Title: Boundary Effects on Quantum Entanglement and its Dynamics in a Detector-Field System

Date: Jun 25, 2012 02:40 PM

URL: http://pirsa.org/12060074

Abstract: We analyze an exactly solvable model consisting of an inertial Unruh-DeWitt detector which interacts linearly with a massless quantum field in Minkowski spacetime with a perfectly reflecting flat plane boundary. This model is related to proposed mirror-field superposition and relevant experiments in macroscopic quantum phenomena, as well as atomic fluctuation forces near a conducting surface. Firstly a coupled set of equations for the detectorâ€™s and the fieldâ€™s Heisenberg operators are derived. After coarse graining the field, the dynamics of the detectorâ€™s internal degreeof freedom is described by a quantum Langevin equation, where the dissipation and noise kernels respectively correspond to the retarded Greenâ€™s functions and Hadamard elementary functions of the free quantum field in half space. We use the linear entropy as measures of entanglement between the detector and the quantum field under mirror reflection, then solve the early-time detector-fieldentanglement dynamics. At late times when the combined system is in a stationary state, we obtain exact expressions for the detectorâ€™s covariance matrix and show that the detector-field entanglement decreases for smaller separation between the detector and the mirror.We explain the behavior of detector-field entanglement qualitatively with the help of a detectorâ€™s mirror image, compare them with the case of two real detectors and explain the differences.

- ## Entanglement Properties of Field-Mediated Local Quantum Systems

"Sudden death" or finite time disentanglement

Residual entanglement, entanglement revival

Finite range of entanglement

Our Target: Effect of Boundary

Effect of boundary on entanglement of localized objects mediated by quantum fields.

Breaks Poincare Invariance.

Curious Queries:

1. Entanglement with Mirror Image?
2. Distance Dependence?

- ## Model:

Inertial Unruh-DeWitt detector with massless scalar field in half space:

$$S[Q,\dot{Q};\Phi,\partial_\mu\Phi] = \frac{1}{2}M_Q \int dt\, (\dot{Q}^2 - \Omega^2 Q^2) + \frac{1}{2}\int dt \int_{x_3>0} d^3x\, \partial_\mu\Phi\partial^\mu\Phi + \lambda_Q \int dt\, Q(t)\Phi(\mathbf{x}_Q, t).$$

$$\Phi(\mathbf{x}_\parallel, x_3 = 0, t) = 0.$$

Initial condition: uncorrelated, both in ground state.

Perfectly reflecting boundary (Dirichlet boundary condition):
can be obtained by taking perfect conducting limit of a microscopic model.

- **Green's Functions of the Field in Half Space**

Field Quantization in Half Space

$$\hat{\Phi}_0(\vec{x}, t) = \int_{k_3>0} d^3k \sqrt{\frac{1}{4\pi^3 \omega_{\vec{k}}}} e^{-i\omega_{\vec{k}} t + i\vec{k}_{\|}\cdot x_{\|}} \sin k_3 x_3 \hat{a}_{\vec{k}} + H.c.$$

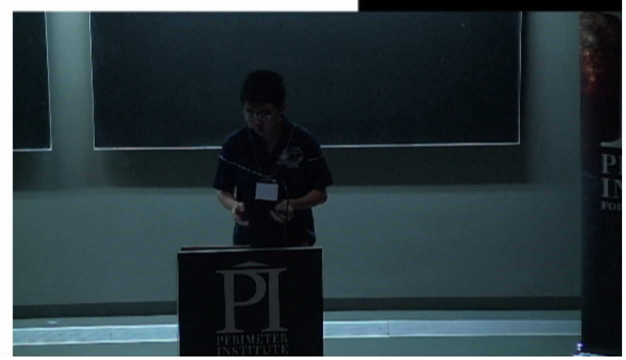
Retarded Propagator and Hadamard Green's Function

$$G^{\Phi}_{ret}(\vec{x}, t; \vec{y}, t') \equiv i\theta(t-t')[\hat{\Phi}_0(\vec{x}, t), \hat{\Phi}_0(\vec{y}, t')] = \theta(t-t') \int^{\infty} d\omega_{\vec{k}} \sin(\omega_{\vec{k}}(t-t')) \cdot I(\omega_{\vec{k}}; \vec{x}, \vec{y})$$

$$G^{\Phi}_{H}(\vec{x}, t; \vec{y}, t') \equiv \langle \{\hat{\Phi}_0(\vec{x}, t), \hat{\Phi}_0(\vec{y}, t')\} \rangle = \int_0^{\infty} d\omega_{\vec{k}} \cos(\omega_{\vec{k}}(t-t')) \cdot I(\omega_{\vec{k}}; \vec{x}, \vec{y})$$

"Spectral Density"

$$I(\omega_{\vec{k}}; \vec{x}, \vec{y}) = \frac{\omega_{\vec{k}}}{2\pi^3} \int_0^{\pi/2} d\theta \int_0^{2\pi} d\phi\, e^{i\vec{k}_{\|}\cdot(\vec{x}_{\|} - \vec{y}_{\|})} \sin(\omega_{\vec{k}} \cos\theta x_3) \sin(\omega_{\vec{k}} \cos\theta y_3)$$



- **Solving Detector-Field System with Mirror**

Equations of motion for Heisenberg operators:

$$M_Q \ddot{\hat{Q}}(t) + M_Q \Omega^2 \hat{Q}(t) = \lambda_Q \int_{x_3>0} d^3x \; \hat{\Phi}(\vec{x}_Q, t)$$

$$\Box \hat{\Phi}(\vec{x}, t) = \lambda_Q \delta^3(\vec{x} - \vec{x}_Q)\hat{Q}(t) \quad \text{and} \quad \hat{\Phi}(\vec{x}_\parallel, x_3 = 0, t) = 0$$

Solution for the field operator:

$$\hat{\Phi}(\vec{x}, t) = \hat{\Phi}_0(\vec{x}, t) + \lambda_Q \int_{t_i}^{t} dt' \, G^{\Phi}_{ret}(t, \vec{x}; t', \vec{x}_Q)\hat{Q}(t')$$

Field-influenced dynamics of the detector:

$$M_Q \ddot{\hat{Q}}(t) + M_Q \Omega^2 \hat{Q}(t) - \lambda_Q^2 \int_{t_i}^{t} dt' \, G^{\Phi}_{ret}(t, \vec{x}_Q; t', \vec{x}_Q)\hat{Q}(t') = \lambda$$

- **Three Temporal Regimes (weak coupling):**

  - **Early-time**:

    Accumulated effect of field-mediated reflected influences is small.

    Influence of field on detector is dominated by vacuum fluctuation of the field.

  - **Intermediate time**: complicated dynamics.

  - **Late-time**: Detector approaches a time-stationary mixed Gaussian state.

    Behavior similar to the entanglement between two inertial detectors coupled to a massless scalar field

    [S.Y.Lin and B.L.Hu, Phys.Rev.D79:085020 (2009)]

- **Three Temporal Regimes (weak coupling):**

  - **Early-time**:
    Accumulated effect of field-mediated reflected influences is small.
    Influence of field on detector is dominated by vacuum fluctuation of the field.

  - **Intermediate time**: complicated dynamics.

  - **Late-time**: Detector approaches a time-stationary mixed Gaussian state.

  Behavior similar to the entanglement between two inertial detectors coupled to a massless scalar field
  [S.Y.Lin and B.L.Hu, Phys.Rev.D79:085020 (2009)]

- **Early-time Evolution of Detector's Correlation Functions**

For factorized initial conditions:

$$
\begin{aligned}
< \hat{Q}(t), \hat{Q}(t) > &= < \hat{Q}(t), \hat{Q}(t) >_{\mathrm{a}} + < \hat{Q}(t), \hat{Q}(t) >_{\mathrm{v}} \\
< \hat{P}(t), \hat{P}(t) > &= < \hat{P}(t), \hat{P}(t) >_{\mathrm{a}} + < \hat{P}(t), \hat{P}(t) >_{\mathrm{v}} \\
< \hat{P}(t), \hat{Q}(t) > &= < \hat{P}(t), \hat{Q}(t) >_{\mathrm{a}} + < \hat{Q}(t), \hat{P}(t) >_{\mathrm{v}}
\end{aligned}
$$

For weak coupling, at early times the influence of the field is governed by its vacuum fluctuations, without reflected influences.

Zeroth-order field-induced correlation functions:

$$
\begin{aligned}
< \hat{Q}(t), \hat{Q}(t) >_{\mathrm{v}} &= \int_{k_3>0} \frac{d^3k}{2\pi^3} \frac{1}{2\omega} |q_+^{(0)}(t, \mathbf{k})|^2 \\
< \hat{P}(t), \hat{P}(t) >_{\mathrm{v}} &= M_Q^2 \int_{k_3>0} \frac{d^3k}{2\pi^3} \frac{1}{2\omega} |\partial_t q_+^{(0)}(t, \mathbf{k})|^2 \\
< \hat{Q}(t), \hat{P}(t) >_{\mathrm{v}} &= M_Q \int_{k_3>0} \frac{d^3k}{2\pi^3} \frac{1}{2\omega} \frac{1}{2} (q_+^{(0)*}(t, \mathbf{k}) \partial_t q_+^{(0)}(t, \mathbf{k}) + q_+^{(0)}(t, \mathbf{k}) \partial_t q_+^{(0)*}(t, \mathbf{k}
\end{aligned}
$$

- **Early-time Evolution of Detector's Correlation Functions**

For factorized initial conditions:

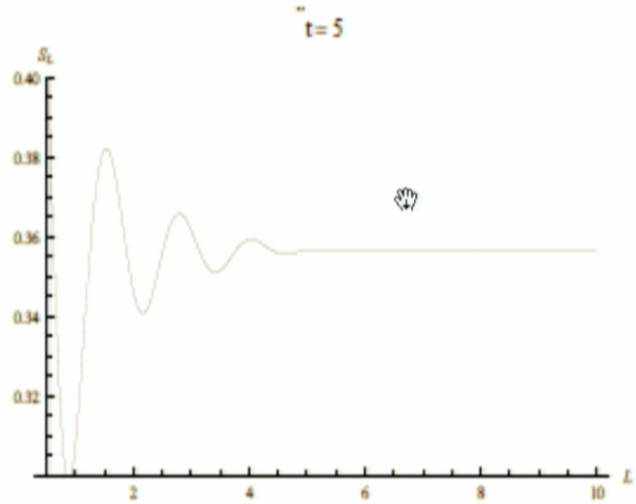
$$
\begin{aligned}
<\hat{Q}(\tau),\hat{Q}(\tau)> &= <\hat{Q}(\tau),\hat{Q}(\tau)>_{\mathrm{a}} + <\hat{Q}(\tau),\hat{Q}(\tau)>_{\mathrm{v}} \\
<\hat{P}(\tau),\hat{P}(\tau)> &= <\hat{P}(\tau),\hat{P}(\tau)>_{\mathrm{a}} + <\hat{P}(\tau),\hat{P}(\tau)>_{\mathrm{v}} \\
<\hat{P}(\tau),\hat{Q}(\tau)> &= <\hat{P}(\tau),\hat{Q}(\tau)>_{\mathrm{a}} + <\hat{Q}(\tau),\hat{P}(\tau)>_{\mathrm{v}}
\end{aligned}
$$

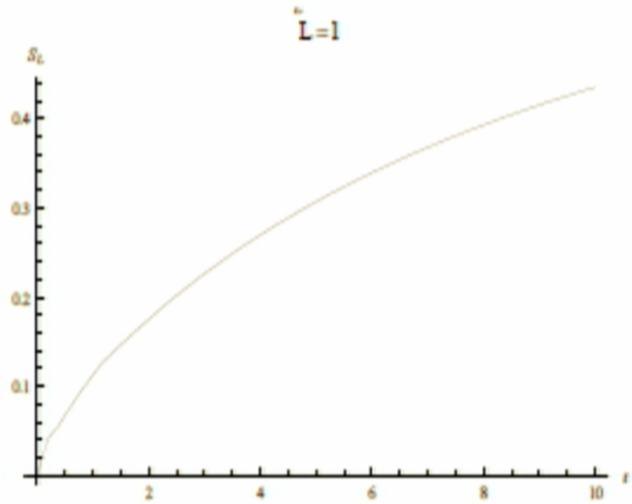
For weak coupling, at early times the influence of the field is governed by its vacuum fluctuations, without reflected influences.

Zeroth-order field-induced correlation functions:

$$
\begin{aligned}
<\hat{Q}(\tau),\hat{Q}(\tau)>_{\mathrm{v}} &= \int_{k_3>0} \frac{d^3k}{2\pi^3}\frac{1}{2\omega}|q_+^{(0)}(\tau,\mathbf{k})|^2 \\
<\hat{P}(\tau),\hat{P}(\tau)>_{\mathrm{v}} &= M_Q^2 \int_{k_3>0} \frac{d^3k}{2\pi^3}\frac{1}{2\omega}|\partial_\tau q_+^{(0)}(\tau,\mathbf{k})|^2 \\
<\hat{Q}(\tau),\hat{P}(\tau)>_{\mathrm{v}} &= M_Q \int_{k_3>0} \frac{d^3k}{2\pi^3}\frac{1}{2\omega}\frac{1}{2}(q_+^{(0)*}(\tau,\mathbf{k})\partial_\tau q_+^{(0)}(\tau,\mathbf{k}) + q_+^{(0)}(\tau,\mathbf{k})\partial_\tau q_+^{(0)*}(\tau,\mathbf{k}
\end{aligned}
$$

- **Early-time Dynamics of Detector-Field Entanglement (2)**





- **The third plot** shows the dependence of linear entropy on L at a given instant of time
- **The fourth plot** exhibits how the linear entropy evolves with time for the detector located at a certain distance.

- **Correction to Linear Entropy due to Mirror Reflection**

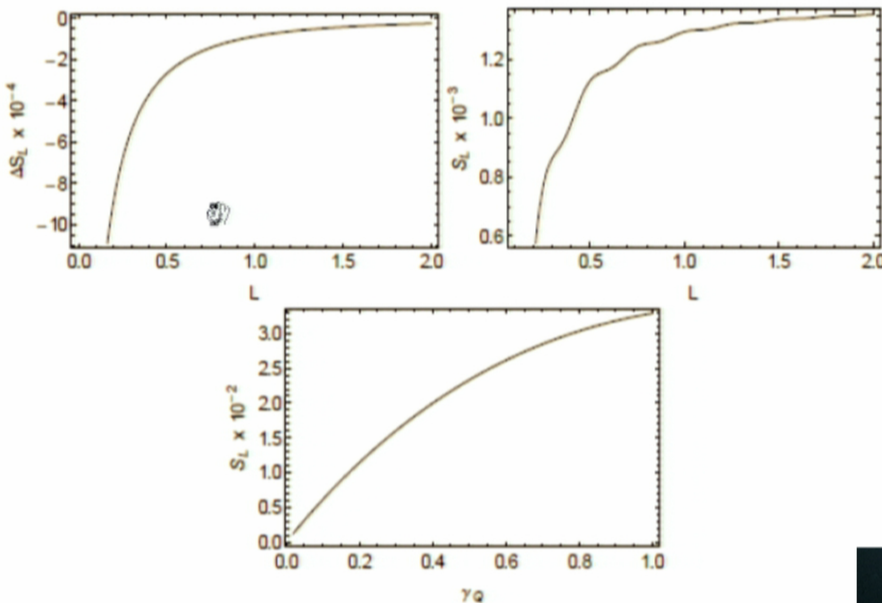
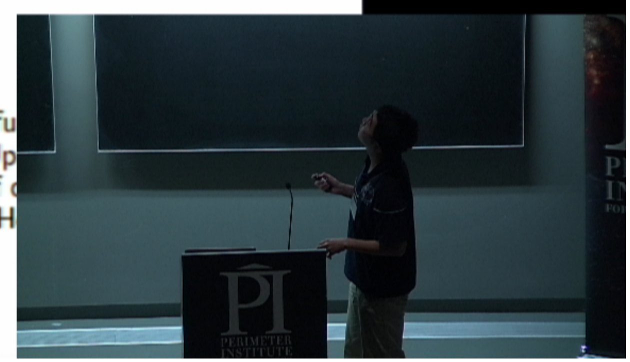


Figure: (Upperleft) Leading-order corrections to linear entropy given in (48) as a fu distance between the detector and its image due to the presence of the mirror. (Up Numerical result of the linear entropy of the detector to all orders as a function of between the detector and its image. (Below) Linear entropy as a function of γ_Q. H $\gamma_Q = 0.02$, $\Omega_r = 5$.



- **Correction to Linear Entropy due to Mirror Reflection**

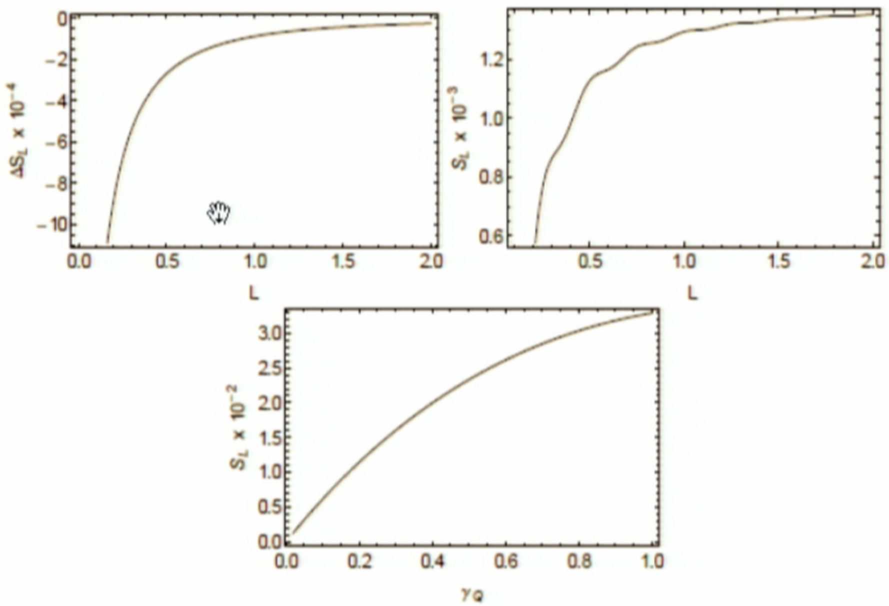


Figure: (Upperleft) Leading-order corrections to linear entropy given in (48) as a function of distance between the detector and its image due to the presence of the mirror. (Upperright) Numerical result of the linear entropy of the detector to all orders as a function of distance between the detector and its image. (Below) Linear entropy as a function of γ_Q. Here $M_Q = 1$, $\gamma_Q = 0.02$, $\Omega_r = 5$.

- **Reduction of Entanglement: Effect of Dirichlet Boundary Condition**

Quantum Langevin equation:

$$\left(\partial_\tau^2 + 2\gamma_Q \partial_\tau + \Omega_r^2\right) \hat{Q}(\tau) = -\frac{2\gamma_Q}{4\pi L}\theta(\tau - L)\hat{Q}(\tau - L) + \lambda_Q \hat{\Phi}(\tau, \vec{x}_Q)$$

Mirror reflection acts on the detector as a time-delayed negative feedback which suppresses the influence of the field on the detector.

The correlation established between the detector and the field arising from their interactions will be effectively reduced, causing the detector to be less entangled with the field.

Possibility of enhancing entanglement:

a.Two adjoining dielectric media with dielectric coefficie $\epsilon_1 > \epsilon_2$

b.Neumann boundary condition

Further Developments:

a. Physical degrees of freedom for the mirror image:

Microscopic model of linear dielectric medium as an interacting array of oscillators linearly coupled to the field:

"Atom-Dielectric Entanglement"

Rong Zhou, R. Behunin, S. Y. Lin and B. L. Hu

b. Topological Effects:

" Spacetime Topology and Entanglement Dynamics"

Rong Zhou, S. Y. Lin, C. H. Chou and B. L. Hu