Title: Integer Quantum Hall Effect for Bosons: A Physical Realization

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Abstract: A simple physical realization of an integer quantum Hall state of interacting two dimensional bosons is provided. This is an example of a "symmetry-protected topological" (SPT) phase which is a generalization of the concept of topological insulators to systems of interacting bosons or fermions. Universal physical properties of the boson integer quantum Hall state are described and shown to correspond to those expected from general classifications of SPT phases.

Integer quantum Hall effect for bosons: A physical realization

T. Senthil (MIT) and Michael Levin (UMCP)
. (arXiv:1206.1604)

Thanks: Xie Chen, Zhengchen Liu, Zhengcheng Gu, **Xiao-gang Wen**, and **Ashvin Vishwanath.**

An obsession in modern condensed matter physics

``Exotic'' Phases of Matter

- Gapped phases with ``topological quantum order'', fractional quantum numbers
(eg, fractional quantum Hall state, gapped quantum spin liquids)

- phases with gapless excitations not required by symmetry alone
(eg, fermi and non-fermi liquid metals, gapless quantum spin liquids)

Emergent non-local structure in ground state wavefunction:

Characterize as ``long range quantum entanglement''

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Emergent non-local structure in ground state wavefunction:

Characterize as ``long range quantum entanglement''

How ``simple'' can ``interesting'' be?

Long range entangled phases have many interesting properties.

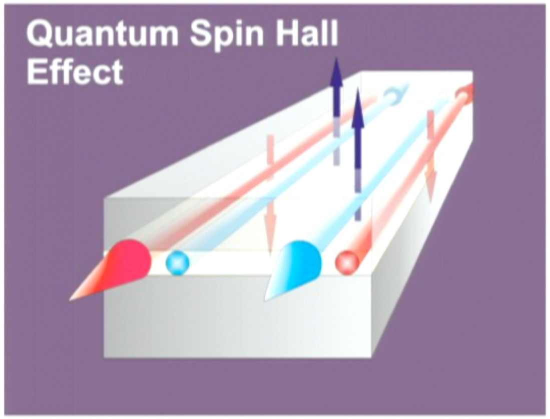
Some of these interesting things may not actually require the long range entanglement.

A sub-obsession: how exotic can a phase with short ranged entanglement be?

Dramatic progress in the context of topological band structures of free fermion models in recent years.

Modern topological insulators

Key characterization: Non-trivial surface states with gapless excitations protected by some symmetry



Interactions?

Current frontier: interaction dominated generalizations of the concept of topological insulators?

Move away from the crutch of free fermion Hamiltonians.

Useful first step: study possibility of topological insulators of bosons

Necessitates thinking more generally about these phases without the aid of a free fermion model.

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Integer Quantum Hall Effect (IQHE) for bosons?

Integer quantum Hall effect of fermions:
Often understand in terms of filling a full Landau level.

For bosons this obviously fails (no Pauli).

Can bosons be in an IQHE state with
(1) a quantized integer Hall conductivity
(2) no fractionalized excitations or topological order (unique ground state on closed manifolds)
(3) a bulk gap

Yes! (according to recent progress in general classification of short ranged entangled phases)

1. Cohomology classification (Chen, Liu, Gu, Wen, 2011)
2. ?? (Kitaev, unpublished)
3. Chern-Simons classification (Lu, Vishwanath, 2012)

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This talk

A physical realization of an integer quantum Hall state of bosons

Simple, possibly experimentally relevant, example of the kind of state the formal classification shows is allowed to exist.

Two component bosons in a strong magnetic field

Two boson species b_I each at filling factor $\nu = 1$

$$H = \sum_I H_I + H_{int} \tag{1}$$

$$H_I = \int d^2x b_I^\dagger \left(-\frac{\left(\vec{\nabla} - i\vec{A}\right)^2}{2m} - \mu \right) b_I \tag{2}$$

$$H_{int} = \int d^2x d^2x' \rho_I(x) V_{IJ}(x - x') \rho_J(x') \tag{3}$$

External magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$.
$\rho_I(x) = b_I^\dagger(x) b_I(x) =$ density of species I

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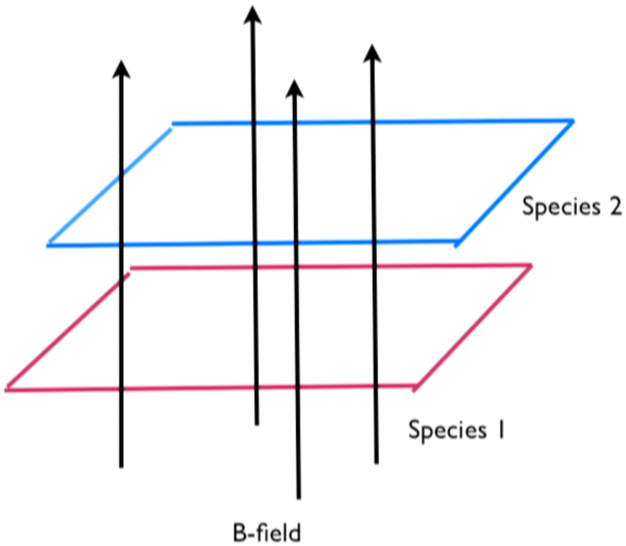
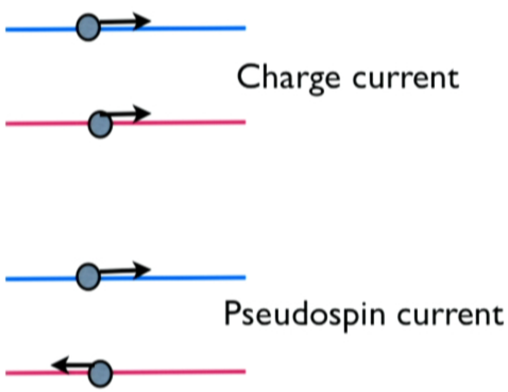
Symmetries and picture

Number of bosons N_1, N_2 of each species separately conserved: two separate global U(1) symmetries.

Total charge = N_1+N_2

Call $N_1 - N_2$ = total ``pseudospin''





Later relax to just conservation of total boson number

Flux attachment mean field theory

$\prod_{i,j}(z_i - w_j)$: particle of each species sees particle of the other species as a vortex.

Flux attachment theory:

Attach one flux quantum of one species to each boson of other species.
"Mutual composite bosons"

$\nu = 1 =>$ on average attached flux cancels external magnetic flux.
Mutual composite bosons move in zero average field.

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Chern-Simons Landau Ginzburg theory

Reformulate in terms of mutual composite bosons.
Implement flux attachment through Chern-Simons gauge fields.

$$\begin{aligned}
\mathcal{L} &= \sum_I \mathcal{L}_I + \mathcal{L}_{int} + \mathcal{L}_{CS} \\
\mathcal{L}_I &= \tilde{b}_I^*(\partial_0 - iA_{I0} + i\alpha_{I0})\tilde{b}_I - \frac{|\vec{\nabla}\tilde{b}_I - i(\vec{A}_I - \vec{\alpha}_I)\tilde{b}_I|^2}{2m} \\
&+ \mu|\tilde{b}_I|^2 \\
\mathcal{L}_{int} &= -V_{IJ}|\tilde{b}_I|^2|\tilde{b}_J|^2 \\
\mathcal{L}_{CS} &= \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\left(\alpha_{1\mu}\partial_\nu\alpha_{2\lambda} + \alpha_{2\mu}\partial_\nu\alpha_{1\lambda}\right) \qquad (1)
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Chern-Simons gauge fields

Probe gauge fields

Mutual Chern-Simons implements flux attachment

Mutual composite fermions see zero average field: condense them.
Internal Chern-Simons gauge fields lock to probe gauge fields.

Physical properties: Hall transport

Effective probe Lagrangian

$$\mathcal{L}_{eff} = \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}\left(A_{1\mu}\partial_\nu A_{2\lambda} + A_{2\mu}\partial_\nu A_{1\lambda}\right) \quad (1)$$

New probe gauge fields that couple to charge and pseudospin currents $A_c = \frac{A_1+A_2}{2}$, $A_s = \frac{A_1-A_2}{2}$.

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Electrical Hall conductivity $\sigma_{xy} = 2$

Pseudospin Hall conductivity $\sigma^s_{xy} = -2$.

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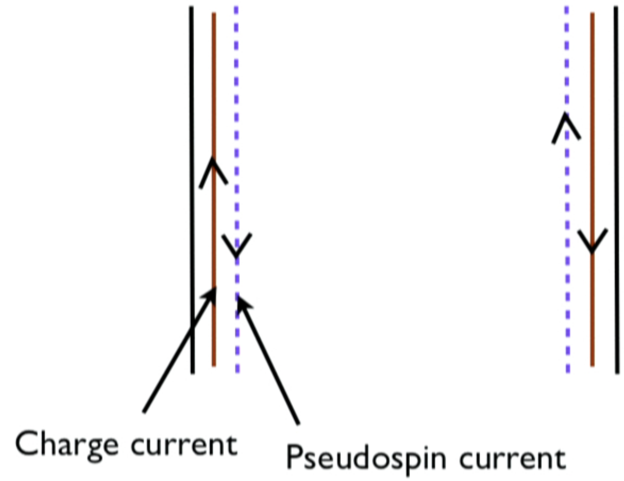
Edge states

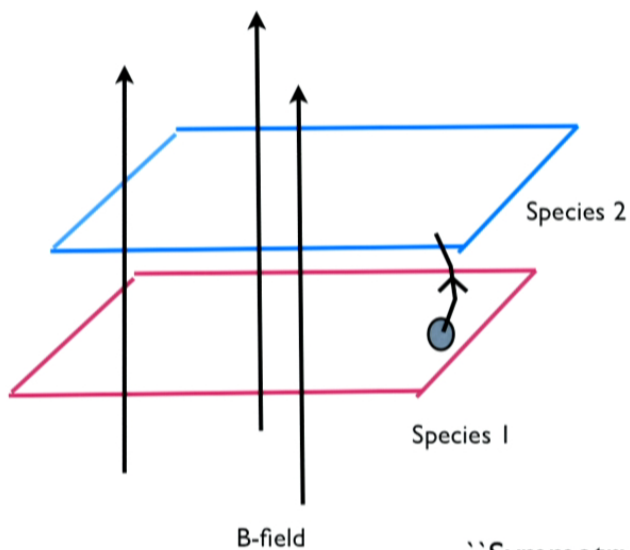


Comments:

1. Counterpropagating edge states but only one branch transports charge.

2. Thermal Hall conductivity = 0

Symmetry protection of edge states



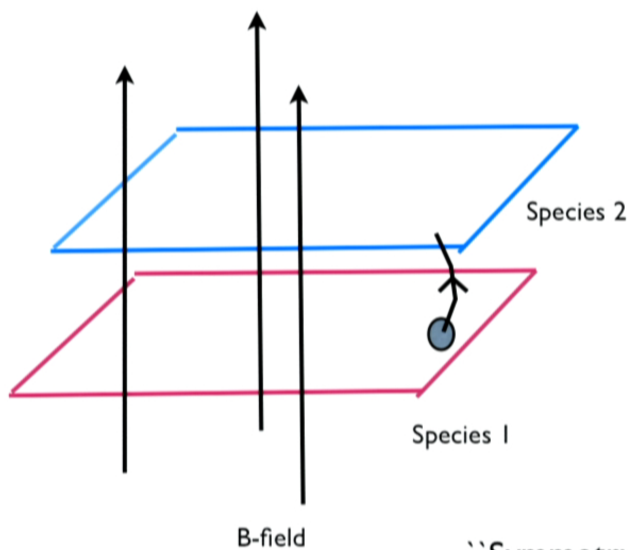
Include interspecies tunneling:
Pseudospin not conserved,
only total particle number conserved.

Counterpropagating edge modes cannot backscatter due to charge conservation.

Edge modes are preserved so long as total charge is conserved.

``Symmetry Protected Topological Phase'' of bosons

Symmetry protection of edge states



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Effective topological field theory

Two component Chern-Simons theory

$$\mathcal{L} = \frac{1}{4\pi}\left(a_1 da_2 + a_2 da_1\right) + \frac{1}{2\pi}\left(da_1 + da_2\right) A_c \qquad (1)$$

"K-matrix" $= \sigma^x$

Unique ground state on closed manifolds as $|DetK| = 1$

Connect to general discussion of Lu, Vishwanath (Ashvin talk)

Ground state wavefunction

(ignore interspecies tunneling)

Naive guess $$\Psi(\{z_i, w_j\}) = \prod_{i,j}(z_i - w_j) \cdot e^{-\sum_i \frac{|z_i|^2+|w_i|^2}{4}} \qquad (1)$$

Problem: Unstable to phase separation (see using Laughlin plasma analogy)

Fix, for example, using ideas initiated by Jain (1993) for some fermionic quantum Hall states

$$\begin{aligned}\Psi_{flux} &= P_{LLL}\prod_{i<j}|z_i - z_j|^2 \cdot \prod_{i<j}|w_i - w_j|^2 \\ &\cdot \prod_{i,j}(z_i - w_j) \cdot e^{-\sum_i \frac{|z_i|^2+|w_i|^2}{4}} \qquad (1)\end{aligned}$$

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Pseudospin properties

The edge theory for this state is identical to the $SU(2)_1$ WZW conformal field theory.

Edge theory has (emergent) pseudospin $SU(2)$ rotation symmetry.

Suggests state itself can be stabilized for a pseudospin $SU(2)$ invariant Hamiltonian.

Can show wavefunction of previous slide is actually a pseudospin singlet

Microscopics: a simple Hamiltonian

$$H = \sum_I H_I + H_{int} \quad (1)$$

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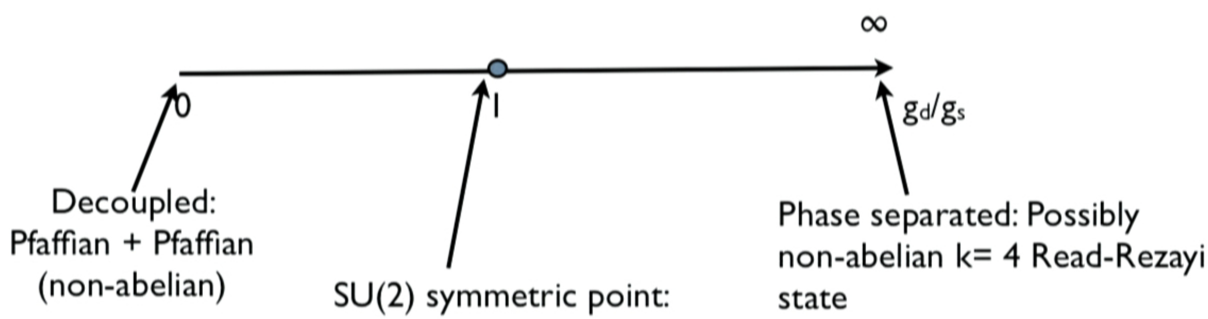
Simple and **realistic** interaction:

$$V_{II}(\vec{x}) = g_s \delta^{(2)}(\vec{x})$$
$$V_{12}(\vec{x}) = g_d \delta^{(2)}(\vec{x})$$

$g_s = g_d$: Pseudospin $SU(2)$ invariance

Possible Phase diagram



Near SU(2) symmetric point, recent exact diagonalization work show an incompressible, spin singlet state (Grass et al, 2012arXiv1204.5423G, Furukawa, Ueda 2012arXiv1205.2169F)

Candidates: 1. Boson IQHE

2. A non-abelian spin singlet state (Ardonne, Schoutens 1999)

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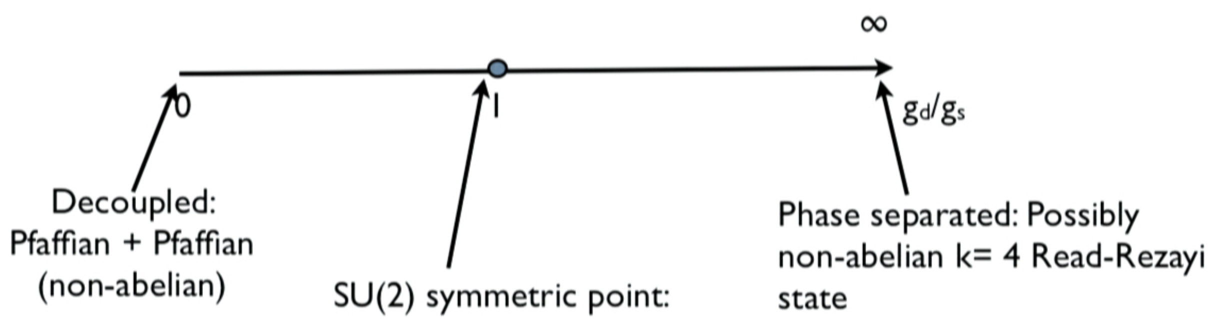

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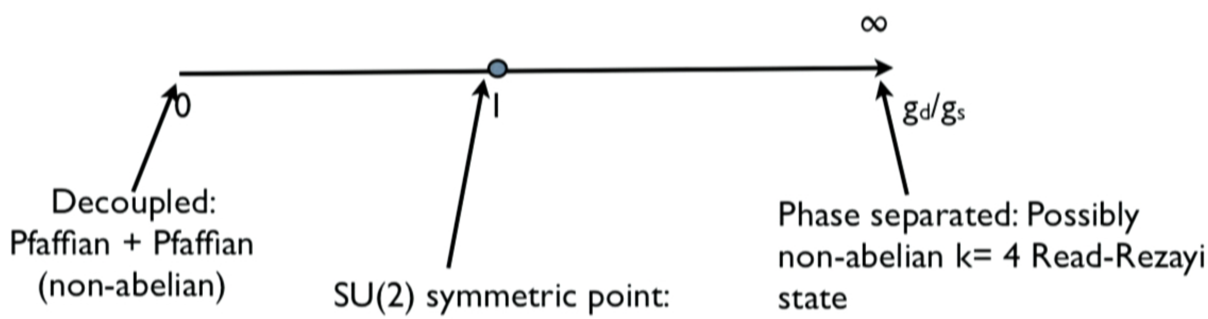

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Prospects - experiments

Obvious place to look is in ultracold atoms in strong artificial magnetic fields.

The delta function repulsion is realistic and controllable.

Challenge: get fields high enough to be in the quantum Hall regime