Title: The Path Integral Interpretation of Quantum Mechanics

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Abstract:

The Path Integral Interpretation of Quantum Mechanics

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Imperial College, London

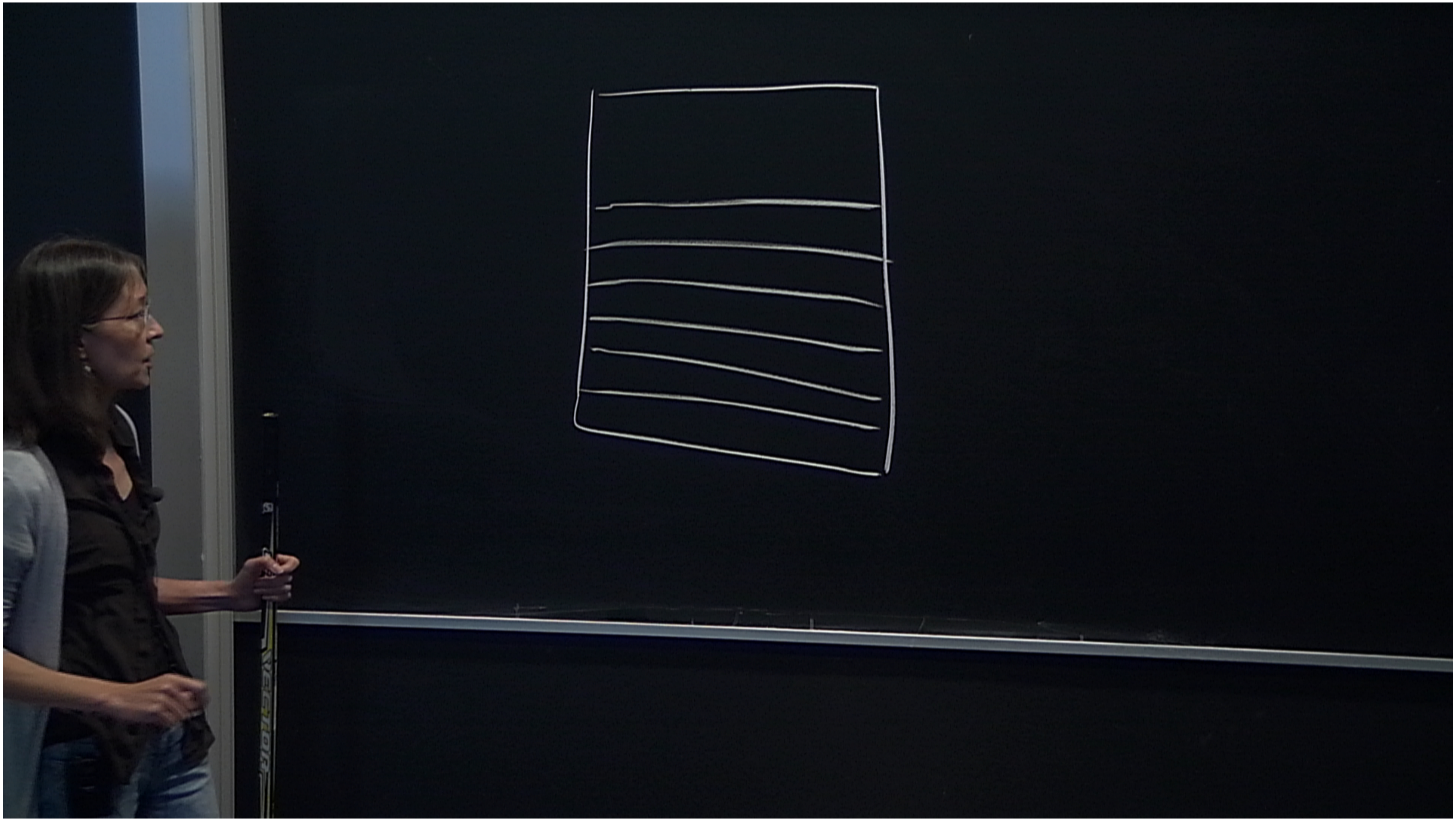
Perimeter Institute 10th July 2012

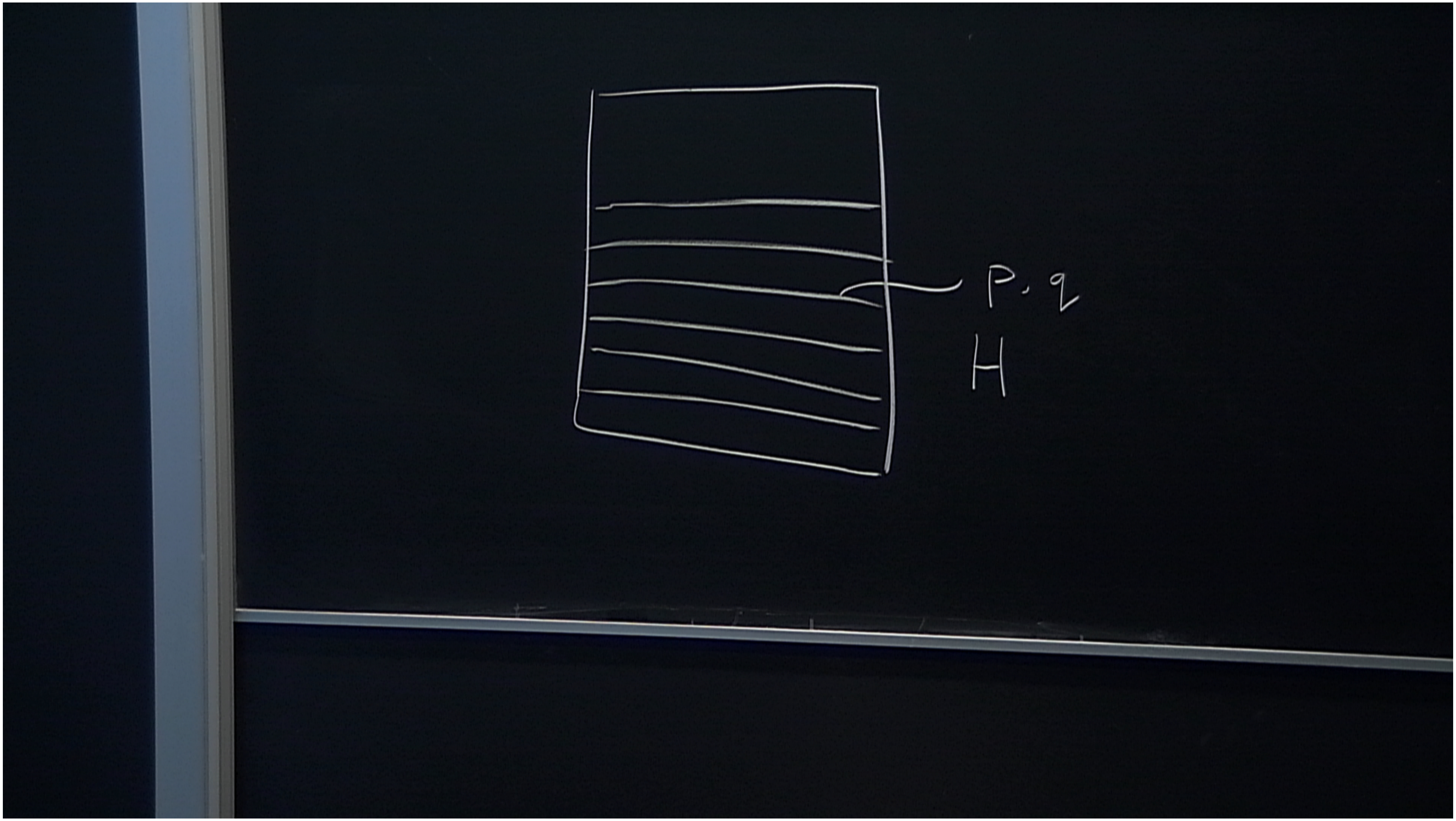
Some take away messages

- The unity of physics demands a spacetime approach to quantum foundations.
- The path integral is the basis of a spacetime theory of closed quantum systems (Dirac, Feynman, Hartle, Sorkin)
- The path integral situates quantum theories in a framework that incorporates classical theories: classical theories are special cases of quantum theories.
- In a path integral approach, the wave function has no fundamental status. It is neither ontic nor epistemic, rather it is a partial summary of enough of the past for future predictions to be able be made.
- As a spacetime approach, the path integral is suited to a quantum theory **of** spacetime (Hartle, Hawking, Loll, Sorkin....)

"The Lagrangian in Quantum Mechanics" Dirac (1932)

- Quantum mechanics was built up on a foundation of analogy with the Hamiltonian theory of classical mechanics
- There is an alternative [..] provided by the Lagrangian. [..T]here are reasons for believing that the Lagrangian one is the more fundamental.
- There is no action principle [..] of the Hamiltonian theory
- The Lagrangian method can easily be expressed relativistically; while the Hamiltonian method is essentially non-relativistic in form, since **it marks out a particular time variable** as the canonical conjugate of the Hamiltonian function.




p, q
H

Against phase space in quantum mechanics

- Phase space is **essentially nonrelativistic**. Position and momentum are **not** on the same footing physically in quantum mechanics – one cannot see this as clearly in the classical theory.
- In a nonrelativistic quantum theory built on the Hamiltonian this shows up, for example, in nonlocal **signalling**.
  Consider two local measurements of position at time $t = -\epsilon$ and $t = \epsilon$. (PICTURE) The earlier projector A is onto a range of position, and so is B, the later. The first measurement is either made or not according to the choice of agent Alya. She can signal to Bai but the signal goes away as $\epsilon \to 0$:

  $$\langle B \rangle - [\langle ABA \rangle + \langle (1-A)B(1-A) \rangle] \to 0$$

  Now add measurement of C, again a projector onto a range of position, at $t = 0$ The signal again goes away as $\epsilon \to 0$.

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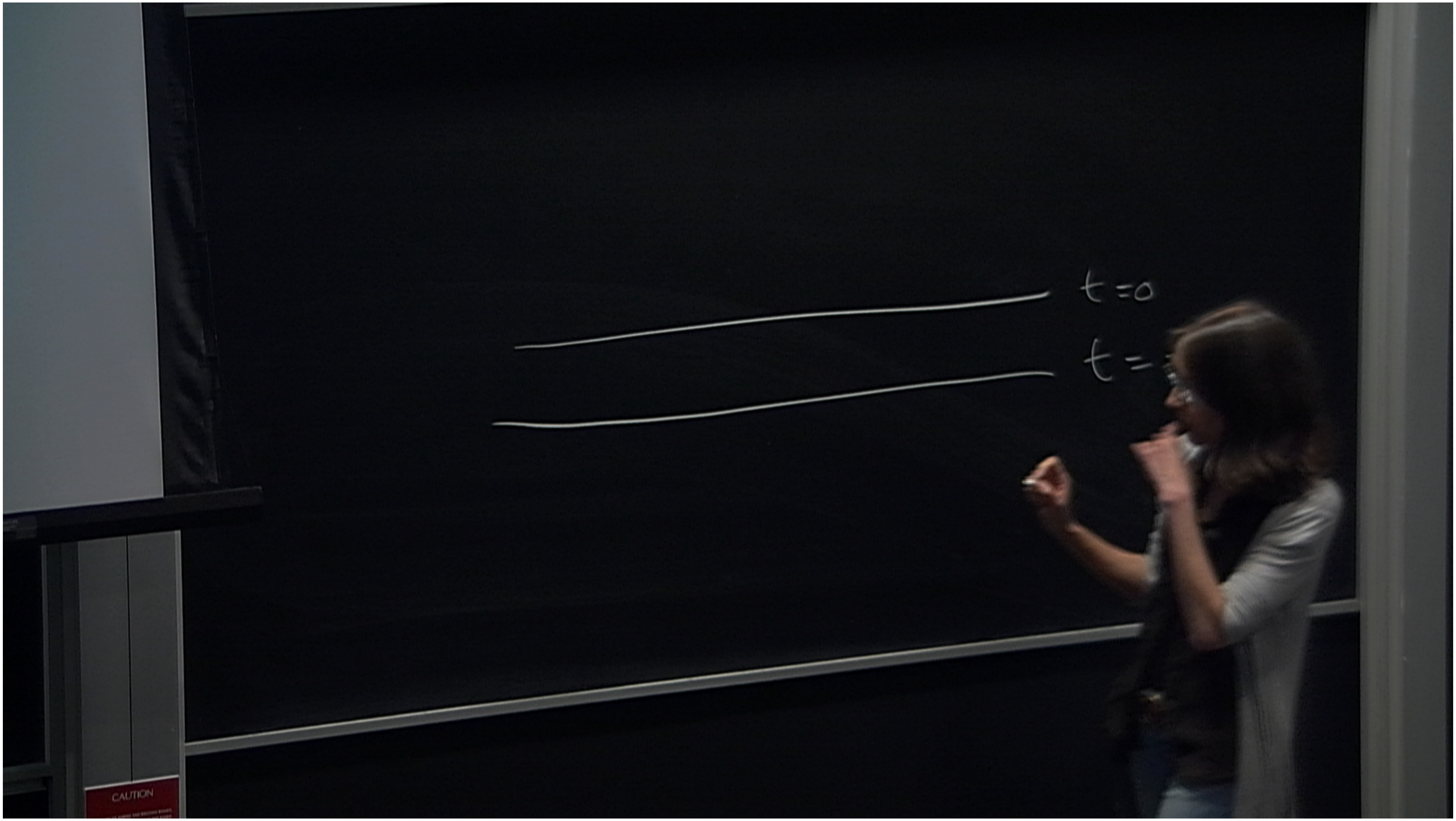
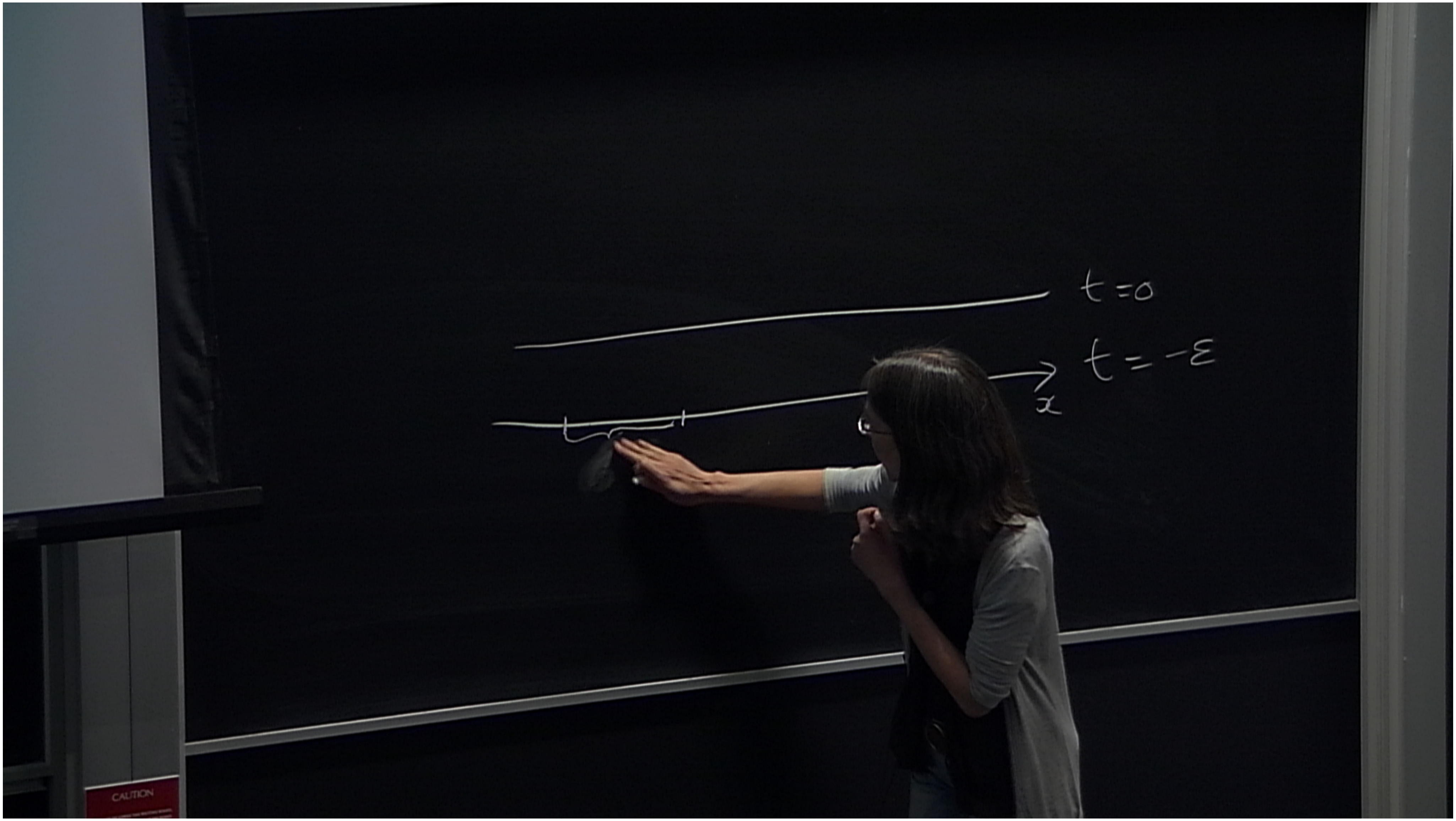
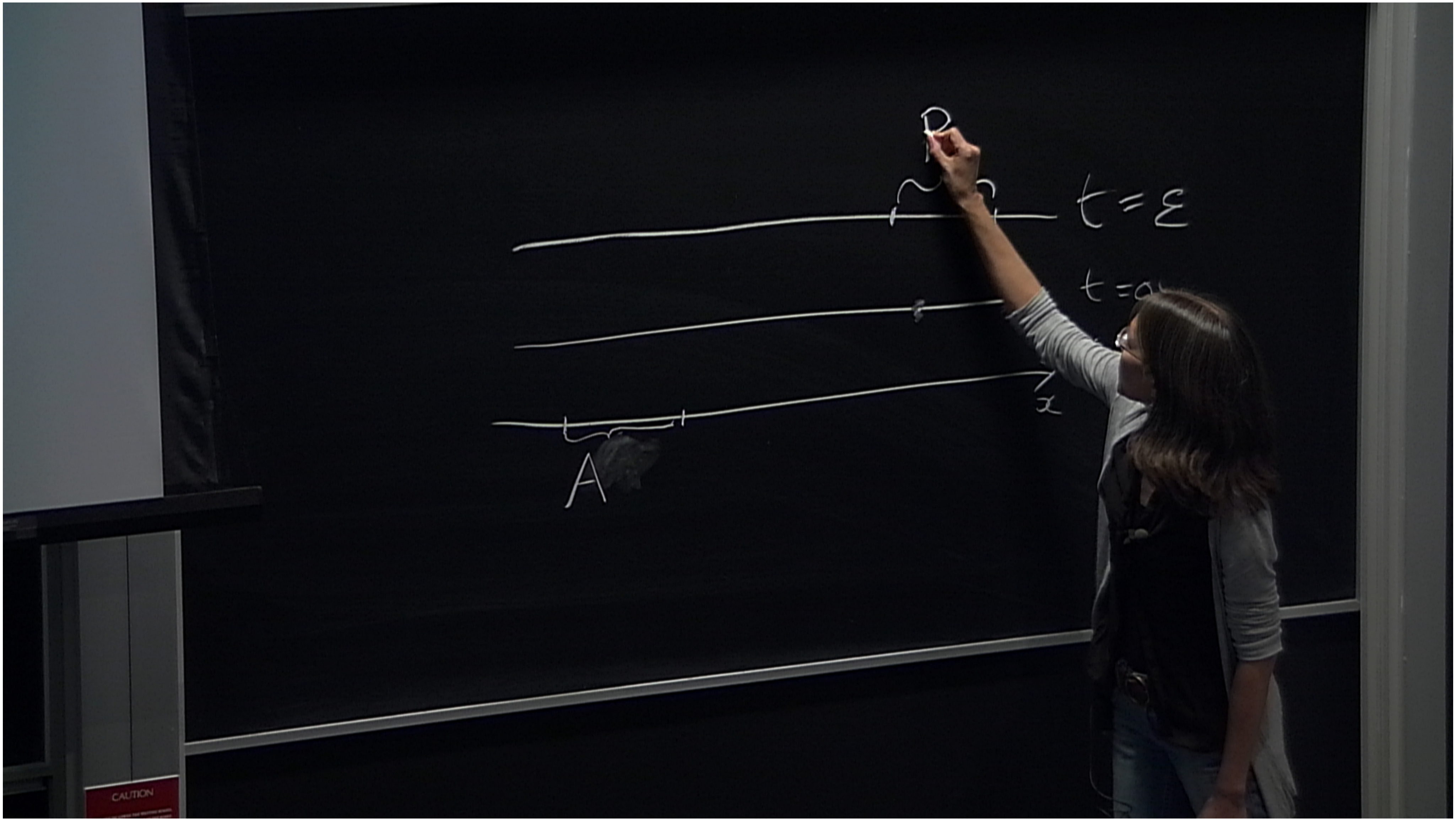
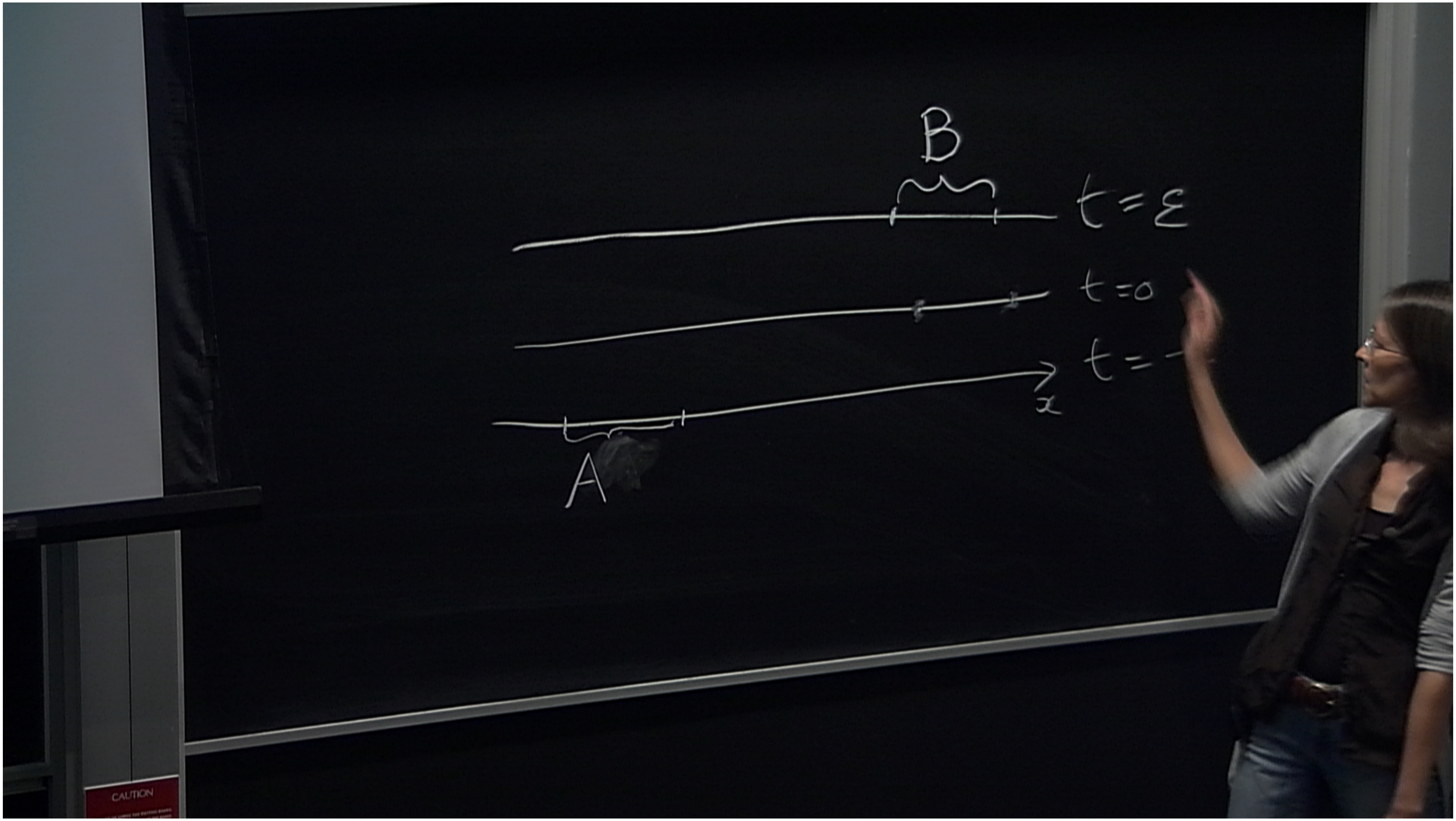
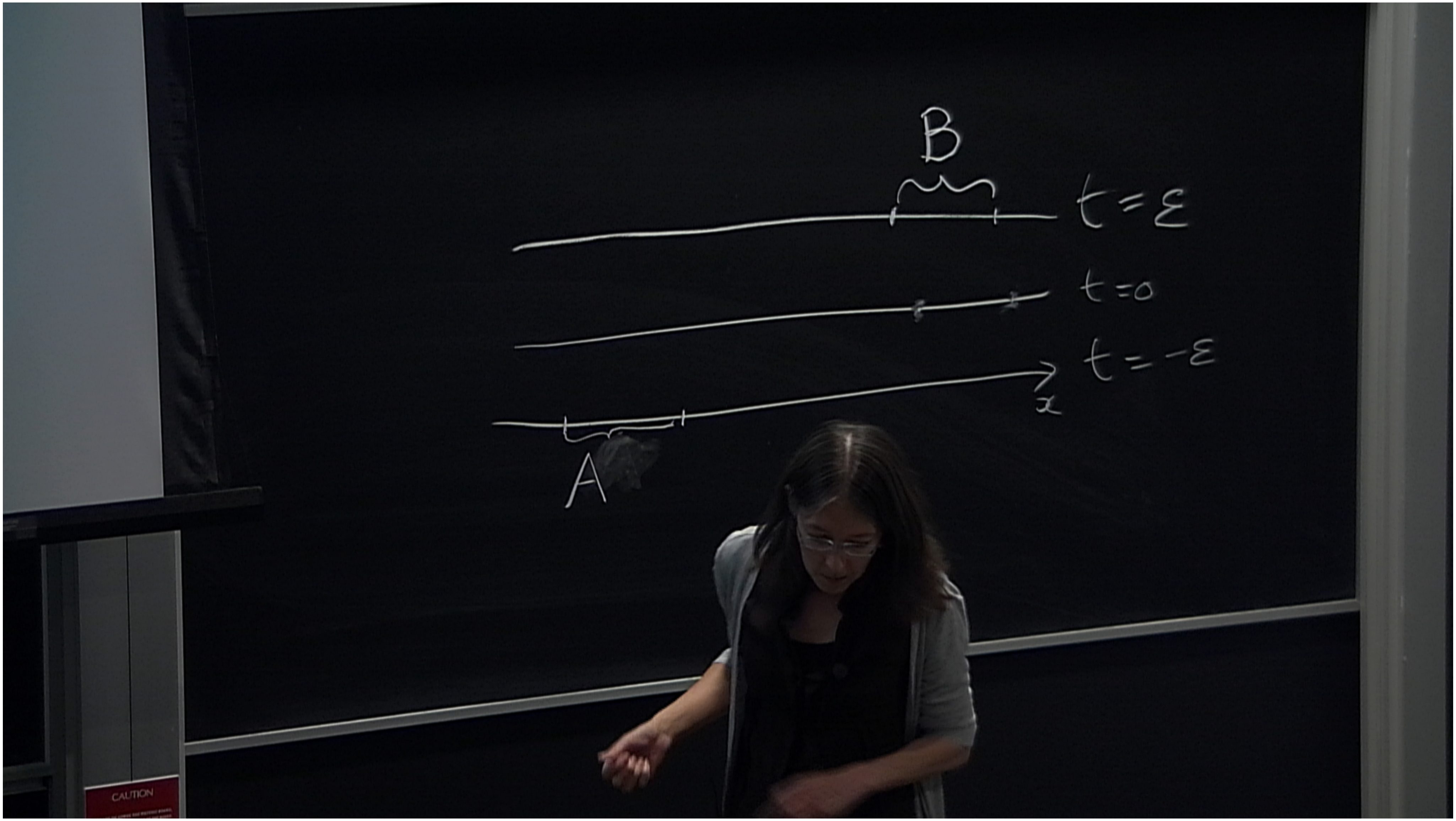
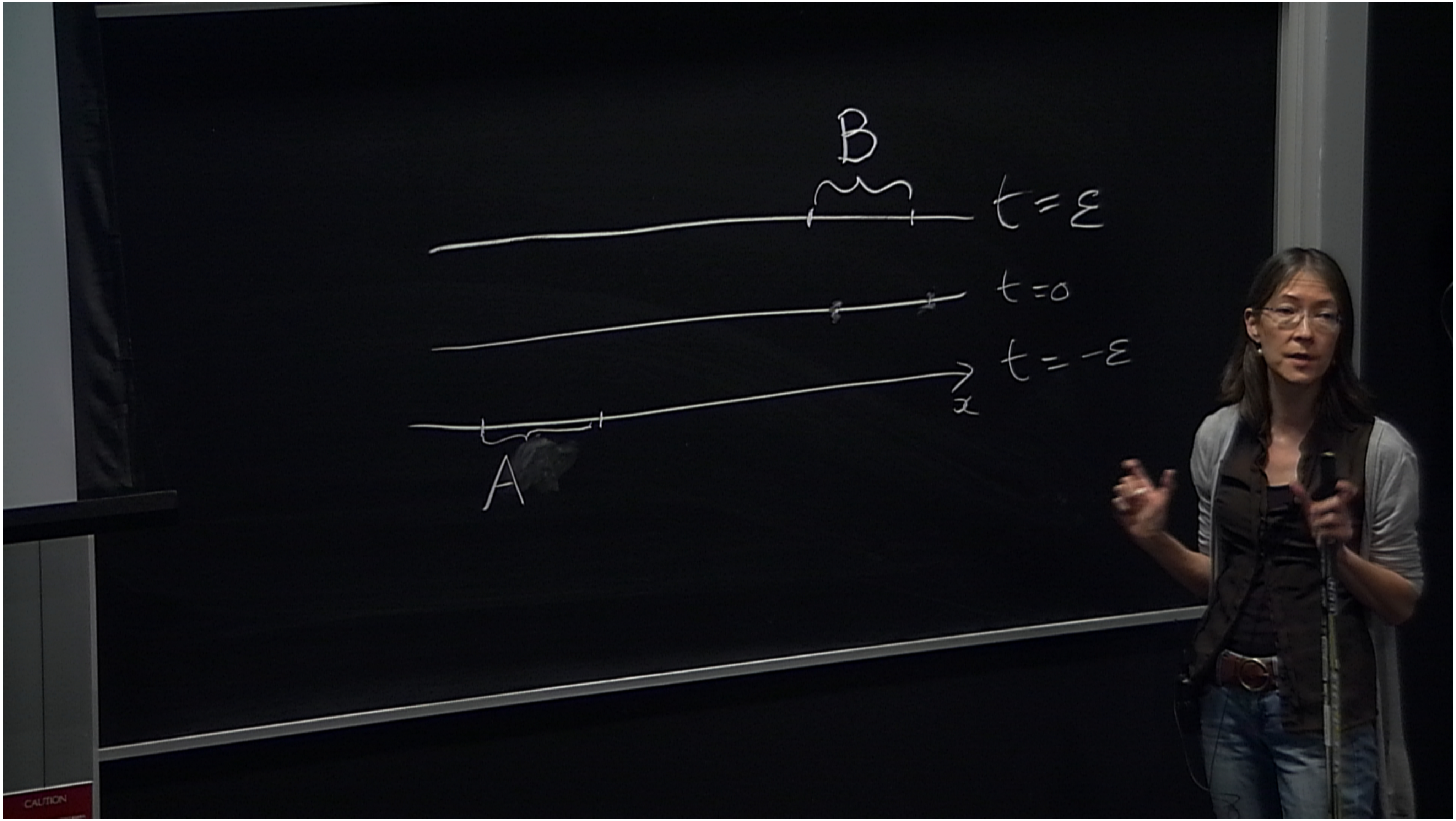

t=0
t=
CAUTION


t = 0
t = -ε
x


B
t = ε
t = 0
x
A
CAUTION


B
t = ε
t = 0
A
x
CAUTION


B
t = ε
t = 0
t = −ε
x
A



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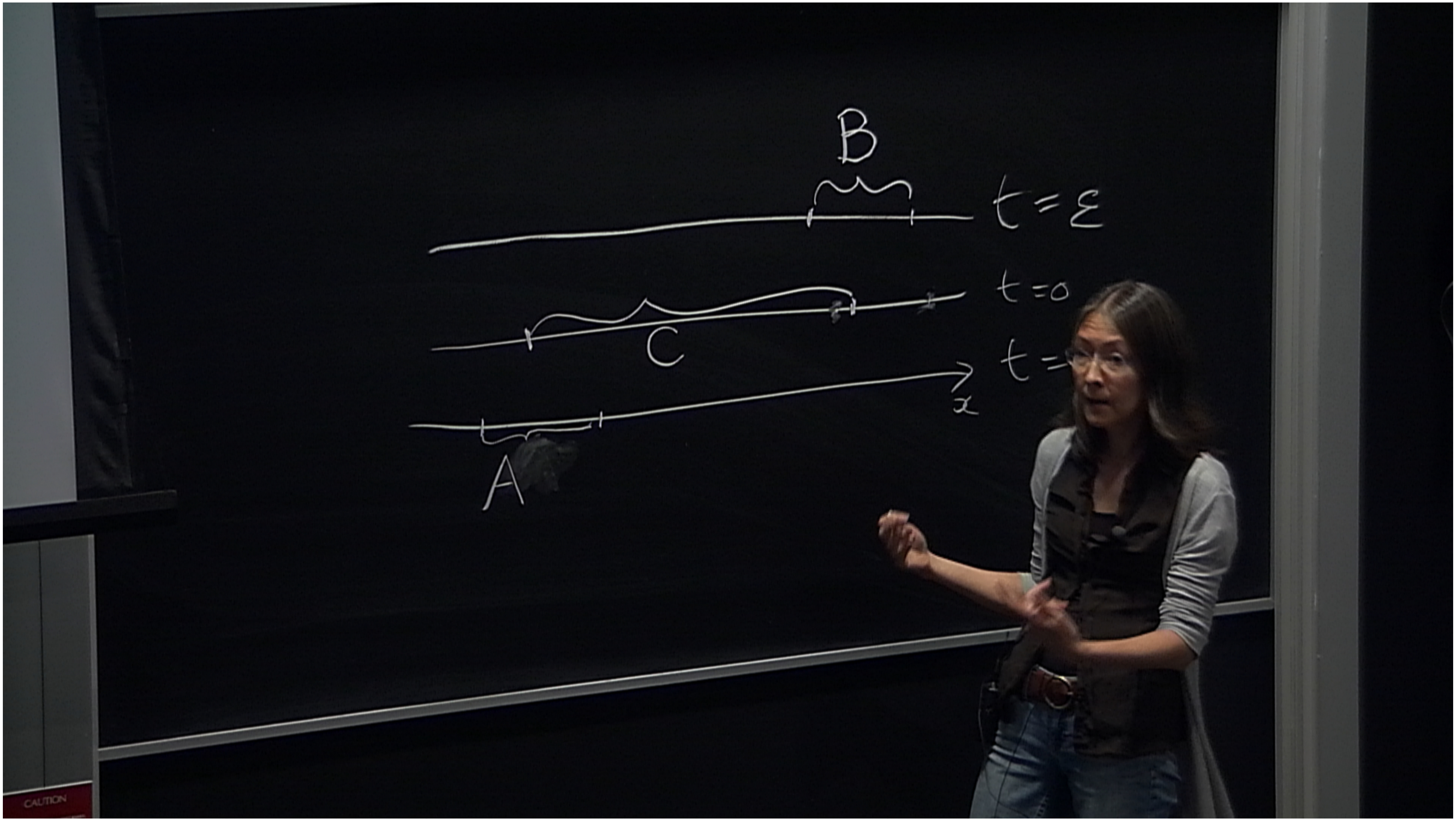
Momentum is nonlocal

- If the intermediate measurement is onto a range of **momentum** the difference in the expected value of B if Alya does or doesn't do her measurement remains finite as $\epsilon \to 0$. There is **nonlocal signalling**
- This simple example can be generalised to the context of a relativistic quantum field theory to demonstrate that, for example, ideal measurements of one particle states enable **superluminal signalling.**
- Sorkin: the beautiful "transformation theory" of the canonical approach which asserts the equivalence of unitarily related bases of the Hilbert space is ruined by the "Law of Locality".
- The essentially non-relativistic nature of the Hamiltonian approach is revealed whenever measurements are analysed because measurements take place in spacetime.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Take the Relativistic Road at Dirac's Fork

- The Lagrangian approach to classical mechanics leads to the path integral for quantum mechanics.
- The Dirac-Feynman path integral for the propagator in 1-d (non-rel) is

$$
\begin{aligned}
K(x', t' \mid x, t) &:= \langle x', t' \mid x, t\rangle \\
&= \int [d\gamma] \exp \frac{i}{\hbar} S[\gamma]
\end{aligned}
$$

$$S[\gamma] = \int_{t}^{t'} dt\, \mathcal{L}(\gamma(t), \dot{\gamma}(t))$$

- First, gives a reformulation of the Copenhagen Rules for Prediction but without Hilbert space.
- Then, a way to go beyond the textbook: a quantum theory for closed systems that is essentially relativistic.

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Sequences of measurements

The Textbook calculation for the probability for a sequence of measurement outcomes gives

$$\mathbb{P}(\Delta_1, \ldots \Delta_n) = \| P_{\Delta_n}(t_n) \ldots P_{\Delta_2}(t_2)\, P_{\Delta_1}(t_1) \mid \Psi\rangle \|^2$$

The projectors are Heisenberg operators

$$P_{\Delta_i}(t_i) = U(t_i, t_0)^{-1} P_{\Delta_i} U(t_i, t_0)$$

and in the Schrödinger picture the formula is

$$\| U(n+1, n)\, P_{\Delta_n} \ldots P_{\Delta_2}\, U(2,1)\, P_{\Delta_1}\, U(1,0) \mid \Psi, t_0\rangle \|^2$$

where t_{n+1} is some arbitary time to the future of t_n.

Sequences of measurements of **positions**

Work with nonrelativistic particle in 1-d (PICTURE). When the measurements are onto ranges of positions

$$P_{\Delta_i} = \int_{a_i}^{b_i} dx \mid x\rangle\langle x \mid$$

we get

$$\mathbb{P}(\Delta_1, \ldots \Delta_n) = \int_{\gamma' \in \Delta} [d\gamma'] \int_{\gamma \in \Delta} [d\gamma] \; D(\gamma'; \gamma)$$

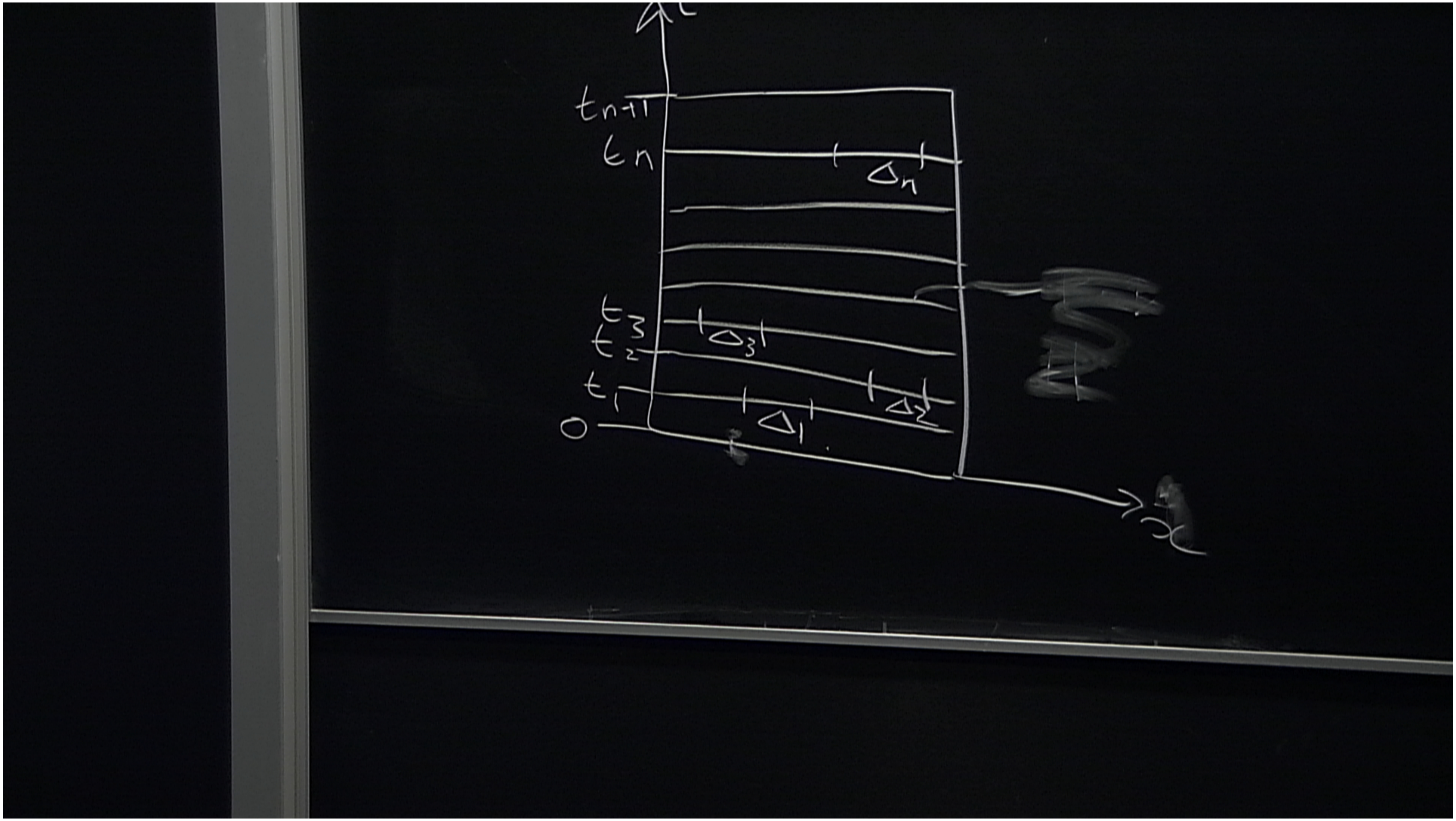
where

Δ is the set of all trajectories such that $\gamma(t_i) \in [a_i, b_i]$ and

$$D(\gamma'; \gamma) = \overline{\mathrm{amplitude}(\gamma')}\mathrm{amplitude}(\gamma) \; \delta(\text{final endpoints})$$

and

$$\mathrm{amplitude}(\gamma) = e^{i\frac{S[\gamma]}{\hbar}} \; \psi(\gamma(0))$$



t
t_{n+1}
t_n
Δ_n
t_3
Δ_3
t_2
Δ_2
t_1
Δ_1
0
x

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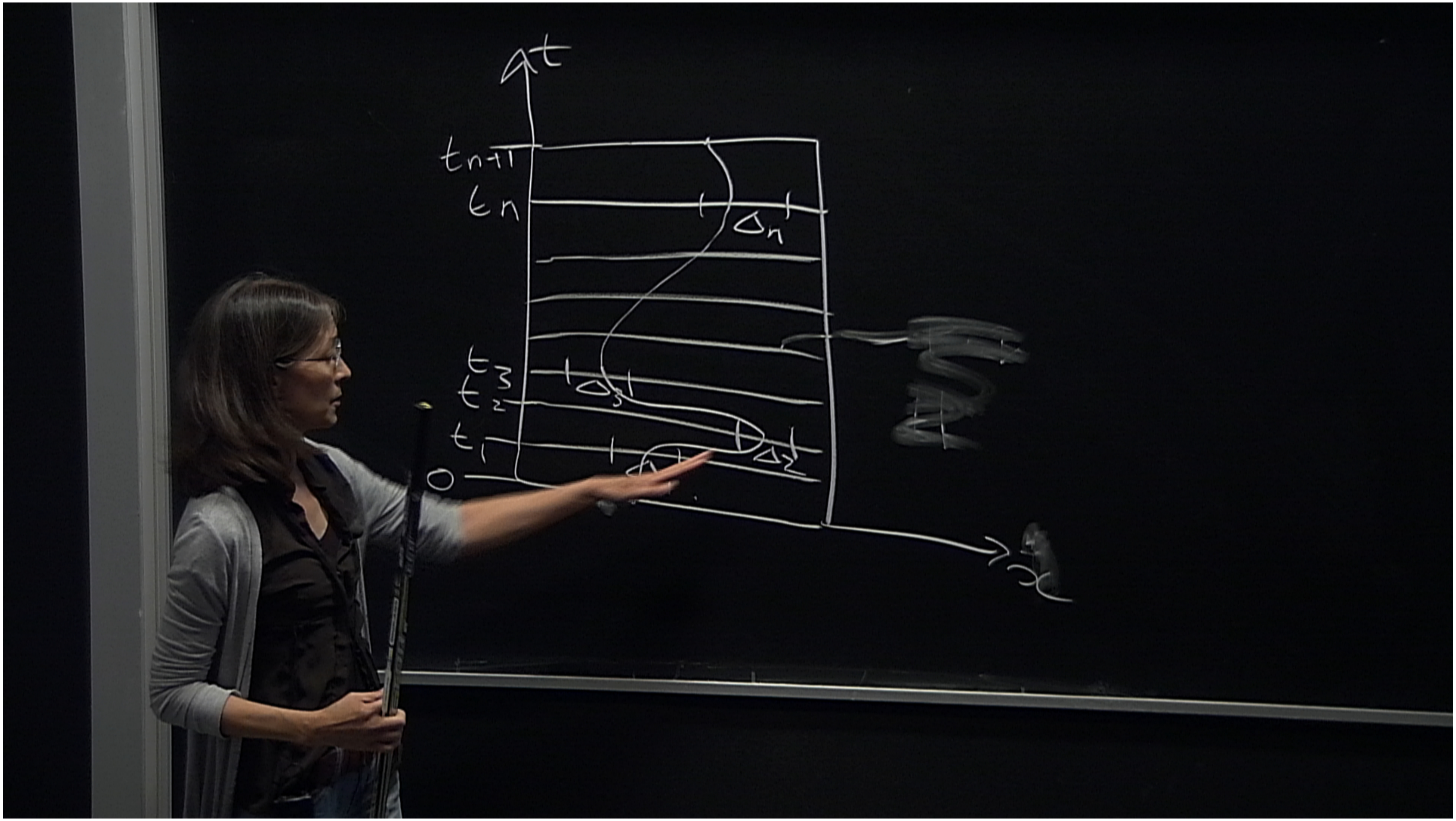

t
t_{n+1}
t_n
Δ_n
t_3
t_2
t_1
Δ_3
Δ_2
Δ_1
0
x

Path integral basis of the textbook formalism

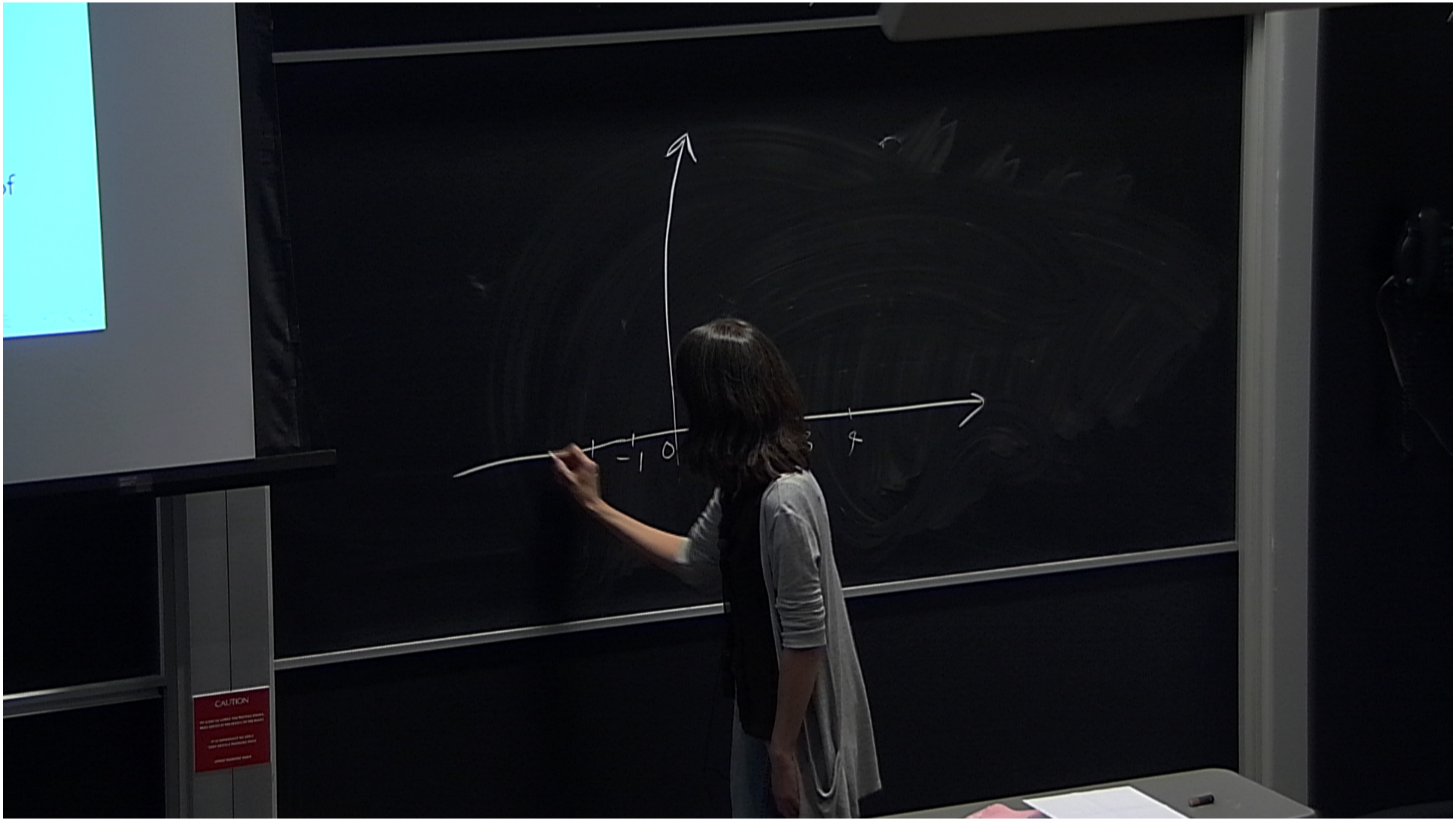
- The path integral (or sum-over-histories)

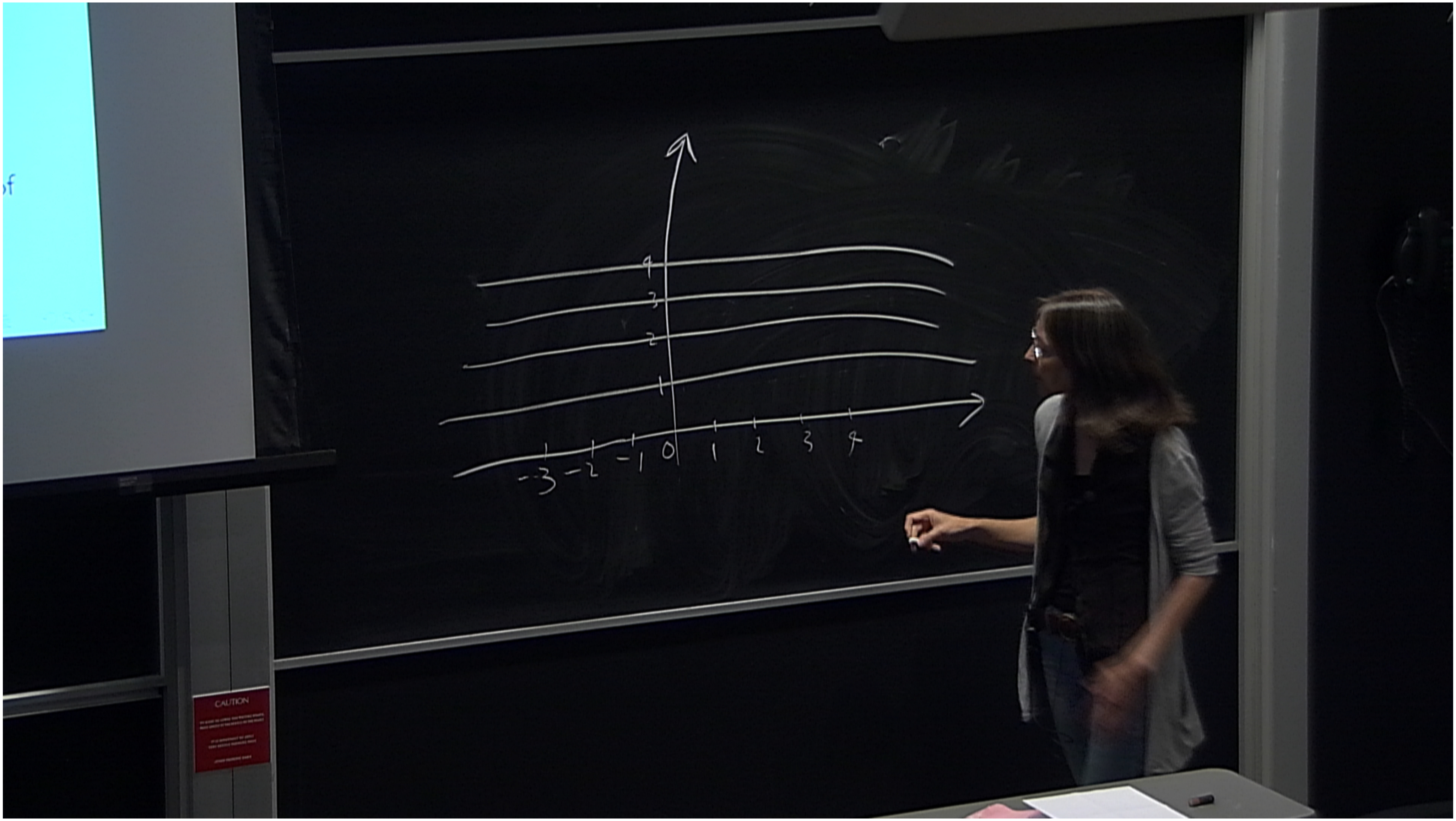
$$\mathbb{P}(\Delta_1, \dots \Delta_n) = \int_{\gamma' \in \Delta} [d\gamma'] \int_{\gamma \in \Delta} [d\gamma]\ [\dots] e^{-\frac{i}{\hbar} S[\gamma']} e^{\frac{i}{\hbar} S[\gamma]}$$

  gives what the textbook theory does: probabilities for outcomes of measurements.
- For nonrelativistic quantum mechanics one can **construct** from it, rigorously, the usual Hilbert space of (equivalence classes) of square integrable complex functions on the configuration space (Dowker, Johnston & Sorkin)
- The wavefunction has no fundamental status but is an executive summary of the measurement outcomes to the past of some hypersurface and one can derive the Schroedinger equation for it (Feynman, Sorkin).

Path integrals beyond the textbook

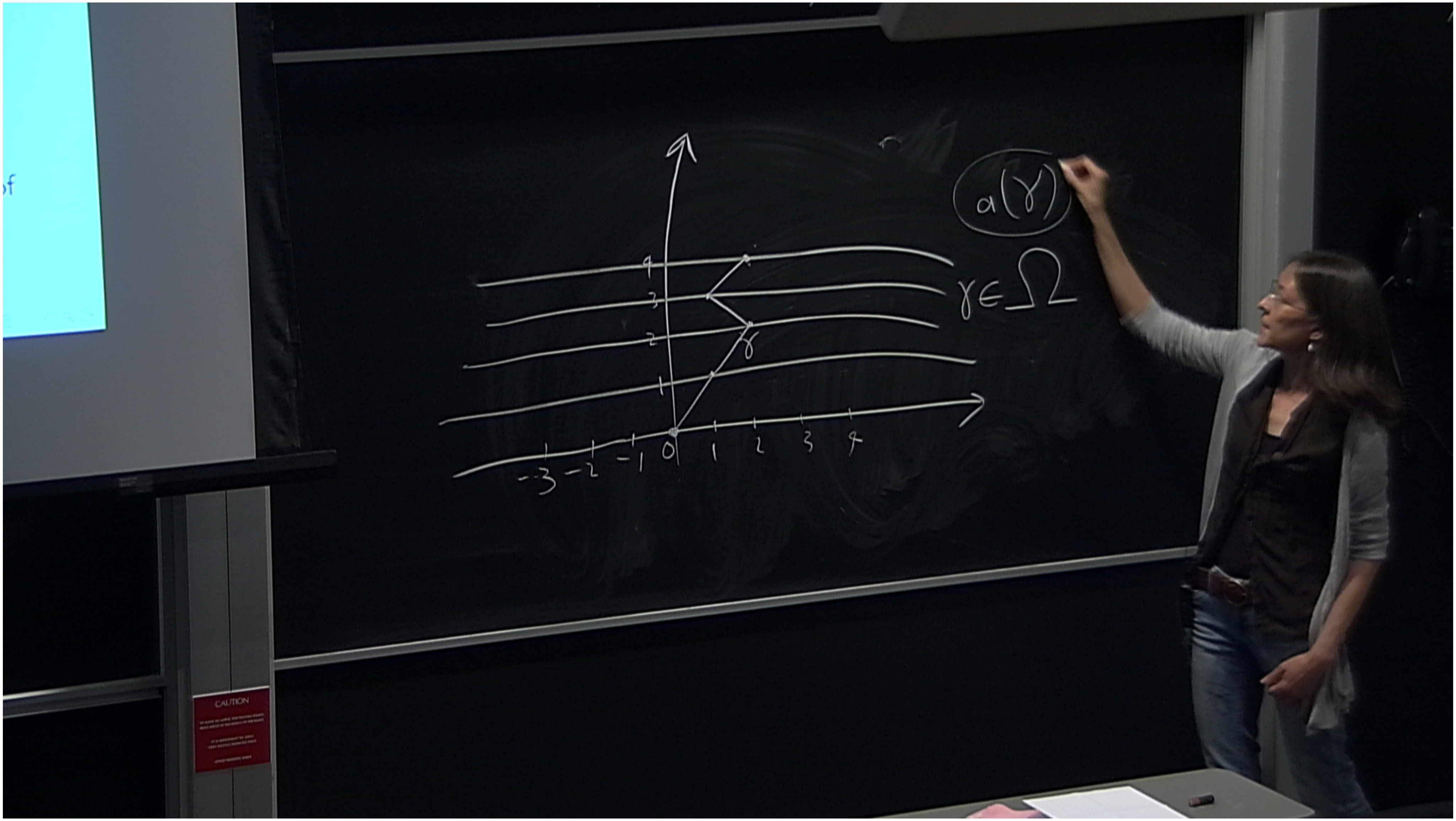
- The path integral is a foundation for quantum theory of **closed** systems: throw away the external measuring apparatus.
- As pointed out already by Dirac in 1932, when $\frac{S}{\hbar}$ is large the only significant contribution to the path integral comes from paths very close to a classical path. If the physical world "corresponds" to the histories that make the most contribution, this would explain when and how classical physics approximates the quantum physics. The question is, what, exactly, is this "correspondence"?
- The rest of the talk is a sketch of a framework for quantum theories, founded on the path integral in which we are working towards answering this question – what is the physical world in a quantum theory? – and making the heuristic argument of Dirac/Feynman rigorous.
- **Note**: the answer crucially depends on kinematics, aka the nature of the histories.

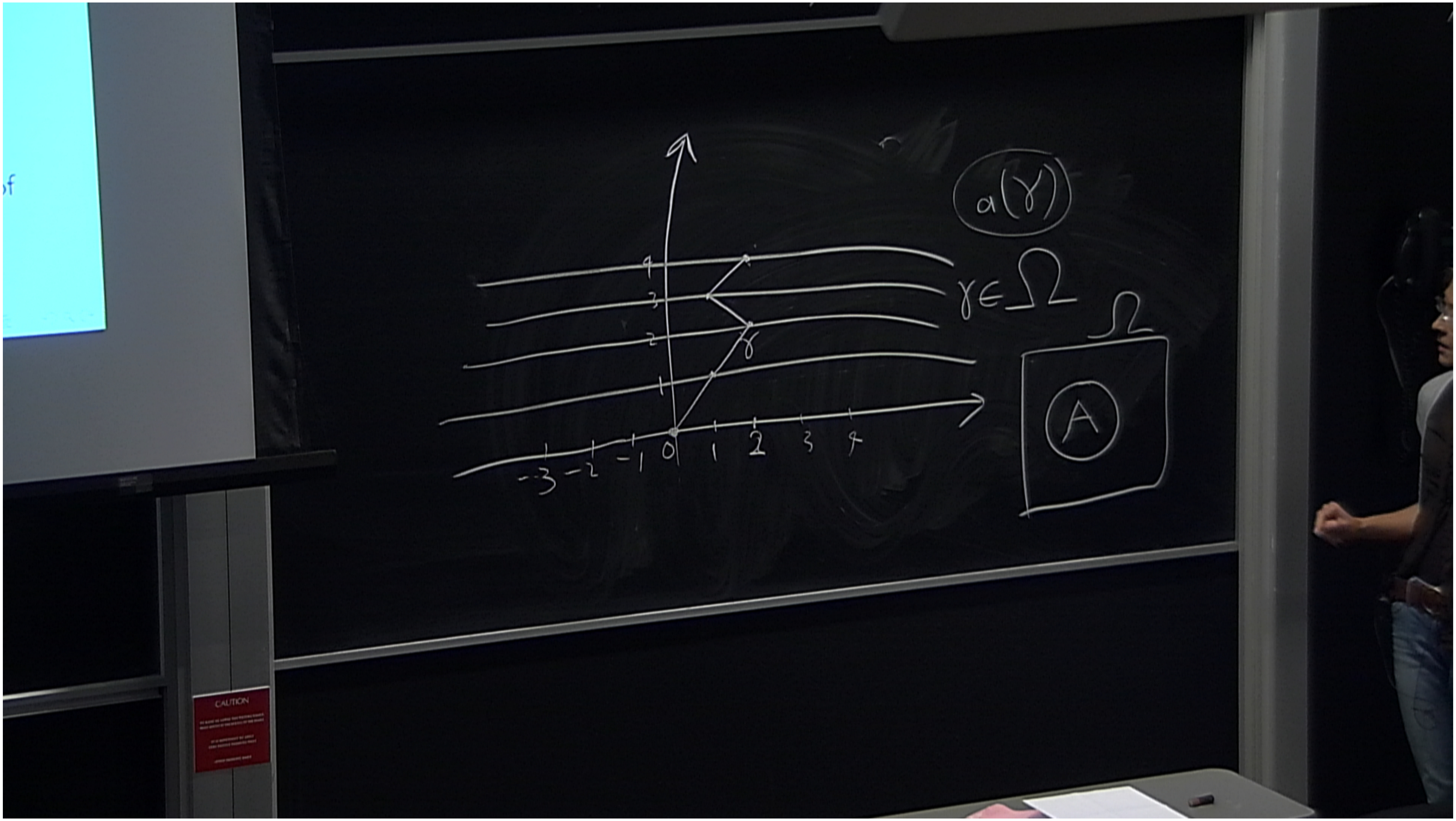

CAUTION

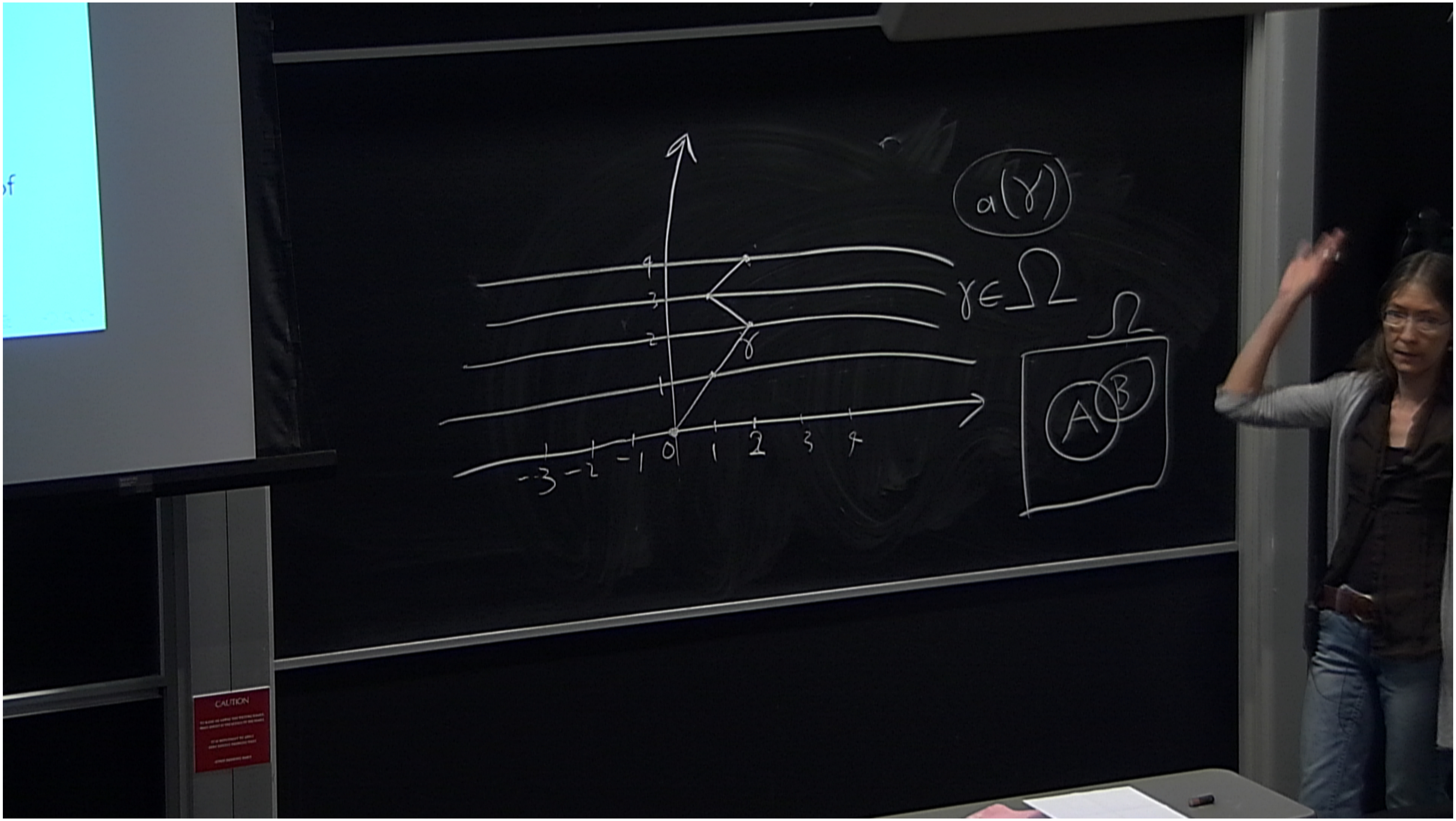


4
3
2
1
-3
-2
-1
0
1
2
3
4
CAUTION

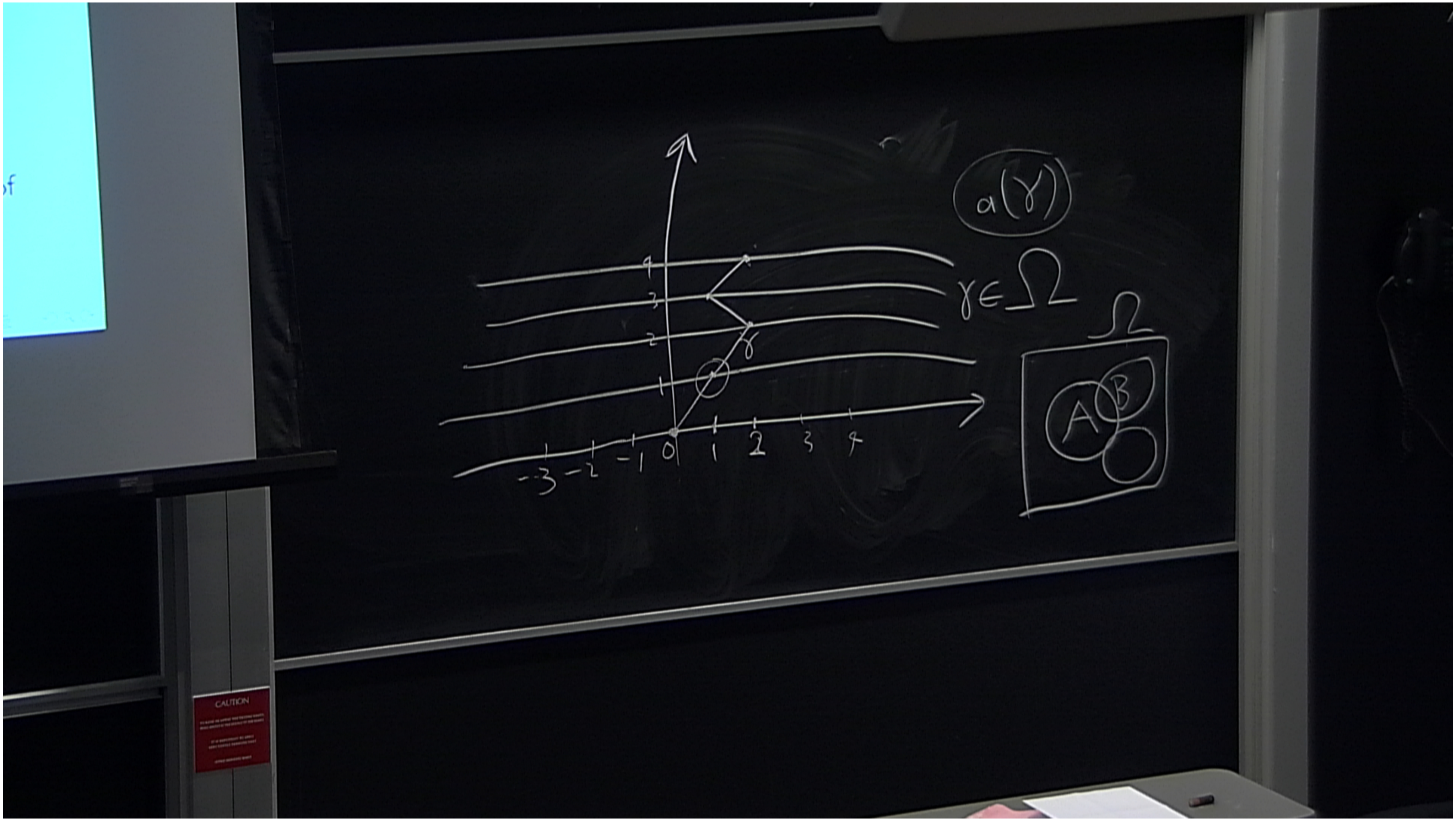
-3
-2
-1
0
1
2
3
4
CAUTION

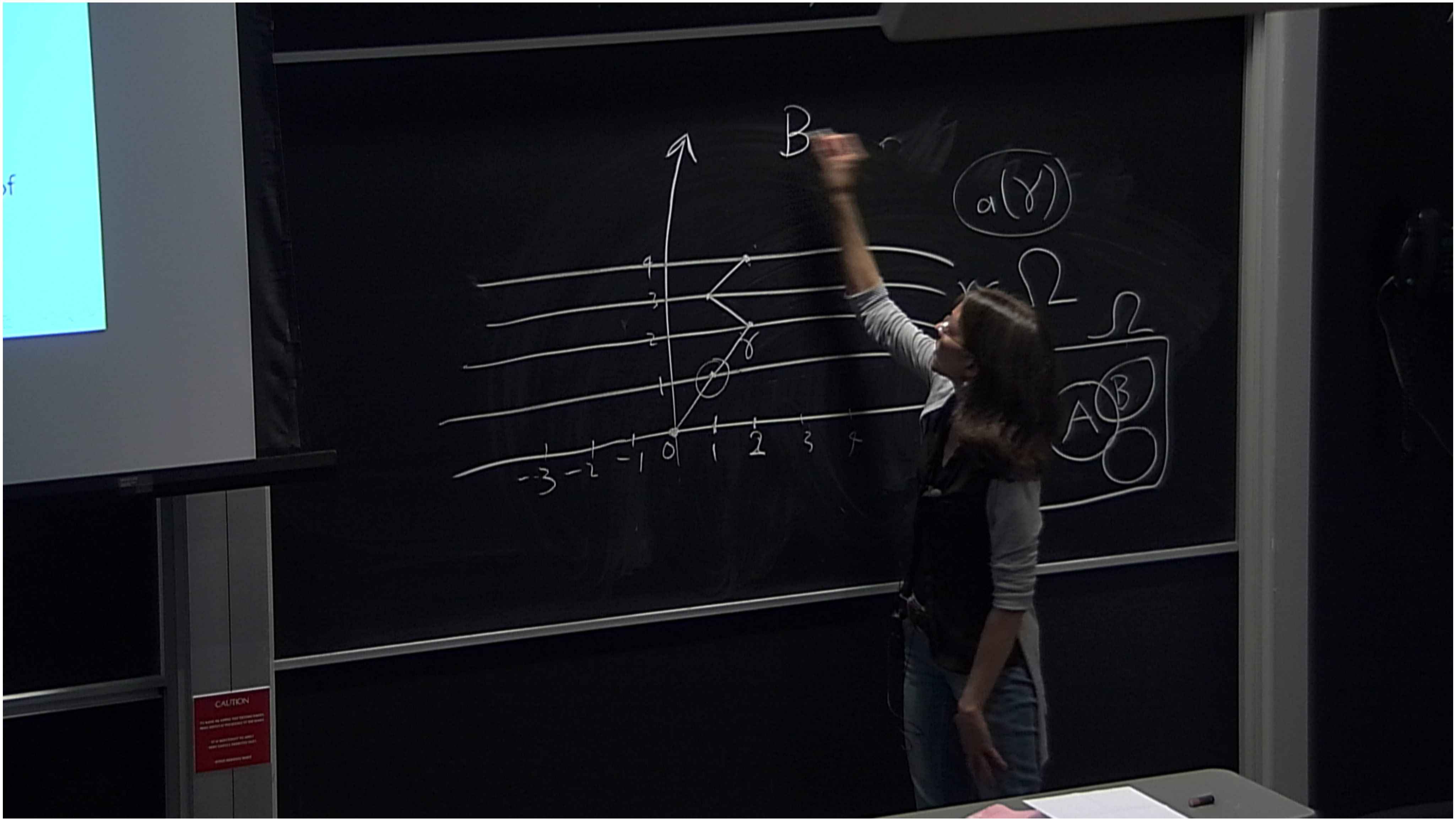

a(γ)
γ ∈ Ω
γ
-3 -2 -1 0 1 2 3 4
1
2
3
4

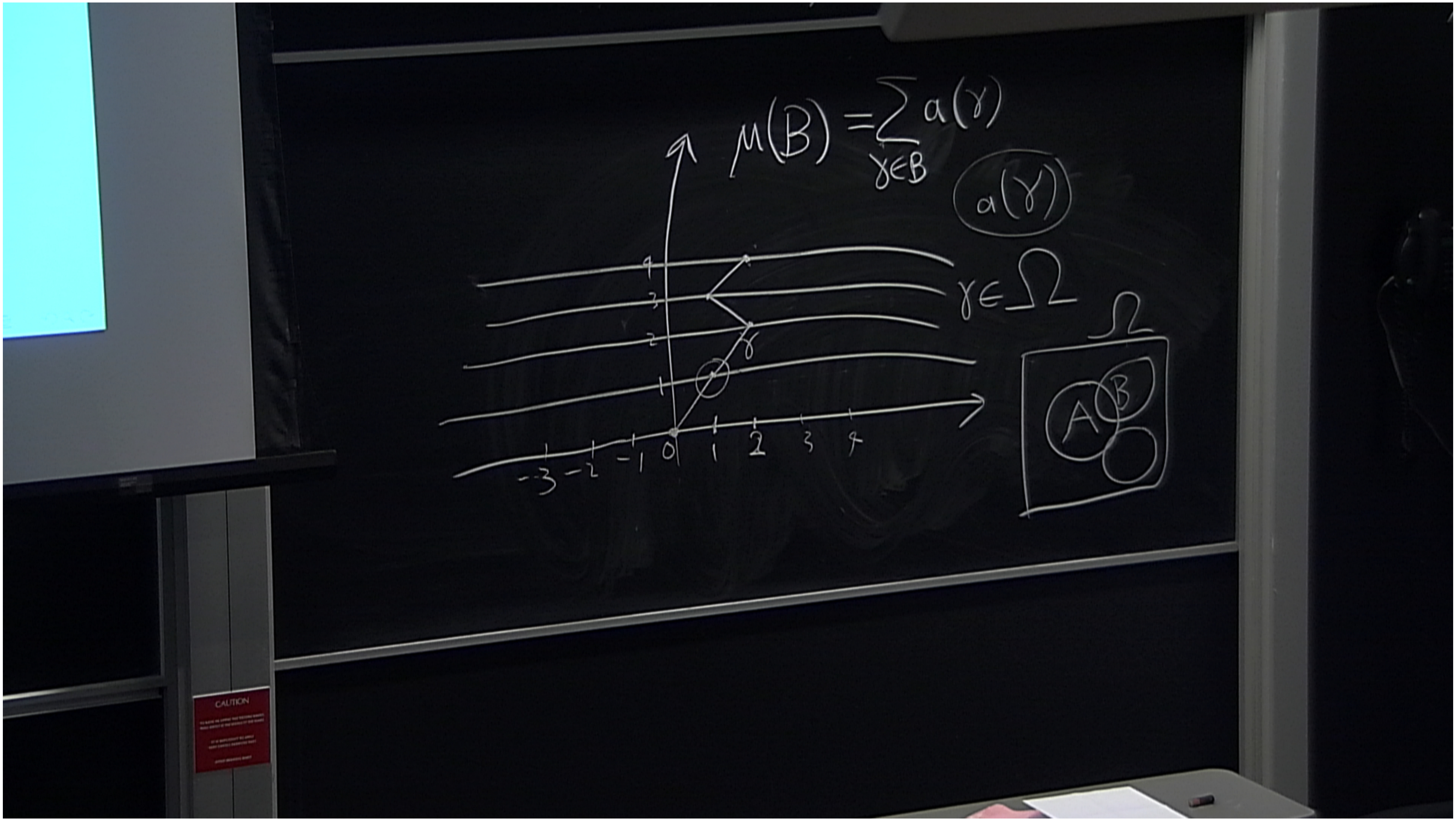

a(γ)
γ∈Ω
Ω
A
γ
-3 -2 -1 0 1 2 3 4
1
2
3
4



a(γ)
γ ∈ Ω
γ
Ω
A
B
-3 -2 -1 0 1 2 3 4
1
2
3
4
CAUTION


a(γ)
γ∈Ω
Ω
A
B
-3 -2 -1 0 1 2 3 4
CAUTION


B
a(γ)
γ
A
B
-3
-2
-1
0
1
2
3
4


$\mu(B) = \sum_{\gamma \in B} a(\gamma)$
$a(\gamma)$
$\gamma \in \Omega$
Ω
γ
A
B
-3 -2 -1 0 1 2 3 4
CAUTION

Quantum mechanics is a "Measure Theory"

Every classical stochastic theory in spacetime has the following structure (assume a finite system):

(1) A set Ω, of possible spacetime histories (e.g. sequences of outcomes of 1000 coin tosses, Wiener paths for a Brownian particle).

(2) A Boolean **event algebra**, $\mathfrak{A} \subseteq 2^{\Omega}$: an event is a subset of Ω. An event is associated to a spacetime region.

(3) A **measure**, μ, on $\mathfrak{A}$ which encodes the dynamics and initial conditions. $\mu(A)$ is the probability that A happens/is affirmed.

(4) The possible **physical worlds**, are the elements γ of Ω. One is realised.

Quantum mechanical systems in spacetime have

(1) Ω: the histories in the SOH (e.g. particle trajectories, field configurations)

(2) Boolean $\mathfrak{A} \subseteq 2^{\Omega}$ e.g. event Δ "the set of all paths that go through $\Delta_1, \ldots \Delta_n$"

(3) A **measure**, μ, on $\mathfrak{A}$ which encodes the dynamics and initial conditions.

(4) The possible physical worlds are ..... what? One is realised.

The Role of the Measure

In a classical stochastic theory, the dynamics and initial state are encoded in a measure, μ which is a positive real function on the event algebra

$$\mu : \mathfrak{A} \to \mathbb{R}$$
$$\mu(A) \geq 0, \quad \forall A \in \mathfrak{A}$$
$$\mu(\Omega) = 1$$
$$\mu(A \sqcup B) = \mu(A) + \mu(B), \quad \forall A, B \in \mathfrak{A}$$

μ is interpreted as a probability measure: the physical world affirms A with probability $\mu(A)$.

The Role of the Quantal Measure?

In a quantum theory, the dynamics and initial state are encoded in a measure, μ. e.g. in non-rel QM the quantal measure $\mu(\Delta)$ is the same path integral we had before

$$\mu(\Delta) := \int_{\gamma' \in \Delta} [d\gamma'] \int_{\gamma \in \Delta} [d\gamma] \; D(\gamma'; \gamma)$$

The measure is a positive real function on the event algebra

$$\mu : \mathfrak{A} \to \mathbb{R}$$
$$\mu(A) \geq 0, \quad \forall A \in \mathfrak{A}$$
$$\mu(\Omega) = 1$$

but, famously, it does **not** satisfy the Kolmogorov sum rules because of interference. μ does, however, satisfy a generalised sum rule for the disjoint union of **three** events. (Sorkin)

So μ cannot be interpreted as a probability measure.

Another Fork in the road:

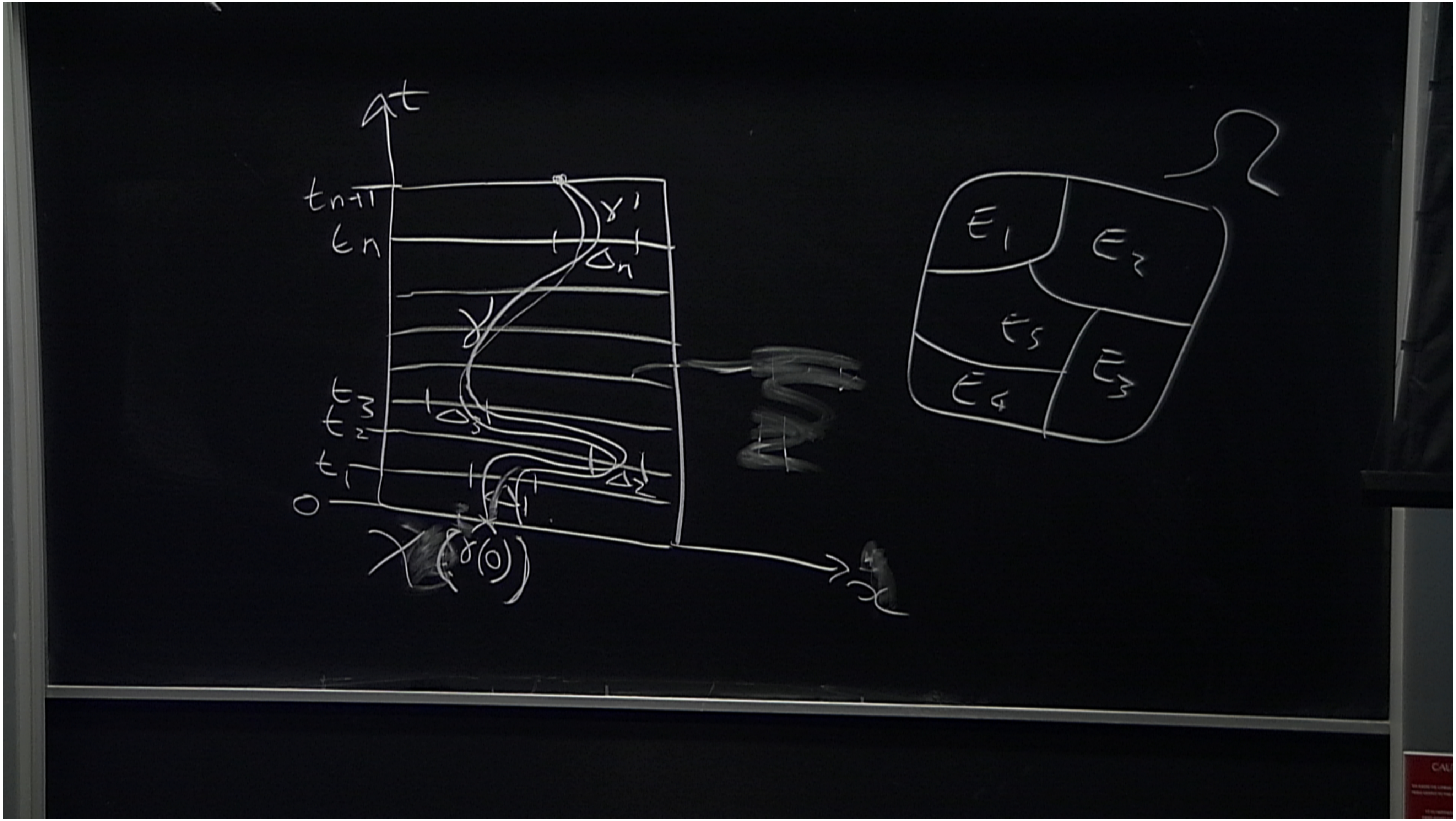
- **Decoherent Histories** (Hartle, Gell-Mann & Hartle)
- **Quantum Measure Theory** (Sorkin)

Decoherent Histories

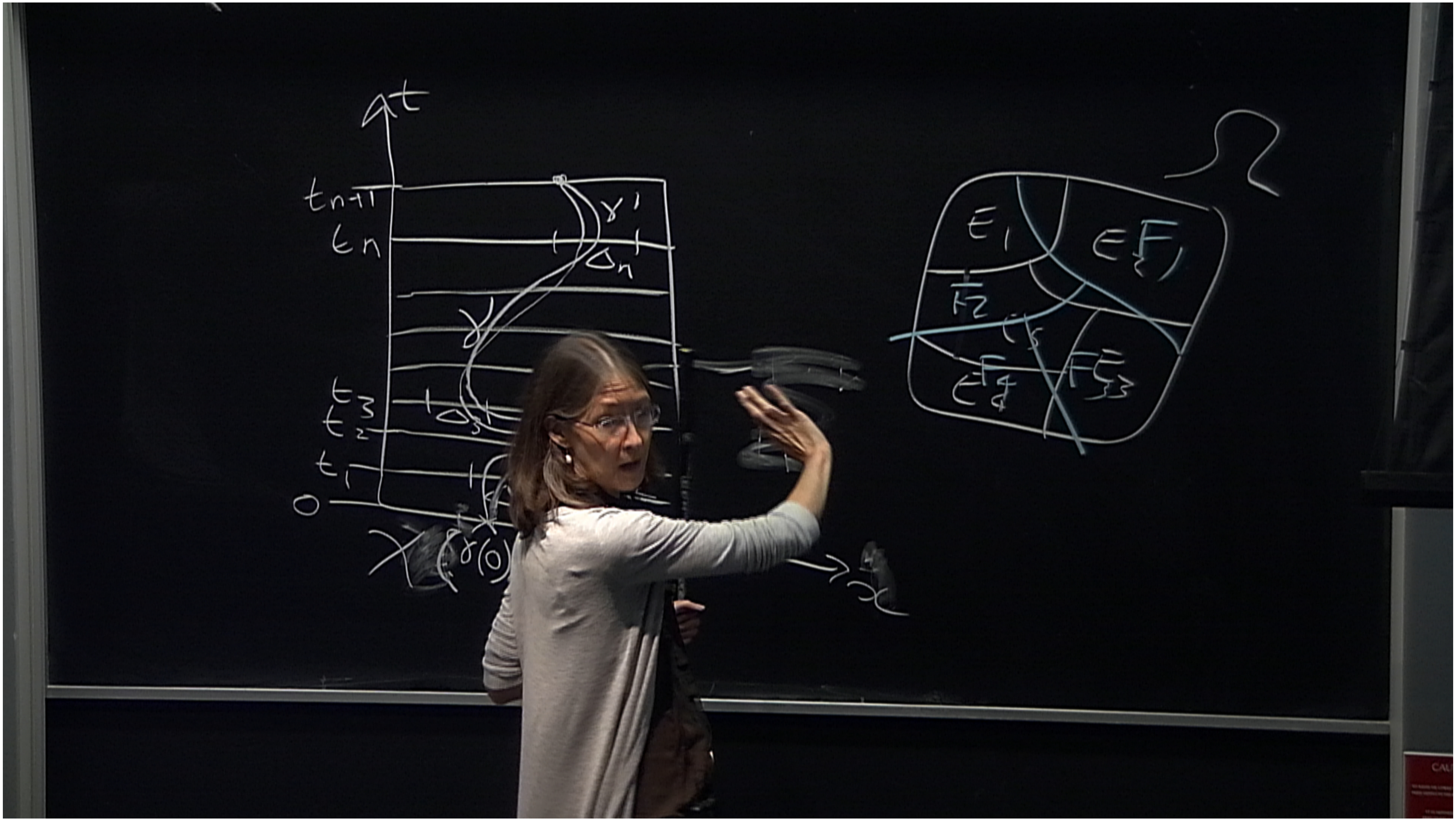
- The Kolmogorov sum rules are necessary to use a measure to make predictions.
- We must restrict the events in the event algebra to a subalgebra for which the sum rule holds.
- Find a partition of $\Omega = \cup_\alpha E_\alpha$ such that the sum rules hold:

$$\mu(E_\alpha \cup E_\beta) = \mu(E_\alpha) + \mu(E_\beta)$$

  and the partition is maximally fine grained.
- Interpret μ on the subalgebra generated by the E_α as a probability measure.
- The physical world is one of the E_α: not a single history but a **coarse grained history**.
- Struggle with the fact that there are many such partitions (Dowker and Kent, Kent)



t
t_{n+1}
t_n
t_3
t_2
t_1
0
γ'
Δ_n
γ
Δ_3
Δ_2
Δ_1
$x(\gamma(0))$
x
E_1
E_2
E_5
E_3
E_4


t
t_{n+1}
t_n
γ'
Δ_n
γ
t_3
t_2
t_1
0
$\gamma(0)$
x
E_1
$\in F_1$
F_2
F_4
F_3

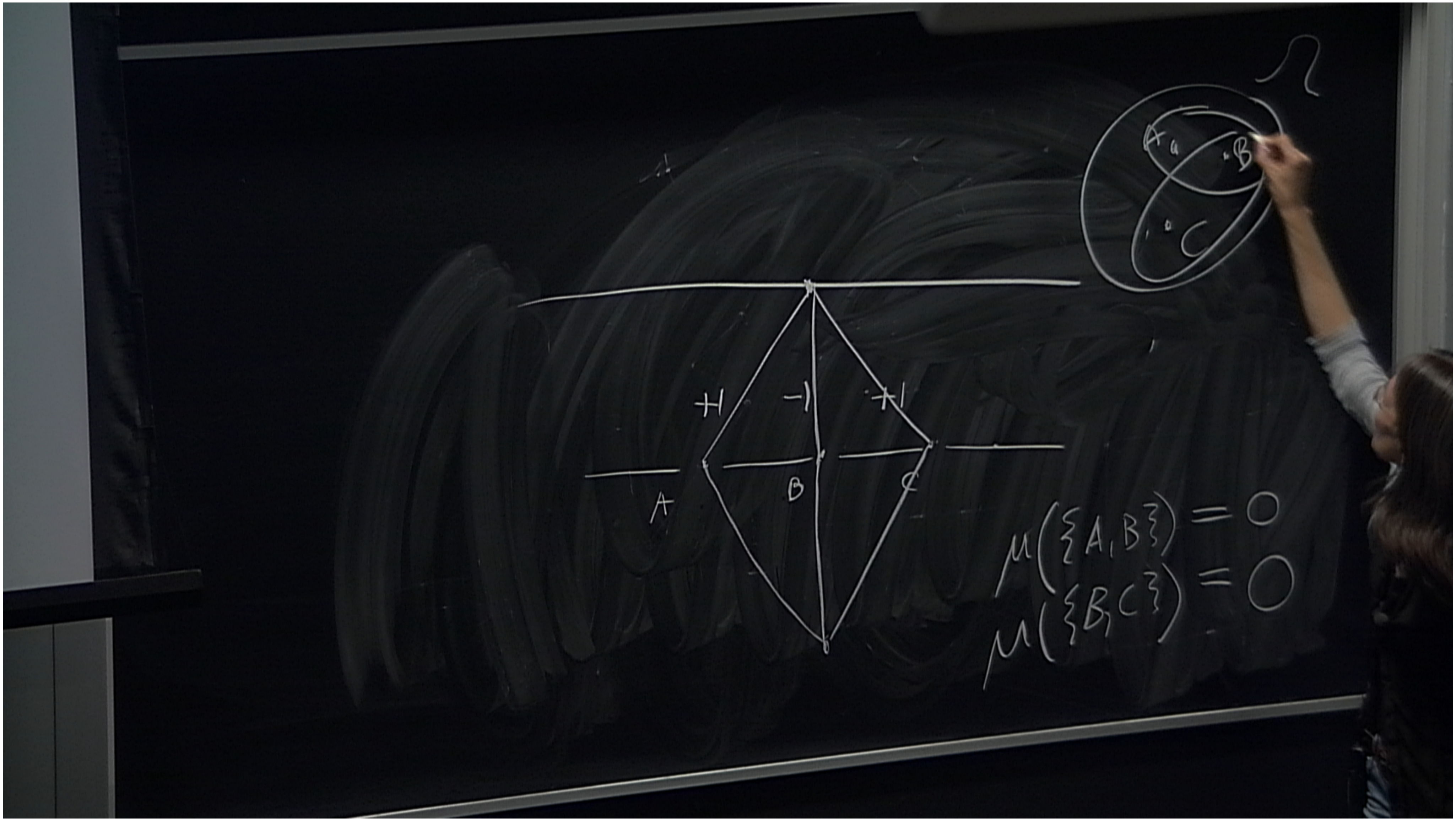
Quantum Measure Theory

- All we really need in science is a notion of **preclusion**: we predict that an event won't happen (typically) if it has zero (or very small) measure. This is called "Principle B" in Kolmogorov's "Grundbegriffe der Wahrscheinlichkeitsrechnung".
- No need for the Kolmogorov sum rules so we adopt the preclusion rule wholesale in the quantum case.
- Struggle with the subjectivity of "approximate preclusion"
- Struggle with the fact that there are "too many" events of measure zero.

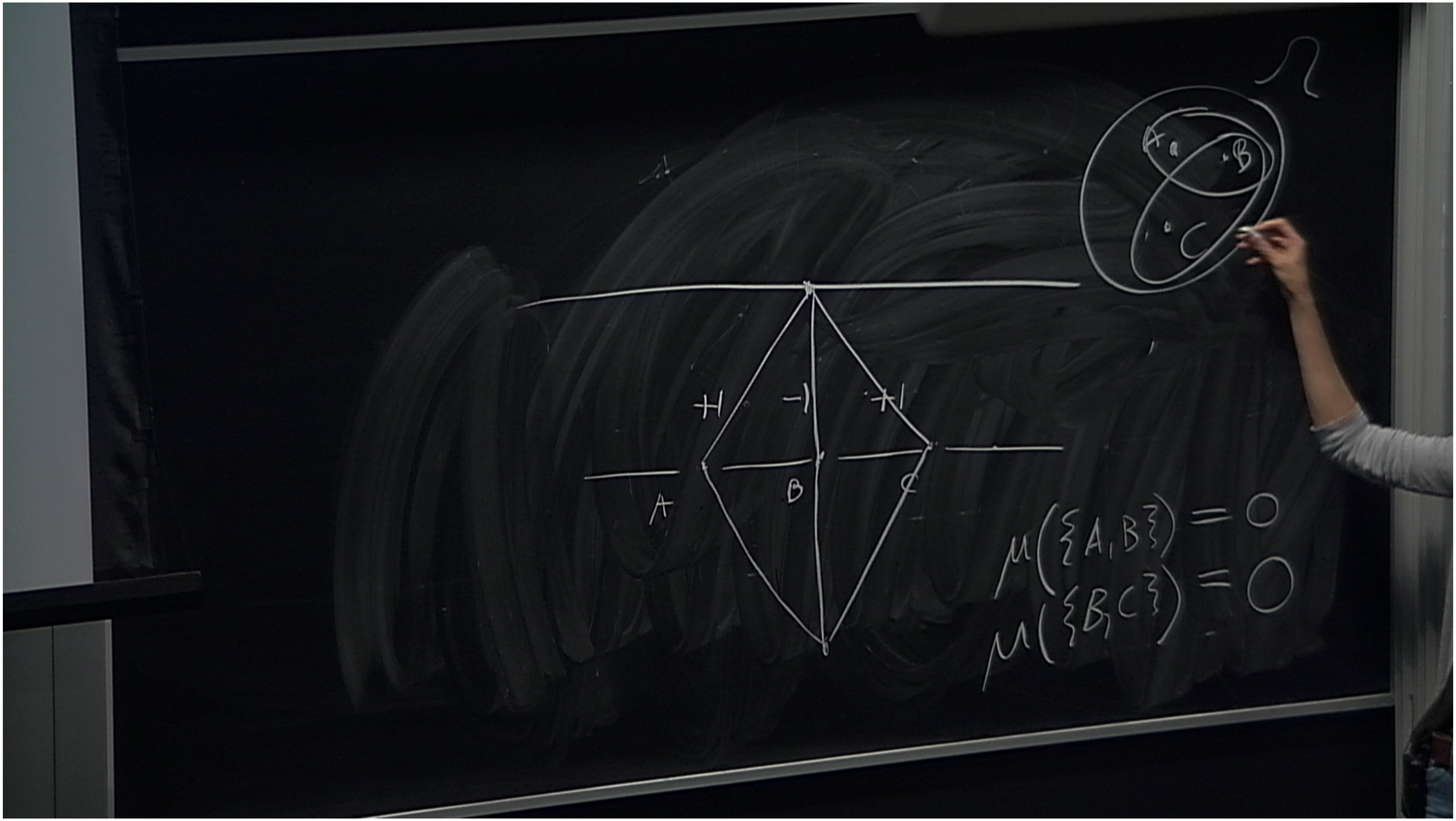
Antinomy 1: The Three Slit Experiment

BLACKBOARD

- Ω consists of three histories, A, B and C (actually A contains many particle trajectories but this simplification preserves the essential point).
- There are two events of measure zero: $\{A, B\}$ and $\{B, C\}$ and their union is the whole of Ω: the physical world cannot be a single history.



Ω
A
B
C
+1
-1
+1
A
B
C
μ({A, B}) = 0
μ({B, C}) = 0



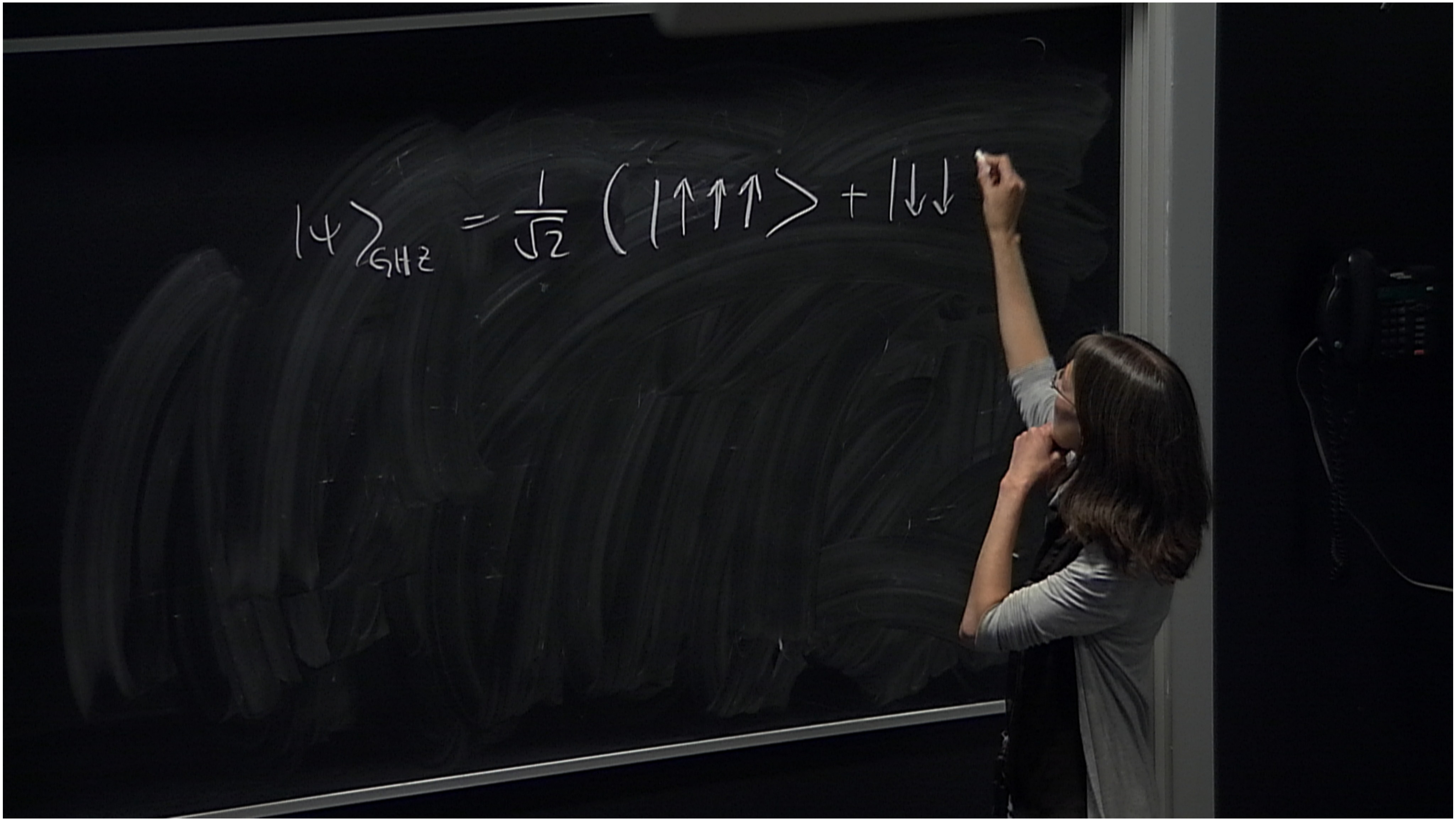
Ω
A
B
C
+1
−1
+1
A
B
C
μ({A, B}) = 0
μ({B, C}) = 0

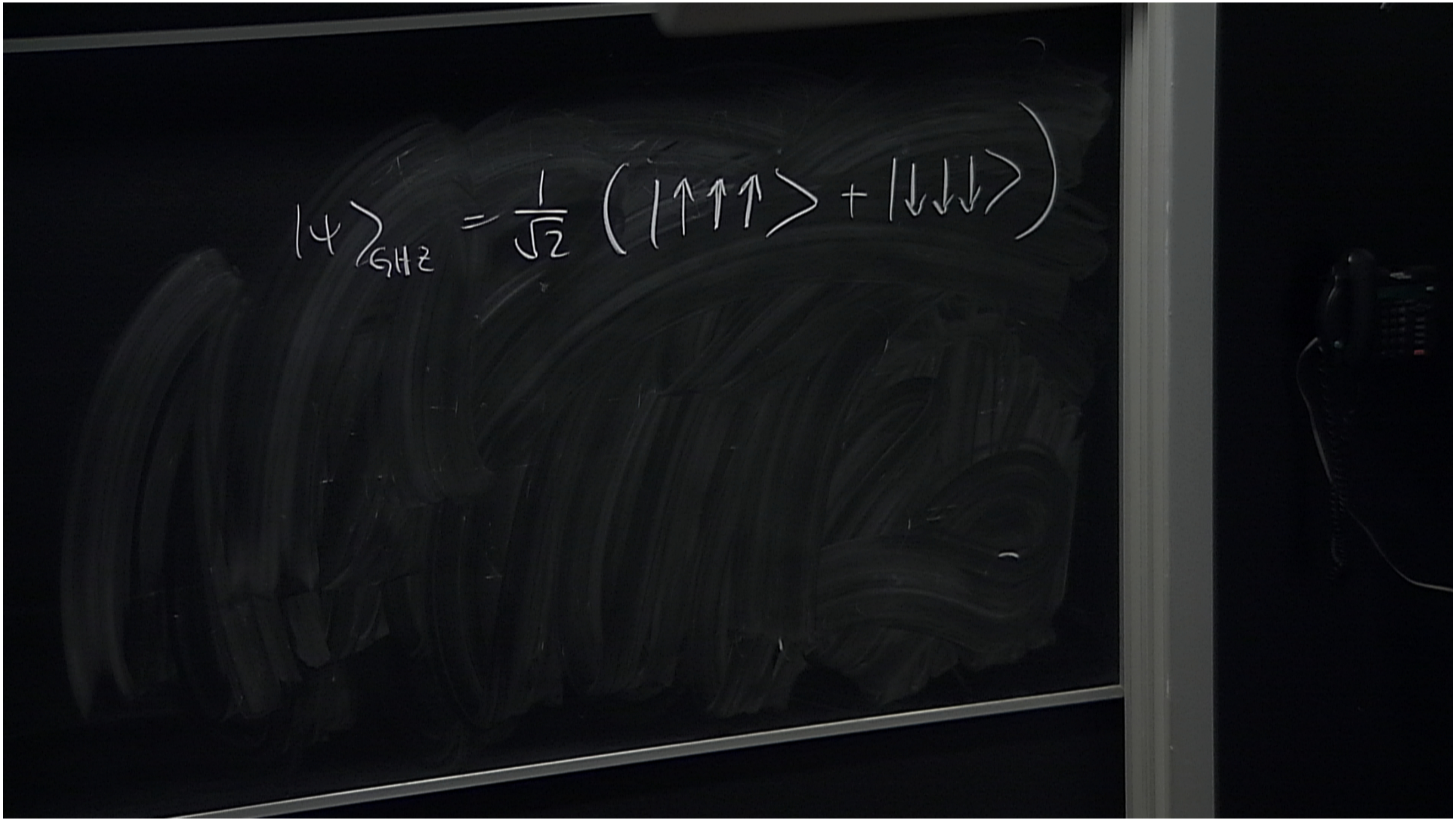
"One History Happens" and Preclusion are in conflict

- Change one or the other or both.
- One Proposal: keep preclusion as is and declare that possible physical worlds are (maximally fine grained) coarse grained histories that respect the preclusions.
  e.g. in three slit case $\{A, C\}$ happens. In the GHZ case, many possible coarse grained histories each with two fine-grained histories in.
- Other proposals: all based on the conception of a physical world as an affirmation or denial of every event: one might say,
  "The physical world is everything that is the case in spacetime" (to paraphrase Wittgenstein)

Guided by:

- reduces to a single history when the measure is classical
- gives "classical" answers about macroscopic events
- gives the Born rule for repeated quantum experiments
- allows the physical world to "evolve"
- throws light on relativistic causality for closed systems


$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}} \left(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow \right.$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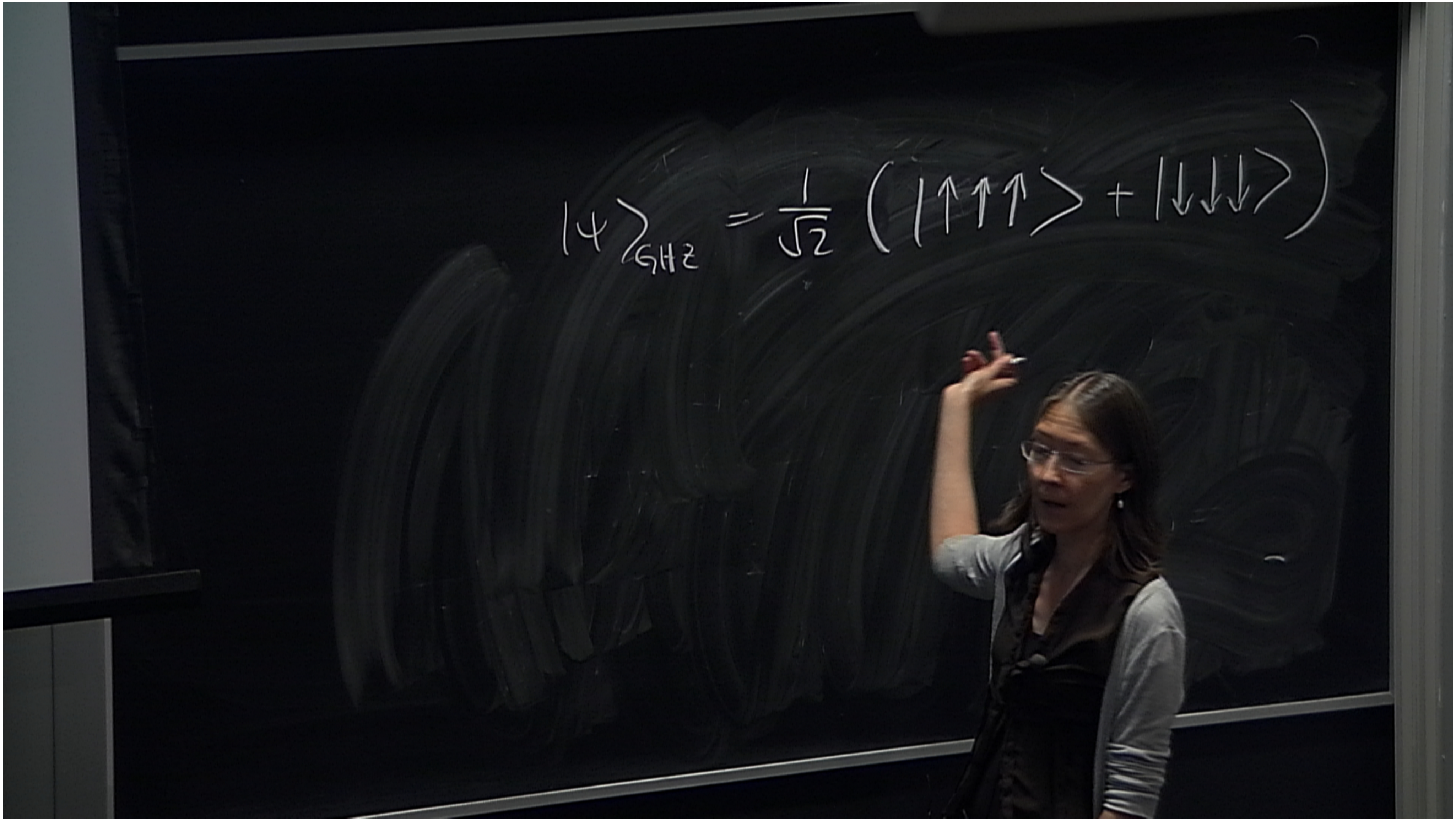
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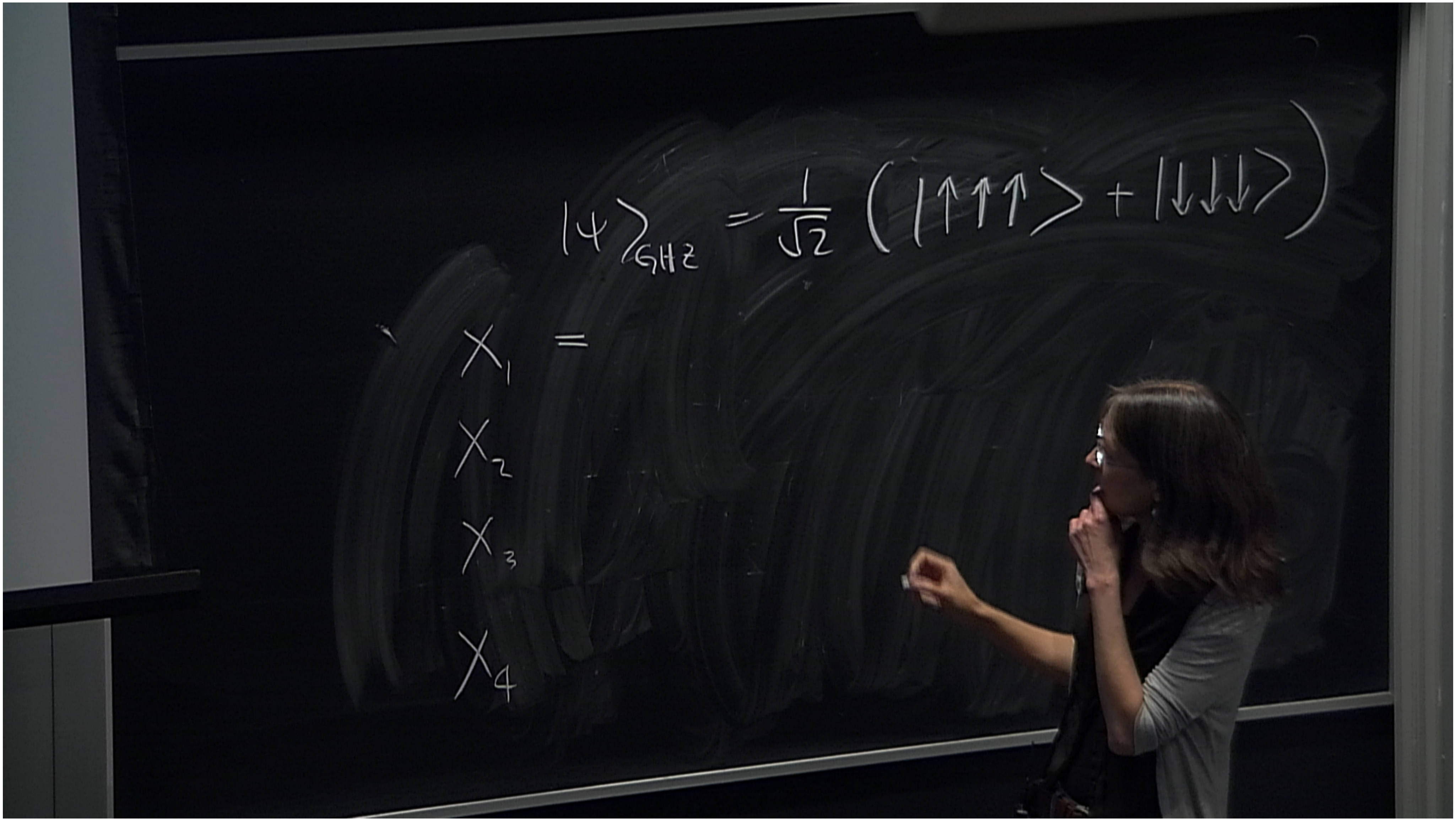
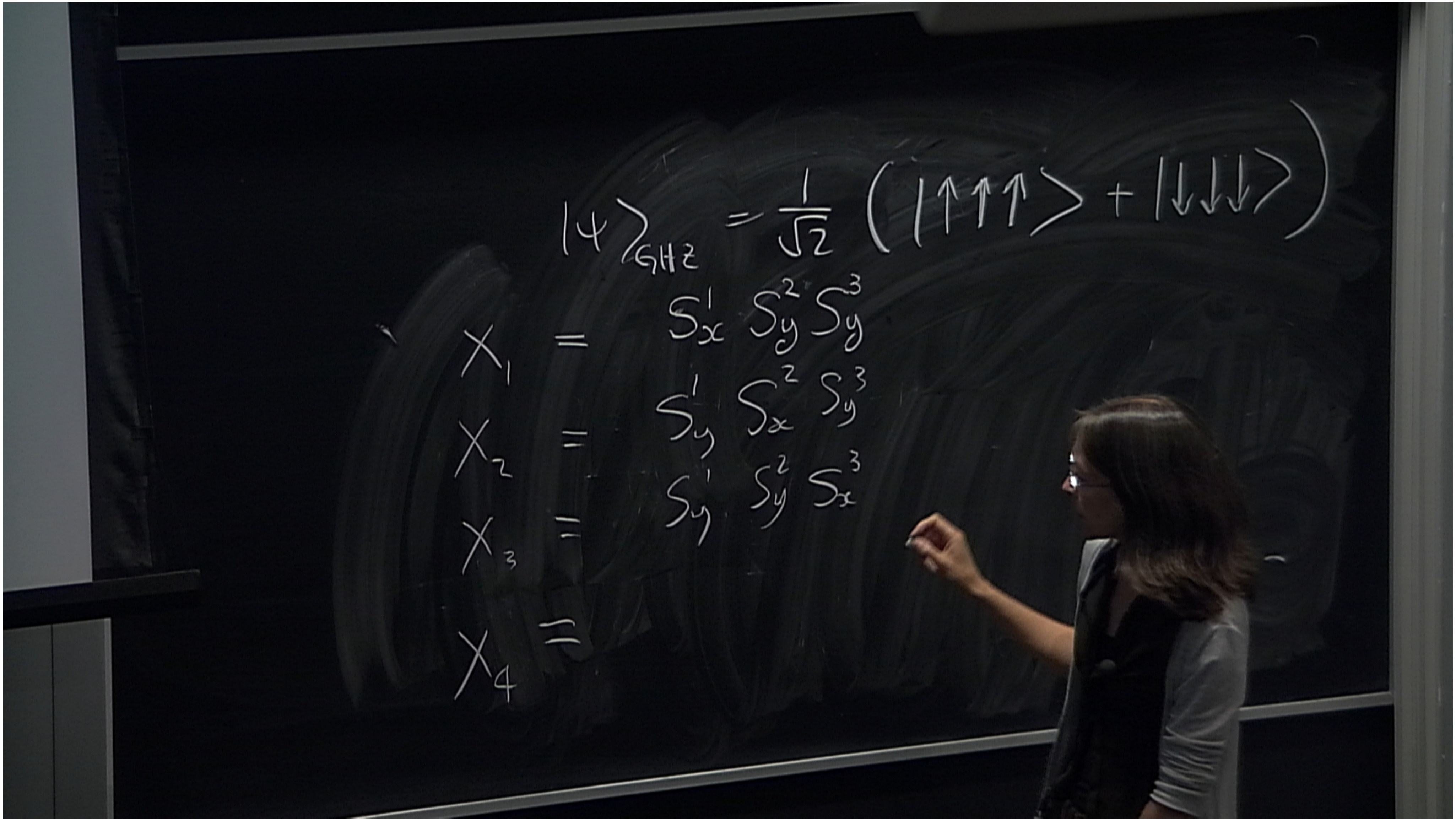
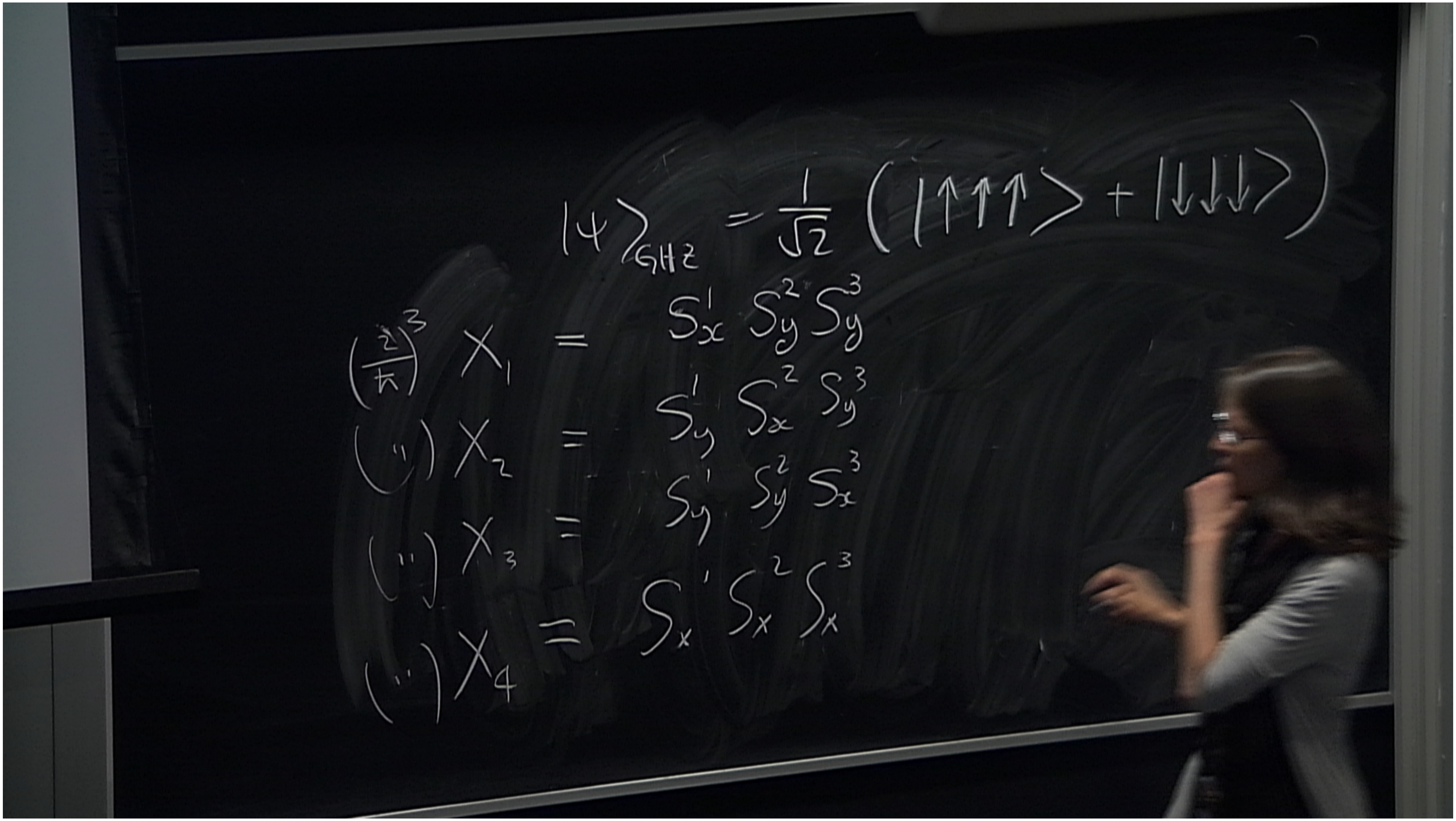
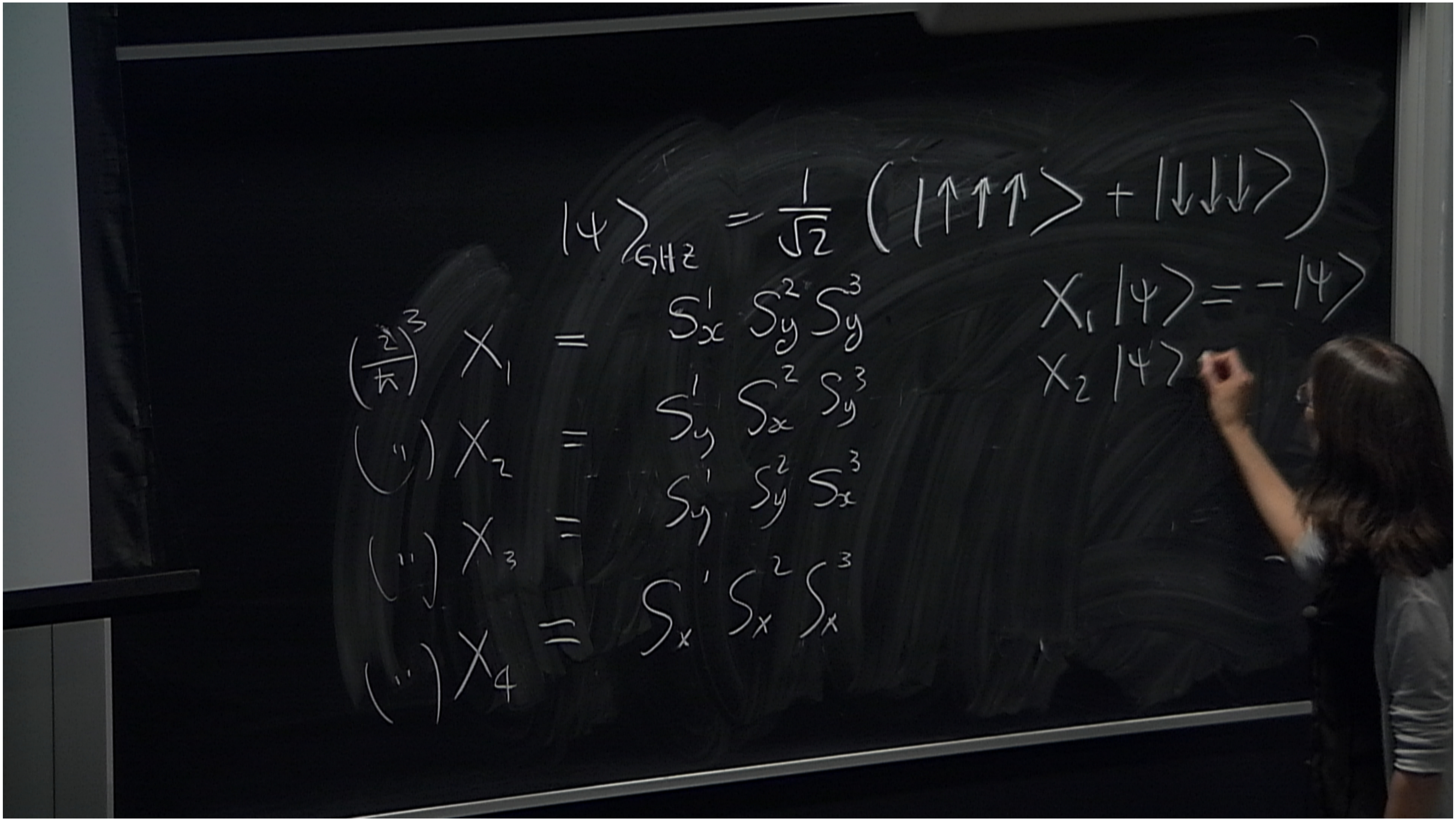
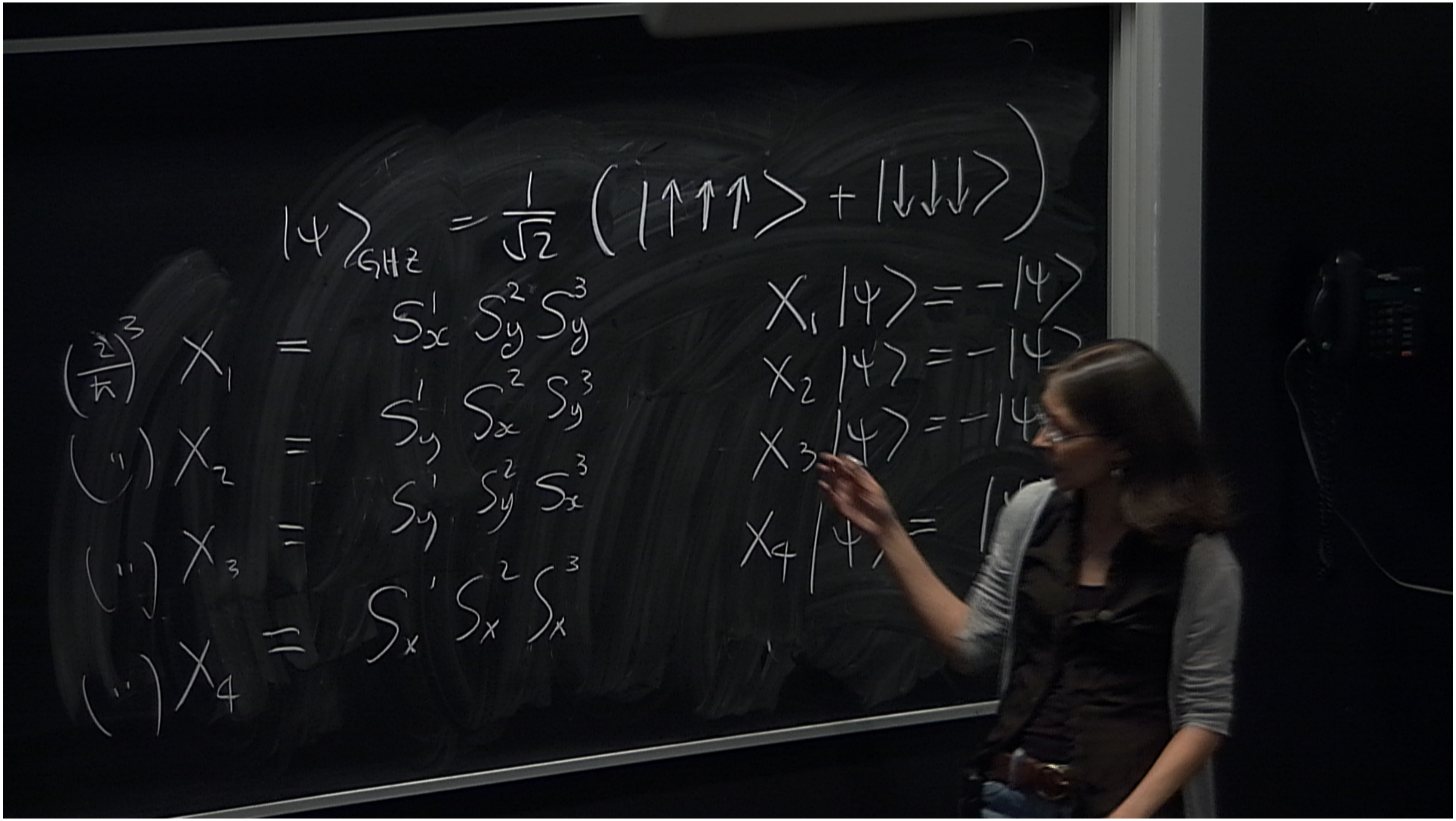
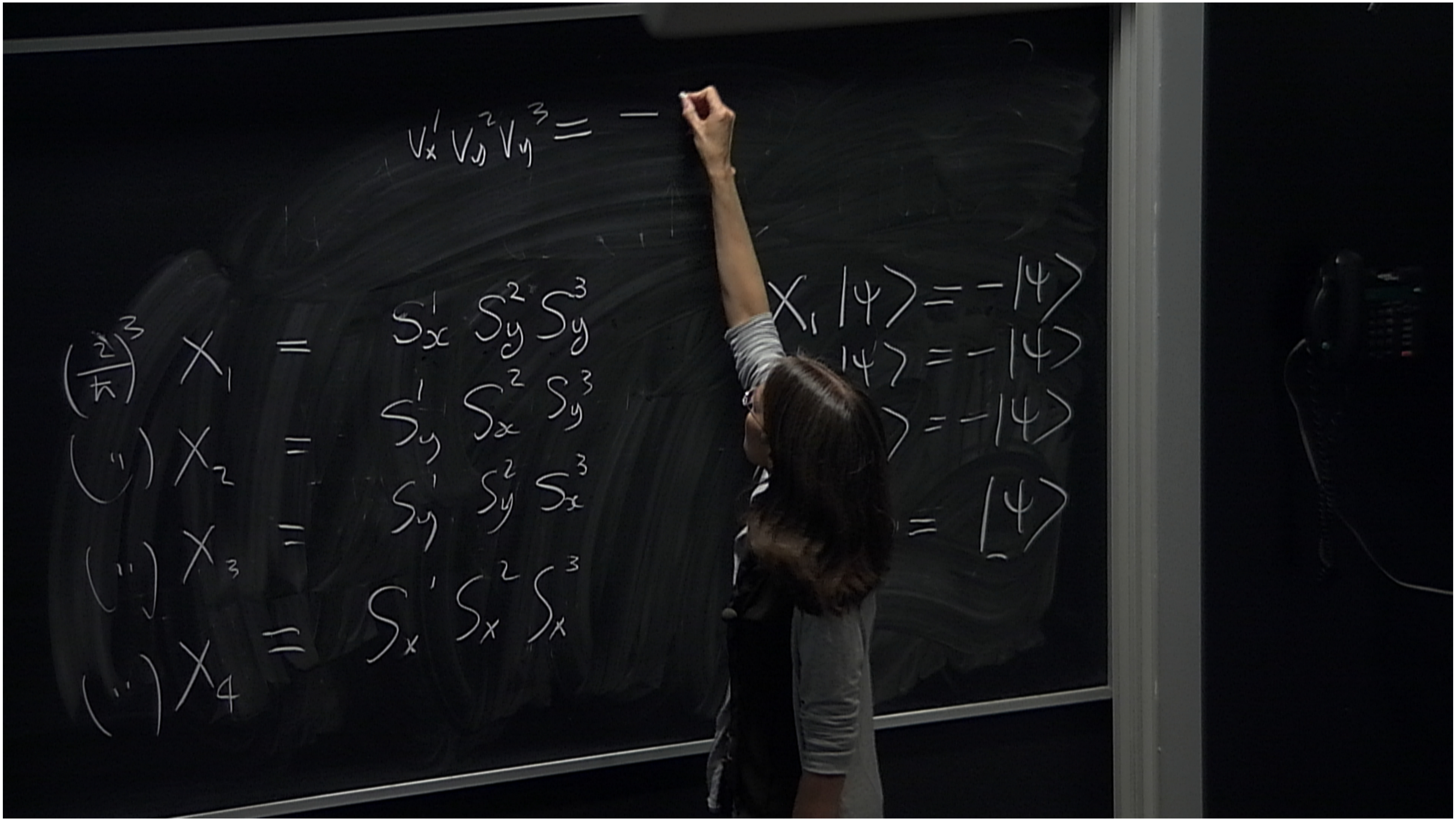

$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle\right)$
$X_1 =$
X_2
X_3
X_4

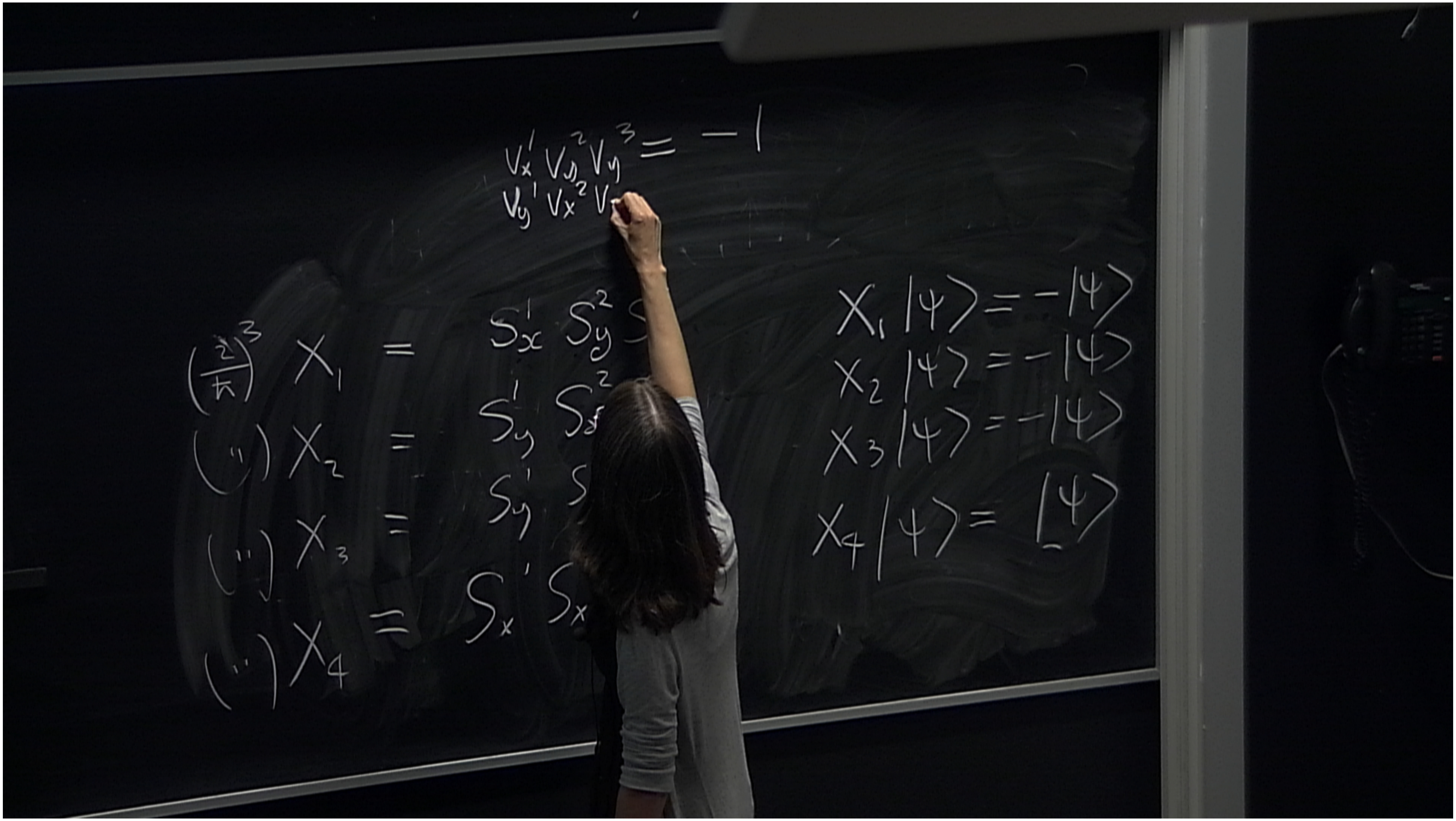

$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle\right)$
$X_1 = S_x^1 S_y^2 S_y^3$
$X_2 = S_y^1 S_x^2 S_y^3$
$X_3 = S_y^1 S_y^2 S_x^3$
$X_4 =$


$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle\right)$
$\left(\frac{2}{\hbar}\right)^3 X_1 = S_x^1 S_y^2 S_y^3$
$(\;''\;) X_2 = S_y^1 S_x^2 S_y^3$
$(\;''\;) X_3 = S_y^1 S_y^2 S_x^3$
$(\;''\;) X_4 = S_x^1 S_x^2 S_x^3$

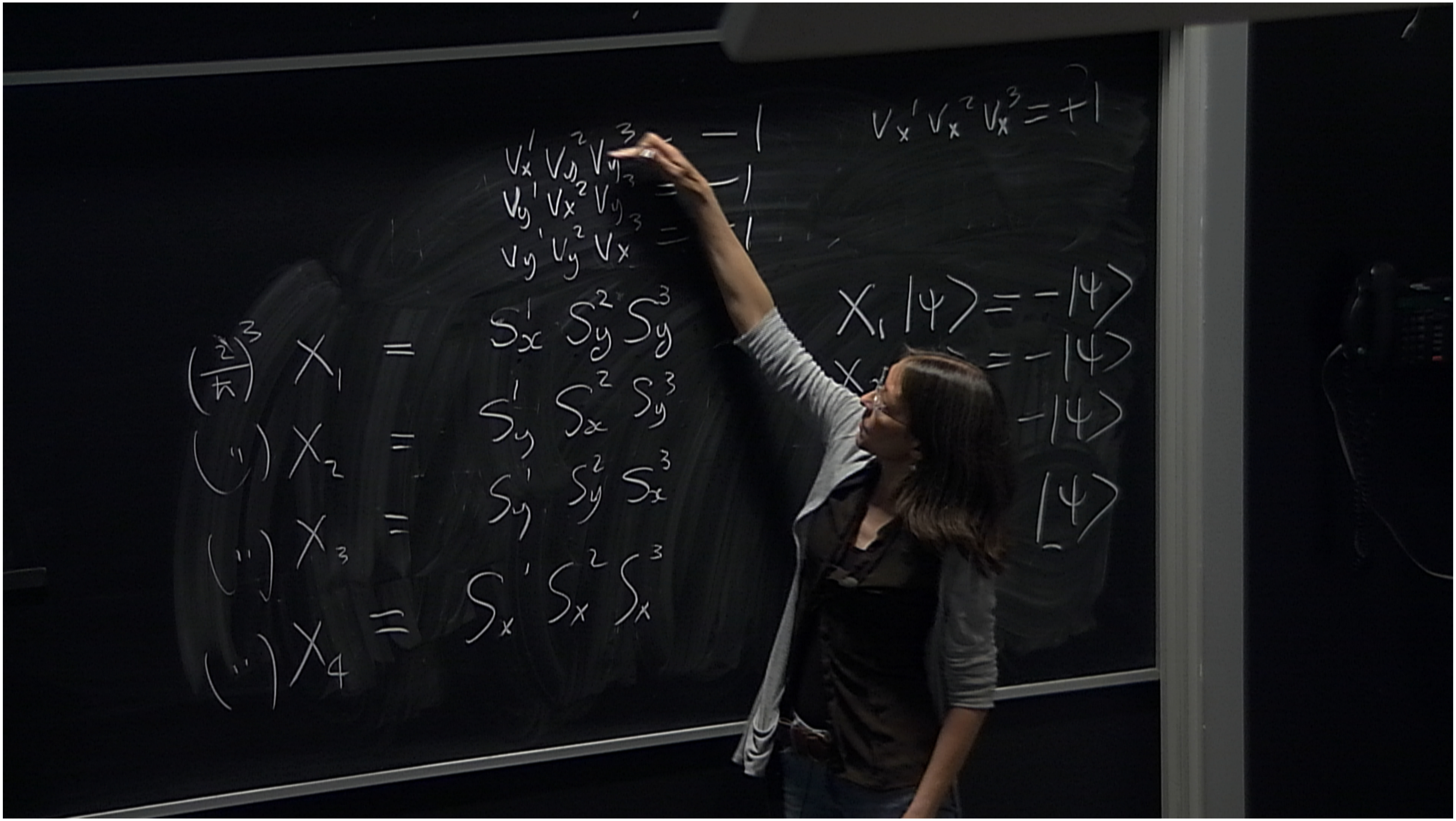

$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle\right)$
$X_1 |\psi\rangle = -|\psi\rangle$
$X_2 |\psi\rangle$
$\left(\frac{2}{\hbar}\right)^3 X_1 = S_x^1 S_y^2 S_y^3$
$(\;''\;) X_2 = S_y^1 S_x^2 S_y^3$
$(\;''\;) X_3 = S_y^1 S_y^2 S_x^3$
$(\;''\;) X_4 = S_x^1 S_x^2 S_x^3$


|ψ⟩_GHZ = 1/√2 (|↑↑↑⟩ + |↓↓↓⟩)
(2/ħ)^3 X_1 = S_x^1 S_y^2 S_y^3
(") X_2 = S_y^1 S_x^2 S_y^3
(") X_3 = S_y^1 S_y^2 S_x^3
(") X_4 = S_x^1 S_x^2 S_x^3
X_1 |ψ⟩ = −|ψ⟩
X_2 |ψ⟩ = −|ψ⟩
X_3 |ψ⟩ = −|ψ⟩
X_4 |ψ⟩ =

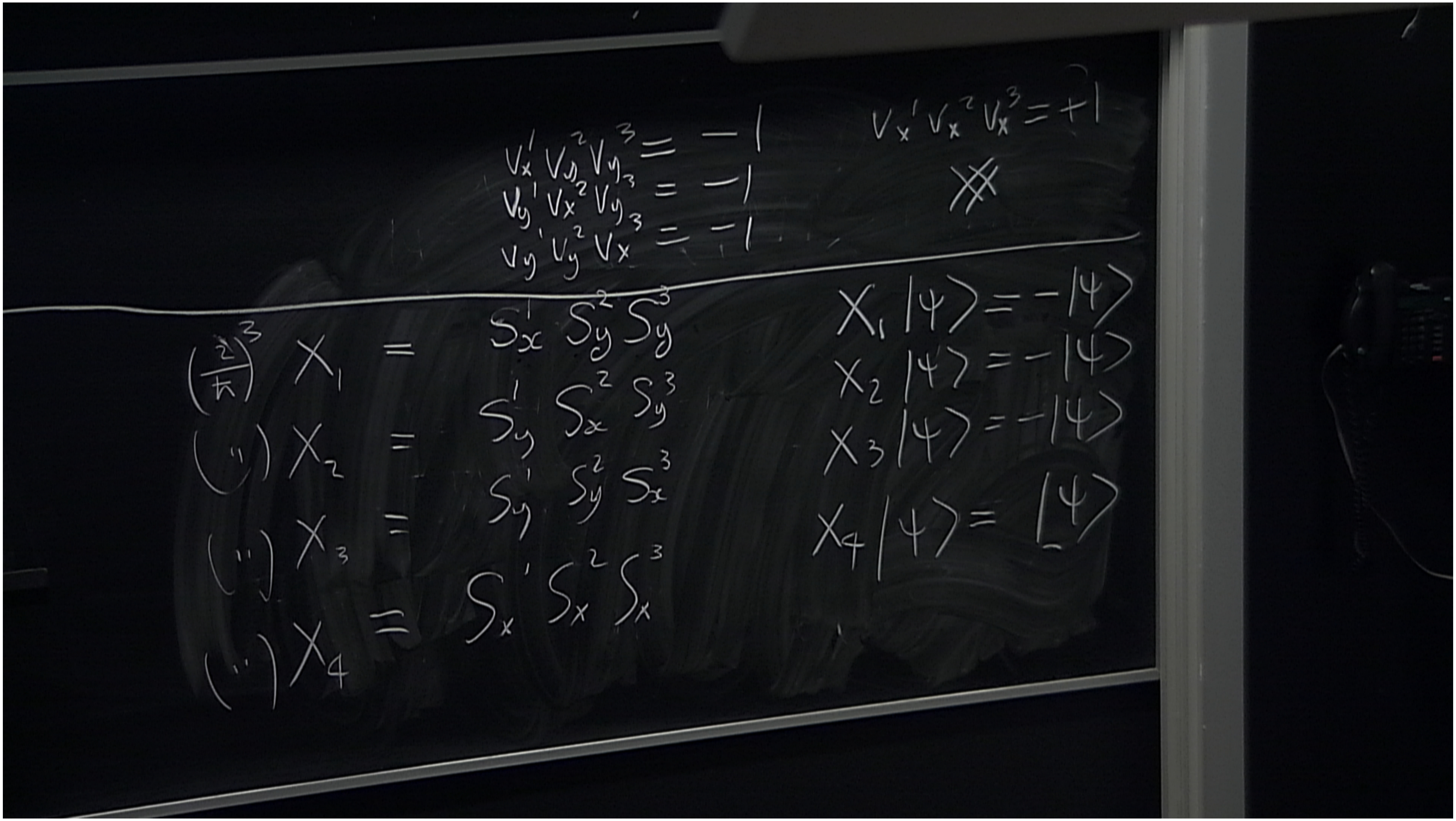

V_x^1 V_y^2 V_y^3 = −
(2/ħ)^3 X_1 = S_x^1 S_y^2 S_y^3
('') X_2 = S_y^1 S_x^2 S_y^3
('') X_3 = S_y^1 S_y^2 S_x^3
('') X_4 = S_x^1 S_x^2 S_x^3
X_1 |ψ⟩ = −|ψ⟩
|ψ⟩ = −|ψ⟩
⟩ = −|ψ⟩
= |ψ⟩



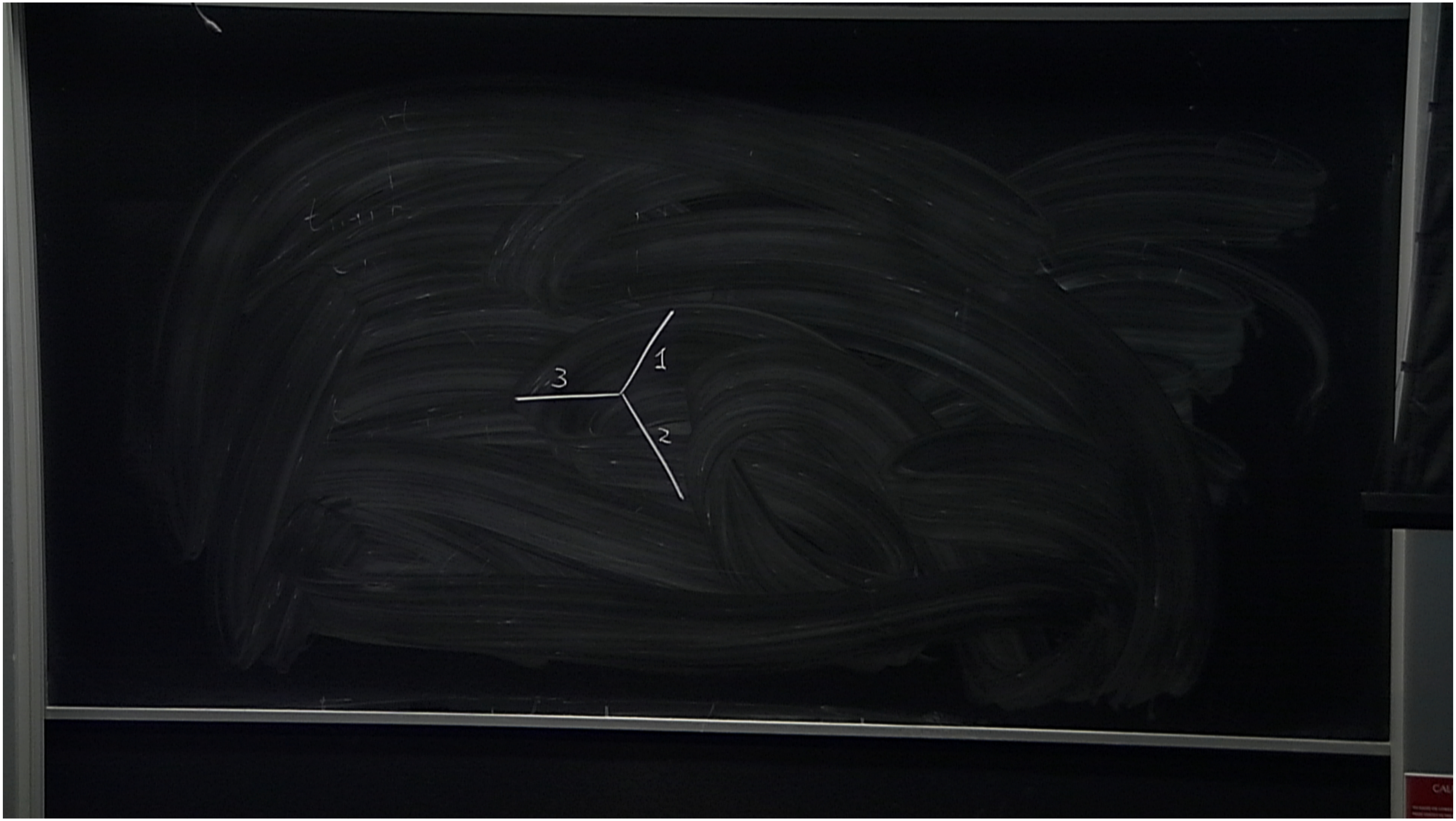
$V_x^1 V_y^2 V_y^3 = -1$
$X_1 |\psi\rangle = -|\psi\rangle$
$X_2 |\psi\rangle = -|\psi\rangle$
$X_3 |\psi\rangle = -|\psi\rangle$
$X_4 |\psi\rangle = |\psi\rangle$
$\left(\frac{2}{\hbar}\right)^3 X_1 =$
$X_2 =$
$X_3 =$
$X_4 =$



Vx1 Vy2 Vy3 = −1
Vy1 Vx2 Vy3 = −1
Vy1 Vy2 Vx3
Vx1 Vx2 Vx3 = +1
(2/ħ)3 X1 = Sx1 Sy2 Sy3
X2 = Sy1 Sx2 Sy3
X3 = Sy1 Sy2 Sx3
X4 = Sx1 Sx2 Sx3
X1 |ψ⟩ = −|ψ⟩
= −|ψ⟩
−|ψ⟩
|ψ⟩



$V_x^1 V_y^2 V_y^3 = -1$
$V_y^1 V_x^2 V_y^3 = -1$
$V_y^1 V_y^2 V_x^3 = -1$
$V_x^1 V_x^2 V_x^3 = +1$
$\left(\frac{2}{\hbar}\right)^3 X_1 = S_x^1 S_y^2 S_y^3$
$(\;''\;) X_2 = S_y^1 S_x^2 S_y^3$
$(\;''\;) X_3 = S_y^1 S_y^2 S_x^3$
$(\;''\;) X_4 = S_x^1 S_x^2 S_x^3$
$X_1 |\psi\rangle = -|\psi\rangle$
$X_2 |\psi\rangle = -|\psi\rangle$
$X_3 |\psi\rangle = -|\psi\rangle$
$X_4 |\psi\rangle = |\psi\rangle$


1
2
3

$$V_x^1 V_y^2 V_y^3 = -1 \qquad V_x^1 V_x^2 V_x^3 = +1$$

$$V_y^1 V_x^2 V_y^3 = -1$$

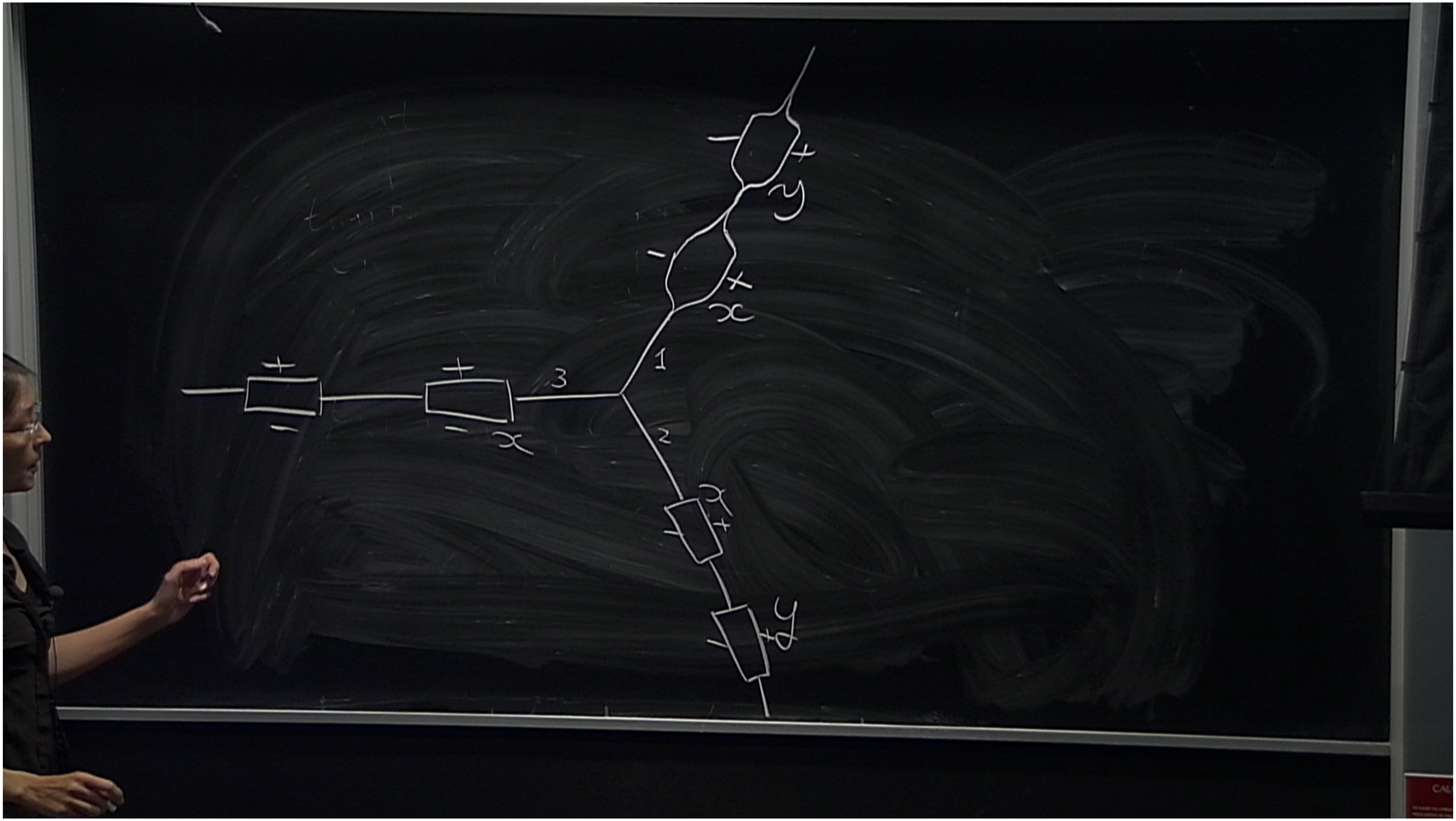
$$V_y^1 V_y^2 V_x^3 = -1$$

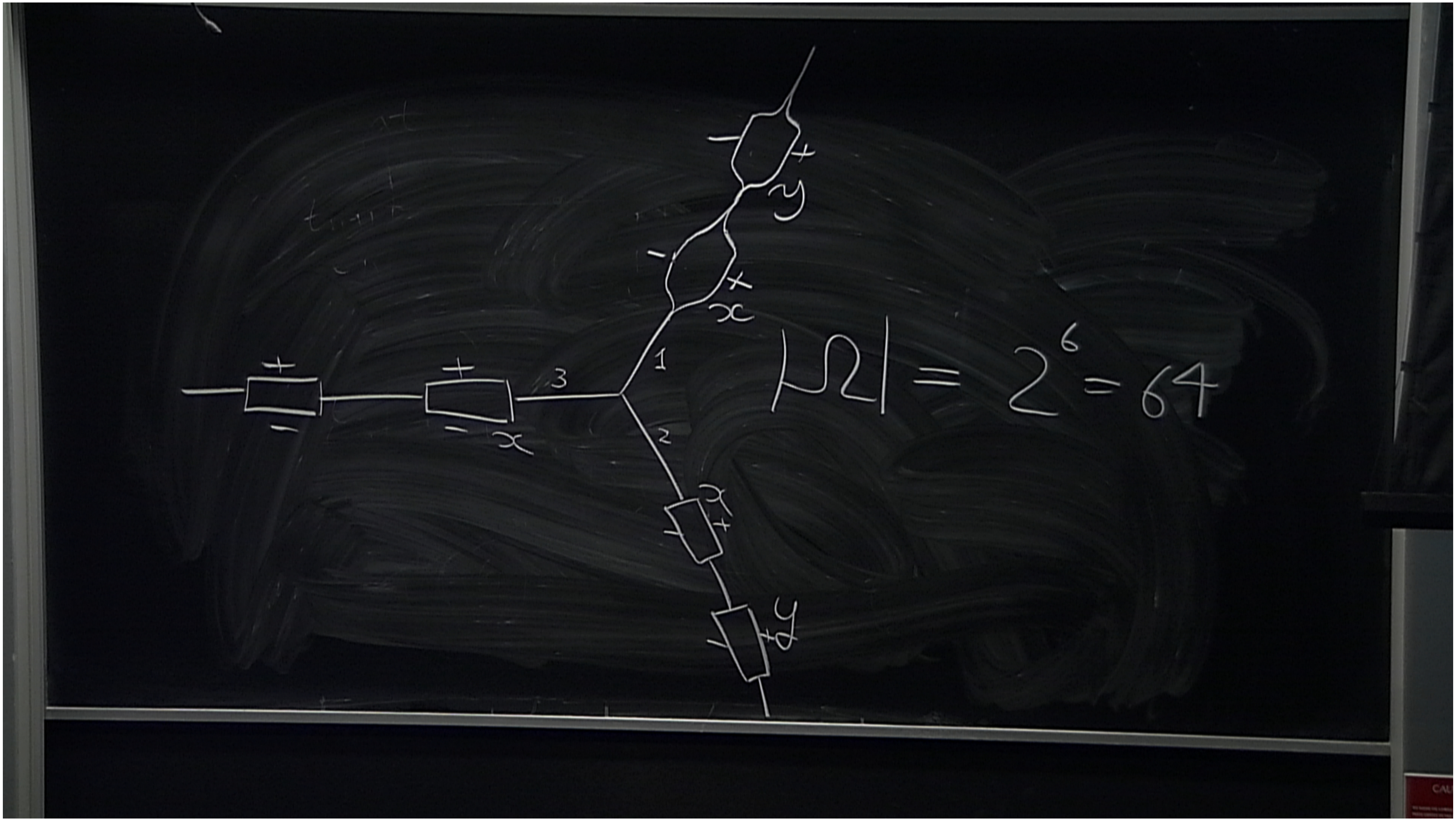
$$\left(\frac{2}{\hbar}\right)^3 X_1 = S_x^1 S_y^2 S_y^3 \qquad X_1|\psi\rangle = -|\psi\rangle$$

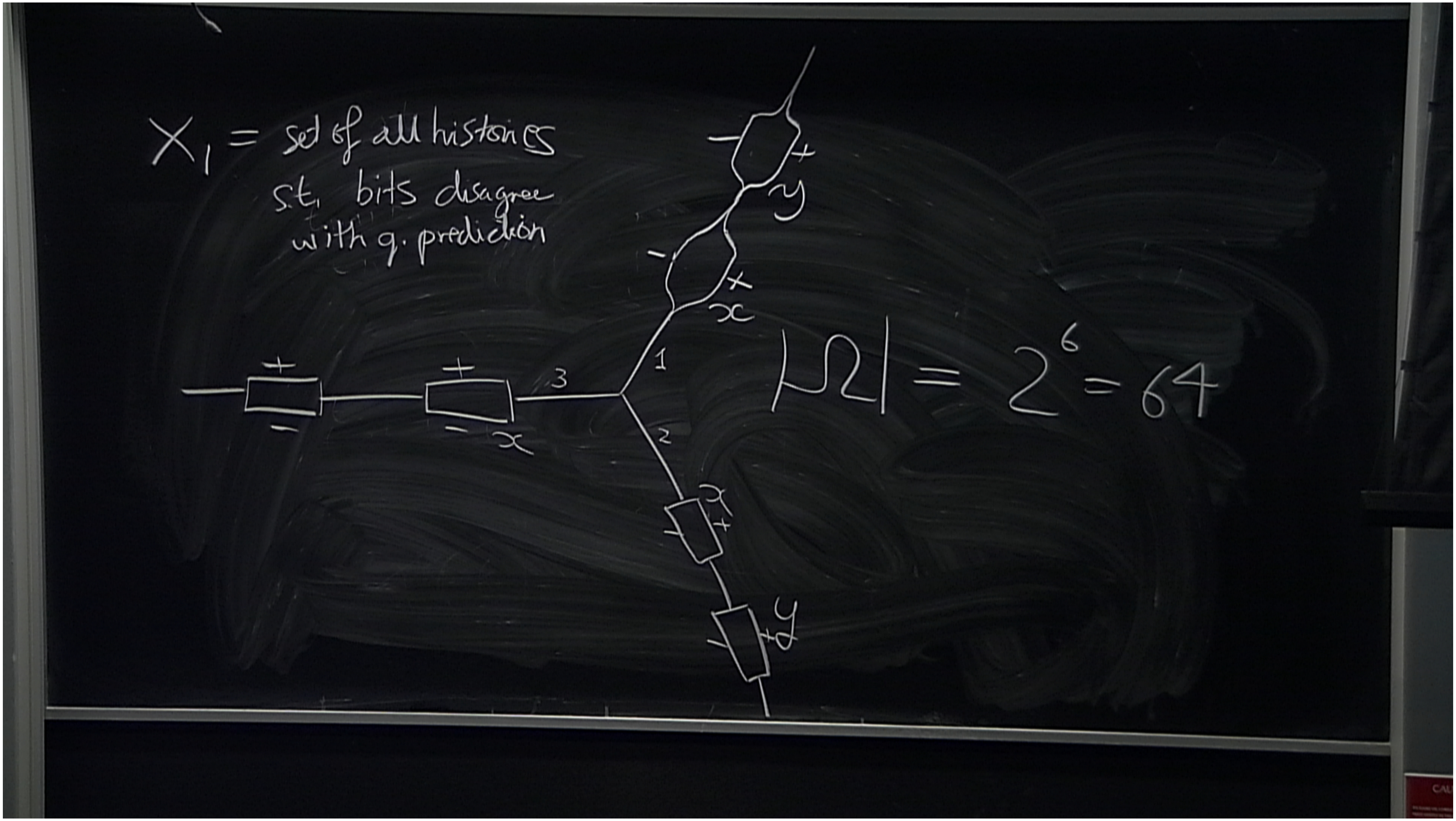
$$(\;''\;)\, X_2 = S_y^1 S_x^2 S_y^3 \qquad X_2|\psi\rangle = -|\psi\rangle$$

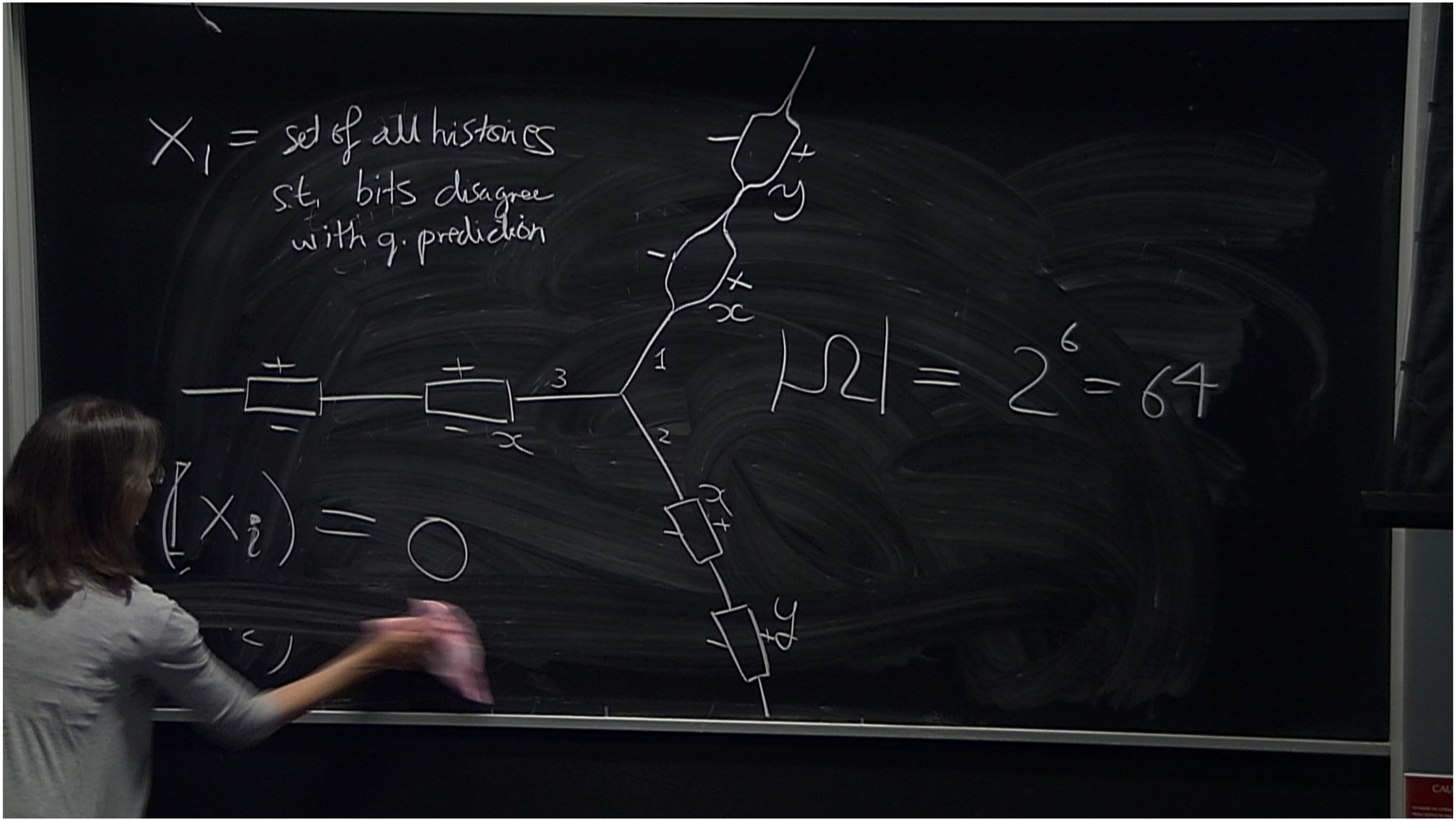
$$(\;''\;)\, X_3 = S_y^1 S_y^2 S_x^3 \qquad X_3|\psi\rangle = -|\psi\rangle$$

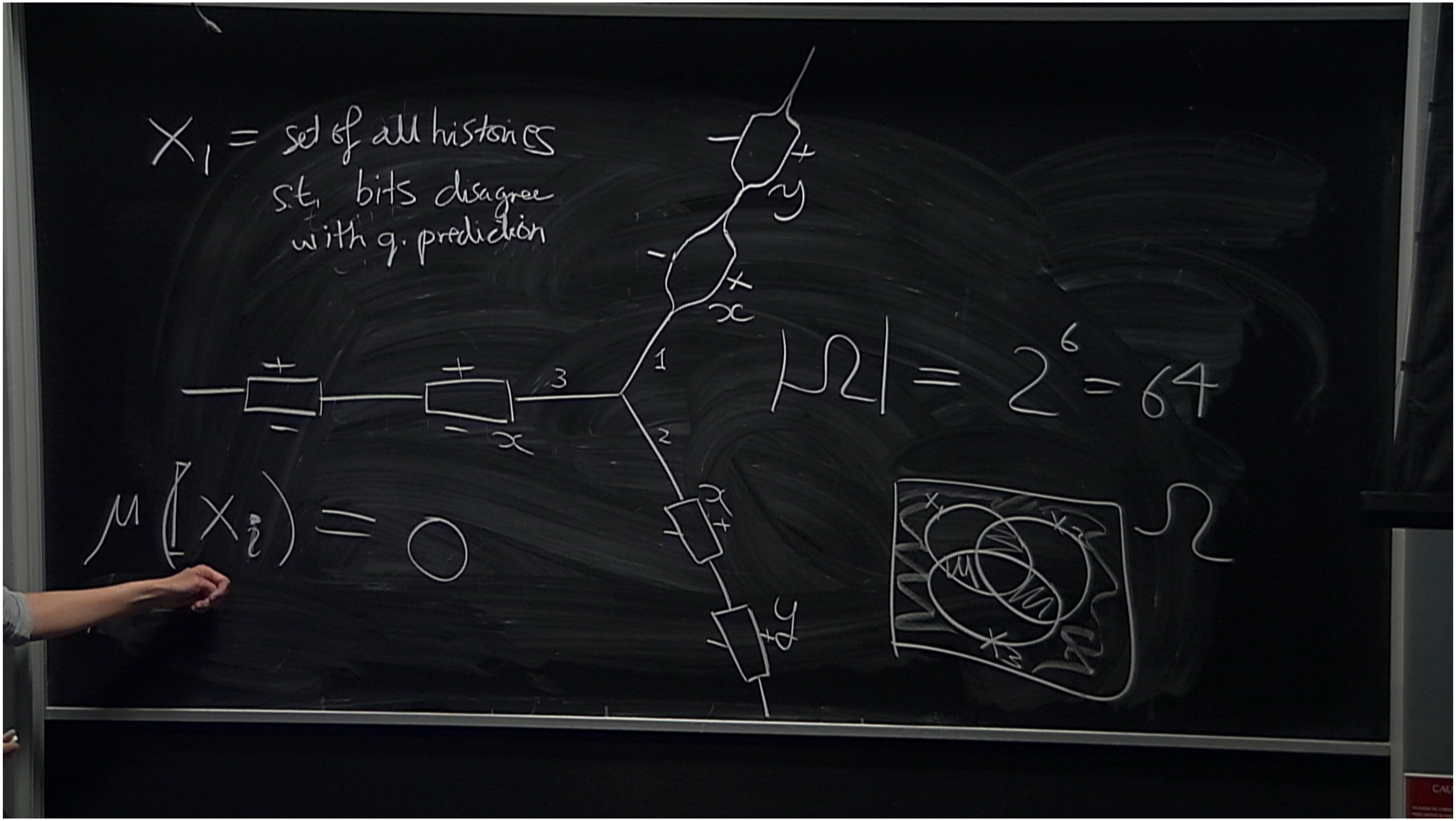
$$(\;''\;)\, X_4 = S_x^1 S_x^2 S_x^3 \qquad X_4|\psi\rangle = |\psi\rangle$$

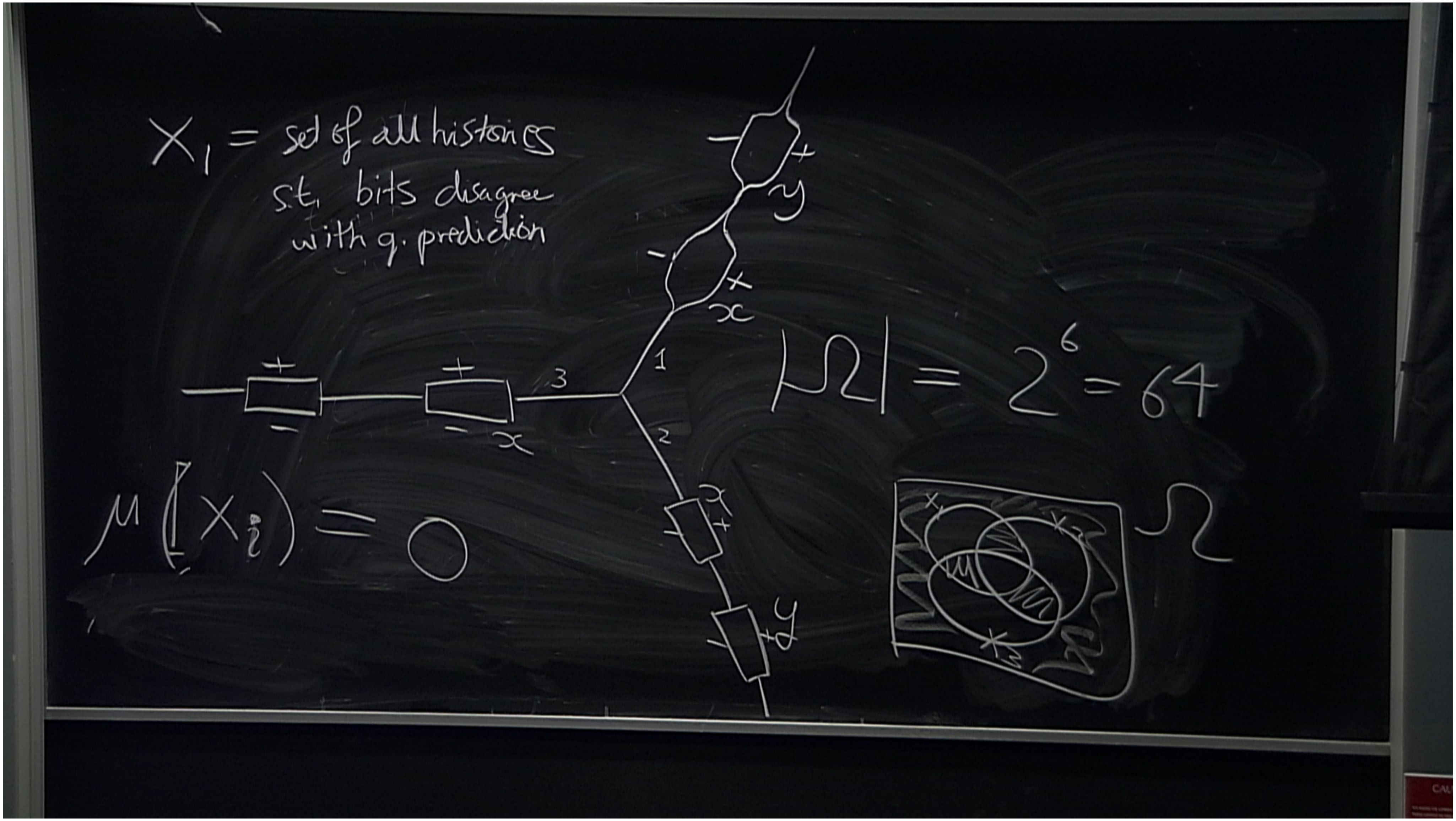

+
+
−
−
3
x
1
2
−
+
x
−
+
y
x
+
−
y
+
−


$|\Omega| = 2^6 = 64$


X1 = set of all histories
s.t. bits disagree
with q. prediction
|Ω| = 2^6 = 64
1
2
3
x
y
+
-


X1 = set of all histories
s.t. bits disagree
with q. prediction
|Ω| = 2^6 = 64
(Xi) = 0


X₁ = set of all histories
s.t. bits disagree
with q. prediction
|Ω| = 2⁶ = 64
μ(Xᵢ) = 0
Ω


X1 = set of all histories
s.t. bits disagree
with q. prediction
|Ω| = 2^6 = 64
μ(Xi) = 0
Ω