

Title: Integral Transforms & Green's Functions - Lecture 4

Date: Aug 30, 2012 09:00 AM

URL: <http://pirsa.org/12080031>

Abstract:

$$\square G(\vec{R}, T) = \delta(\vec{R}) \delta(T)$$

$$G(\vec{R}, T) = \frac{1}{(2\pi)^4} \int d^4k \, G(\vec{R}, \omega) e^{i k \cdot x}$$

$$\square G(\vec{R}, T) = \delta(\vec{R}) \delta(T)$$

$$G(\vec{R}, T) = \frac{1}{(2\pi)^4} \int d^4 k \, G(\vec{k}, \omega) e^{i k \cdot x}$$

$$k = (\omega, \vec{k})$$

$$x = (T, \vec{R})$$

$$k \cdot x = -k^0 x^0 + k^1 x^1 + k^2 x^2 + k^3 x^3 \\ = -\omega T + \vec{k} \cdot \vec{R}$$

$$\square G(\vec{R}, T) = \delta(\vec{R}) \delta(T)$$

$$G(\vec{R}, T) = \frac{1}{(2\pi)^4} \int d^4 k \, G(\vec{k}, \omega) e^{i k \cdot x}$$

$$\square G = \frac{1}{(2\pi)^4} \int d^4 k \, G \, \square e^{-i\omega T + i\vec{k} \cdot \vec{R}} = \frac{1}{(2\pi)^4} \int d^4 k \, e^{i k \cdot x} \left[-x^0 + k^1 x^1 + k^2 x^2 + k^3 x^3 + \vec{k} \cdot \vec{R} \right]$$

$$k = (\omega, \vec{k})$$

$$x = (T, \vec{R})$$

$$k \cdot x = x^0 + k^1 x^1 + k^2 x^2 + k^3 x^3 + \vec{k} \cdot \vec{R}$$

$$\square G(\vec{R}, T) = \delta(\vec{R}) \delta(T)$$

$$G(\vec{R}, T) = \frac{1}{(2\pi)^4} \int d^4 k \, G(\vec{k}, \omega) e^{i k \cdot x}$$

$$k = (\omega, \vec{k})$$

$$x = (T, \vec{R})$$

$$k \cdot x = -k^0 x^0 + k^1 x^1 + k^2 x^2 + k^3 x^3$$

$$\square G = \frac{1}{(2\pi)^4} \int d^4 k \, G \, \square e^{-i\omega T + i\vec{k} \cdot \vec{R}} = \frac{1}{(2\pi)^4} \int d^4 k \, e^{i(-\omega T + \vec{k} \cdot \vec{R})}$$

PROPAGATOR

QM:

$$Z = \int \underbrace{D(q, \dot{q})}_{\text{PI MEASURE}} e^{i/\hbar \int_0^T dt L(q, \dot{q})}$$

PROPAGATOR

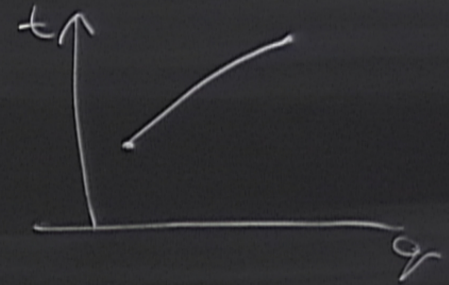
$$S = \int_0^T dt \left(\frac{1}{2} m \dot{q}^2 - V(q) \right)$$

QM:

$$Z = \int \underbrace{D(q, \hbar)}_{\text{PI MEASURE}} e^{i/\hbar \underbrace{\int_0^T dt L(q, \dot{q})}_{\text{CLASSICAL ACTION-S}}}$$

PROPAGATOR

$$S = \int_0^T dt \left(\frac{1}{2} m \dot{q}^2 - V(q) \right)$$



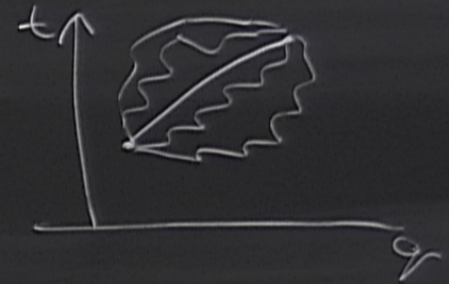
QM:

$$Z = \int \mathcal{D}q \, e^{i/\hbar \int_0^T dt L(q, \dot{q})}$$

CLASSICAL ACTION - S

PROPAGATOR

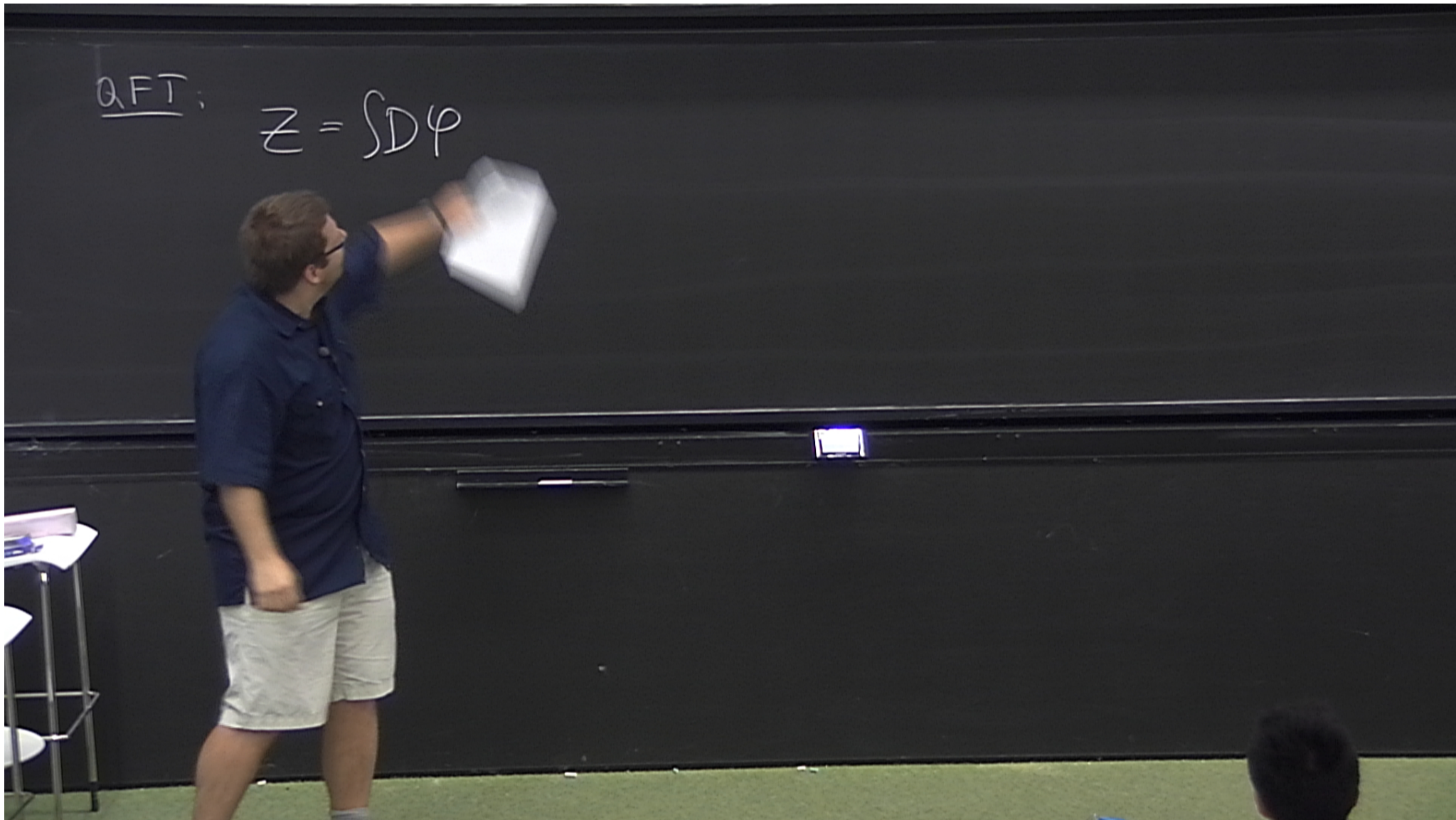
$$S = \int_0^T dt \left(\frac{1}{2} m \dot{q}^2 - V(q) \right)$$



QM:

$$Z = \int_{PI} D[q] e^{i \int_0^T dt L(q, \dot{q})}$$

CLASSICAL ACTION - S



QFT: $Z = \int D\varphi e^{\frac{i}{\hbar} \underbrace{\int d^4x \mathcal{L}(\varphi)}_{\text{CLASSICAL ACTION FOR THE FIELD } \varphi}}$

QFT:

$$Z = \int D\varphi e^{\frac{i}{\hbar} \int d^4x \mathcal{L}(\varphi)}$$

CLASSICAL ACTION FOR THE FIELD φ

$$i \int d^4x \left[-\frac{1}{2} (\partial\varphi)^2 - \frac{1}{2} m^2 \varphi^2 - V(\varphi) + J(x) \varphi(x) \right]$$

QFT:

$$Z = \int D\varphi e^{\frac{i}{\hbar} \int d^4x \mathcal{L}(\varphi)}$$

CLASSICAL ACTION FOR THE FIELD φ

$$\stackrel{\hbar=1}{=} \int D\varphi e^{i \int d^4x \left[\underbrace{-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - V(\varphi)}_{\text{FREE FIELD}} + J(x)\varphi(x) \right]}$$

QFT:

$$Z = \int D\varphi e^{\frac{i}{\hbar} \int d^4x \mathcal{L}(\varphi)}$$

CLASSICAL ACTION FOR THE FIELD φ

$$= \int D\varphi e^{i \int d^4x \left[\underbrace{-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2}_{\text{FREE FIELD}} - \underbrace{V(\varphi)}_{\text{INTERACTIONS}} + \int d^4x \varphi(x) J(x) \right]}$$

QFT:

$$Z = \int D\varphi e^{\frac{i}{\hbar} \int d^4x \mathcal{L}(\varphi)}$$

CLASSICAL ACTION FOR THE FIELD φ

$$\stackrel{|\hbar=1|}{=} \int D\varphi e^{i \int d^4x \left[\underbrace{-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2}_{\text{FREE FIELD}} - \underbrace{V(\varphi)}_{\text{INTERACTIONS}} + \int d^4x \varphi(x) J(x) \right]}$$

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QFT:

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CLASSICAL ACTION FOR THE FIELD φ

$$\stackrel{\hbar=1}{=} \int D\varphi e^{i \int d^4x \left[\underbrace{-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2}_{\text{FREE FIELD}} - \underbrace{V(\varphi)}_{\text{INTERACTIONS}} + \underbrace{J(x)\varphi(x)}_{\text{SOURCE}} \right]}$$

$\mathcal{L}$ LAGRANGIAN DENSITY

∂

QFT:

$$Z = \int D\varphi e^{\frac{i}{\hbar} \int d^4x \mathcal{L}(\varphi)}$$

CLASSICAL ACTION FOR THE FIELD φ

$$\stackrel{\hbar=1}{=} \int D\varphi e^{i \int d^4x \left[\underbrace{-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2}_{\text{FREE FIELD}} - \underbrace{V(\varphi)}_{\text{INTERACTIONS}} + \underbrace{j(x)\varphi(x)}_{\text{SOURCE}} \right]}$$

$\mathcal{L}$ LAGRANGIAN DENSITY, $\partial = \partial_M = (\partial_t, \nabla)$

• CLASSICAL EOM. $\partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \varphi)} - \frac{\delta \mathcal{L}}{\delta \varphi} = 0 \quad (E-L)$

FREE FIELD: $\mathcal{L} = -\frac{1}{2} (\partial \varphi)^2 - \frac{1}{2} m^2 \varphi^2$

• CLASSICAL EOM. $\partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \varphi)} - \frac{\delta \mathcal{L}}{\delta \varphi} = 0$ (E-L)

FREE FIELD: $\mathcal{L} = -\frac{1}{2} (\partial \varphi)^2 - \frac{1}{2} m^2 \varphi^2$

$$-\partial \cdot \partial \varphi + m^2 \varphi = 0$$

$$\boxed{(\partial^2 - m^2) \varphi = 0}$$

K

QFT, $Z = \int D\varphi e^{\frac{i}{\hbar} \int d^4x \mathcal{L}(\varphi)}$

CLASSICAL ACTION FOR THE FIELD φ

$$|_{\hbar=1} = \int D\varphi e^{i \int d^4x \left[\underbrace{-\frac{1}{2}(\partial\varphi)^2}_{\text{FREE FIELD}} - \underbrace{\frac{1}{2}m^2\varphi^2}_{\text{INTERACTIONS}} - \underbrace{V(\varphi)}_{\text{SOURCE}} + \int d^4x J(x)\varphi(x) \right]}$$

$\mathcal{L}$ LAGRANGIAN DENSITY, $\partial = \partial_M = (\partial_t, \nabla)$

• CLASSICAL EOM, $\partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \varphi)} - \frac{\delta \mathcal{L}}{\delta \varphi} = 0$ (E-L)

FREE FIELD, $\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$

$$-\partial^2 \varphi + m^2 \varphi = 0$$

$$(\partial^2 - m^2)$$

KLEIN-GORDON EQ

• CLASSICAL EOM. $\partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \varphi)} - \frac{\delta \mathcal{L}}{\delta \varphi} = 0$ (E-L)

FREE FIELD: $\mathcal{L} = -\frac{1}{2} (\partial \varphi)^2 - \frac{1}{2} m^2 \varphi^2$

$$\boxed{(\partial^2 - m^2) \varphi = 0}$$

KLEIN-GORDON EQ

$-\partial \cdot \partial \varphi + m^2 \varphi = 0$ $(\partial \varphi) \cdot (\partial \varphi)$

• WANT Z FOR FREE FIELD

BY PARTS

$$Z = \int D\varphi \, e^{i \int d^4x \left[+\frac{1}{2} \varphi (\partial^2 - m^2) \varphi + J\varphi \right]}$$

(I)

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2}$

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} J}$

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} \cdot J}$

↑
INVERSE MATRIX

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} J}$

↑
INVERSE MATRIX

• $\langle \psi | A | \psi \rangle =$

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} \cdot J}$

$\uparrow$ INVERSE MATRIX

• $\langle \psi | A | \psi \rangle = \sum_{x, z} \overset{\psi(x)}{\langle \psi | x \rangle} \langle x | A | z \rangle \overset{\psi(z)}{\langle z | \psi \rangle}$

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} J}$

↑
INVERSE MATRIX

• $\langle \psi | A | \psi \rangle = \sum_{x,z} \overbrace{\langle \psi | x \rangle}^{\psi(x)} \underbrace{\langle x | A | z \rangle}_{A(x,z)} \overbrace{\langle z | \psi \rangle}^{\psi(z)}$

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} \cdot J}$

$\uparrow$ INVERSE MATRIX

• $\langle \psi | A | \psi \rangle = \sum_{x,z} \underbrace{\langle \psi | x \rangle \langle x | A | z \rangle \langle z | \psi \rangle}_{\delta(x-z) (\partial_x^2 - m^2)} =$

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} J}$

$\uparrow$ INVERSE MATRIX

• $\langle \varphi | A | \varphi \rangle = \sum_{x,z} \underbrace{\langle \varphi | x \rangle \langle x | A | z \rangle \langle z | \varphi \rangle}_{\delta(x-z) (\partial_{(x)}^2 - m^2)} = \int d^4x \varphi (\partial^2 - m^2) \varphi$

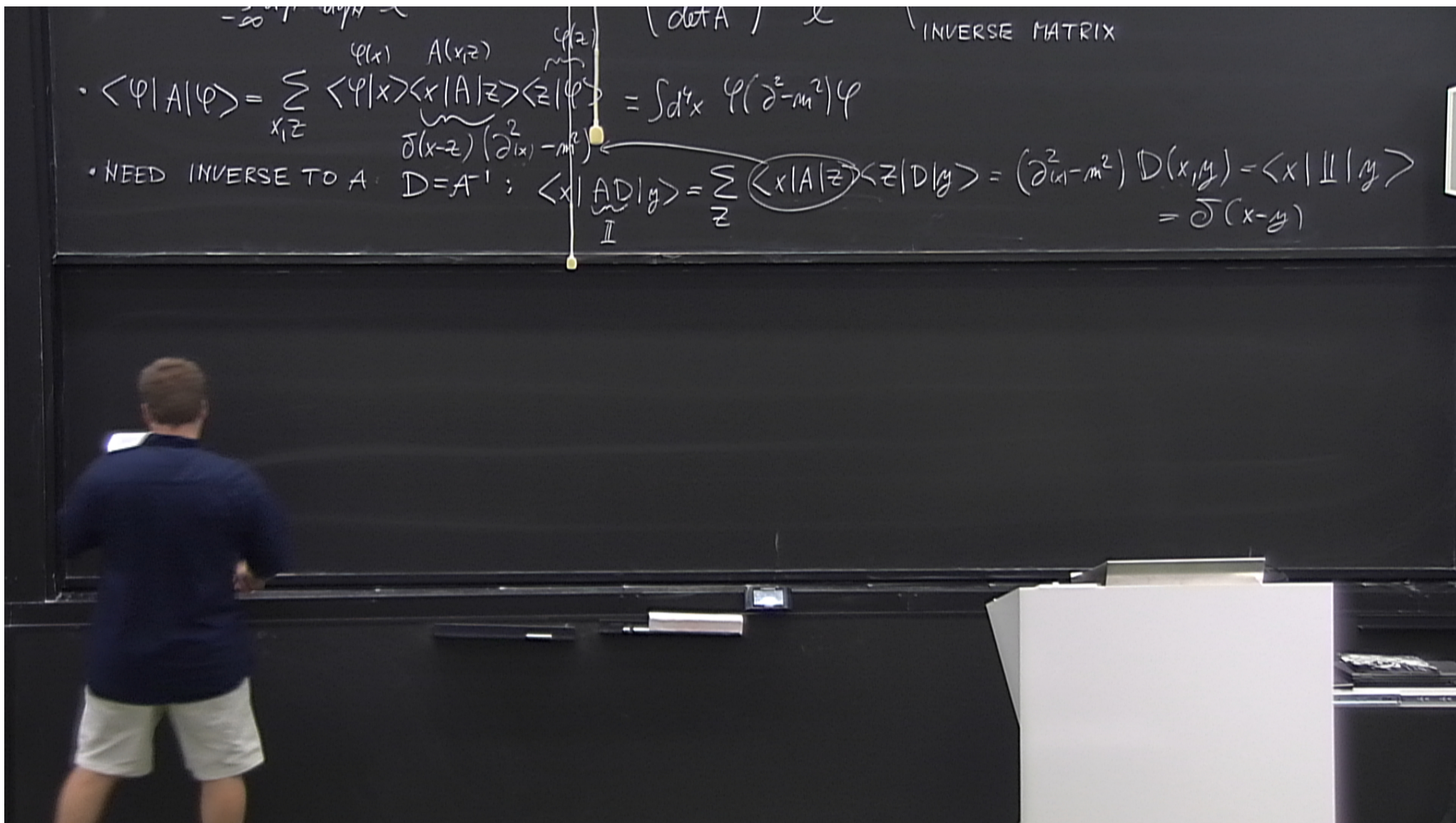
KLING-GORDON EQ

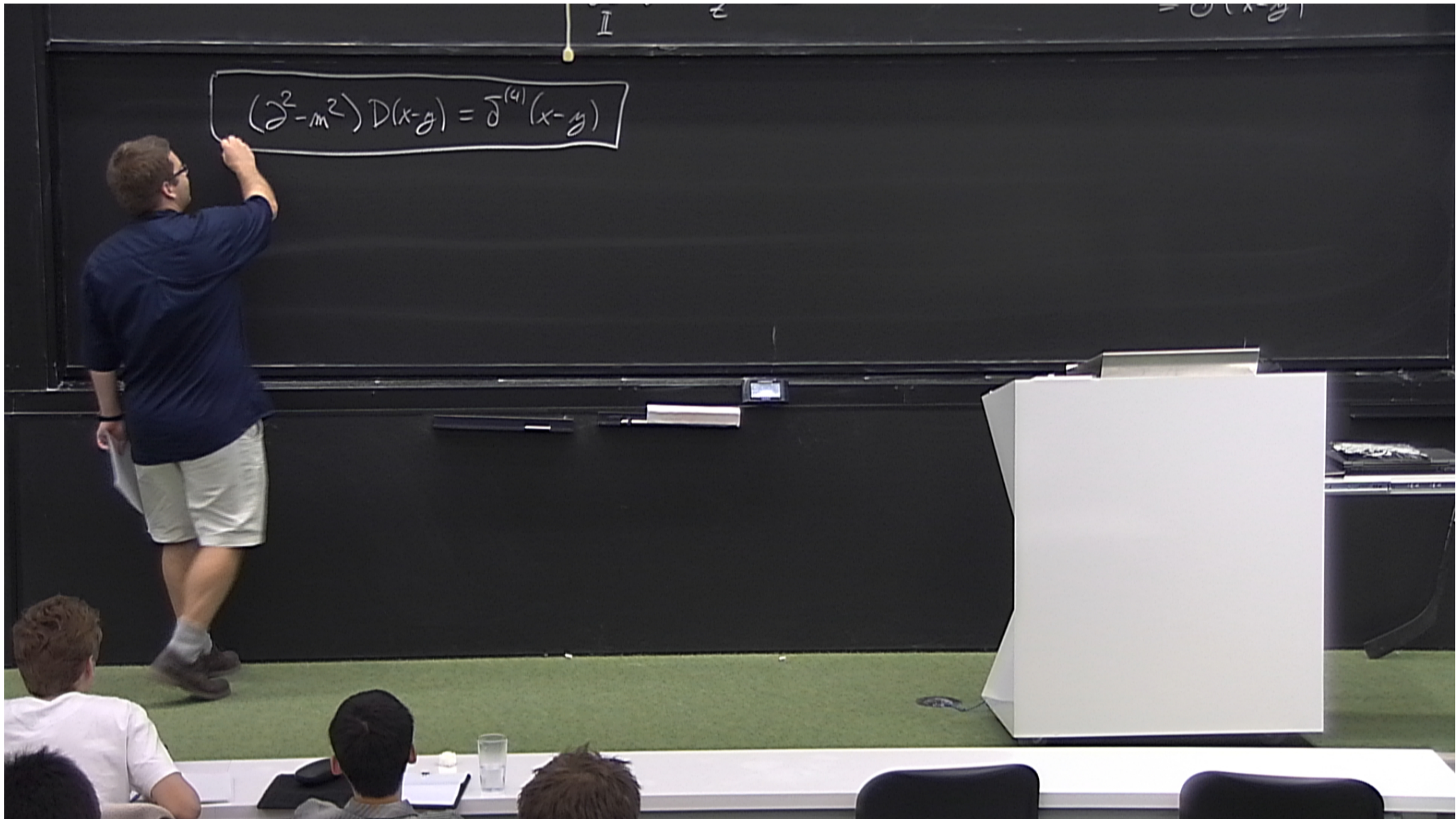
• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_n \exp \left(\frac{i}{2} q \cdot A \cdot q + i J \cdot q \right) = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} \exp \left(-\frac{i}{2} J \cdot A^{-1} \cdot J \right)$ ↑ INVERSE MATRIX

• $\langle \varphi | A | \varphi \rangle = \sum_{x,z} \underbrace{\langle \varphi | x \rangle \langle x | A | z \rangle \langle z | \varphi \rangle}_{\delta(x-z) (\partial_{(x)}^2 - m^2)} = \int d^4x \varphi (\partial_{(x)}^2 - m^2) \varphi$

• NEED INVERSE TO A

- REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{-\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} \cdot J}$
 $\uparrow$ INVERSE MATRIX
- $\langle \psi | A | \psi \rangle = \sum_{x,z} \underbrace{\langle \psi | x \rangle \langle x | A | z \rangle}_{\delta(x-z) (\partial_{(x)}^2 - m^2)} \langle z | \psi \rangle = \int d^4x \psi (\partial^2 - m^2) \psi$
- NEED INVERSE TO A: $D = A^{-1}$; $\langle x | \underbrace{A D}_{\mathbb{I}} | y \rangle$





$$\boxed{(\partial^2 - m^2) D(x-y) = \delta^{(4)}(x-y)}$$

↑
PROPAGATOR (GREEN'S FUNCTION)

REIND $\int_{-\infty}^{\infty} dq_1 \dots dq_n e^{i \int \varphi \cdot \Pi \cdot \varphi + i \int J \cdot \varphi} = \left(\frac{(2\pi i)^n}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} \cdot J}$ ↑ INVERSE MATRIX

$$\langle \varphi | A | \varphi \rangle = \sum_{x,z} \underbrace{\langle \varphi | x \rangle \langle x | A | z \rangle \langle z | \varphi \rangle}_{\delta(x-z) (\partial_{(x)}^2 - m^2)} = \int d^4x \varphi (\partial^2 - m^2) \varphi$$

• NEED INVERSE TO A $D = A^{-1}$; $\langle x | \underbrace{A D}_{I} | y \rangle = \sum_z \langle x | A | z \rangle \langle z | D | y \rangle = (\partial_{(x)}^2 - m^2) D(x,y) = \langle x | I | y \rangle = \delta(x-y)$

$$(\partial^2 - m^2) D(x-y) = \delta^{(4)}(x-y)$$

↑ PROPAGATOR (GREEN'S FUNCTION)

ONCE D IS KNOWN

$$Z[J] = Z(0) e^{-\frac{i}{2} \int d^4x d^4y J(x) D(x-y) J(y)}$$

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{-\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} \cdot J}$
 $\uparrow$ INVERSE MATRIX

• $\langle \varphi | A | \varphi \rangle = \sum_{x, z} \underbrace{\langle \varphi | x \rangle}_{\varphi(x)} \underbrace{\langle x | A | z \rangle}_{A(x, z)} \underbrace{\langle z | \varphi \rangle}_{\varphi(z)} = \int d^4x \varphi (\partial^2 - m^2) \varphi$

• NEED INVERSE TO A $D = A^{-1}$: $\langle x | A D | y \rangle = \sum_z \underbrace{\langle x | A | z \rangle}_{\delta(x-z)} \langle z | D | y \rangle = (\partial_{(x)}^2 - m^2) D(x, y) = \langle x | \mathbb{1} | y \rangle = \delta(x - y)$

ONCE D IS KNOWN
 $Z() = Z(0) e^{-\frac{i}{2} \int d^4x d^4y J(x) D(x-y) J(y)}$
 PROPAGATOR (GREEN'S FUNCTION)

• REMIND $\int_{-\infty}^{\infty} dq_1 \dots dq_N e^{\frac{i}{2} q \cdot A \cdot q + i J \cdot q} = \left(\frac{(2\pi i)^N}{\det A} \right)^{1/2} e^{-\frac{i}{2} J \cdot A^{-1} \cdot J}$
 $\uparrow$ INVERSE MATRIX

• $\langle \varphi | A | \varphi \rangle = \sum_{x,z} \langle \varphi | x \rangle \underbrace{\langle x | A | z \rangle}_{\delta(x-z) (\partial_{(x)}^2 - m^2)} \langle z | \varphi \rangle = \int d^4x \varphi (\partial^2 - m^2) \varphi$

• NEED INVERSE TO A $D = A^{-1}$: $\langle x | \underbrace{A D}_{I} | y \rangle = \sum_z \langle x | A | z \rangle \langle z | D | y \rangle = (\partial_{(x)}^2 - m^2) D(x,y) = \langle x | 1 | y \rangle = \delta(x-y)$

ONCE D IS KNOWN
 $Z[J] = Z(0) e^{-\frac{i}{2} \int d^4x d^4y J(x) D(x-y) J(y)}$
 PROPAGATOR (GREEN'S FUNCTION)

$\langle J | D | J \rangle = \sum_{x,y} \langle J | x \rangle \langle x | D | y \rangle \langle y | J \rangle$



• FOR (I) TO CONVERGE, WE PUT $m^2 \rightarrow m^2 - i\epsilon$

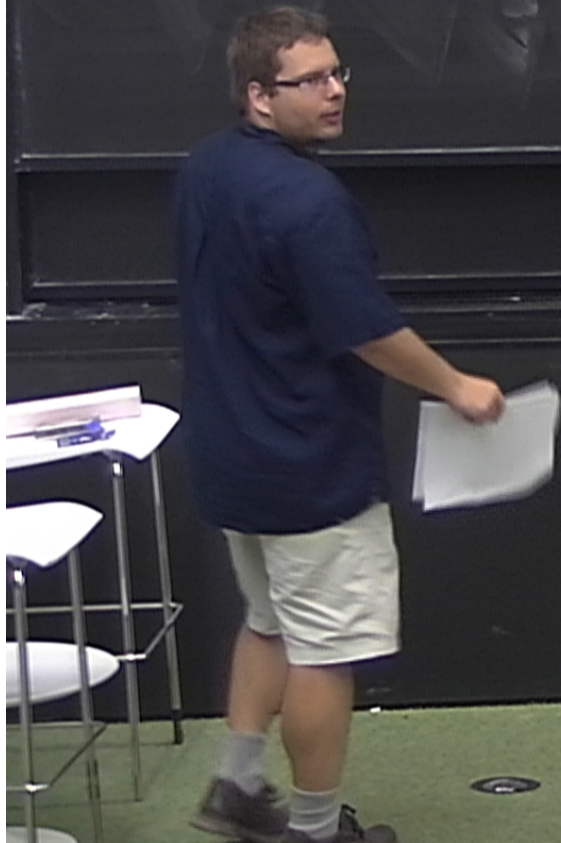
• SOLVE FOR (D) USING FT

$$\frac{1}{(2\pi)^4} \int d^4k D_k (\partial^2 - m^2 + i\epsilon) e^{ik \cdot x}$$

• FOR (I) TO CONVERGE, WE PUT $m^2 \rightarrow m^2 - i\epsilon$

• SOLVE FOR (D) USING FT

$$\frac{1}{(2\pi)^4} \int d^4k D_k (\partial^2 - m^2 + i\epsilon) e^{ik \cdot x} = \frac{1}{(2\pi)^4} \int d^4k e^{ik \cdot x}$$





- FOR (I) TO CONVERGE, WE PUT $m^2 \rightarrow m^2 - i\epsilon$

- SOLVE FOR (D) USING FT

$$\frac{1}{(2\pi)^4} \int d^4k \, D_k (\underbrace{\partial^2 - m^2 + i\epsilon}_{-k^2 - m^2 + i\epsilon}) e^{ik \cdot x} = \frac{1}{(2\pi)^4} \int d^4k \, e^{ik \cdot x}$$

$$\Rightarrow \boxed{D_k = \frac{1}{-k^2 - m^2 + i\epsilon}}$$

- FOR (I) TO CONVERGE, WE PUT $m^2 \rightarrow m^2 - i\epsilon$

- SOLVE FOR (D) USING FT

D(x):

$$\frac{1}{(2\pi)^4} \int d^4k \, D_k (\underbrace{\partial^2 - m^2 + i\epsilon}_{-k^2 - m^2 + i\epsilon}) e^{ik \cdot x} = \frac{1}{(2\pi)^4} \int d^4k \, e^{ik \cdot x}$$

$$\Rightarrow \boxed{D_k = \frac{1}{-k^2 - m^2 + i\epsilon}}$$

- GO BACK TO X-SPACE

$$k = (k^0, \vec{k}), \quad x = (t, \vec{x})$$

$$k^2 = -(k^0)^2 + \vec{k}^2, \quad \text{and}$$

- FOR (I) TO CONVERGE, WE PUT $m^2 \rightarrow m^2 - i\epsilon$

- SOLVE FOR (D) USING FT

D(x):

$$\frac{1}{(2\pi)^4} \int d^4k \, Dk \, (\underbrace{\partial^2 - m^2 + i\epsilon}_{-k^2 - m^2 + i\epsilon}) e^{ik \cdot x} = \frac{1}{(2\pi)^4} \int d^4k \, e^{ik \cdot x}$$

$$\Rightarrow \boxed{Dk = \frac{1}{-k^2 - m^2 + i\epsilon}}$$

- GO BACK TO X-SPACE

$$k = (k^0, \vec{k}), \quad x = (t, \vec{x})$$

$$k^2 = -(k^0)^2 + \vec{k}^2, \quad \omega_k = \sqrt{\vec{k}^2 + m^2}$$

$$\boxed{D(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik \cdot x}}{k^2 - \omega_k^2 + i\epsilon}}$$

INTEGRATE OVER Q^0

INTEGRATE OVER $\vec{q}$

$$D(x) = -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left[e^{-i(\omega_k t - \vec{k} \cdot \vec{x})} \Theta(t) \right]$$



INTEGRATE OVER q^0

$$D(x) = -i \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2\omega_k} \left[e^{-i(\omega_k t - \vec{k} \cdot \vec{x})} \Theta(t) + e^{i(\omega_k t + \vec{k} \cdot \vec{x})} \Theta(-t) \right]$$

INTEGRATE OVER d^3k

$$D(x) = -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left[e^{-i(\omega_k t - \vec{k} \cdot \vec{x})} \Theta(t) + e^{i(\omega_k t + \vec{k} \cdot \vec{x})} \Theta(-t) \right]$$

AMPLITUDE FOR A DI

INTEGRATE OVER $\vec{k}$

$$D(x) = -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left[e^{-i(\omega_k t - \vec{k} \cdot \vec{x})} \Theta(t) + e^{i(\omega_k t + \vec{k} \cdot \vec{x})} \Theta(-t) \right]$$

AMPLITUDE FOR A DISTURBANCE IN THE FIELD TO PROPAGATE FROM THE ORIGIN
TO x .

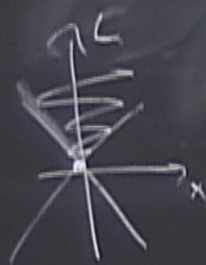
INTEGRATE OVER $\vec{k}$



$$D(x) = -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left[e^{-i(\omega_k t - \vec{k} \cdot \vec{x})} \Theta(t) + e^{i(\omega_k t + \vec{k} \cdot \vec{x})} \Theta(-t) \right]$$

AMPLITUDE FOR A DISTURBANCE IN THE FIELD TO PROPAGATE FROM THE ORIGIN
TO $\vec{x}$. DEPENDS ON WHETHER

INTEGRATE OVER k^0



$$D(x) = -i \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2\omega_k} \left[e^{-i(\omega_k t - \vec{k} \cdot \vec{x})} \Theta(t) + e^{i(\omega_k t + \vec{k} \cdot \vec{x})} \Theta(-t) \right]$$

AMPLITUDE FOR A DISTURBANCE IN THE FIELD TO PROPAGATE FROM THE ORIGIN TO $\underline{x}$. DEPENDS ON WHERE ON THE LIGHT CONE.



INTEGRATE OVER d^3k



$$D(x) = -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left[e^{-i(\omega_k t - \vec{k} \cdot \vec{x})} \Theta(t) + e^{i(\omega_k t + \vec{k} \cdot \vec{x})} \Theta(-t) \right]$$

AMPLITUDE FOR A DISTURBANCE IN THE FIELD TO PROPAGATE FROM THE ORIGIN TO $\underline{x}$. DEPENDS ON WHERE ON THE LIGHT CONE.

$$\bullet \quad x = (t, 0), t > 0$$

INTEGRATE OVER d^3k



$$D(x) = -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left[e^{-i(\omega_k t - \vec{k} \cdot \vec{x})} \Theta(t) + e^{i(\omega_k t + \vec{k} \cdot \vec{x})} \Theta(-t) \right]$$

AMPLITUDE FOR A DISTURBANCE IN THE FIELD TO PROPAGATE FROM THE ORIGIN TO $\underline{x}$. DEPENDS ON WHERE ON THE LIGHT CONE.

• $\underline{x} = (t, 0), t > 0$: SUPERPOSITION OF PLANE WAVES $\sim$ OSCILLATORY BEHAVIOR

TO x . DEPENDS ON WHERE ON THE LIGHT CONE.

• $x = (t, 0), t > 0$: SUPERPOSITION OF PLANE WAVES $\sim$ OSCILLATORY BEHAVIOR

• $x = (0, x)$. $D \sim e^{-m|\vec{x}|}$ (SEE ZEE I.3)

CLASSICALLY, A PARTICLE CANNOT GET OUTSIDE THE LIGHT CONE,

TO x . DEPENDS ON WHERE ON THE LIGHT CONE.

• $x = (t, 0), t > 0$: SUPERPOSITION OF PLANE WAVES $\sim$ OSCILLATORY BEHAVIOR

• $x = (0, x)$. $D \sim e^{-m|x|}$ (SEE ZEE I.3)

CLASSICALLY, A PARTICLE CANNOT GET OUTSIDE THE LIGHT CONE,
BUT QF CAN "LEAK" OUT OVER A DISTANCE $\frac{1}{m}$.

$$(\partial^2 - m^2) D(x-y) = \delta^{(4)}(x-y)$$

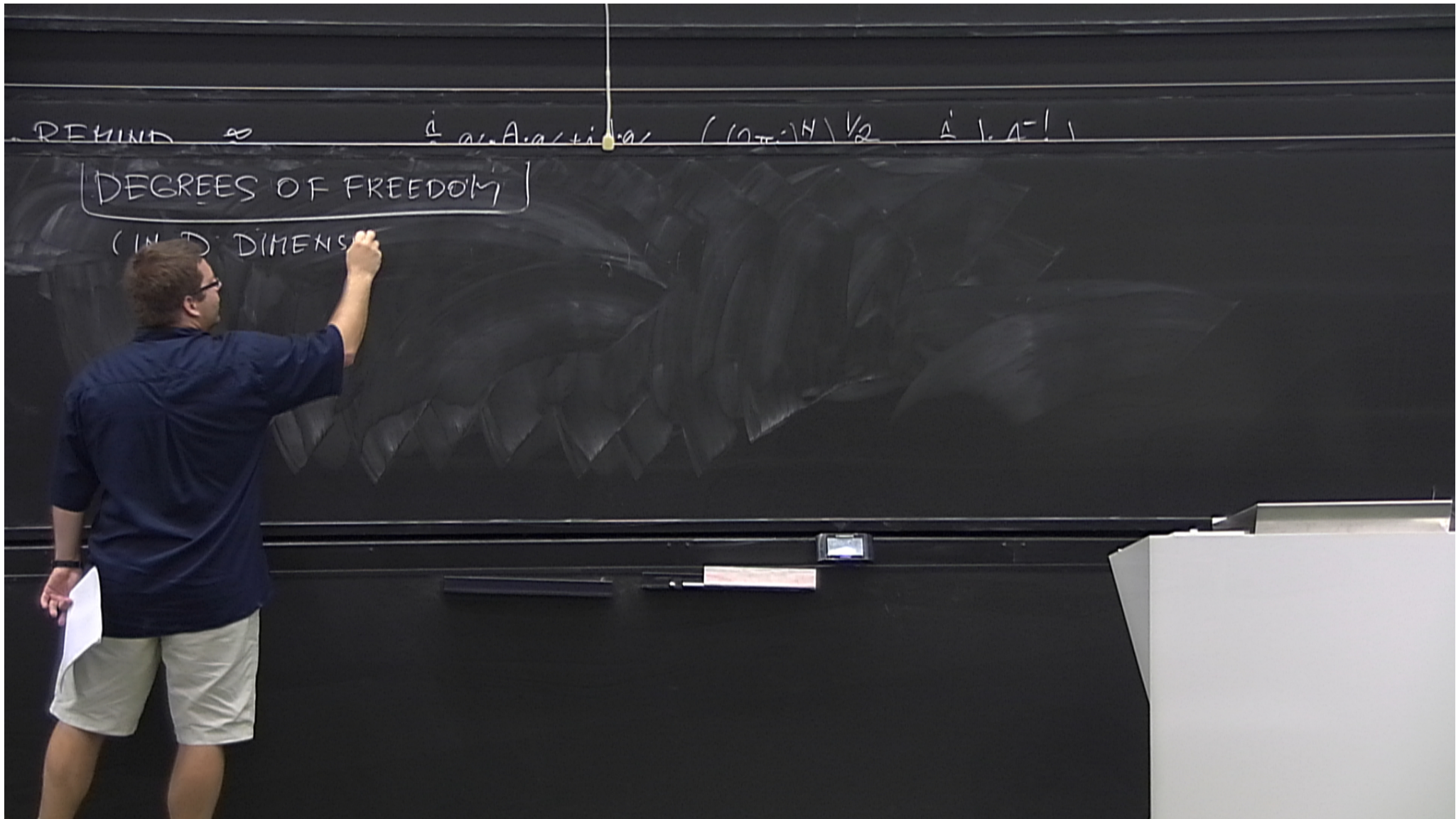
PROPAGATOR (GREEN'S FUNCTION)

ONCE D IS KNOWN

$$Z[J] = Z(0) \exp\left[-\frac{i}{2} \int d^4x d^4y J(x) D(x-y) J(y)\right]$$

$$\langle J | D | J \rangle = \sum_{x,y} \langle J | x \rangle \langle x | D | y \rangle \langle y | J \rangle$$

$$i D(x) = \langle 0 | T(\phi(x) \phi(0)) | 0 \rangle$$



REHIND ∞

$\frac{1}{2} \text{ action } (10\pi/4)^{1/2} \frac{1}{2} 1.4^{-1}$

DEGREES OF FREEDOM

(IN D DIMENSION)

a) SCALAR FIELD,

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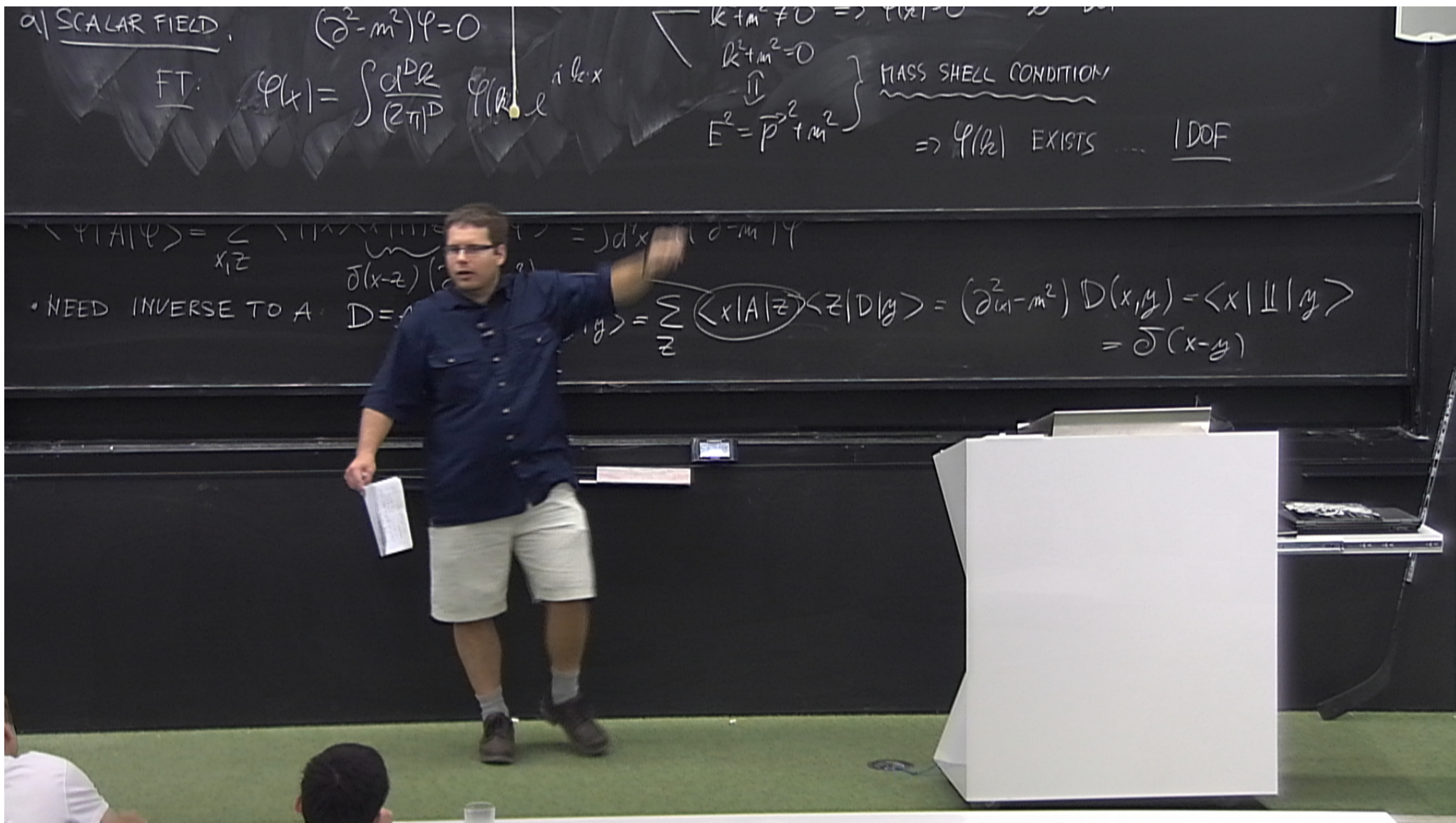
$$(\partial^2 - m^2)\varphi = 0$$

FT:

$$\varphi(x) = \int \frac{d^D k}{(2\pi)^D} \varphi(k) e^{i k \cdot x}$$

$$\Rightarrow \boxed{(k^2 + m^2) \varphi(k) = 0}$$

$$- k^2 + m^2 \neq 0 \Rightarrow \varphi$$



b) GAUGE FIELD

ROLE: IN ANY GAUGE TH.

DOF = # APPARENT DOF - 2x DOF OF THE GAUGE FUNCTION

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DOF = # APPARENT DOF - 2x DOF OF THE GAUGE FUNCTION

TO SHOW THIS EXPLICITLY, LET'S USE LIGHTCONE COORDINATES

$$A = (A^+, A^-, A^\perp), \quad A \cdot B = -A^+ B^- - A B^+ + A^\perp B^\perp \Leftrightarrow \mathcal{M} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & \mathbb{I} \end{pmatrix}$$

c) MAXWELL EM POTENTIAL A^μ

$$\partial^\mu \partial_\mu A^\nu - \partial^\nu (\partial_\mu A^\mu) = 0$$

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INVARIANT UNDER $\delta A_\mu = \partial_\mu \chi$

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GAUGE F.

INVARIANT UNDER $\delta A_\mu = \partial_\mu \chi$

$$\# \text{DOF} = D - 2 \times 1 = 2$$

• WRITING

$$A^\mu(x) = \int \frac{d^D p}{(2\pi)^D} A^\mu(p) e^{i p \cdot x}$$

c) MAXWELL EM POTENTIAL A^μ D

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GAUGE F,

INVARIANT UNDER $\boxed{\delta A_\mu = \partial_\mu \chi}$

$$\bullet \# \text{DOF} = D - 2 \times 1 = \underline{\underline{D-2}}$$

• WRITING

$$A^\mu(x) = \int \frac{d^D p}{(2\pi)^D} A^\mu(p) e^{i p \cdot x}$$

$$\boxed{p^2 A^\mu - p^\mu (p \cdot A) = 0}$$

$$\boxed{\partial \cdot A = i p_\mu \chi}$$

TO x . DEPENDS ON WHERE ON THE LIGHT CONE.
 $x = (t, \vec{x}) \rightarrow 0$ SUPERPOSITION OF PLANE WAVES & OSCILLATORY REMAINDER
 $\bullet p^+ \neq 0$ (WLOG) : $\partial A^+ = i p^+ \chi \Rightarrow$ CHOOSE χ SO THAT $\underline{A^+ = 0}$

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- $-p^+ A^- - p^- A^+ + p^I A^I = 0$

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 \end{aligned}$$

BUT QF CAN "LEAK" OUT OVER A DISTANCE $\frac{1}{m}$.

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 & \text{• } \boxed{p^2 A^I = 0} \quad \begin{cases} \text{ON SHELL } (p^2=0) \dots (D-2) \text{ DOF} \\ \text{OFF-11- } (p^2 \neq 0) \dots \text{ } \text{ } \text{ DOF} \end{cases}
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REMARKS: •

$-p^+ A^- - \cancel{p^- A^+} + p^I A^I = 0$

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ON SHELL ($p^2 = 0$) ... (D-2) DOF
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REMARKS: • WHEN $p^2 \neq 0$, DIVIDE
 $\Rightarrow A^\mu = \frac{1}{p^2} p^\mu (p \cdot A)$

• $p^2 A^I = 0$ $\begin{cases} \text{ON SHELL } (p^2 = 0) \dots (D-2) \text{ DOF} \\ \text{OFF-SHELL } (p^2 \neq 0) \dots 0 \text{ DOF} \end{cases}$

AMPLITUDE FOR A DISTURBANCE IN THE FIELD TO PROPAGATE FROM THE ORIGIN

• $p^+ \neq 0$ (WLOG) : $\partial A^+ = i p^+ \chi \Rightarrow$ CHOOSE χ SO THAT $A^+ = 0$

REMARKS : • WHEN $p^2 \neq 0$, DIVIDE BY IT

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PURE GAUGE ($\chi = -i \frac{p \cdot A}{p^2}$)

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ON SHELL
OFF-SHELL

(D-2) DOF
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REMARKS: • WHEN $p^2 \neq 0$, DIVIDE BY IT

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PURE GAUGE ($\chi = -i \frac{p \cdot A}{p^2}$)

• 1 PHOTON STATE

$$\sum_{I=2}^D \sum_{\mu=1}^2 \langle A_{\mu I}^I | \dots \rangle$$

POLARIZATION