large with h_2 small
- **Conclusion: very tall trees should have many branches very high up and few below**



“spherical chicken in a vacuum”

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Canada Geese:
Physics of Formation Flight
Miriam Diamond
Perimeter Scholars International
August 17 2012
perimeter scholars international
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Flock of Canada Geese

Branta canadensis

Motivation

- Canada Geese are a national symbol!
- On the long migration south each autumn and north each spring, flock flies in a V-formation (skein)
- Much multi-disciplinary research, both theoretical and observational, has been conducted to determine why
- Uncovering the answers helps us improve our understanding of aerodynamics

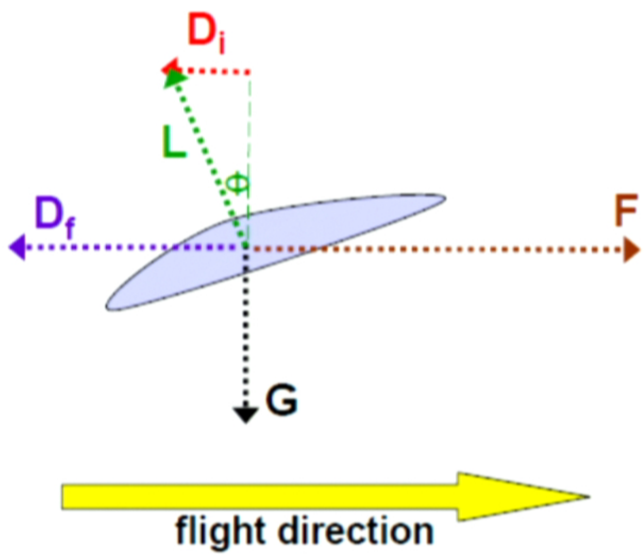
Outline

- Relevant aerodynamics for flight of a single bird
- Aerodynamic effects of formation flight
- Skein parameters
- Additional factors

Relevant Aerodynamics: Single Bird Flight

Begin with basic case of fixed-wing aircraft. Major forces at work:

- Gravity **G**
- Forward drive **F**
- Friction drag $\mathbf{D_f}$
- Lift **L**
- Induced drag $\mathbf{D_i}$

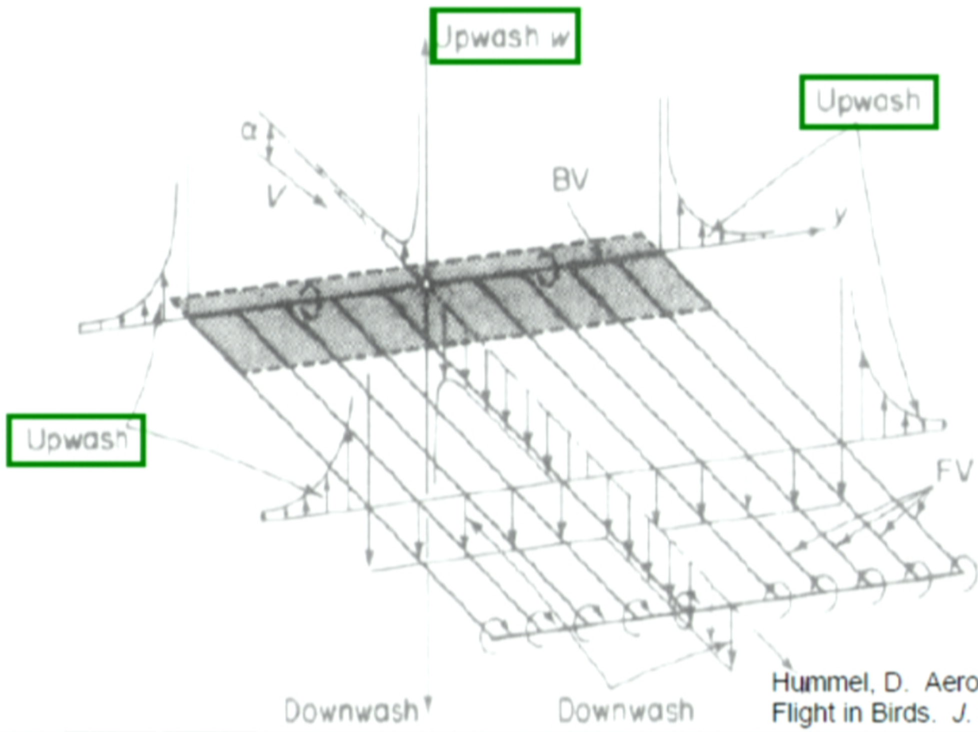


Relevant Aerodynamics: Single Bird Flight

- Power demand N for constant flight speed V:
  $N = (D_f + D_i)V$
- What's most relevant here is the induced drag: flying in formation will reduce it
- "Fixed-wing analog": use fixed-wing aircraft to model a flapping bird
  - Flapping losses (extra power required to flap the wing) are low, so long as speed of flapping wing tip is low compared to V

Relevant Aerodynamics: Single Bird Flight

- Wing acts as system of free and bound vortices

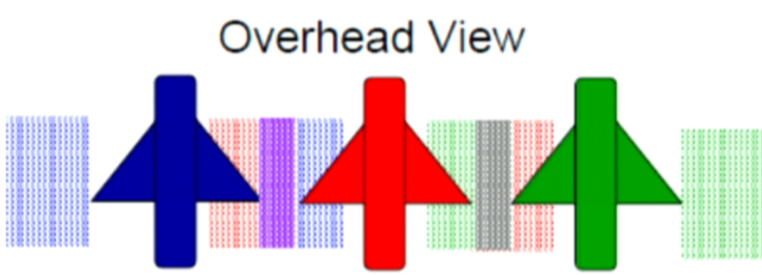


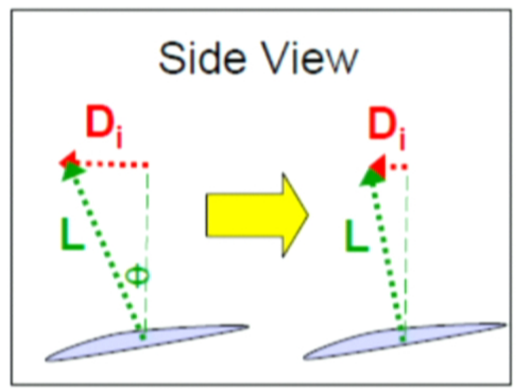
Vortex: tube of circulating air, with spinning low pressure core

Hummel, D. Aerodynamic Aspects of Formation Flight in Birds. *J. theor. Biol.* (1983) **104**, 321-347

Aerodynamic Effects of Formation Flight

- First consider multiple birds, flying beside each other in linear formation
- Each bird flies in the upwash field of its neighbours
- This decreases Φ, reducing induced drag





Aerodynamic Effects of Formation Flight

- Drag reduction: $-\Delta D = \frac{L\bar{w}}{V}$

  Value of upwash field present, averaged over wing surface

- Power demand reduction:

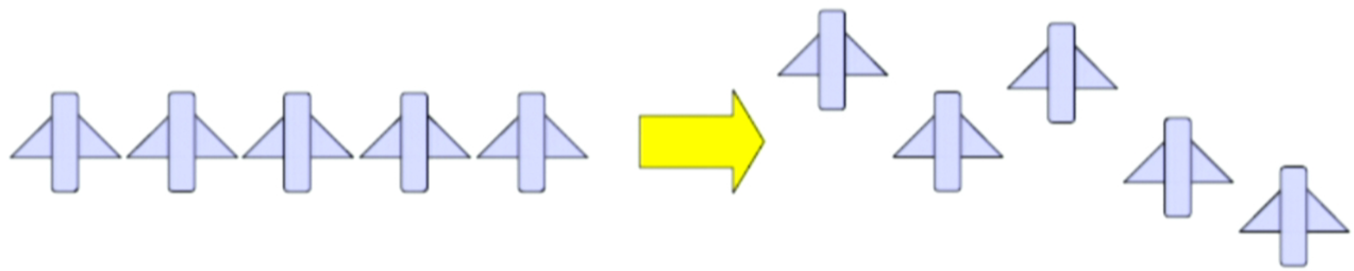
$$-\Delta N = (-\Delta D)V = L\bar{w}$$

- Relative flight power reduction:

$$e = \frac{-\Delta N}{N} = \frac{L\bar{w}}{VD}$$

Aerodynamic Effects of Formation Flight

- Same benefit applies to many other formations, including V
- Due to Munk's Stagger Theorem: For a system of lifting surfaces, elements may be displaced parallel to flight path without affecting total induced drag on the system

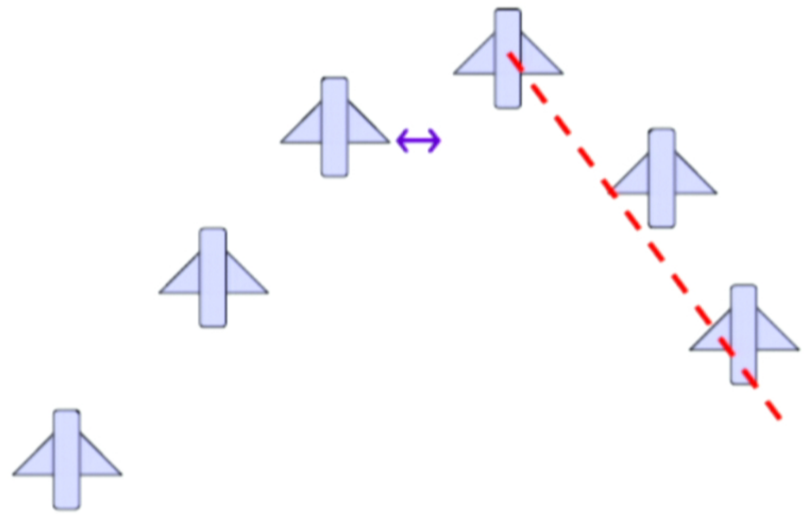


Aerodynamic Effects of Formation Flight

- V formation in particular has additional benefit of distributing the drag savings more equally amongst all members
  - Members in centre get upwash from neighbours on both sides
  - But members near tip gets more fully-developed upwash on the one side

Skein Parameters

- Tip spacing
  - Average value
  - Variation
- V angle
- Curvature
- # members
  - Total
  - Asymmetry



Skein Parameters

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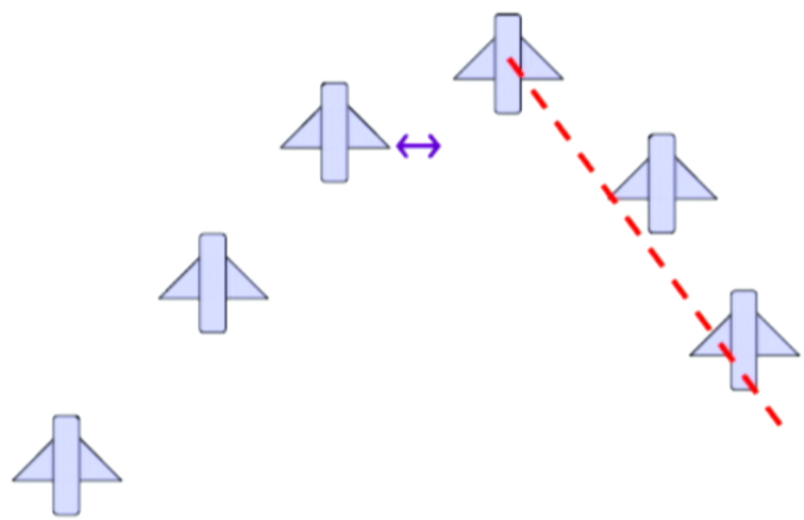
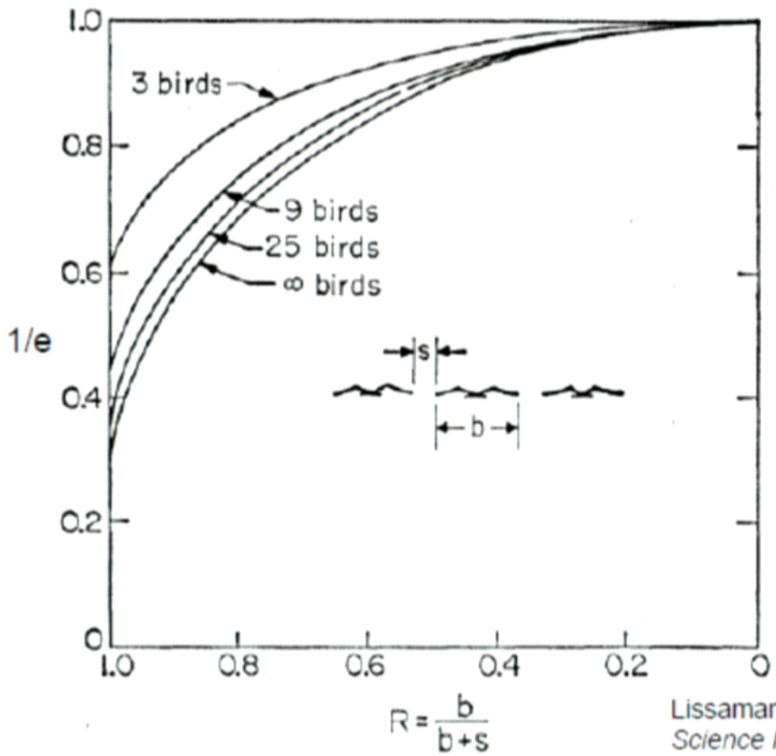


Skein Parameters



- Theoretical calculations have been performed using various parameter values
- Efforts made to observe parameter values in nature, and compare

Lissaman and Schollenberger, Formation Flight of Birds. *Science New Series* (1970) **168**:3934, 1003-1005.

Additional Factors

Aerodynamic considerations regarding individual birds in the formation:

- Differing wing properties amongst flock members
  - Span, weight, aspect ratio
- Angle, "twist" at which each member keeps its wings
  - Attaining elliptical loading
  - Avoiding rolling moment
- Wing-flapping phase (coordinated amongst flock members, or essentially random?)

Additional Factors

Issues that arise in nature:

- Shape of formation tends to change (and possibly break down) with time
- Any deviations from single plane of flight reduce aerodynamic benefits
- Non-aerodynamic considerations in flock behaviour
  - Visibility of fellow flock members
  - Avoiding collisions amongst flock members

Conclusions

- Flying in V formation provides aerodynamic advantages for Canada Geese
  - Reduces induced drag (each bird flying in upwash field of its neighbours)
  - Distributes drag savings fairly equally amongst flock members
- Skein has several parameters, which can be quantified and studied
- Aerodynamic advantage likely not the only factor in adoption of formation shape