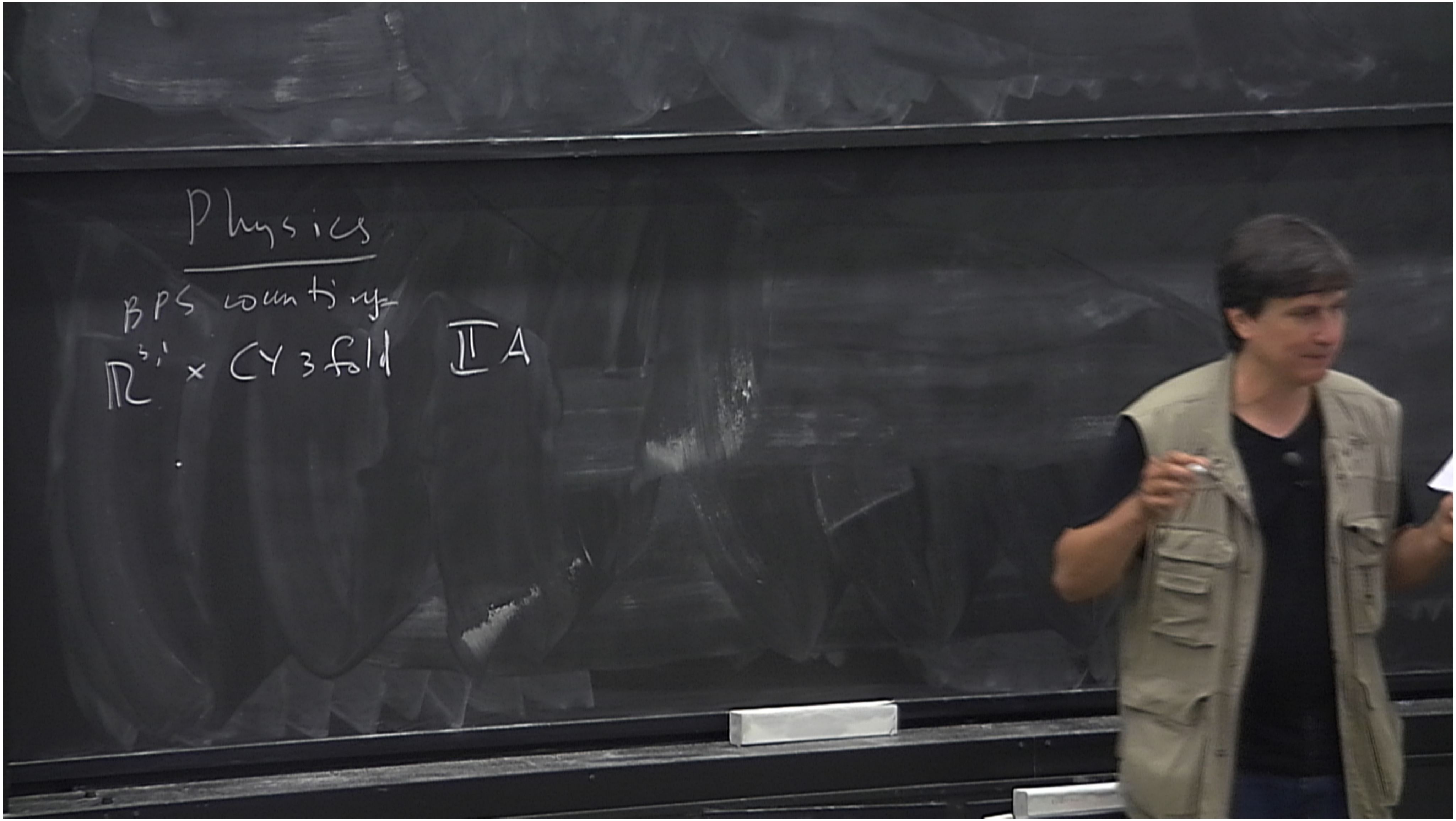
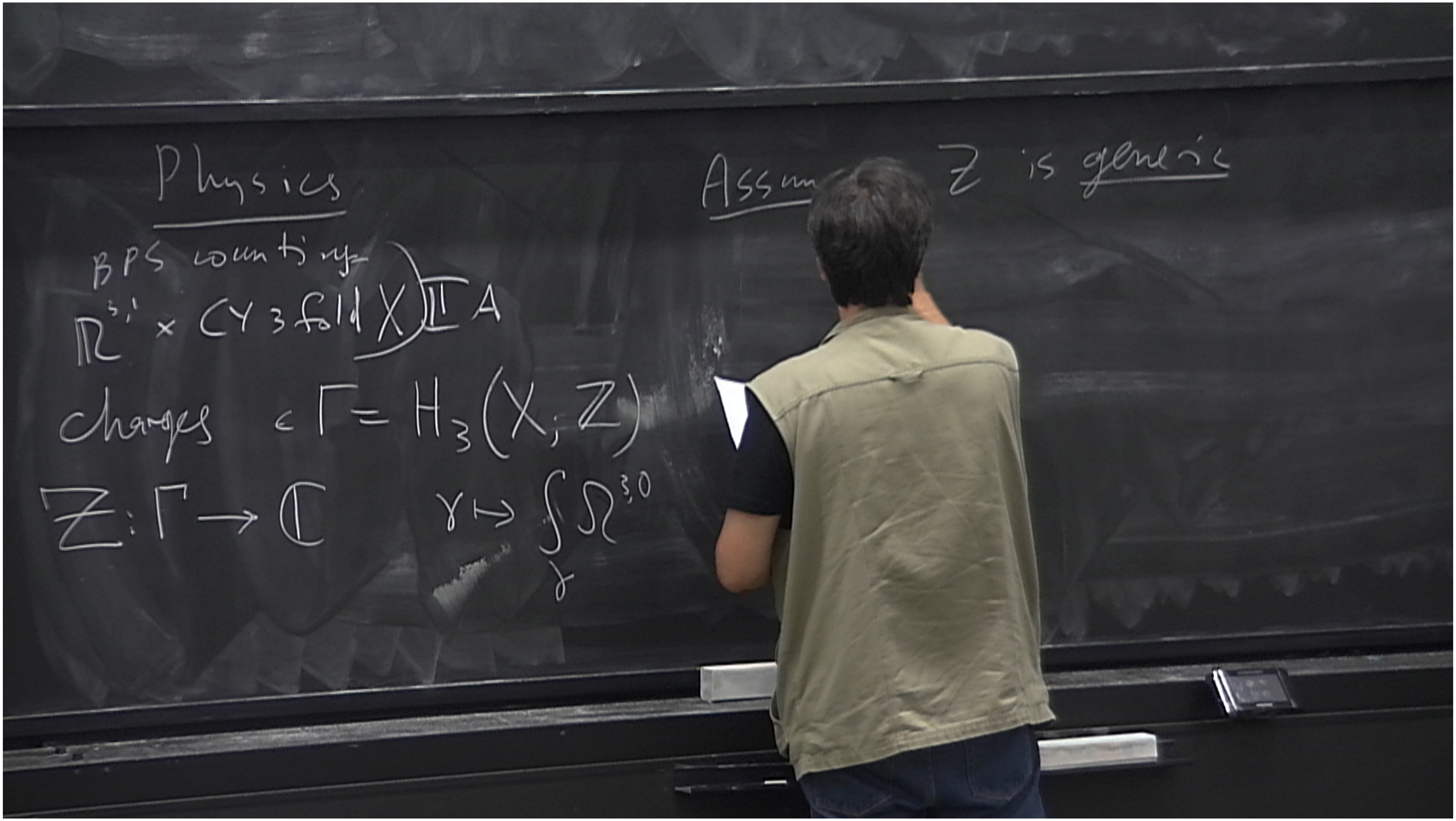
Title: DT-invariants and K_2

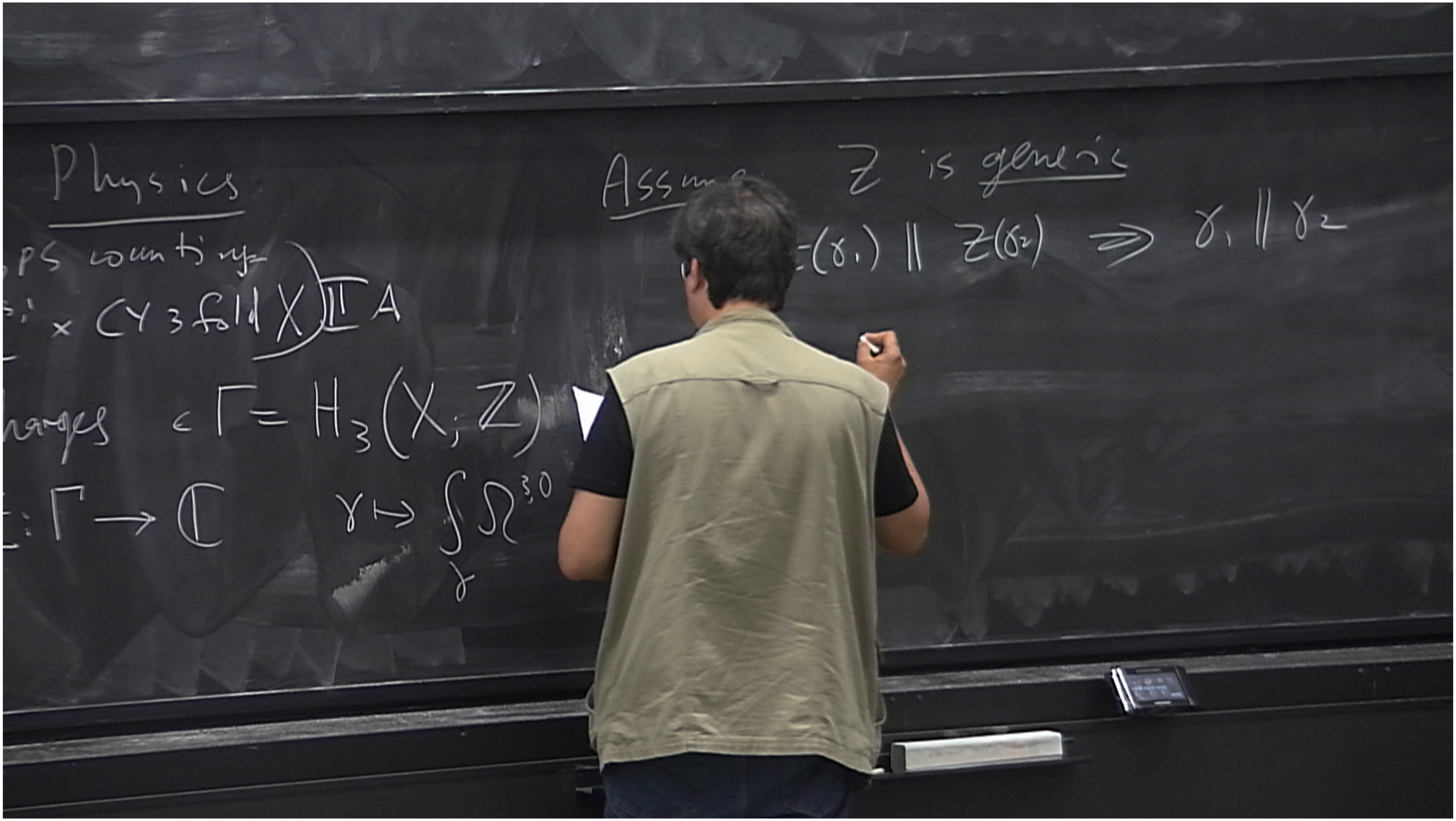
Date: Aug 23, 2012 02:00 PM

URL: http://pirsa.org/12080009

Abstract: TBA


Physics
BPS counting
$\mathbb{R}^{3,1} \times$ CY 3 fold IIA


Physics
BPS counting
$\mathbb{R}^{3,1} \times$ CY 3 fold X IIA
charges $\in \Gamma = H_3(X; \mathbb{Z})$
$Z : \Gamma \to \mathbb{C}$
$\gamma \mapsto \int_\gamma \Omega^{3,0}$
Z is generic


Physics
counting
CY 3 fold X
IIA
$\Gamma = H_3(X; \mathbb{Z})$
$\Gamma \to \mathbb{C}$
$\gamma \mapsto \int_\gamma \Omega^{3,0}$
Z is generic
$\| \; Z(\gamma_2) \Rightarrow \gamma_1 \| \gamma_2$

Physics

PS counting

$\times$ CY 3fold X IIB

harges $\in \Gamma = H_3(X;\mathbb{Z})$

$: \Gamma \to \mathbb{C} \qquad \gamma \mapsto \int_\gamma \Omega^{3,0}$

Assume: Z is generic

if $Z(\gamma_1) \parallel Z(\gamma_2) \Rightarrow \gamma_1 \parallel \gamma_2$

$\dashrightarrow \Omega(\gamma) \in \mathbb{Z}$ (index)

"# of BPS states

$\chi($ $\mathbb{Z}$-graded vector space $)$

Z is generic

$Z(\gamma_1) \parallel Z(\gamma_2) \Rightarrow \gamma_1 \parallel \gamma_2$

$\Omega(\gamma) \in \mathbb{Z}$ (index)

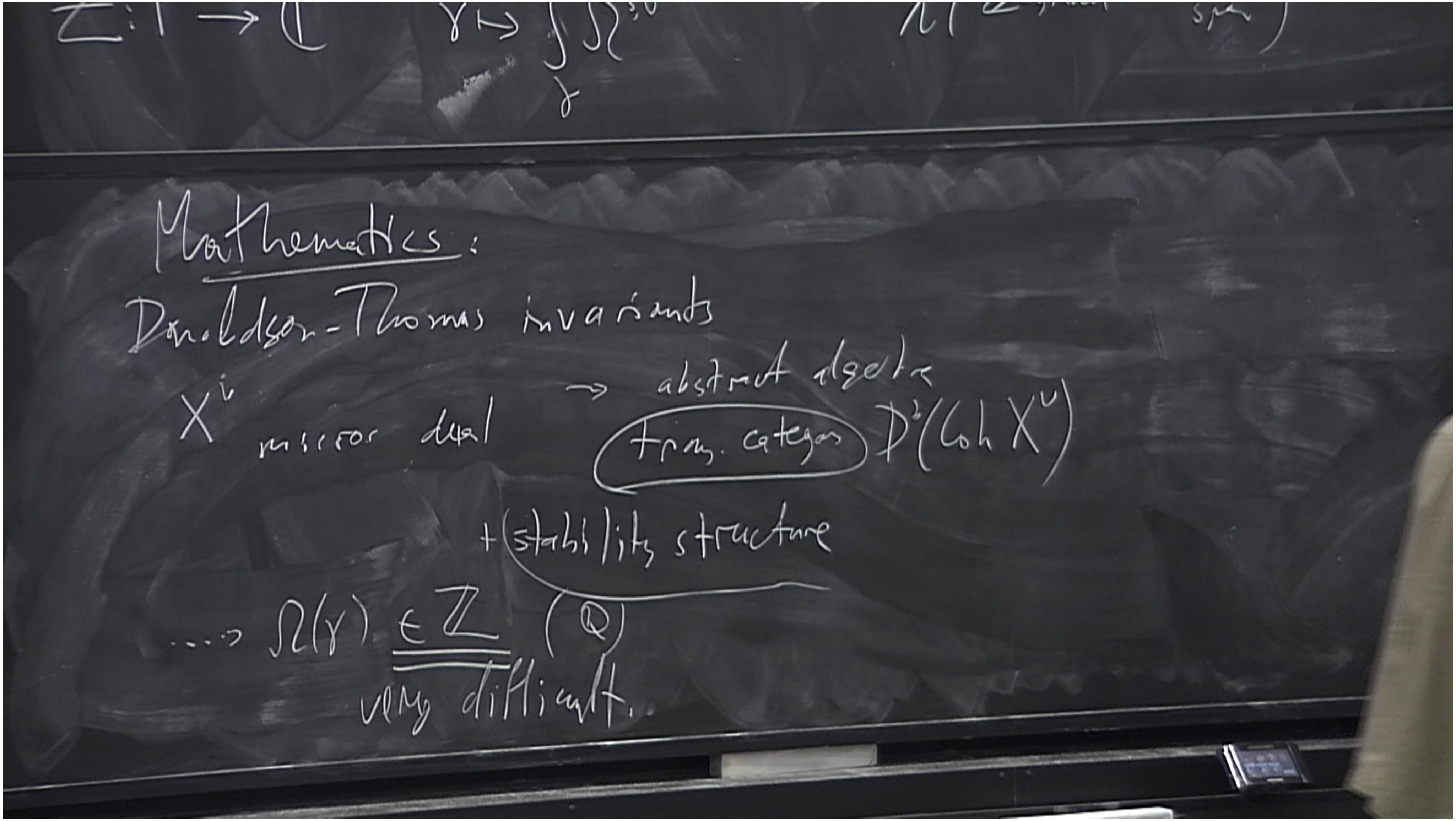
"# of BPS states"

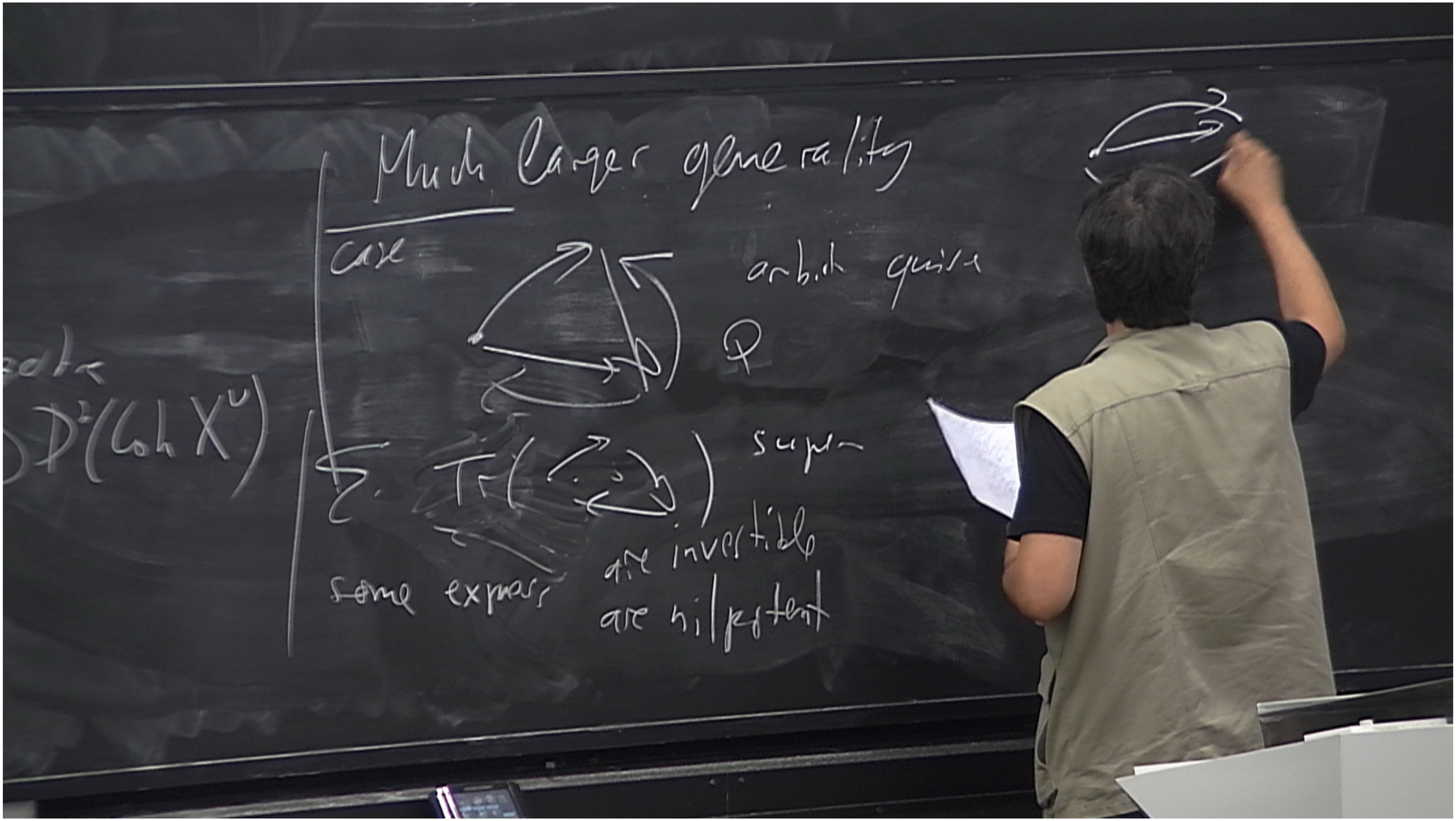
χ($\mathbb{Z}$-graded vector space)

Facts:

- $\Omega(\gamma)$ does not depend on $[\omega^{1,1}] \in H^2$
- locally constant in Moduli of $\mathbb{C}$-str.

jumps.

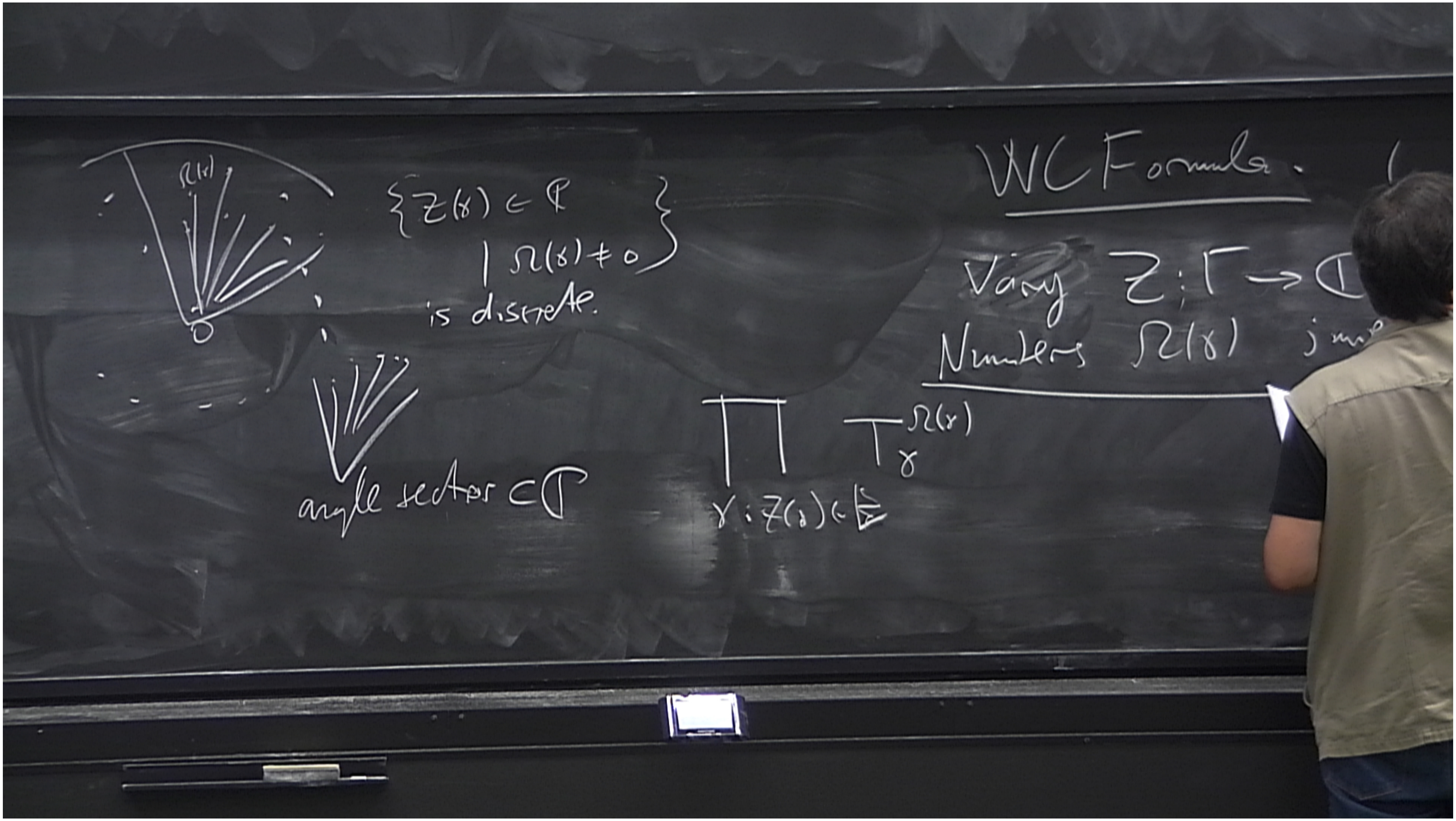

Mathematics:
Donaldson-Thomas invariants
X^v → abstract algebra
mirror dual
triang. category D^b(Coh X^v)
+ stability structure
... → Ω(γ) ∈ ℤ (ℚ)
very difficult...


Much larger generality
case
super
some express
are invertible
are nilpotent

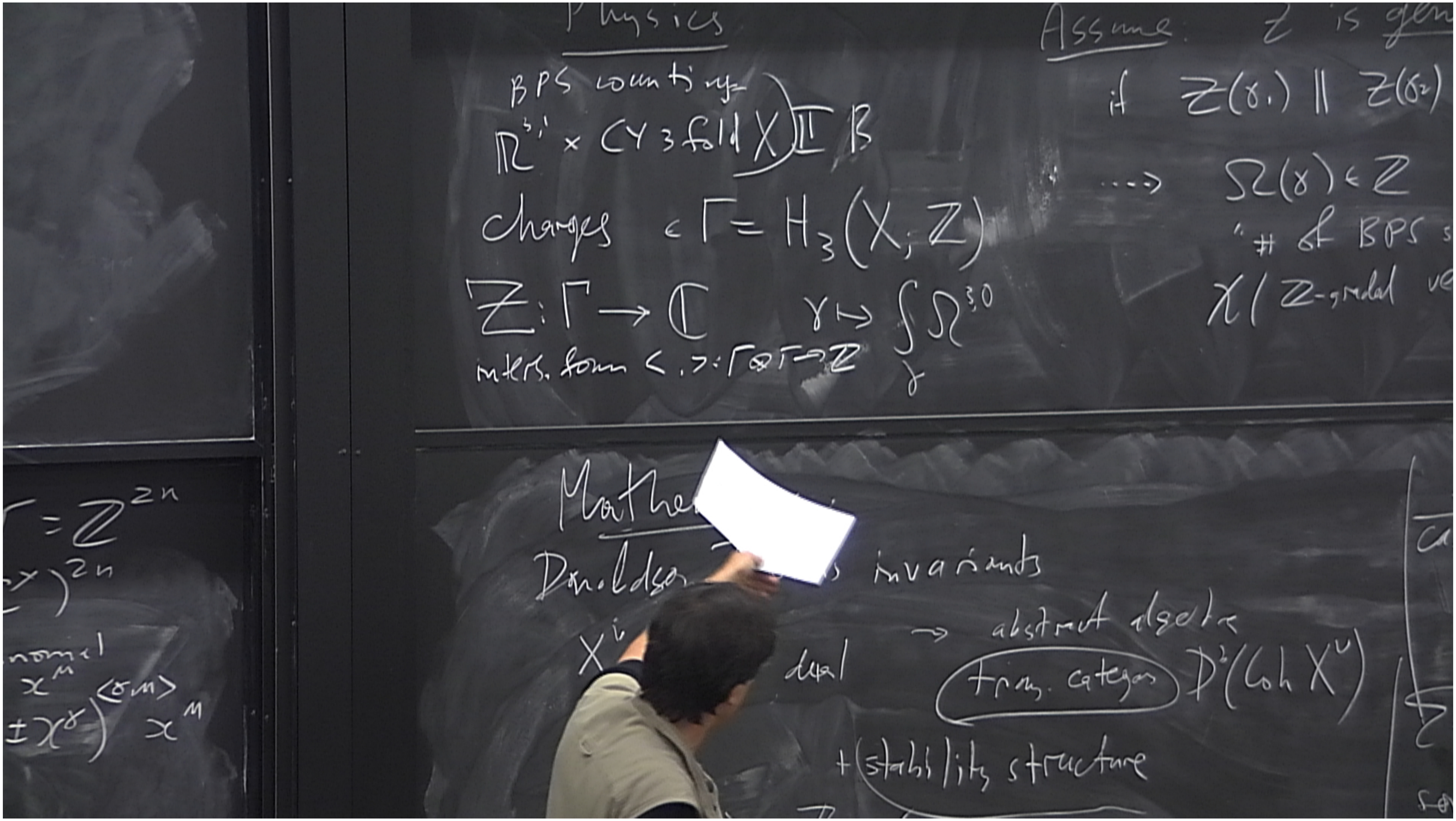
WC Formula.

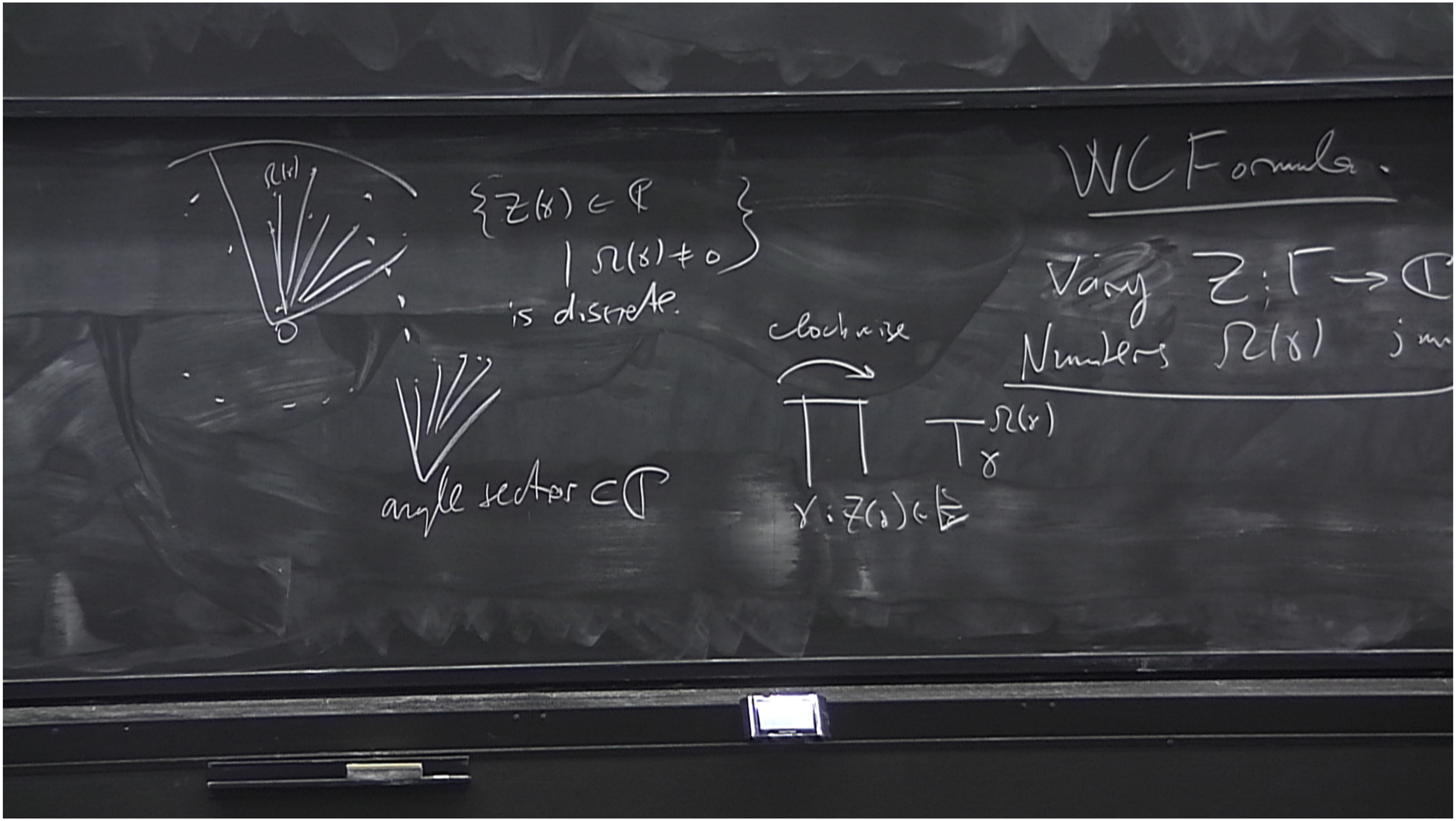
Vary $Z : \Gamma \to \mathbb{C}$

Numbers $\Omega(\gamma)$ jump.



WC Formula.
Vary $Z : \Gamma \to \mathbb{C}$
Numbers $\Omega(\gamma)$
$\{Z(\gamma) \in \mathbb{C} \mid \Omega(\gamma) \neq 0\}$ is discrete.
$\Omega(\gamma)$
0
angle sector $\subset \mathbb{C}$
$\prod_{\gamma : Z(\gamma) \in \angle} T_\gamma^{\Omega(\gamma)}$


Physics
BPS counting
CY 3 fold
charges
inters. form
Assume:
is gen
of BPS
Z-graded
Mathe
invariants
abstract algebra
dual
triang. category
+ stability structure
normal


WC Formula.
Vary Z : Γ → ℂ
Numbers Ω(γ)
{Z(γ) ∈ ℂ | Ω(γ) ≠ 0} is discrete.
clockwise
∏ T_γ^Ω(γ)
angle sector ⊂ ℂ

$Z(\gamma) \in \ell$ }
$\mid \Omega(\gamma) \neq 0$ }
is discrete.

...ctors $\subset \mathbb{C}$

WC Formula.

Vary $Z : \Gamma \to \mathbb{C}$

Numbers $\Omega(\gamma)$ jump.

clockwise

$$\prod_{\gamma : Z(\gamma) \in \ell} T_\gamma^{\Omega(\gamma)}$$

Recall: if change Z ⇝ birational map
but no $Z(\gamma)$, $\Omega(\gamma) \neq 0$ crosses $\partial \ell$

Then $\overrightarrow{\prod}$ stays the same.

Torus $\Gamma = \mathbb{Z}^{2n}$

$\mathrm{Hom}(\Gamma, \mathbb{C}^\times) = (\mathbb{C}^\times)^{2n}$

$\gamma \rightsquigarrow$ monomial x^μ

$T_\gamma : x^\mu \longmapsto (1 - \pm x^\gamma)^{\langle \gamma, \mu \rangle} x^\mu$

$\{ Z(\gamma) \in \mathbb{C}$
$\mid \Omega(\gamma) \neq 0 \}$
is discrete.

...ctors $\subset \mathbb{C}$

clockwise

$$\prod_{\gamma : Z(\gamma) \in \angle} T_\gamma^{\Omega(\gamma)}$$

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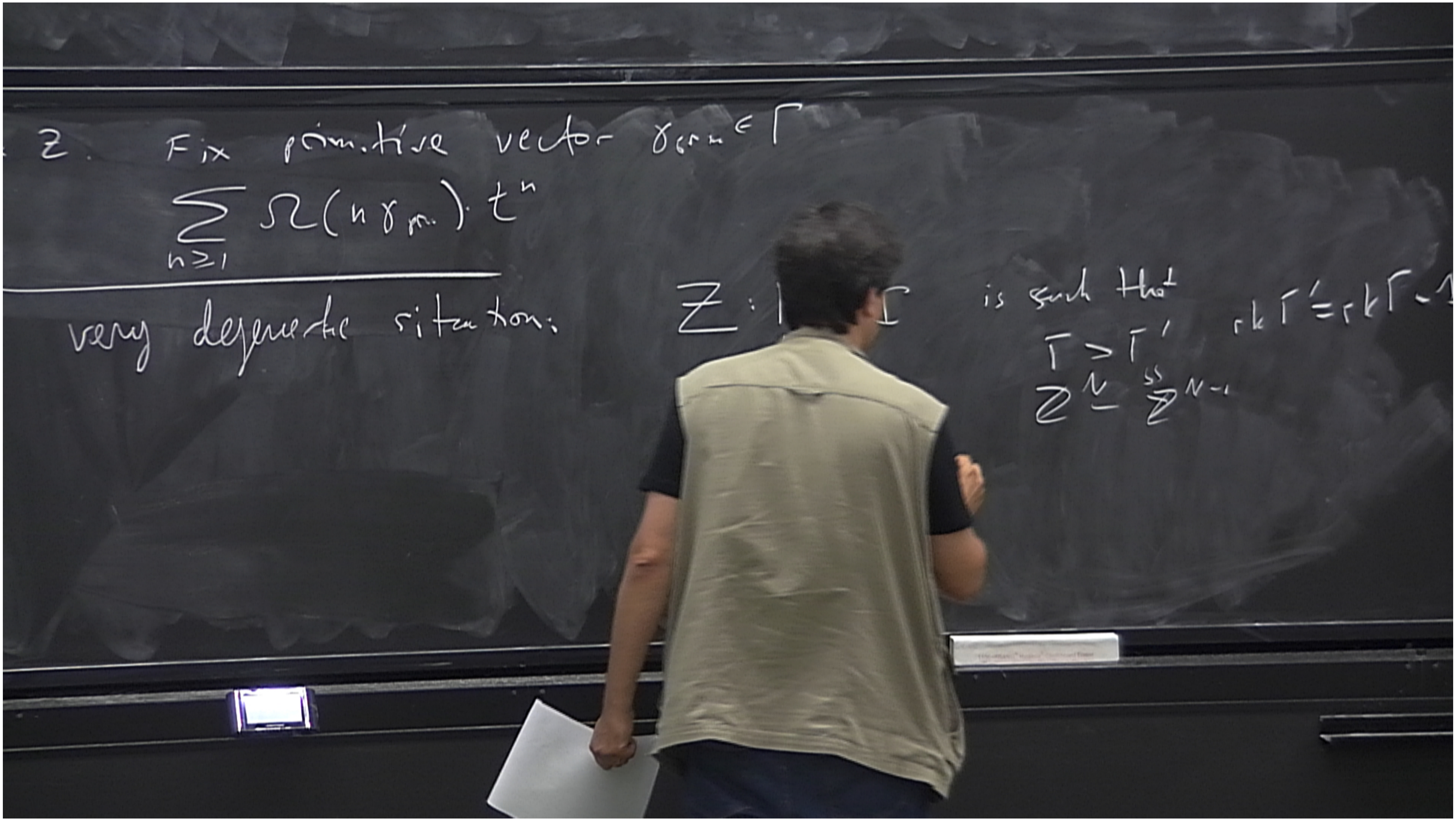
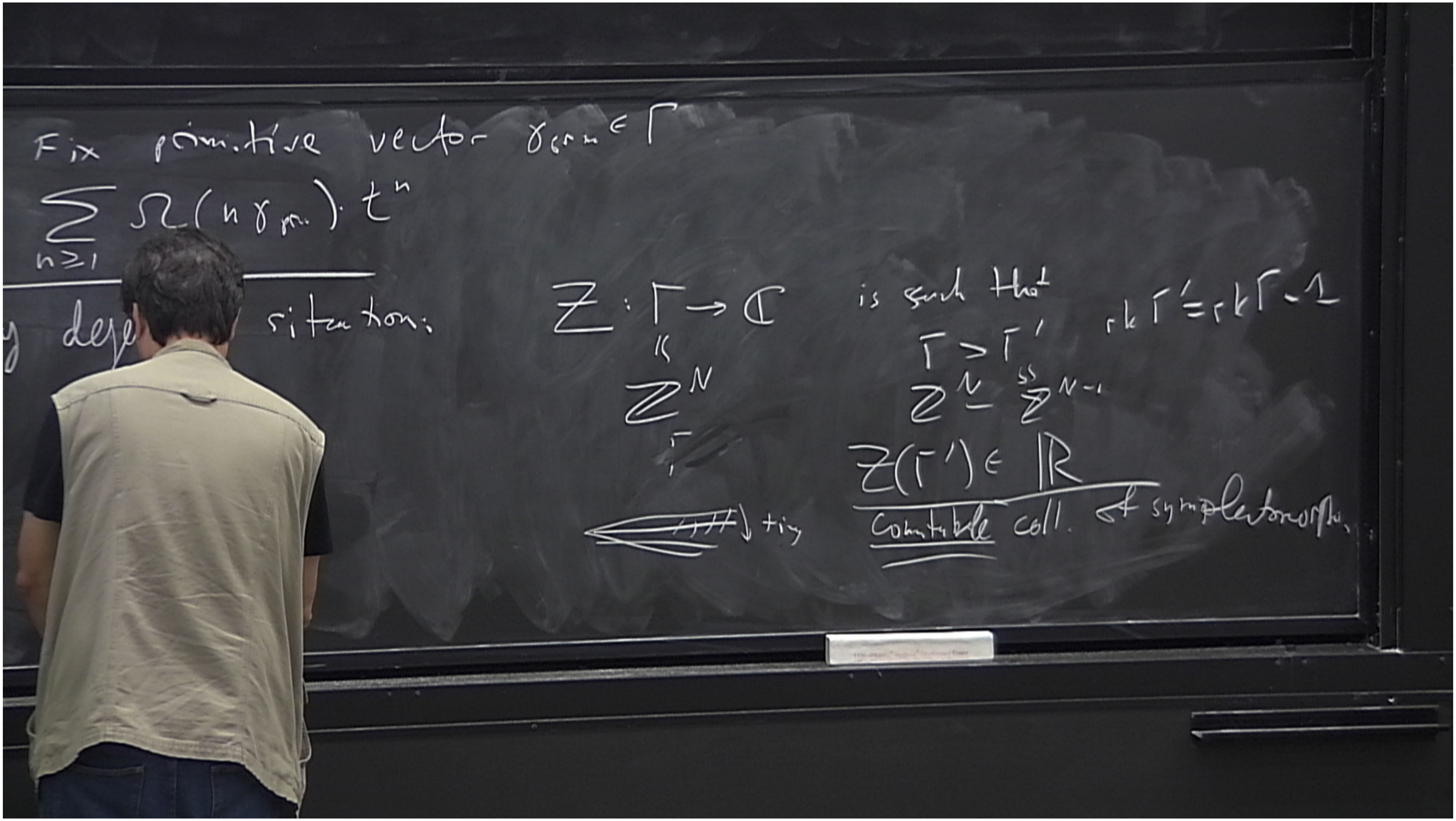
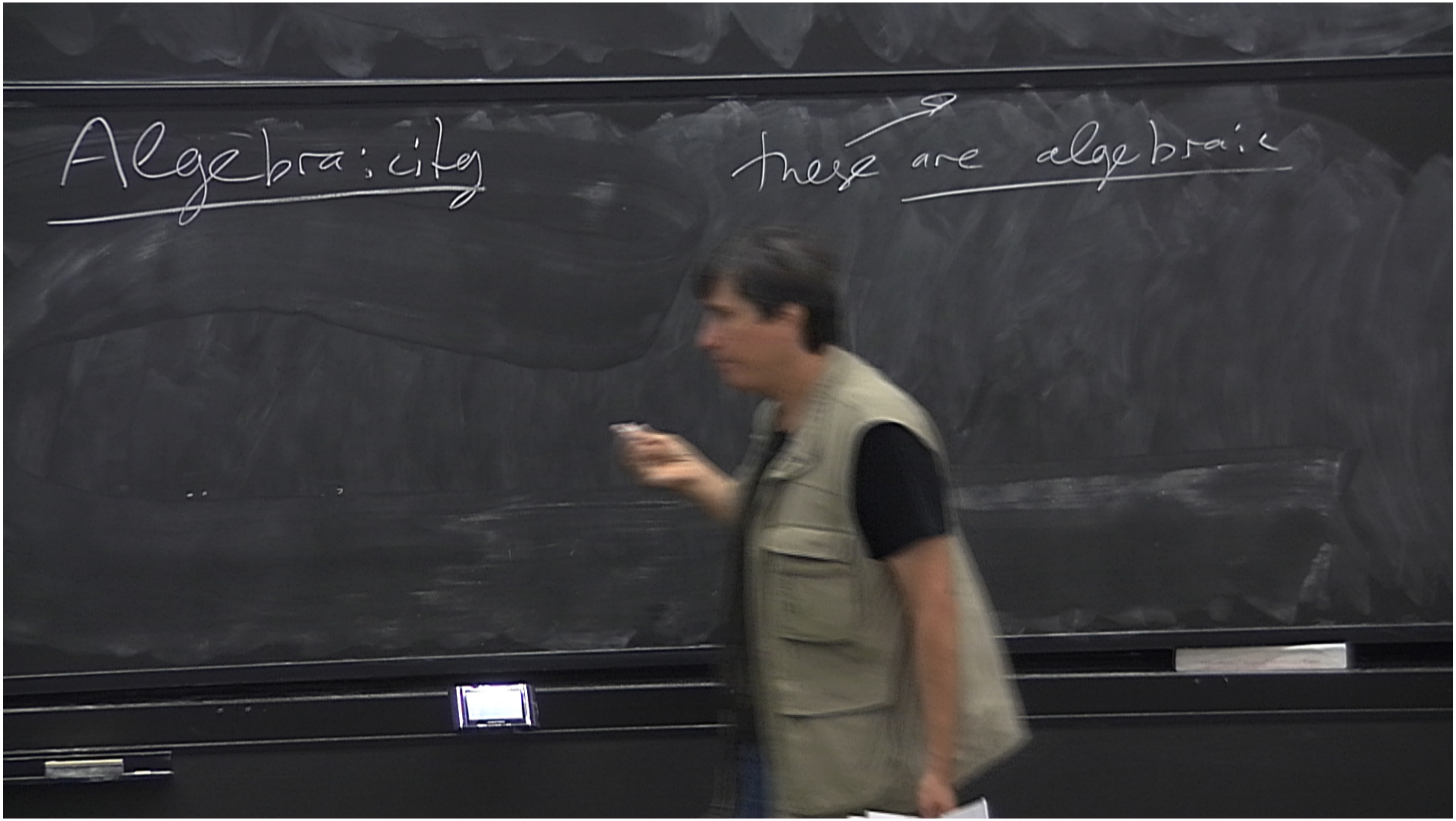
birational map

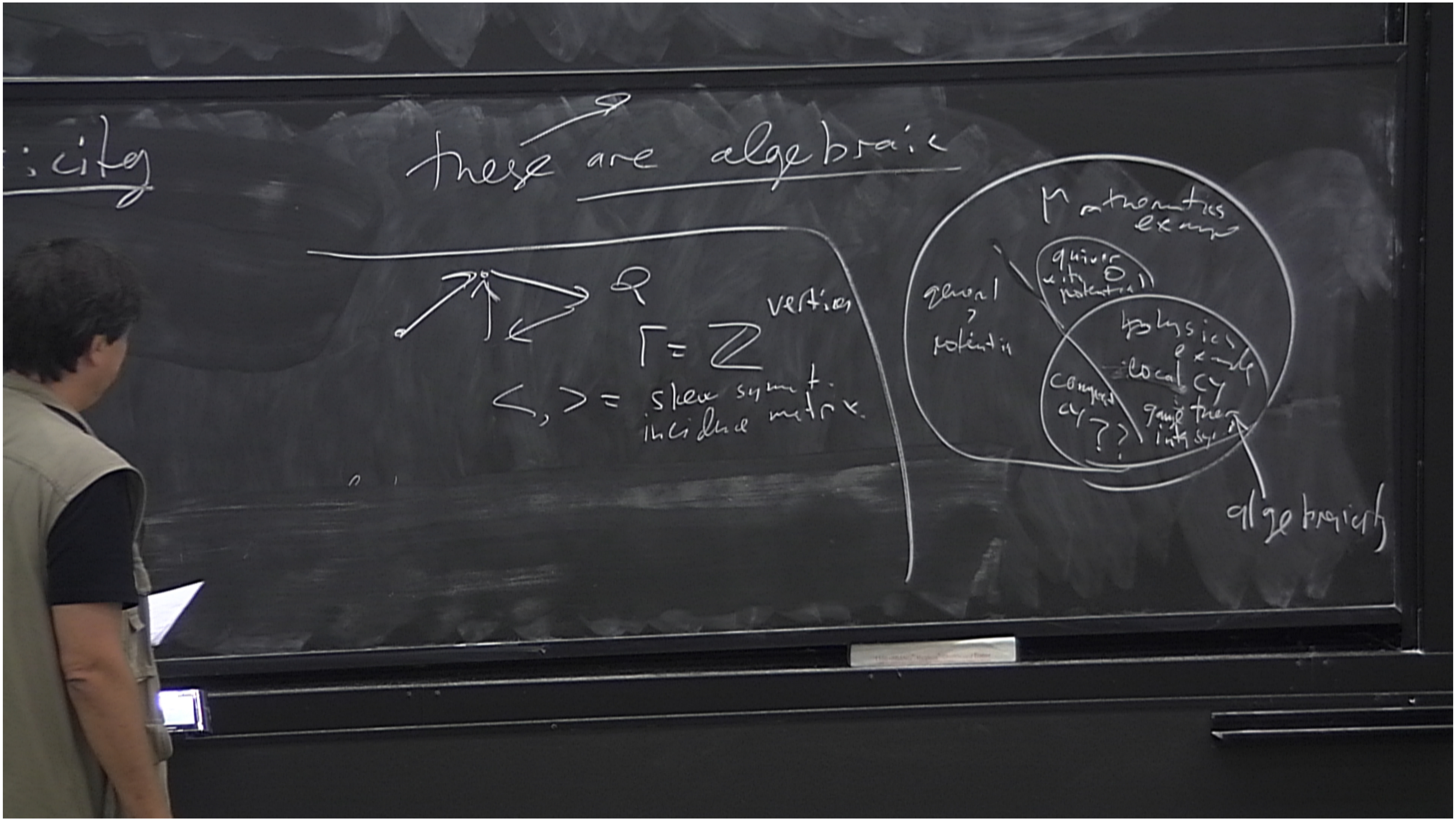
Fix Z. Fix primitive vector $\gamma_{prim} \in \Gamma$

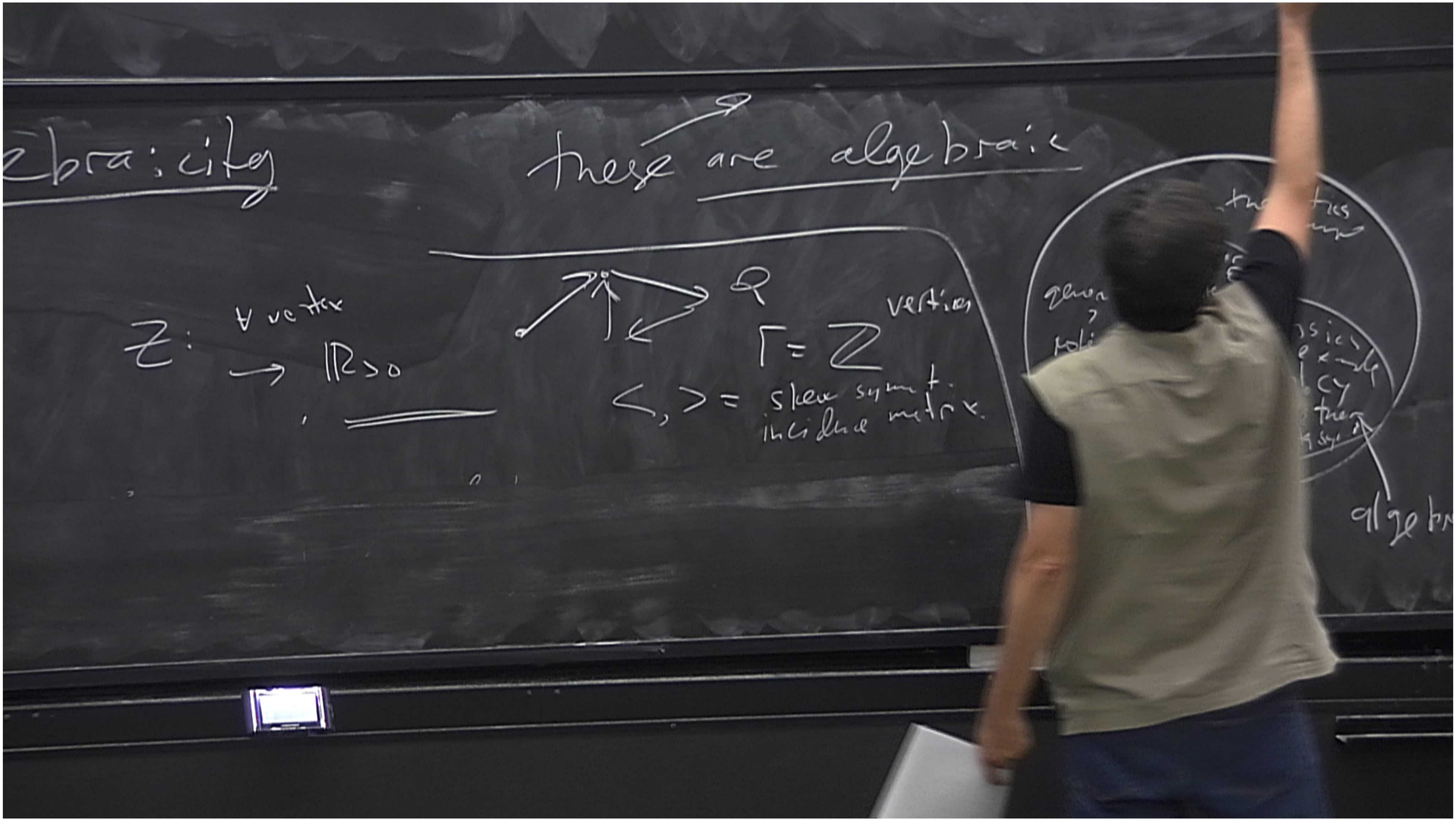
$$\sum_{n \geq 1} \Omega(n\gamma_{prim}) \cdot t^n$$

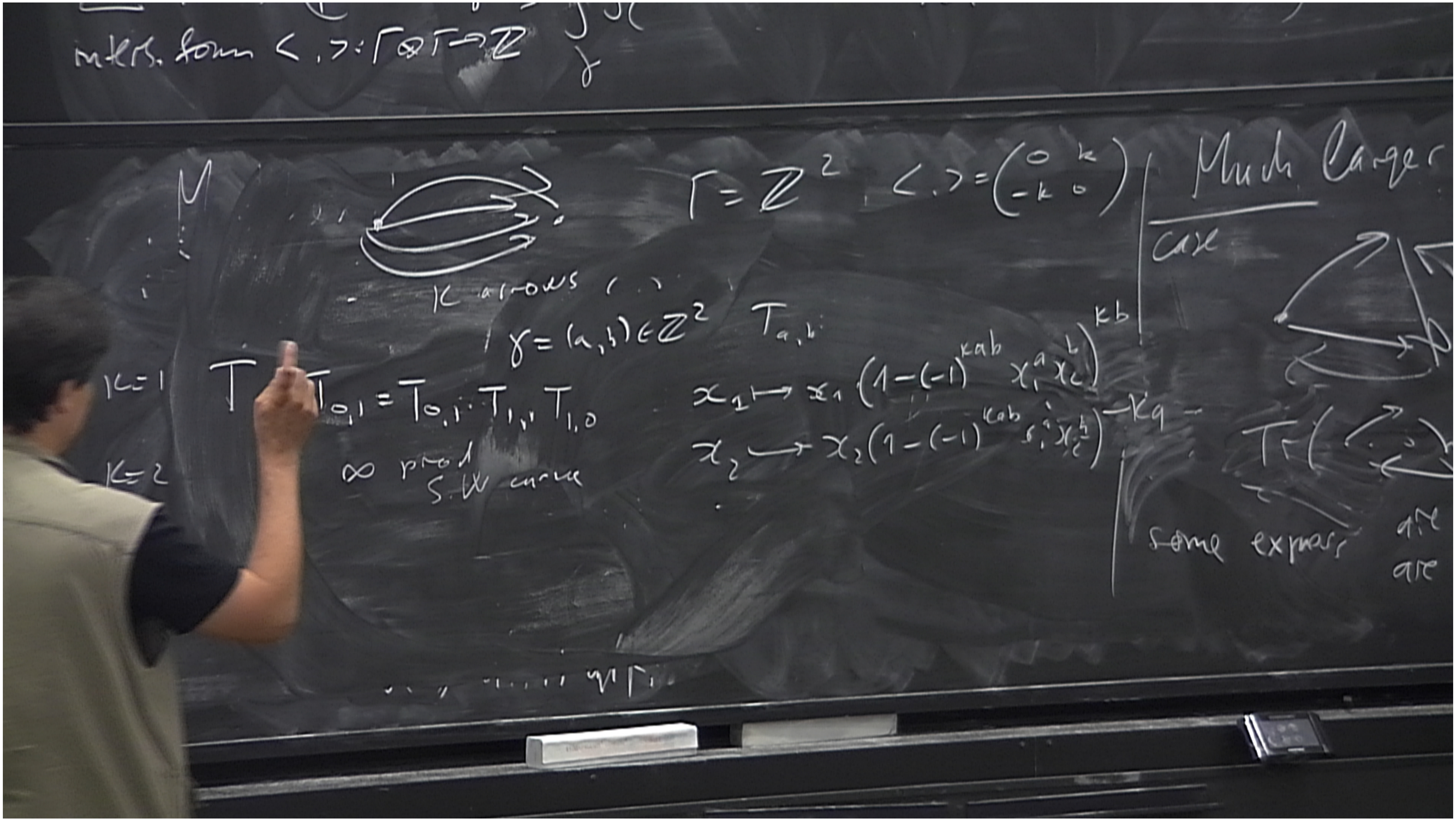

2. Fix primitive vector $\gamma_{prim} \in \Gamma$
$\sum_{n \geq 1} \Omega(n \gamma_{prim}) \cdot t^n$
very degenerate situation:
is such that
$\Gamma \supset \Gamma'$
$\mathrm{rk}\, \Gamma' = \mathrm{rk}\, \Gamma - 1$

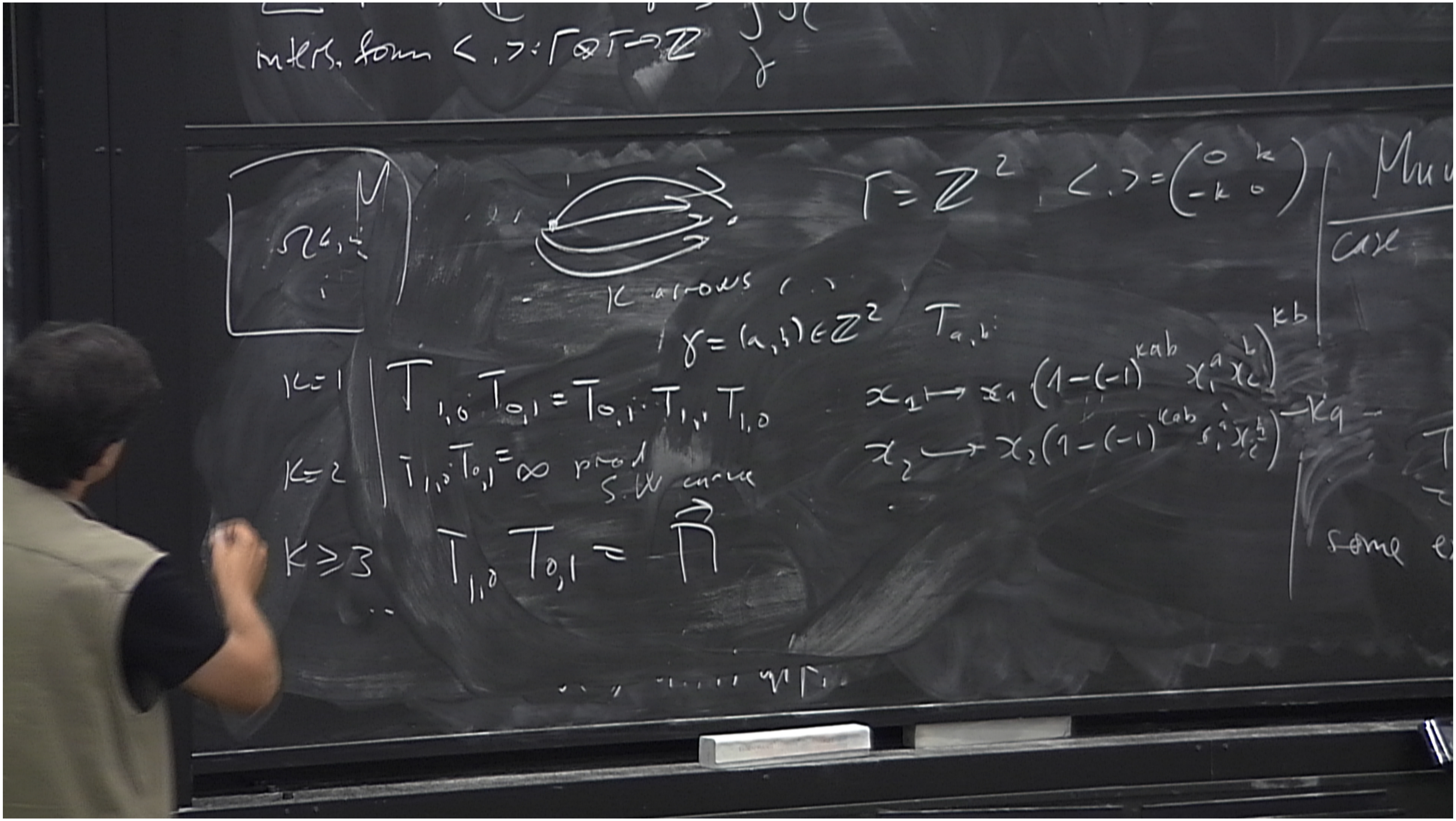

Fix primitive vector $\gamma_{prim} \in \Gamma$
$\sum_{n\geq 1} \Omega(n\gamma_{prim}) \cdot t^n$
situation:
$Z : \Gamma \to \mathbb{C}$
$\Gamma \simeq \mathbb{Z}^N$
is such that
$\Gamma \supset \Gamma'$
$\mathrm{rk}\,\Gamma' = \mathrm{rk}\,\Gamma - 1$
$\mathbb{Z}^N \supset \mathbb{Z}^{N-1}$
$Z(\Gamma') \in \mathbb{R}$
Countable coll. of

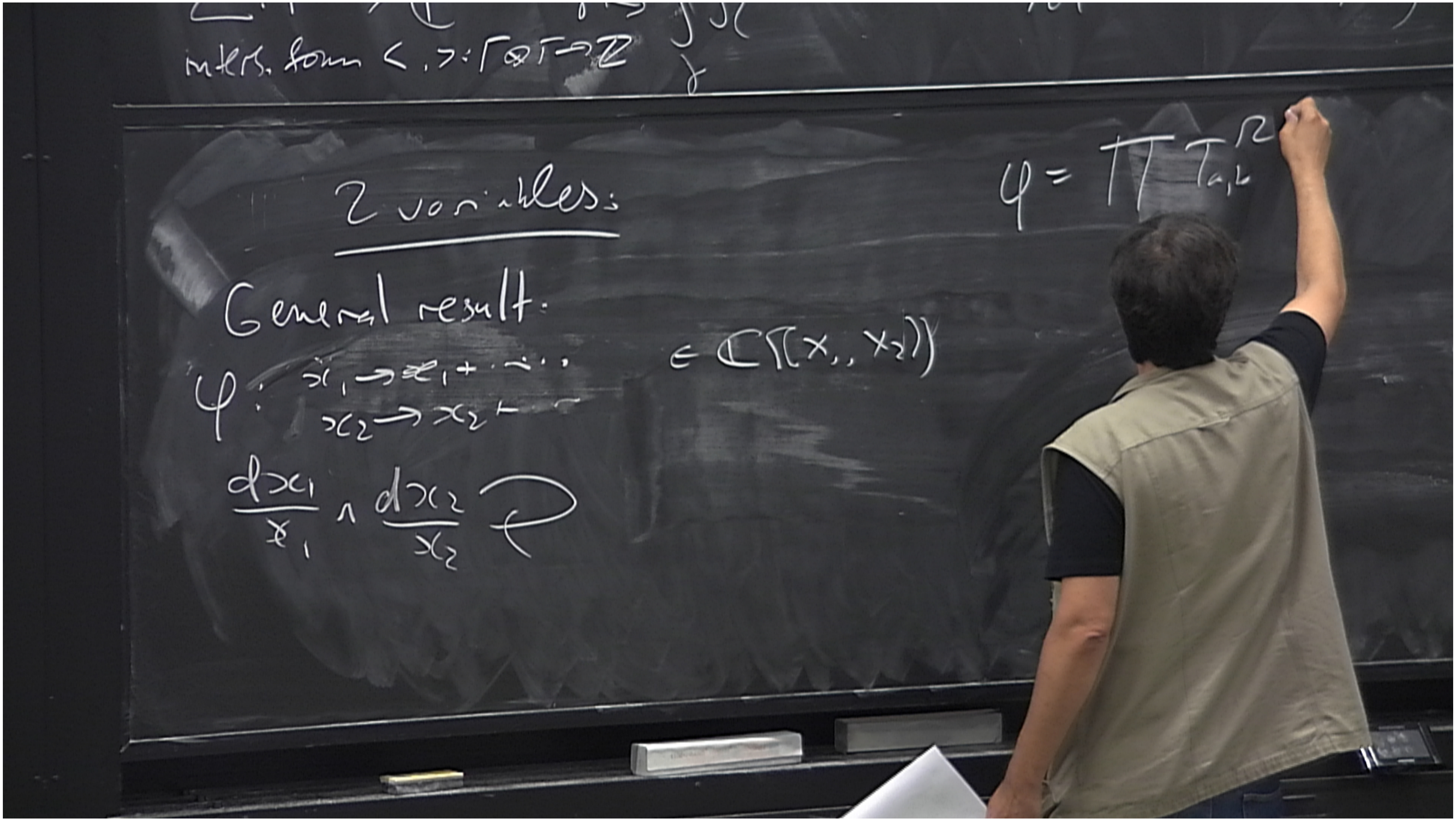

Algebraicity
these are algebraic


these are algebraic
Q
vertices
Γ = Z
<, > = skew symmt. incidence matrix
mathematics examples
quiver with potential
general potential
physics examples
local CY
compact CY ??
gauge theory
algebraicity


these are algebraic
∀ vertex
Z:
→ ℝ>0
Γ = Z
vertices
<,> = skew symmt.
incidence matrix

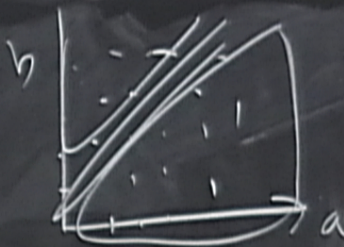

intrs. form ⟨ , ⟩ : Γ⊗Γ→Z
Γ = Z^2   ⟨ , ⟩ = (0 k / -k 0)
Much larger case
k arrows
γ = (a,b) ∈ Z^2   T_{a,b}
k=1
T_{0,1} = T_{0,1} · T_{1,1} T_{1,0}
∞ prod
S.W curve
k=2
x_1 ↦ x_1 (1-(-1)^{kab} x_1^a x_2^b)^{kb}
x_2 ↦ x_2 (1-(-1)^{kab} x_1^a x_2^b)^{-ka}
some express


inters. form < , > : Γ⊗Γ → Z
Γ = Z^2   < , > = (0 k; -k 0)
K arrows
γ = (a,b) ∈ Z^2   T_{a,b}
x_1 ↦ x_1 (1 − (−1)^{kab} x_1^a x_2^b)^{kb}
x_2 ↦ x_2 (1 − (−1)^{kab} x_1^a x_2^b)^{−ka}
k=1   T_{1,0} T_{0,1} = T_{0,1} T_{1,1} T_{1,0}
k=2   T_{1,0} T_{0,1} = ∞ prod
S.W curve
k≥3   T_{1,0} T_{0,1} =
case
some


inters. form < , >: Γ⊗Γ → ℤ
2 variables:
General result:
φ: x1 → x1 + ...
x2 → x2 + ...
∈ ℂ[[x1, x2]]
dx1/x1 ∧ dx2/x2
φ = ∏ T_{a,b}

$$\varphi = \overrightarrow{\prod} T_{a,b}^{R(a,b)}$$

$$= \underset{a<b}{\varphi_-} \cdot \underset{a=b}{\varphi_0} \cdot \underset{a>b}{\varphi_+}$$



$\in \mathbb{C}[[x_1, x_2]]$

φ is algebraic

$\iff \varphi_-, \varphi_0, \varphi_+$ are algebraic.

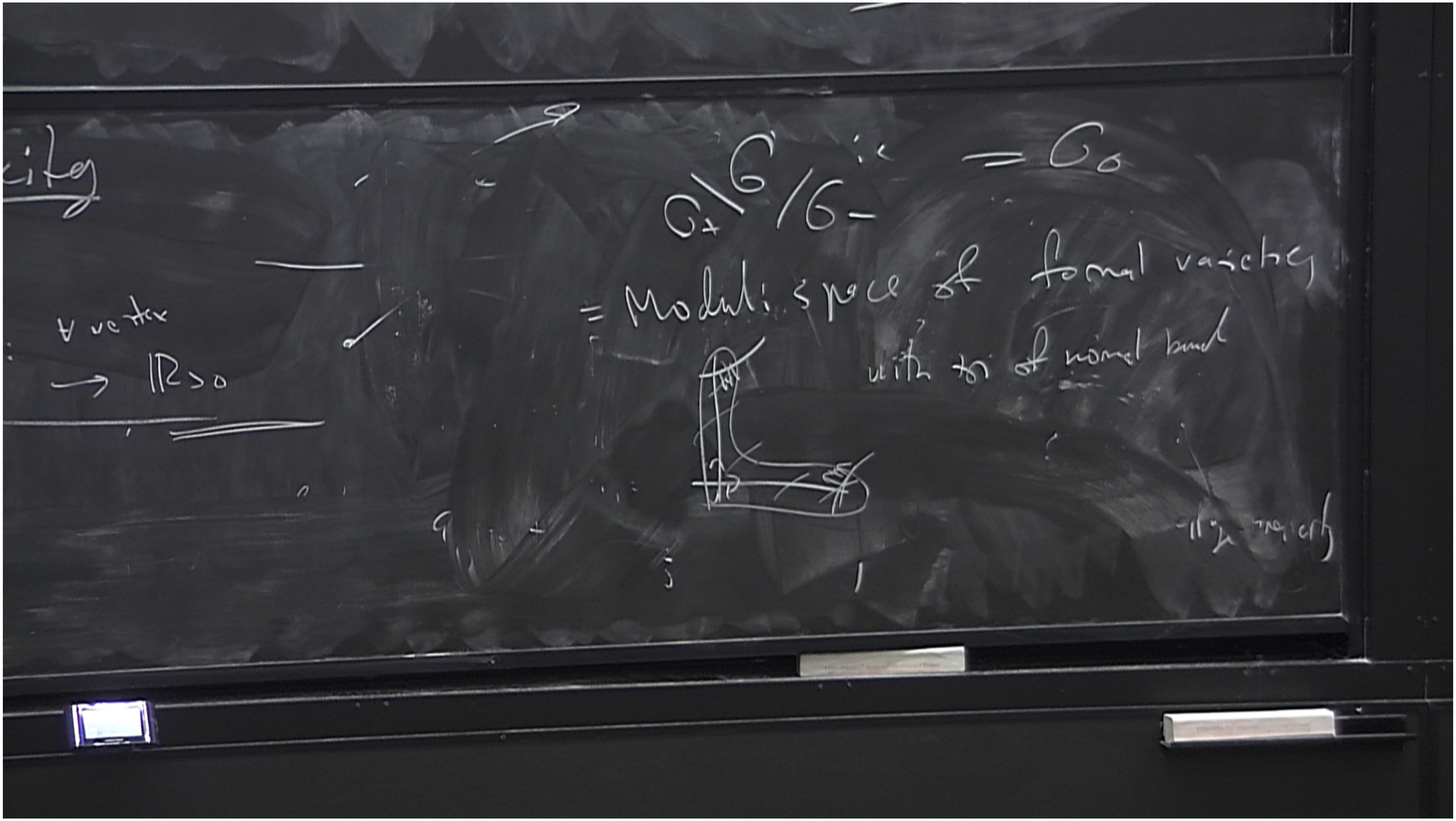
city

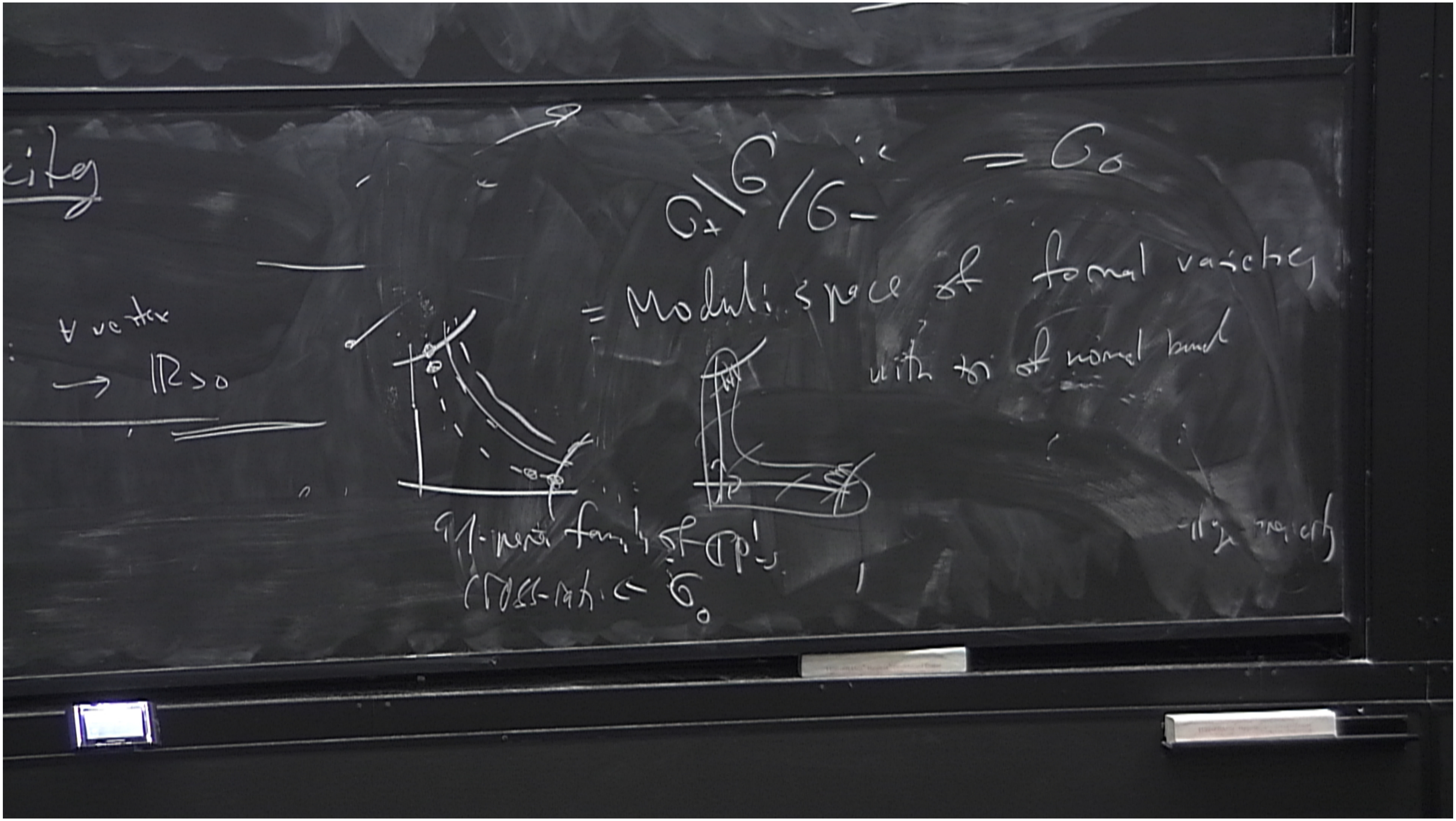
∀ vertex

$\to \mathbb{R}_{>0}$

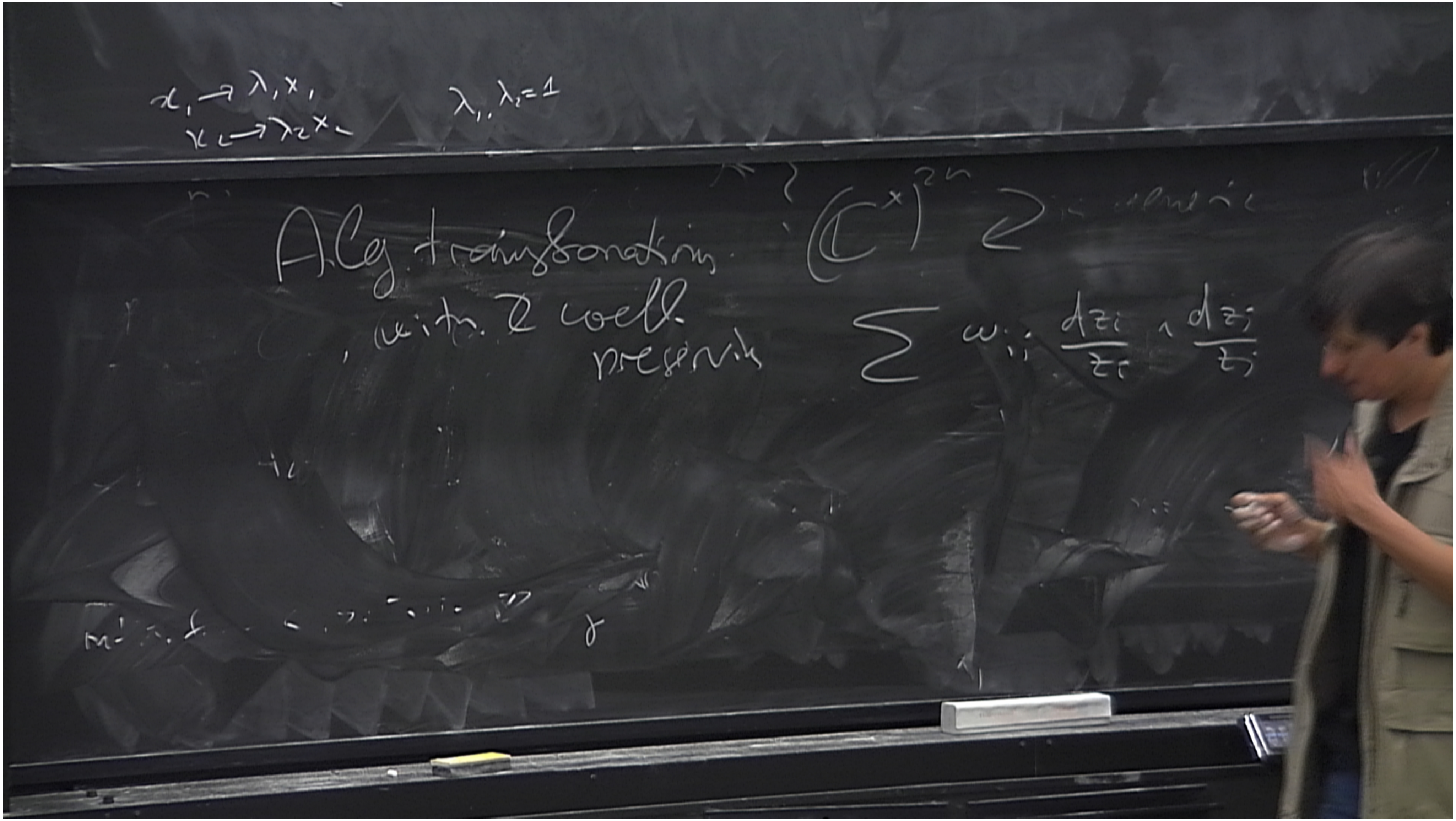
$G_+ \backslash G / G_-$

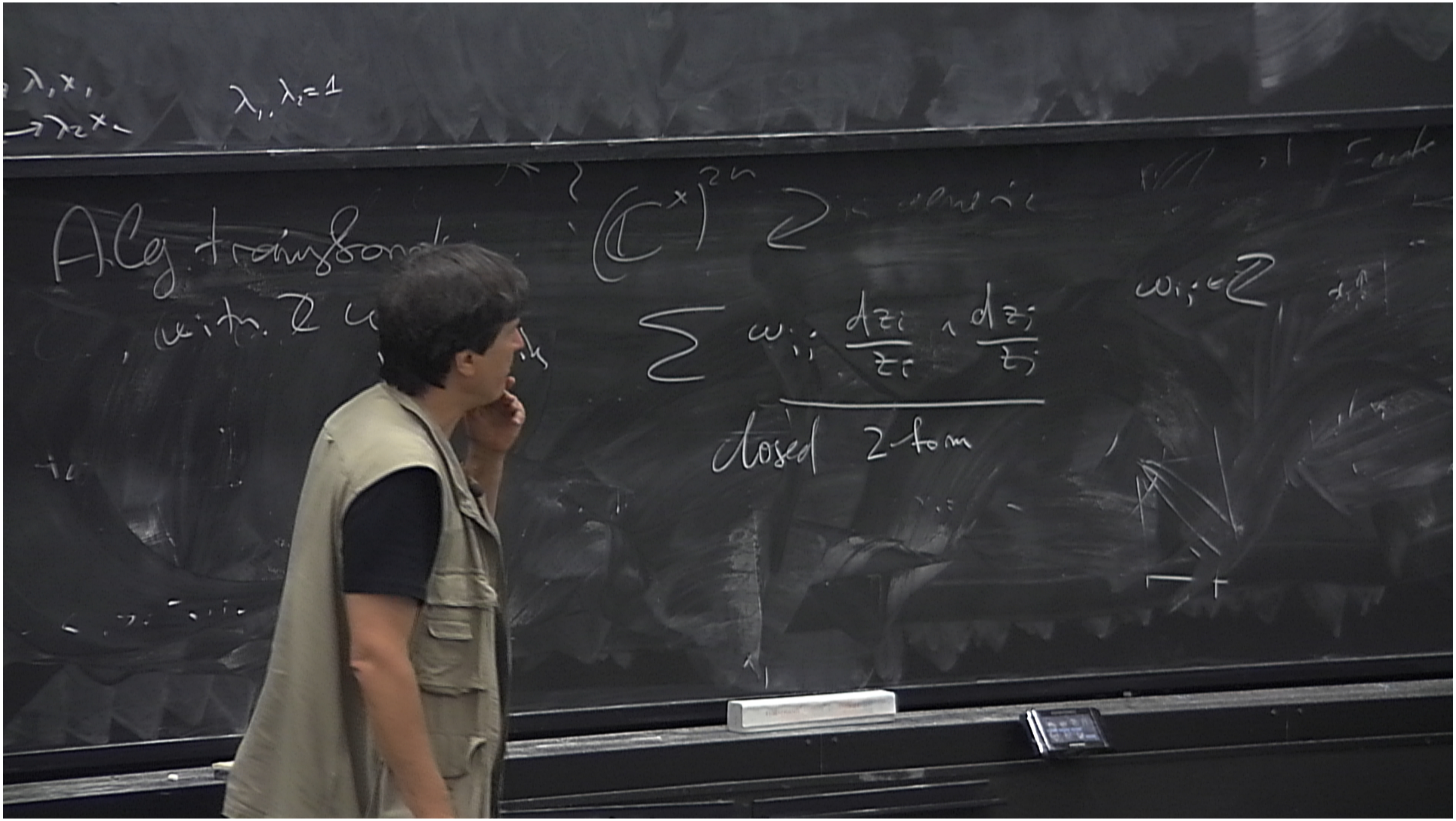
= Moduli space of formal variety

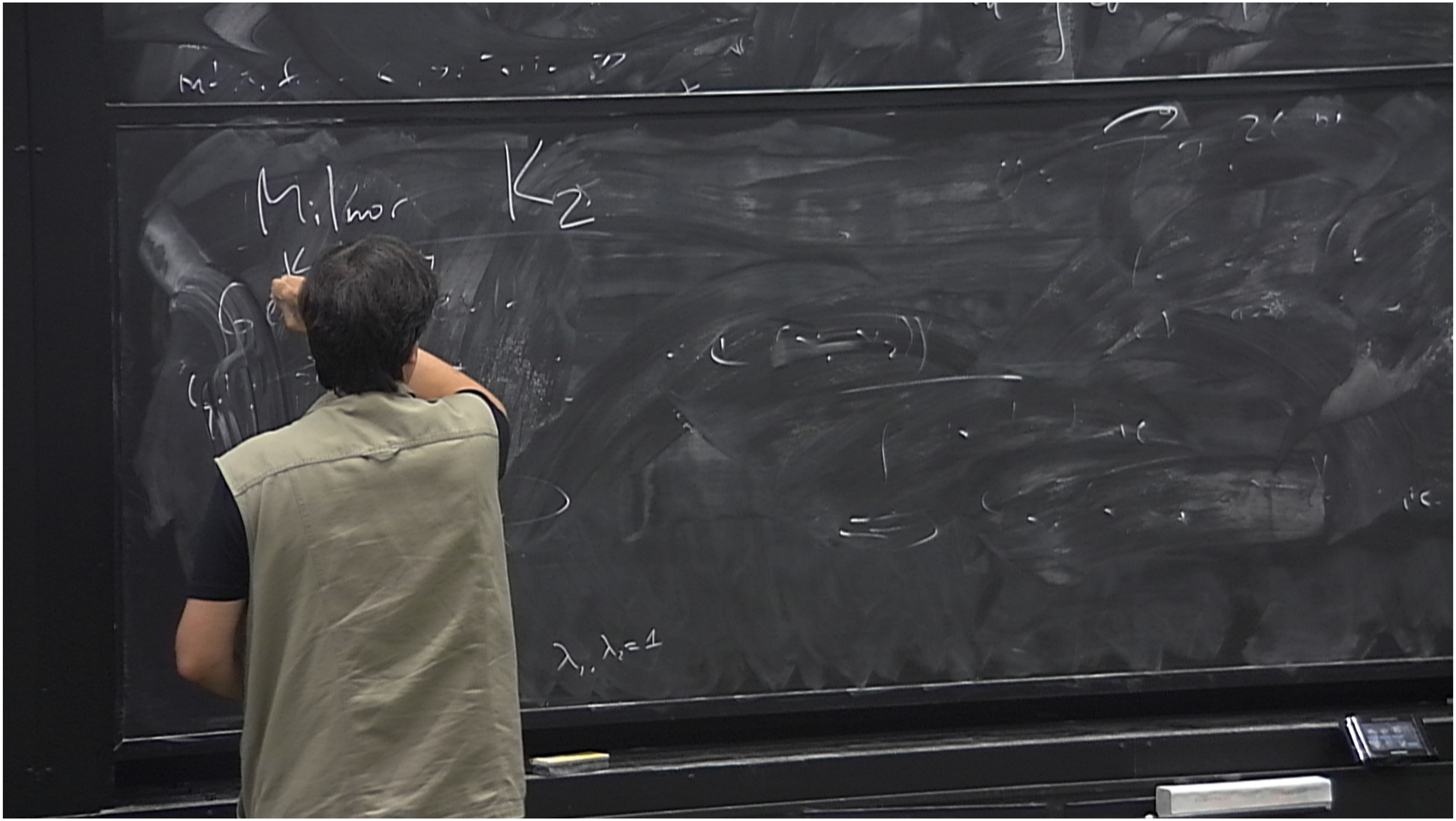

$G_+ \backslash G / G_- = G_0$
= Moduli space of formal varieties
with ... of normal bund
∀ vertex
→ $\mathbb{R}_{>0}$



∀ vertex
→ ℝ>0
= G0
= Moduli space of formal variety
with triv of normal bund
1-param family of ℂP¹s
cross-ratio ∈ G0


$x_1 \to \lambda_1 x_1$
$x_2 \to \lambda_2 x_2$
$\lambda_1 \lambda_2 = 1$
Alg transformation
$(\mathbb{C}^\times)^{2n}$
with 2 well preserving
$\sum \omega_{ij} \frac{dz_i}{z_i} \wedge \frac{dz_j}{z_j}$


λ1 λ2 = 1
(C×)^2n
Σ ω_ij dz_i/z_i ∧ dz_j/z_j
closed 2-form
ω_ij ∈ Z


Milnor K2
λ1. λ2=1

field

here $F = \mathbb{Q}(x_1, \dots, x_{2n})$

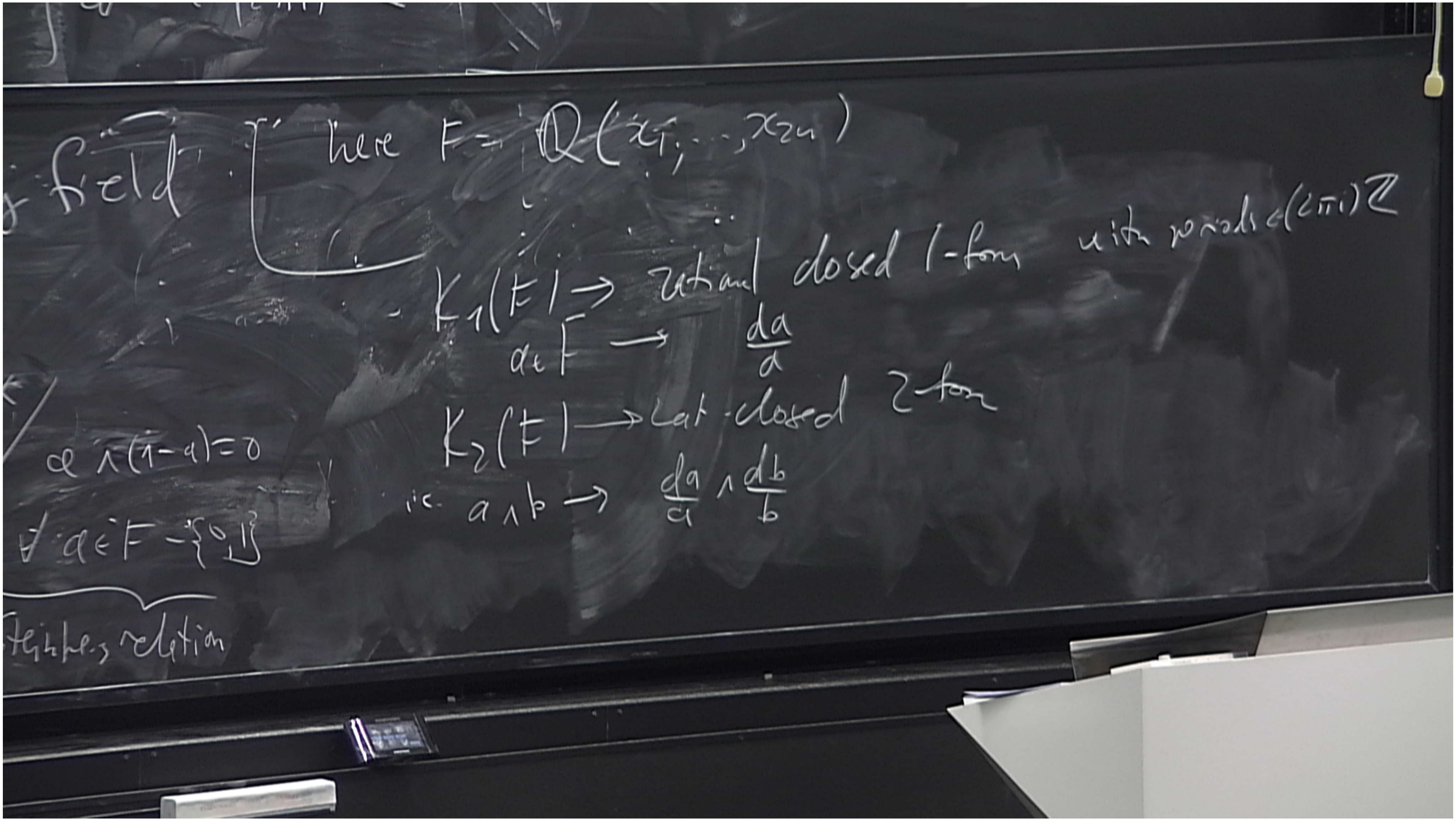
$K_1(F) \to$ rational closed 1-form with periods $(2\pi i)\mathbb{Z}$

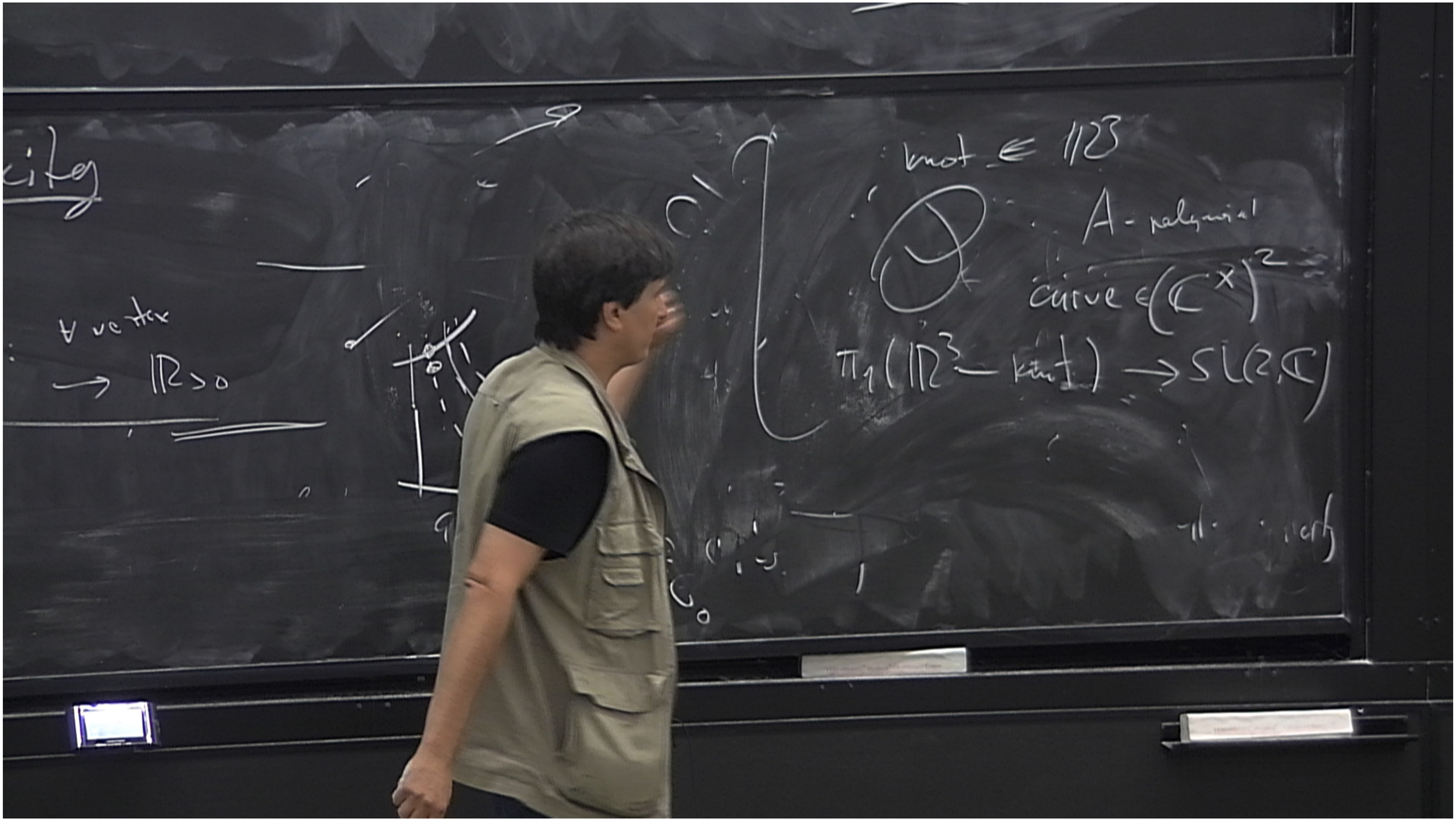
$a \in F \to \frac{da}{a}$

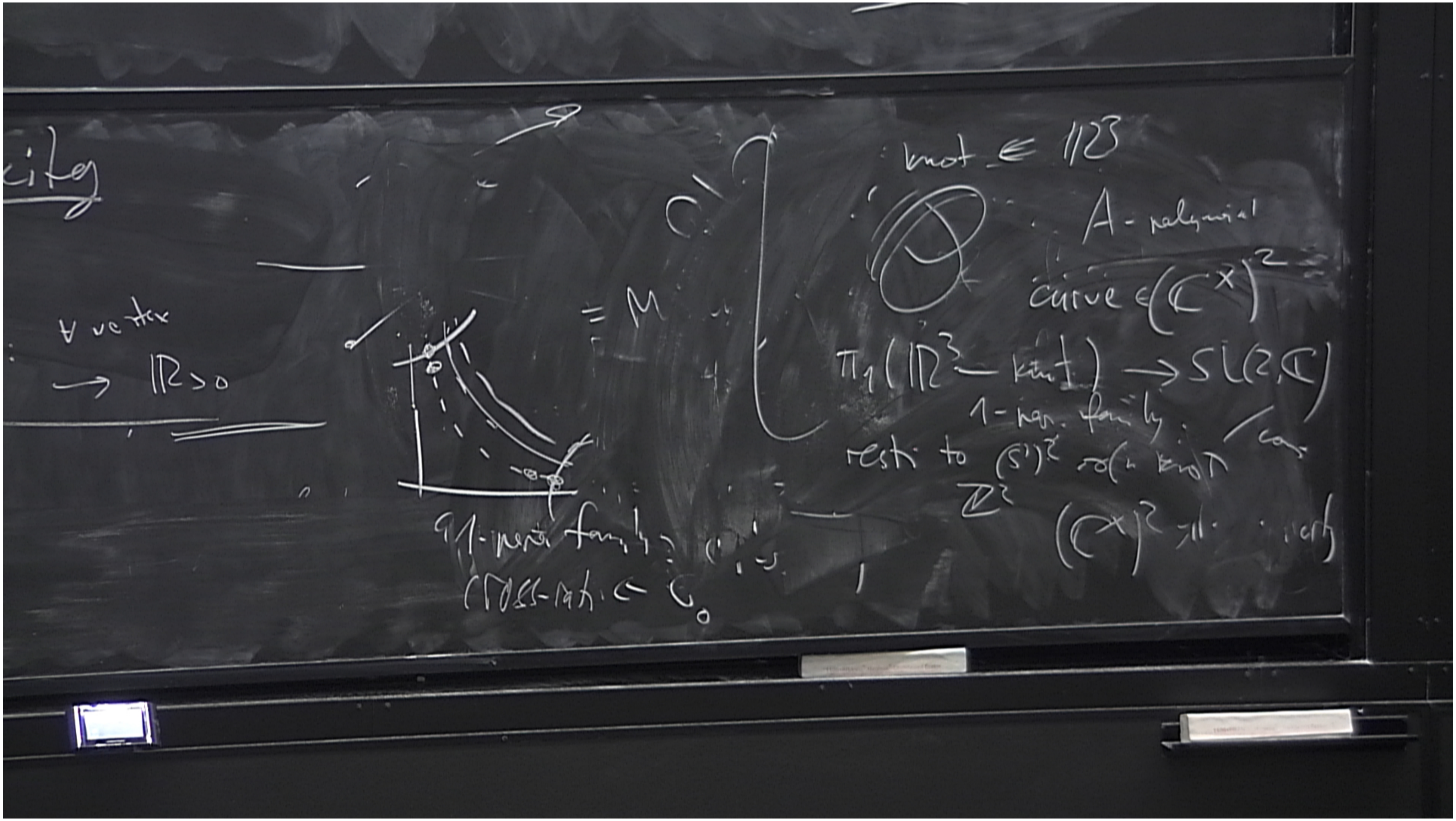
$K_2(F) \to$ rat. closed 2-form

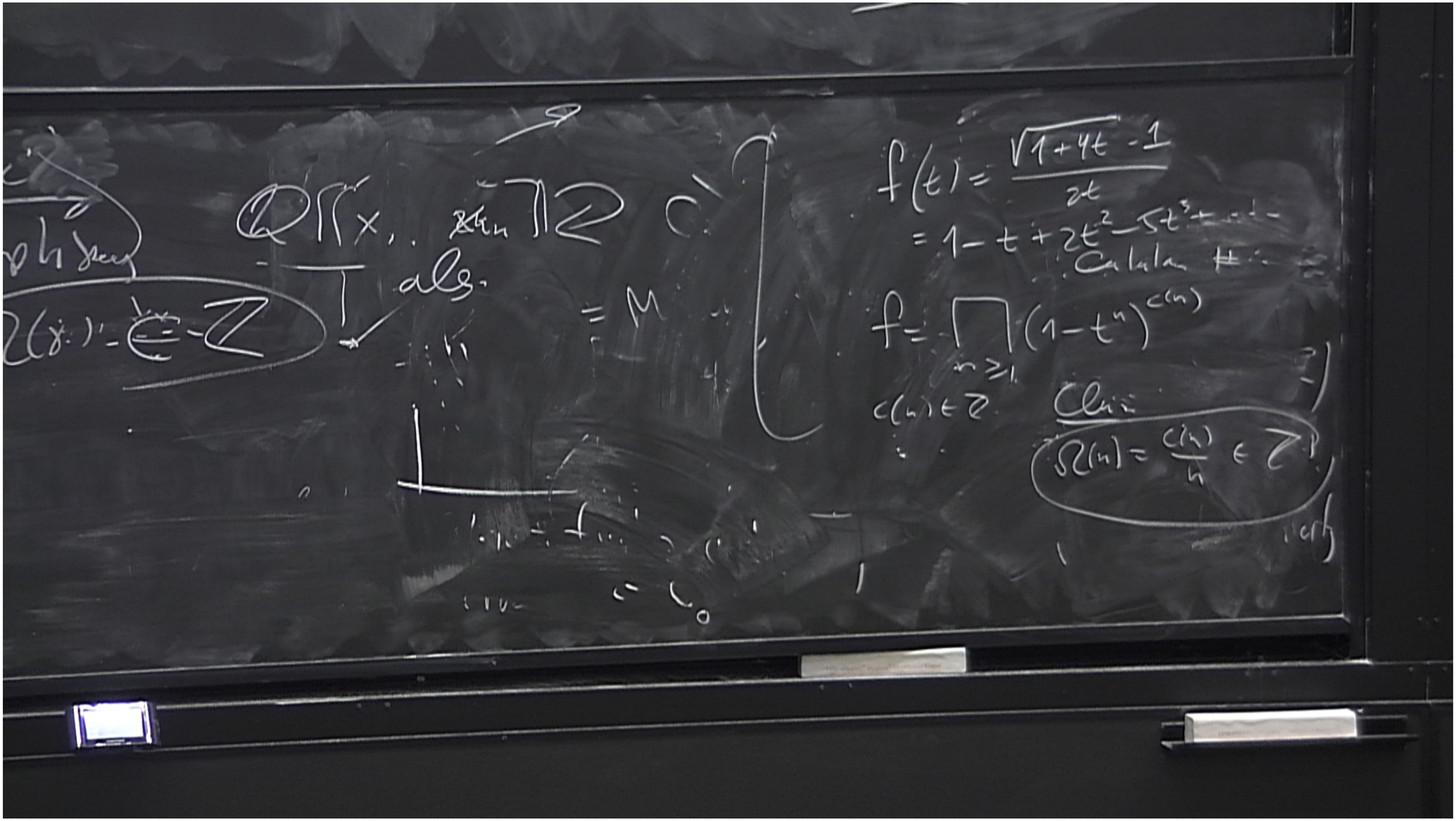
i.e. $a \wedge b \to \frac{da}{a} \wedge \frac{db}{b}$

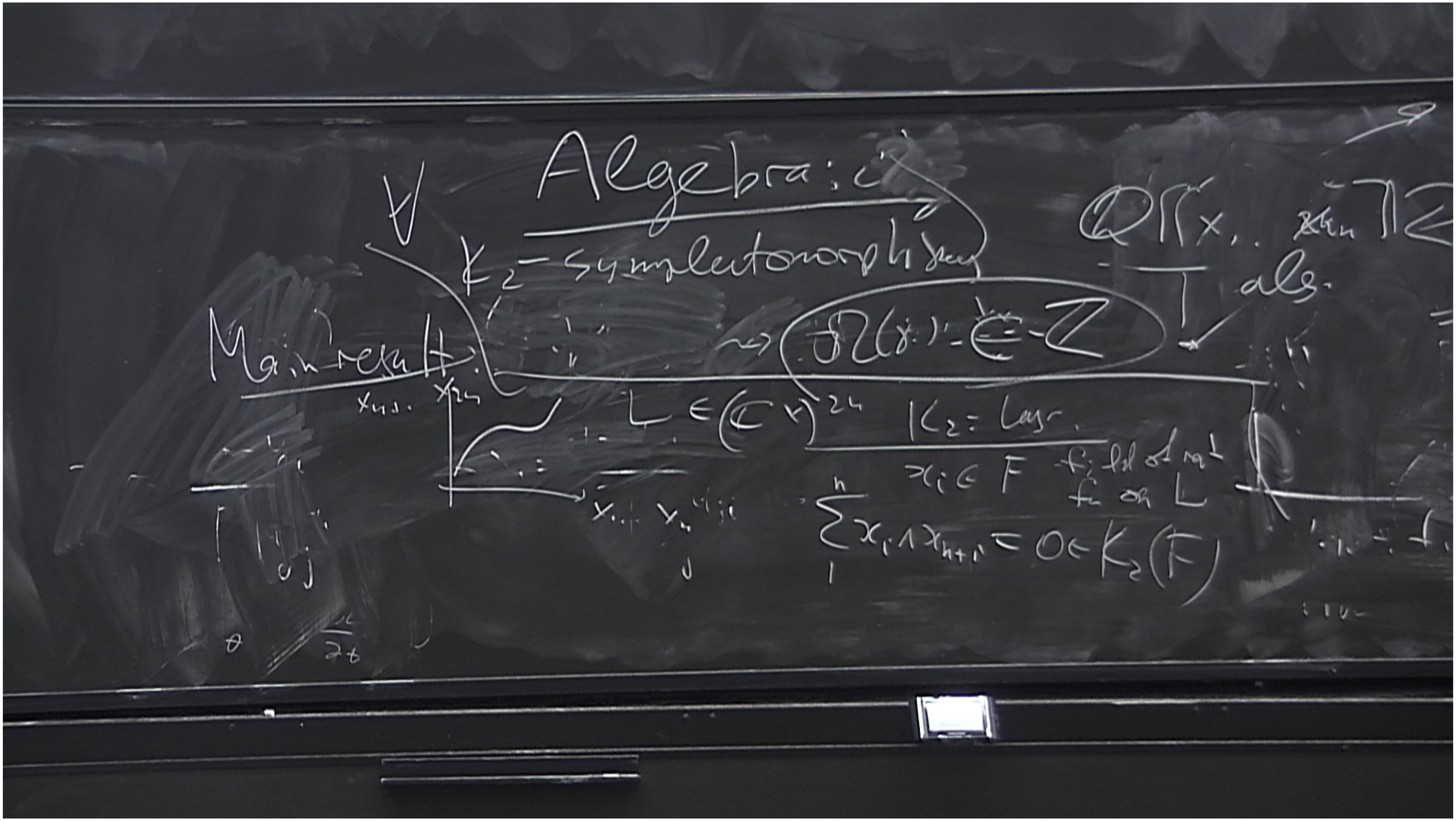
$\underbrace{a \wedge (1-a) = 0 \quad \forall\, a \in F - \{0,1\}}_{\text{Steinberg relation}}$


field
here F = Q(x1, ..., x2n)
K1(F) → rational closed 1-form with periods (2πi)Z
a ∈ F → da/a
K2(F) → rat. closed 2-form
i.e. a ∧ b → da/a ∧ db/b
a ∧ (1 − a) = 0
∀ a ∈ F − {0,1}
Steinberg relation

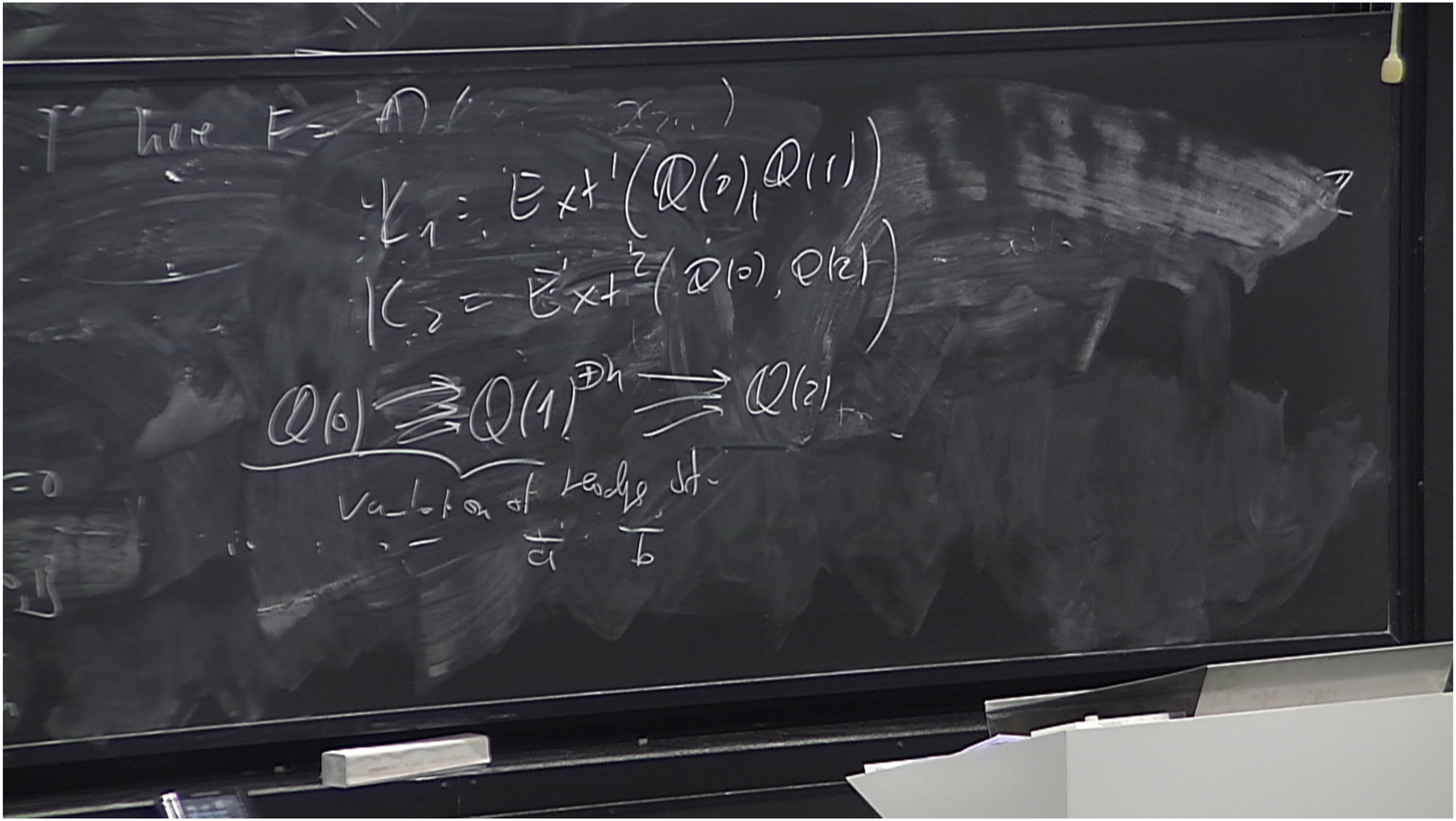

knot ∈ ℝ³
A-polynomial
curve ∈ (ℂˣ)²
π₁(ℝ³ − knot) → SL(2,ℂ)
∀ vertex
→ ℝ>0


knot ∈ ℝ³
A-polynomial
curve ∈ (ℂ^×)²
π₁(ℝ³ − knot) → SL(2,ℂ)
1-par. family
∀ vertex
→ ℝ>0
cross-ratio


Q[[x_1, .. x_n]]Z
alg.
f(t) = (√(1+4t) − 1) / 2t
= 1 − t + 2t² − 5t³ + ...
Calculate
f = ∏_{n≥1} (1 − tⁿ)^{c(n)}
c(n) ∈ Z
Claim:
Ω(n) = c(n)/n ∈ Z ?



Algebraic
Main result
alg.
K₂ = Lagr.


here F =
$K_1 = \mathrm{Ext}^1(Q(0), Q(1))$
$K_2 = \mathrm{Ext}^2(Q(0), Q(2))$
$Q(0) \Rrightarrow Q(1) \Rrightarrow Q(2)$
variation of Hodge str.