Title: Entanglement, Holography, and the Quantum Phases of Matter

Date: Jul 13, 2012 02:30 PM

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Abstract:

"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

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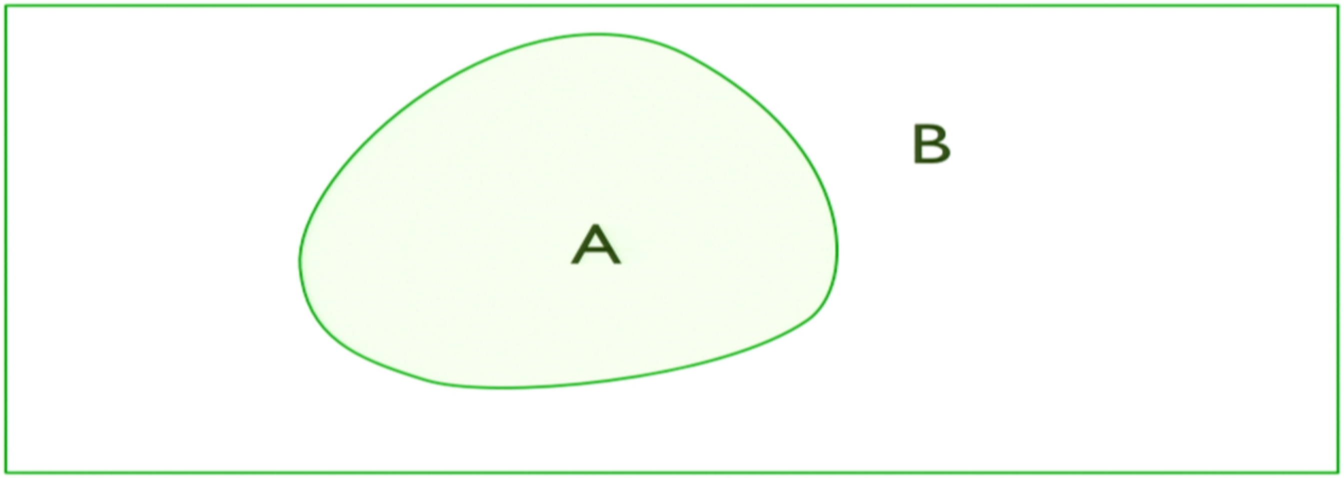
?

Entanglement entropy



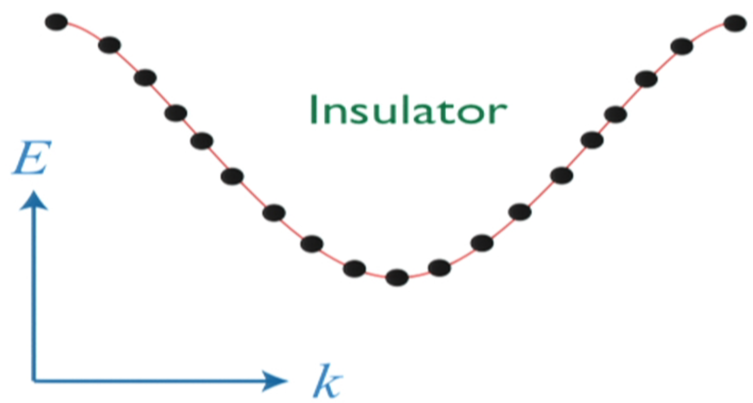
$|\Psi\rangle \Rightarrow$ Ground state of entire system,
$\rho = |\Psi\rangle\langle\Psi|$

$\rho_A = \mathrm{Tr}_B \rho =$ density matrix of region A

Entanglement entropy $S_E = -\mathrm{Tr}\,(\rho_A \ln \rho_A)$

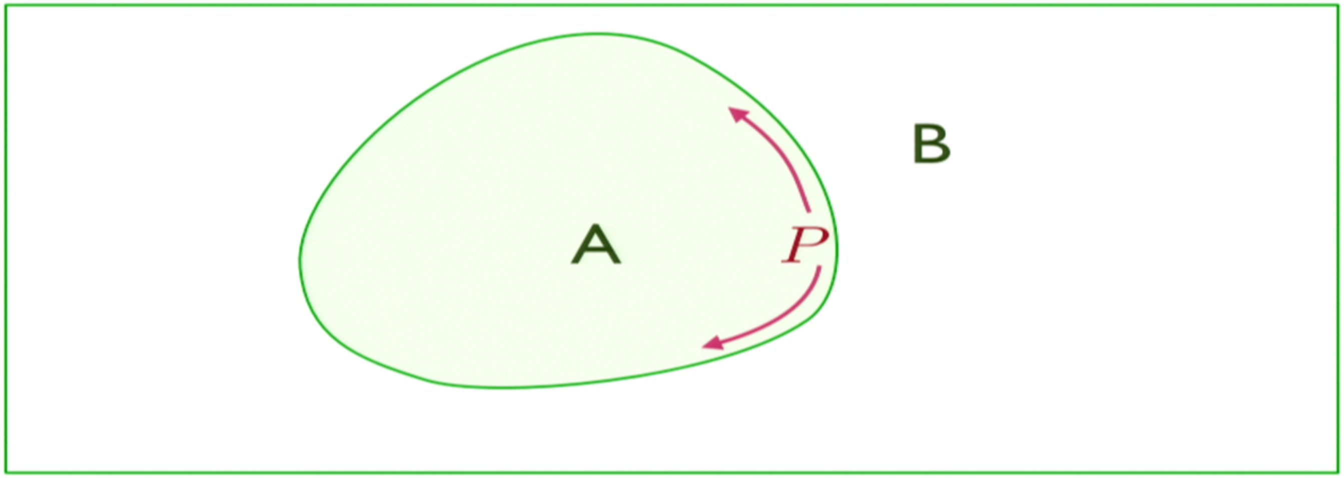
M. Srednicki, Phys. Rev. Lett. **71**, 666 (1993)

Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator

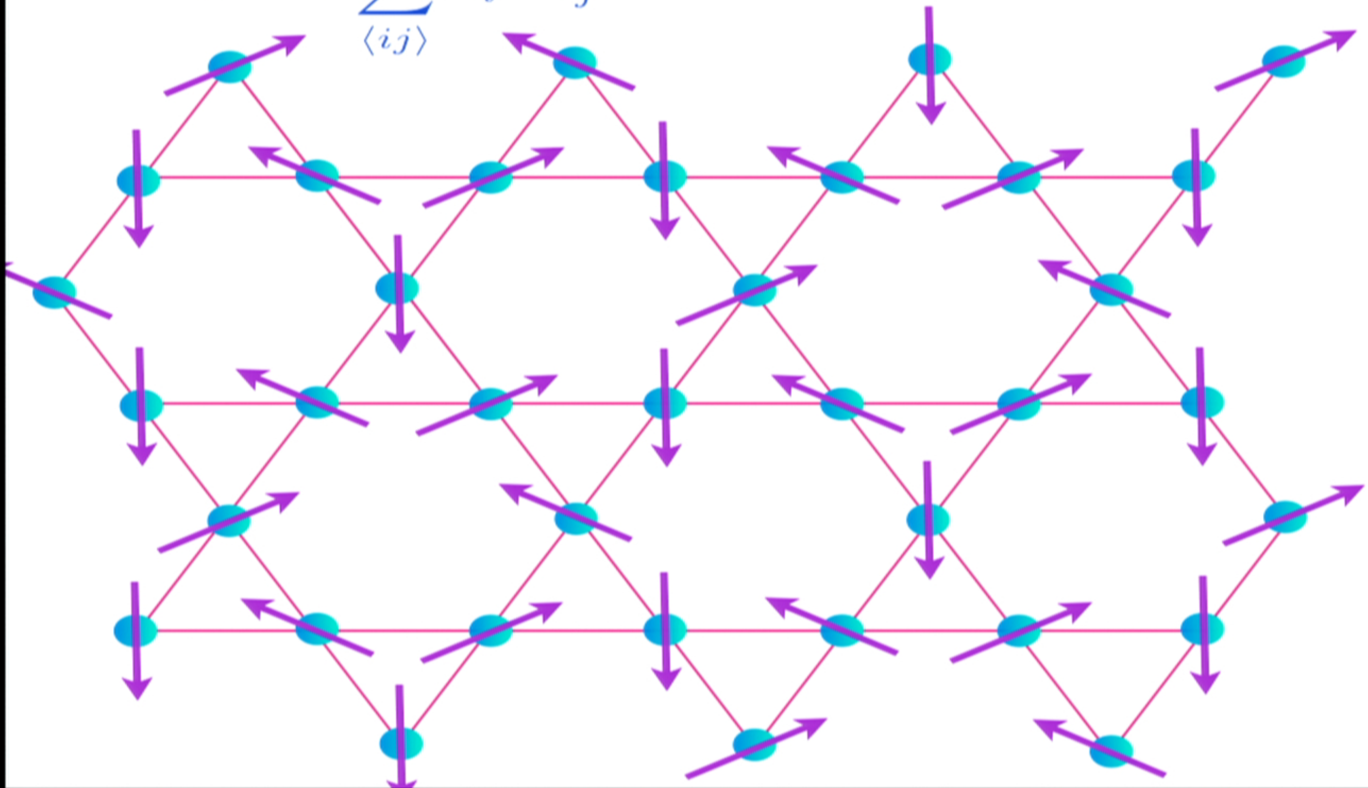


$$S_E = aP - b\exp(-cP)$$

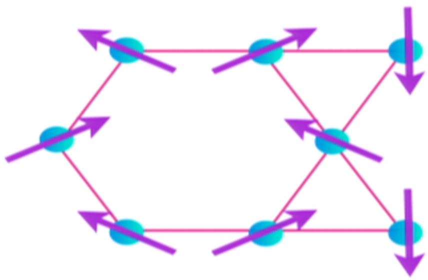
where P is the surface area (perimeter) of the boundary between A and B.

Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Kagome antiferromagnet

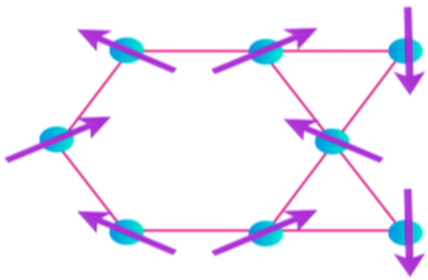


non-collinear Néel state

Quantum "disordered" state with exponentially decaying spin correlations.

s_c s

Kagome antiferromagnet: Z_2 spin liquid



non-collinear Néel state

Entangled quantum state:
A stable "$\mathbb{Z}_2$ spin liquid".
The excitations carry 'electric' and 'magnetic' charges of an emergent $\mathbb{Z}_2$ gauge field.

s_c s

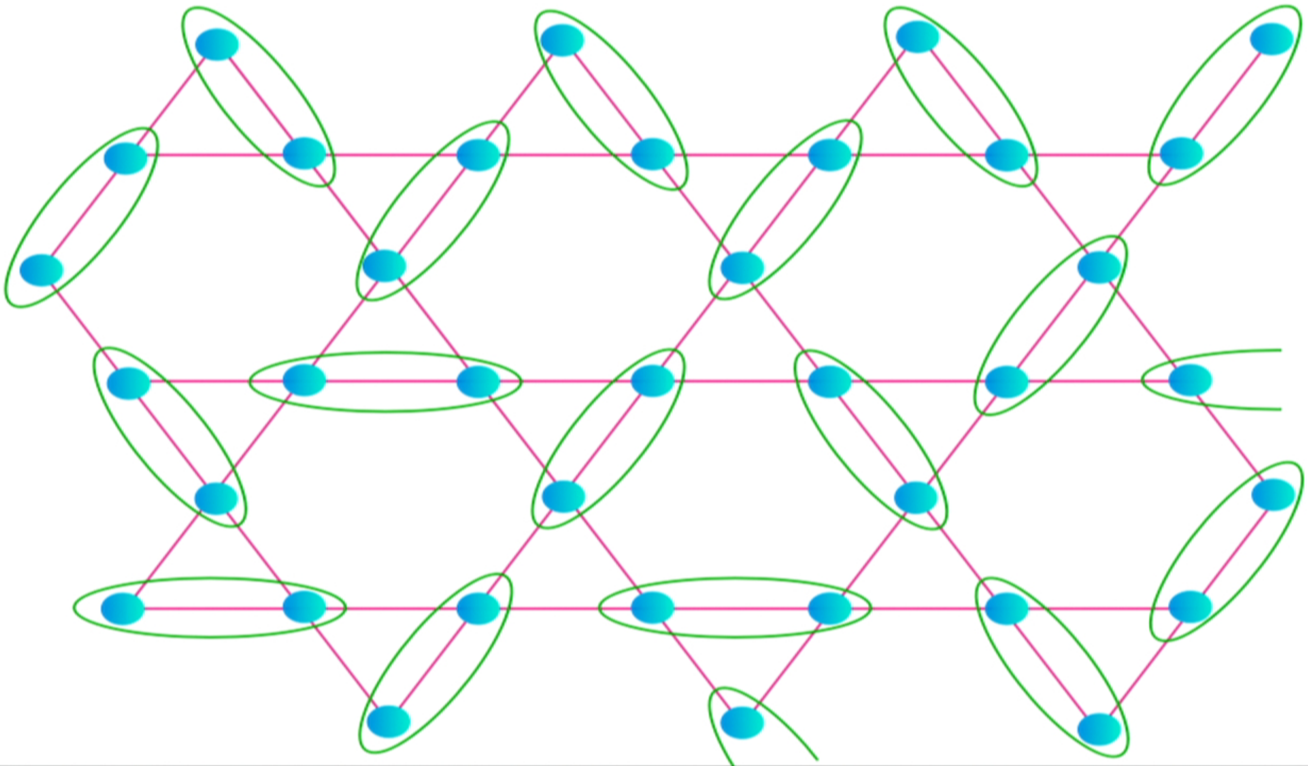
S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)
Y. Huh, M. Punk, and S. Sachdev, *Phys. Rev.* B **84**, 094419 (2011)

The Z_2 spin liquid was introduced in
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991),
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Mott insulator: Kagome antiferromagnet

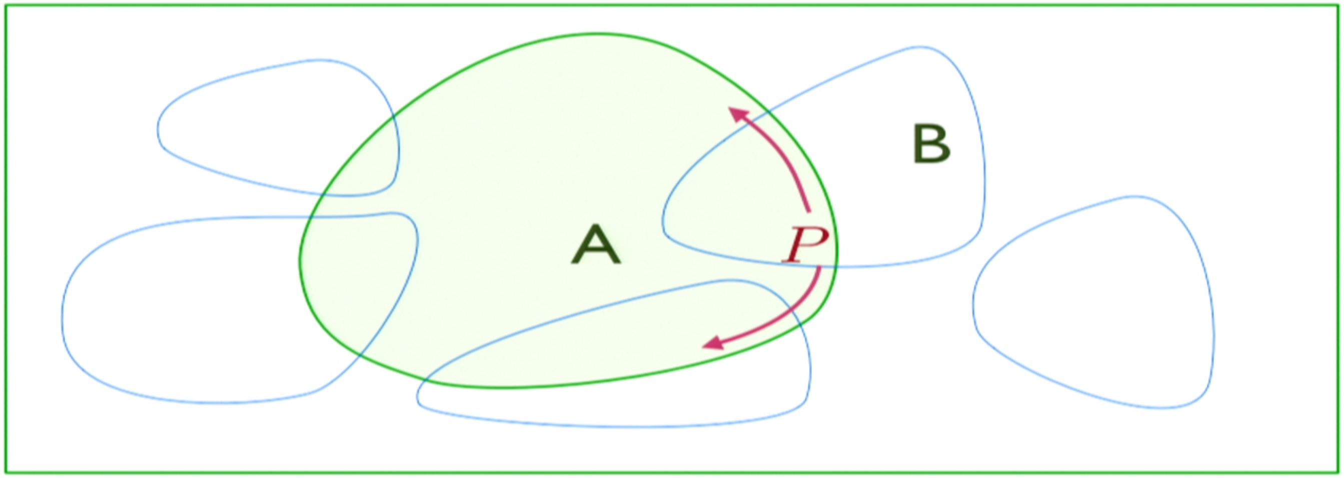
Alternative view

Pick a reference configuration



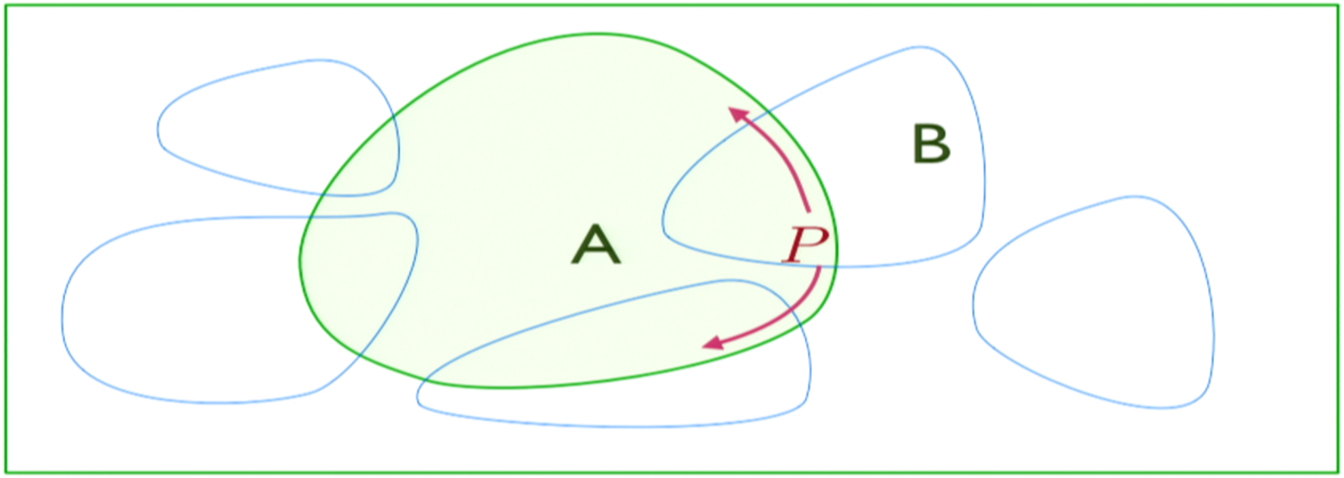
D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Entanglement in the Z_2 spin liquid



Sum over closed loops: only an even number of links cross the boundary between A and B

Entanglement in the Z_2 spin liquid

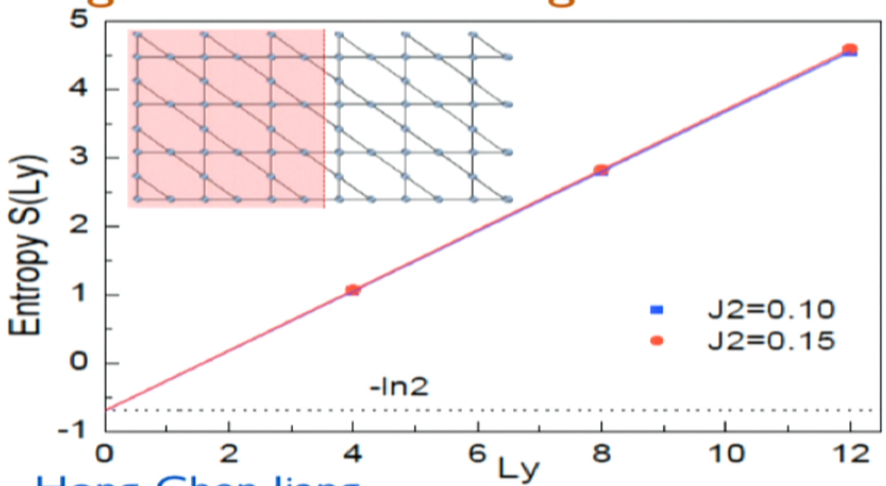


$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

A. Hamma, R. Ionicioiu, and P. Zanardi, Phys. Rev. A **71**, 022315 (2005)
M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006); A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006)
Y. Zhang, T. Grover, and A. Vishwanath, Phys. Rev. B **84**, 075128 (2011)

Kagome antiferromagnet

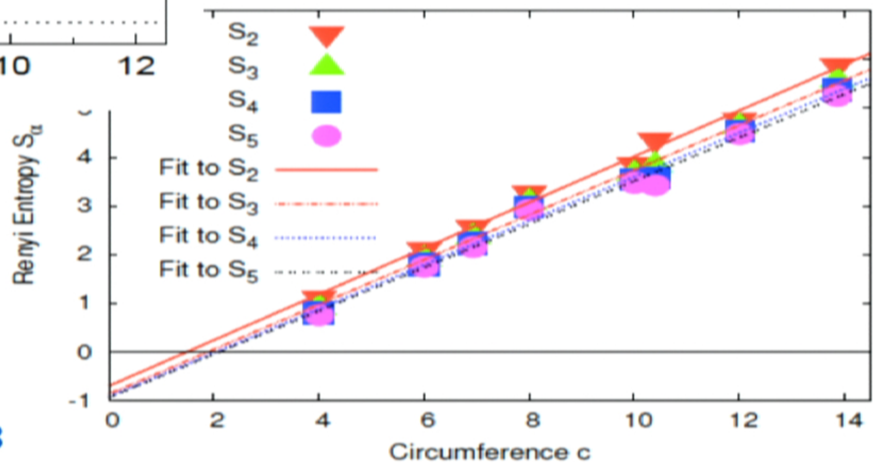


Strong numerical evidence for a Z_2 spin liquid

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Hong-Chen Jiang, Z. Wang, and L. Balents, arXiv:1205.4289

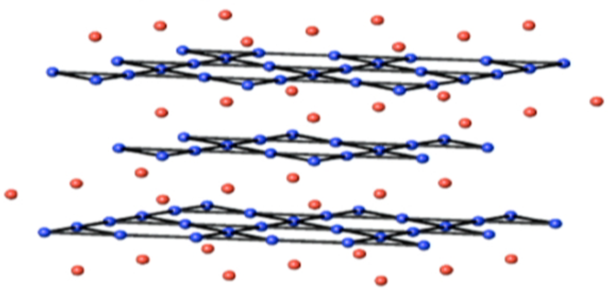
S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, arXiv:1205.4858

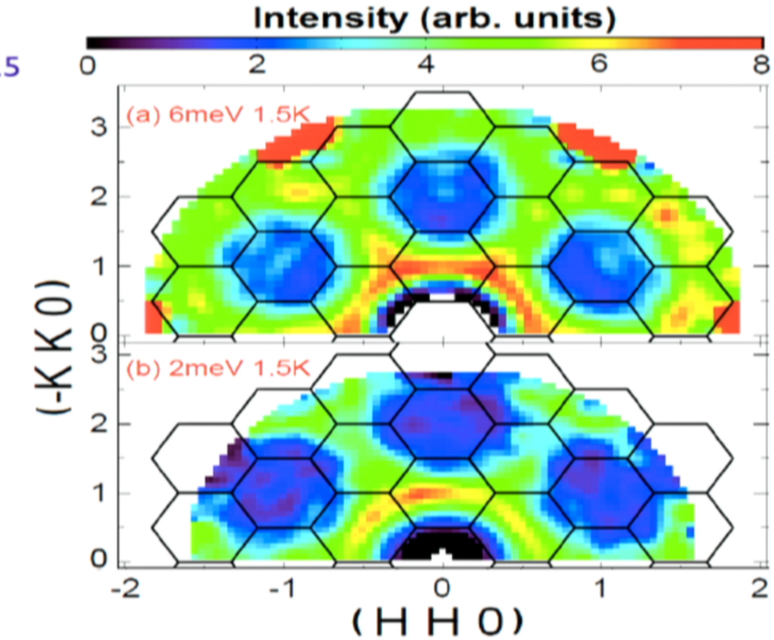


Kagome antiferromagnet

Evidence for spinons
Young Lee,
APS meeting, March 2012
http://meetings.aps.org/link/BAPS.2012.MAR.H8.5

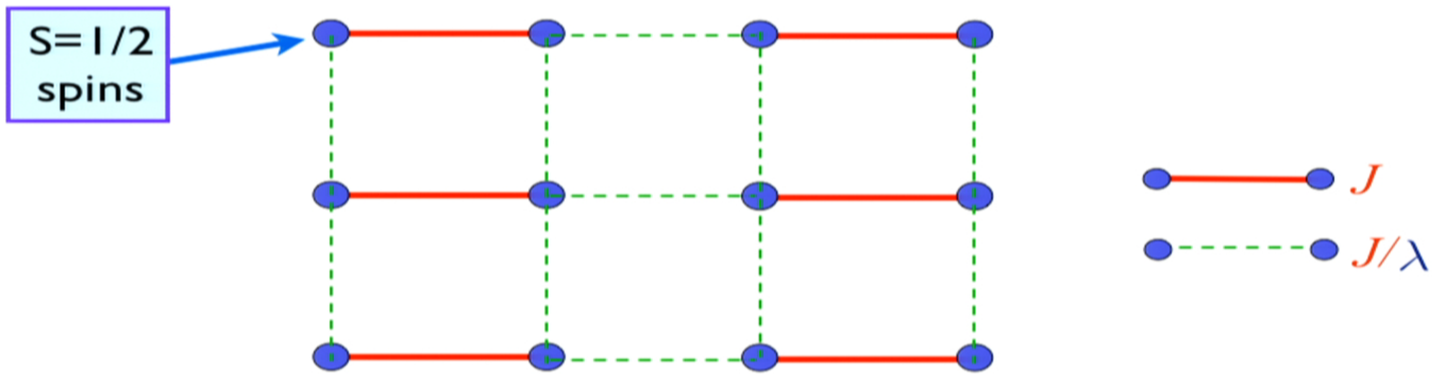
$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





Coupled dimer antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

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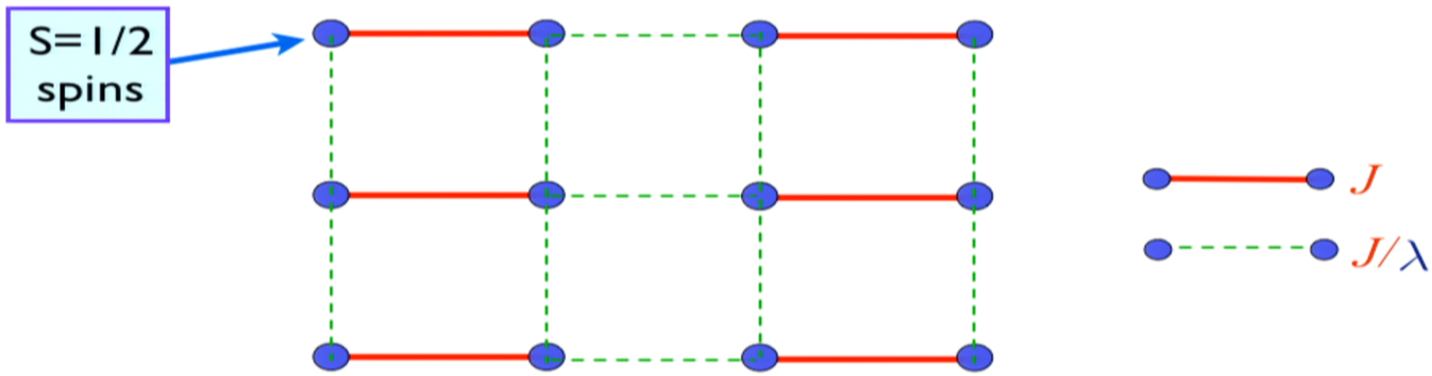

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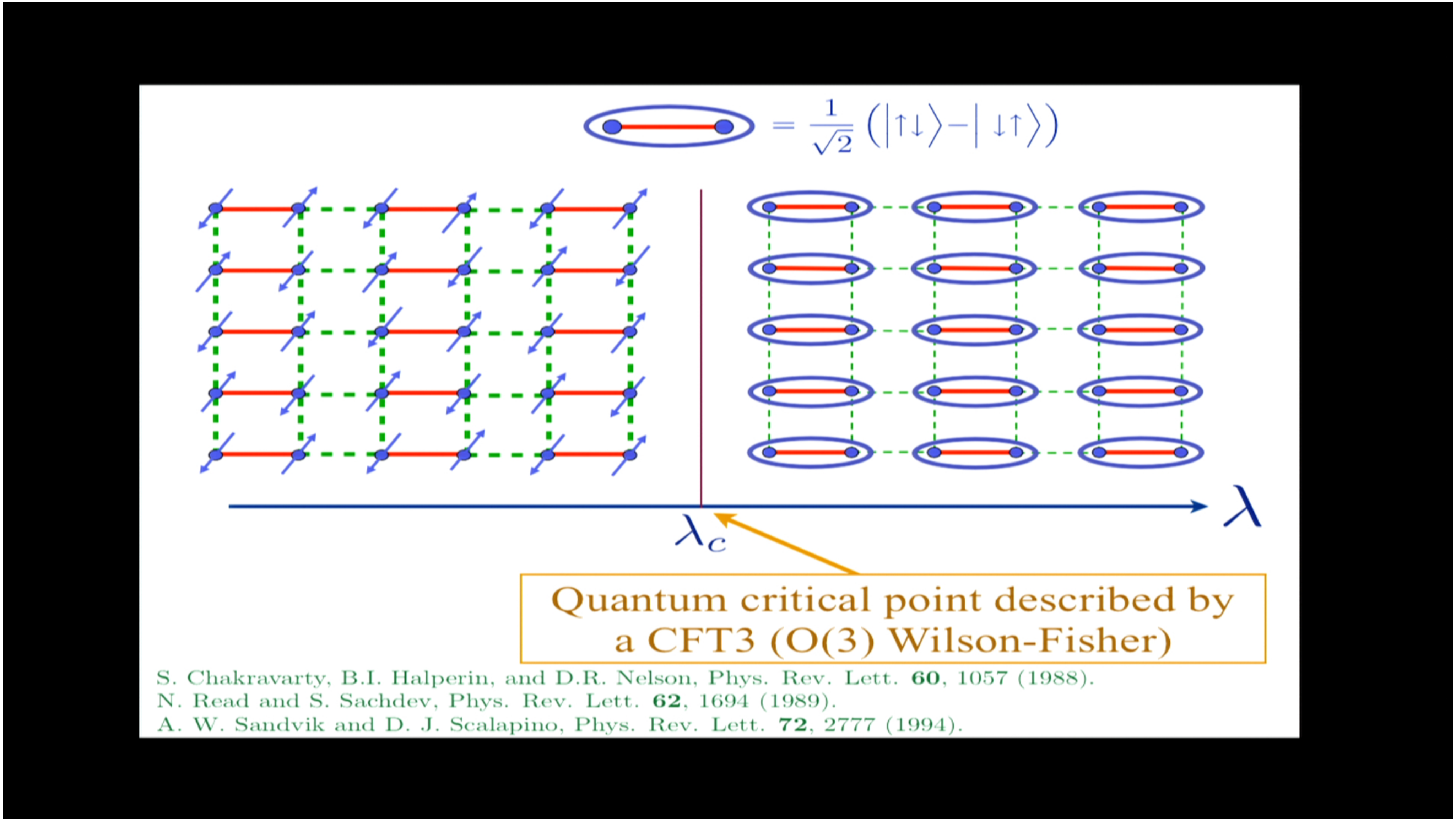
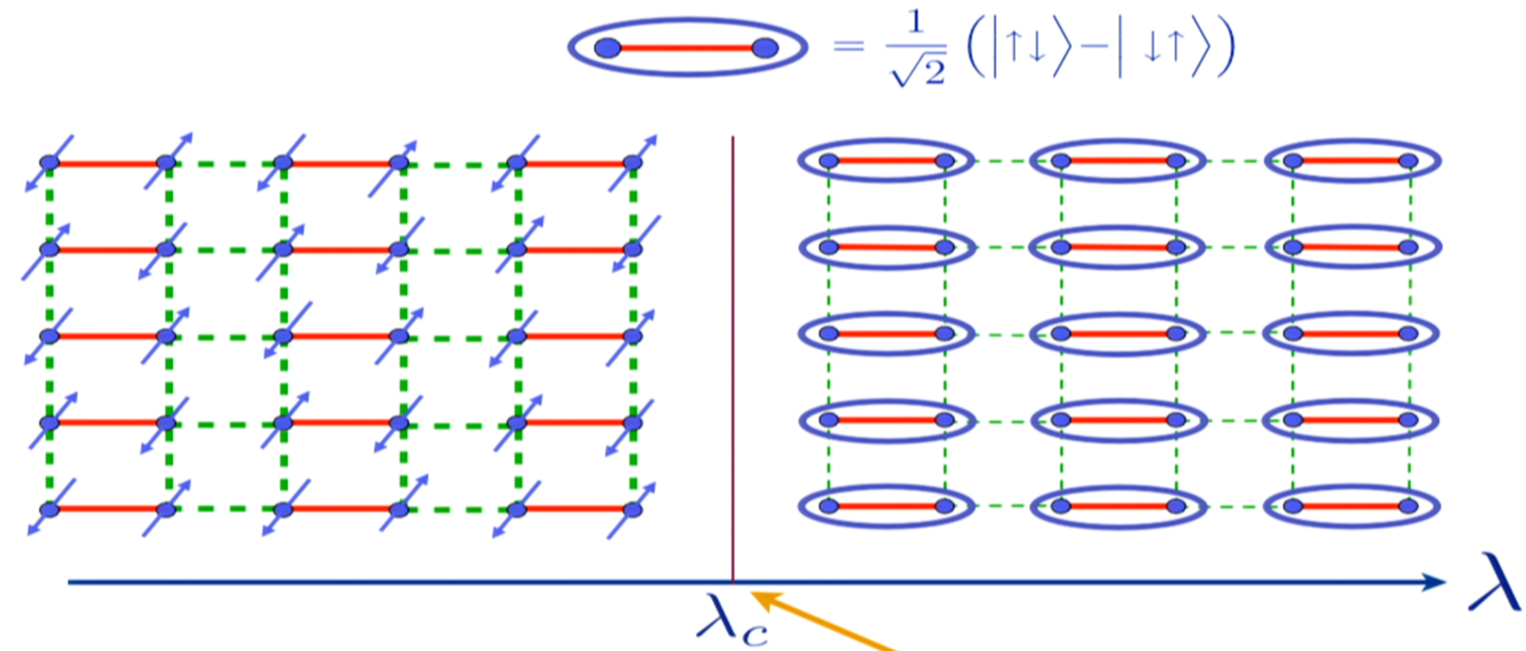


$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$
λ_c
λ
Quantum critical point described by a CFT3 (O(3) Wilson-Fisher)
S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. Lett. **60**, 1057 (1988).
N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).
A. W. Sandvik and D. J. Scalapino, Phys. Rev. Lett. **72**, 2777 (1994).



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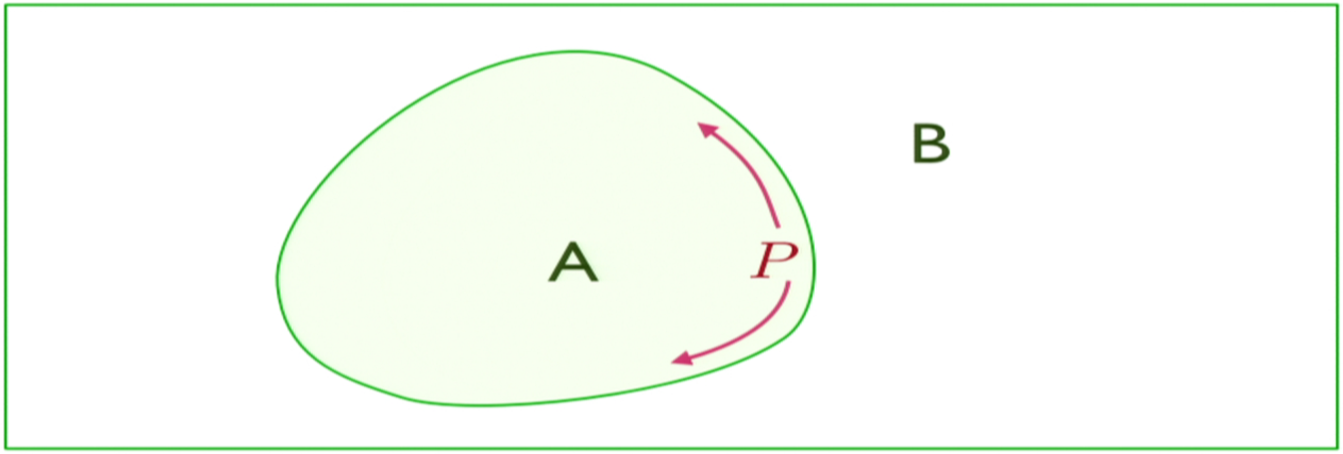
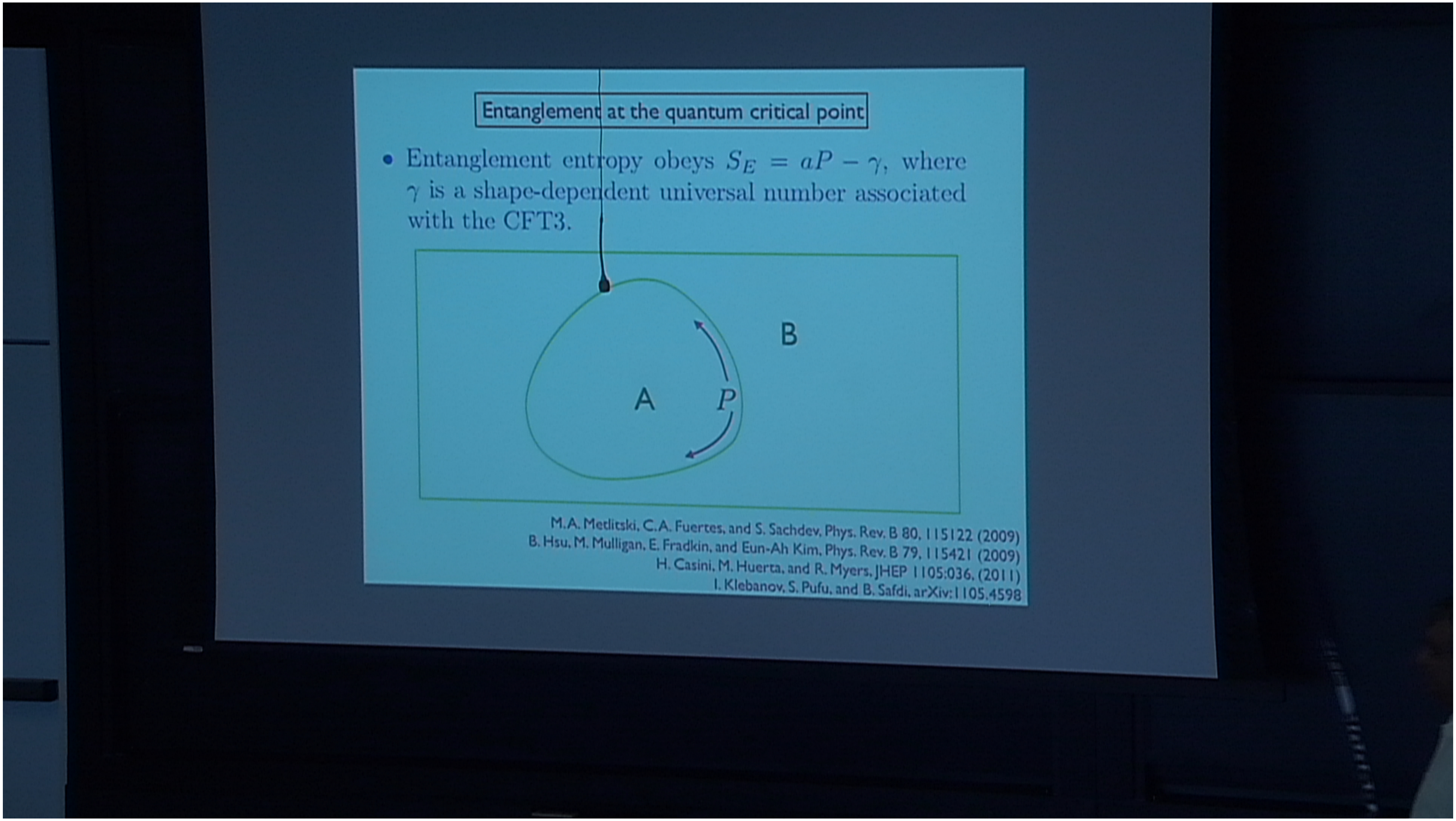
Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.



M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Phys. Rev. B 80, 115122 (2009)
B. Hsu, M. Mulligan, E. Fradkin, and Eun-Ah Kim, Phys. Rev. B 79, 115421 (2009)
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598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B
A
P
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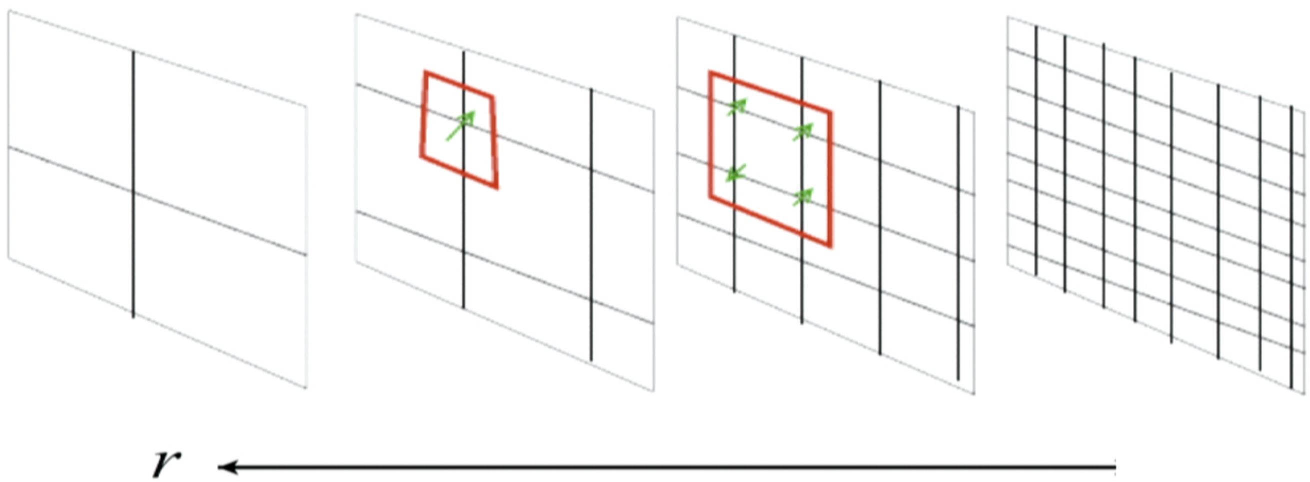
Holography



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \rightarrow \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

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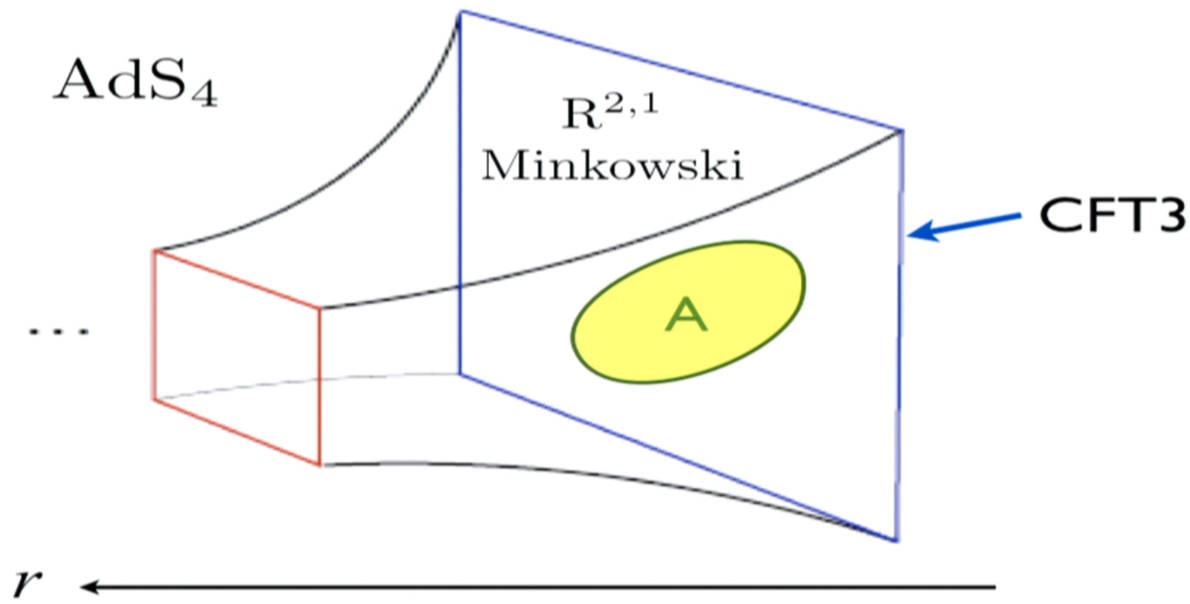
AdS/CFT correspondence



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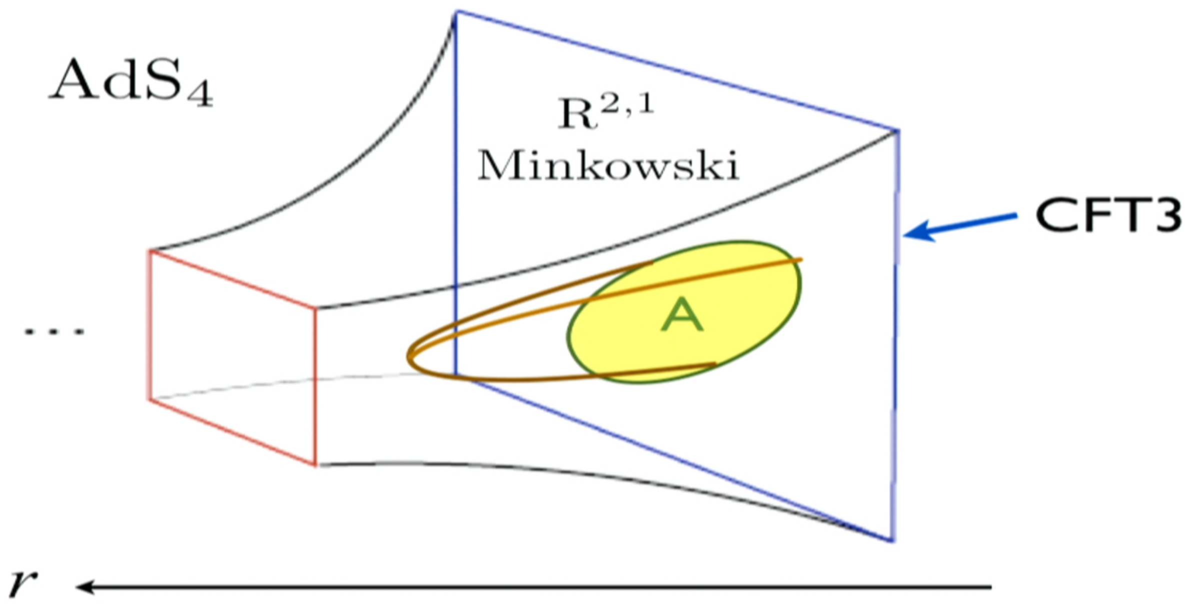
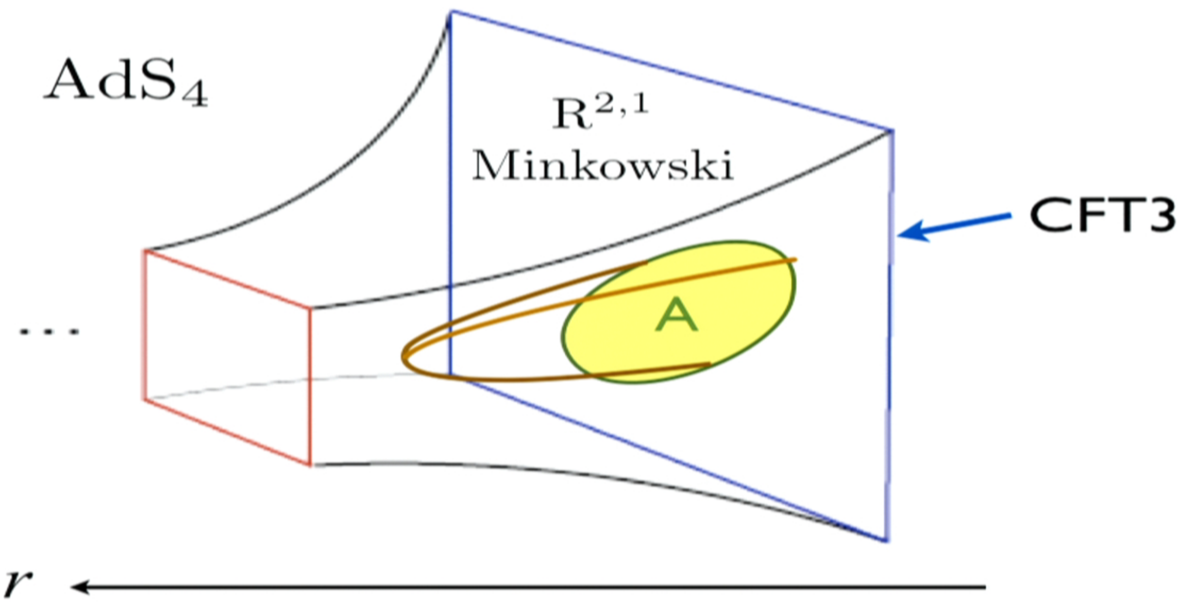


Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

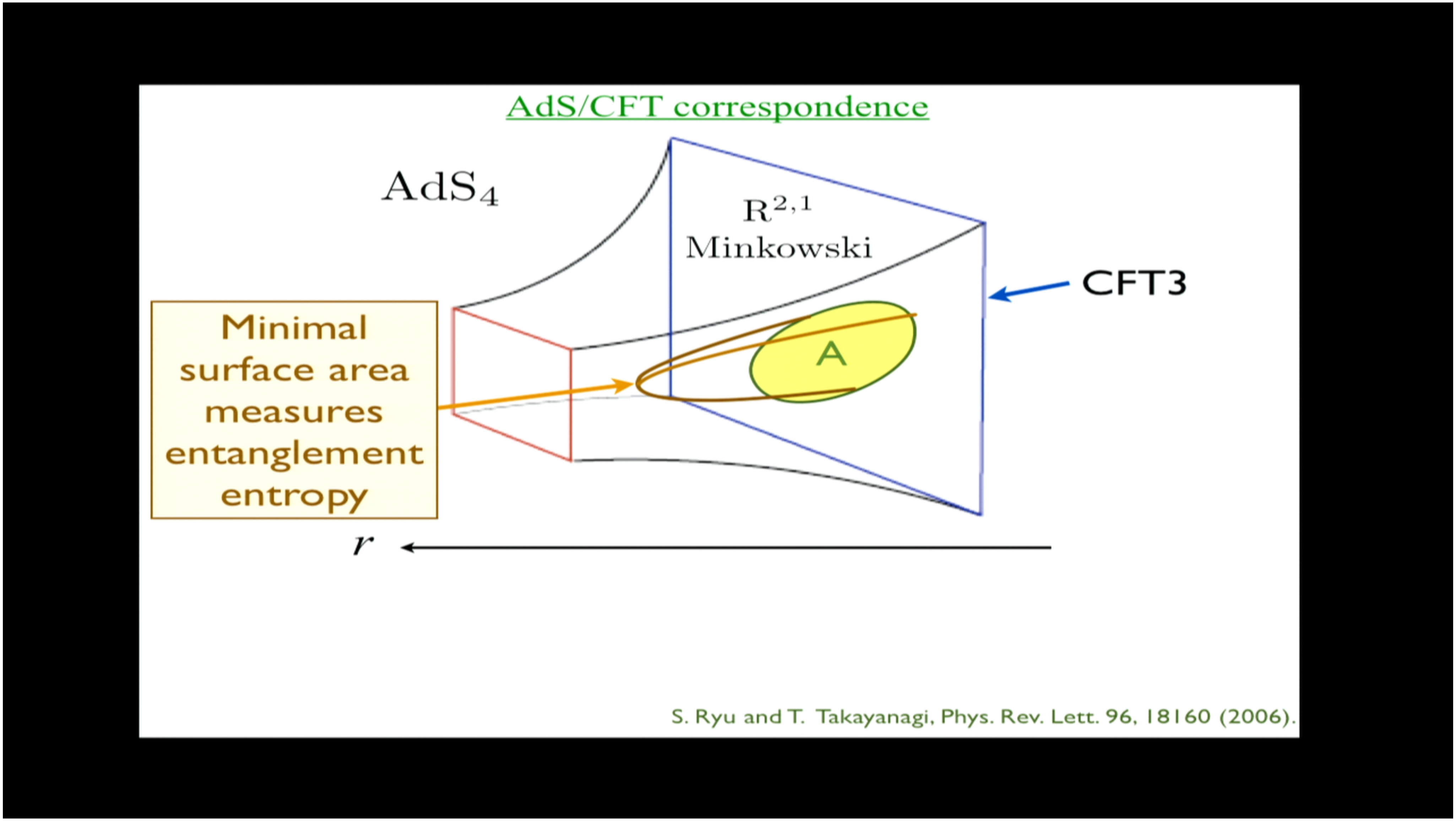
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence

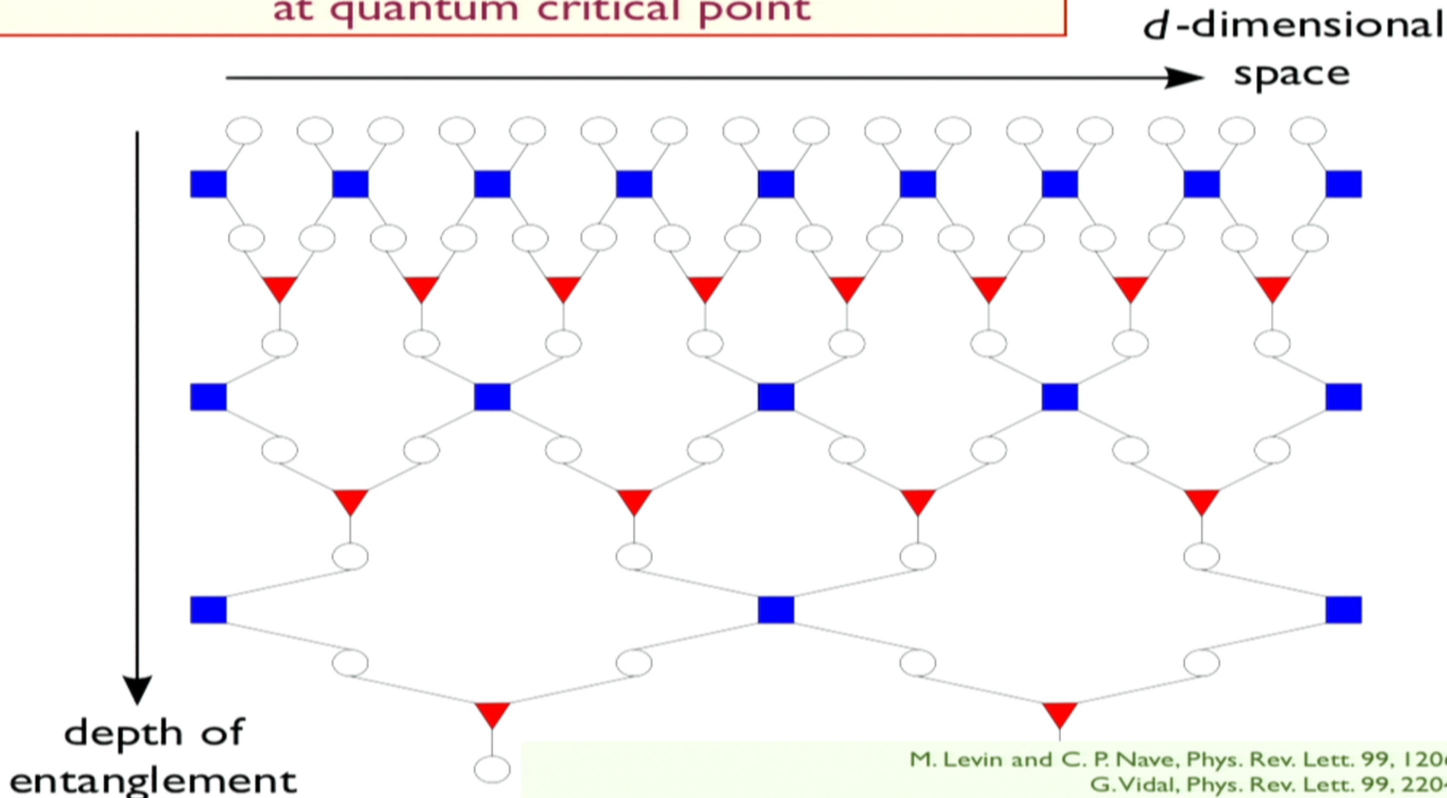


The entropy of this region is bounded by its surface area
(Bekenstein-Hawking-'t Hooft-Susskind)

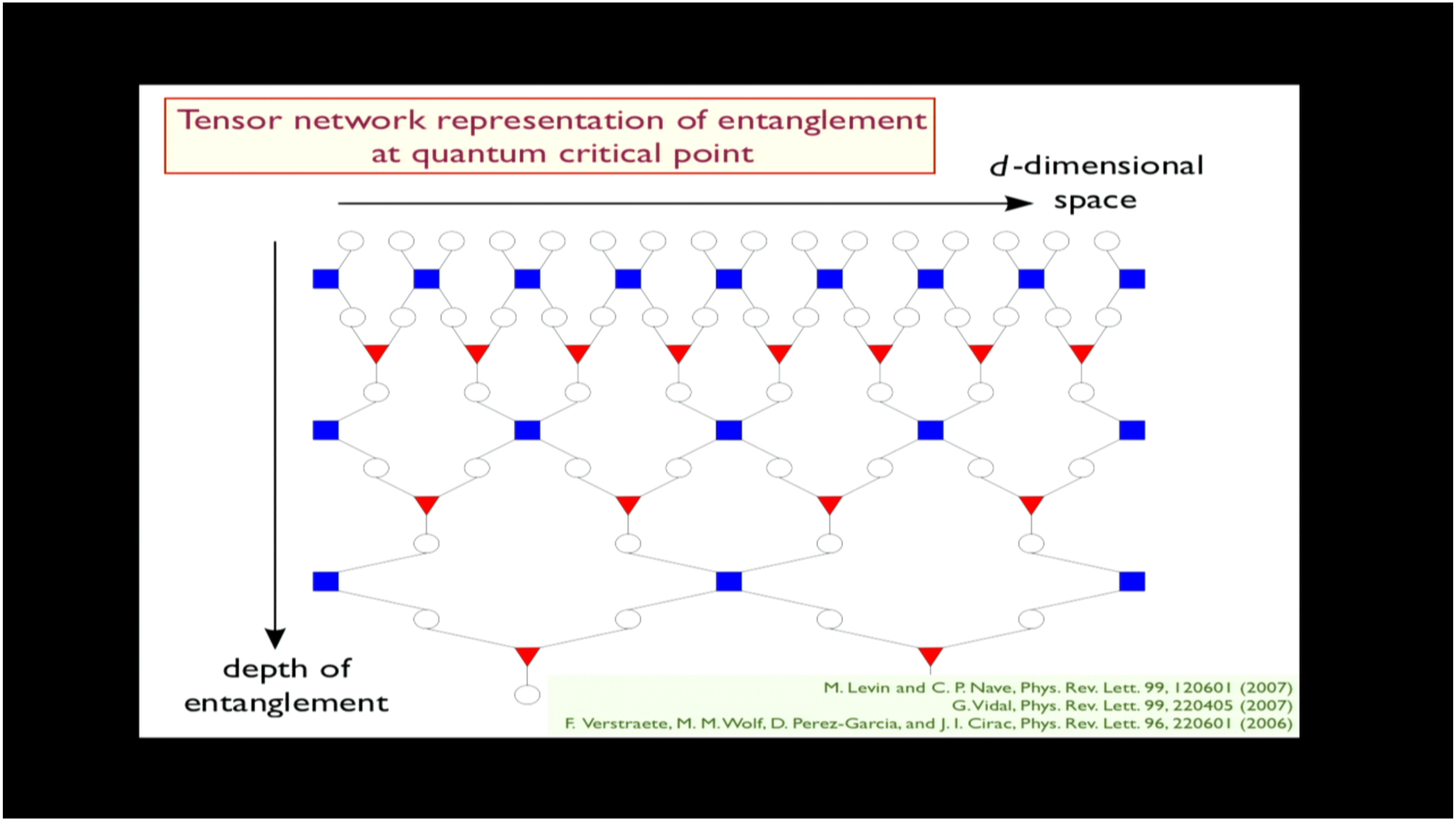
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).


AdS/CFT correspondence
AdS$_4$
$R^{2,1}$
Minkowski
CFT3
Minimal surface area measures entanglement entropy
A
r
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

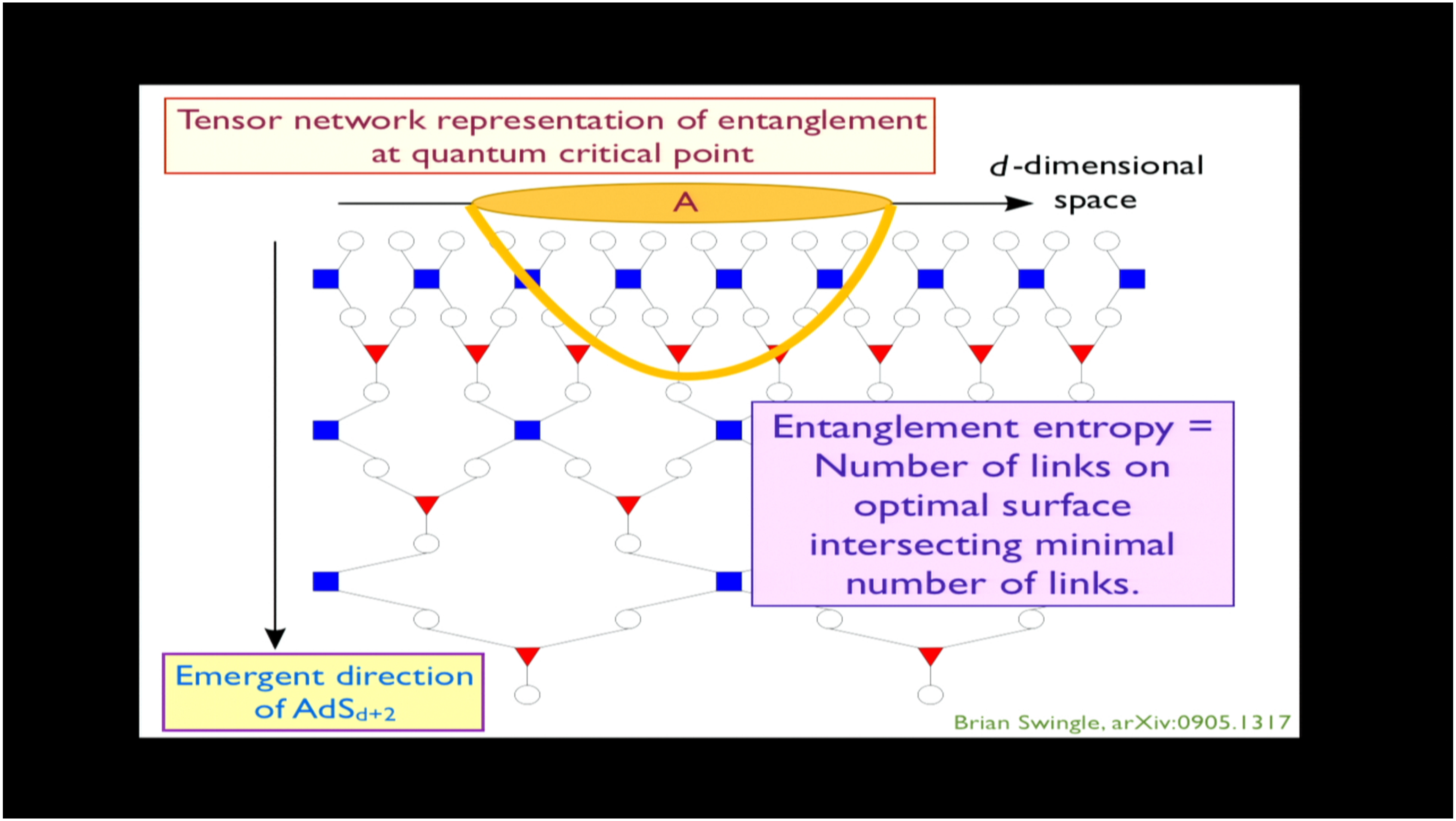
Tensor network representation of entanglement at quantum critical point

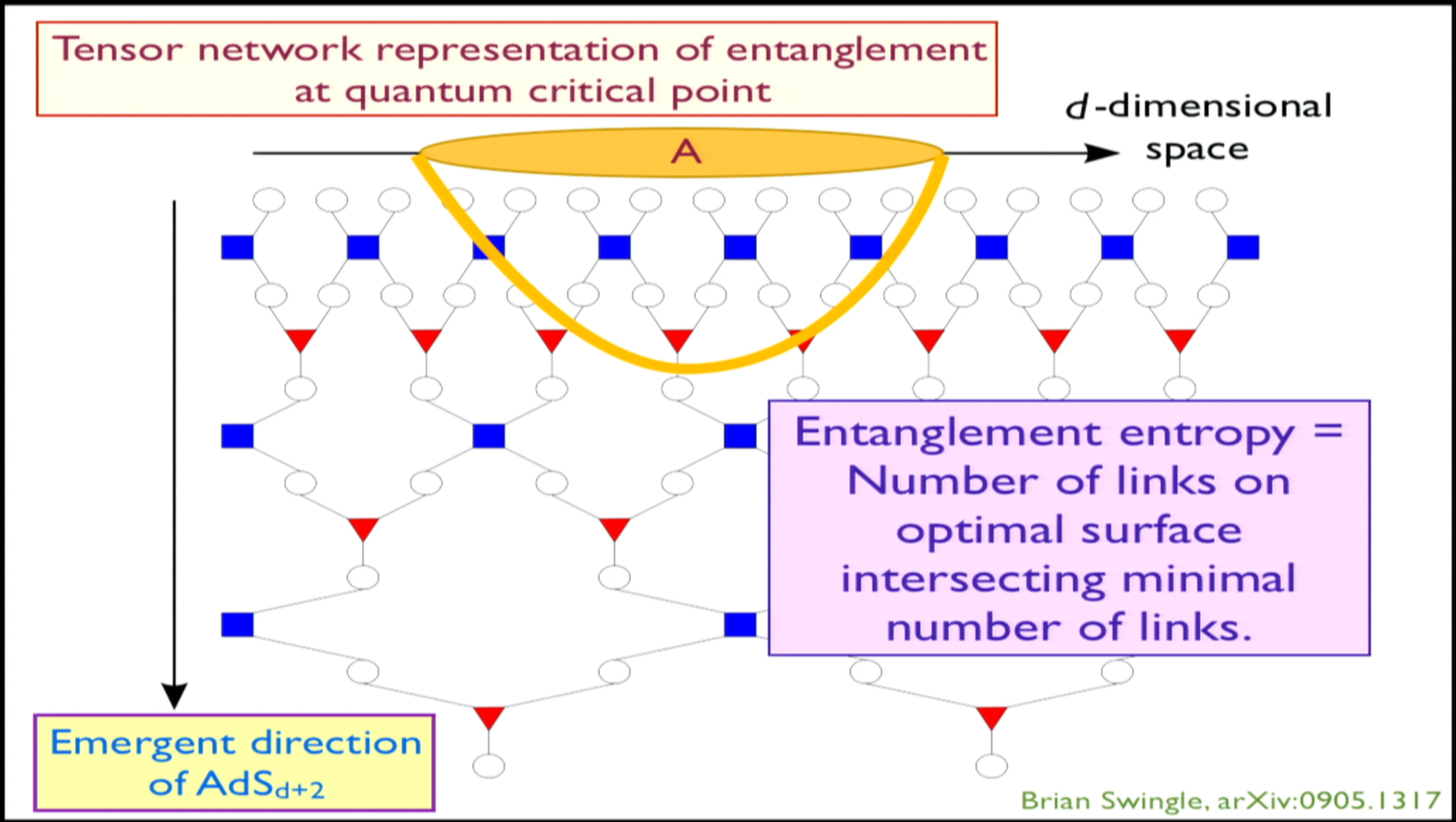


M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)



Tensor network representation of entanglement at quantum critical point
d-dimensional space
depth of entanglement
M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)


Tensor network representation of entanglement at quantum critical point
d-dimensional space
A
Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.
Emergent direction of AdSd+2
Brian Swingle, arXiv:0905.1317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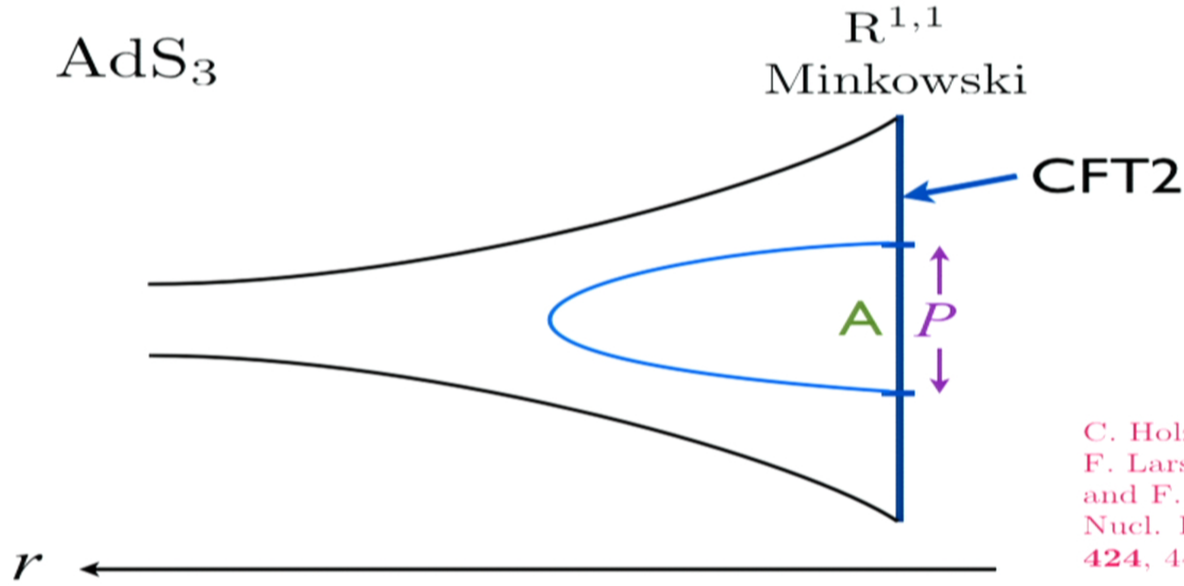

C. Holzhey, F. Larsen and F. Wilczek, Nucl. Phys. B **424**, 443 (1994).

- Computation of minimal surface area, or direct computation in CFT2, yield $S_E = (c/6)\ln P$, where c is the central charge.

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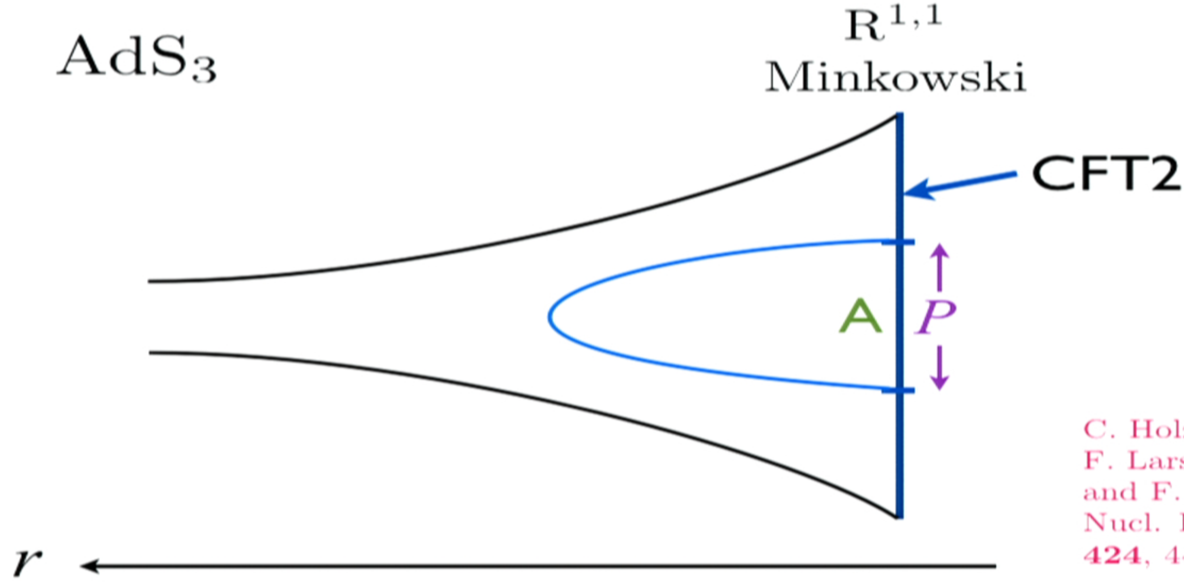

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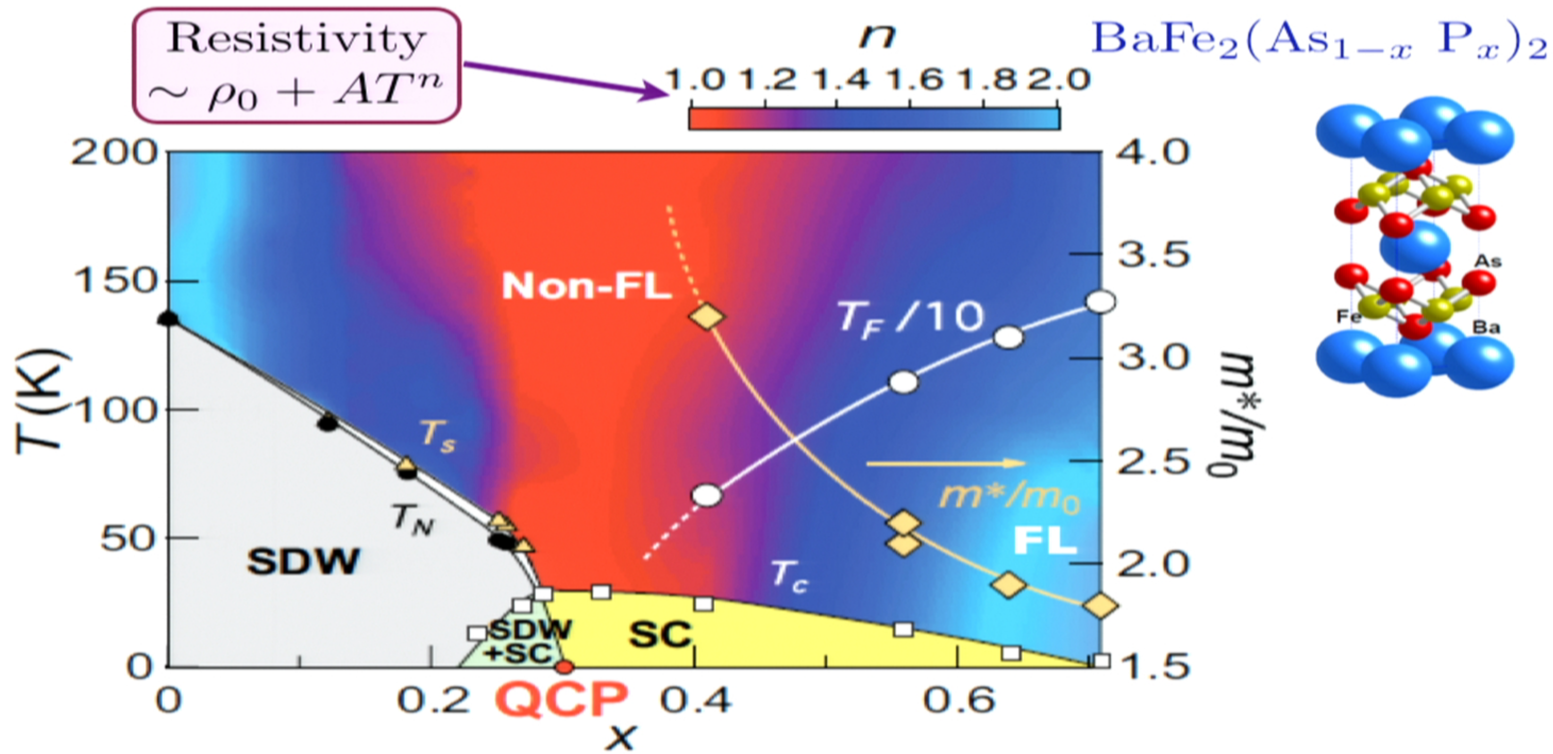
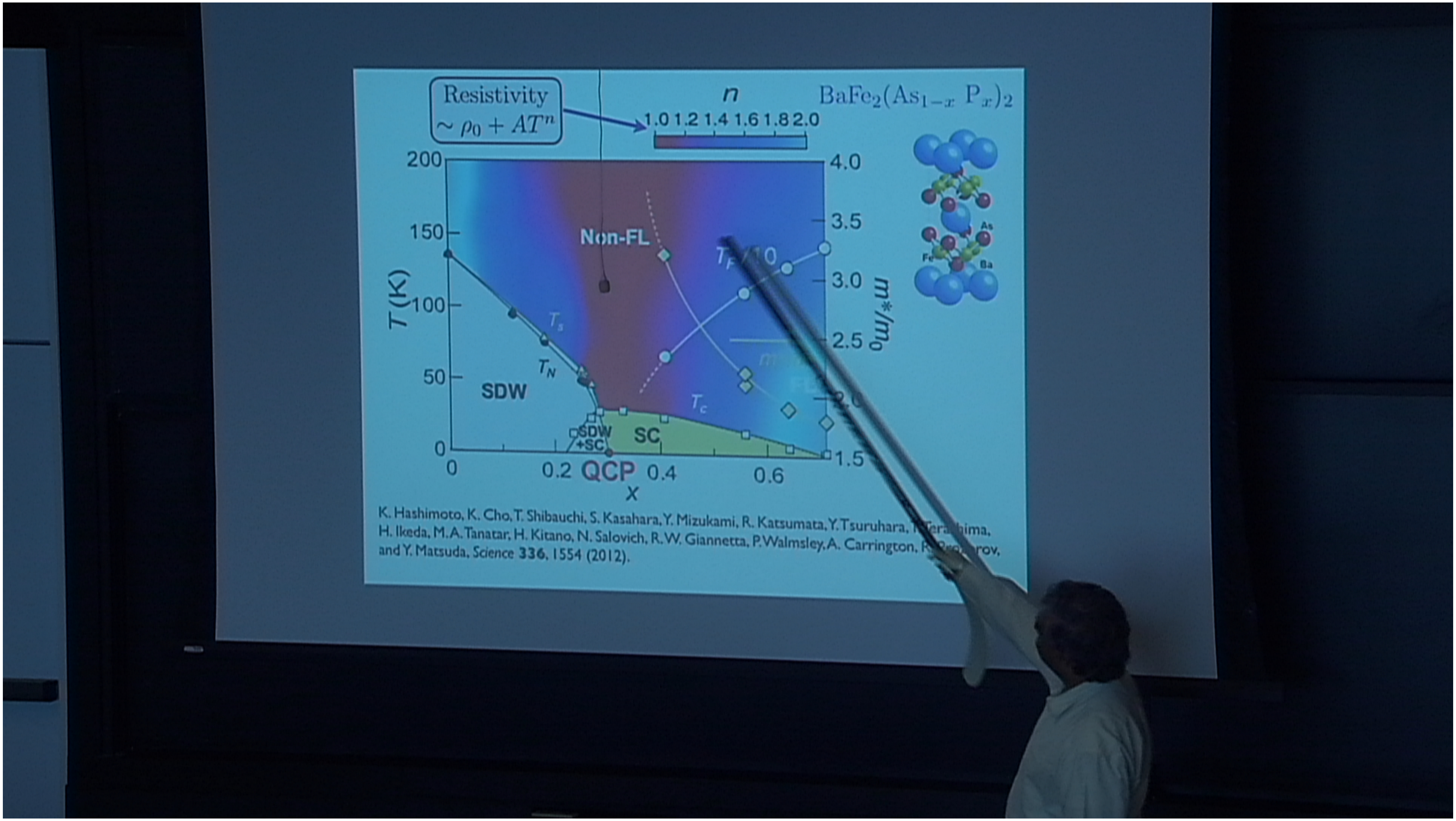


K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).


Resistivity
~ ρ0 + AT^n
n
1.0 1.2 1.4 1.6 1.8 2.0
BaFe2(As1−x Px)2
Non-FL
T (K)
m*/m0
SDW
SC
SDW +SC
QCP
x
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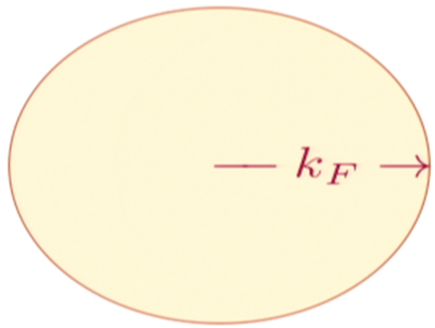
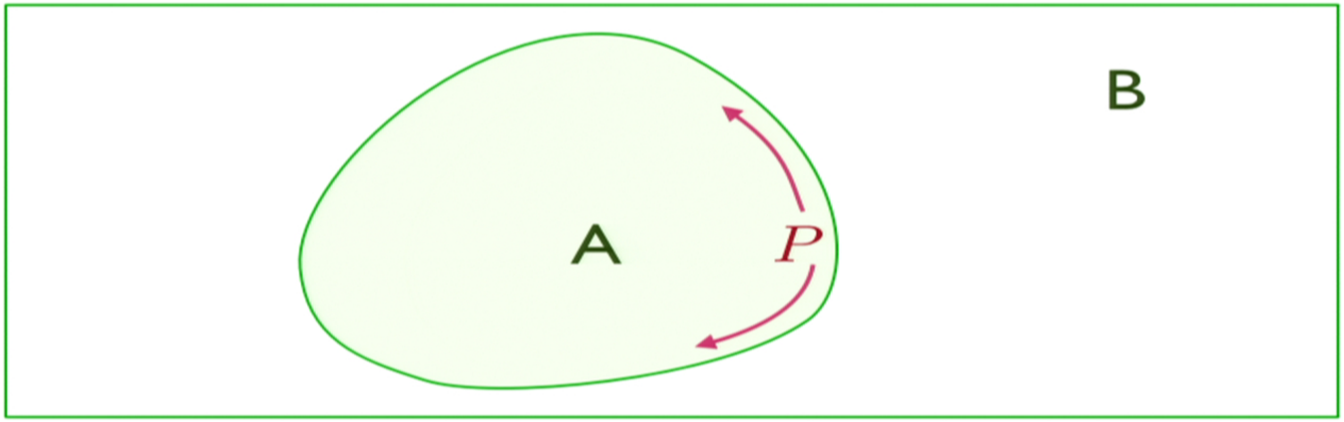
The Fermi liquid



- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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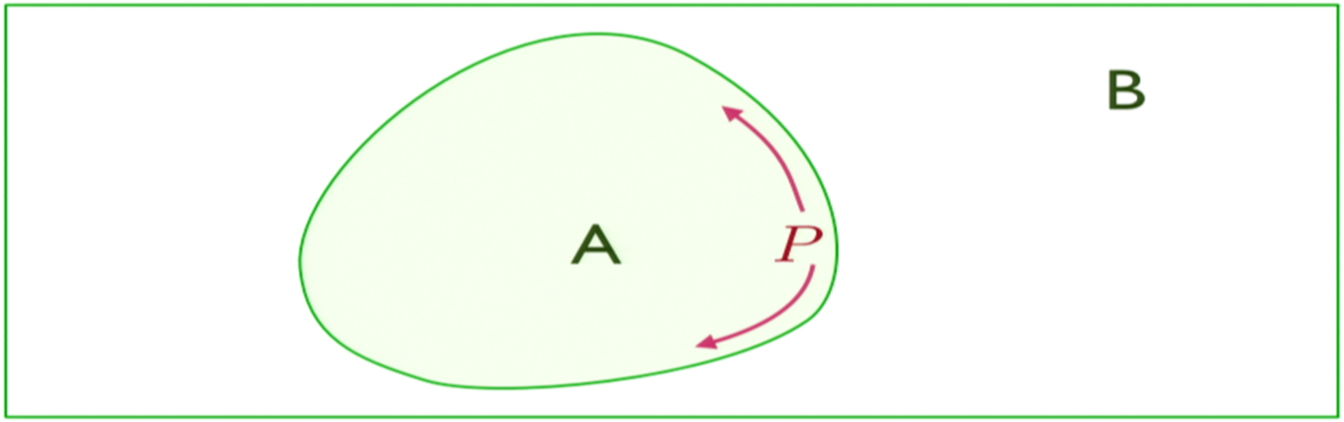

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Consider a model of interacting bosons, b, whose density is $\mathcal{Q} = b^\dagger b$ is conserved. We want a ground state which does not break any symmetries (and so solids and superfluids are excluded). The only known possibilities are:

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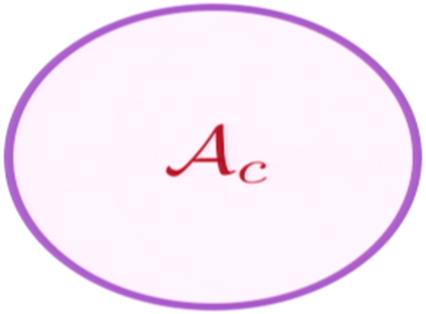
- **FL:** The boson forms a bound state, c, with some spectator fermions, and there is a Fermi liquid of the fermionic c molecules.



$$\mathcal{A}_c = \langle \mathcal{Q} \rangle$$

S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)
P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

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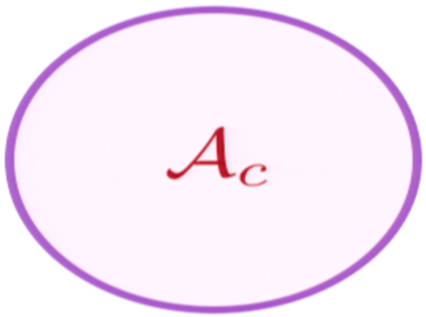

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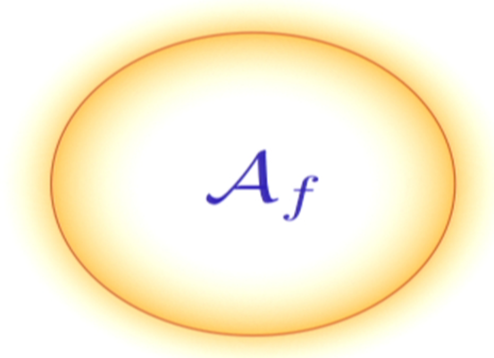
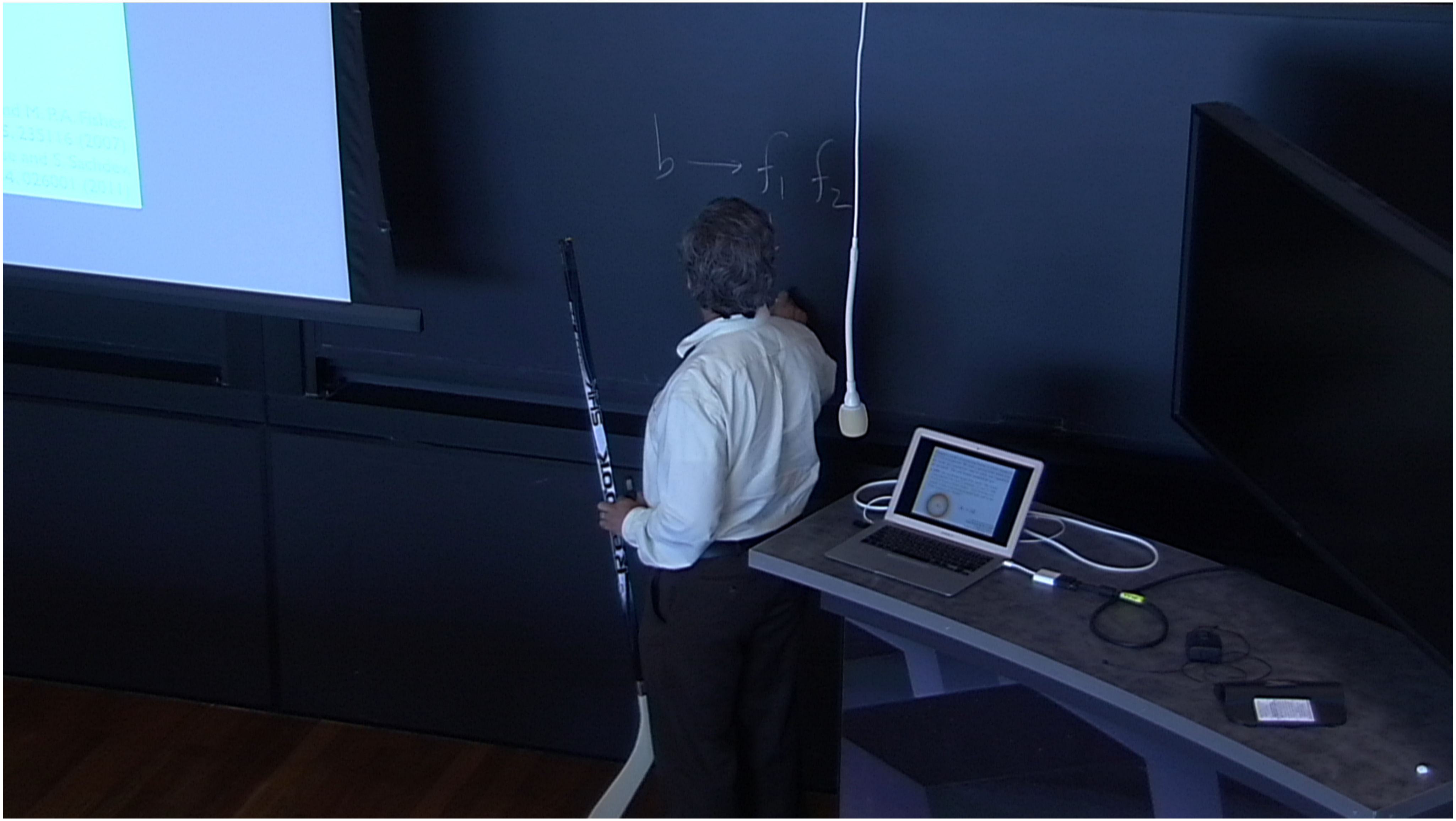
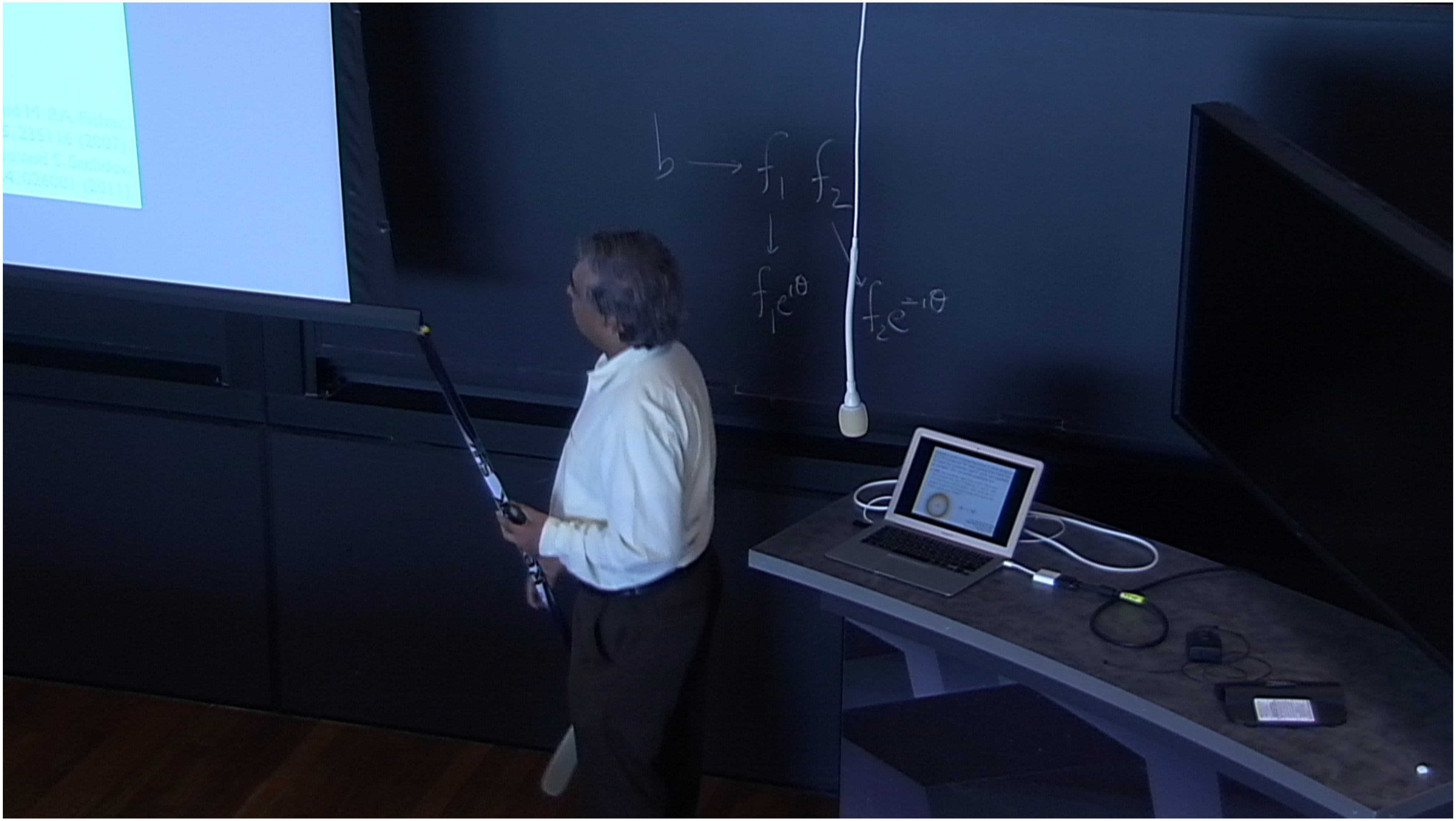
- **NFL**, the non-Fermi liquid *Bose metal*. The boson fractionalizes into (say) 2 fermions, f_1 and f_2, each of which forms a Fermi surface. Both fermions necessarily couple to an emergent gauge field, and so the Fermi surfaces are "*hidden*".



$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

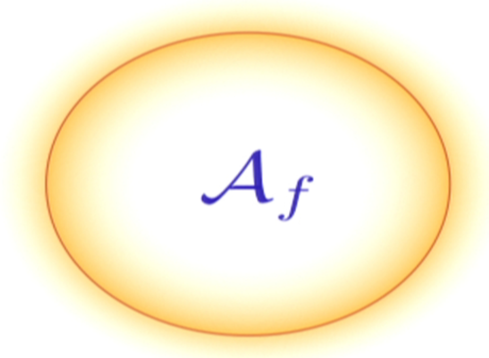
O. I. Motrunich and M. P. A. Fisher, *Physical Review* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011)


$b \to f_1 f_2$


b → f1 f2
f1 e^iθ
f2 e^-iθ

Consider a model of interacting bosons, b, whose density is $\mathcal{Q} = b^\dagger b$ is conserved. We want a ground state which does not break any symmetries (and so solids and superfluids are excluded). The only known possibilities are:

- **NFL**, the non-Fermi liquid *Bose metal*. The boson fractionalizes into (say) 2 fermions, f_1 and f_2, each of which forms a Fermi surface. Both fermions necessarily couple to an emergent gauge field, and so the Fermi surfaces are "*hidden*".



$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

O. I. Motrunich and M. P. A. Fisher, *Physical Review* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011)

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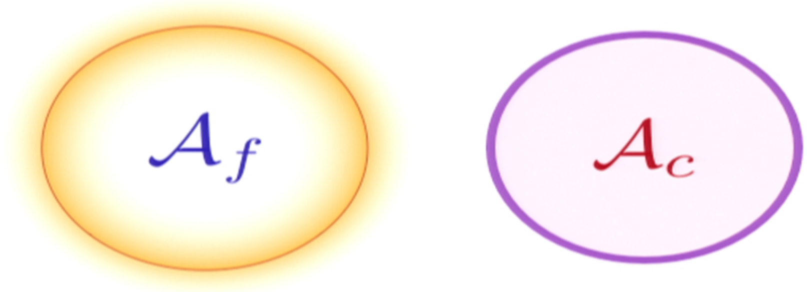
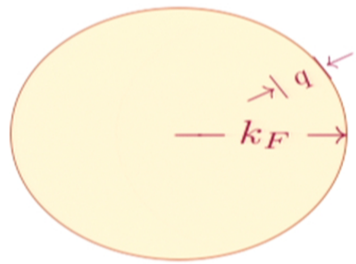
- **FL*** Partially fractionalized state, with co-existence of visible and hidden Fermi surfaces.



$$\mathcal{A}_c + \mathcal{A}_f = \langle \mathcal{Q} \rangle$$

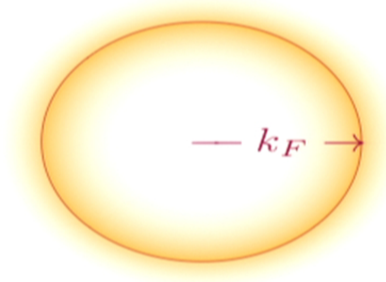
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)
L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011).

The Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

Bose metal



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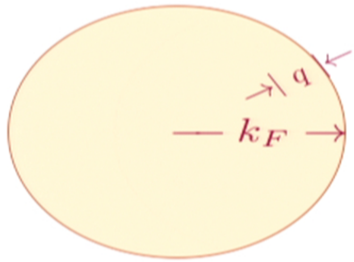

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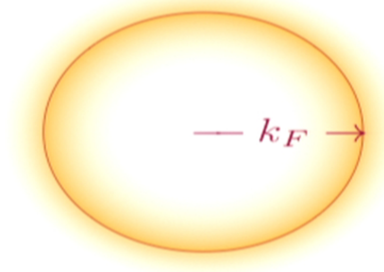

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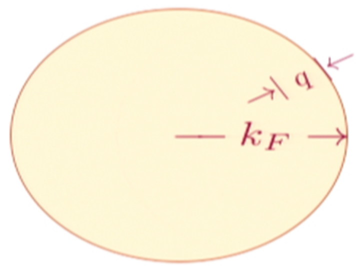

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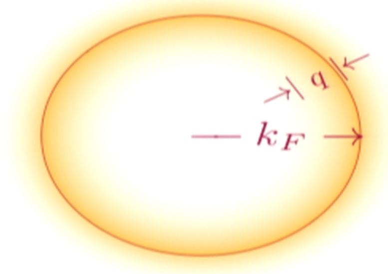
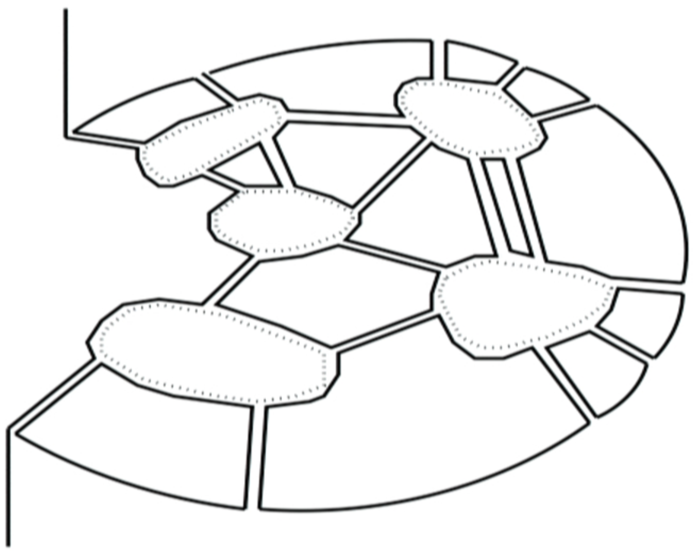
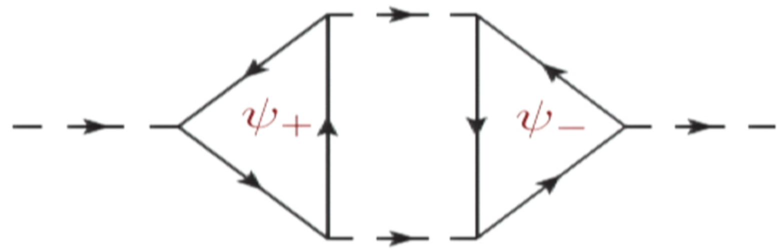


- $k_F^d \sim \mathcal{Q}$.
- Diffuse fermionic excitations with $z = 3/2$ to three loops.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Computations in the $1/N$ expansion





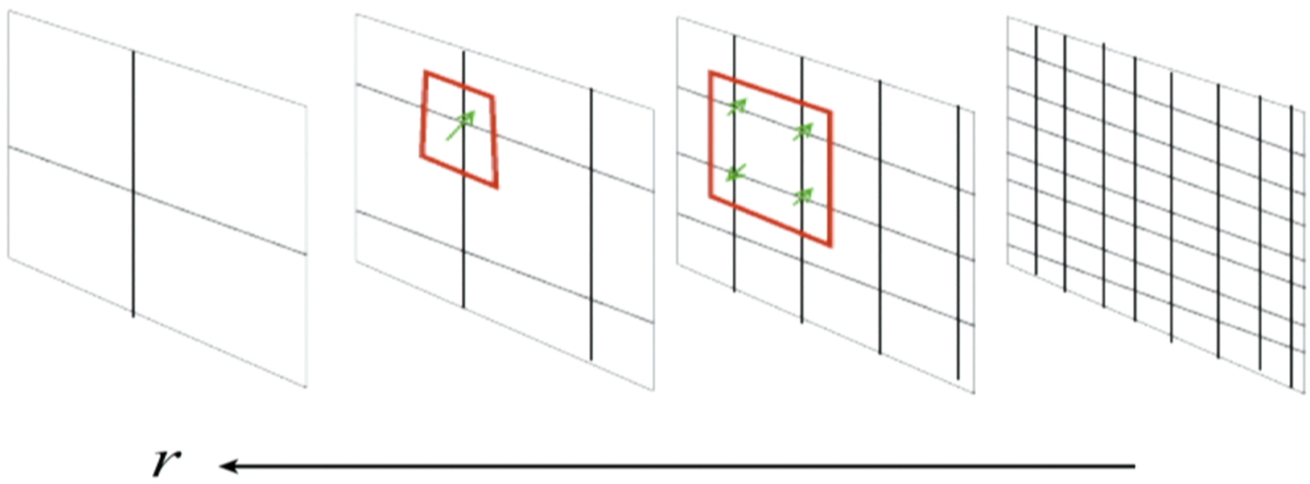
Graph mixing antipodal arcs is $\mathcal{O}\left(N^{3/2}\right)$ (instead of $\mathcal{O}\left(N\right)$), violating genus expansion

All planar graphs of fermions on an arc of the Fermi surface are as important as the leading term

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Holography



Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$
ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)
$$

We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

At $T > 0$, there is a *horizon*, and computation of its Bekenstein-Hawking entropy shows

$$S \sim T^{(d-\theta)/z}.$$

So θ is indeed the violation of hyperscaling exponent as claimed. For a compressible quantum state we should therefore *choose* $\theta = d - 1$.

No additional choices will be made, and all subsequent results are consequences of the assumption of the existence of a holographic dual.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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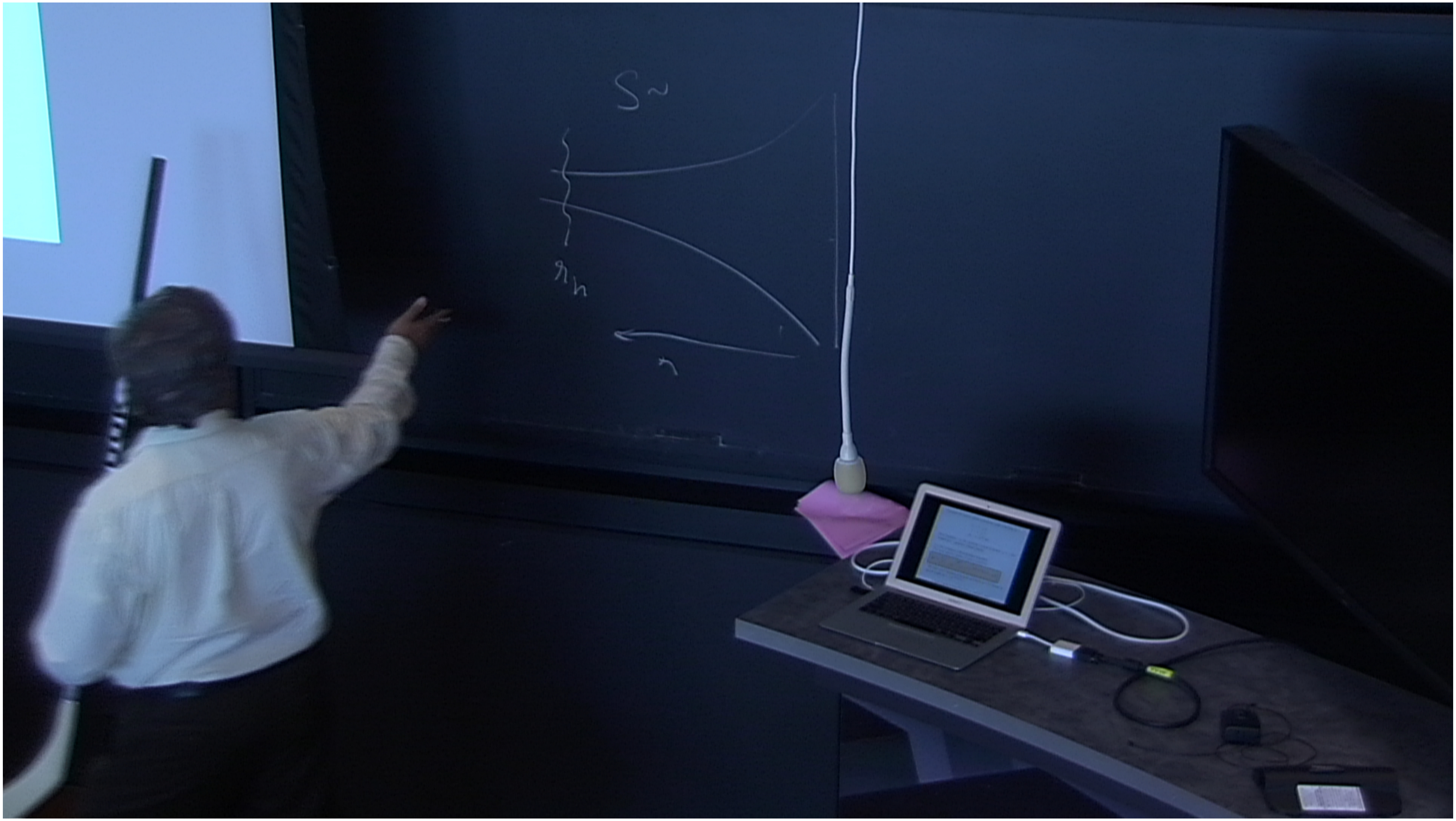

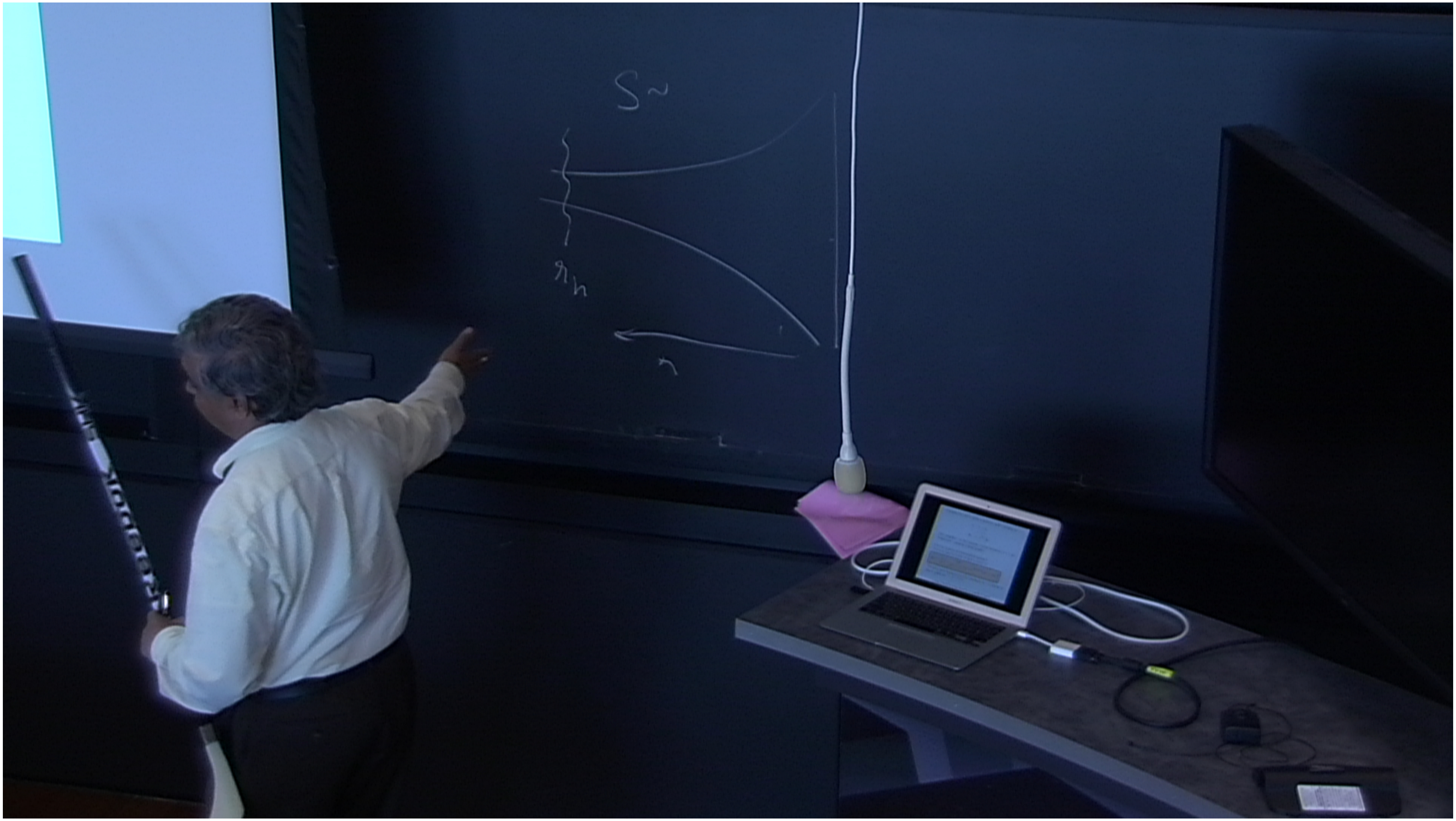

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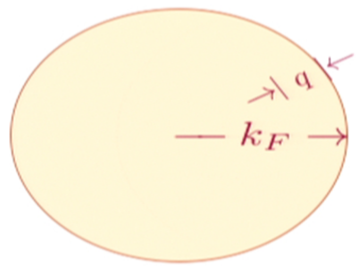
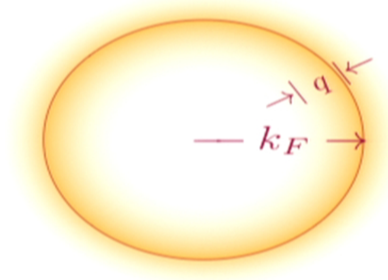
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

The Fermi liquid	Bose metal
	
• $k_F^d \sim \mathcal{Q}$, the fermion density	• $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \lvert q \rvert^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
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Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. Lett.* **107**, 067202 (2011)

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L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim \mathcal{Q}^{(d-1)/d} P \ln P$$

with a co-efficient *independent* of UV details and of the shape of the entangling region. These properties are just as expected for a circular Fermi surface with $\mathcal{Q} \sim k_F^d$.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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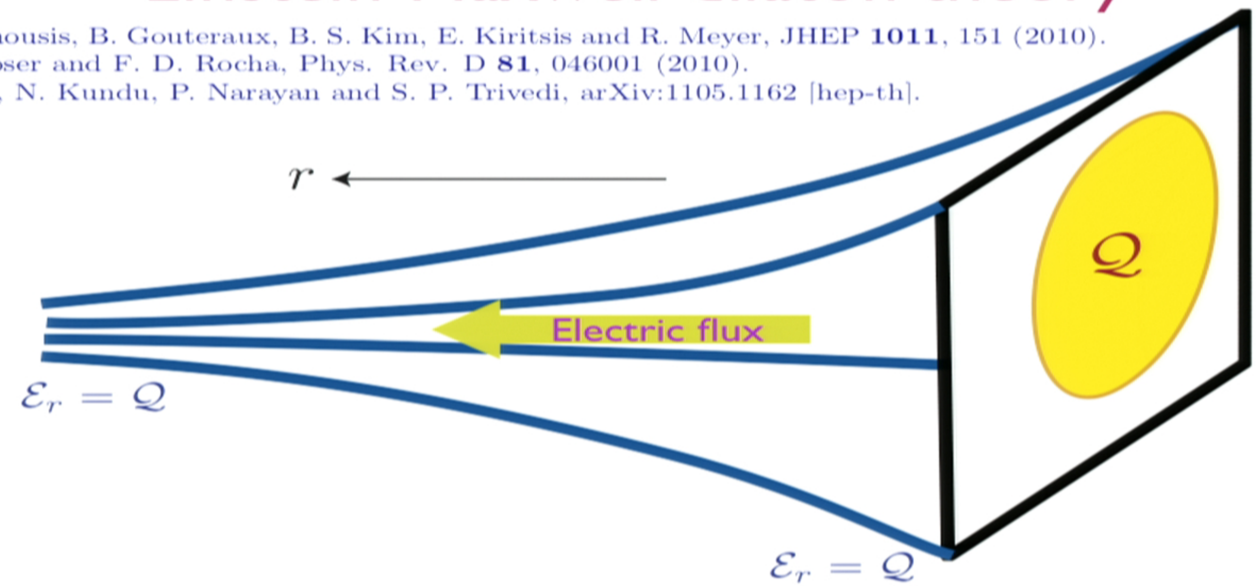
Holographic theory of a non-Fermi liquid (NFL)

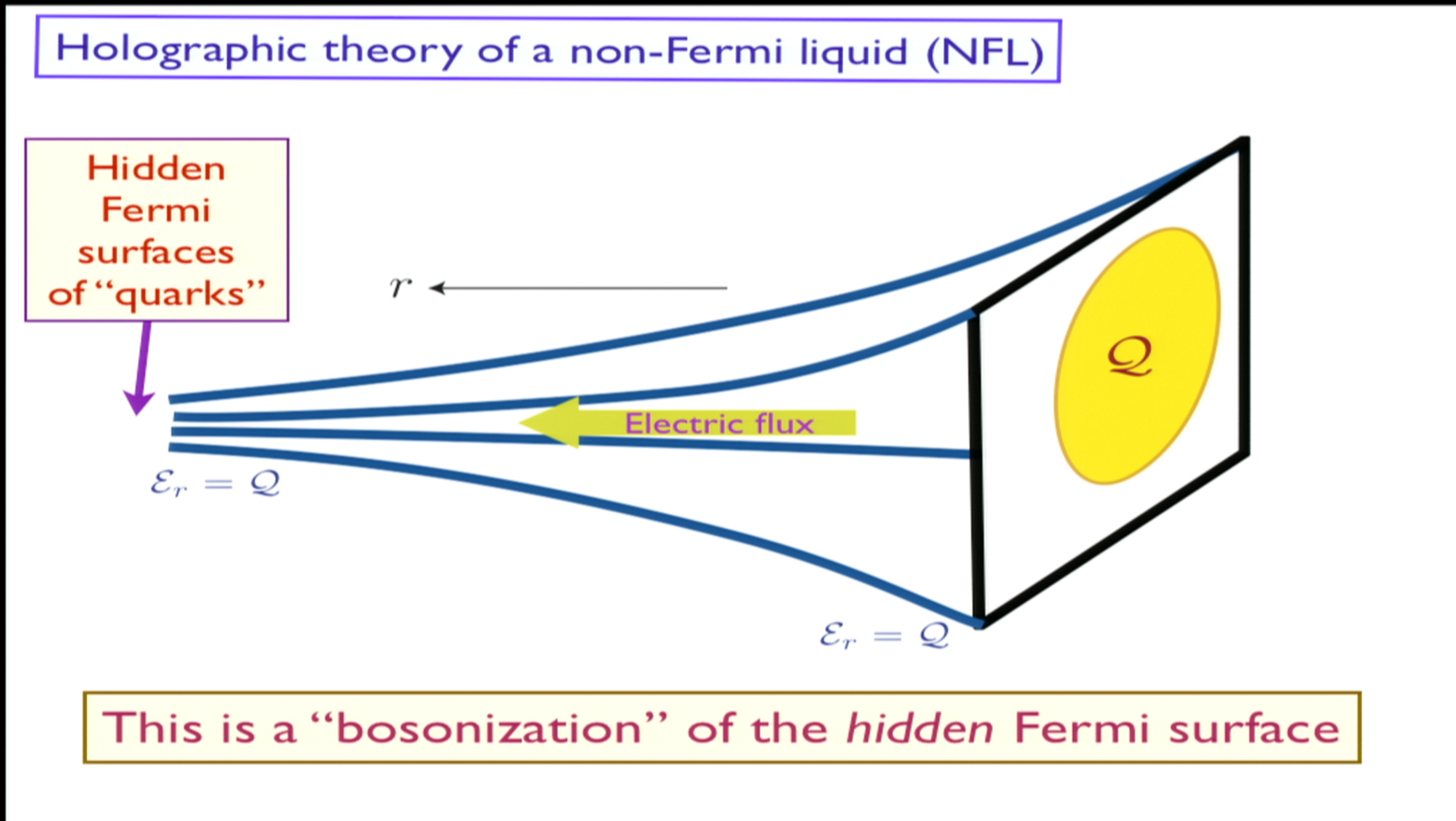
Einstein-Maxwell-dilaton theory

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].



$$\mathcal{S} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2}\right) - \frac{Z(\Phi)}{4e^2}F_{ab}F^{ab}\right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.


Holographic theory of a non-Fermi liquid (NFL)
Hidden Fermi surfaces of "quarks"
r
Electric flux
$\mathcal{Q}$
$\mathcal{E}_r = \mathcal{Q}$
$\mathcal{E}_r = \mathcal{Q}$
This is a "bosonization" of the *hidden* Fermi surface

Holography of strange metals

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L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography, fractionalization, and hidden Fermi surfaces

- Electric flux exiting the horizon corresponds to fractionalized component of the conserved density $\mathcal{Q}$, which is proposed to be associated with "hidden" Fermi surfaces of gauge-charged particles.

- Gauss Law and the "attractor" mechanism in the bulk
$\Leftrightarrow$ Luttinger theorem on the boundary theory.

Conclusions

Gapped quantum matter

- Numerical and experimental observation of a spin liquid on the kagome lattice. Likely a Z_2 spin liquid.