Title: Entanglement of a Relativistic Field in the Vacuum State

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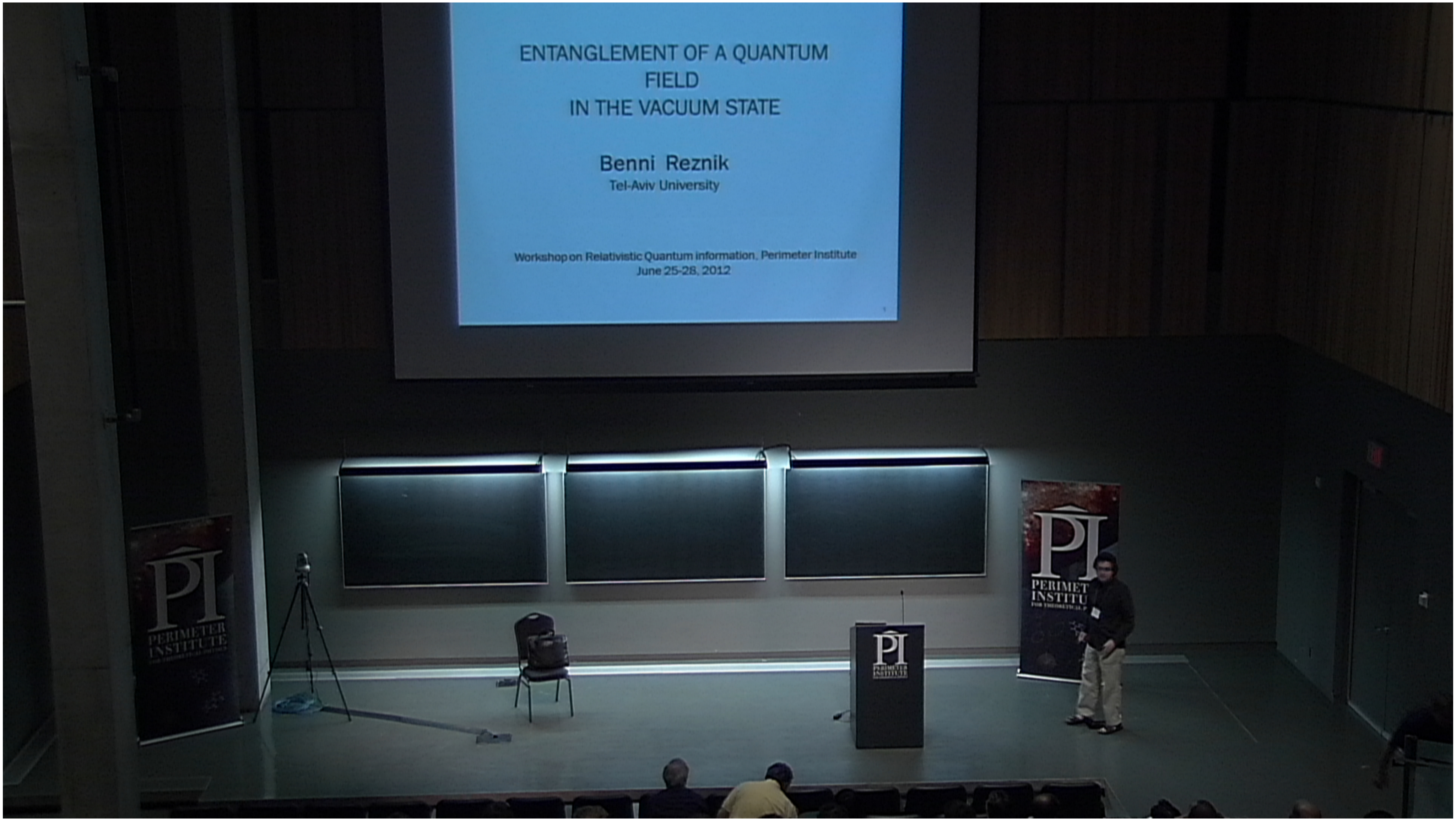
Abstract: We discuss gedanken experiments for measuring local and non-local observables in QFT that
 respect causality, and can by used to test the entanglement between two spatially distant regions in the vacuum. It is shown that the entanglement decays exponentially with the distance between the regions and does not vanish, in contrast to the case of lattice models. We discuss in this respect a possible mechanism which might explain this persistence effect, and a connection between the Reeh-Schlieder theorem and superoscillations.


ENTANGLEMENT OF A QUANTUM FIELD
IN THE VACUUM STATE
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Workshop on Relativistic Quantum information, Perimeter Institute
June 25-28, 2012
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OUTLINE

- (more) Problems with measurements and causality.

- Entanglement of the vacuum (free FT):

  --entanglement "probes"

  --a discretized lattice approach.

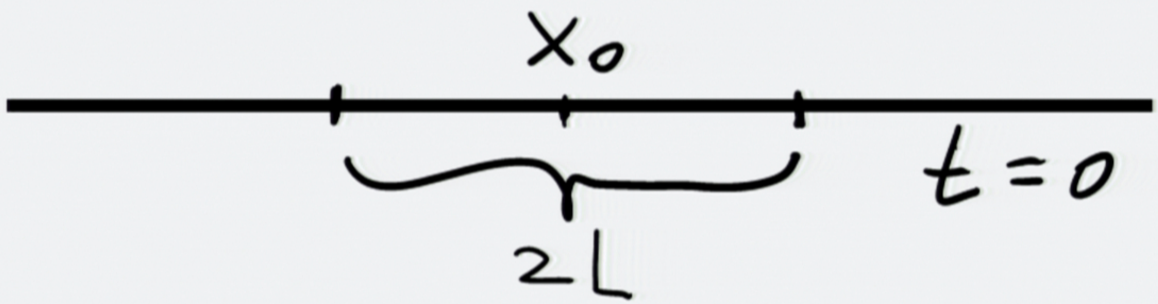
- mechanisms preserving vacuum entanglement:

  --- spatial structure.

  --- Reeh-Schlieder and superoscillations.

MEASUREMENTS

- "smeared" observables



$$\bar{\phi} = \int g(x' - x_0)\phi(x',0)dx'$$
$$\bar{\pi} = \int g(x' - x_0)\pi(x',0)dx'$$

Such that $[\bar{\phi}, \bar{\pi}] = i$

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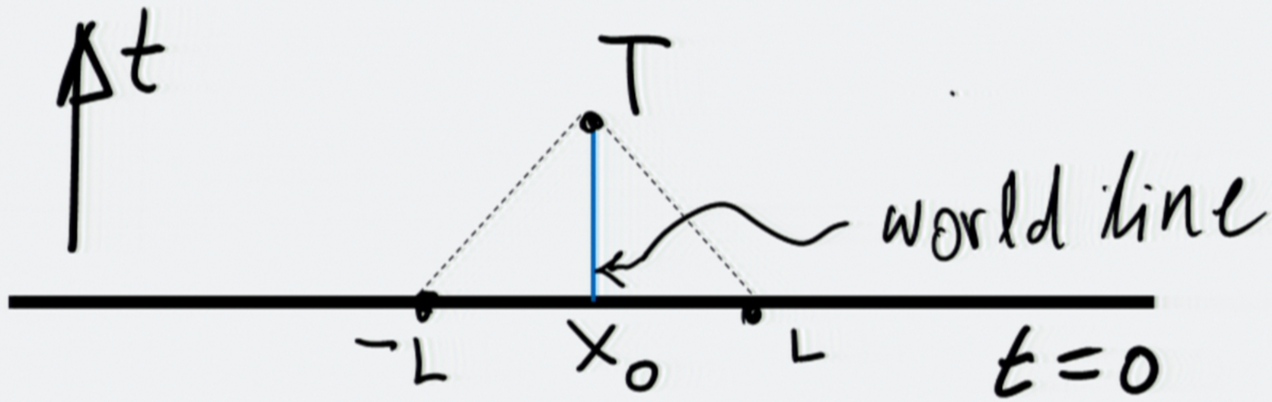
TEMPORAL AVERAGE



$$\int \phi(x_0, t')f(t')dt' = \alpha\,\bar{\phi} + \beta\,\bar{\pi}$$

spatial smearing can be extracted from temporal average

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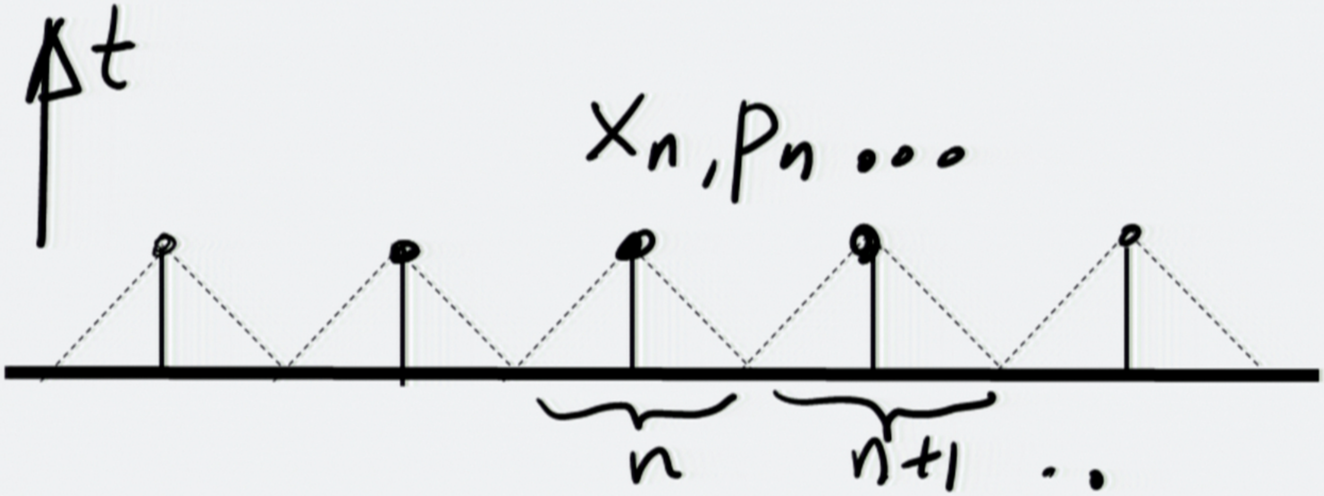
IDEALLY: DO A "SWAP"

And "freeze" the field averages on a set of mechanical oscillators.



Always consistent with causality,.

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TOY EXAMPLE

- $H = \frac{p^2+x^2}{2} + g(t)Qx$
- $\ddot{x} + \mathrm{x} = \mathrm{g(t)Q}$ $\quad Q$ is a constant of motion
- $\dot{P}$=-$g(t)x(t)$
- MD affects the oscillator (field) => P's record includes an error due to Q.

COMPENSATION (BOHR AND ROSENFELD)

add the "spring" term:

- $H \rightarrow H + kQ^2$
- Q is unknown (for a sharp measurement) but the coefficient k can be computed.
- As a result : The "fields" (x, and p) are disturbed but for the MD the unknown disturbance is exactly compensated.

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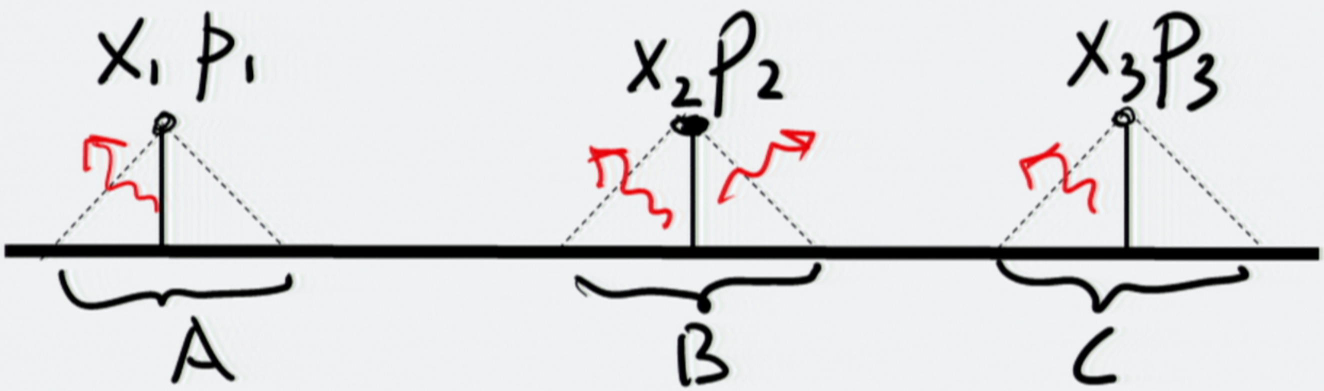
SWAP?

- $H = \frac{p^2+x^2}{2} - g(t)Qp + g(t)Px$
- Q and P are **not** constants of motion,
- problems with self/interacting fields.
- extra field+MD compensation tems needed at the scale of the "block", which are (again) nonlocal.

ENTANGLEMENT

"Partial" entanglement swap



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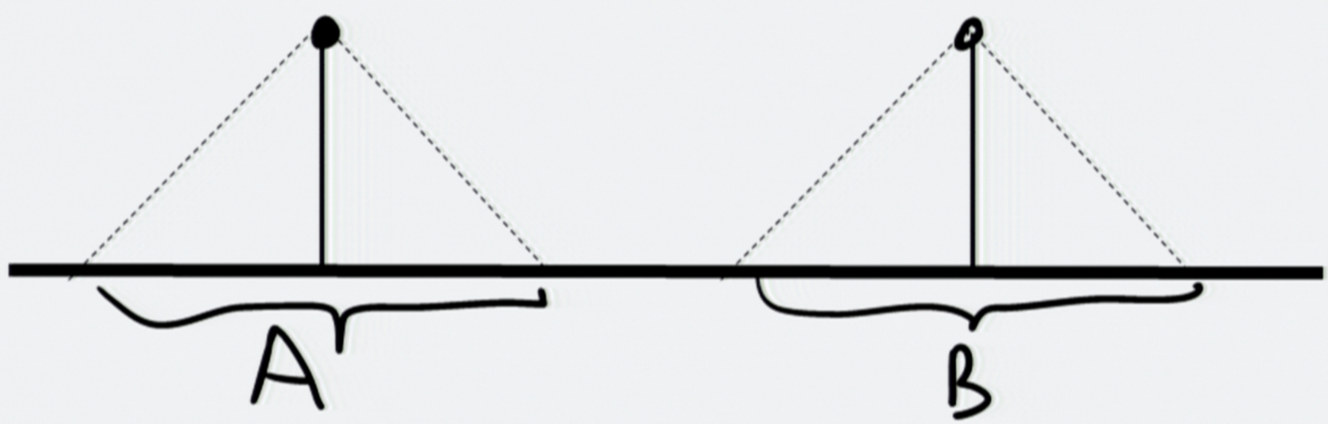
DISJOINT REGIONS



$$L > 2T$$

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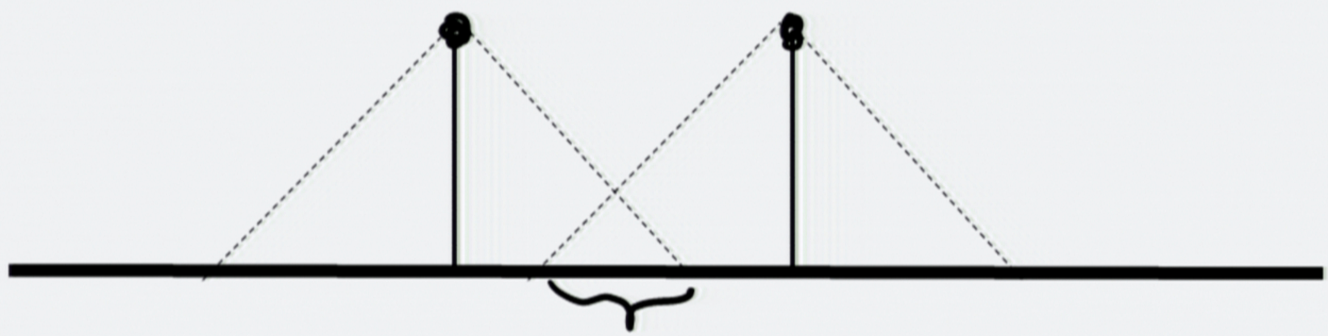
OVERLAPPING REGIONS



Common "Source"

$L < 2T$

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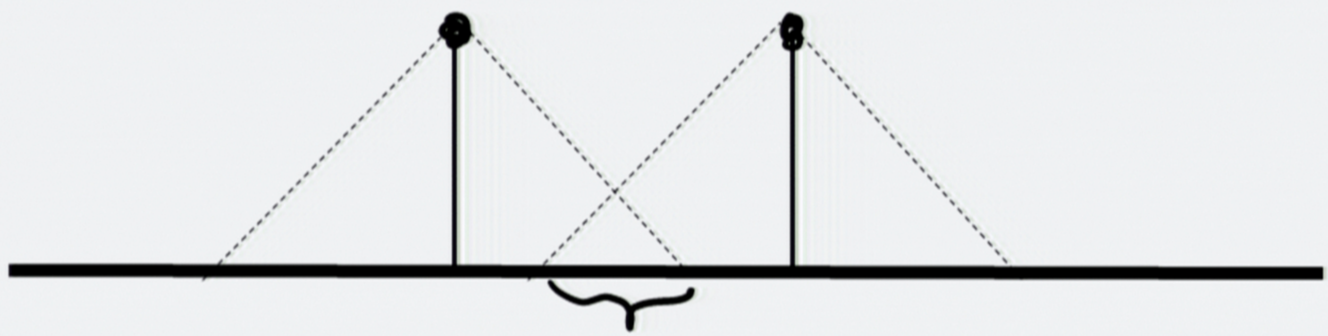
OVERLAPPING REGIONS



Common "Source"

$$L < 2T$$

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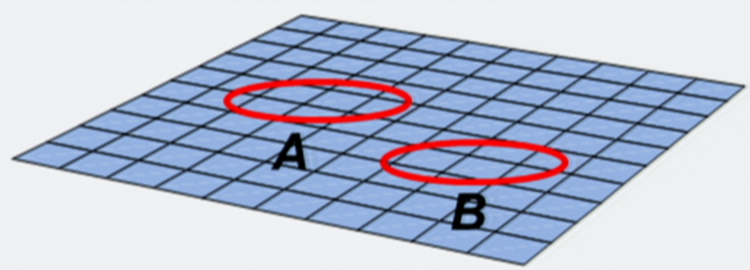
ENTANGLEMENT PROBS

LOCC + built in causal structure

L> 2T



Detectors' final entanglement -> Lower bound.
on field's entanglement

Reznik, Found. Phys. 2003 , arXive:quant-ph/0008006

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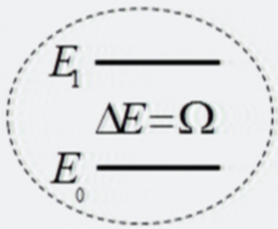
UNRUH-DEWITT DETECTOR

:



Two-level system

$$H_A = \varepsilon_A(t)\left(e^{+i\Omega t}\sigma_A^{+} + e^{-i\Omega t}\sigma_A^{-}\right)\phi(x_A, t)$$

↑

"Window Function"

Initial state:

$$|\Psi(0)\rangle = |\Downarrow_A\rangle\, |\Downarrow_B\rangle |VAC\rangle$$

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PROBES' ENTANGLEMENT

$$\rho_{AB}^{(4\times 4)} = \mathrm{Tr}_F\ \rho^{(\infty\times\infty)}$$

$$\neq \sum_i p_i\ \rho_A^{(2\times2)}\rho_B^{(2\times2)}$$

For two hyperbolically accelerated detectors analytic methods are available. However $L > 2T$ not satisfied. $(T \to \infty, L \to 0)$.

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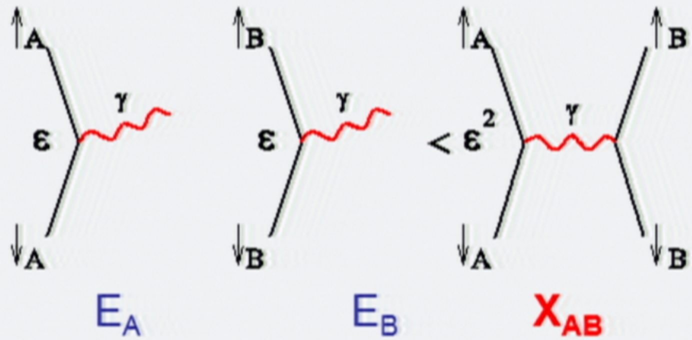
ENTANGLEMENT TEST



$$\|E_A\|\|E_B\| < \left|\langle 0 | X_{AB} \rangle\right|$$

Intuition :

$$\Psi_{final} \sim |\downarrow\downarrow\rangle + \langle X_{AB}|0\rangle|\uparrow\uparrow\rangle + \ \dots\text{"}noise\text{"}$$

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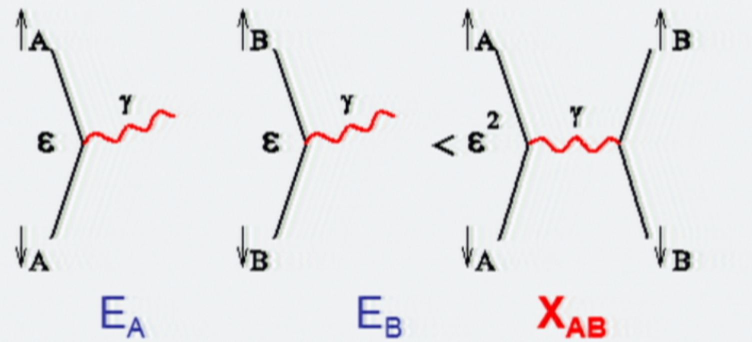
ENTANGLEMENT TEST



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ENTANGLEMENT TEST

$$\int_0^\infty \omega d\omega \epsilon_A(\Omega+\omega)\epsilon_B(\Omega+\omega) < \frac{1}{L}\int_0^\infty d\omega \sin(\omega L)\, \epsilon_A(\Omega+\omega)\epsilon_B(\Omega-\omega)$$

For a certain choice of ϵ_A and ϵ_B entanglement is nonzero for arbitrary $L \gg T$

Negativity then decays exponentially, but slower then

$E_{Ln} \approx e^{-L^2/D^2}$ where $D = cT$

Reznik, Found. Phys. 2003 , arXive:quant-ph/0008006

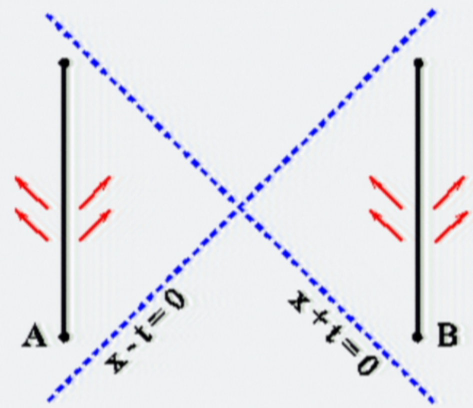
Reznik, Silman Reztker , PRA, 2005

BELL'S INEQUALITIES IN VACUUM

---Final state can be "filtered" using extra ancilla, and manifest violation of Bells' inequalities Reznik, Silman, Retzker 2005.

---Related result using algebraic FT methods by R. Werner.

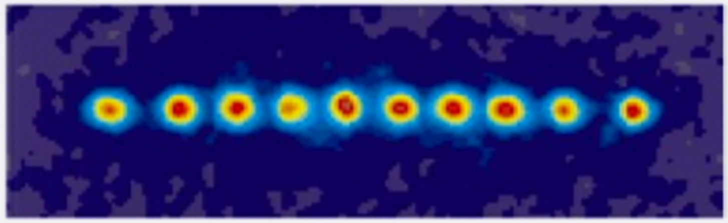
SOME EXTENSIONS



-Exact results for harmonic detectors
, Massar and Spindel (2006)
-Adiabatic method (massive field)
Cliche and A. Kempf (2010)
-Entanglement in curved background
(cosmological case) Steeg and Menicucci (2009).

-Fermion field entanglement
Silman and Reznik (2005) (distilation but no proof of BIV)
-Many regions (W-state Entanglement, not GHZ)
Silman and Reznik (2007)

ANALOG SIMULATION



@Blatt

"Vacuum" field entanglement ➔ motional degrees of the ions

$$\hat{\phi}(x) \longrightarrow \hat{X}_i$$

$$x \longrightarrow i$$

Retzker, Cirac, Reznik, PRL 2005

RESULTS FROM THE LATTICE

GAUSSIAN STATES

1D HARMONIC LATTICE

$$H = \frac{E_0}{2} \sum p_i^2 + q_i^2 - \alpha q_i q_{i-1}$$

$0 < \alpha(K, M, \omega) < 1$ $\alpha \to 1$ gives the continuum limit

$$\psi(q) \propto \exp[-q^T H\, q]$$

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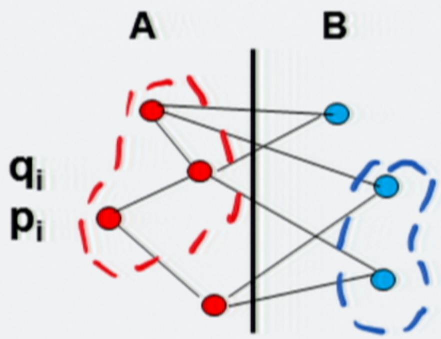
MIXED GAUSSIAN ENTANGLEMENT



$\rho_{AB} \rightarrow \rho_{AB}^{\text{p.t}}$

$\lambda_i < \frac{1}{2}$ after partial transposition for CM:

Logarithmic negativity measure: $E_{Ln}(\rho_{AB}) = -\sum_{2\lambda_i<1}^{N} \ln(2\lambda_i)$

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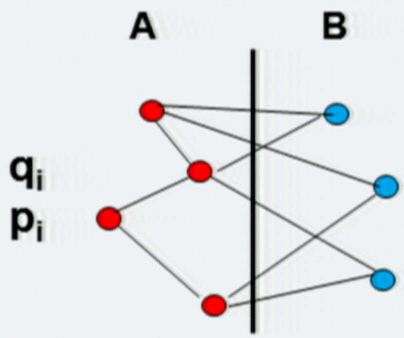
BI-PARTITE GAUSSIAN ENTANGLEMENT



$$M = \begin{bmatrix} M_A & K \\ K^T & M_B \end{bmatrix} \rightarrow W$$

⇨ Symplectic eigenvalues λ_i

$$S(\rho) = -tr(\rho_A log \rho_A)$$

$$S(\rho) = \sum_{i=1}^{N} (\lambda_i + 1/2)\ln(\lambda_i + 1/2) - (\lambda_i - 1/2)\ln(\lambda_i - 1/2)$$

ADJACENT OSCILLATORS

negativity $\neq 0$

negativity $\neq 0$

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SEPARATION OF TWO OR MORE SITES

Entanglement vanishes even at criticality

Intuition: the correlations between two sites decays with with the distance. However the decoherence with respect to the environement (near sites) remains roughly unchanged. Thus at finite separation the density matrix becomes separable.

Is it in conflict with the “probe approach”

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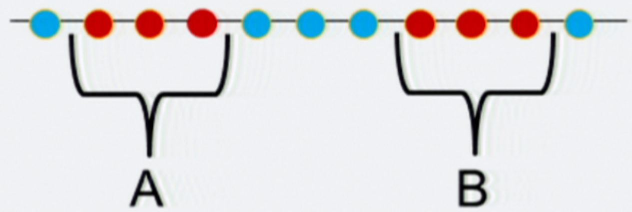
ENTANGLEMENT BETWEEN BLOCKS



Maximal separation before entanglement vanishes increases rapidly with the number of oscillators in each block

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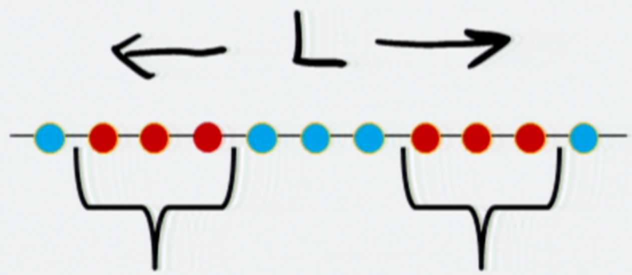
SCALING AT CRITICALITY



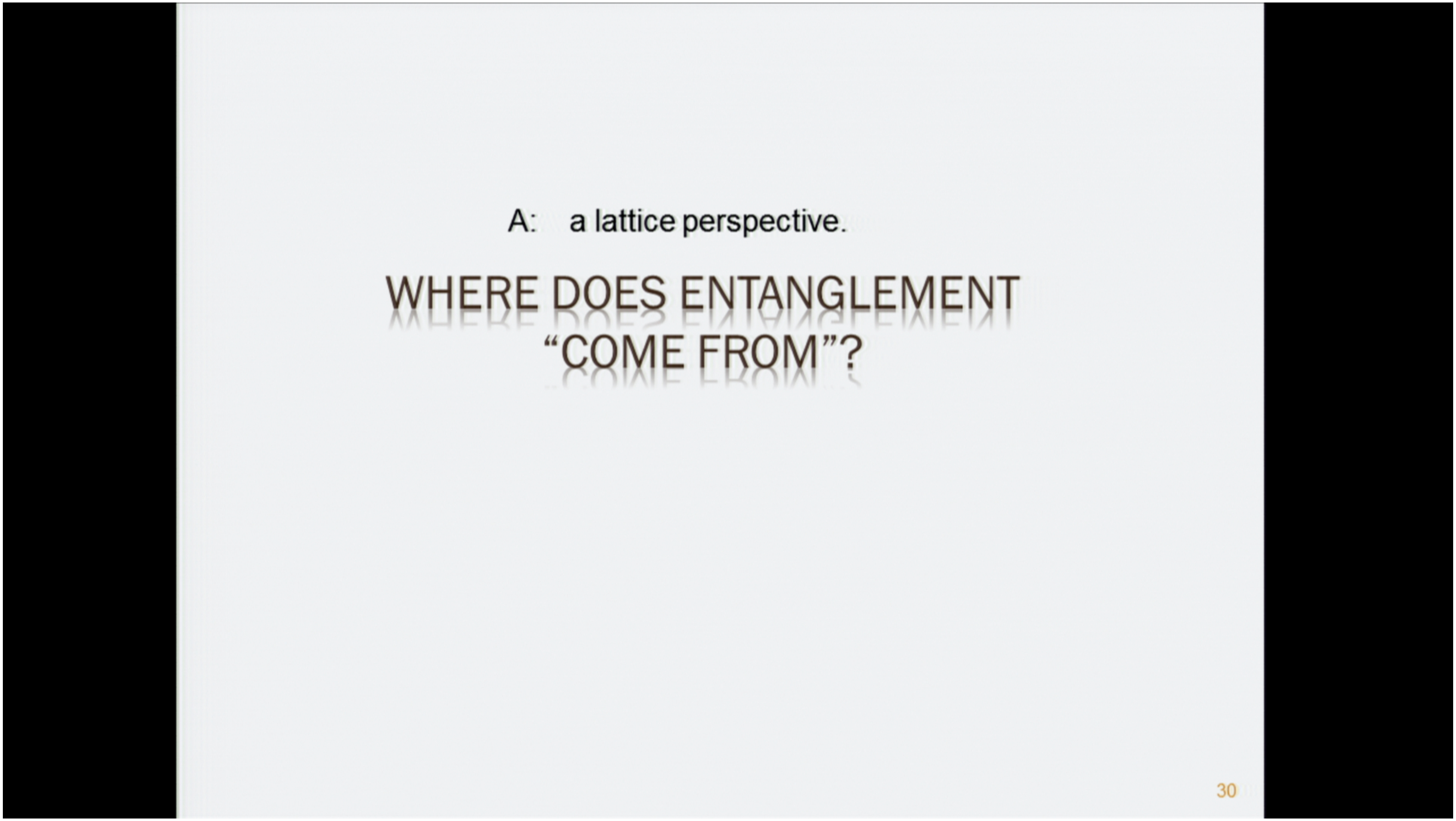
with

$$E_{Ln} = f(r)e^{-\beta r} \qquad r= \frac{L}{D} >>1$$

$$\beta \approx 2\sqrt{2}$$

. Markovich, A. Retzker, M. Plenio, B. Reznik, PRA 2009.
very recently computed analytically for CFT :Calabrese, Cardy, Tonni, arXive 2012.

WHAT GIVES RISE TO VACUUM ENTANGLEMENT AT ARBITRARY SEPARATION?


A: a lattice perspective.
WHERE DOES ENTANGLEMENT
"COME FROM"?
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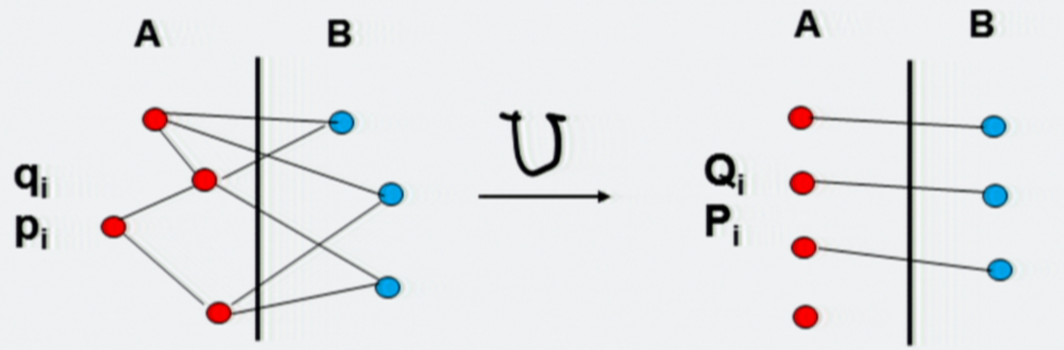
MODE-WISE DECOMPOSITION



$$\psi_{AB} = \sum c_i |A_i\rangle |B_i\rangle \xrightarrow{U} \psi_{AB} = \psi_{11}\psi_{22}\cdots\psi_{kk}\ \psi_{0,..}$$

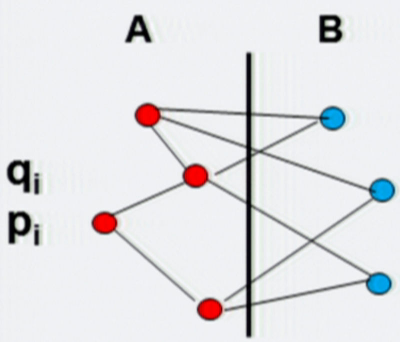
Schmidt — **Mode-Wise decomposition**

$$\psi_{kk} = \sum e^{-\beta_k n} |n\rangle |n\rangle$$

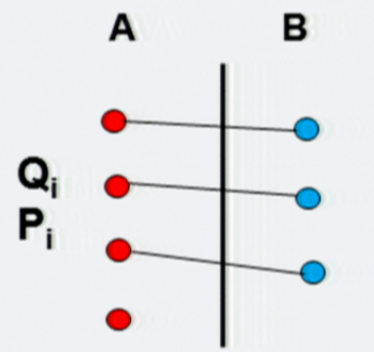
Two modes squeezed state

Botero, Reznik, PRA, 2003, (bosonic Gaussian states)
Botero, Reznik, PL, 2004. (fermionic Gaussian states)

QUALITATIVE SPATIAL STRUCTURE



local		collective
q_i	$\rightarrow$	$Q_m = \sum u_i \, q_i$
p_i	$\rightarrow$	$P_m = \sum v_i \, p_i$

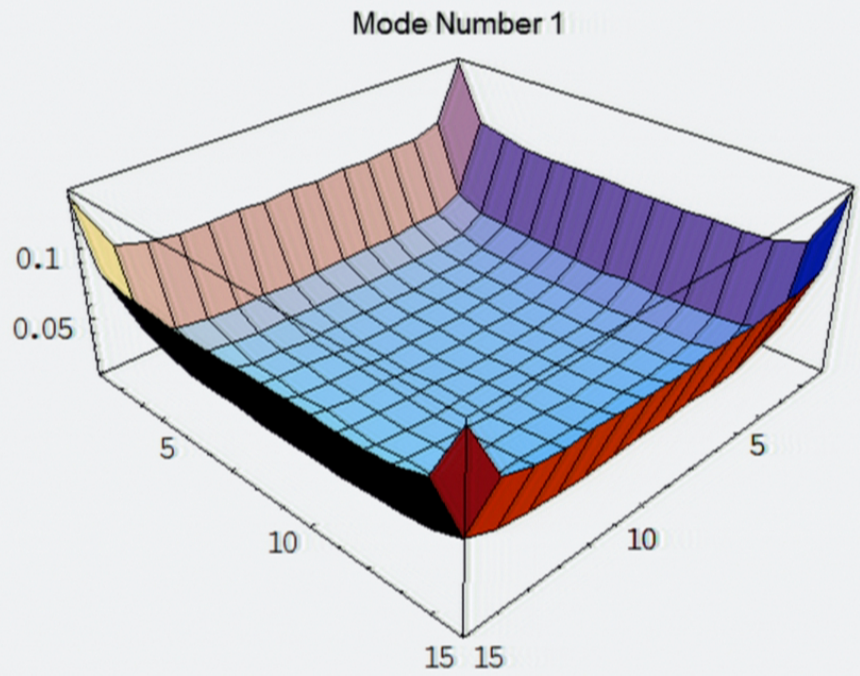


Participation function :=
$P_i = u_i \, v_{i,}$ $\sum P_i = 1$

Quantifies the contribution of local (q_i, p_i) oscillators to the collective coordinates (Q_i, P_i)

ENTANGLEMENT STRUCTURE: 2D LATTICE

harmonic



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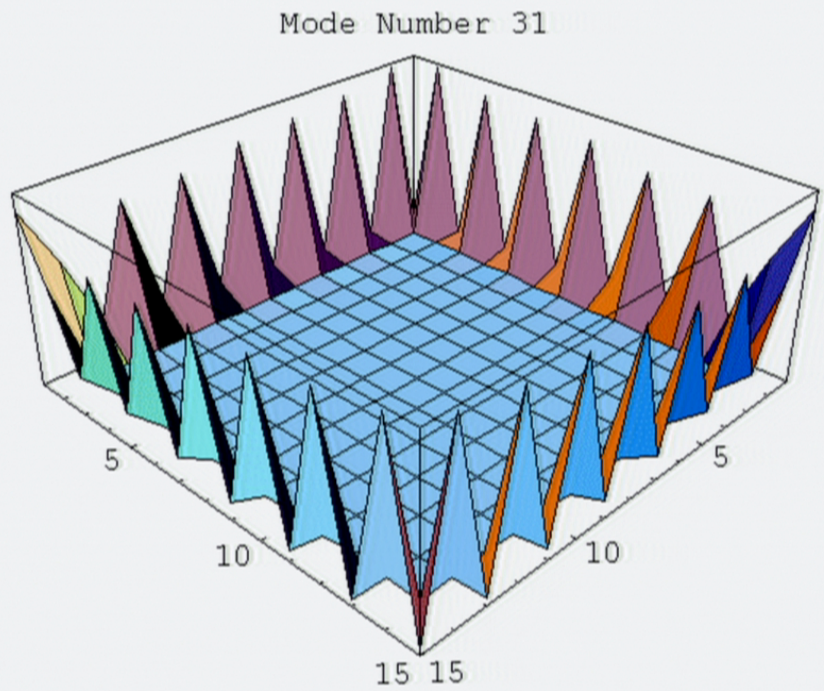


SUPEROSCILLATIONS ARE NEEDED

$$\int_0^\infty \omega d\omega \epsilon_A(\Omega+\omega)\epsilon_B(\Omega+\omega) < \frac{1}{L}\int_0^\infty d\omega \sin(\omega L)\,\epsilon_A(\Omega+\omega)\epsilon_B(\Omega-\omega)$$

Off resonance

Vacuum "window function"

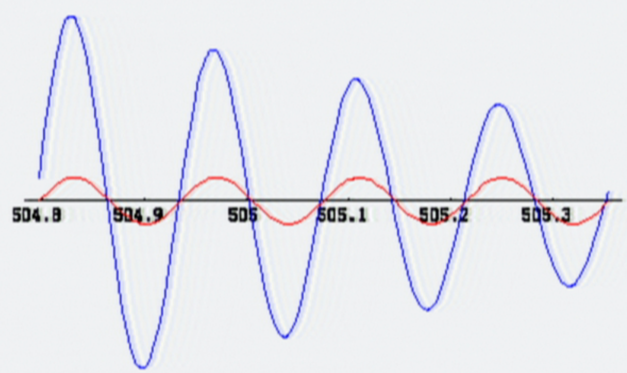
But $\epsilon(\omega)$ oscillates (in ω) like 1/T >>L for a normal function this leads to an Exponential decay of the r.h.s !!

We pick a very special superoscillatory function for $\epsilon_A(\Omega+\omega)$.

Superocillatory functions: *Aharonov (88).*

SUPEROSCILLATORY WINDOW

- In frequency space:
- $\epsilon_A(\omega + \Omega)$ where $0 < \omega < \infty, \quad \Omega > 0$
- Superoscillations of order ~1 found at the tail of an infinitely diverging function "hidden" in non-physical region. $\omega < 0$.



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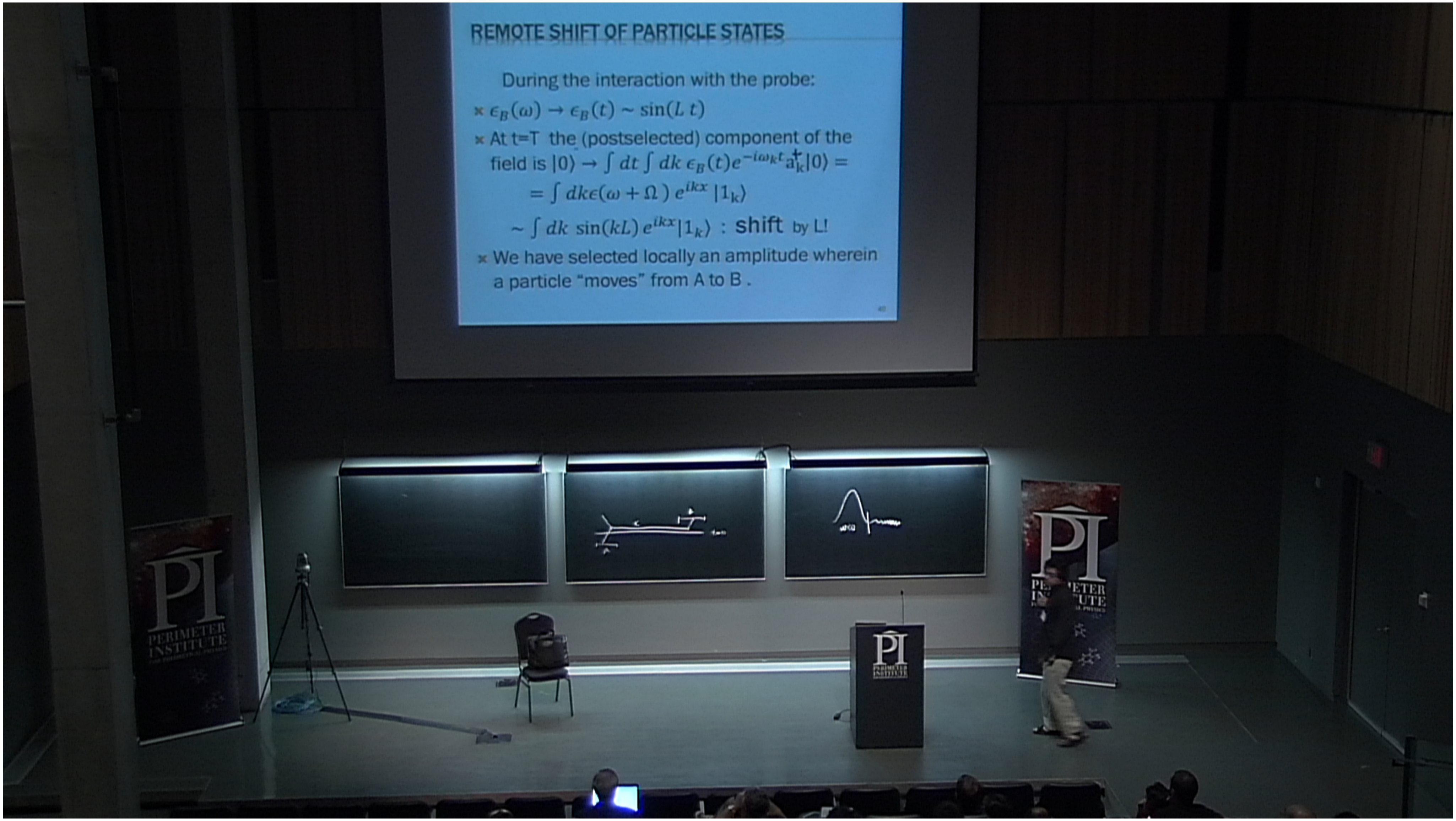
REMOTE SHIFT OF PARTICLE STATES
During the interaction with the probe:
$\epsilon_B(\omega) \to \epsilon_B(t) \sim \sin(L\,t)$
At t=T the (postselected) component of the field is $|0\rangle \to \int dt \int dk\, \epsilon_B(t) e^{-i\omega_k t} a_k^{+}|0\rangle =$
$= \int dk \epsilon(\omega + \Omega)\, e^{ikx}\, |1_k\rangle$
$\sim \int dk\, \sin(kL)\, e^{ikx} |1_k\rangle$: shift by L!
We have selected locally an amplitude wherein a particle "moves" from A to B .
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  $$= \int dk \epsilon(\omega + \Omega)\, e^{ikx}\, |1_k\rangle$$

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- We have selected locally an amplitude wherein a particle “moves” from A to B .