Title: Causal Constraints on Possible Measurements

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Abstract: A
crucial question in any approach to quantum information processing
is: first, how are classical bits

encoded
physically in the quantum system, second, how are they then manipulated and,
third, how are they finally read out?

These
questions are particularly challenging when investigating quantum
information processing in a relativistic spacetime. An obvious
framework for such an investigation is relativistic quantum field
theory. Here, progress is hampered by the lack of a universally
applicable rule for calculating the probabilities of the outcomes of ideal
measurements on a relativistic quantum field in a collection of spacetime
regions. Indeed,
a straightforward relativistic generalisation of the non-relativistic formula
for these probabilities leads to superluminal signalling.

Motivated
by these considerations we ask what interventions/ideal measurements can we in
principle make, taking causality as our guiding criterion. In the course
of this analysis we reconsider various aspects of ideal measurements in QFT,
detector models and the probability rules themselves. In particular, it is

shown that an ideal measurement of a oneâ€“particle wave packet state of a relativistic quantum field in Minkowski spacetime enables superluminal signalling. The result holds for a measurement that takes place over an intervention region in spacetime whose extent in time in some frame is longer than the light crossing time of the packet in that frame.

Causal constraints on measurement

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Show me the bits!

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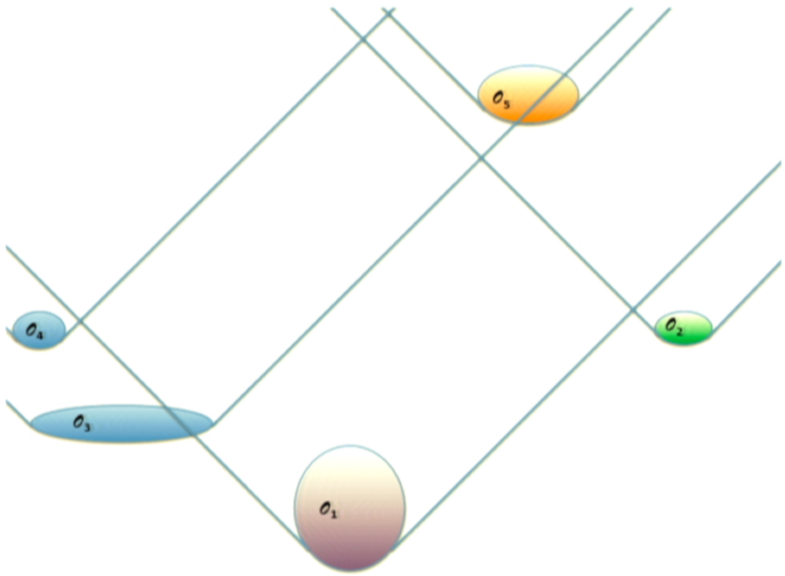
Particularly challenging/interesting in the relativistic setting of quantum field theory taking into account the locations in spacetime of the actions of the external agents.

A straightforward relativistic generalisation of the non-relativistic formula for probabilities

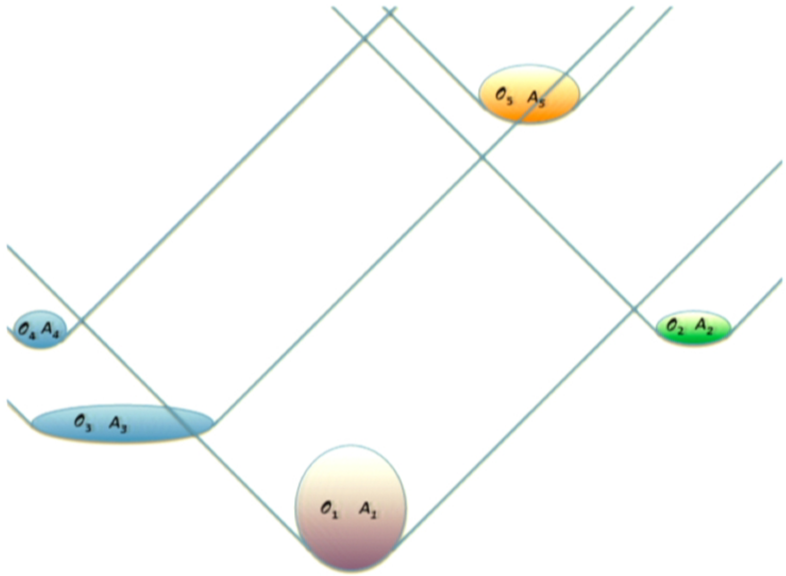
- $\mathcal{O}_i$, $i = 1, \dots n$: regions in a globally hyperbolic spacetime.
- $\mathcal{O}_j \preceq \mathcal{O}_k$ iff some point in $\mathcal{O}_j$ is in the causal past of some point in $\mathcal{O}_k$.



- Can then label regions $i = 1, \dots n$ such that $\mathcal{O}_j \preceq \mathcal{O}_k$ implies $j \leq k$.

A straightforward relativistic generalisation of the formula for probabilities

- Consider, for each i, the measurement of an observable A_i in region $\mathcal{O}_i$

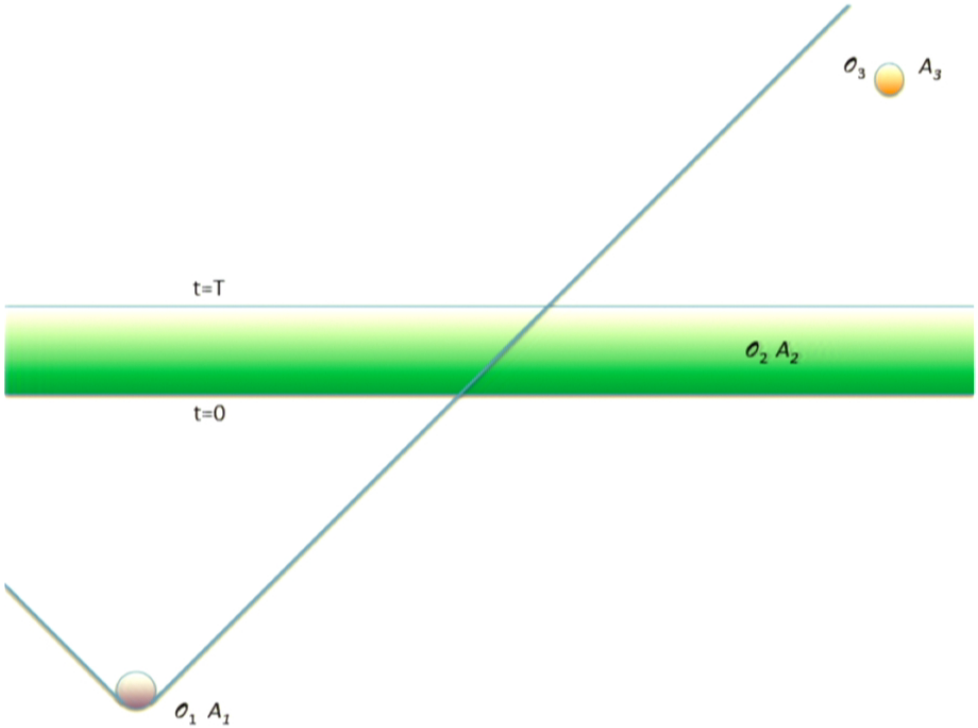


- The probability of obtaining those particular outcomes to the sequence of measurements in the regions $\mathcal{O}_i$ is proposed to be [Sorkin:1993]

$$\mathrm{Tr}\left(P_n \ldots P_1 \rho P_1 \ldots P_n\right)$$

Impossible measurements of wave packets: The general setup

$(d+1)$-dim Minkowski space and a free massless scalar field $\hat{\phi}(x)$ [Sorkin:1993]:



$\mathcal{O}_1 = X^\mu, X^0 \leq 0$ $\qquad$ $\mathcal{O}_2 = 0 \leq t < T$ $\qquad$ $\mathcal{O}_3 = Y^\mu, Y^0 \geq T$

$A_1 = e^{i\lambda\hat{\phi}(X)}$ $\qquad$ $A_2 = |1\rangle\langle 1|$ $\qquad$ $A_3 = \phi(Y)$

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- Measure of superluminal signal

$$S(Y) := \mathrm{Im}[\psi(Y)]$$

where $\psi(Z) := \langle 0|\phi(Z)|1\rangle$ is the "one–particle wavefunction".

- When $|1\rangle$ is a one particle state with a precise d-momentum, $\mathbf{k}$,

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- Moreover,

$$\psi(Y^\mu + \xi^\mu) = \psi(Y^\mu)$$

where ξ^μ is any null vector proportional to the 4-momentum, $k^\mu = (|\mathbf{k}|, \mathbf{k})$.

- Superluminal signal remains no matter how large T.
- Not surprising, given the nonlocal character of a fixed momentum state: it is defined on an entire spacelike hypersurface.

Other impossible interventions: Unitary rotations

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$$U = e^{i\theta}(C|1\rangle\langle 1| + D|1\rangle\langle 1'| - D^*|1'\rangle\langle 1| + C^*|1'\rangle\langle 1'|) + \mathbb{1}^{\perp}, \quad |C|^2 + |D|^2 = 1$$

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Let's work in a box of side length L, so that

$$\hat{\phi}(X) = L^{-\frac{d}{2}}\sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_{\mathbf{k}}}}\left[a_{\mathbf{k}}e^{ik_\mu X^\mu} + a^\dagger_{\mathbf{k}}e^{-ik_\mu X^\mu}\right]$$

$|1\rangle = a^\dagger_{\mathbf{p}}|0\rangle$ and $|1'\rangle = a^\dagger_{-\mathbf{p}}|0\rangle \implies$ signal:

$$-\frac{2L^{-d}}{\omega_{\mathbf{k}}}\sin\left[\omega_{\mathbf{k}}(Y^0 - X^0)\right]\cos\left[\mathbf{k}\cdot(\mathbf{Y}+\mathbf{X})\right]$$

Only $e^{i\int d^d x f(x)\hat{\phi}(t,x)}$ type unitaries ensure no sulu signal

Different rules

1. *Restricting the regions*: For example, one might require that for every pair $(\mathcal{O}_j, \mathcal{O}_k)$ such that $j < k$, either the two regions are entirely spacelike to each other or every point of $\mathcal{O}_j$ is in the causal past of every point of $\mathcal{O}_k$.

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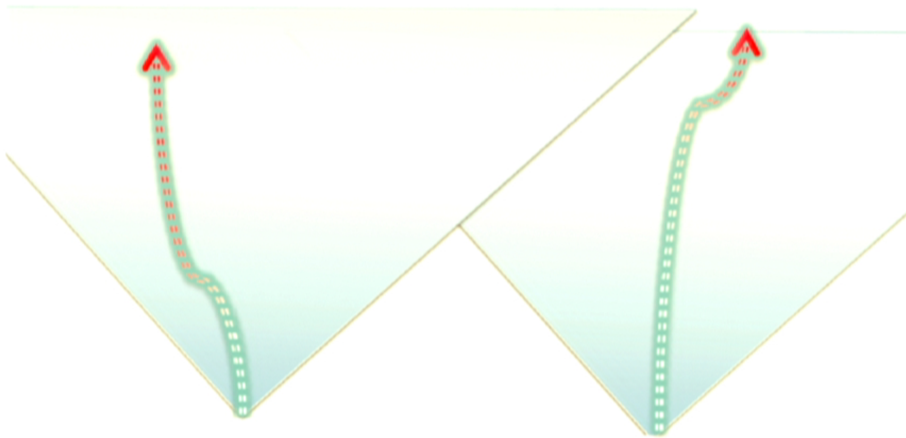
2. *Restricting the observables:* Alternatively, one could restrict the observables, begging the question why some observables are measurable and others not.

Field operators smeared with real functions over subsets of spacelike hypersurfaces are essentially local. See [Brukner, Costa, Kofler, and Zych: 2010]

Detector models

Detector models

Unruh-Dewitt (UD) detector [Unruh:1976,DeWitt:1980]:



$H_{I,1} \sim \lambda_1 Q_1 \phi(X(\tau))$ $H_{I,2} \sim \lambda_1 Q_2 \phi(X(\sigma))$

Detector models: finite mode couplings

(1+1)-dim 2-detector example:

$$\hat{H}(t) = \sum_{i=1}^{2} \frac{w_i}{2}(\hat{q}_i^2 + \hat{p}_i^2) + \frac{\Omega}{2}(\hat{Q}^2 + \hat{P}^2) + 2\sum_{i=1}^{2} \lambda_i(t)\hat{q}_i[\hat{Q}\cos(\Omega x_i) - \hat{P}\sin(\Omega x_i)]$$

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- Interaction Hamiltonians do not commute $[H_{1,\mathrm{int}}(t_1), H_{2,\mathrm{int}}(t_2)] =$

$$-2i\lambda_1(t_1)\lambda_2(t_2)(\hat{d}_1 e^{-i\omega_1 t_1} + \hat{d}_1^\dagger e^{i\omega_1 t_1})(\hat{d}_2 e^{-i\omega_2 t_2} + \hat{d}_2^\dagger e^{i\omega_2 t_2}) \sin \Omega_\mu (X_1 - X_2)^\mu$$

nonzero for almost every pair of spacetime points along spacelike trajectories

$$\left[\hat{d}_i = \frac{1}{\sqrt{2}}(\hat{q}_i + i\hat{p}_i), \qquad \Omega^\mu := (\Omega, \Omega)\right]$$

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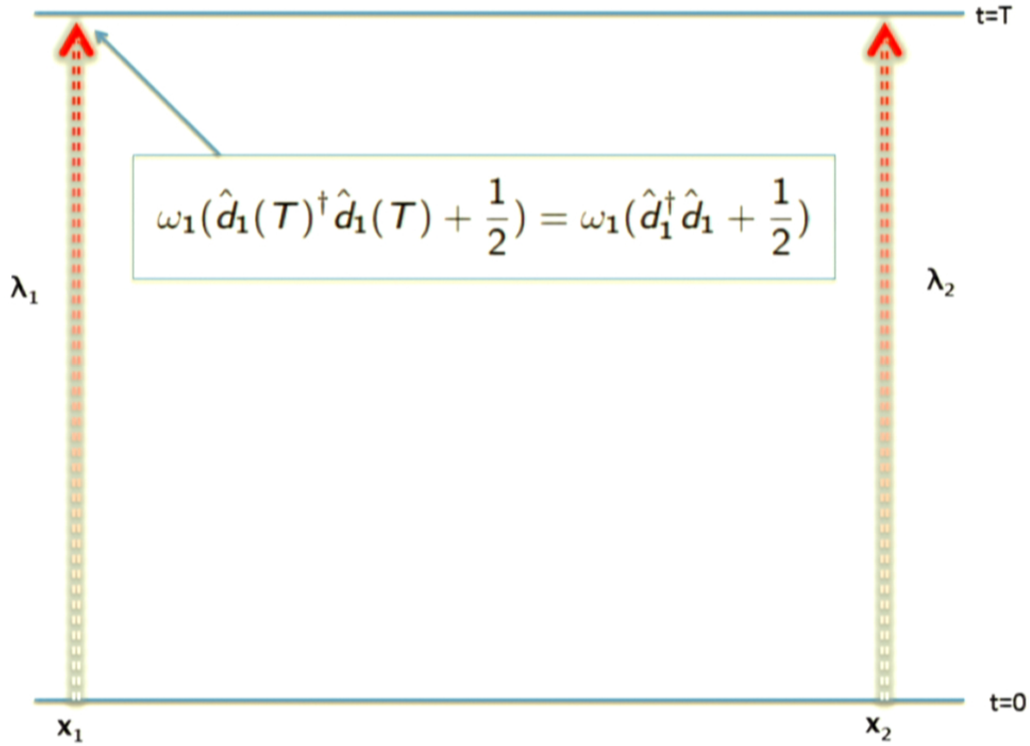

$$\omega_1(\hat{d}_1(T)^\dagger \hat{d}_1(T) + \frac{1}{2}) = \omega_1(\hat{d}_1^\dagger \hat{d}_1 + \frac{1}{2})$$

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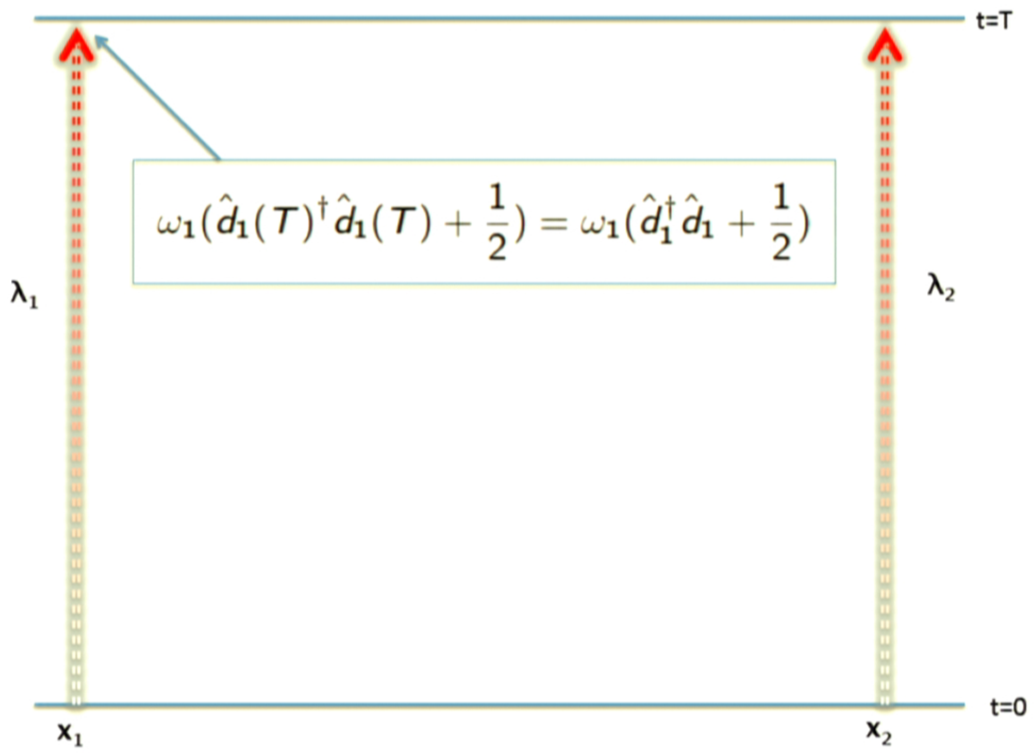

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(1+1)-dim 2-detector example:

This signalling becomes more comprehensible if the detector is interpreted as a variant of an UD detector in which the detector couples both to the field and to its conjugate momentum.

$$\hat{f}e^{i\Omega(x_1-t)} + \hat{f}^\dagger e^{-i\Omega(x_1-t)} = \int dx\hat{\phi}(t,x)F_1(x) + \int dx\hat{\pi}(t,x)G_1(x)$$

where

$$F_1(x) = (2\Omega)^{\frac{1}{2}}L^{-\frac{1}{2}}\cos[\Omega(x-x_1)], \quad G_1(x) = \left(\frac{2}{\Omega}\right)^{\frac{1}{2}}L^{-\frac{1}{2}}\sin[\Omega(x-x_1)]$$

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Cavities+Conclusions

- In a cavity, ideal measurements of observables such as particle number *can* be done [Johnson et al:2010]
- Jaynes-Cummings model ([Miller et al:2005] for a review): successful model of an atom-qubit interacting with QED in a cavity which is of the form investigated above in which a detector couples to a single mode of the field
- The Jaynes-Cummings model is a phenomenological model which applies only on time scales many orders of magnitude larger than the light crossing time of the cavity
- The Jaynes-Cummings Hamiltonian and its relatives cannot model an atom-qubit coupled to a quantum field in Minkowski spacetime, or in any spacetime where two atom-qubits can be placed at distances larger than the timescale on which the detector model is valid

Cavities+Conclusions

Foundational issues

The questions are interesting from the point of view of QI but also because the attempt to answer them pushes us to address more foundational issues:

- Perhaps a more physical approach is needed: we require a framework for closed relativistic quantum systems including detectors, in which experimental, measurement-like situations can be analysed fully quantum mechanically in an essentially relativistic way.

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- The path integral roots quantum theory firmly in spacetime – rather than Hilbert space – as the arena for physics.
- As a framework for closed quantum systems which deals directly with spacetime events, the path integral approach is eminently suitable for the investigation of measurements on relativistic quantum fields in Minkowski spacetime.