Title: The closest cousins of quantum theory from three simple principles

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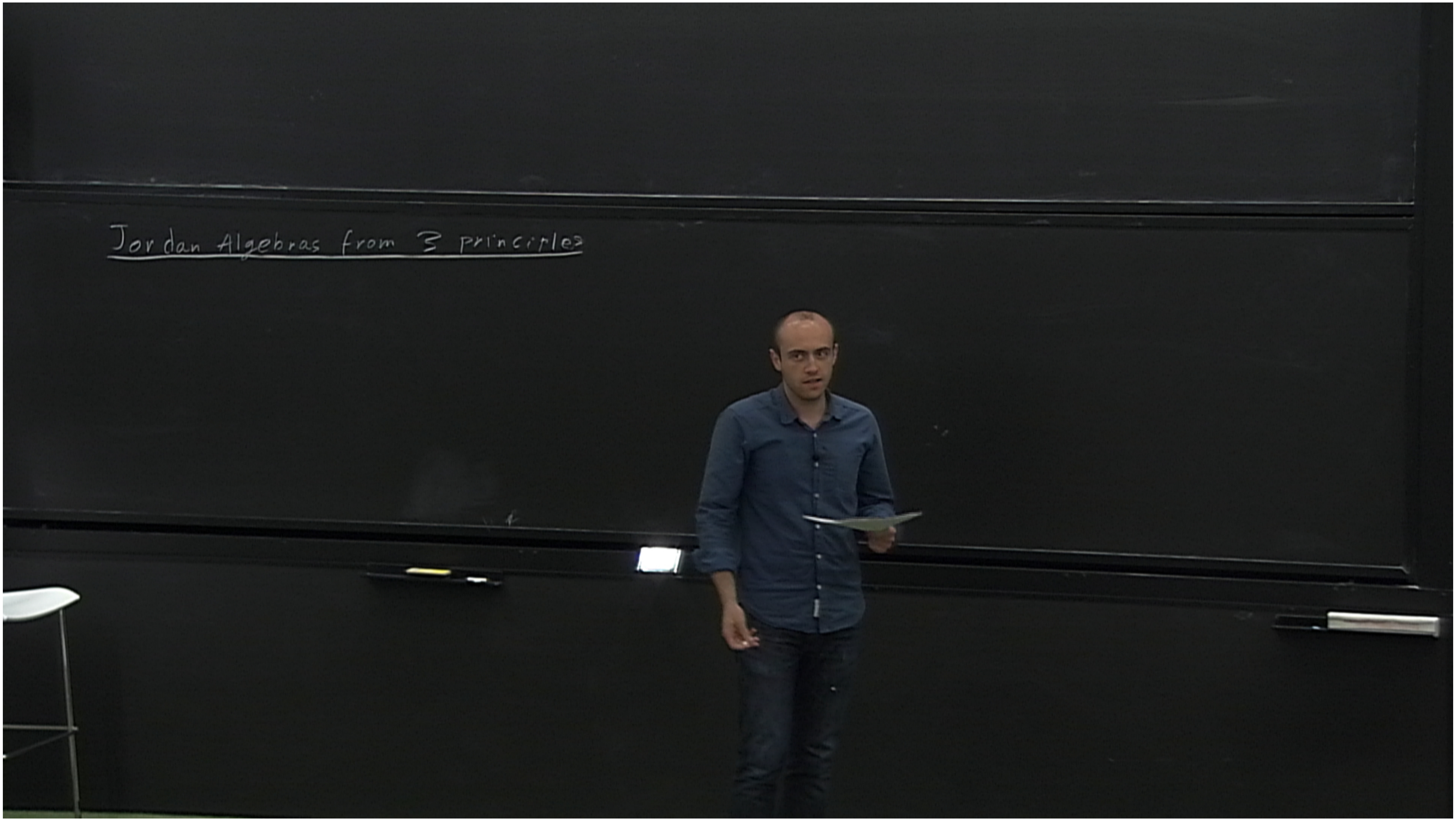
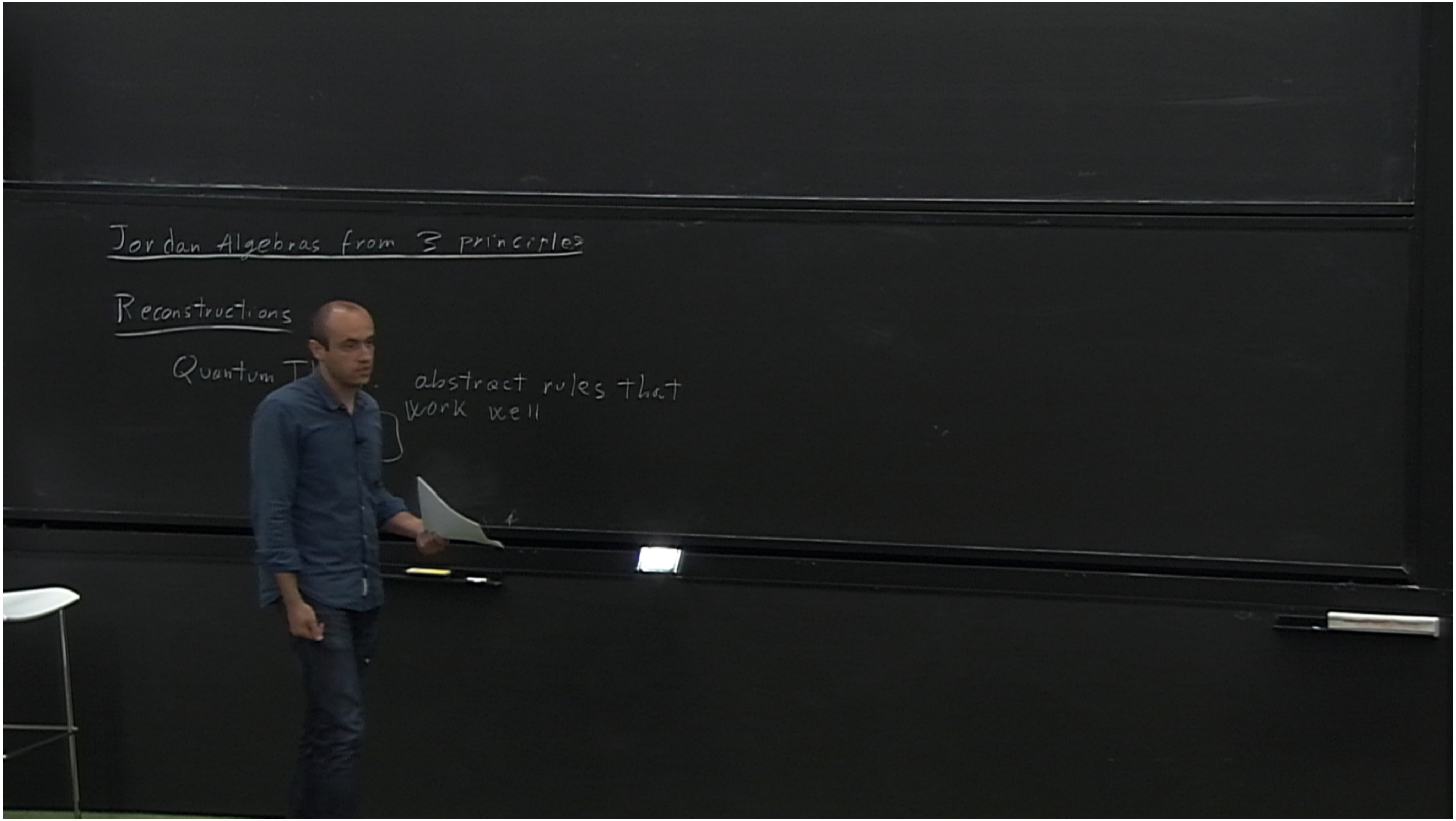
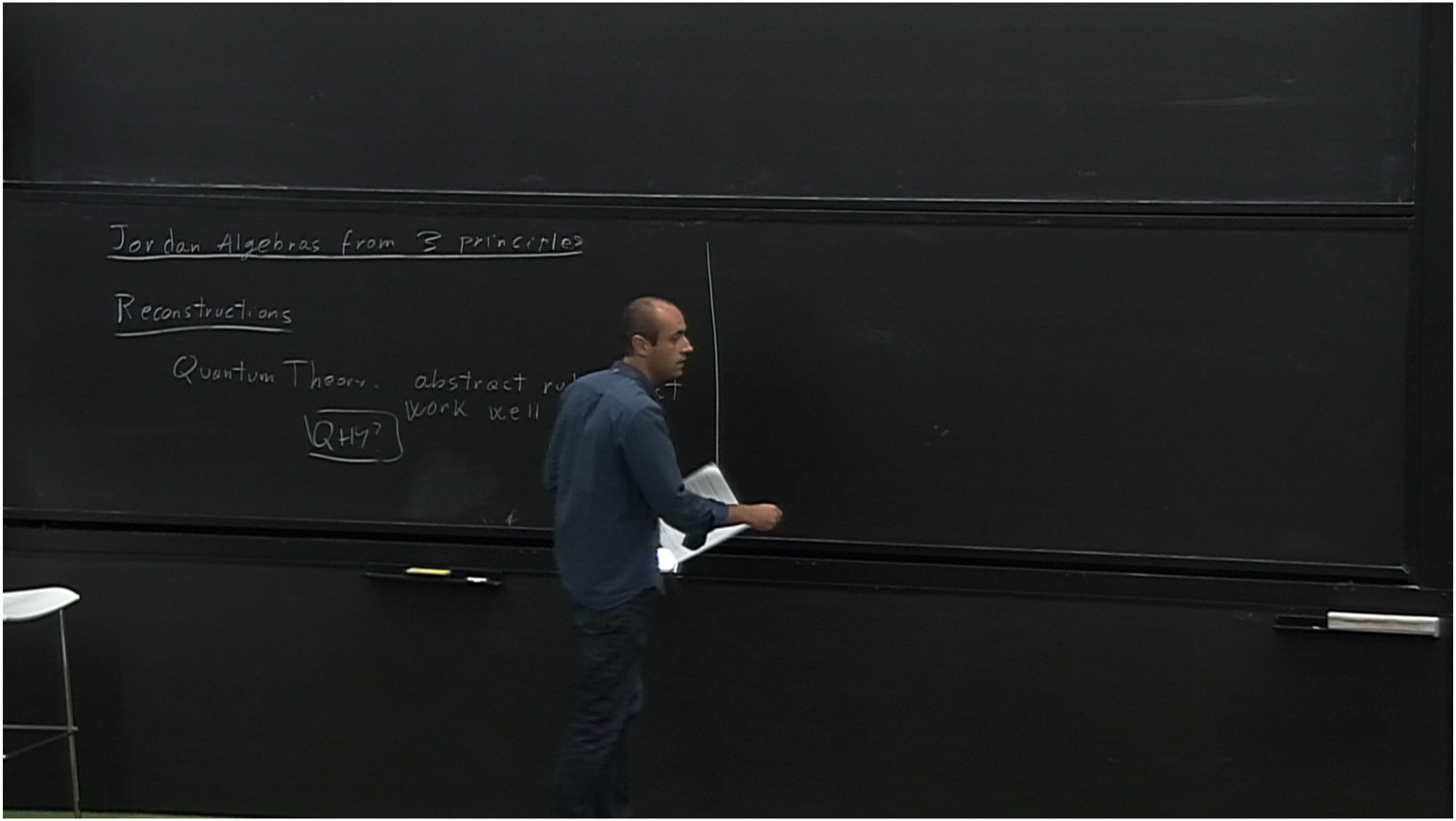
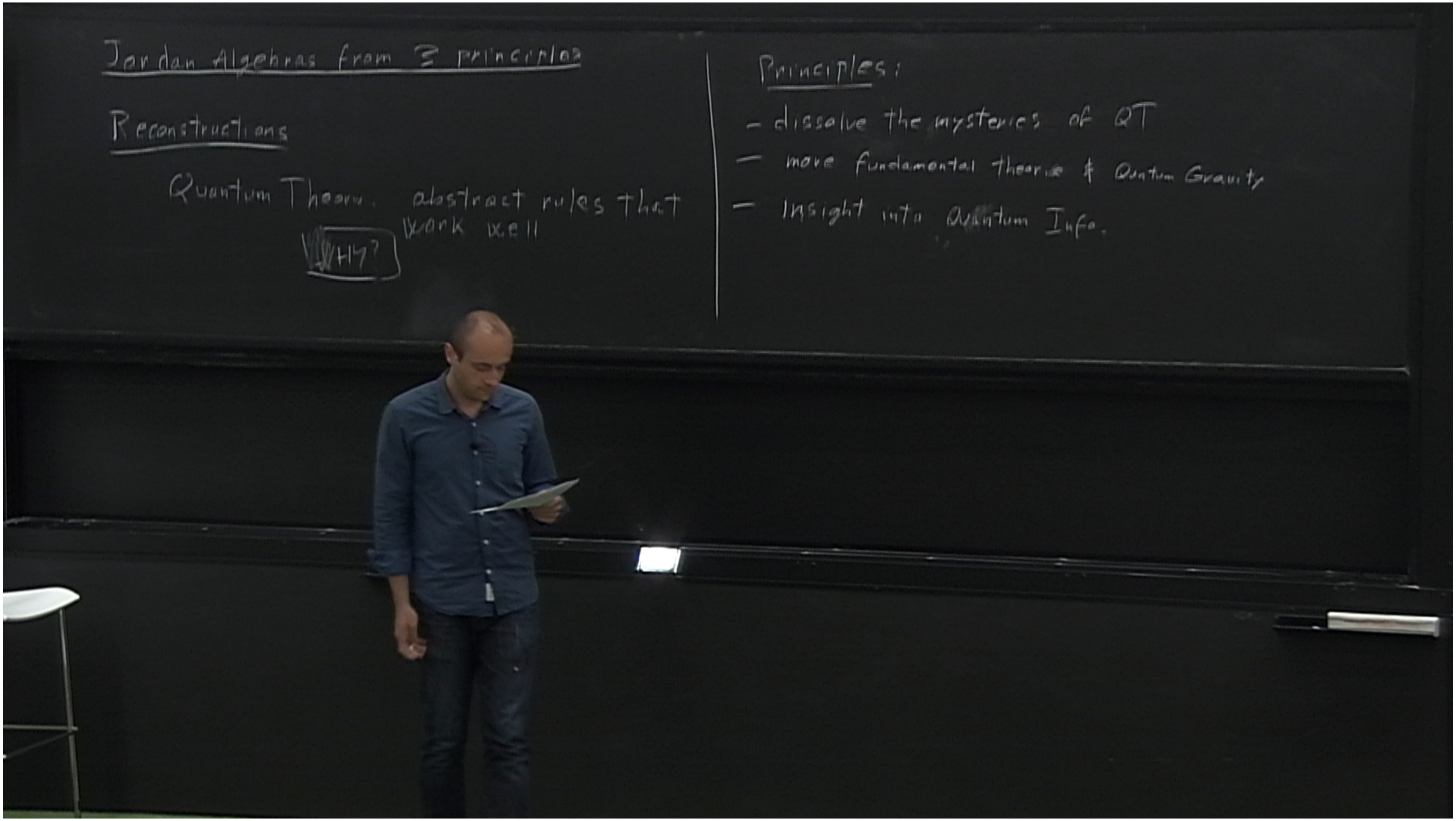
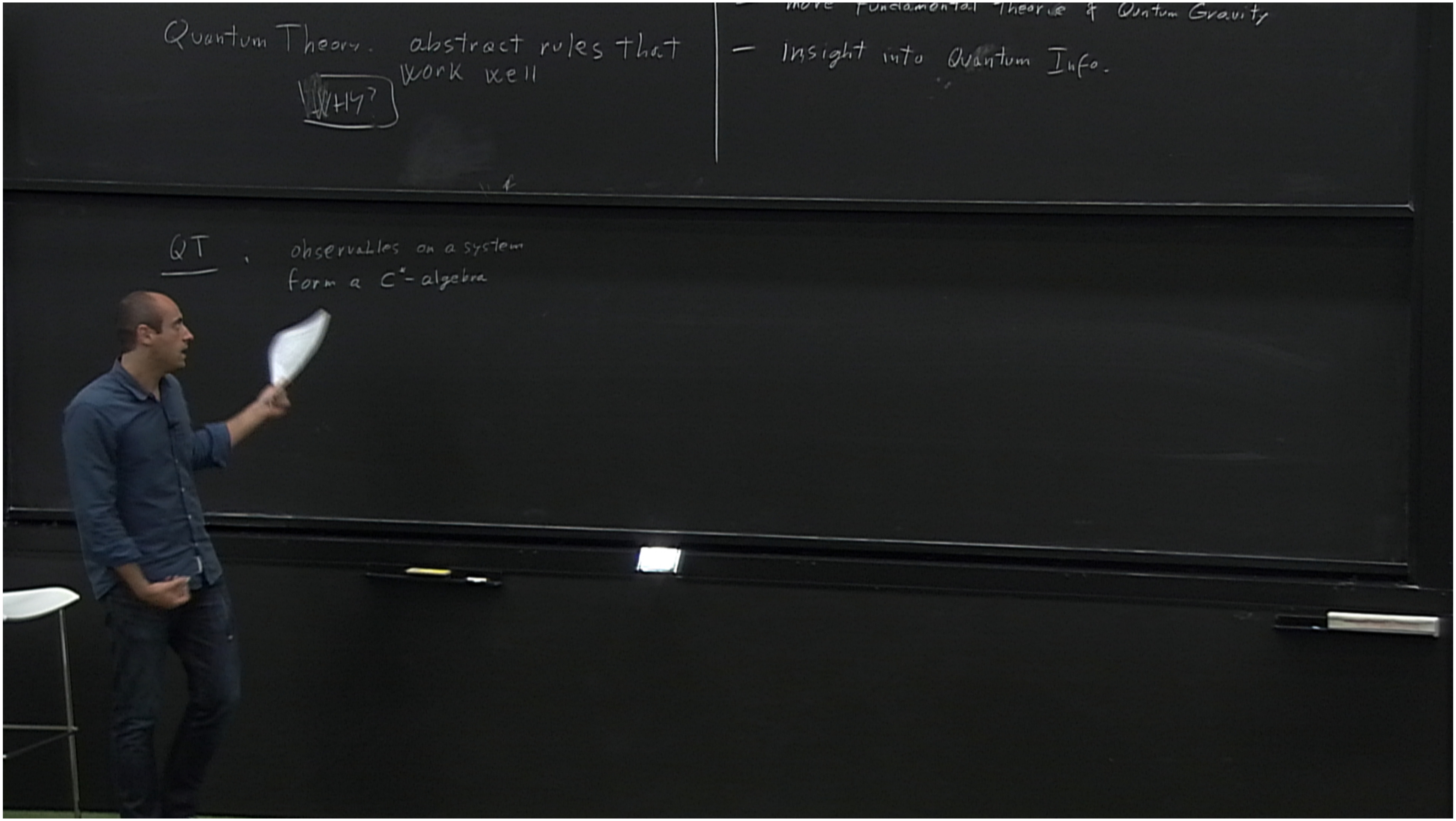
Abstract: A very general way of describing the abstract structure of quantum theory is to say that the set of observables on a quantum system form a C*-algebra. A natural question is then, why should this be the case - why can observables be added and multiplied together to form any algebra, let alone of the special C* variety? I will present recent work with Markus Mueller and Howard Barnum, showing that the closest algebraic cousins to standard quantum theory, namely the Jordan-algebras, can be characterized by three principles having an informational ï¬‚avour, namely: (1) a generalized spectral decomposition, (2) a high degree of symmetry, and (3) a requirement on conditioning on the results of observations.   I'll then discuss alternatives to the third principle, as well as the possibility of dropping it as a way of searching for natural post-quantum theories.

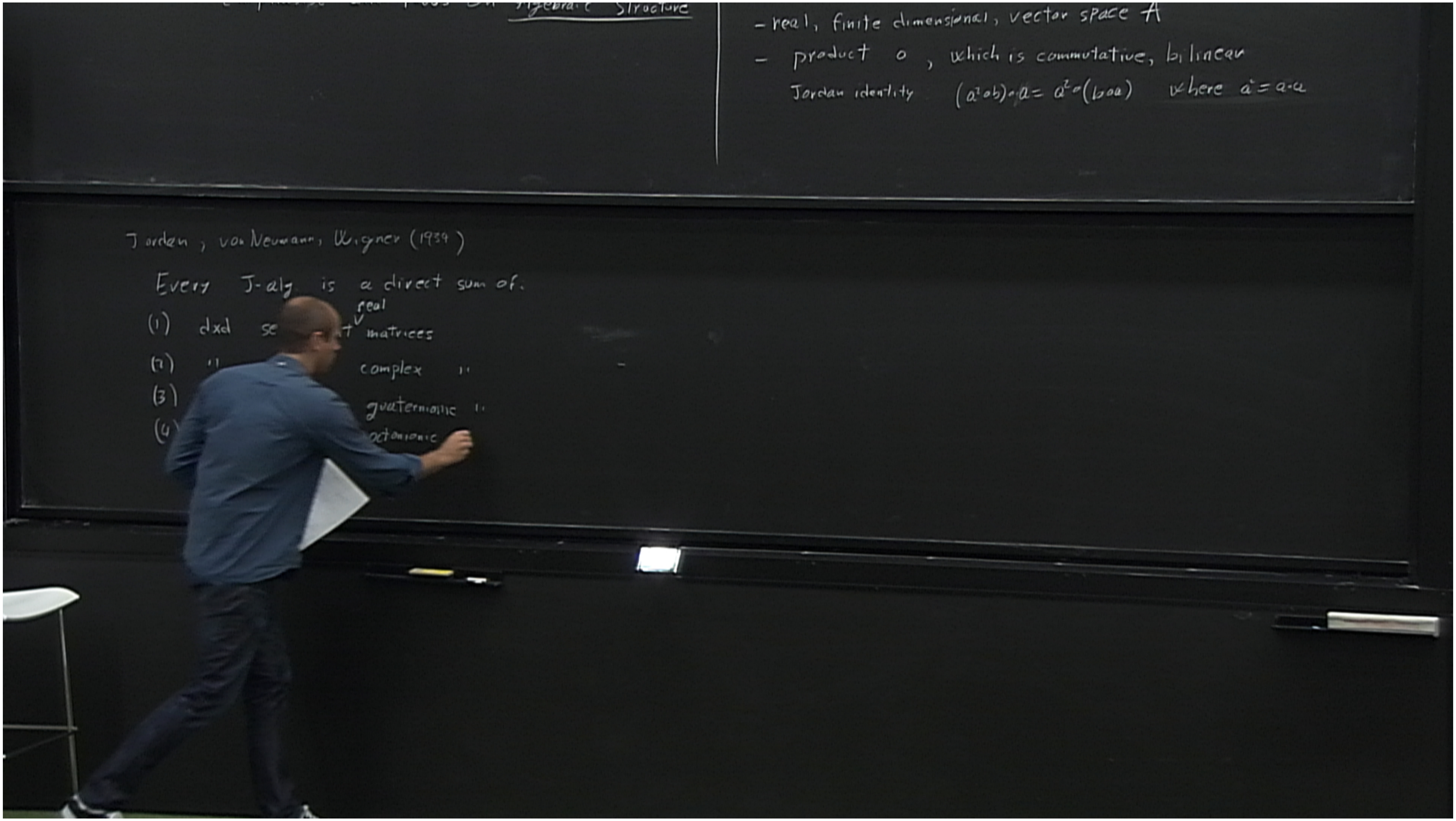

Jordan Algebras from 3 principles

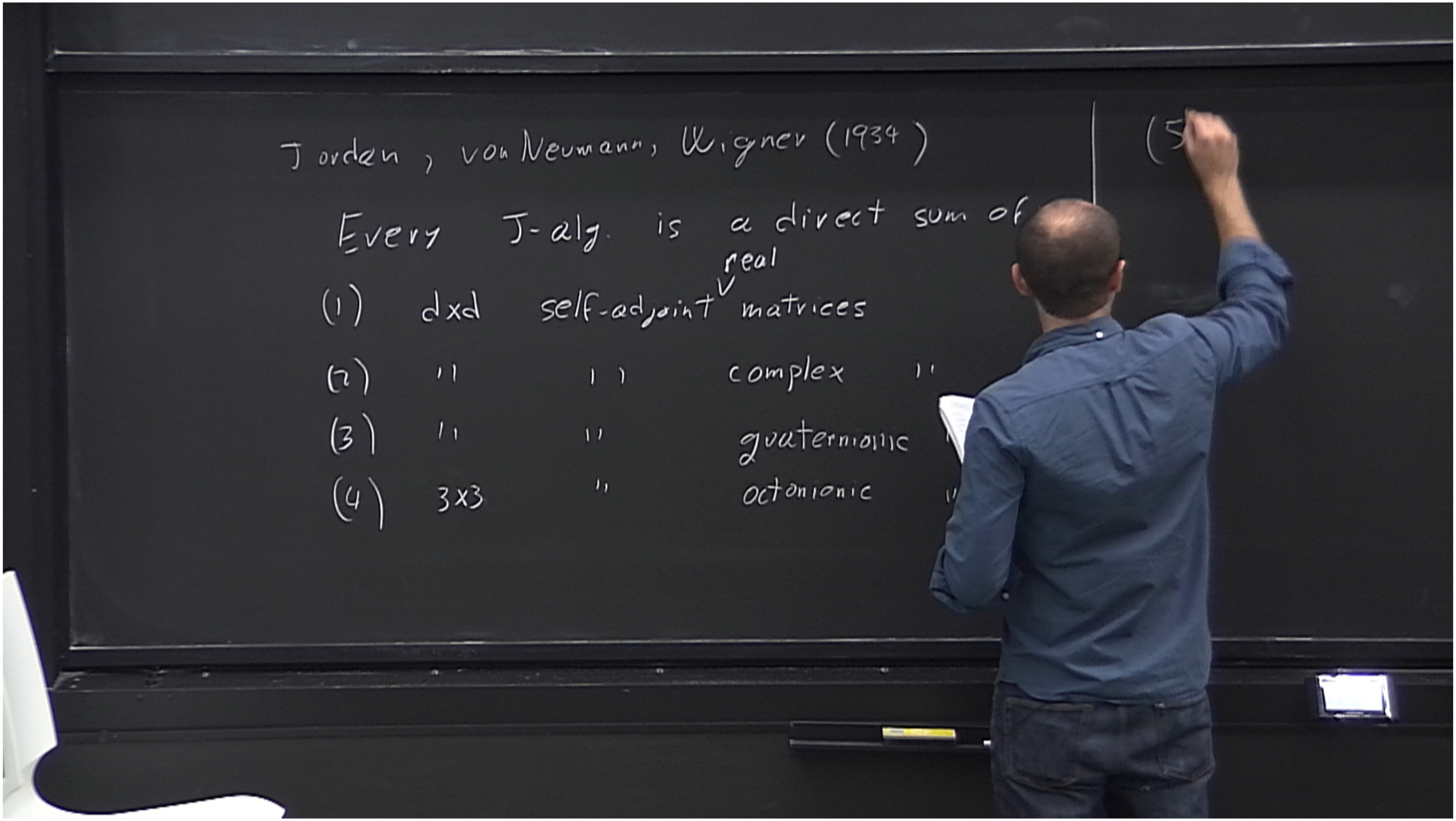

Jordan Algebras from 3 principles
Reconstructions
Quantum T
abstract rules that
work well

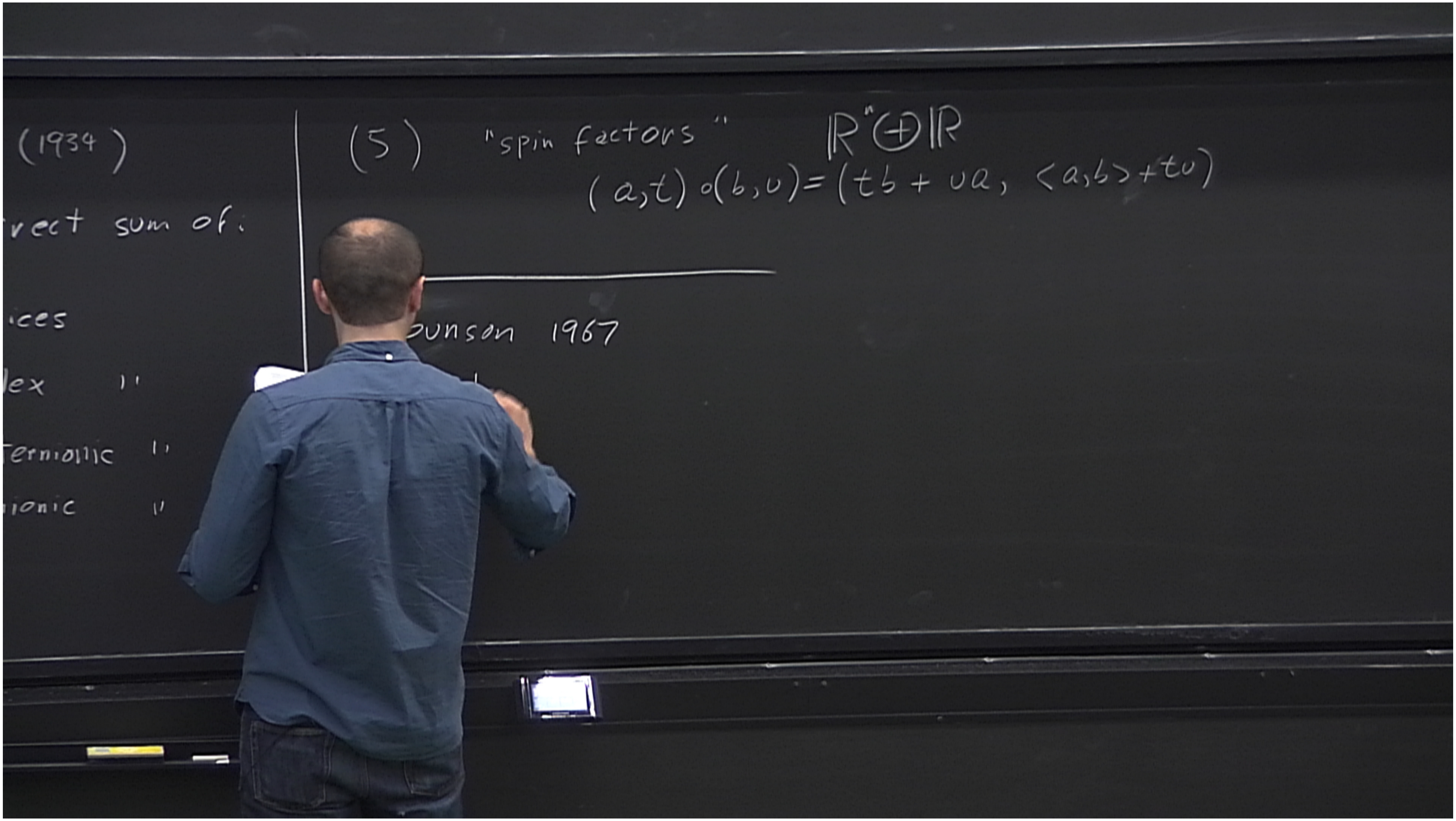

Jordan Algebras from 3 principles
Reconstructions
Quantum Theory.
abstract
work well


Jordan Algebras from 3 principles
Reconstructions
Quantum Theory: abstract rules that work well
WHY?
Principles:
– dissolve the mysteries of QT
– more fundamental theories & Quantum Gravity
– Insight into Quantum Info.

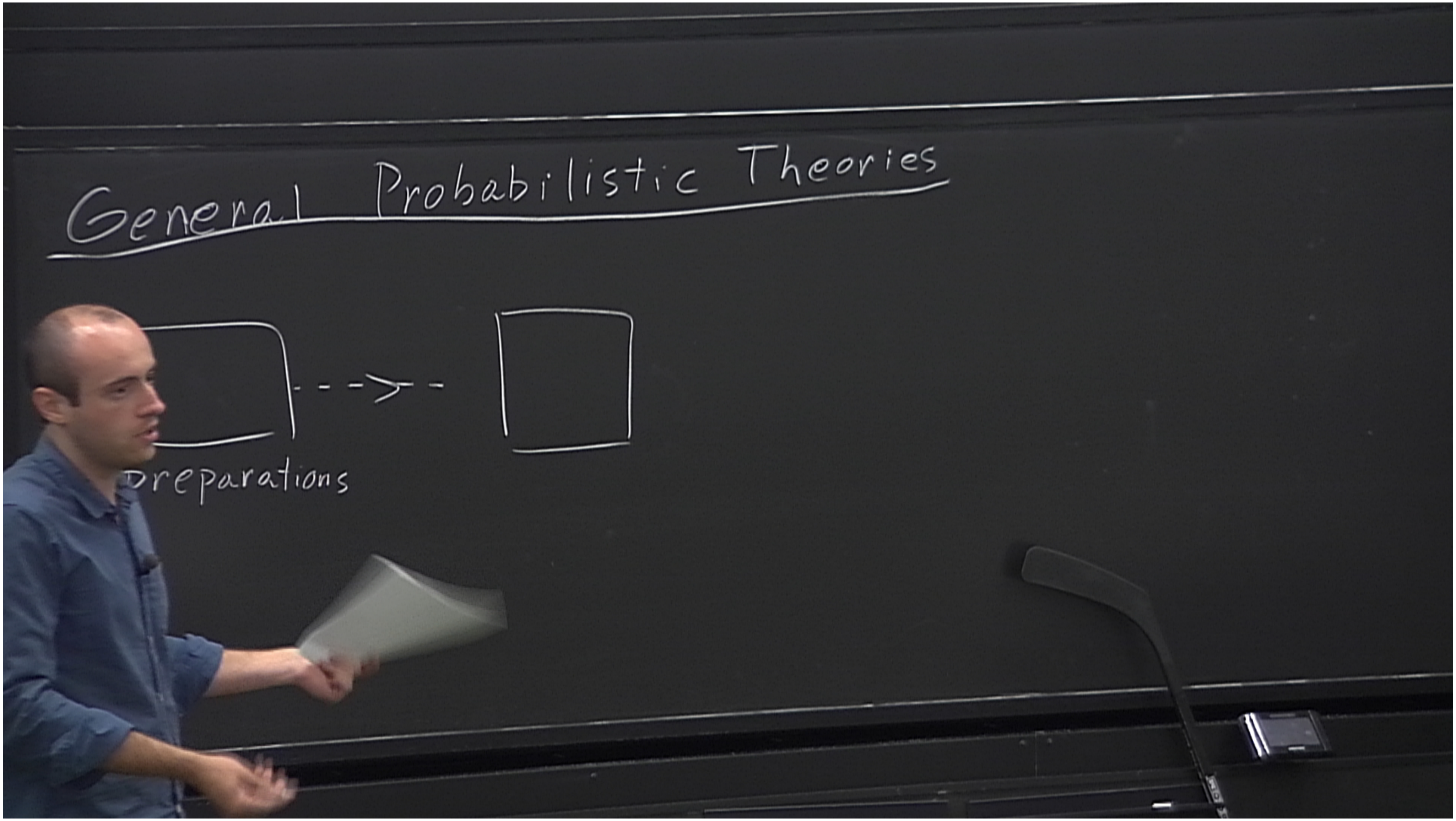

Quantum Theory. abstract rules that work well
WHY?
Theorie & Quantum Gravity
— Insight into Quantum Info.
QT : observables on a system form a C*-algebra

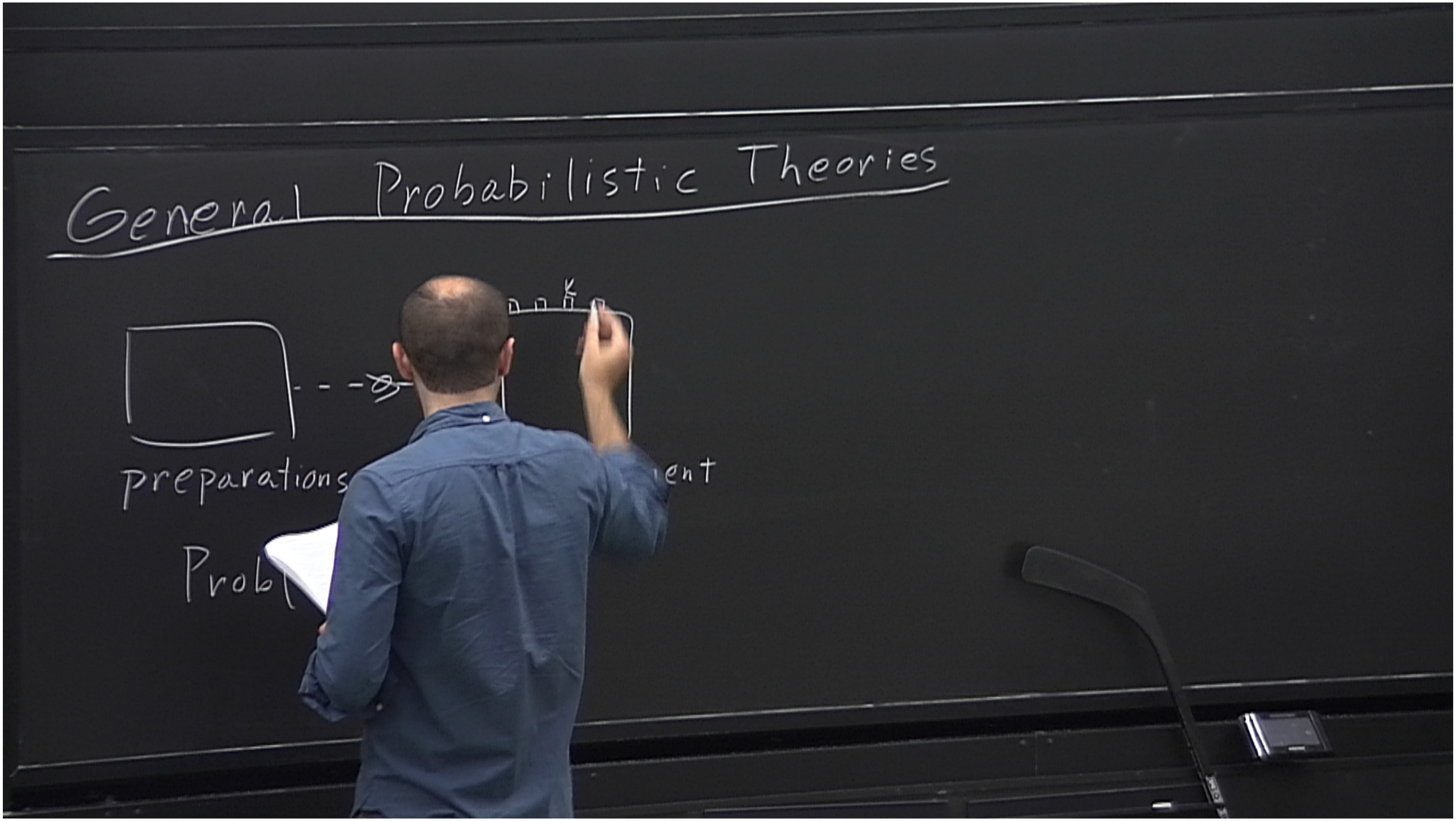

Structure
- real, finite dimensional, vector space A
- product ∘ , which is commutative, bilinear
Jordan identity (a²∘b)∘a = a²∘(b∘a) where a² = a∘a
Jordan, von Neumann, Wigner (1934)
Every J-alg is a direct sum of.
real
(1) dxd
matrices
(2) "
complex "
(3)
quaternionic "
(4)
octonionic


Jorden, von Neumann, Wigner (1934)
Every J-alg. is a direct sum of
(1) dxd self-adjoint real matrices
(2) " " complex "
(3) " " quaternionic "
(4) 3x3 " octonionic "
(5



(1934)
vect sum of:
ices
lex ''
Ternionic ''
ionic ''
(5) "spin factors" $\mathbb{R}^n \oplus \mathbb{R}$
$(a,t) \circ (b,u) = (tb + ua, \langle a,b \rangle + tu)$
ounson 1967


General Probabilistic Theories
preparations


General Probabilistic Theories
preparations
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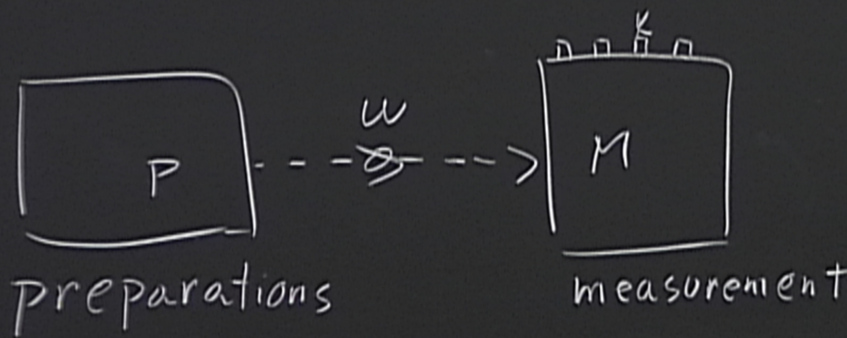
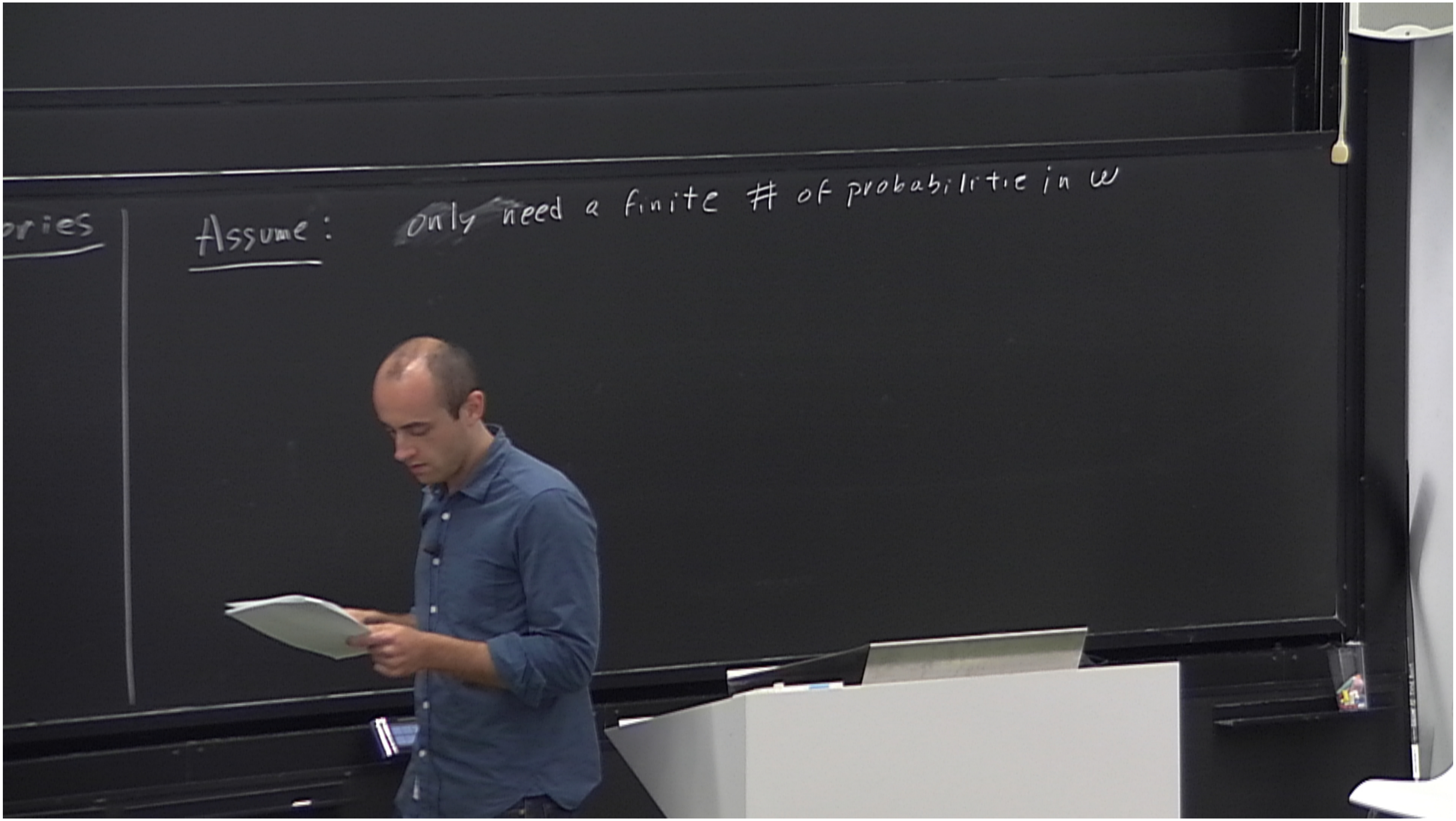
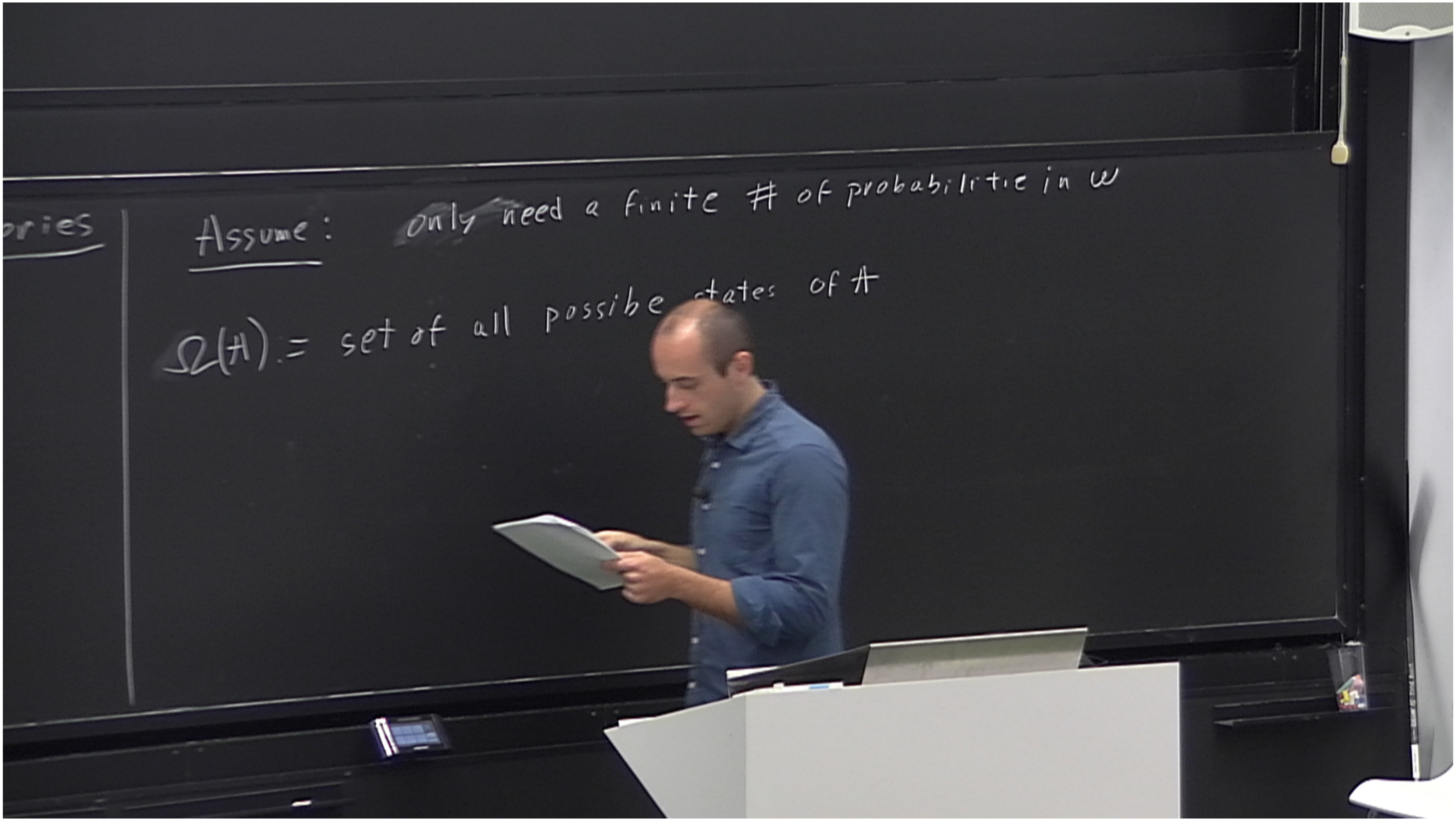
Prob

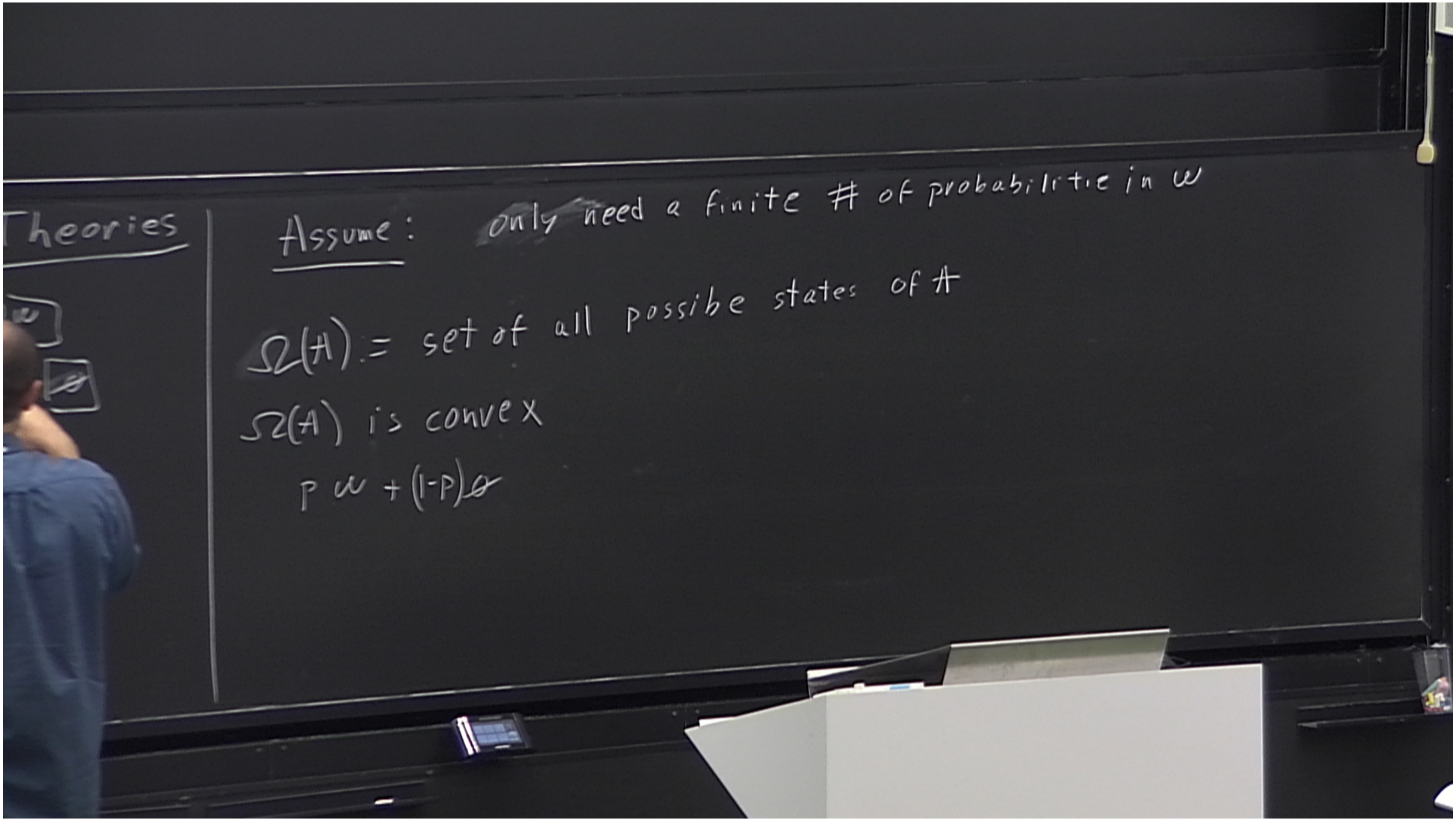
General Probabilistic Theories

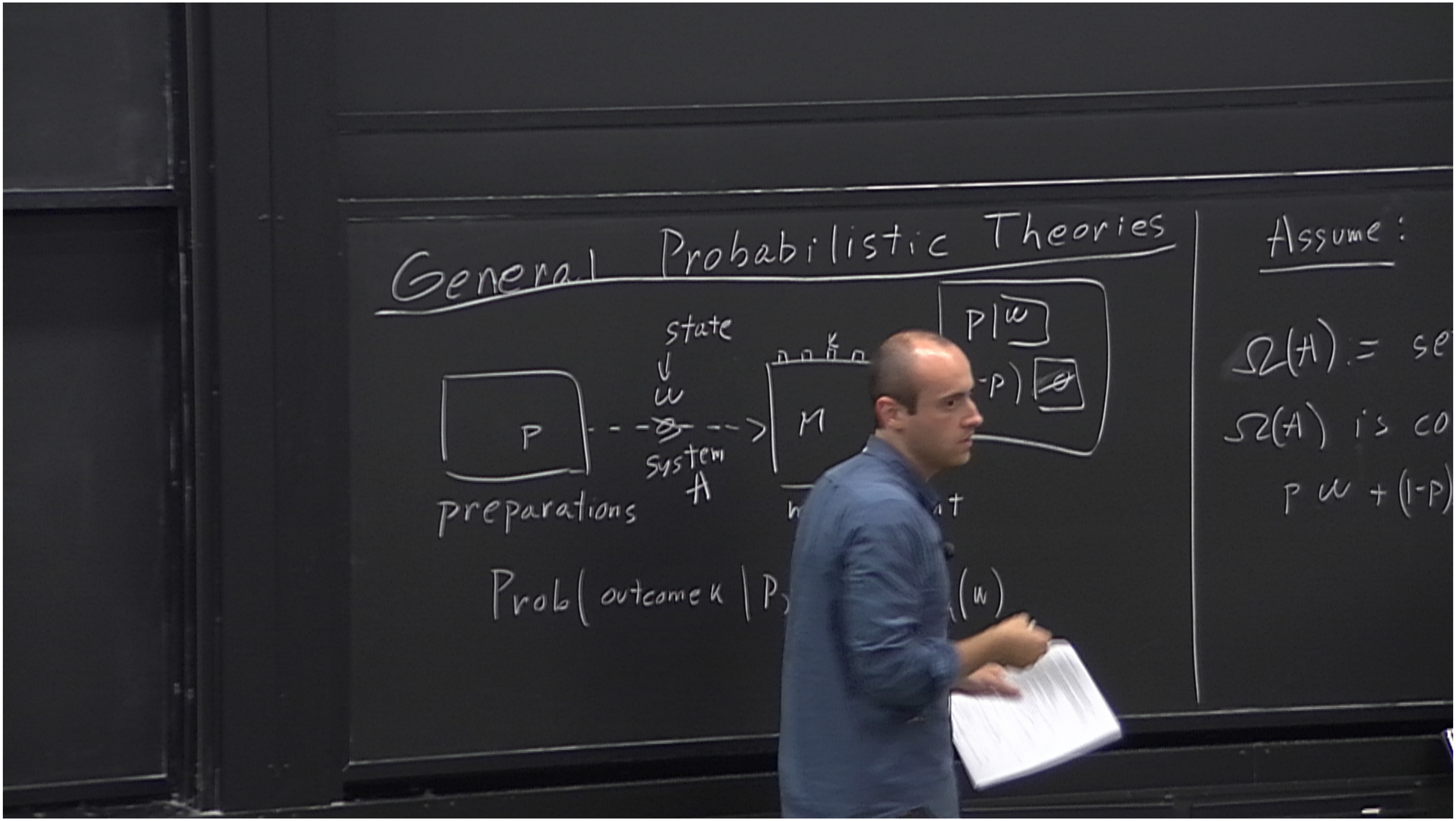


$$\mathrm{Prob}(\text{outcome } k \mid P, M)$$


ories
Assume: only need a finite # of probabilitie in ω


ories
Assume: only need a finite # of probabilitie in ω
Ω(A) = set of all possibe states of A


Theories
Assume: only need a finite # of probabilitie in ω
Ω(A) = set of all possibe states of A
Ω(A) is convex
p ω + (1-p)σ


General Probabilistic Theories
Assume:
state
system
A
P
M
preparations
Prob(outcome k | P,
Ω(A) := se
Ω(A) is co
pω + (1-p)

...ic Theories

$p \, \boxed{\omega}$

$(1-p) \, \boxed{\sigma}$ $\longrightarrow$

...ment

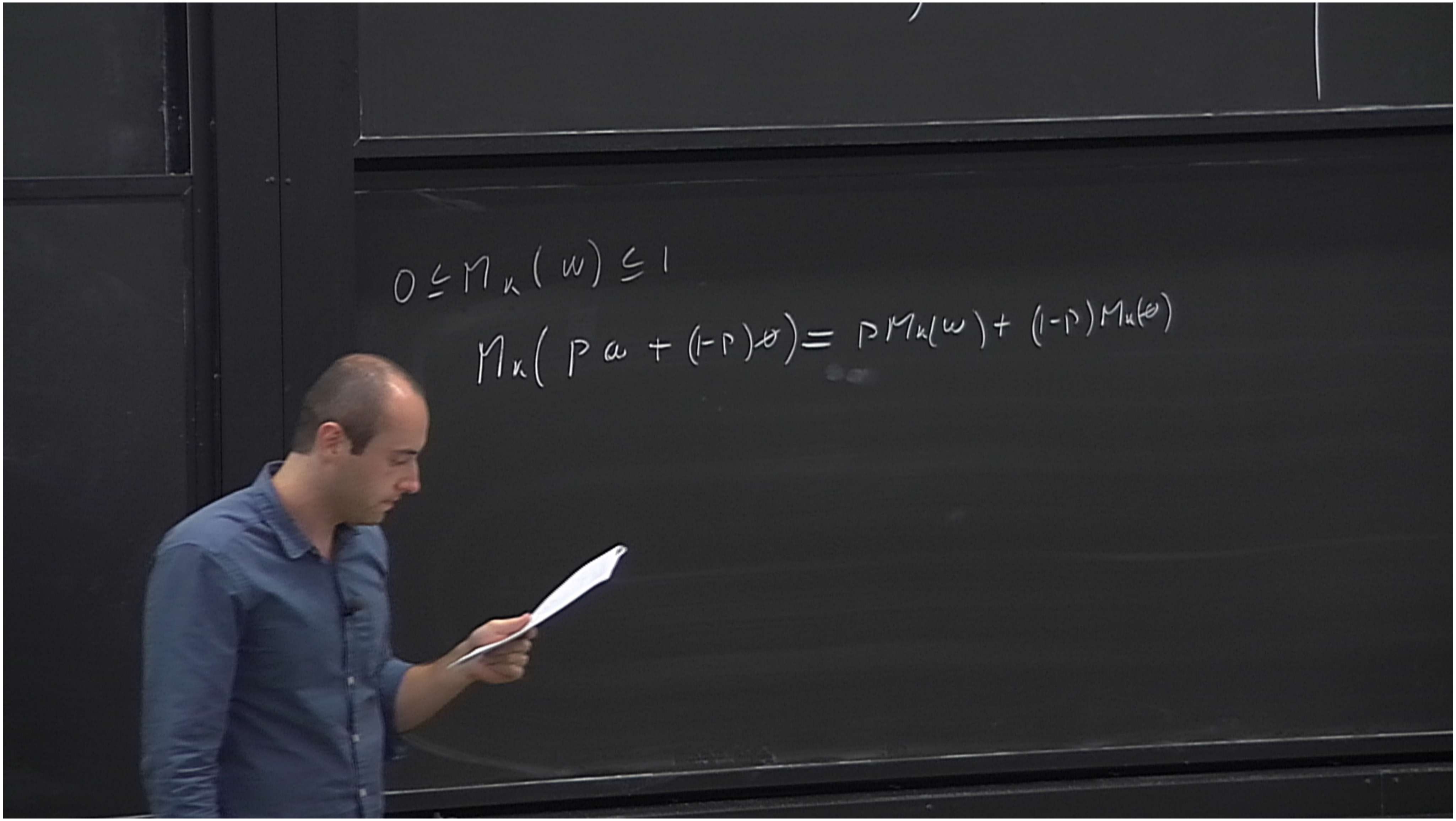
$:= M_n(\mathbb{C})$

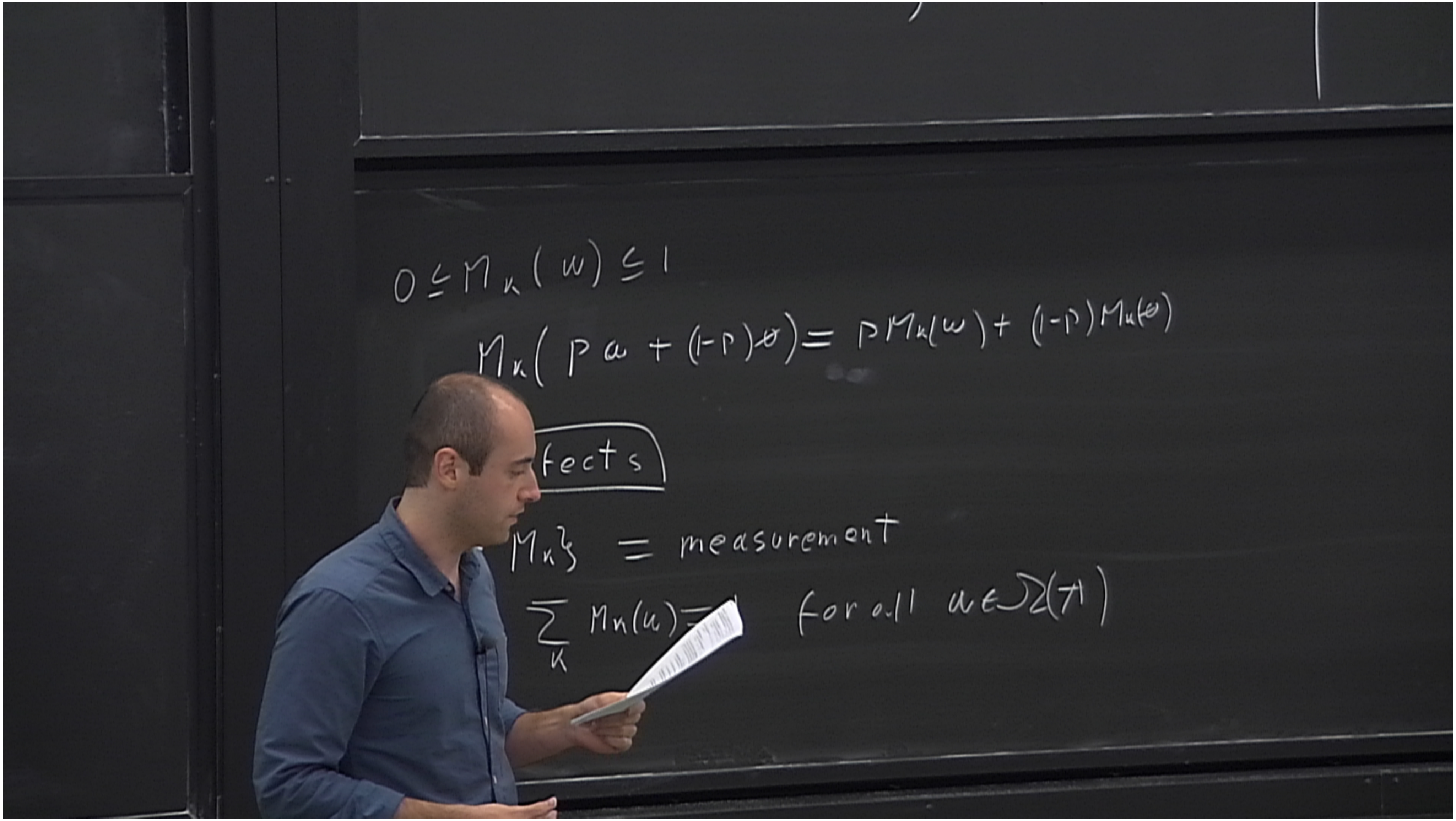
Assume: only need a finite # of probabilities in ω

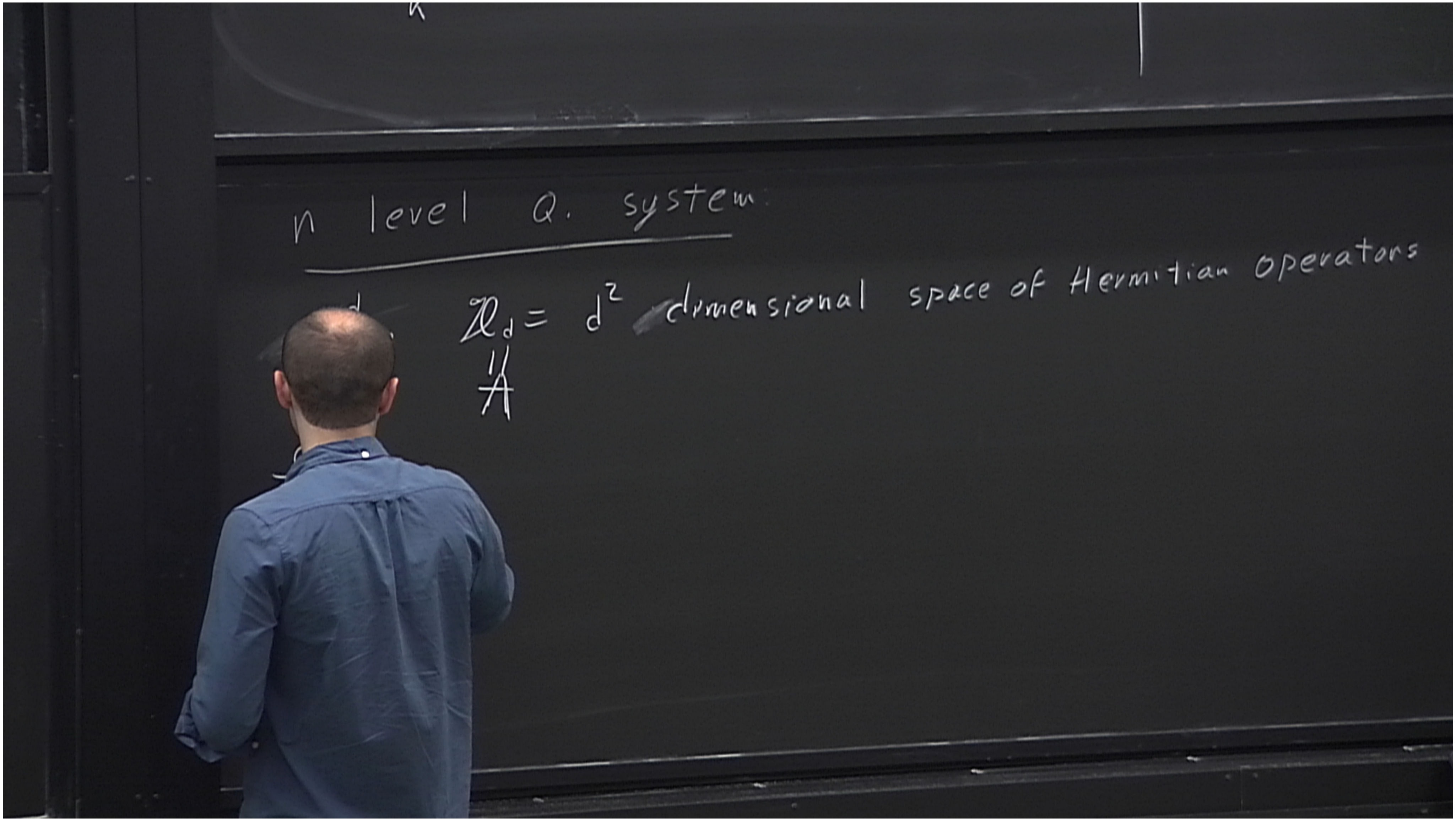
$\Omega(A)$ = set of all possible states of A

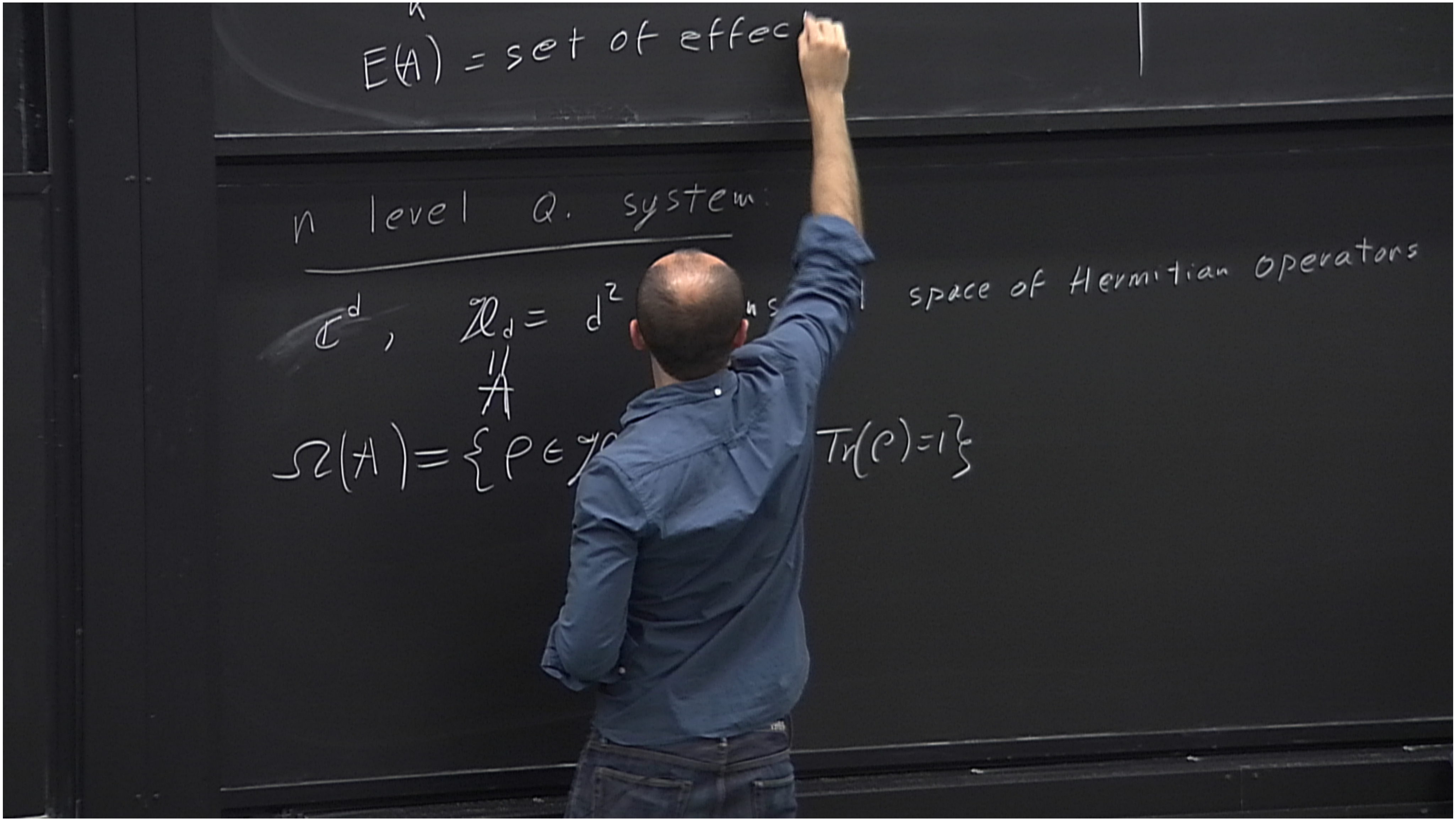
$\Omega(A)$ is convex, compact, $\subseteq \mathbb{R}^n =: A$

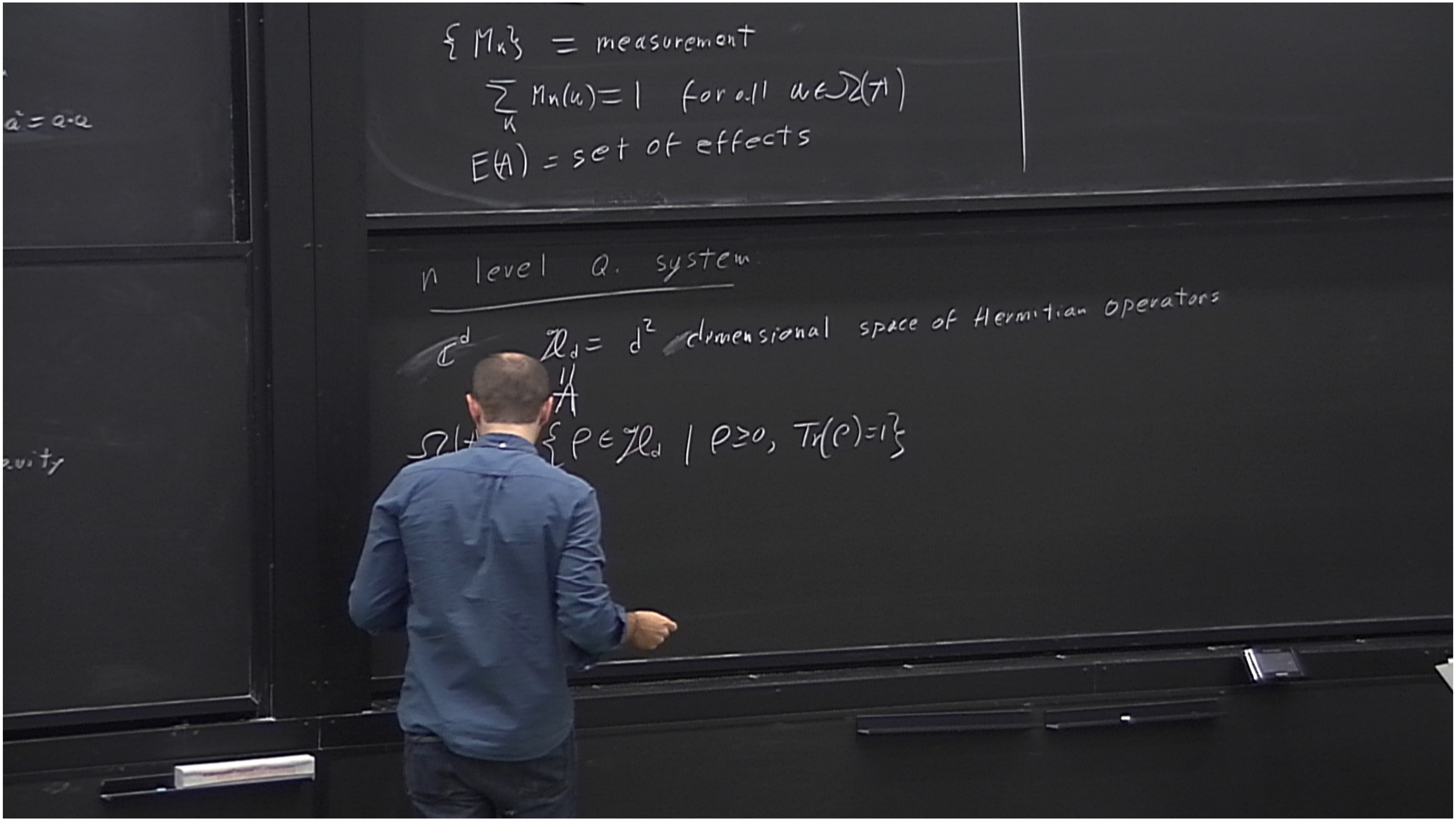
$p\omega + (1-p)\sigma$

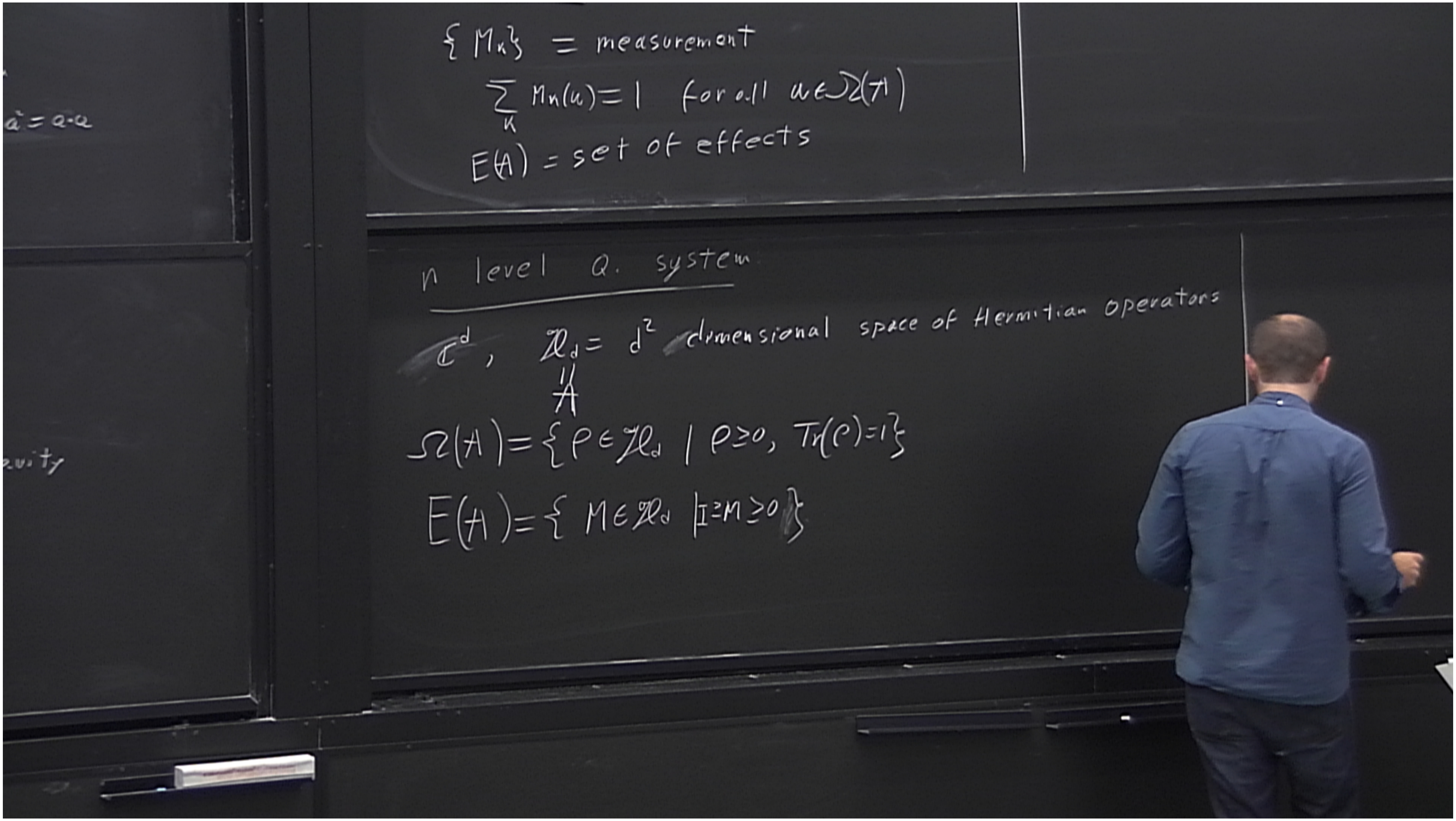

$0 \leq M_k(w) \leq 1$
$M_k(p\omega + (1-p)\theta) = pM_k(\omega) + (1-p)M_k(\theta)$

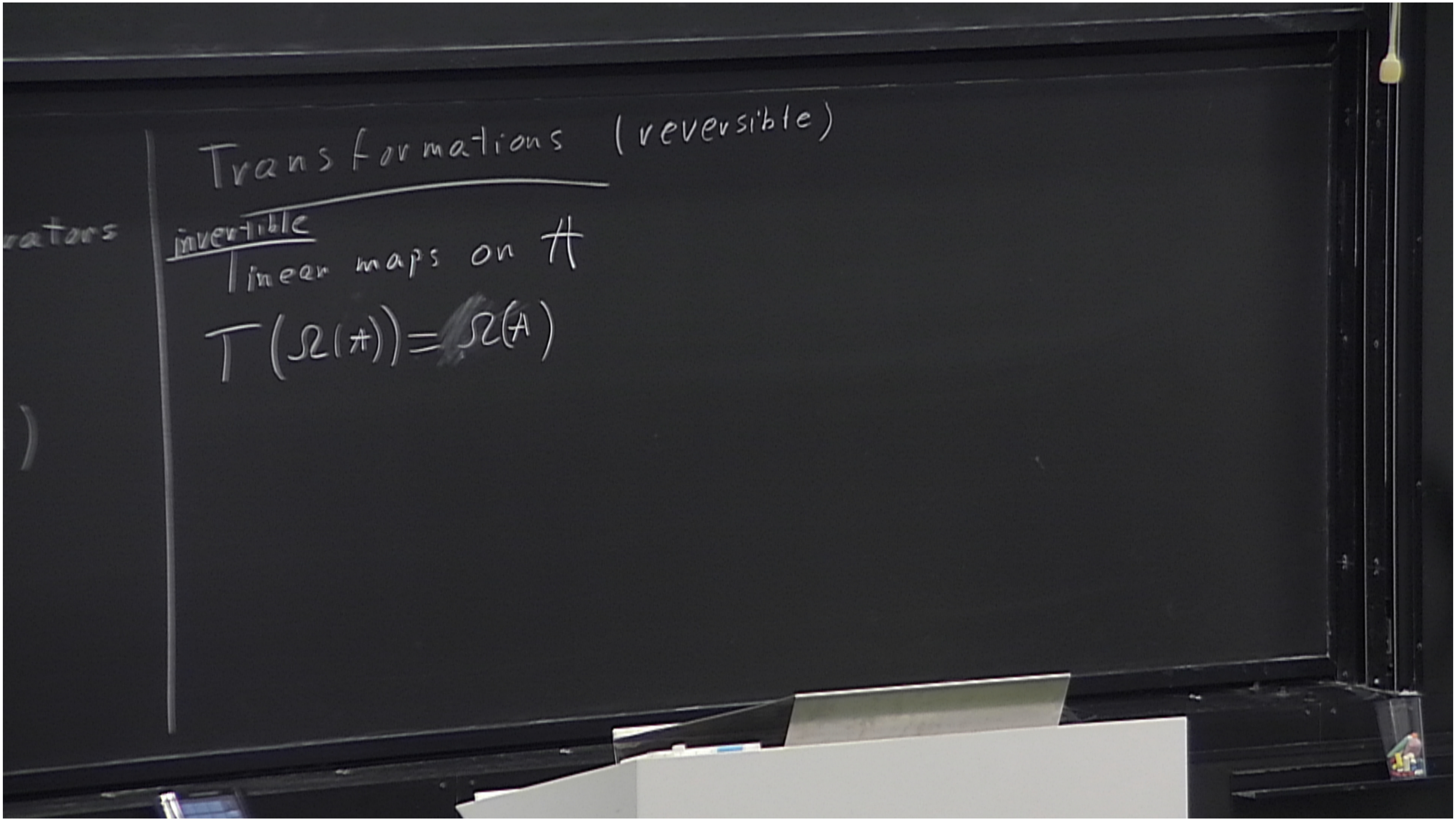

0 ≤ M_k(ω) ≤ 1
M_k(pω + (1-p)σ) = pM_k(ω) + (1-p)M_k(σ)
fects
M_k} = measurement
∑_k M_k(ω) = for all ω ∈ Ω(A)

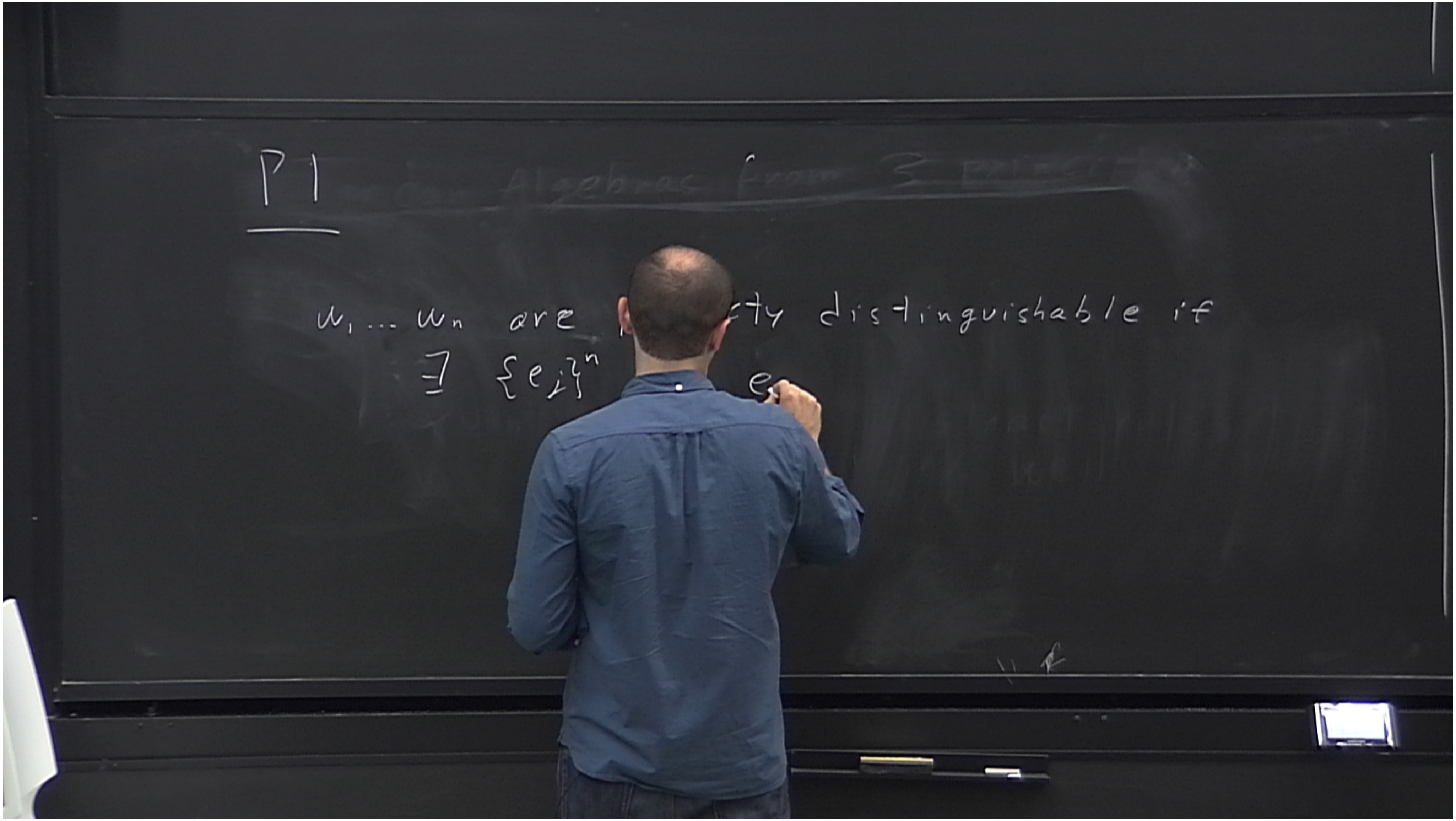

n level Q. system:
$\mathcal{H}_d = d^2$ dimensional space of Hermitian operators

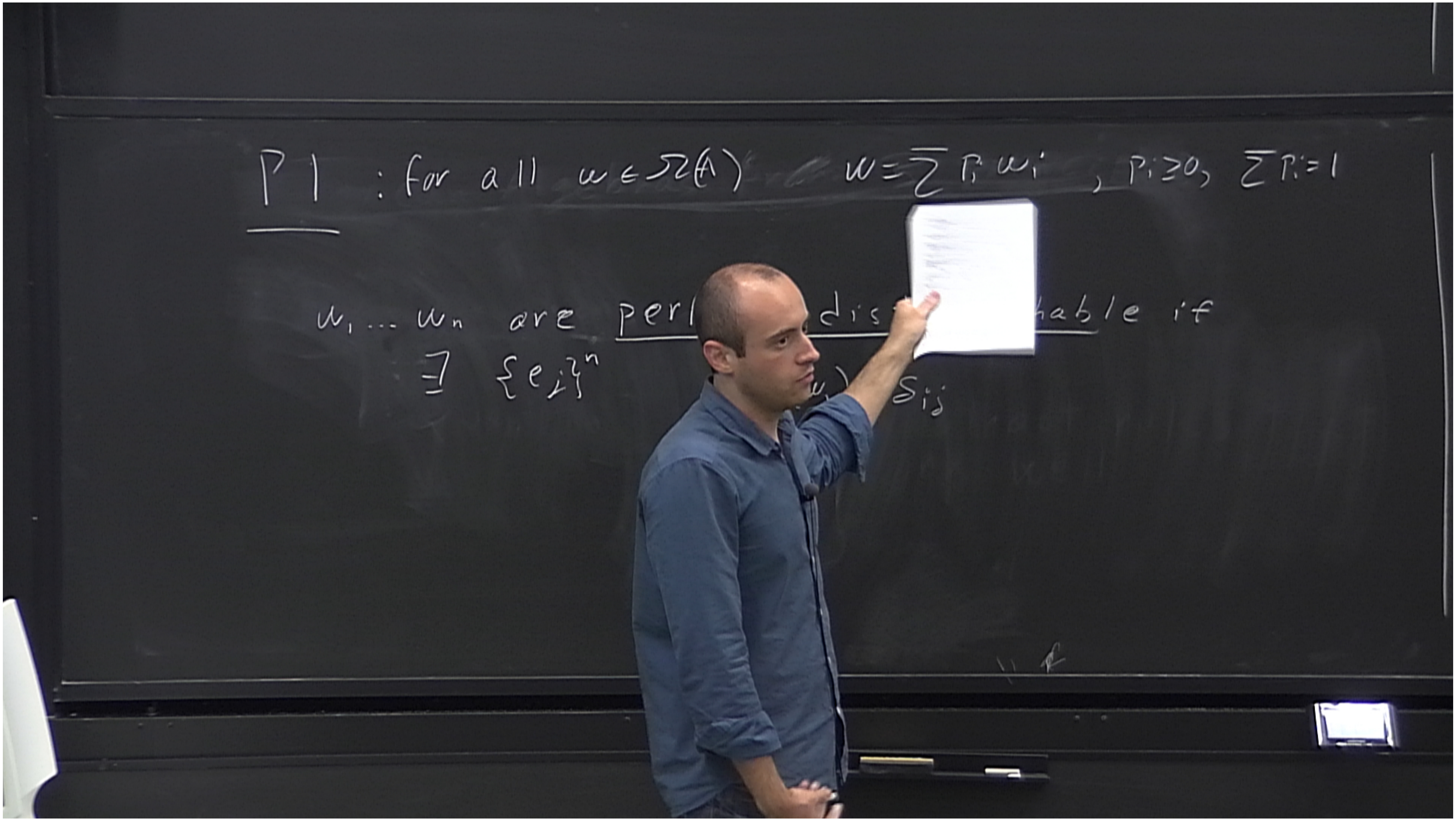

E(A) = set of effec
n level Q. system:
space of Hermitian operators
Tr(ρ)=1}

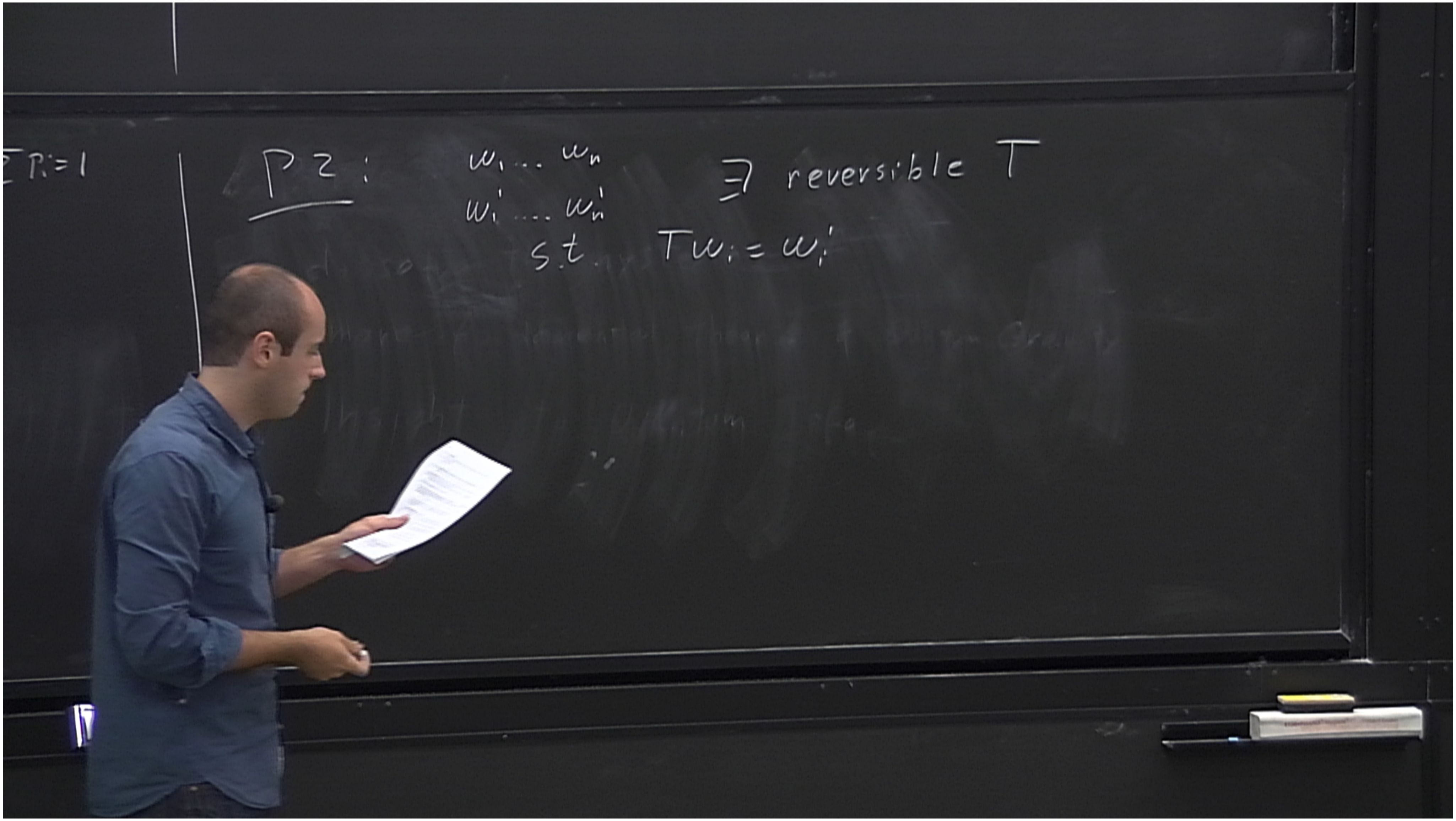

{M_k} = measurement
∑_k M_k(ω) = 1 for all ω ∈ Ω(A)
E(A) = set of effects
n level Q. system
ℂ^d ℋ_d = d^2 dimensional space of Hermitian operators
Ω(A) = {ρ ∈ ℋ_d | ρ ≥ 0, Tr(ρ) = 1}
a² = a·a

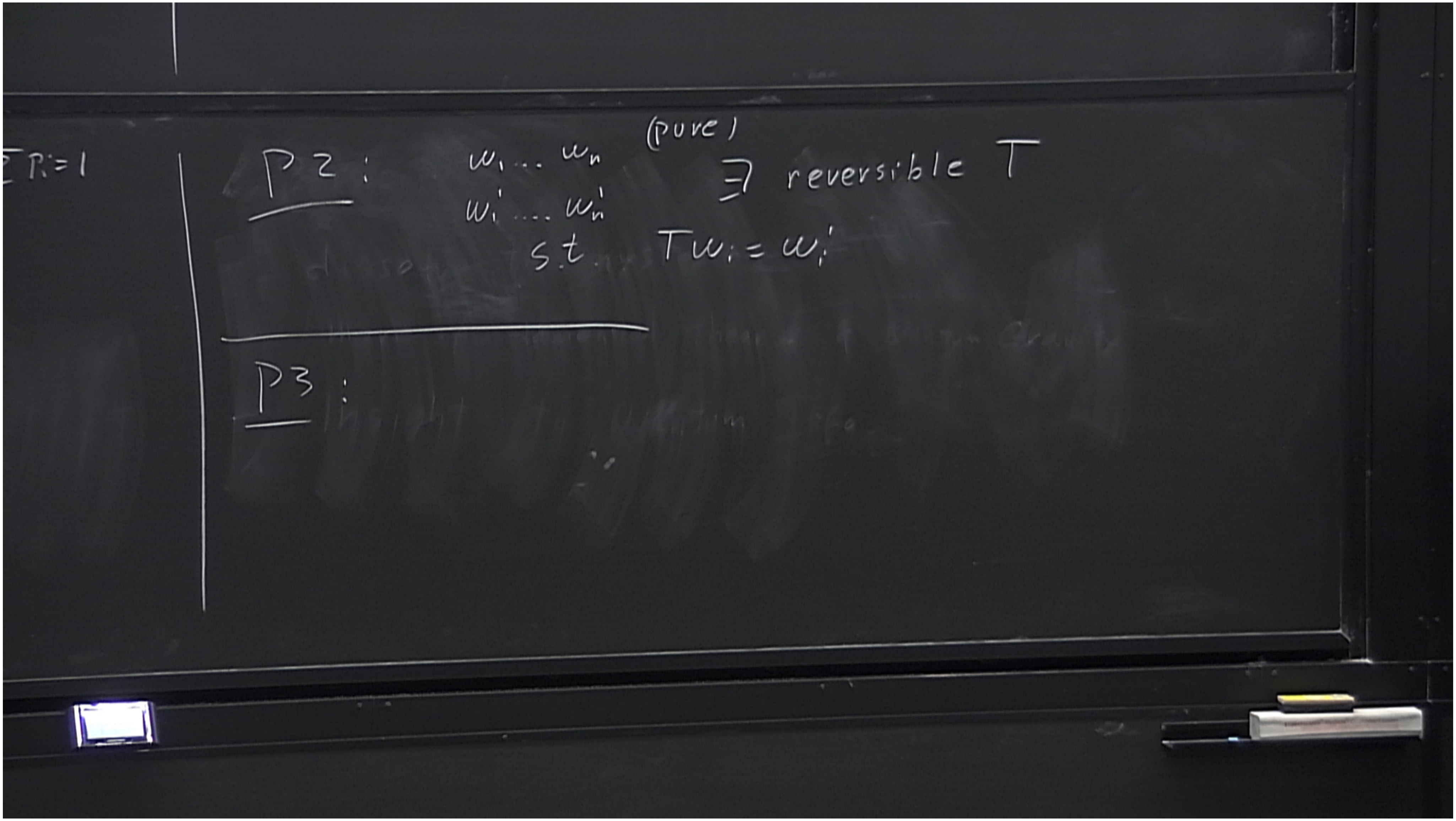

$\{M_k\}$ = measurement
$\sum_k M_k(\omega) = 1$ for all $\omega \in \Omega(A)$
$E(A)$ = set of effects
n level Q. system
$\mathbb{C}^d$, $\mathcal{H}_d$ = d^2 dimensional space of Hermitian operators
$\Omega(A) = \{\rho \in \mathcal{H}_d \mid \rho \geq 0, \mathrm{Tr}(\rho) = 1\}$
$E(A) = \{M \in \mathcal{H}_d \mid I \geq M \geq 0\}$

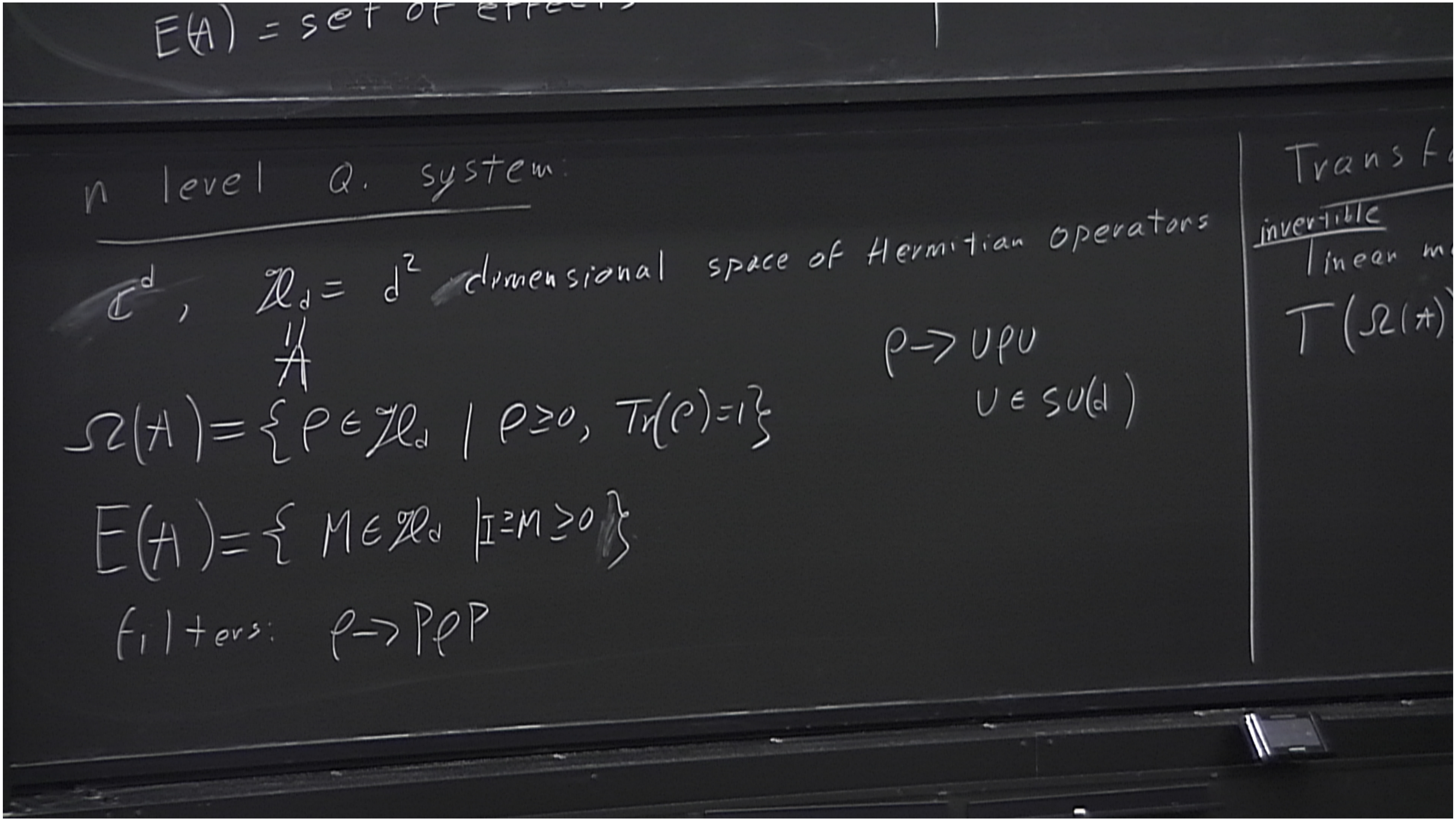

Transformations (reversible)
invertible
linear maps on $\mathcal{H}$
$T(\Omega(\mathcal{H})) = \Omega(\mathcal{H})$


P1
w1 ... wn are ctly distinguishable if
∃ {ei}n e


P1 : for all ω ∈ Ω(A)   ω = Σ pᵢ ωᵢ , pᵢ ≥ 0, Σ pᵢ = 1
ω₁ ... ωₙ are
∃ {eᵢ}ⁿ
δᵢⱼ


$\Sigma P_i = 1$
P2 :
$w_1 \ldots w_n$
$w_1' \ldots w_n'$
∃ reversible T
s.t. $Tw_i = w_i'$


$\sum P_i = 1$
P2 : $\omega_1 \ldots \omega_n$ (pure) $\exists$ reversible T
$\omega_1' \ldots \omega_n'$
s.t. $T\omega_i = \omega_i'$
P3 :


E(A) = set of
n level Q. system:
$\mathbb{C}^d$, $\mathcal{H}_d$ = d^2 dimensional space of Hermitian operators
$\Omega(A) = \{\rho \in \mathcal{H}_d \mid \rho \geq 0, Tr(\rho) = 1\}$
$E(A) = \{M \in \mathcal{H}_d \mid I \geq M \geq 0\}$
filters: $\rho \to P\rho P$
$\rho \to U\rho U$
$U \in SU(d)$
Trans
invertible
linear
$T(\Omega(A)$

$\sum P_i = 1$

P2: $\omega_1 \dots \omega_n$ (pure) $\exists$ reversible T

$\omega_1' \dots \omega_n'$

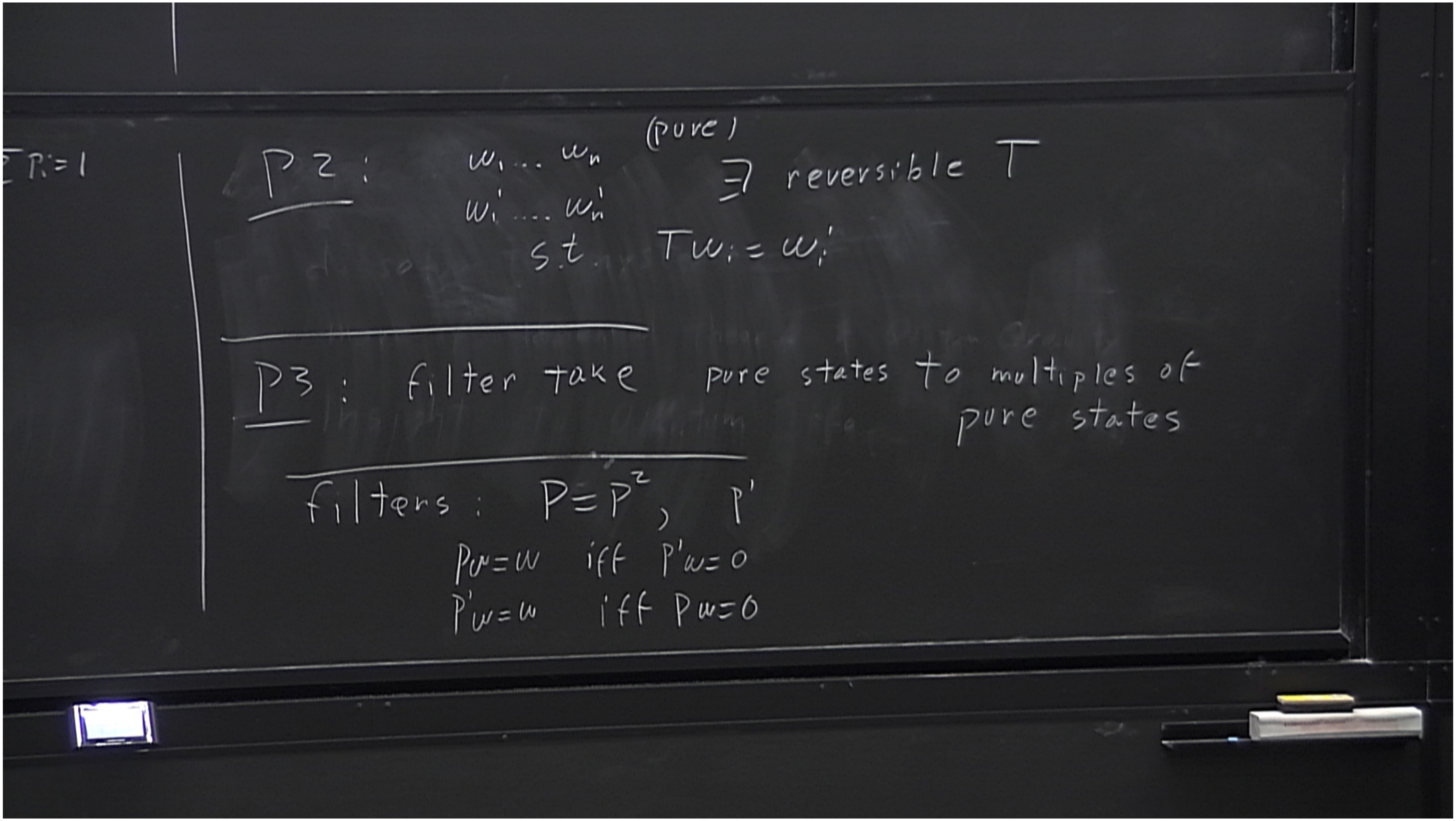
s.t. $T\omega_i = \omega_i'$

P3:

filters: $P = P^2$, P'

$P\omega = \omega$ iff $P'\omega = 0$

$P'\omega = \omega$ iff $P\omega = 0$


$\sum P_i = 1$
P2: $\omega_1 \dots \omega_n$ (pure) $\exists$ reversible T
$\omega_1' \dots \omega_n'$
s.t. $T\omega_i = \omega_i'$
P3: filter take pure states to multiples of pure states
filters: $P = P^2$, P'
$P\omega = \omega$ iff $P'\omega = 0$
$P'\omega = \omega$ iff $P\omega = 0$

$E(A) = \text{set of eff}\ldots$

n level Q. system:

$\mathbb{C}^d$, $\mathcal{H}_d = d^2$ dimensional space of Hermitian operators

$\mathbb{1}$

A

$\rho \to U \rho U$

$U \in SU(d)$

$$\Omega(A) = \{ \rho \in \mathcal{H}_d \mid \rho \geq 0,\ Tr(\rho) = 1 \}$$

$$E(A) = \{ M \in \mathcal{H}_d \mid I \geq M \geq 0 \}$$

$P |\varphi\rangle\langle\varphi| P = \lambda |\theta\rangle\langle\theta|$

$0 \leq \lambda \leq 1$

filters: $\rho \to P \rho P$

$\rho \to (1-P)\rho(1-P)$

Transf…

invertible

linear m…

$T(\Omega(A)$…

$E(A) = $ set of eff…

n level Q. system:

$\mathbb{C}^d$, $\mathcal{H}_d = d^2$ dimensional space of Hermitian operators

$\mathcal{H}_d = A$

$$\Omega(A) = \{ \rho \in \mathcal{H}_d \mid \rho \geq 0,\ Tr(\rho) = 1 \}$$

$$E(A) = \{ M \in \mathcal{H}_d \mid I \geq M \geq 0 \}$$

filters: $\rho \to P\rho P$

$\rho \to (1-P)\rho(1-P)$

$\rho \to U\rho U$

$U \in SU(d)$

$P|\varphi\rangle\langle\varphi|P = \lambda|\varphi\rangle\langle\varphi|$

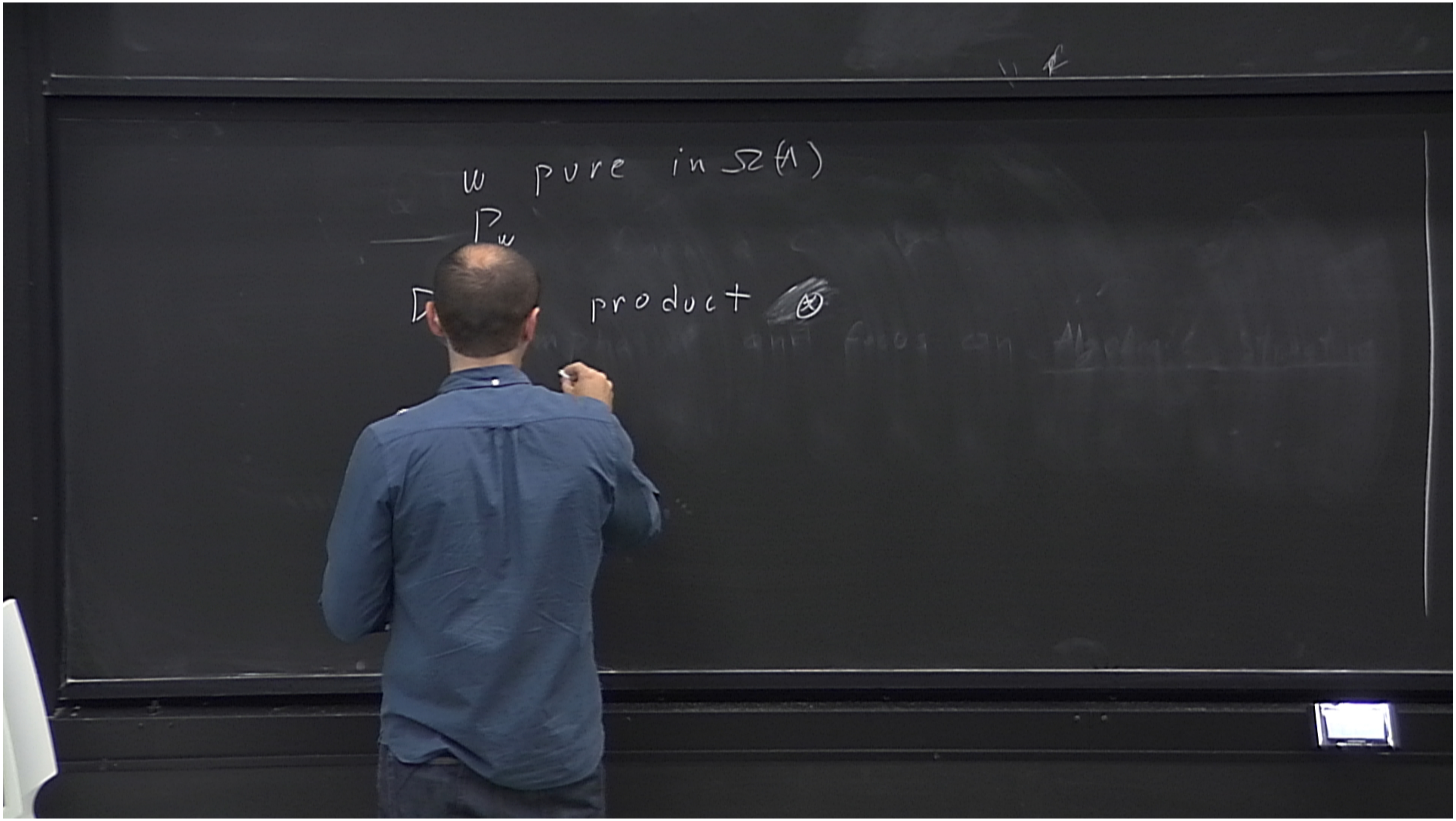
$0 \leq \lambda \leq 1$

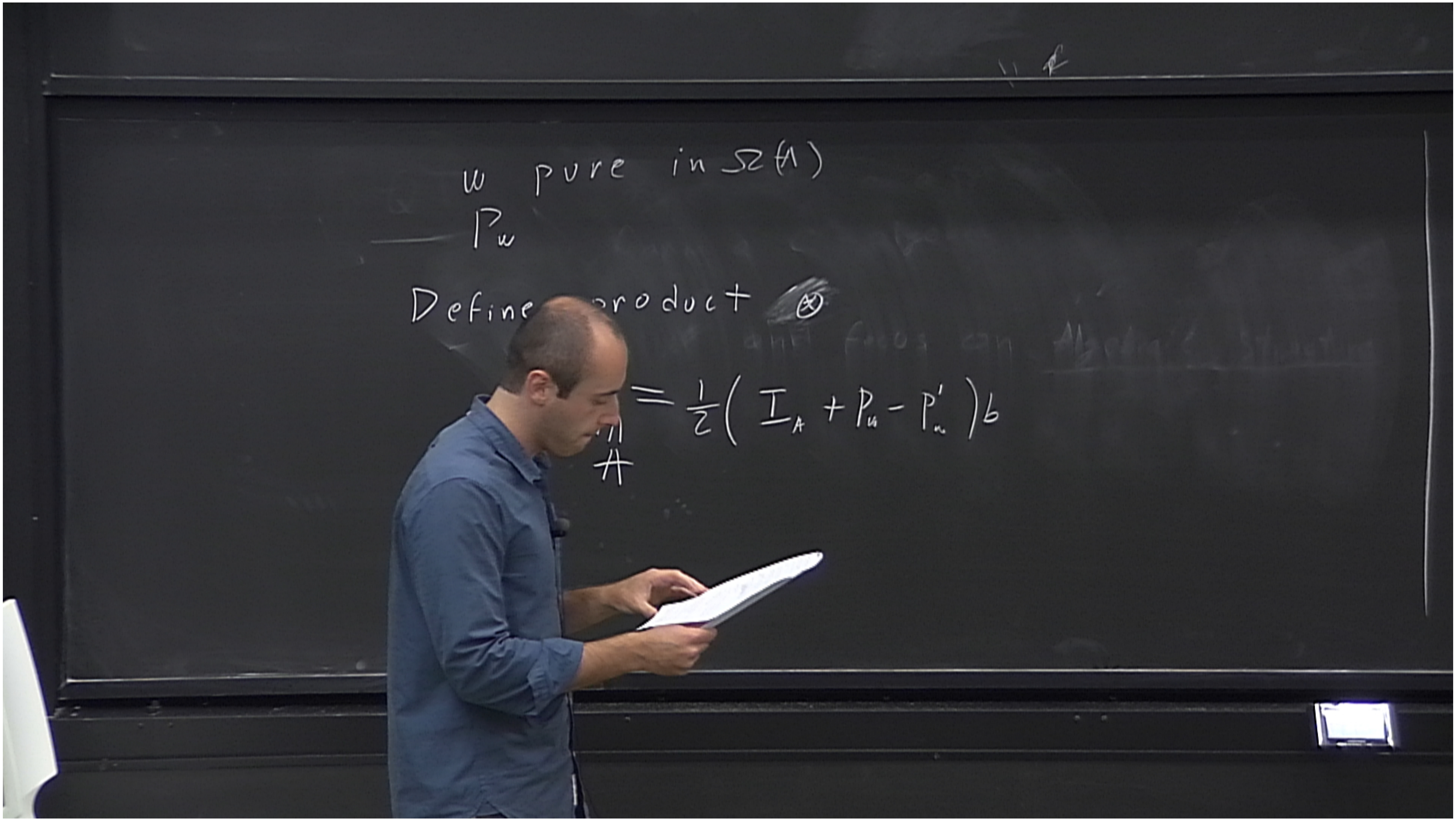
Transf…

invertible

linear m…

$T(\Omega(A)$…


ω pure in Ω(A)
P_ω
product ⊗



ω pure in Ω(A)
P_ω
Define product ⊗
$= \frac{1}{2}(I_A + P_\omega - P'_\omega)b$