Title: Making the Case for Conformal Gravity

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Abstract: We discuss the shortcomings of Einstein gravity at both the classical and quantum levels. We discuss the motivation for replacing Einstein gravity by conformal gravity. We show how the conformal gravity theory is able to naturally solve the quantum gravity problem, the vacuum zero-point energy problem, the vacuum zero-point pressure problem, the cosmological constant problem, and the dark matter problem. Central to its viability as aquantum theory is that the conformal theory is both renormalizable and unitary, with unitarity being obtained because the theory is a PTsymmetric rather than a Hermitian theory. We show that in the conformal theory there can be no a priori classical curvature, with all curvature having to result from quantization. In the conformal theory gravity requires no independent quantization of its own, with it being quantized solely by virtue of its being coupled to a quantized matter source. In the absence of quantum mechanics then there would thus be no gravity, with it being the desire to start with a classical gravity theory and then quantize it
that has prevented the construction of a sensible quantum gravity theory. We show that the macroscopic classical theory that results from the quantum conformal theory incorporates global physics effects coming from the material outside of galaxies (viz. the rest of theuniverse), global physics effects that are found to provide for a detailed accounting of a comprehensive set of 138
galactic rotation curves with no adjustable parameters other than galactic mass to light ratios, and with the need for no dark matter whatsoever. With these global effects eliminating the need for dark matter, we see that invoking dark matter in galaxies could potentially be nothing more than an attempt to describe global physics effects in purely local galactic terms.

GHOST PROBLEMS, UNITARITY OF FOURTH-ORDER THEORIES AND PT QUANTUM MECHANICS

1. P. D. Mannheim and A. Davidson, *Fourth order theories without ghosts*, January 2000 (arXiv: 0001115 [hep-th]).
2. P. D. Mannheim and A. Davidson, *Dirac quantization of the Pais-Uhlenbeck fourth order oscillator*, Phys. Rev. A **71**, 042110 (2005). (arXiv: 0408104 [hep-th]).
3. P. D. Mannheim, *Solution to the ghost problem in fourth order derivative theories*, Found. Phys. **37**, 532 (2007). (arXiv:0608154 [hep-th]).
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7. C. M. Bender and P. D. Mannheim, *PT symmetry and necessary and sufficient conditions for the reality of energy eigenvalues*, Phys. Lett. A 374, 1616 (2010). (arXiv: 0902.1365 [hep-th])
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1 The Einstein Equations: what kind of equations are they?

$$-\frac{1}{8\pi G}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R^{\alpha}{}_{\alpha}\right)=T^{\mu\nu}_{\rm M}. \qquad (1)$$

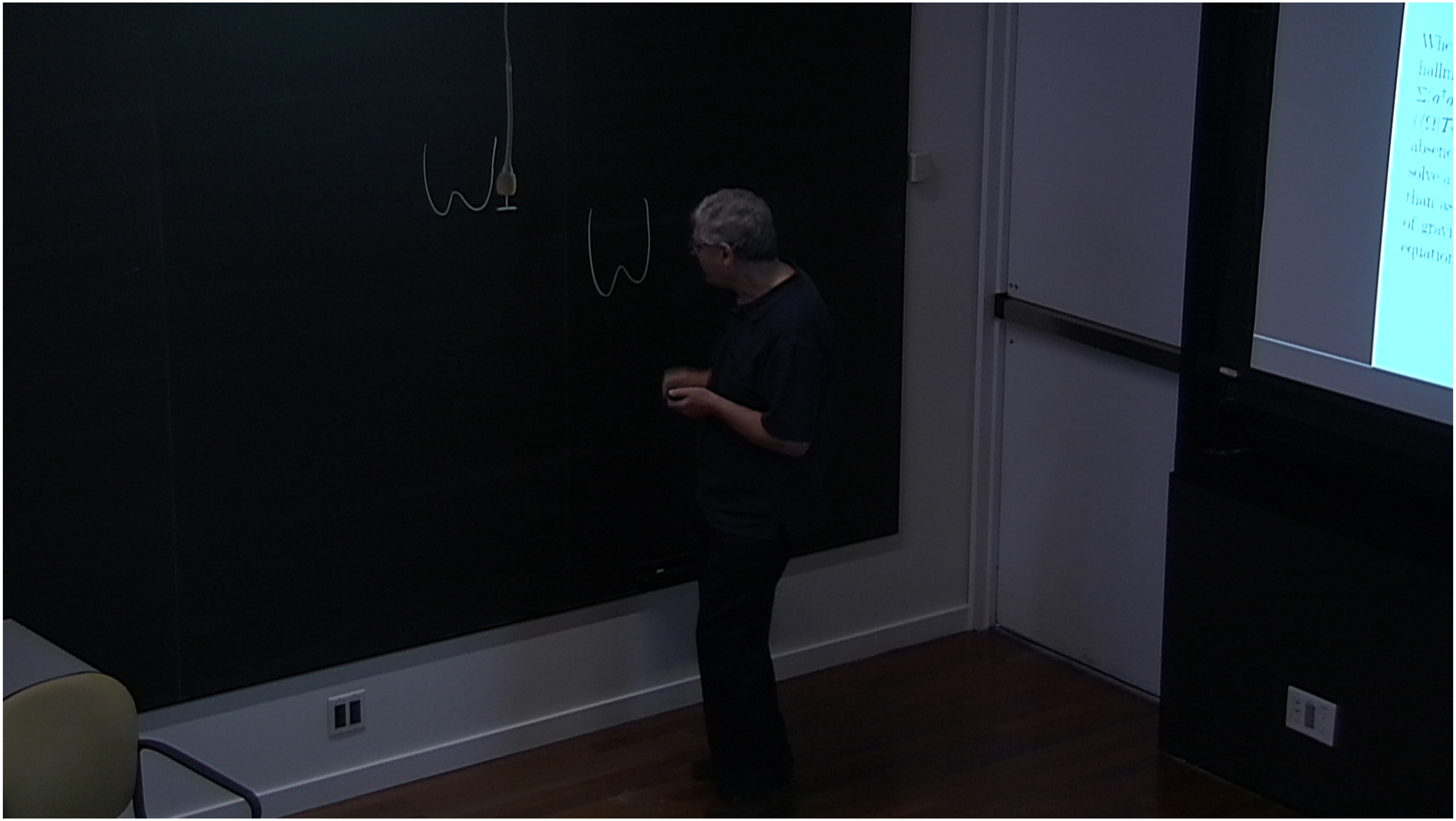
Classical equals classical? Quantum equals quantum? But sources include quantum-mechanical Pauli pressure in white dwarf stars and energy density ρ and pressure $p=\rho/3$ of blackbody radiation in cosmology. So try:

$$-\frac{1}{8\pi G}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R^{\alpha}{}_{\alpha}\right)=\langle T^{\mu\nu}_{\rm M}\rangle. \qquad (2)$$

But right-hand side not finite. So subtract off infinite part and try:

$$-\frac{1}{8\pi G}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R^{\alpha}{}_{\alpha}\right)=(\langle T^{\mu\nu}_{\rm M}\rangle)_{\rm FIN}\,. \qquad (3)$$

Where does this equation come from? As yet not derived from a fundamental theory. Also it violates the hallmark of Einstein gravity, namely that gravity couple to energy rather than to energy difference. Thus in $\Sigma(a^\dagger a+1/2)\hbar\omega$ ignore zero-point energy density. Also ignore zero-point pressure. And when all this is done, $(\langle\Omega|T^{\mu\nu}_{\rm M}|\Omega\rangle)_{\rm FIN}\sim-\Lambda g_{\mu\nu}$ with $p=-\rho$. Then since in flat spacetime $(\langle\Omega|T^{\mu\nu}_{\rm M}|\Omega\rangle)_{\rm FIN}\sim-\Lambda g_{\mu\nu}$ even in the absence of gravity (c.f. Higgs double-well potential, which induces huge Λ), how can gravity be expected to solve a problem it did not cause. Answer: put both sides on same footing and expand as power series in $\hbar$ rather than as a power series in gravitational coupling constant. Cancel zero-point of matter field against zero-point of gravitational field. But then need a renormalizable theory of gravity, and then can write a gravitational equation of motion in which quantum equals quantum. Hence conformal gravity.



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2 Einstein gravity: what must be kept

$$\Gamma^{\mu}_{\nu\sigma} = \frac{1}{2} g^{\mu\lambda}\left[\partial_\nu g_{\lambda\sigma} + \partial_\sigma g_{\lambda\nu} - \partial_\lambda g_{\nu\sigma}\right], \tag{4}$$

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu \tag{5}$$

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^{\mu}_{\nu\sigma} \frac{dx^\nu}{ds}\frac{dx^\sigma}{ds} = 0. \tag{6}$$

$$R^{\lambda}{}_{\mu\nu\kappa} = \frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^\kappa} + \Gamma^{\lambda}_{\kappa\eta}\Gamma^{\eta}_{\mu\nu} - \frac{\partial \Gamma^{\lambda}_{\mu\kappa}}{\partial x^\nu} - \Gamma^{\lambda}_{\nu\eta}\Gamma^{\eta}_{\mu\kappa}. \tag{7}$$

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \tag{8}$$

On solar distance scales outside the sun have $R_{\mu\nu} = 0$, i.e.

$$B(r) = A^{-1}(r) = 1 - 2\beta/r, \qquad \beta = MG/c^2, \tag{9}$$

3 Einstein gravity: what could be changed

$$I_{\rm UNIV} = I_{\rm EH} + I_{\rm M} = -\frac{1}{16\pi G}\int d^4x(-g)^{1/2}R^{\alpha}{}_{\alpha} + I_{\rm M}. \quad (10)$$

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} + \frac{1}{6}R^{\alpha}{}_{\alpha}\left[g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}\right] - \frac{1}{2}\left[g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}\right].$$
$$g_{\mu\nu}(x) \rightarrow e^{2\alpha(x)}g_{\mu\nu}(x), \qquad C_{\lambda\mu\nu\kappa} \rightarrow e^{2\alpha(x)}C_{\lambda\mu\nu\kappa} \quad \text{remarkable geometric property} \quad (11)$$

$$\begin{aligned} I_{\rm UNIV} = I_{\rm W} + I_{\rm M} &= -\alpha_g\int d^4x(-g)^{1/2}C_{\lambda\mu\nu\kappa}C^{\lambda\mu\nu\kappa} + I_{\rm M} \\ &= -\alpha_g\int d^4x(-g)^{1/2}\left[R_{\lambda\mu\nu\kappa}R^{\lambda\mu\nu\kappa} - 2R_{\mu\kappa}R^{\mu\kappa} + \frac{1}{3}(R^{\alpha}{}_{\alpha})^2\right] + I_{\rm M} \\ &\equiv -2\alpha_g\int d^4x(-g)^{1/2}\left[R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}(R^{\alpha}{}_{\alpha})^2\right] + I_{\rm M}. \end{aligned} \quad (12)$$

$$\text{Lanczos}: \quad L_L = (-g)^{1/2}\left[R_{\lambda\mu\nu\kappa}R^{\lambda\mu\nu\kappa} - 4R_{\mu\kappa}R^{\mu\kappa} + (R^{\alpha}{}_{\alpha})^2\right] = \text{ total divergence} \quad (13)$$

α_g is dimensionless, so conformal theory is renormalizable

$$W^{\mu\nu} = \frac{1}{2}g^{\mu\nu}(R^{\alpha}{}_{\alpha})^{;\beta}{}_{;\beta} + R^{\mu\nu;\beta}{}_{;\beta} - R^{\mu\beta;\nu}{}_{;\beta} - R^{\nu\beta;\mu}{}_{;\beta} - 2R^{\mu\beta}R^{\nu}{}_{\beta}$$
$$+\frac{1}{2}g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - \frac{2}{3}g^{\mu\nu}(R^{\alpha}{}_{\alpha})^{;\beta}{}_{;\beta} + \frac{2}{3}(R^{\alpha}{}_{\alpha})^{;\mu;\nu} + \frac{2}{3}R^{\alpha}{}_{\alpha}R^{\mu\nu} - \frac{1}{6}g^{\mu\nu}(R^{\alpha}{}_{\alpha})^2 = \frac{1}{4\alpha_g}T^{\mu\nu}_{\rm M}. \quad (14)$$

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$$\begin{aligned} I_{\rm UNIV} = I_{\rm W} + I_{\rm M} &= -\alpha_g\int d^4x(-g)^{1/2}C_{\lambda\mu\nu\kappa}C^{\lambda\mu\nu\kappa} + I_{\rm M} \\ &= -\alpha_g\int d^4x(-g)^{1/2}\left[R_{\lambda\mu\nu\kappa}R^{\lambda\mu\nu\kappa} - 2R_{\mu\kappa}R^{\mu\kappa} + \frac{1}{3}(R^{\alpha}{}_{\alpha})^2\right] + I_{\rm M} \\ &\equiv -2\alpha_g\int d^4x(-g)^{1/2}\left[R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}(R^{\alpha}{}_{\alpha})^2\right] + I_{\rm M}. \end{aligned} \quad (12)$$

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α_g is dimensionless, so conformal theory is renormalizable

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$$+\frac{1}{2}g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} - \frac{2}{3}g^{\mu\nu}(R^{\alpha}{}_{\alpha})^{;\beta}{}_{;\beta} + \frac{2}{3}(R^{\alpha}{}_{\alpha})^{;\mu;\nu} + \frac{2}{3}R^{\alpha}{}_{\alpha}R^{\mu\nu} - \frac{1}{6}g^{\mu\nu}(R^{\alpha}{}_{\alpha})^2 = \frac{1}{4\alpha_g}T^{\mu\nu}_{\rm M}. \quad (14)$$

$W_{\mu\nu} = 0$ if $R_{\mu\nu} = 0$. **Einstein sufficient to give Schwarzschild and Newton, but not necessary.**

3 Einstein gravity: what could be changed

$$I_{\rm UNIV} = I_{\rm EH} + I_{\rm M} = -\frac{1}{16\pi G}\int d^4x(-g)^{1/2}R^{\alpha}{}_{\alpha} + I_{\rm M}. \qquad (10)$$

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} + \frac{1}{6}R^{\alpha}{}_{\alpha}\left[g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}\right] - \frac{1}{2}\left[g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}\right].$$

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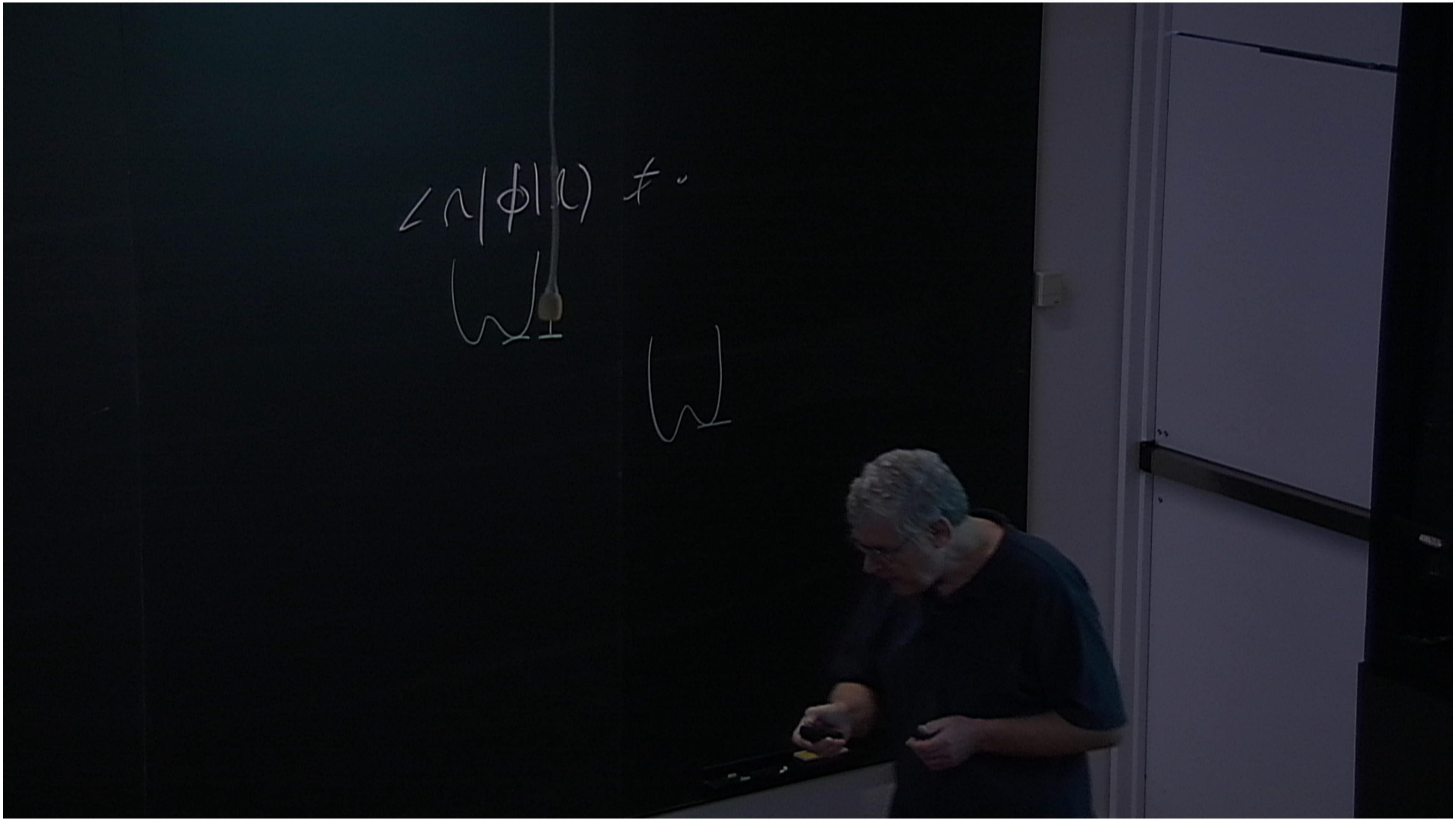
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3 Einstein gravity: what could be changed

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4 Conformal gravity: an ab initio approach

$$\psi(x) \to e^{i\beta(x)}\psi(x), \quad A_\mu(x) \to A_\mu(x) + \partial_\mu\beta(x), \quad I_{\rm M} = -\int d^4x \bar{\psi}(x)\gamma^\mu[i\partial_\mu + A_\mu(x)]\psi(x). \quad (15)$$

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$\psi(x) \to e^{-3\alpha(x)/2+i\beta(x)}\psi(x), \quad V^\mu_a(x) \to e^{-\alpha(x)}V^\mu_a(x), \quad A_\mu(x) \to A_\mu(x)$. Get local conformal invariance for free. (17)

4.1 Global invariances of the flat spacetime light cone $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = 0$

Symmetry of gauge/fermion sector of Yang-Mills since fermions and gauge bosons have no kinematical mass. 15 invariances:

$x^\mu \to x^\mu + \epsilon^\mu$	4 translations	$P^\mu = -i\partial^\mu$
$x^\mu \to \Lambda^\mu_\nu x^\nu$	6 rotations	$M^{\mu\nu} = -i(x^\mu\partial^\nu - x^\nu\partial^\mu)$
$x^\mu \to \lambda x^\nu$	1 dilatation	$D = ix^\mu\partial_\mu$
$x^\mu \to \dfrac{x^\mu + c^\mu x^2}{1 + 2c\cdot x + c^2x^2}$	4 conformal	$C^\mu = -i(x^\mu x^\nu\partial_\nu - x^2\partial^\mu)$. (18)

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(\eta_{\mu\sigma}M_{\nu\rho} - \eta_{\mu\rho}M_{\nu\sigma} + \eta_{\nu\rho}M_{\mu\sigma} - \eta_{\nu\sigma}M_{\mu\rho})$$
$$[M_{\mu\nu}, P_\sigma] = i(\eta_{\mu\sigma}P_\nu - \eta_{\nu\sigma}P_\mu), \quad [M_{\mu\nu}, C_\sigma] = i(\eta_{\mu\sigma}C_\nu - \eta_{\nu\sigma}C_\mu), \quad [M_{\mu\nu}, D] = 0,$$
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$$[D, P_\mu] = -iP_\mu, \quad [D, C_\mu] = iC_\mu \quad (19)$$

Close on SO(4,2). However equivalent to SU(2,2) invariance since 4+6+1+4=15 Dirac matrices: γ_μ, $i[\gamma_\mu, \gamma_\nu]$, γ_5, $\gamma_\mu\gamma_5$ close on same algebra. 4-component fermions reducible under Lorentz, but irreducible under conformal group. Thus all fermions 4-component, and right-handed neutrinos must exist. Hence both strong and weak interactions are chiral.

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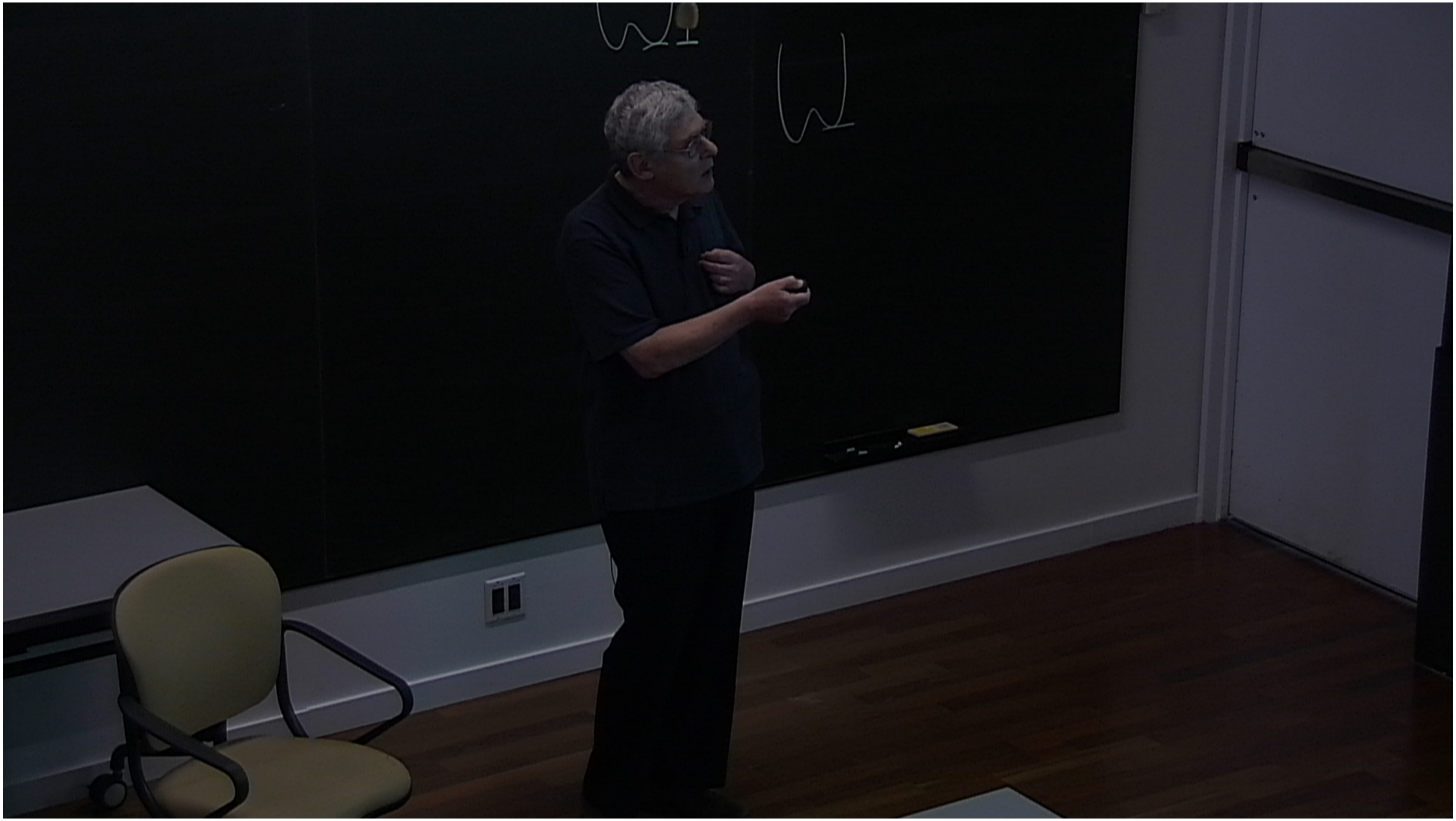

4 Conformal gravity: an ab initio approach

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4.4 Connecting Conformal Gravity and Einstein Gravity

Under $g_{\mu\nu}(x) = \omega^2(x)\hat{g}_{\mu\nu}(x)$

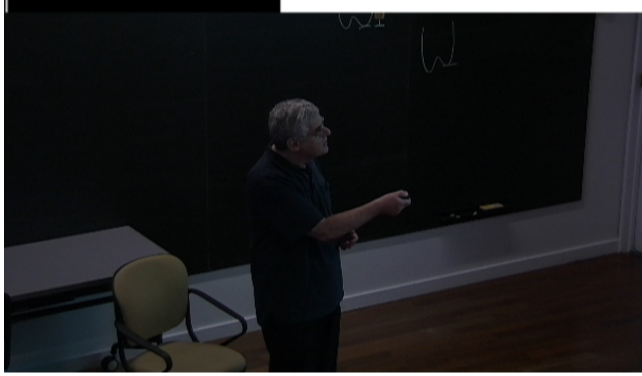
$$I_{\rm EH} = -\frac{1}{16\pi G}\int d^4x(-g)^{1/2}R^\alpha{}_\alpha = -\frac{1}{16\pi G}\int d^4x(-\hat{g})^{1/2}\left(\omega^2\hat{R}^\alpha{}_\alpha - 6\hat{g}^{\mu\nu}\partial_\mu\omega\partial_\nu\omega\right), \qquad (26)$$

't Hooft (2011): Do path integral over conformal factor $\omega(x)$

$$I_{\rm EFF} = {\rm Tr}\ \ln[\hat{g}^{\mu\nu}\hat{\nabla}_\mu\hat{\nabla}_\nu + \frac{1}{6}\hat{R}^\alpha{}_\alpha] \longrightarrow \frac{C}{120}\int d^4x(-\hat{g})^{1/2}[\hat{R}^{\mu\nu}\hat{R}_{\mu\nu} - \frac{1}{3}(\hat{R}^\alpha{}_\alpha)^2], \qquad (27)$$

't Hooft (2011): go from Einstein to conformal. Maldacena (2011): go from conformal to Einstein.

4.5 Particle physics motivation for conformal invariance



conformal invariance at energies much greater than particle masses. Violated by radiative
ored at a renormalization group fixed point where we get scaling with anomalous dimensions.
(1964, 1967): if QED at Gell-Mann-Low fixed point then

$$(p,p,0) = (-p^2/m^2)^{\gamma_{\bar{\psi}\psi}/2}, \qquad d_{\bar{\psi}\psi} = 3 + \gamma_{\bar{\psi}\psi}, \qquad m_0 = m(\Lambda^2/m^2)^{\gamma_{\bar{\psi}\psi}/2}. \qquad (28)$$

zero, and all mass is dynamical, with physical mass obeying a homogeneous equation. However,
ot be zero too, since zero is a solution to a homogeneous equation. So could non-zero solution
eakdown. So need to adapt Nambu-Jona-Lasinio to theories with a fixed point.

5 Quantization of gravity through coupling

5.1 Canonical quantization of matter fields

For fermion fields with $I_{\rm M} = \int d^4x i\hbar\bar{\psi}\gamma^\mu\partial_\mu\psi$, stationary fields obey $\delta I_{\rm M}/\delta\bar{\psi} = i\hbar\gamma^\mu\partial_\mu\psi = 0$. We introduce creation and annihilation operators according to

$$\begin{aligned}\psi(x,t) &= \sum_s \int \frac{d^3k}{(2\pi)^{3/2}}\left[b(k,s)u(k,s)e^{-ik\cdot x} + d^\dagger(k,s)v(k,s)e^{ik\cdot x}\right],\\ \psi^\dagger(x,t) &= \sum_s \int \frac{d^3k}{(2\pi)^{3/2}}\left[b^\dagger(k,s)u^\dagger(k,s)e^{ik\cdot x} + d(k,s)v^\dagger(k,s)e^{ik\cdot x}\right].\end{aligned} \tag{34}$$

Quantizing the fermion field according to $\{\psi_\alpha(x,t),\psi^\dagger_\beta(x',t)\} = \delta^3(x-x')\delta_{\alpha,\beta}$ then requires that its creation and annihilation operators obey

$$\{b(k,s),b^\dagger(k',s')\} = \delta^3(k-k')\delta_{s,s'}, \qquad \{d(k,s),d^\dagger(k',s')\} = \delta^3(k-k')\delta_{s,s'}. \tag{35}$$

For the energy-momentum tensor we obtain

$$\langle\Omega|\left[2g^{-1/2}\frac{\delta I_{\rm M}}{\delta g^{\mu\nu}}\right]|\Omega\rangle = \langle\Omega|T^{\rm M}_{\mu\nu}|\Omega\rangle = \langle\Omega|\bar{\psi}i\hbar\gamma_\mu\partial_\nu\psi|\Omega\rangle = -\frac{2\hbar}{(2\pi)^3}\int\frac{d^3k}{k_0}k_\mu k_\nu. \tag{36}$$

$$\rho_{\rm M} = \langle\Omega|T^{\rm M}_{00}|\Omega\rangle = -\frac{2\hbar}{(2\pi)^3}\int d^3k\omega_k, \qquad p_{\rm M} = \langle\Omega|T^{\rm M}_{11}|\Omega\rangle = -\frac{2\hbar}{(2\pi)^3}\int\frac{d^3k}{\omega_k}k_1^2 = \frac{1}{3}\langle\Omega|T^{\rm M}_{00}|\Omega\rangle = \frac{1}{3}\rho_{\rm M}. \tag{37}$$

$\langle\Omega|T^{\rm M}_{0i}|\Omega\rangle$ and the total matter vacuum momentum $P^{\rm M}_i = \int d^3x\langle\Omega|T^{\rm M}_{0i}|\Omega\rangle$ are both zero, with $\langle\Omega|T^{\rm M}_{11}|\Omega\rangle = \langle\Omega|T^{\rm M}_{22}|\Omega\rangle = \langle\Omega|T^{\rm M}_{33}|\Omega\rangle$ being a vacuum pressure. Thus have two zero-point infinities to deal with, viz. $\rho_{\rm M}$ and $p_{\rm M}$, and neither behaves like a cosmological constant since $p_{\rm M}$ and $\rho_{\rm M}$ have same sign.

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5 Quantization of gravity through coupling

5.1 Canonical quantization of matter fields

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But if M massless gauge bosons and N massless two-component spinors

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For $SU(3)\times SU(2)\times U(1)$ M=12, N=16 per generation. For SO(10) M=45, N=16, so need 3 generations. For all generations in a common multiplet need $SO(2n)$ where $2n\geq 16$. (No solution for $SU(N)$, so triangle-anomaly-free grand-unifying groups preferred.) For SO(16) M=120, N=128, so 8 generations. Have asymptotic freedom up to $SO(20)$ if all fermions in same multiplet (Wilczek and Zee (1982)). So just SO(16), SO(18) and SO(20).

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$$T^{\mu\nu}_{\rm UNIV} = T^{\mu\nu}_{\rm GRAV} + T^{\mu\nu}_{\rm M} = 0. \tag{45}$$

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$$Z(k) = 1. \tag{47}$$

But if M massless gauge bosons and N massless two-component spinors

$$Z(k) = (N - M)/2, \qquad Z(k) > 0 \longrightarrow N > M. \tag{48}$$

For $SU(3)\times SU(2)\times U(1)$ M=12, N=16 per generation. For SO(10) M=45, N=16, so need 3 generations. For all generations in a common multiplet need $SO(2n)$ where $2n \geq 16$. (No solution for $SU(N)$, so triangle-anomaly-free grand-unifying groups preferred.) For SO(16) M=120, N=128, so 8 generations. Have asymptotic freedom up to $SO(20)$ if all fermions in same multiplet (Wilczek and Zee (1982)). So just SO(16), SO(18) and SO(20).

In general

$$(T^{\mu\nu}_{\rm GRAV})_{\rm DIV} + (T^{\mu\nu}_{\rm M})_{\rm DIV} = 0, \qquad (T^{\mu\nu}_{\rm GRAV})_{\rm FIN} + (T^{\mu\nu}_{\rm M})_{\rm FIN} = 0. \tag{49}$$

So zero-point fluctuations of gravity and matter take care of each other, and conformal trace anomalies take care of each other since $g_{\mu\nu}T^{\mu\nu}_{\rm UNIV} = 0$ is not a conformal Ward identity. Ability to solve trace-anomaly problem is because we do not use $(-1/8\pi G)G^{\mu\nu} = \langle T^{\mu\nu}_{\rm M}\rangle$ and try to show that $\langle T^{\mu\nu}_{\rm M}\rangle$ is conformal-anomaly-free all on its own.

If all mass and length scales come from symmetry breaking, then all scales come from quantum mechanics. I.e. without length scales there cannot be any curvature.

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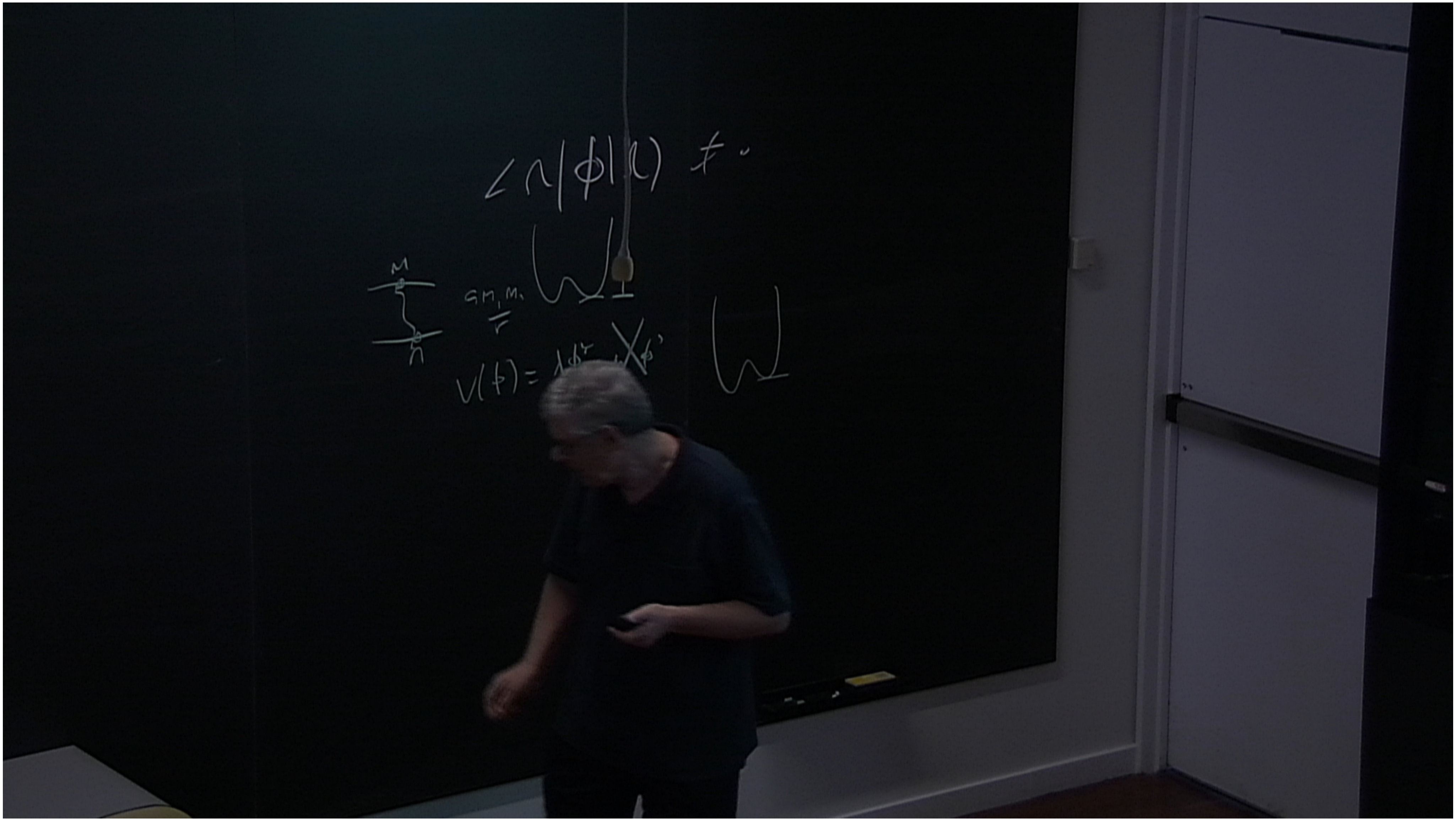
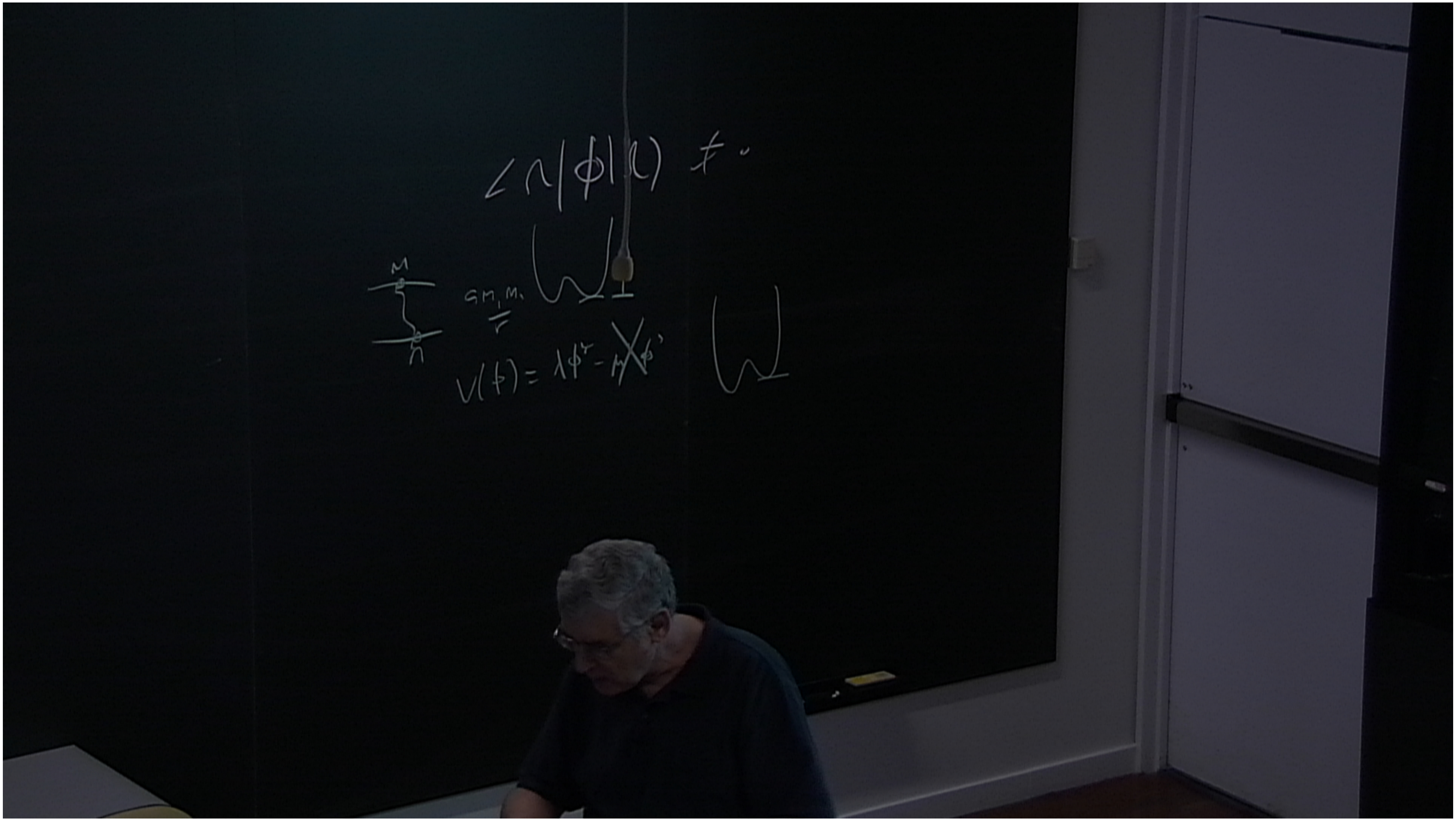




QUANTUM MECHANICS IS A GLOBAL THEORY. NEED TO SUPPLY GLOBAL INFORMATION. NEED TO LOOK AT WAVE FUNCTIONS. FIND THAT $H_{\rm PU}$, z, p_z ARE NOT HERMITIAN

$$H_{\rm PU} = \frac{p_x^2}{2\gamma} + p_z x + \frac{\gamma}{2}\left(\omega_1^2 + \omega_2^2\right) x^2 - \frac{\gamma}{2}\omega_1^2\omega_2^2 z^2, \tag{74}$$

$$[x, p_x] = i, \qquad p_x = -i\frac{\partial}{\partial x}, \qquad [z, p_z] = i, \qquad p_z = -i\frac{\partial}{\partial z} \tag{75}$$

$$\psi_0(z, x) = \exp\left[\frac{\gamma}{2}(\omega_1 + \omega_2)\omega_1\omega_2 z^2 + i\gamma\omega_1\omega_2 zx - \frac{\gamma}{2}(\omega_1 + \omega_2)x^2\right] \tag{76}$$

The states of negative norm are states of INFINITE norm since $\int dx dz \psi_0^*(z, x)\psi_0(z, x)$ and thus $\langle\Omega|\Omega\rangle$ are divergent, and when acting on such states, one CANNOT set $p_z = -i\partial/\partial z$.

$$\left[e^{i\theta}z, -\frac{i}{e^{i\theta}}\frac{\partial}{\partial z}\right]\psi(e^{i\theta}z) = i\psi(e^{i\theta}z), \qquad z \to -iz, \qquad p_z \to \frac{\partial}{\partial z} \tag{77}$$

p_z and z not Hermitian – they are anti-Hermitian.

$$y = e^{\pi p_z z/2} z e^{-\pi p_z z/2} = -iz, \qquad q = e^{\pi p_z z/2} p_z e^{-\pi p_z z/2} = ip_z \tag{78}$$

$$H = \frac{p^2}{2\gamma} - iqx + \frac{\gamma}{2}\left(\omega_1^2 + \omega_2^2\right) x^2 + \frac{\gamma}{2}\omega_1^2\omega_2^2 y^2 \neq H^\dagger, \qquad p = p_x \tag{79}$$

$$\text{Hermitian} \quad [x, p] = i, \quad \text{Hermitian} \quad [y, q] = i, \quad \text{non} - \text{Hermitian} \;\; H \tag{80}$$

....THE NORM IS NOT THE DIRAC NORM

$$\tilde{H} = e^{-Q/2} H e^{Q/2} = \frac{p^2}{2\gamma} + \frac{q^2}{2\gamma\omega_1^2} + \frac{\gamma}{2}\omega_1^2 x^2 + \frac{\gamma}{2}\omega_1^2\omega_2^2 y^2 \quad (84)$$

$$\tilde{H}|\tilde{n}\rangle = E_n|\tilde{n}\rangle, \qquad H|n\rangle = E_n|n\rangle, \qquad |n\rangle = e^{Q/2}|\tilde{n}\rangle \quad (85)$$

$$\langle\tilde{n}|\tilde{H} = E_n\langle\tilde{n}|, \qquad \langle n| \equiv \langle\tilde{n}|e^{Q/2}, \qquad \langle n|e^{-Q}H = \langle n|e^{-Q}E_n \quad (86)$$

The energy eigenbra $\langle n|e^{-Q}$ is not the Dirac conjugate of the energy eigenket $|n\rangle$, since $\langle n|H^\dagger = \langle n|E_n$ is not an eigenvalue equation for H.

$$\langle\tilde{n}|\tilde{m}\rangle = \delta_{m,n}, \qquad \Sigma|\tilde{n}\rangle\langle\tilde{n}| = \mathbf{1}, \qquad \tilde{H} = \Sigma|\tilde{n}\rangle E_n\langle\tilde{n}| \quad (87)$$

$$\langle n|e^{-Q}|m\rangle = \delta_{m,n}, \qquad \Sigma|n\rangle\langle n|e^{-Q} = \mathbf{1}, \qquad H = \Sigma|n\rangle E_n\langle n|e^{-Q} \quad (88)$$

The e^{-Q} norm is positive and so theory is unitary. Since $e^{-Q} = PC$ where $C^2 = I$, the relative plus and minus signs in the fourth-order propagator are due to the fact that the two sets of poles have opposite-signed eigenvalues (± 1) of C.

7.2 The general situation

$$H^\dagger = VHV^{-1}, \qquad H|R\rangle = E|R\rangle, \quad \langle R|H^\dagger = \langle R|E,$$
$$\langle R|VH = \langle R|VE, \qquad \langle L| = \langle R|V, \qquad \langle L|H = \langle L|E, \tag{89}$$

$$\langle R(t)|V|R(t)\rangle = \langle L(t)|R(t)\rangle = \langle L(t=0)|e^{iHt}e^{-iHt}|R(t=0)\rangle$$
$$= \langle L(t=0)|R(t=0)\rangle = \langle R(t=0)|V|R(t=0)\rangle. \tag{90}$$

$$\int d^4ke^{-ik\cdot x}D(k^2) = \langle\Omega_L|T(\phi(x)\phi(0)|\Omega_R\rangle = \langle\Omega_R|VT(\phi(x)\phi(0)|\Omega_R\rangle \neq \langle\Omega_R|T(\phi(x)\phi(0)|\Omega_R\rangle \tag{91}$$

7.3 The Lehmann representation

Set $\int d^4ke^{-ik\cdot x}D(k^2) = \langle\Omega_R|T(\phi(x)\phi(0)|\Omega_R\rangle$, $\quad \Sigma\,|n\rangle\langle n| = I$,
$\rho(q^2) = (2\pi)^3\,\Sigma_n\,\delta^4(k^n_\mu - q_\mu)|\langle\Omega_R|\phi(0)|k^n_\mu\rangle|^2\theta(q_0)$,

$$D(k^2) = \int dq^2\frac{\rho(q^2)}{k^2+q^2} \tag{92}$$

Get contradiction at large k^2 if $\rho(q^2)$ is positive and $D(k^2) \to 1/k^4$. Hence need a cancellation, and obtain by identifying $D(k^2) = \int d^4ke^{ik\cdot x}\langle\Omega_R|VT(\phi(x)\phi(0)|\Omega_R\rangle$. Problem is never met in second-order theories, so fourth-order conformal gravity is first time problem is encountered. Same conformal invariance that makes Yang-Mills second-order (and thus renormalizable) makes gravity fourth-order both renormalizable and PT.

7.4 The pure fourth-order limit is singular

In limit $\omega_2 \to \omega_1$ we find that $e^{-Q/2}$ becomes singular. And likewise if $M^2 \to 0$. In the limit the Hamiltonian becomes non-diagonalizable Jordan-block Hamiltonian. Thus do not have two observable gravitons.

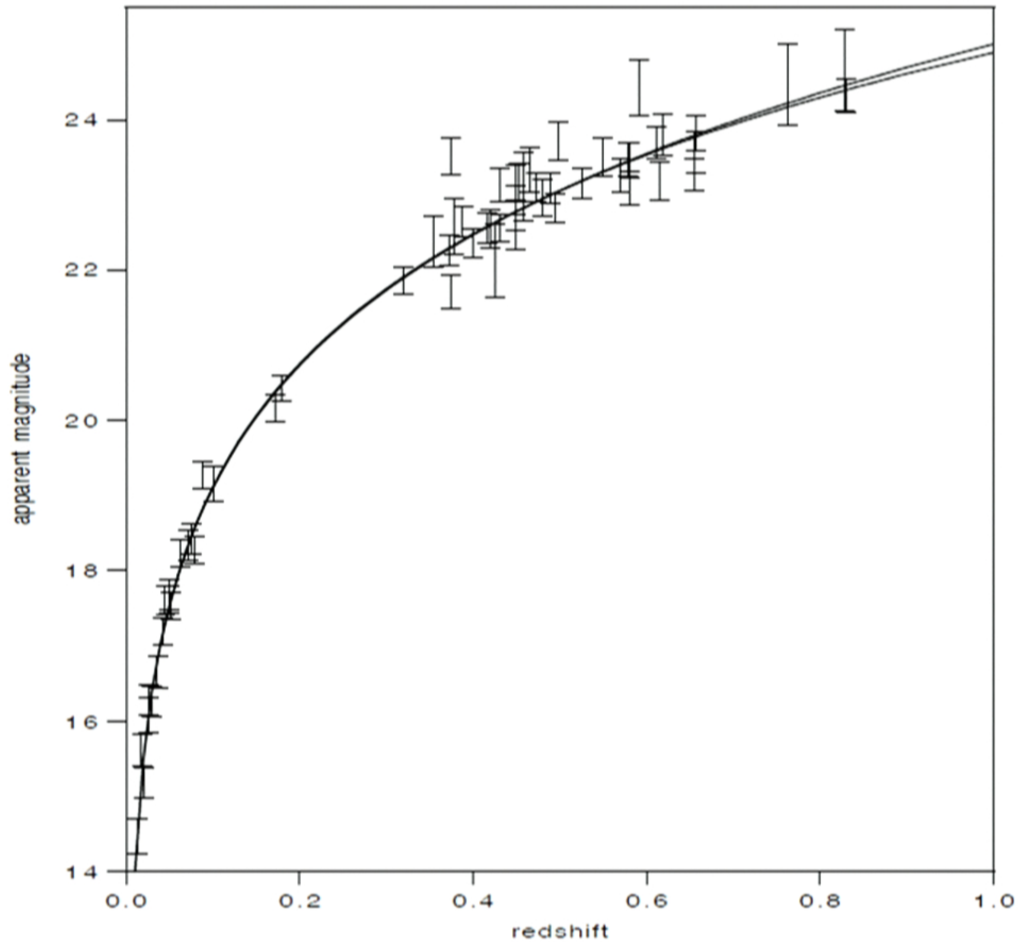
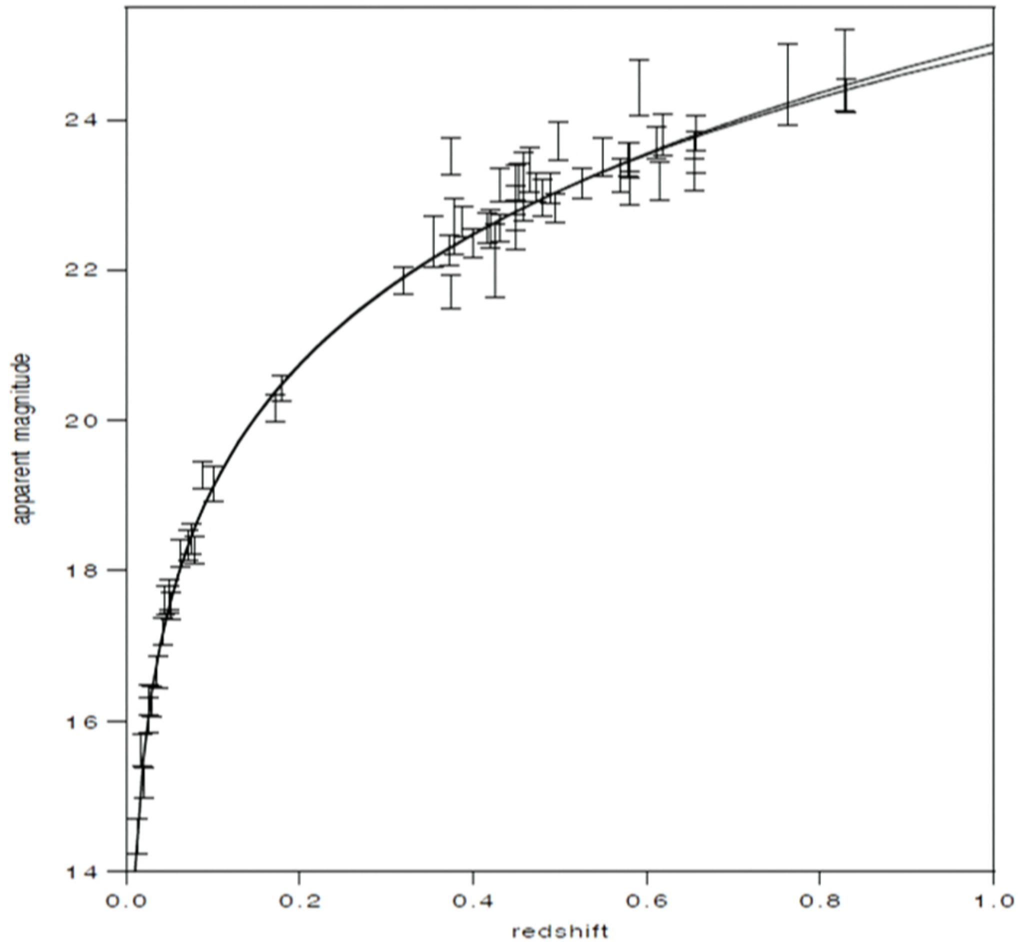


Figure 1: The $q_0 = -0.37$ conformal gravity fit (upper curve) and the $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ standard model fit (lower curve) to the $z < 1$ supernovae Hubble plot data.



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$$\rho = \frac{4r}{2(1+\gamma_0 r)^{1/2} + 2 + \gamma_0 r}, \qquad \tau = \int dt R(t) \tag{115}$$

$$-(1+\gamma_0 r)c^2dt^2 + \frac{dr^2}{(1+\gamma_0 r)} + r^2 d\Omega_2 = \frac{1}{R^2(\tau)}\left(\frac{1+\gamma_0\rho/4}{1-\gamma_0\rho/4}\right)^2 \left[-c^2 d\tau^2 + \frac{R^2(\tau)}{[1-\gamma_0^2\rho^2/16]^2}\left(d\rho^2 + \rho^2 d\Omega_2\right)\right] \tag{116}$$

$$\frac{v_{\rm TOT}^2}{R} = \frac{v_{\rm LOC}^2}{R} + \frac{\gamma_0 c^2}{2}. \tag{117}$$

$$\gamma^* = 5.42 \times 10^{-41}{\rm cm}^{-1}, \qquad \gamma_0 = 3.06 \times 10^{-30}{\rm cm}^{-1}. \tag{118}$$

$$\frac{v_{\rm TOT}^2}{R} = \frac{v_{\rm LOC}^2}{R} + \frac{\gamma_0 c^2}{2} - \kappa c^2 R, \quad \frac{v_{\rm TOT}^2}{R} \to \frac{N^*\beta^* c^2}{R^2} + \frac{N^*\gamma^* c^2}{2} + \frac{\gamma_0 c^2}{2} - \kappa c^2 R, \tag{119}$$

$$\kappa = 9.54 \times 10^{-54}\ {\rm cm}^{-2} \approx (100\ {\rm Mpc})^{-2}. \tag{120}$$

Fit 138 galaxies with VISIBLE N^* of each galaxy as only variable, β^*, γ^*, γ_0^* and κ are all universal, and with NO DARK MATTER, and with 276 fewer free parameters than in dark matter calculations. Works since $(v^2/c^2R)_{\rm last} \sim 10^{-30}{\rm cm}^{-1}$ for every galaxy.

8 The dark matter problem

$$ds^2 = -B(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega_2, \tag{106}$$

$$\frac{3}{B(r)}(W^0{}_0 - W^r{}_r) = \nabla^4 B = B'''' + \frac{4B'''}{r} = \frac{(rB)''''}{r} = \frac{3}{4\alpha_g B(r)}(T^0{}_0 - T^r{}_r) \equiv f(r) \tag{107}$$

$$B(r > r_0) = 1 - \frac{2\beta}{r} + \gamma r, \tag{108}$$

$$2\beta = \frac{1}{6}\int_0^{r_0} dr' r'^4 f(r'), \qquad \gamma = -\frac{1}{2}\int_0^{r_0} dr' r'^2 f(r'). \tag{109}$$

$$V^*(r) = -\frac{\beta^* c^2}{r} + \frac{\gamma^* c^2 r}{2} \tag{110}$$

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$$\phi(r) = -\frac{1}{r}\int_0^r dr' r'^2 g(r') - \int_r^\infty dr' r' g(r'), \qquad \frac{d\phi(r)}{dr} = \frac{1}{r^2}\int_0^r dr' r'^2 g(r'). \tag{112}$$

Newtonian Gravity is LOCAL

$$\phi(r) = -\frac{r}{2}\int_0^r dr' r'^2 h(r') - \frac{1}{6r}\int_0^r dr' r'^4 h(r') - \frac{1}{2}\int_r^\infty dr' r'^3 h(r') - \frac{r^2}{6}\int_r^\infty dr' r' h(r') \tag{113}$$

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Conformal Gravity is GLOBAL. So cannot ignore the rest of the universe. Rest of universe has homogeneous Hubble flow and inhomogeneous clusters of galaxies.

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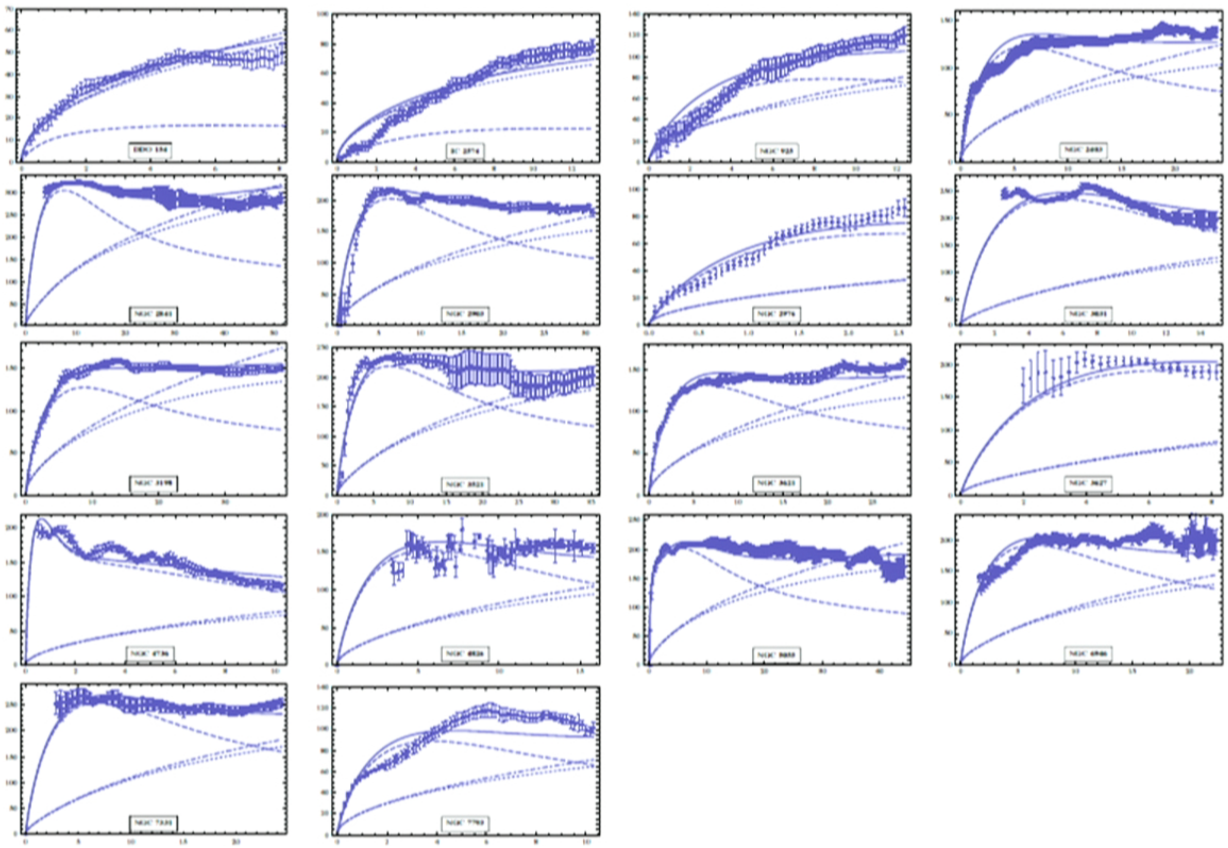


FIG. 1: Fitting to the rotational velocities (in km sec^{-1}) of the THINGS 18 galaxy sample

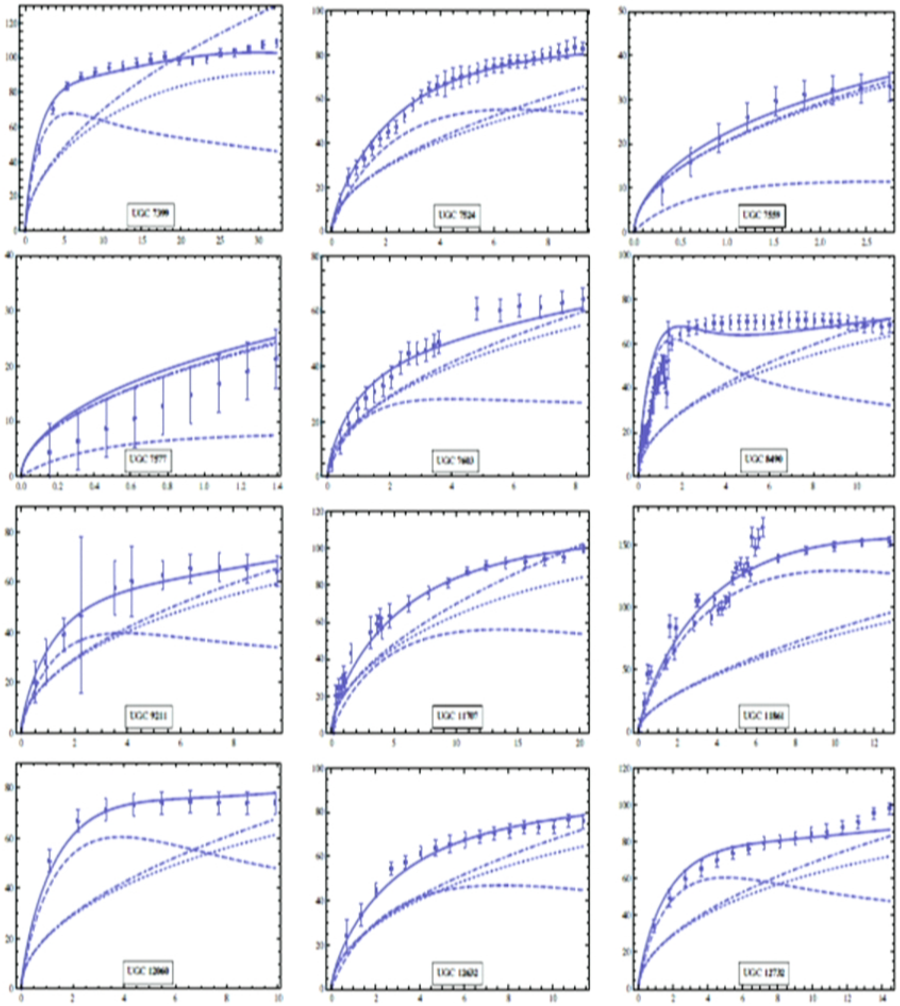


FIG. 8: Fitting to the rotational velocities of the 24 dwarf galaxy sample – Part 2

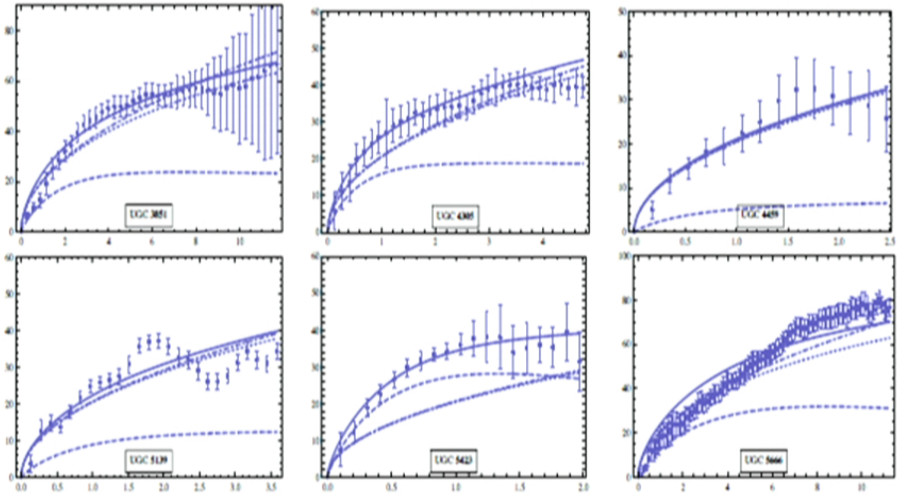


FIG. 9 Fitting to the rotational velocities of the 6 dwarf galaxy sample

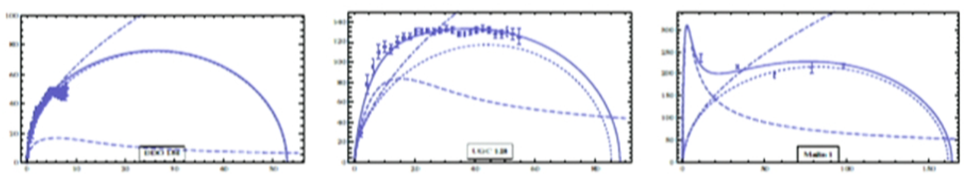
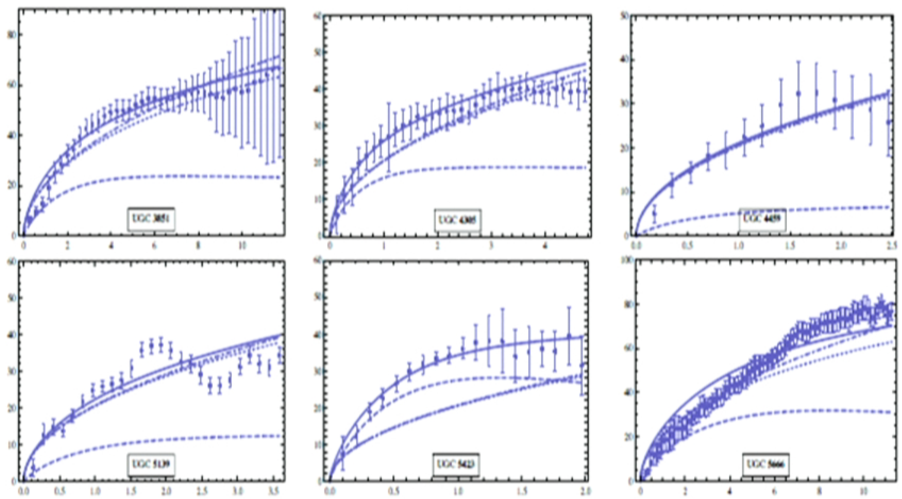


FIG. 10: Extended distance predictions for DDO 154, UGC 128, and Malin 1



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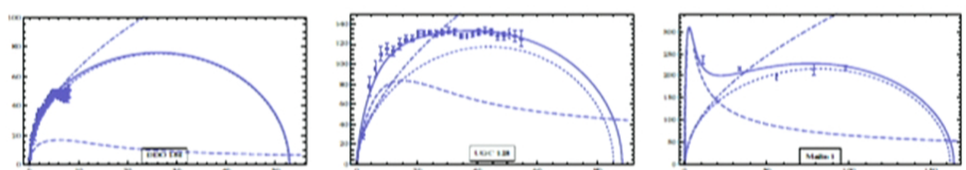
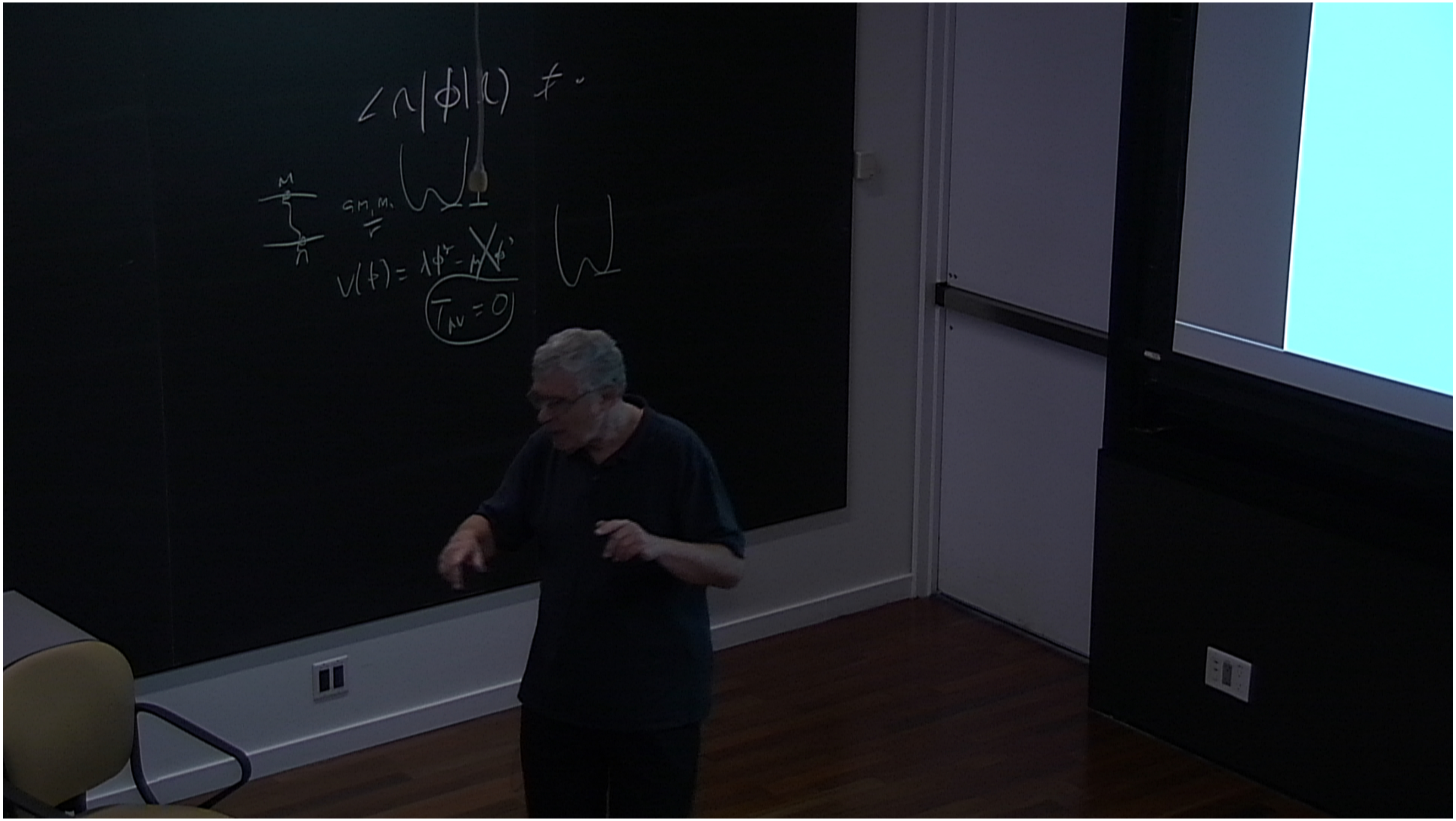
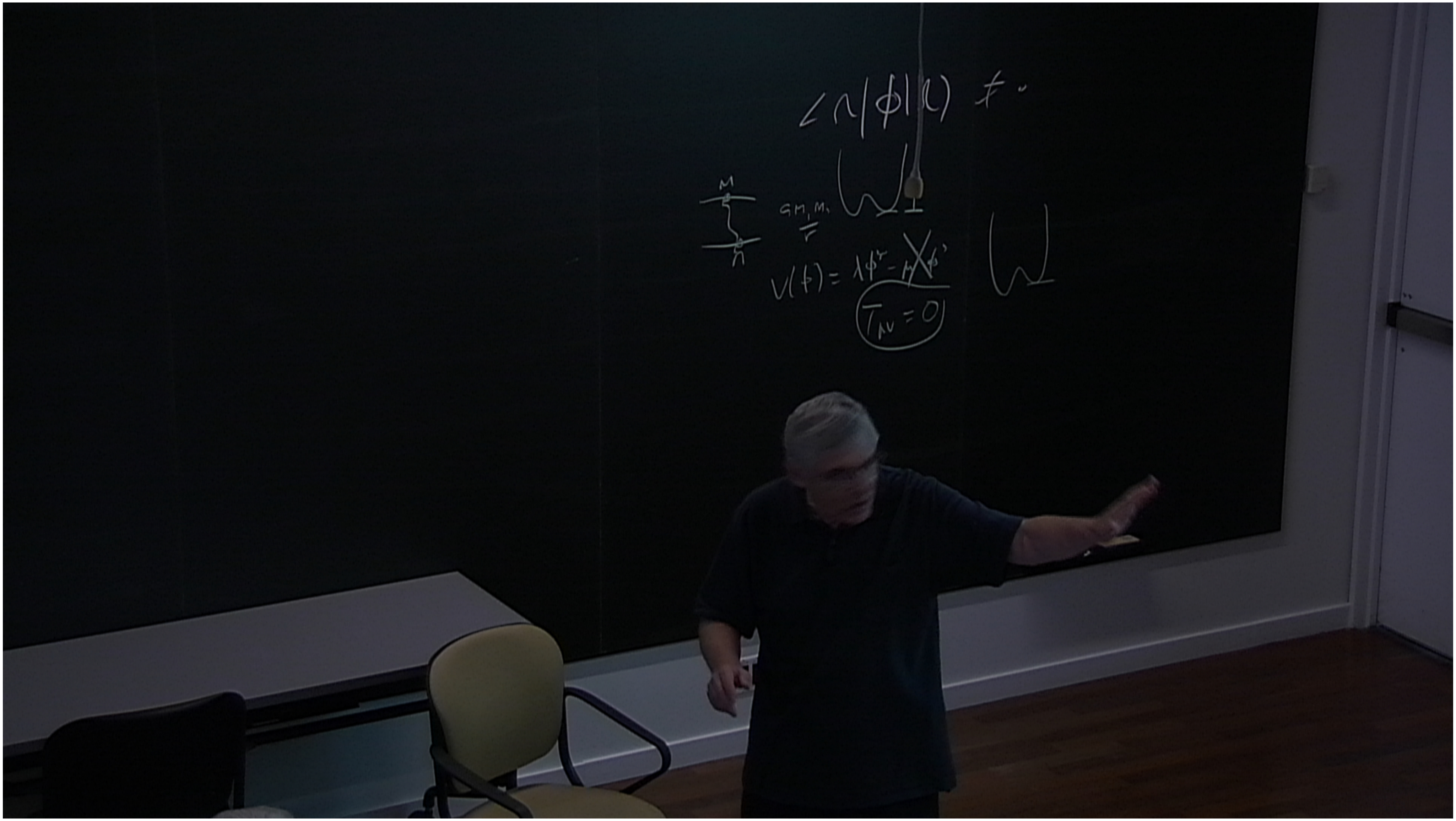


FIG. 10: Extended distance predictions for DDO 154, UGC 128, and Malin 1

9 Summary

To conclude, we note that at the beginning of the 20th century studies of black-body radiation on microscopic scales led to a paradigm shift in physics. Thus it could that at the beginning of the 21st century studies of black-body radiation, this time on macroscopic cosmological scales, might be presaging a paradigm shift all over again.




$T_{\mu\nu} = 0$