

Title: Holographic Mutual Information is Monogamous

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URL: <http://pirsa.org/12060070>

Abstract: I'll describe a special information-theoretic property of quantum field theories with holographic duals: the mutual informations among arbitrary disjoint spatial regions A,B,C obey the inequality $I(A:BC) \geq I(A:B)+I(A:C)$, provided entanglement entropies are given by the Ryu-Takayanagi formula. Inequalities of this type are known as monogamy relations and are characteristic of measures of quantum entanglement. This suggests that correlations in holographic theories arise primarily from entanglement rather than classical correlations. Moreover, monogamy property implies that the Ryu-Takayanagi formula is consistent with all known general inequalities obeyed by the entanglement entropy, including an infinite set recently discovered by Cadney, Linden, and Winter; this constitutes significant evidence in favour of its validity.

Holographic Mutual Information is Monogamous

Patrick Hayden with
Matthew Headrick and Alex Maloney



McGill

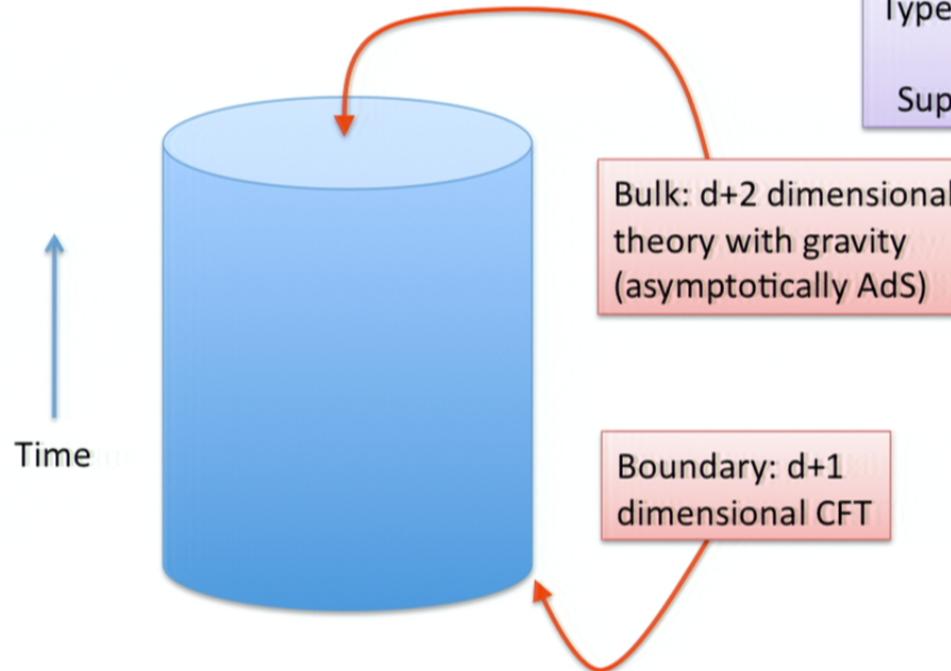


Relativistic Quantum Information 2012

Outline

- Ryu-Takayanagi proposal for entanglement entropy in holographic field theories
- Mutual information
 - Non-Extensivity and Monogamy
 - Behaviour in general field theories
- Mutual information in holographic theories
 - Picture proof of monogamy
 - Consequences (Cadey-Linden-Winter inequalities)
- Natural generalizations?
- Rigorous proof
- A conjecture

Entanglement entropy in AdS/CFT

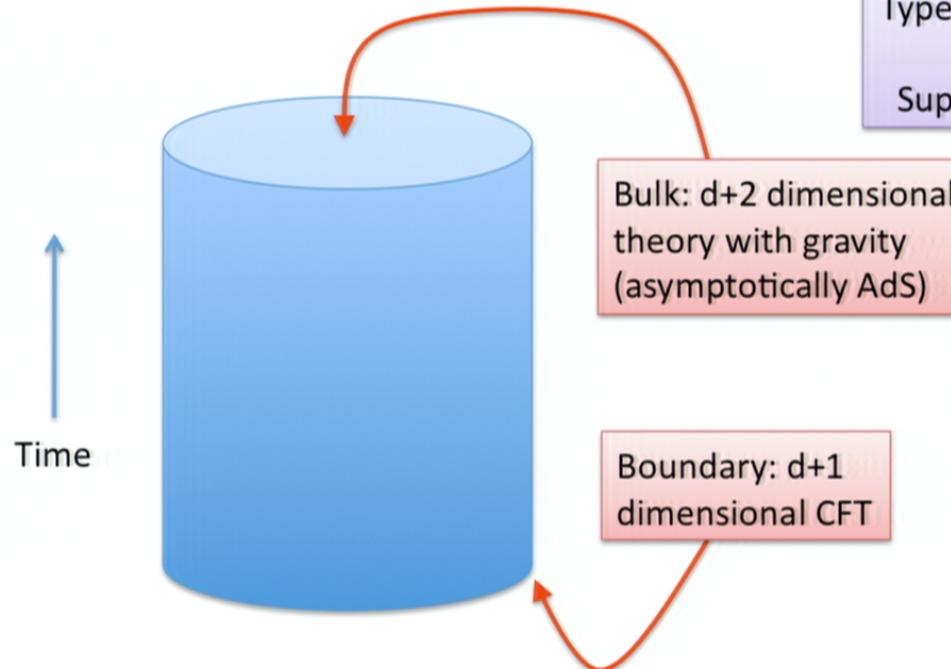


Example:

Type IIB string theory on $\text{AdS}_5 \times S^5$
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Supersymmetric N=4 Yang-Mills

Conjecture: Equivalence of string (gravity) theory in bulk with CFT on boundary [Maldacena'97]

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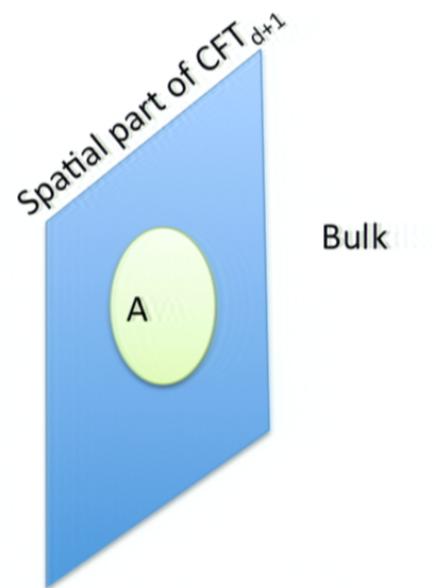
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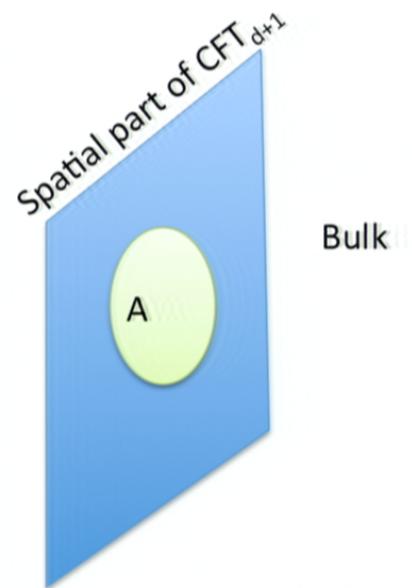
Restrict to static spacetimes. Work in a timeslice.

$$S(A) = - \text{tr} \rho_A \log \rho_A$$



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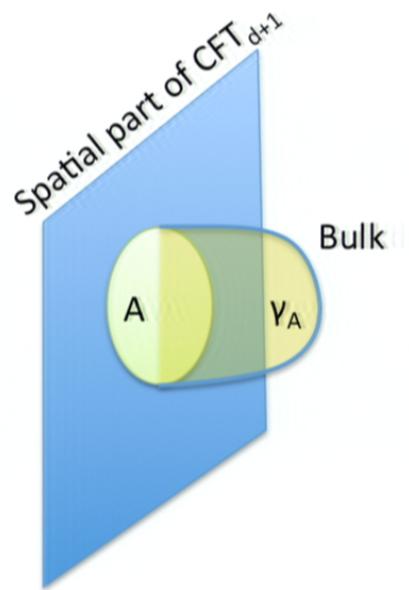
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$$S(A) = \frac{1}{4G_N} \min_{\gamma_A} (\text{area}(\gamma_A))$$

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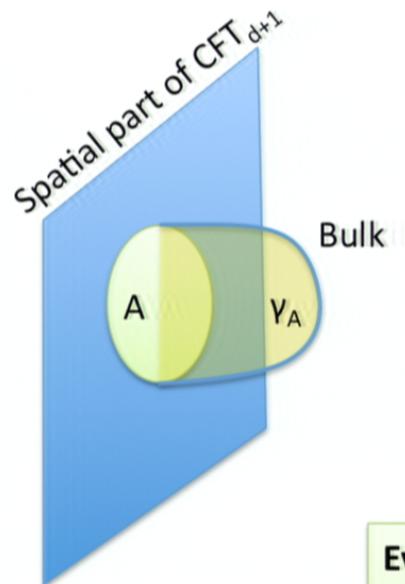
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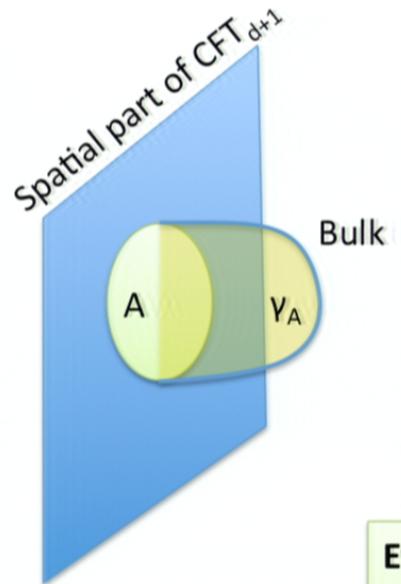
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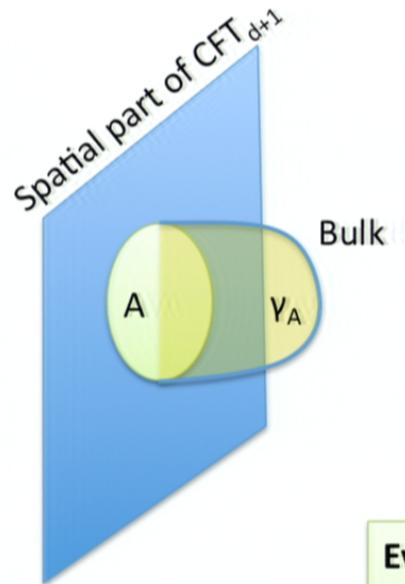
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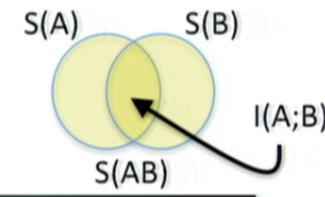
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Mutual information



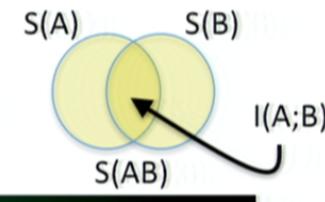
$$I(A;B) = S(A) + S(B) - S(AB)$$

What's to like?

For an information theorist:

- * Zero if and only if state is product
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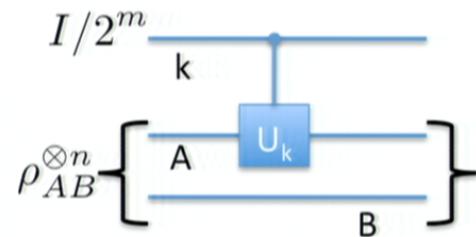


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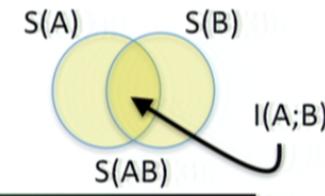
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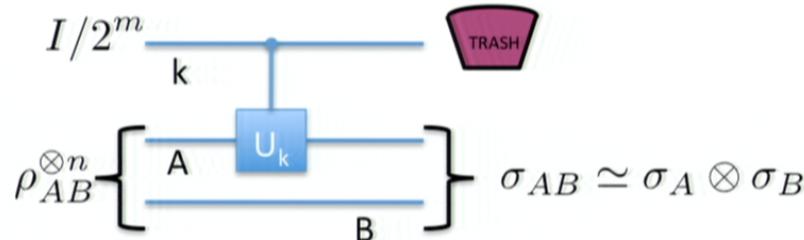


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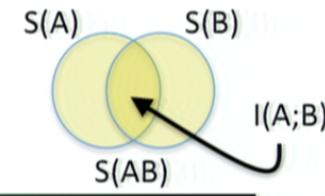
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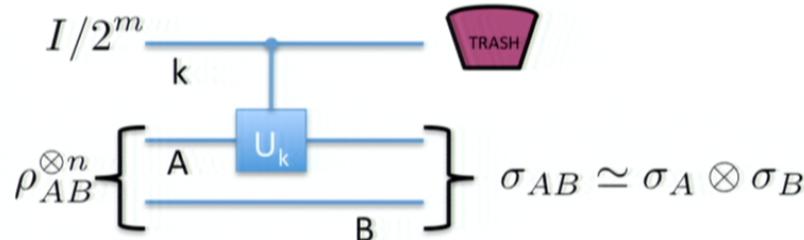


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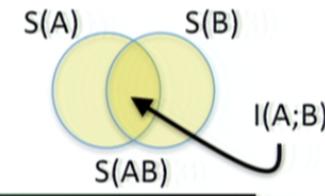
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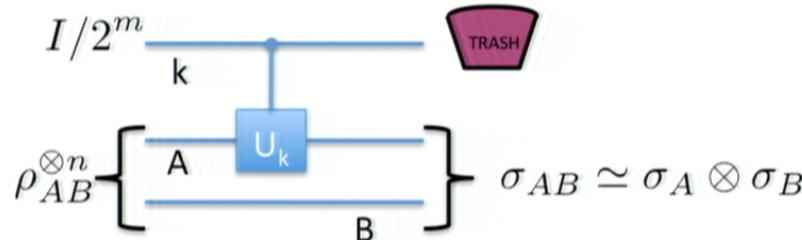


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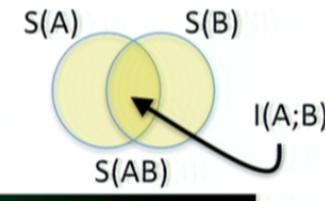
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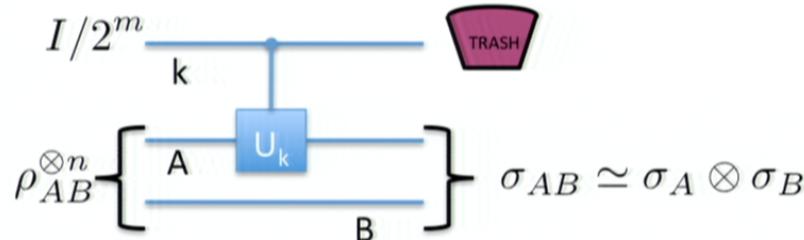


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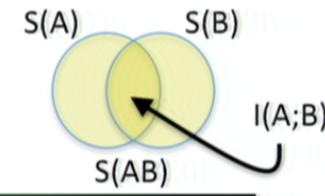
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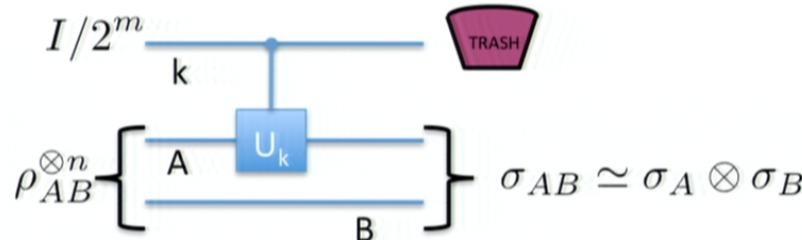


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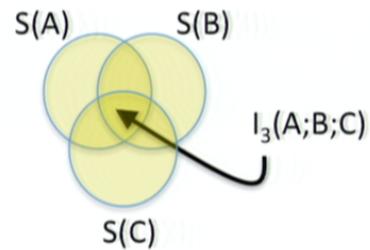
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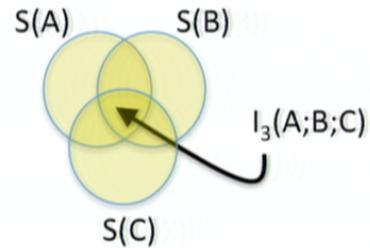
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Mutual information and extensivity



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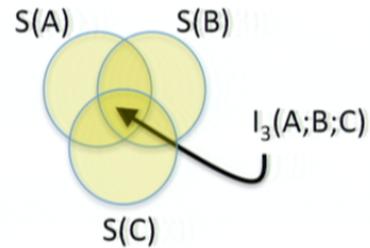


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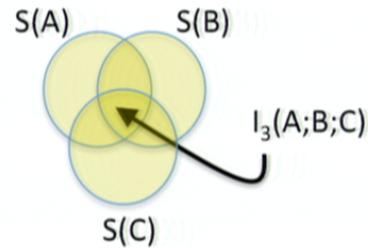
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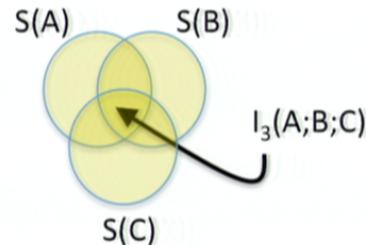
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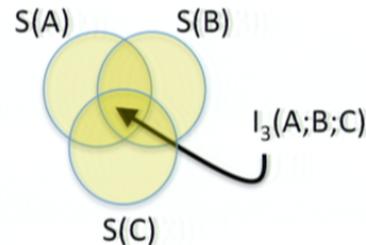
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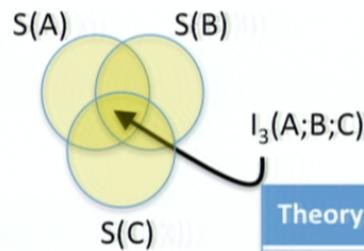
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Extensivity in field theories

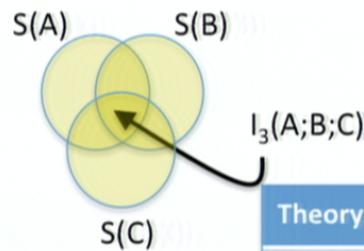


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1+1 dim CFT with smallest scaling dimension $\Delta = 1/2$	Extensive $I_3 = 0$	Free fermion
1+1 dim CFT with smallest scaling dimension $\Delta > 1/2$	Superextensive $I_3 < 0$	Free boson on a small circle
1+1 dim massive fermion	$I_3 > 0$ for small sep $I_3 < 0$ for large sep	
2+1 massive theory with topological order	Superextensive $I_3 < 0$	Toric code, Laughlin state, Moore-Read state

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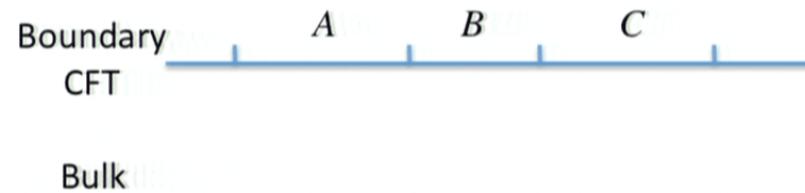
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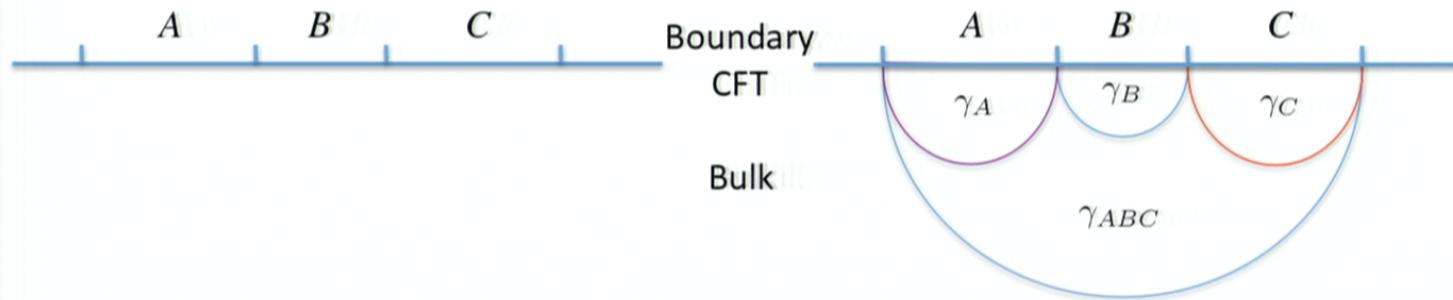


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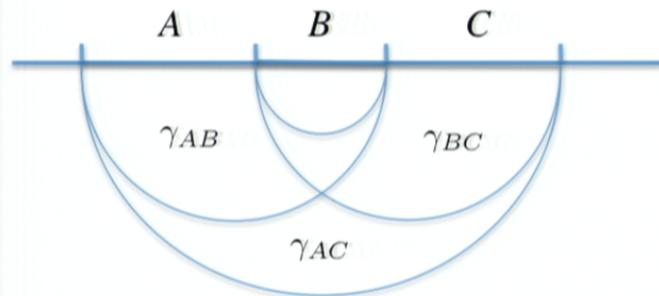
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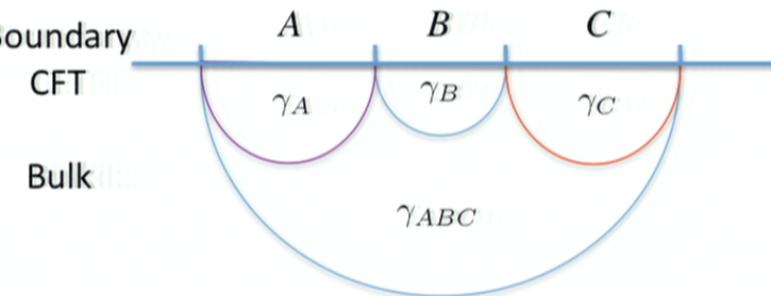
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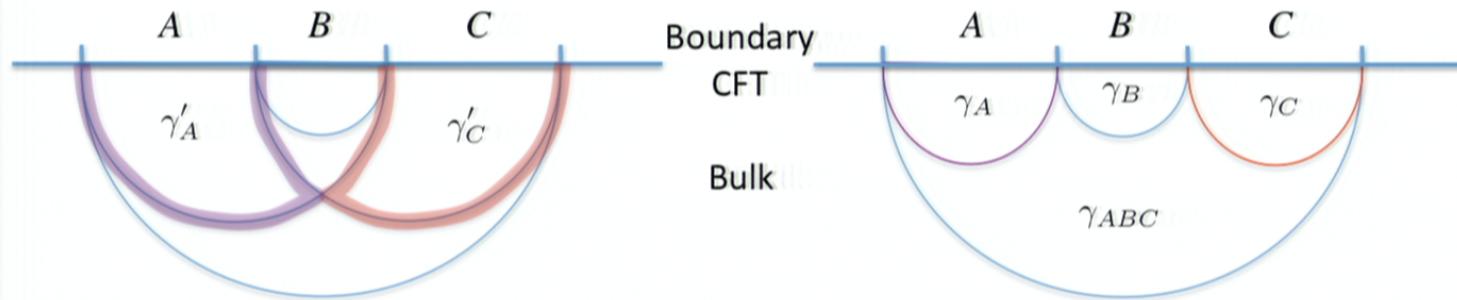
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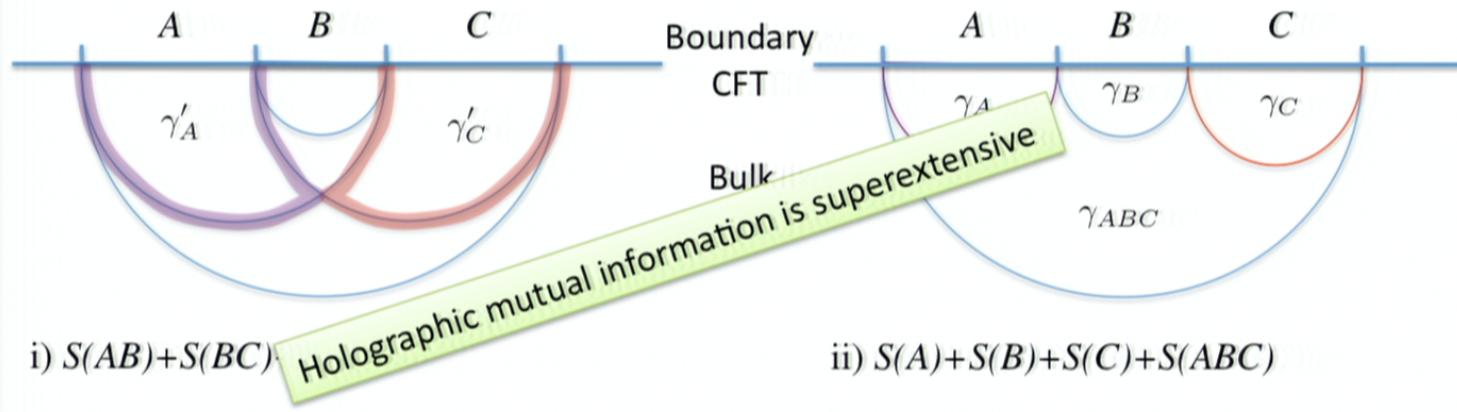
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$$\begin{aligned} I_3(A; B; C)_\rho &:= S(A)_\rho + S(B)_\rho + S(C)_\rho - \\ &\quad - S(AB)_\rho - S(BC)_\rho - S(AC)_\rho + S(ABC)_\rho \end{aligned}$$

Consequences

Implies that the Ryu-Takayanagi formula is consistent with **all** known inequalities satisfied by the von Neumann entropy

Gives a check on validity of the conjecture that the formula really does calculate entanglement entropy

Strong subadditivity:

$$S(AB) + S(BC) \geq S(ABC) + S(B)$$

Superextensivity:

$$\begin{aligned} S(AB) + S(BC) &\geq S(ABC) + S(B) + [S(A) + S(C) - S(AC)] \\ &= S(ABC) + S(B) + I(A; C) \end{aligned}$$

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If $I(A;C|B) = I(A;B|C) = I(B;C|D) = 0$ then $I(C;D) \geq I(AB;C)$.

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Superextensivity and the constraint imply $0 = I(B;C|D) \geq I(B;C) = 0$

$$\begin{aligned} I(AB;C) &= I(A;C|B) + I(B;C) && \text{Chain rule (trivial identity)} \\ &= 0 \end{aligned}$$

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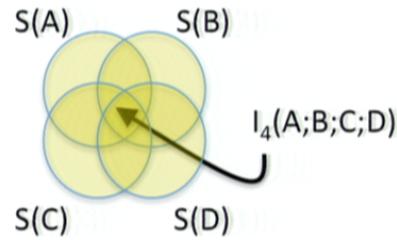
Cadney-Linden-Winter inequality:

If $I(A; C|B) = I(B; C|A) = 0$, then

$$S(X_1 \cdots X_n) + (n-1)I(AB; C) \leq \sum_{i=1}^n S(X_i) + \sum_{i=1}^n I(A; B|X_i)$$

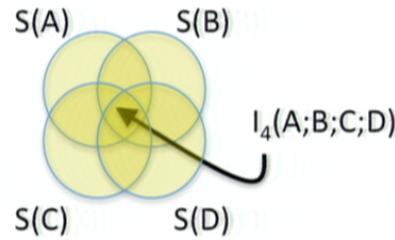
for any quantum state and disjoint subsystems $\{A, B, C, X_1, \dots, X_n\}$.

$I_4(A;B;C;D)?$



$$I_4(A; B; C; D) = - \sum_{J \subseteq \{A, B, C, D\}} (-1)^{|J|} S(J)$$

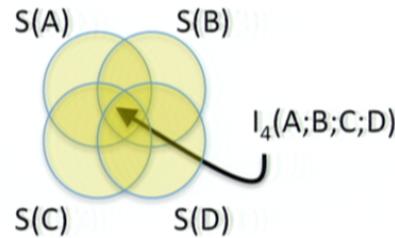
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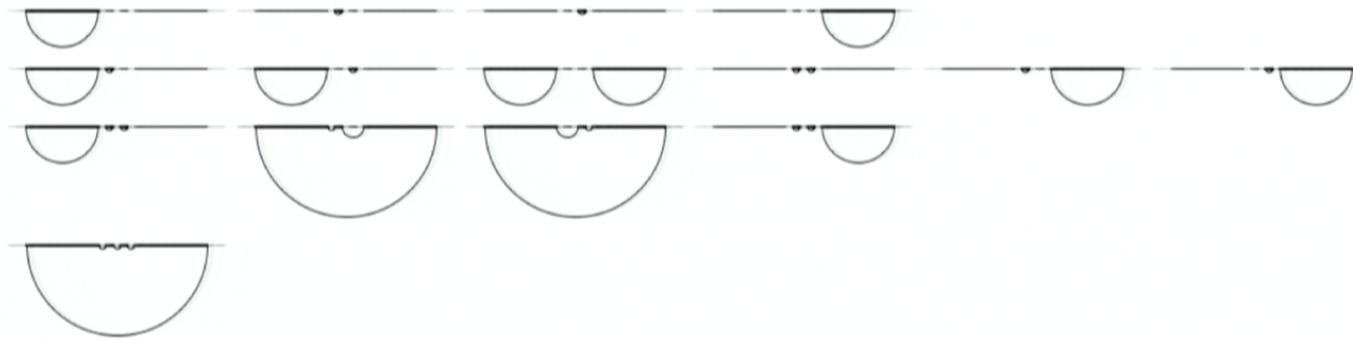
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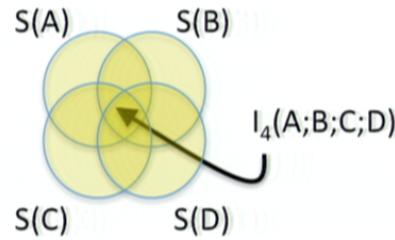
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Counterexample: $A=[0,1]$, $B=[1.1,1.2]$, $C=[1.3,1.4]$, $D=[1.5,2.5]$



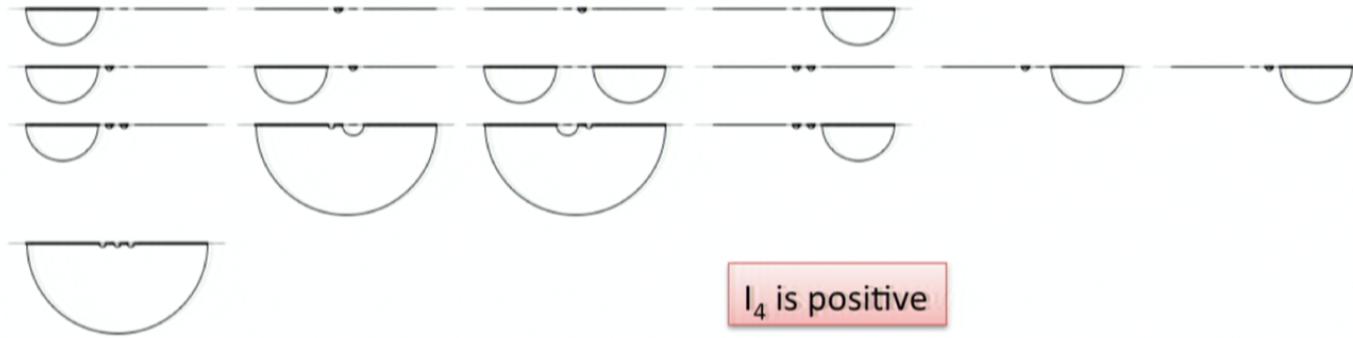
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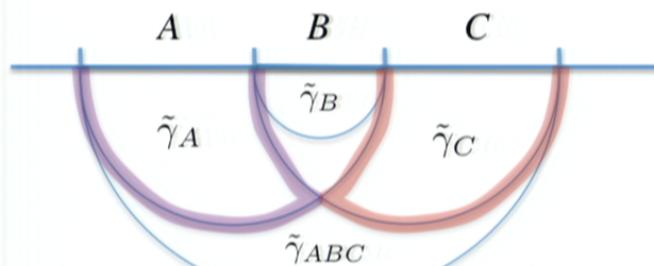
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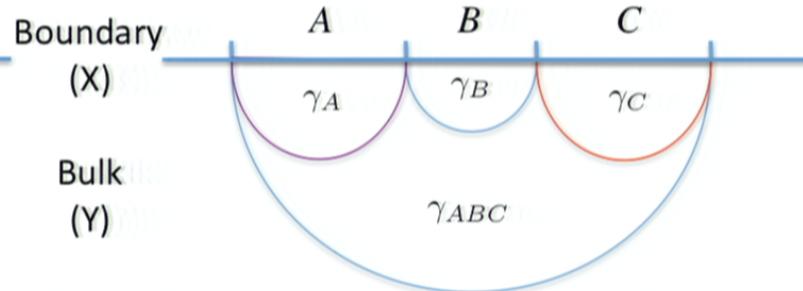
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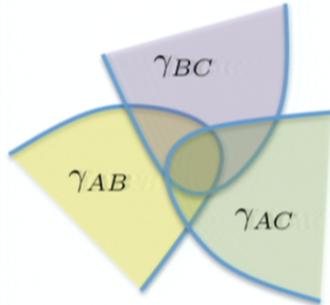


i) $S(AB) + S(BC) + S(AC)$

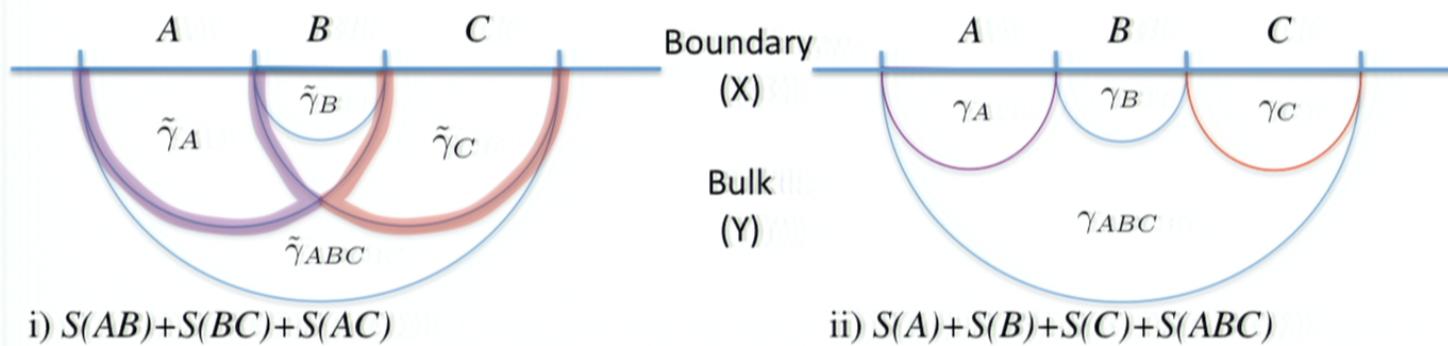


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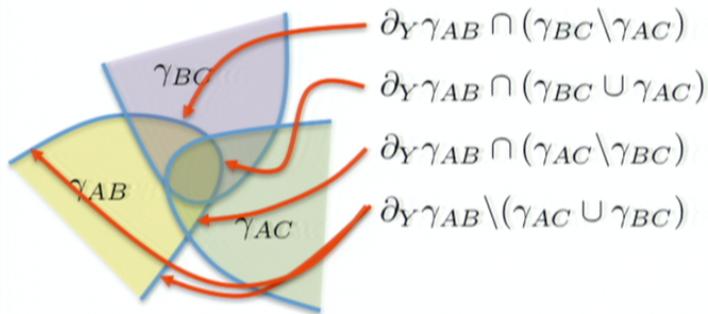
Break $\partial_Y \gamma_{AB}$ into four pieces:



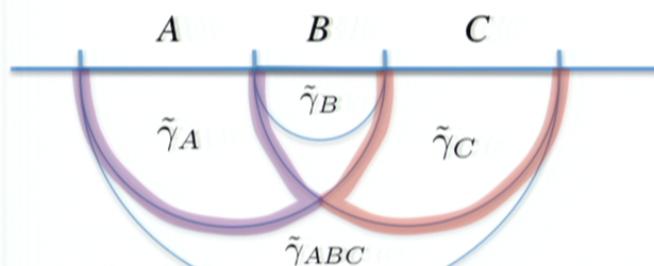
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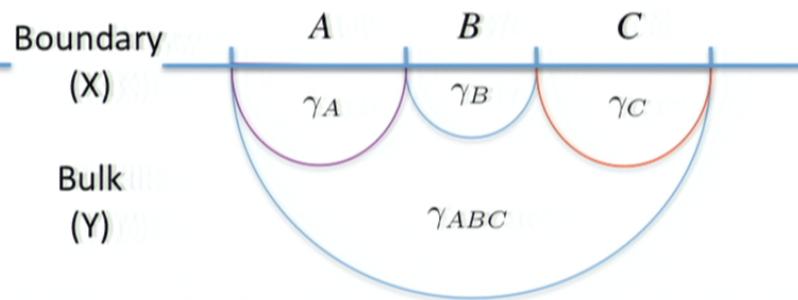
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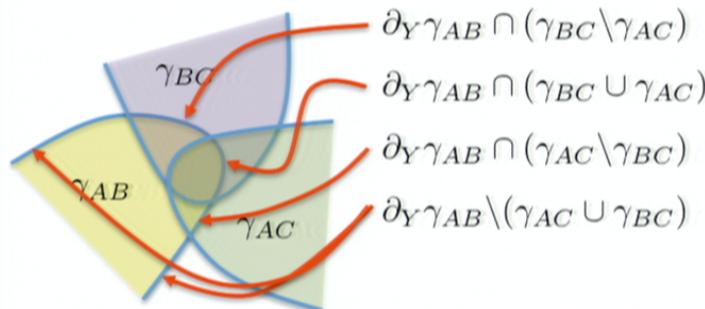


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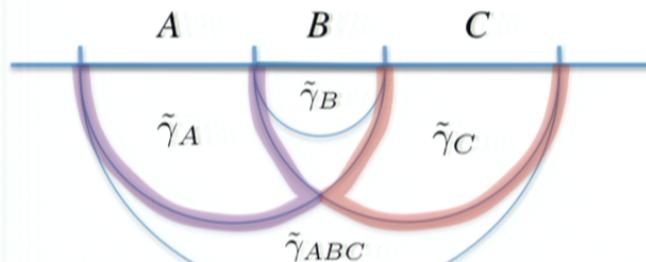


Do the same for $\partial_Y \gamma_{BC}$ and $\partial_Y \gamma_{AC}$

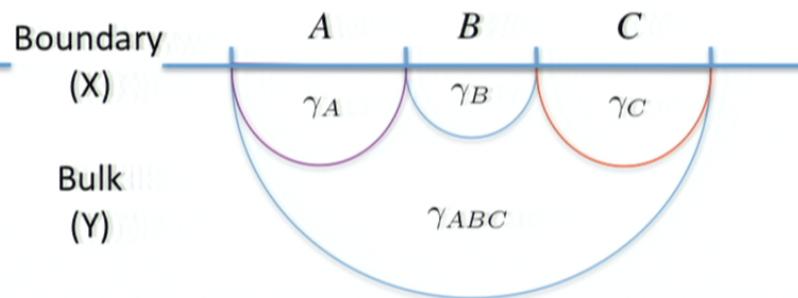
Bulk surface broken into 12 pieces

$$\begin{aligned}\tilde{\gamma}_A &:= \gamma_{AB} \cap \gamma_{AC} \setminus \gamma_{BC} \\ \tilde{\gamma}_B &:= \gamma_{AB} \cap \gamma_{BC} \setminus \gamma_{AC} \\ \tilde{\gamma}_C &:= \gamma_{AC} \cap \gamma_{BC} \setminus \gamma_{AB} \\ \tilde{\gamma}_{ABC} &:= \gamma_{AB} \cup \gamma_{BC} \cup \gamma_{AC}\end{aligned}$$

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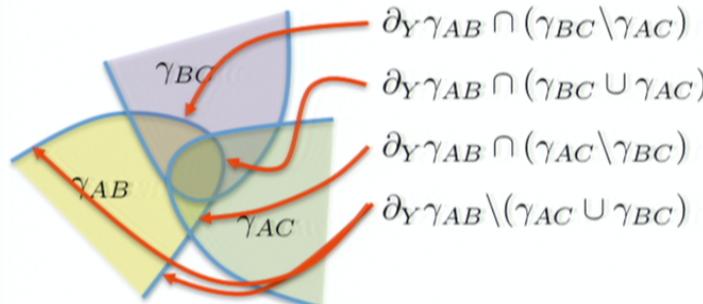


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$$\begin{aligned}\partial_Y \tilde{\gamma}_A &= \partial_Y (\gamma_{AB} \cap \gamma_{AC} \setminus \gamma_{BC}) \\ &= \left[(\partial_Y \gamma_{AB} \cap \gamma_{AC} \setminus \gamma_{BC}) \right. \\ &\quad \left. \cup (\gamma_{AB} \cap \partial_Y \gamma_{AC} \setminus \gamma_{BC}) \right. \\ &\quad \left. \cup (\gamma_{AB} \cap \gamma_{AC} \cap \partial_Y \gamma_{BC}) \right]\end{aligned}$$

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- The more entangled A is with B, the less entangled A can be with C

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- Compare to holographic mutual information:

$$I(A; B) + I(A; C) \leq I(A; BC)$$

Monogamy suggests mutual information is detecting entanglement

A conjecture (i.e. open problem)

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(Wildey optimistic) conjecture: In a holographic theory, the infimum in the definition of E_{sq} can be restricted to spatial extensions

Consequence: $E_{sq}(A; B) = I(A; B)/2$