Title: Symmetry Protected Topological Phases; from Quantum Entanglement to Interaction Effects

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Abstract: I will briefly review topological phases of non interacting fermions, such as topological insulators, and discuss how ideas from quantum information, in particular the entanglement spectrum, can be used to characterize them.

For topological phases protected by inversion symmetry I argue that this is the ideal tool, and discuss how it leads to a classification of 3D topological phases. Next, I will discuss topological phases that only appear in the presence of interactions, although they share essential characteristics with non-interacting topological phases. A general theory of such states of bosons in 2+1 dimensions will be given, and specific examples, the bosonic analogs of topological superconductors and insulators, will be discussed.

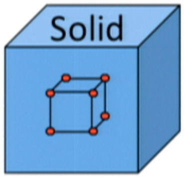
`Integer' Topological Phases: Entanglement and Interaction Effects

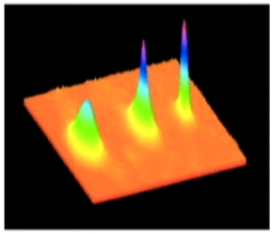
Ashvin Vishwanath
UC Berkeley

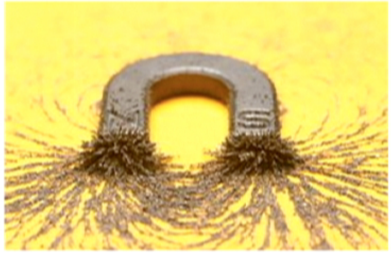
Conventional (Landau) Phases

Distinguished by spontaneous symmetry breaking.

- **Solid** (broken translation)
- **Superfluids** ψ
- **Magnets** (broken spin symmetry) **M**



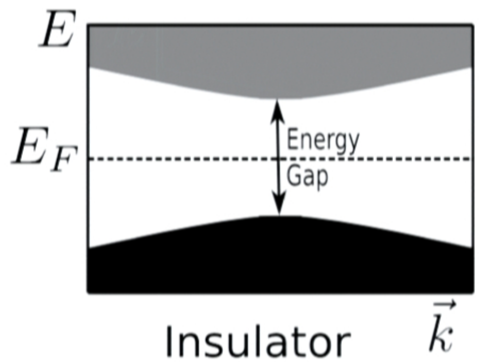




In contrast –topological phases…

Band insulators

- A band insulator:



Eg.

Band Insulators with the same symmetry - the same phase?

Band insulators

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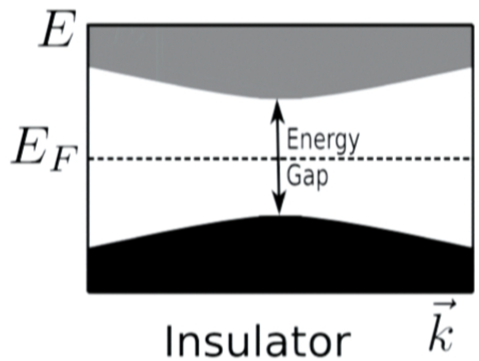


Eg.

Band Insulators with the same symmetry - the same phase?

No! Distinction at the level of topology.

Topological Band Insulators - Review

- Topology characterizes the identity of objects up to deformation, e.g. genus of surfaces

Figure courtesy C. Kane

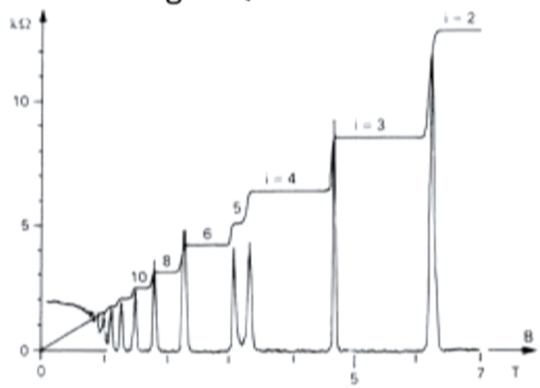
- Genus **g** (#of handles) by integrating Gaussian curvature ***K***

$$\oint \boldsymbol{K}\, d\boldsymbol{S} = 4\pi(1-\boldsymbol{g})$$

Topological Band Insulators

2D Chern Insulators or Integer Quantum Hall States (Klitzing, Laughlin, Halperin, Thouless, Kohmoto, Nightingale, den Nijs, Avron, Haldane)

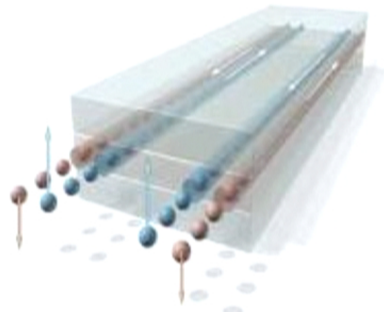
Integer QHE



$$C = \frac{1}{2\pi}\int F dk_x\, dk_y$$

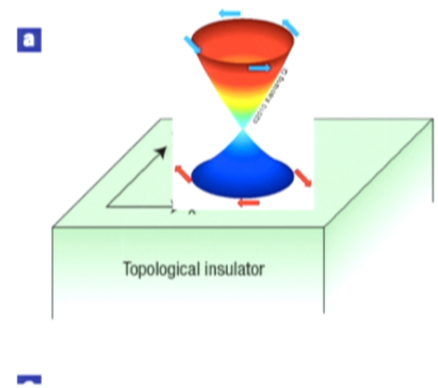
Topological Band Insulators

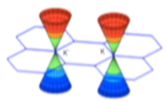
Quantum Spin Hall Insulators (Kane, Mele; Bernevig, Zhang, Hughes; Konig et al.)



3D Topological Insulators

(Fu, Kane, Mele; Moore, Balents; Fu, Kane; Roy; Hsieh, Hasan, Cava et al.; Xia et al.; Chen et al.; Qi, Hughes, Zhang)





Graphene
–2 nodes

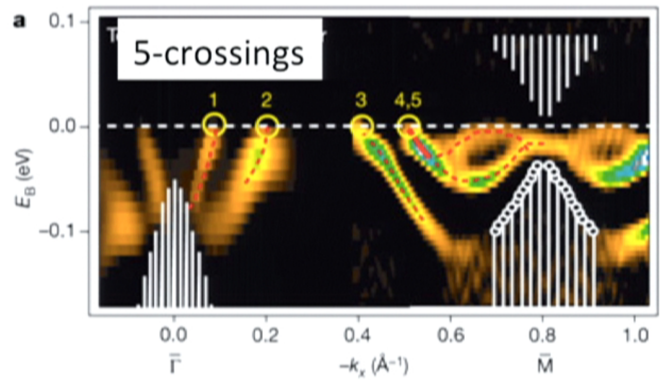
- Bulk gapped *but* exotic surface states.
- Protected by Symmetry. (Time reversal)

Experiments

- $Bi_{1-x}Sb_x$ (x=0.1)

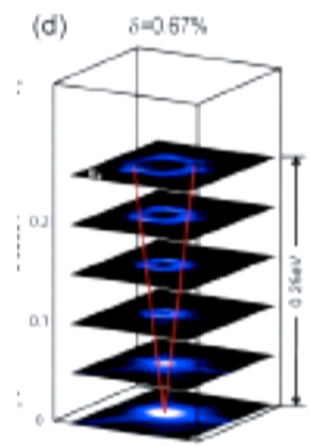
  Theory: L. Fu & C. L. Kane (07)

  Surface modes ARPES confirms strong T-I.



Experiment: D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava and M. Z. Hasan, Nature (08)

- Newer materials: Bi_2Se_3, Bi_2Te_3
  - Eg. Y. Chen et al. (2009). Gap=1 ,000K



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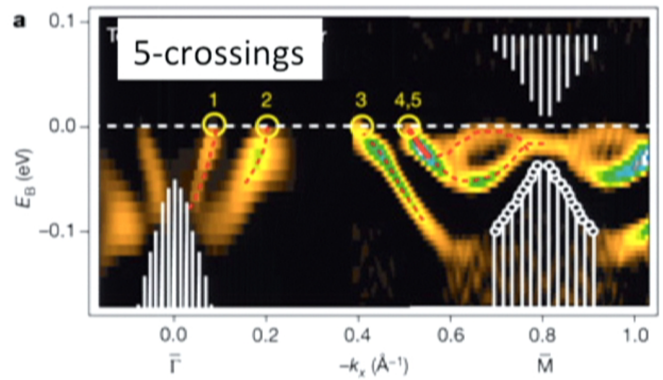

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- Newer materials: Bi_2Se_3, Bi_2Te_3
  - Eg. Y. Chen et al. (2009). Gap=1 ,000K
  - Well known thermoelectric

Peltier - Polymerase Chain Reaction Unit

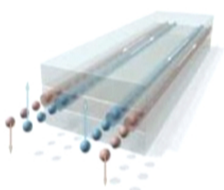
Topological Phases

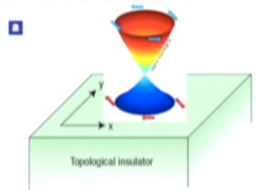
Integer topological phases

- Integer Quantum Hall & Topological insulators

- Interacting analogs in D=2,3 (Kitaev, Chen-Gu-Wen, Lu&AV)

Non-trivial surface states





How to tell – given ground state wave-function(s)?
Entanglement as topological `order parameter'.

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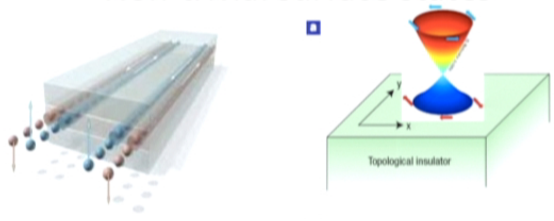

Fractional topological phases

- Fractional Quantum Hall
- Gapped spin liquids

Topological Order:

1. Fractional statistics excitations (anyons).
2. Topological degeneracy on closed manifolds.

How to tell – given ground state wave-function(s)?
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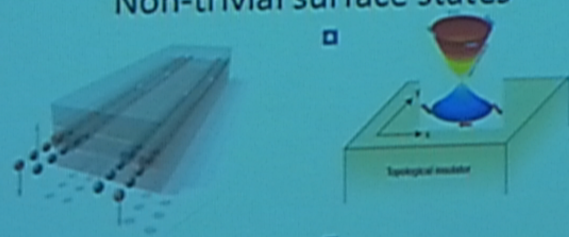

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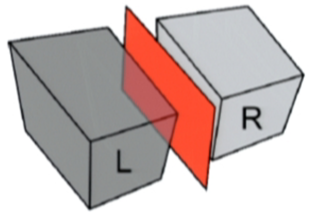
Entanglement as topological `order parameter'.

Entanglement Signatures of Topological Insulators

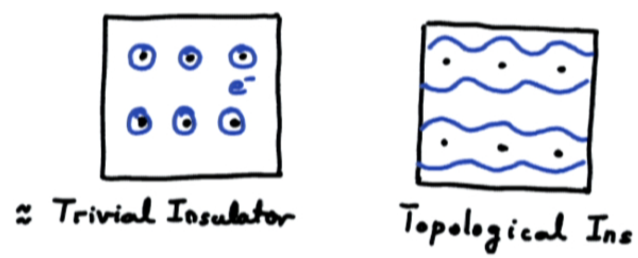
- Bipartite Entanglement of Quantum Systems
  - Ground state property (no excitations involved)



$$\mathbf{Gnd.\,State:}\ |\Psi_0\rangle$$

$$\rho_R = Tr_L\left[\,|\Psi_\mathbf{0}\rangle\langle\Psi_\mathbf{0}|\,\right]$$

- Topological Insulator must have signature



(Turner, Yi Zhang, AV arXiv/0909.3119 & refs. therein)

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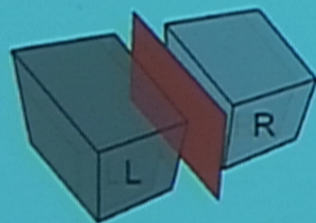

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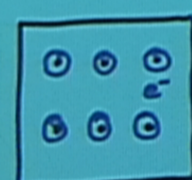
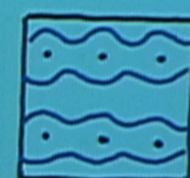


≈ Trivial Insulator



Topological Ins.

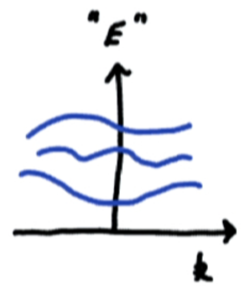
(Tur[illegible] Yi Zhang, AV arXiv/0909.3119 & refs. therein)

Entanglement Spectrum and Topological Insulators

- Entanglement entropy: Does ***not*** diagnose topological insulators. (Topological Entanglement Entropy only for Topologically ordered phases)
- Entanglement Spectrum. Li and Haldane (FQHE).

$$\rho_R = \sum_i \frac{\exp(-E_i)}{Z} |\varphi_i\rangle\langle\varphi_i|$$

With translation symmetry along the cut $E(k_x,k_y)$



Li and Haldane (FQHE)

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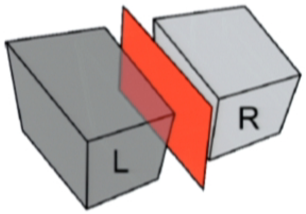

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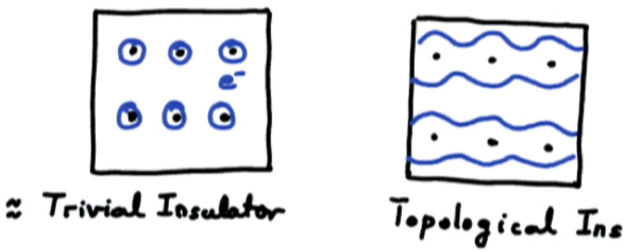

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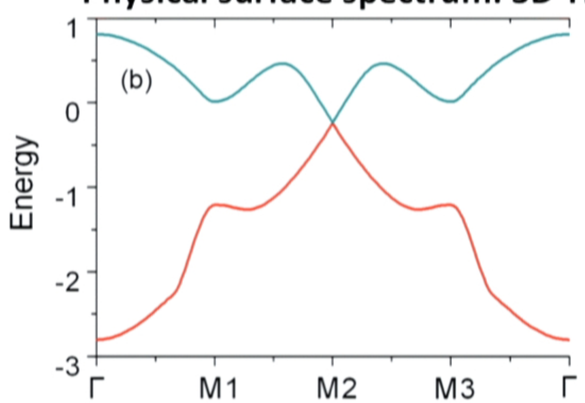
Entanglement and Edge States

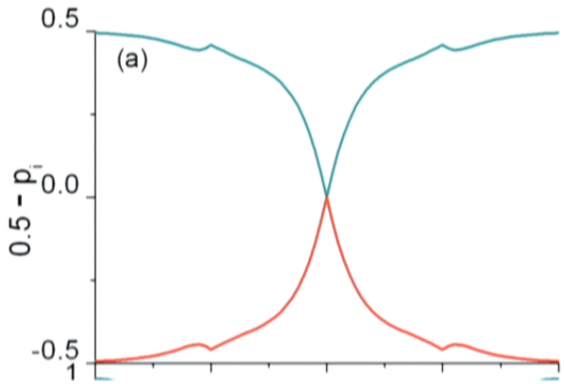
- For non-interacting fermions: If physical edge has protected modes, entanglement spectrum also has same type of modes. (Turner, Zhang, AV 2009, Fidkowskii)

Entanglement and Edge States

- For non-interacting fermions: If physical edge has protected modes, entanglement spectrum also has same type of modes. (Turner, Zhang, AV 2009, Fidkowski)

Physical surface spectrum: 3D TI

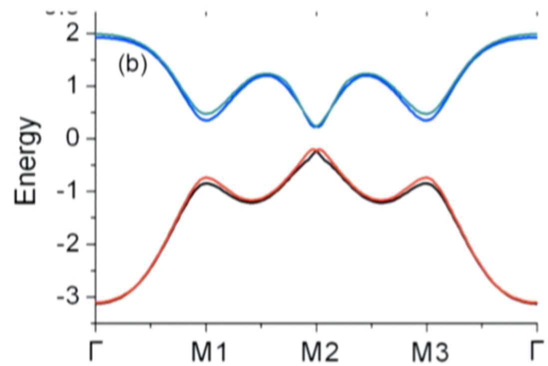




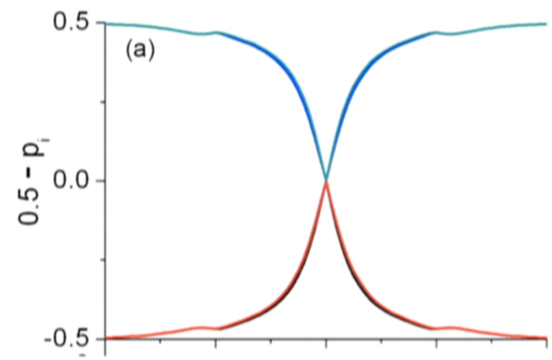
Beyond Surface States

Converse: Protected Entanglement Modes -> Surface state?

Physical surface spectrum



Entanglement 'spectrum'



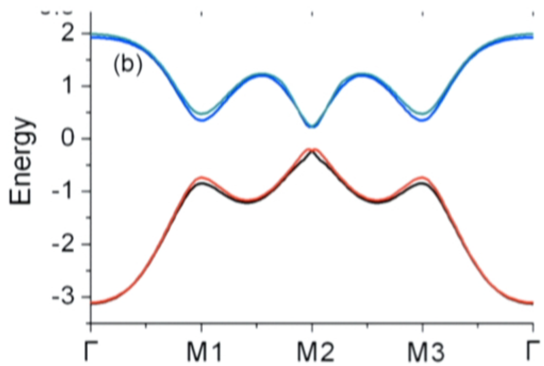
Break Time Reversal Symmetry

Physical Surface is gapped, but entanglement spectrum gapless.

Beyond Surface States

Converse: Protected Entanglement Modes -> Surface state? NO

Physical surface spectrum



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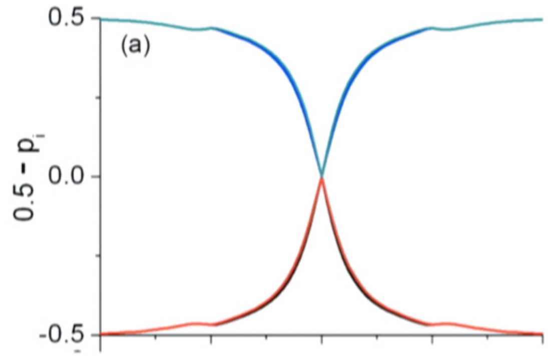


Break Time Reversal Symmetry

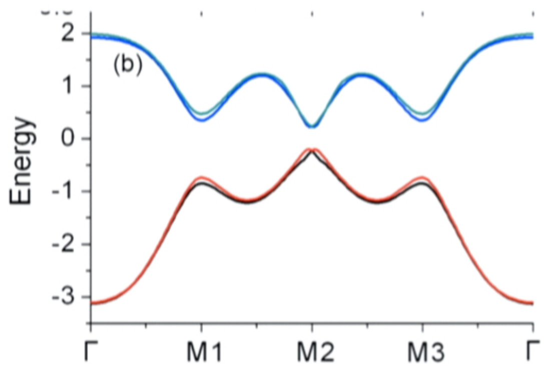
Physical Surface is gapped, but entanglement spectrum gapless.

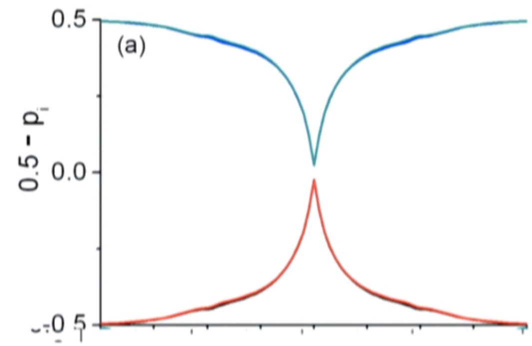
Due to **Inversion** symmetry

Beyond Surface States

Converse: Protected Entanglement Modes -> Surface state? **NO**

Physical surface spectrum **Entanglement 'spectrum'**





Break Time Reversal Symmetry

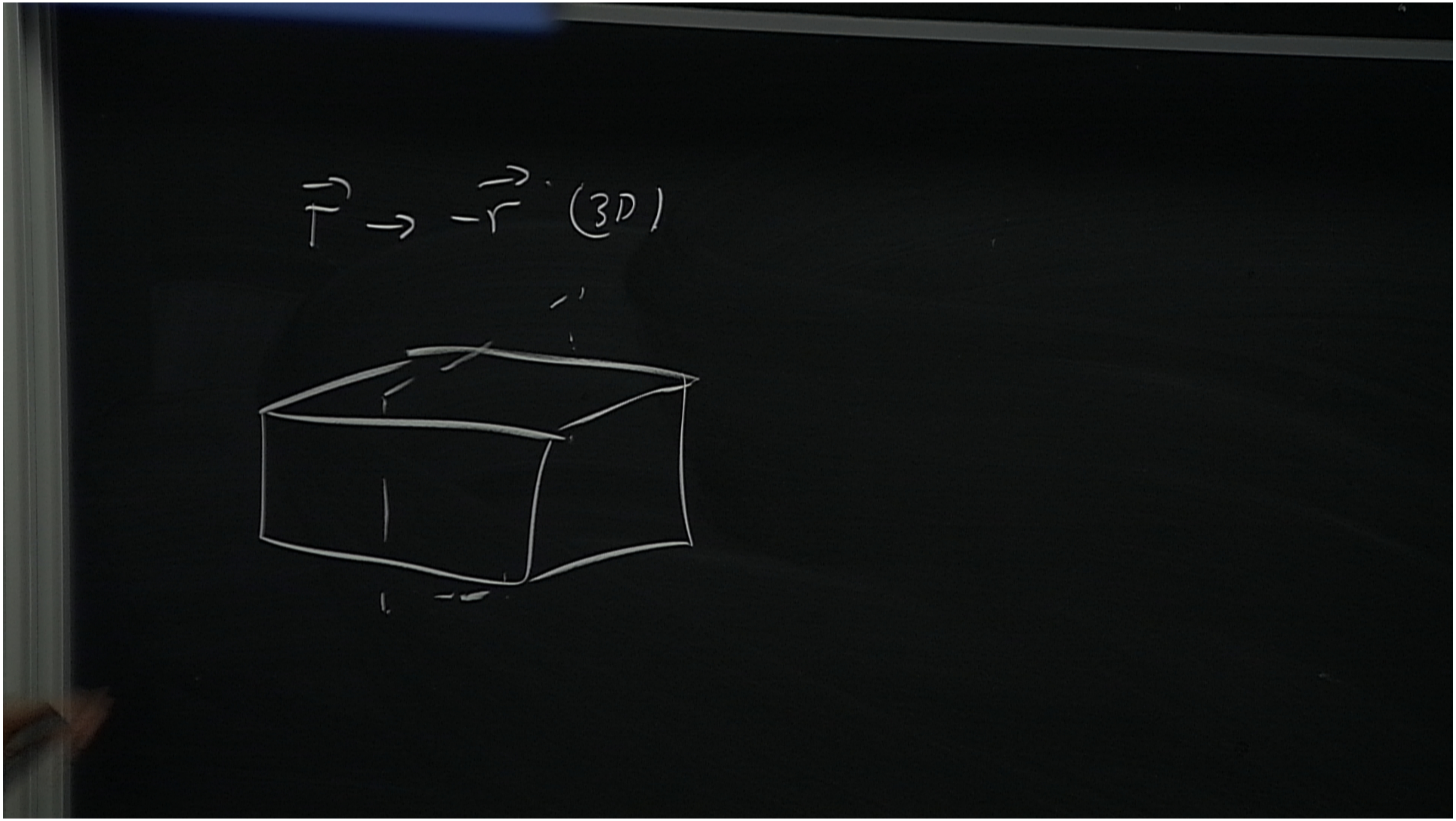
Physical Surface is gapped, but entanglement spectrum gapless.

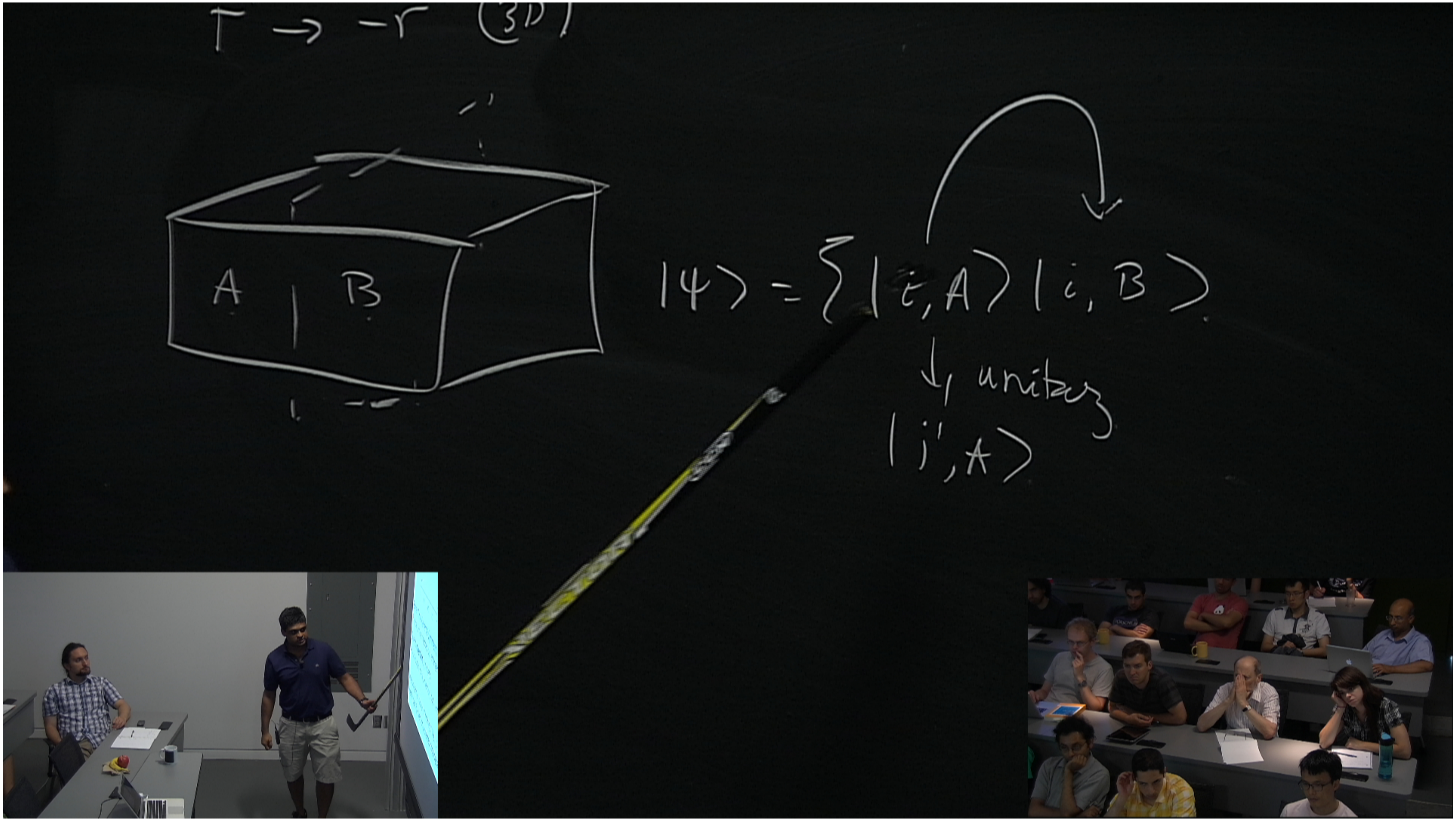
Due to **Inversion** symmetry

Physical surface breaks inversion, but entanglement cut does not.

Inversion anti-unitary and "*Kramers*" *like*. $I^2 = -1$

Used to classify inversion protected topological states. (Zhang, Turner, AV)


$\vec{r} \to -\vec{r}$ (3D)


r → −r (3D)
A
B
|ψ> = Σ |i, A>|i, B>
unitary
|j', A>

Part 2: Correlated Topological Phases

- Current topological phases - band theory works well.

- Going beyond the `band'-wagon?

Interacting Topological Phases?

Integer Quantum Hall → Strong Interactions → **Fractional Quantum Hall**

Bulk: like trivial insulator
Edge: protected edge modes

Bulk: nontrivial! **Topological order.**

Fractional excitations,
Topological degeneracy.
Topological entanglement entropy

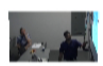
Interacting Topological Phases?

Interacting ('integer') Topological Insulators?

- NO fractional excitations BUT non-trivial edge states.
- Interactions essential. Eg. *Bosons*
- Do they exist?

Integer Quantum Hall —Strong Interactions→ **Fractional Quantum Hall**

Bulk: like trivial insulator
Edge: protected edge modes



Bulk: nontrivial! **Topological order.**

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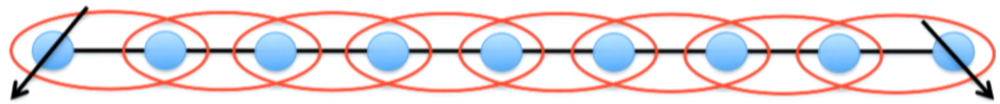
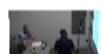
‘Integer’ Topological Phases of Bosons

- ***Yes.*** Integer topological phases of bosons (or spins) known in 1D.
- Eg. Haldane chain (spin ½ edge state of spin 1 chain)



- What about D=2?
  - Kitaev (example of bosonic ‘integer’ topological phase, no symmetry)
  - Chen-Gu-Wen (cohomology classification of symmetry protected phases).

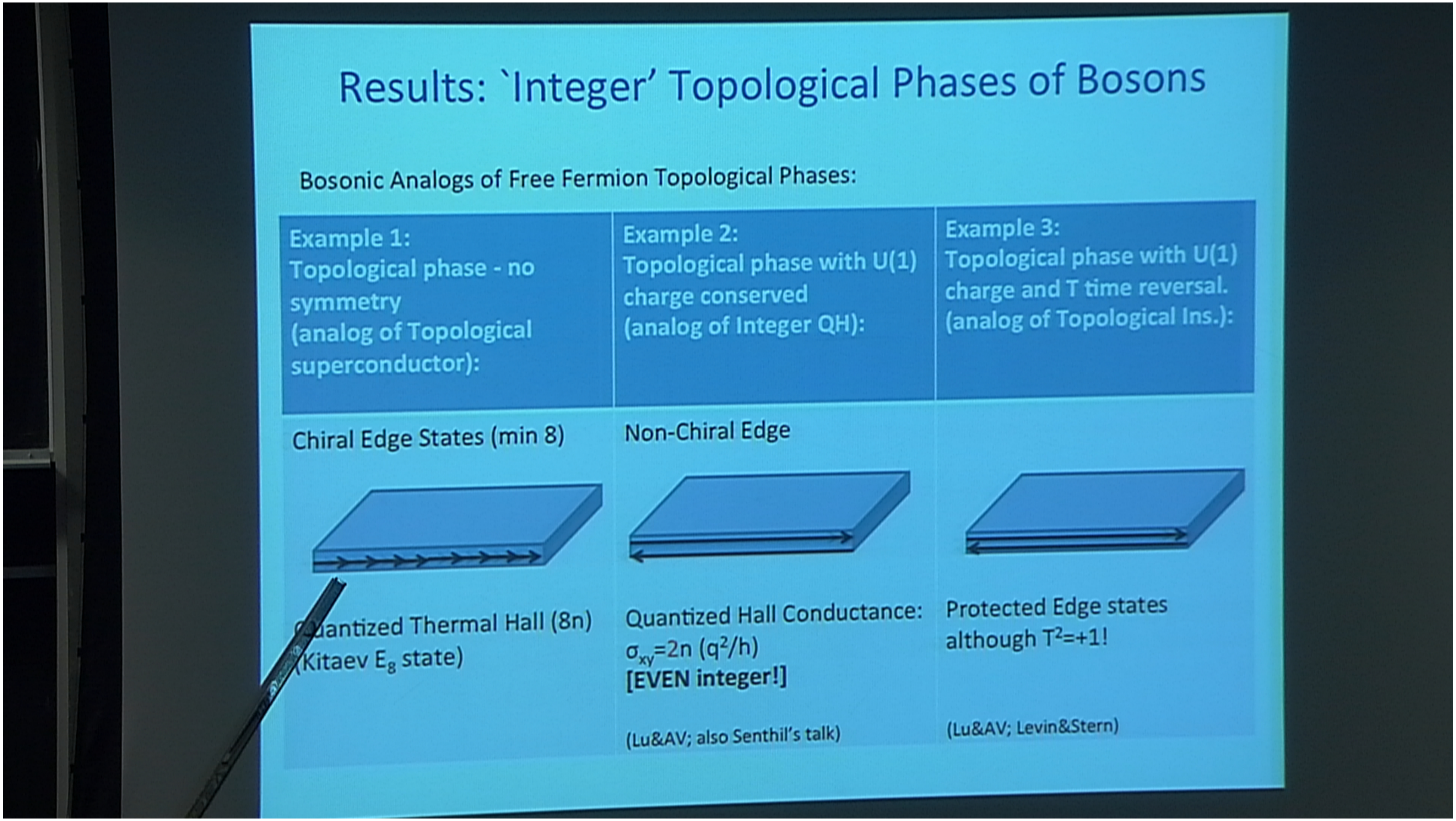


  - **Here**: A field theory [Chern-Simons] approach that reveals physical properties of edge and quantized responses. (Lu&AV. arXiv:1205.3156)

Results: `Integer' Topological Phases of Bosons

Bosonic Analogs of Free Fermion Topological Phases:

Example 1: **Topological phase - no symmetry** **(analog of Topological superconductor):**	**Example 2:** **Topological phase with U(1) charge conserved** **(analog of Integer QH):**	**Example 3:** **Topological phase with U(1) charge and T time reversal.** **(analog of Topological Ins.):**



Results: `Integer' Topological Phases of Bosons
Bosonic Analogs of Free Fermion Topological Phases:
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Example 3: Topological phase with U(1) charge and T time reversal. (analog of Topological Ins.):
Chiral Edge States (min 8)
Non-Chiral Edge
Quantized Thermal Hall (8n)
(Kitaev E_8 state)
Quantized Hall Conductance:
$\sigma_{xy}=2n\ (q^2/h)$
[EVEN integer!]
(Lu&AV; also Senthil's talk)
Protected Edge states although $T^2=+1$!
(Lu&AV; Levin&Stern)

Chern-Simons Approach

- K-matrix description of Abelian quantum Hall. Currents -> gauge flux:

$$L = \frac{K}{4\pi} a \cdot \nabla \times a \qquad \nabla = (\partial_\tau, \partial_x, \partial_y)$$

K=1 Integer quantum Hall (fermions). K=3 Laughlin state. In general, K symmetric integer matrix.

1. For bosons, diagonals are *even* integers.

Example 1: Chiral Boson State

- Construct bosonic K matrix, with Det=1, and all positive eigenvalues.

  Smallest matrix 8x8.

$$K = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Cartan Matrix of the group E8.
Identical to Kitaev state (obtained differently)

Example 2: Integer Quantum Hall Effect of Bosons

- Need U(1) charge conservation. Symmetry Protected Edge states. $K = \sigma_x$

$$L = \frac{a_1 \cdot \nabla \times a_2}{2\pi} + \frac{(a_1 + n a_2) \cdot \nabla \times A}{2\pi}$$

$\sigma_{xy}=2n$

- Integer quantum Hall of Bosons

- Incorporate general symmetries:

Symmetry Protected Topological Phases

- K-matrix formulation+ Symmetry. (like classifying space groups of crystals). Check for edge states.

Symmetry	*Topological Classification*
No symmetry (chiral)	$\mathbb{Z}$
Z_2^T	$\mathbb{Z}_1$
U(1)	$\mathbb{Z}$
$U(1) \rtimes Z_2^T$	$\mathbb{Z}_2$
$U(1) \times Z_2^T$	$\mathbb{Z}_1$
Z_n	$\mathbb{Z}_n$
$Z_n \rtimes Z_2^T$	$\mathbb{Z}_{(n,2)}^2$
$Z_n \times Z_2^T$	$\mathbb{Z}_{(n,2)}^2$
$U(1) \times Z_2$	$\mathbb{Z} \times \mathbb{Z}_2^2$

Agrees with Wen et al. cohomology classification

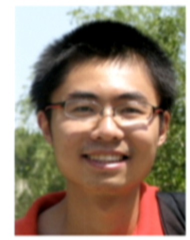
Summary

- New Theme in Solid State Physics:
  - Topological distinction between phases
  - Many exciting things to do…
  - Entanglement as the new ‘order parameter’?

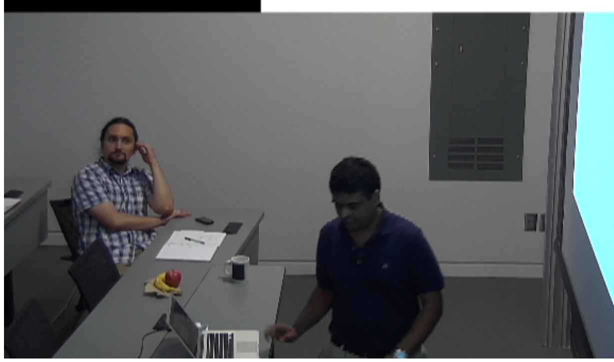
Acknowledgements

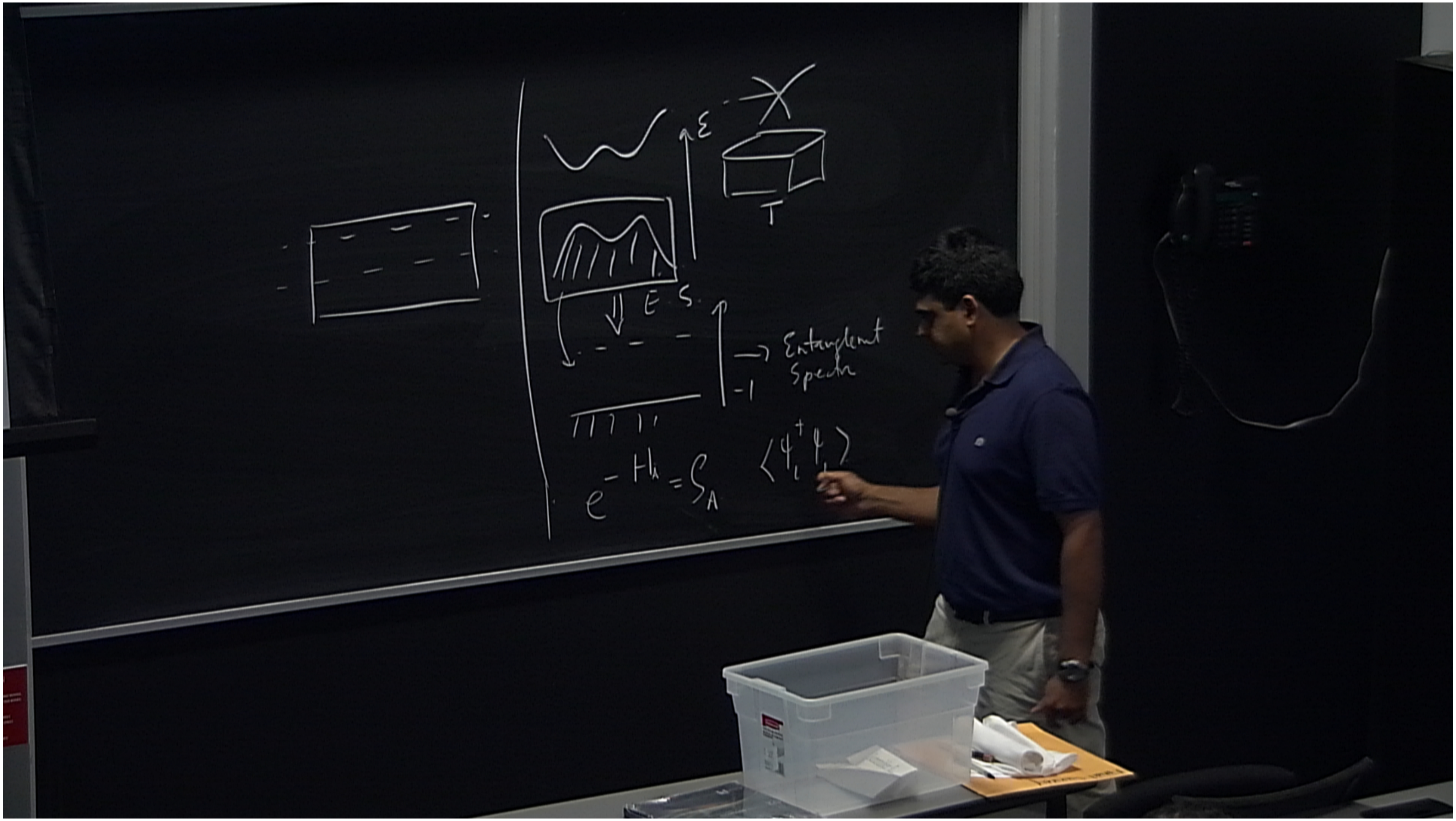
Ari Tuner (Berkeley-> Amsterdam)

Frank Yi Zhang
(Berkeley-> Stanford)



Yuan-Ming Lu (Berkeley)




E S
Entanglemt
Spectr
$e^{-H_A} = \rho_A$
$\langle \psi_i^{\dagger} \psi_j \rangle$