Title: Photon Location in Rindler Coordinates

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Abstract: Bases of orthonormal localized states are constructed in Rindler coordinates and applied to an Unruh detector with good time resolution and an accelerated rod-like array detector.

INTRODUCTION

Localized states are controversial in relativistic QM. The corresponding field is not localized and it has been claimed that there are problems with invariance and causality and that there is no photon position operator or number density. (Hegerfeldt, Bialnicki-Birula, Sipe, Scully and Zubairy *QO*, and Birrell and Davies.) But experimentalists count photons every day. It is straightforward to define an orthonormal and complete basis $u_{t,x}$ on a hypersurface that describes a hypothetical particle counting experiment (even for photons, see arXiv/quant-ph Hawton). Following Newton and Wigner I will call these the localized states.

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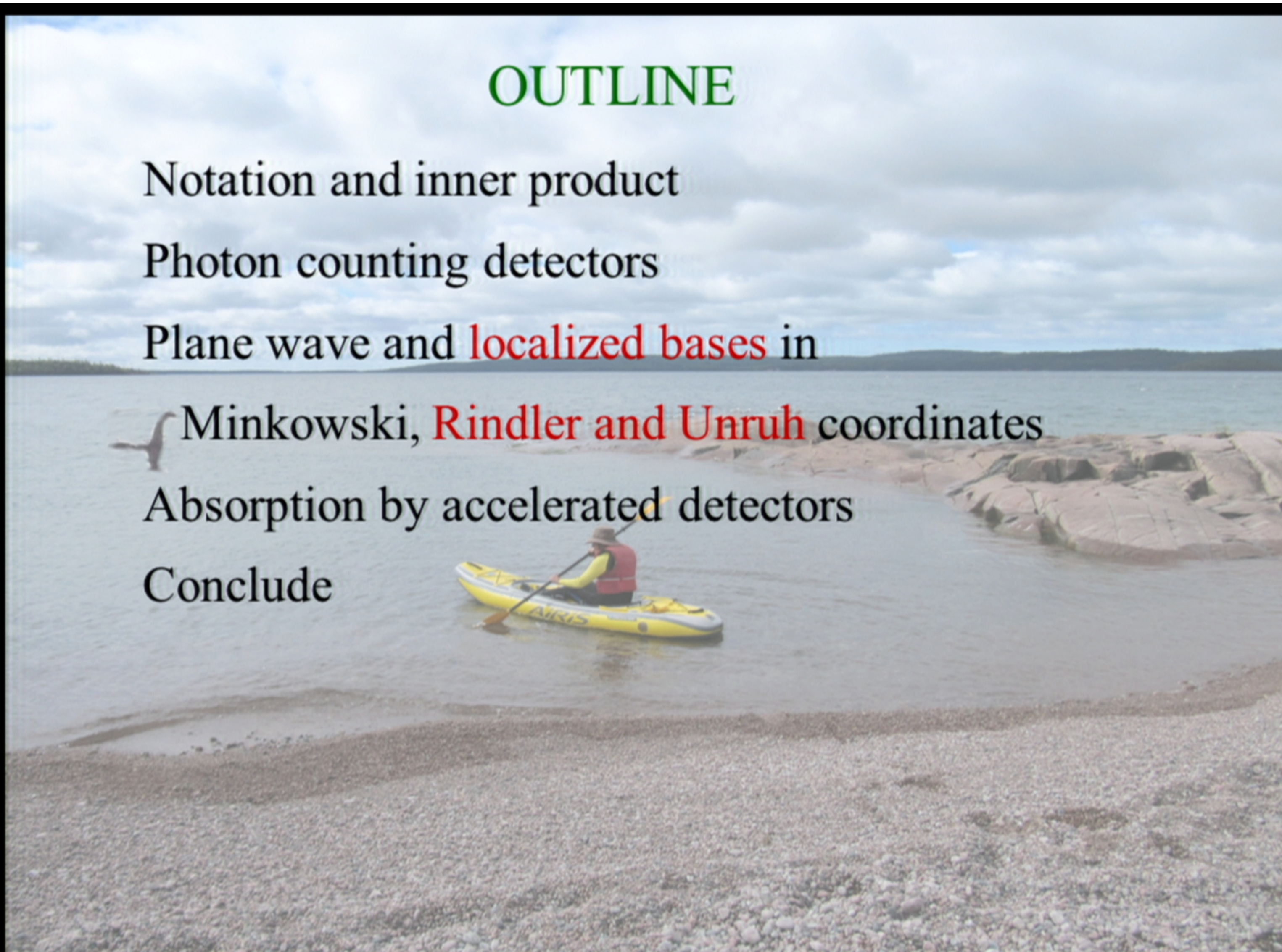


OUTLINE

Notation and inner product

Photon counting detectors

Plane wave and localized bases in

Minkowski, Rindler and Unruh coordinates

Absorption by accelerated detectors

Conclude

NOTATION, FIELD OPERATORS

$(-,+)$ metric signature in $2D$

natural units $\hbar = c = 1$

$x^\mu = (t, x)$ or (η, ξ) are the spacetime coordinates

$k^\mu = (\omega, k)$ or (Ω, K) are frequency and wave vector

$k^\mu x_\mu = kx - \omega t$ or $K\xi - \Omega\eta$

The $+ve$ ω part of the vector potential with polarization λ is

$\widehat{a}_\lambda^{\mu(+)}(t, x) = \int_{-\infty}^{\infty} dk \frac{\exp(ikx - i\omega t)}{(2\omega)^{1/2}(2\pi)^{1/2}} e_\lambda^\mu(\omega, k)\, \widehat{a}_\lambda(\omega, k)$

while the electric field is

$\widehat{\mathbf{E}}_\lambda^{(+)}(t, x) = \int_{-\infty}^{\infty} dk \frac{(2\omega)^{1/2}\exp(ikx - i\omega t)}{(2\pi)^{1/2}} \mathbf{e}_\lambda(\omega, k)\, \widehat{a}_\lambda(\omega, k)$

INDEFINITE INNER PRODUCT in *x*-space and *k*-space on hypersurface Σ

$$(\phi,\psi) = i\int_\Sigma d\Sigma^\mu \phi^{\nu *}(t,x)\overleftrightarrow{\partial}_\mu \psi_\nu(t,x)$$

The covariant inverse Fourier transform is

$$\psi^\mu(t,x) = \sum_\lambda \int_\Sigma \frac{d\Sigma}{2k_\Sigma}\frac{\exp(ikx - i\omega t)}{(2\pi)^{1/2}} e^\mu_\lambda(\omega,k)\,\psi_\lambda(\omega,k)$$

with $\epsilon \equiv k_\Sigma / |k_\Sigma|$ so the inner product can be written as

$$(\phi,\psi) = \sum_{\lambda,\epsilon}\int_\Sigma \frac{d\Sigma}{2k_\Sigma}\phi^*_\lambda(\omega,k)\,\psi_\lambda(\omega,k)\,.$$

If Σ is a $t = const$ hypersurface

$$(\phi,\psi) = \sum_{\lambda,\epsilon}\int_{-\infty}^{\infty}\frac{dk}{2\omega}\phi^*_\lambda(\omega,k)\,\psi_\lambda(\omega,k)$$

PHOTON COUNTING POVM

A semiconductor detector counts photons if it is thick enough to absorb all incident photons. Absorption probability is $\propto \omega$ but penetration depth is $\propto 1/\omega$. The Glauber probability for a atom to absorb a photon is $\propto$

$$\left\langle \psi \left| \widehat{E}_{\lambda}^{\dagger}(t,x)\, \widehat{E}_{\lambda}(t,x) \right| \psi \right\rangle \propto \omega \exp\left(-2\alpha_{\omega} x\right)$$

probability$\propto \int_0^{\infty} dx \omega \exp\left(-2\alpha_{\omega} x\right) = \frac{\omega}{2\alpha_{\omega}}$ where $\alpha_{\omega} \propto \omega$.

The ω dependence cancels so that probability is proportional to photon number. This is straightforward for a plane wave since there is a single frequency and thus there are no interference terms.

The x-basis will always count the same number of photons as the k-basis. Ideally a photon would be counted at the (t,x) coordinates where it crossed Σ, but this is an approximation.

If $|\psi\rangle$ is a pulse with center frequency ω' and width $\Delta\omega$, to $2nd$ order is $(\omega - \omega')/\Delta\omega$ the probability density to count a photon at time t is $\langle\psi|\widehat{n}_\lambda(t - \Delta t, x)|\psi\rangle$ where Δt is a $1st$ order correction of a few optical periods,

$$\widehat{n}_\lambda(t,x) = i\widehat{a}_\lambda^{\nu\dagger}(t,x)\overleftrightarrow{\partial}_t\widehat{a}_{\lambda,\nu}(t,x)$$

This $\widehat{n}$ is the integrand of the inner product converted to an operator. A noncovariant number operator was used in the published version.

Photon number and probability density are meaningf at least in the context of a photon counting experimer

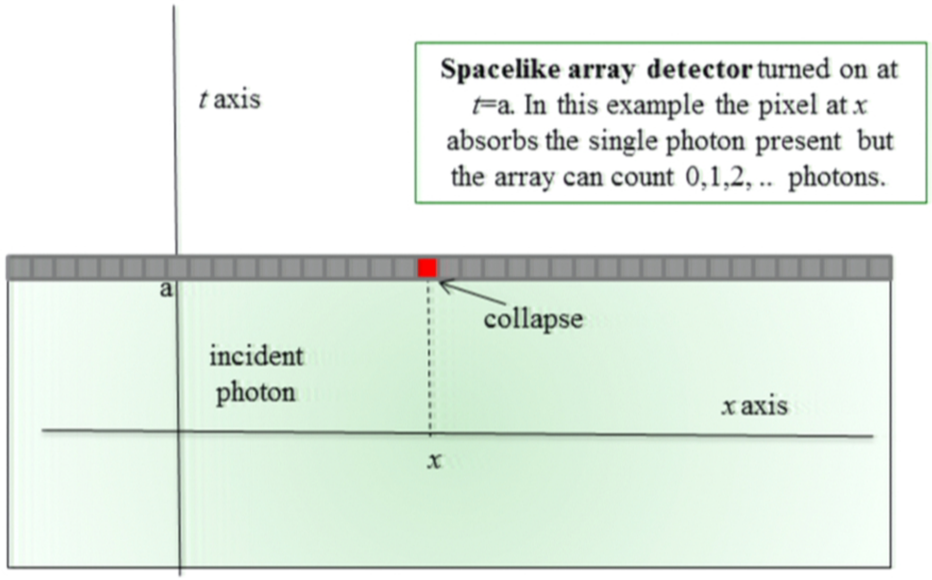
TO AVOID MULTIPLE COUNTING, PHOTONS SHOULD BE COUNTED ON A HYPERSURFACE

Spacelike gedanken experiment: At any time a photon must be somewhere in space. Imagine an array of transparent photon counting detectors throughout space turned on at time t=a with timelike normal n=(1,0). The photon will be detected at some position x.

Timelike real experiment: A photon is detected when it arrives at the detector at x=b with spacelike normal n=(0,1) at time t. Since all ω's are required, the basis does not distinguish between absorption and emission but this may be known from the initial or final state.

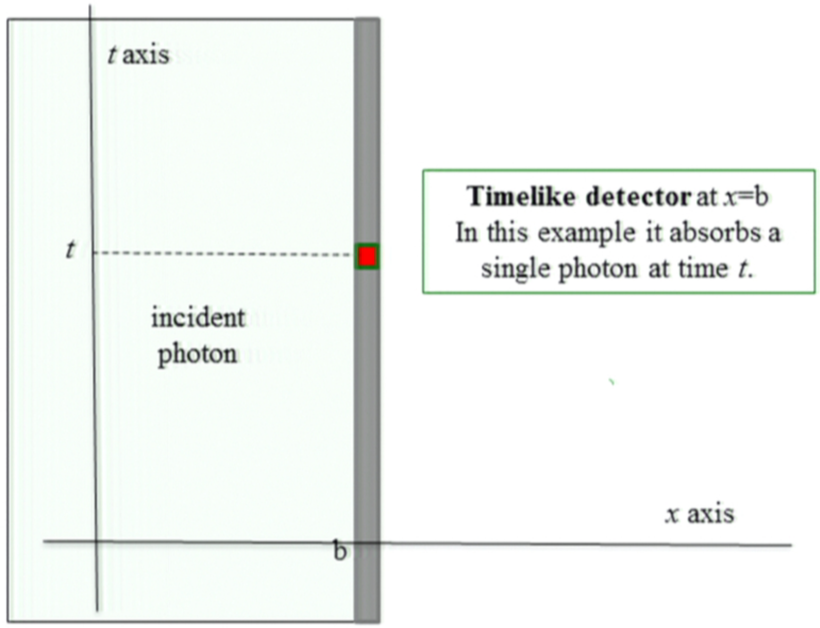
SPACELIKE DETECTOR AT REST

If ω>0 a photon is absorbed, while if ω<0 it is emitted.



TIMELIKE DETECTOR AT REST

This is not a Cauchy surface and there is no Killing vector to separate positive and negative frequencies.



Here $|\psi\rangle$ is the Minkowski plane wave with wave vector k and $+ve$ ω.

The probability density to count a photon at (t, b) is $\left|\frac{\exp(ikb - i\omega t)}{(2\pi)^{1/2}}\right|^2 = \frac{1}{2\pi}$.

$\omega > 0$ ensures that the photon is absorbed rather than emitted.

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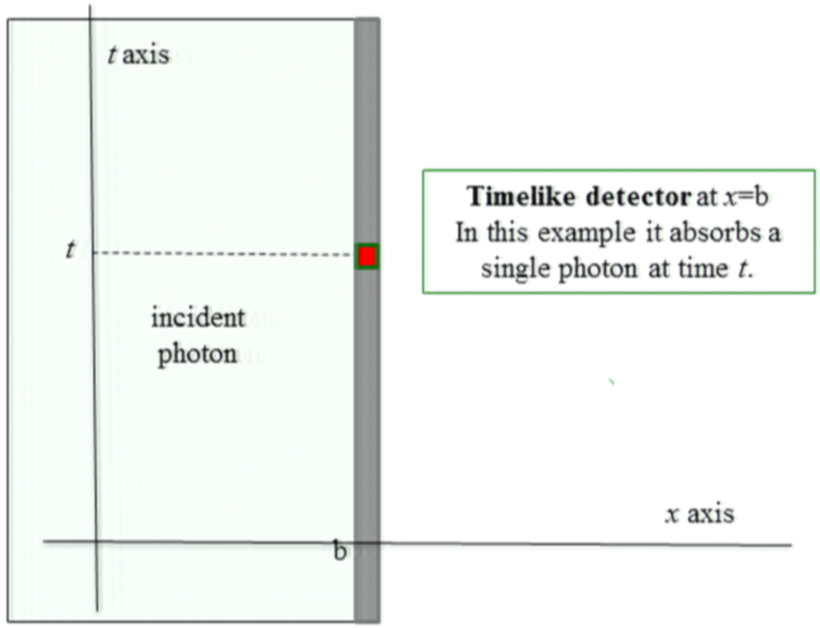

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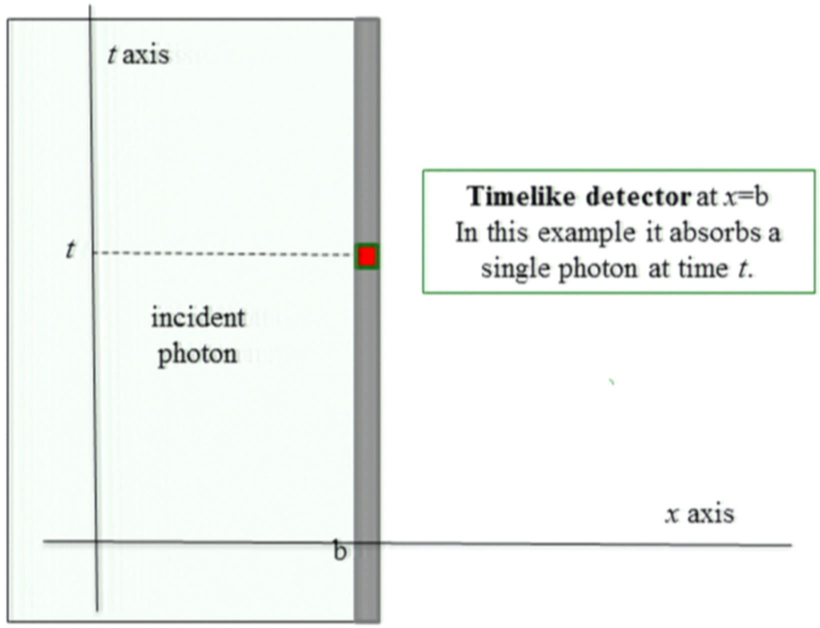

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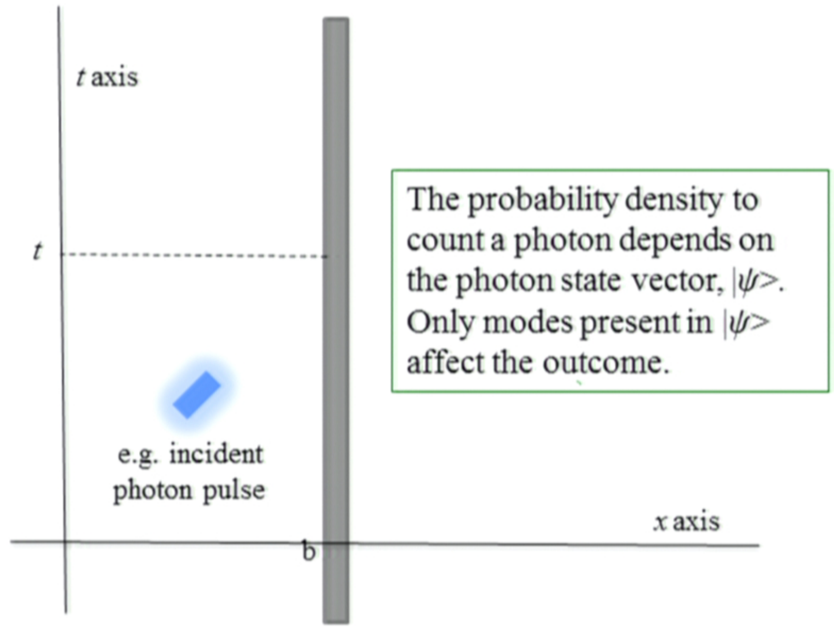
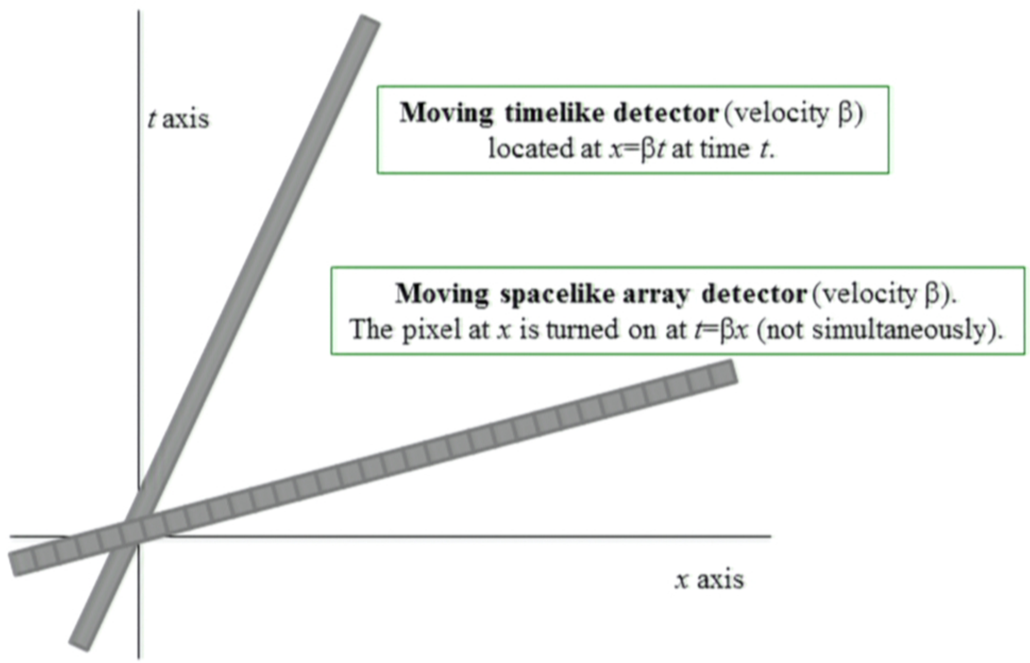
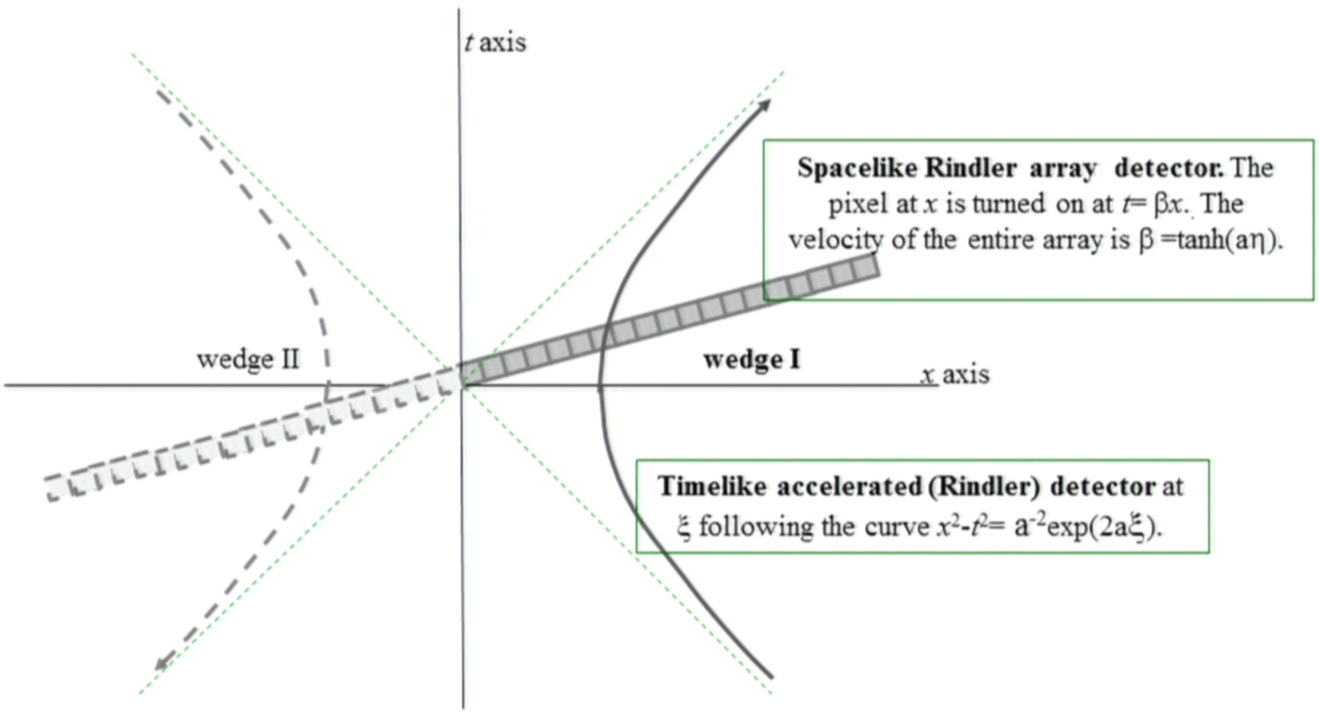


The propagating $|\psi>$ is not exactly localized so the Hegerfeldt theorem is not a problem. The photon counting experiment and its associated localized basis (POVM) never leave Σ.

SPACELIKE AND TIMELIKE DETECTORS WITH VELOCITY β RELATIVE TO THE OBSERVER



RINDLER SPACELIKE AND TIMELIKE DETECTORS



Consider a single λ', $u_{\omega',k',M}(t,x) \equiv u_{\omega',k',\lambda';\lambda',M}(t,x)$.
Prime denotes an fixed value, no prime a variable.

The positive frequency Minkowski plane waves in x-space

$$u_{\omega',k',M}(t,x) = \frac{\exp\left(-i\omega' t + ik' x\right)}{(2\omega')^{1/2}(2\pi)^{1/2}}$$

are orthonormal and complete on Σ. Their complex conjugate negative frequency waves are also orthonormal and complete, but with negative inner product. Mixed inner products are zero.

On Σ defined by $t' = const$ with $\omega' = |k'|$

$$(u_{\omega',k',M}, u_{\omega'',k'',M}) = \delta(k' - k'')$$

$$\left(u^*_{\omega',k',M}, u^*_{\omega'',k'',M}\right) = -\delta(k' - k'')$$

$$\left(u_{\omega',k',M}, u^*_{\omega'',k'',M}\right) = 0$$

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RECIPE TO CONVERT A PLANE WAVE IN x-SPACE TO A LOCALIZED STATE IN k-SPACE

Move the $\sqrt{\omega}$ factor to the numerator. Interchange wave vector and position coordinates. Change the sign in the exponent. All k are then included with equal weight so it is δ–function localized.

$$\frac{\exp\big(-i\omega' t+ik'x\big)}{(2\omega')^{1/2}(2\pi)^{1/2}} \rightarrow \frac{(2\omega)^{1/2}\exp\big(i\omega t'-ikx'\big)}{(2\pi)^{1/2}}$$

I'll define the Minkowski $+ve$ ω localized states in k-space as

$$u_{t',x',M}(\omega,k)=\frac{(2\omega)^{1/2}\exp\left(i\omega t'-ikx'\right)}{(2\pi)^{1/2}}$$

so that they are orthonormal. This can be verified by substitution.

On Σ defined by $t'=const$

$$\left(u_{t',x',M},u_{t',x'',M}\right)=\int_{-\infty}^{\infty}\frac{dk}{2\omega}u^{*}_{t',x',M}(\omega,k)\,u_{t',x'',M}(\omega,k)$$
$$=\delta\left(x'-x''\right)$$

$$\left(u^{*}_{t',x',M},u^{*}_{t',x'',M}\right)=-\delta\left(x'-x''\right)$$

$$\left(u_{t',x',M},u^{*}_{t',x'',M}\right)=0$$

The field (potential) described in x-space,

$$u_{t',x',M}(t,x)=\int_{-\infty}^{\infty}\frac{dk}{2\omega}\frac{(2\omega)^{1/2}\exp\left[-i\omega\left(t-t'\right)+ik\left(x-x'\right)\right]}{(2\pi)^{1/2}}$$

is nonlocal due to the factor $\sqrt{\omega}$. This expression also explains the choice of sign in the exponent in the definition above. But it's easier to work in k-space when using localized states.

Rindler plane waves in wedges I and II are analogous to Minkowski plane waves. For Rindler frequency Ω' and wave vector K' , e.g. on the η=const hypersurface with $-\infty<\xi<\infty$,

$$u_{\Omega',K',I}\,(\eta,\xi) = \frac{\exp\left(-i\Omega'\eta + iK'\xi\right)}{(2\Omega')^{1/2}(2\pi)^{1/2}}$$

$$u_{\Omega',K',II}\,(\eta,\xi) = \frac{\exp\left(i\Omega'\eta + iK'\xi\right)}{(2\Omega')^{1/2}(2\pi)^{1/2}}$$

$$\left(u_{\Omega',K',I}, u_{\Omega',K'',I}\right) = \delta\left(K' - K''\right)\ .\ .\ .$$

The Rindler localized states at (ξ',η') will be defined as

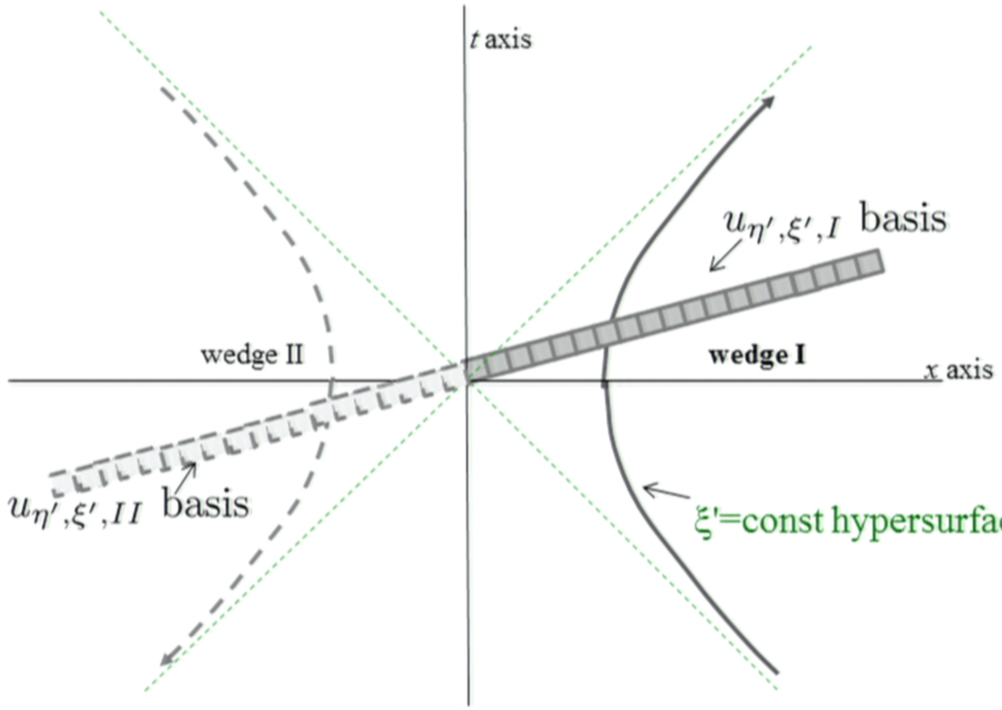
$$u_{\eta',\xi',I}\,(\Omega,K) = (2\Omega)^{1/2}\,\frac{\exp\left(i\Omega\eta' - iK\xi'\right)}{(2\pi)^{1/2}}$$

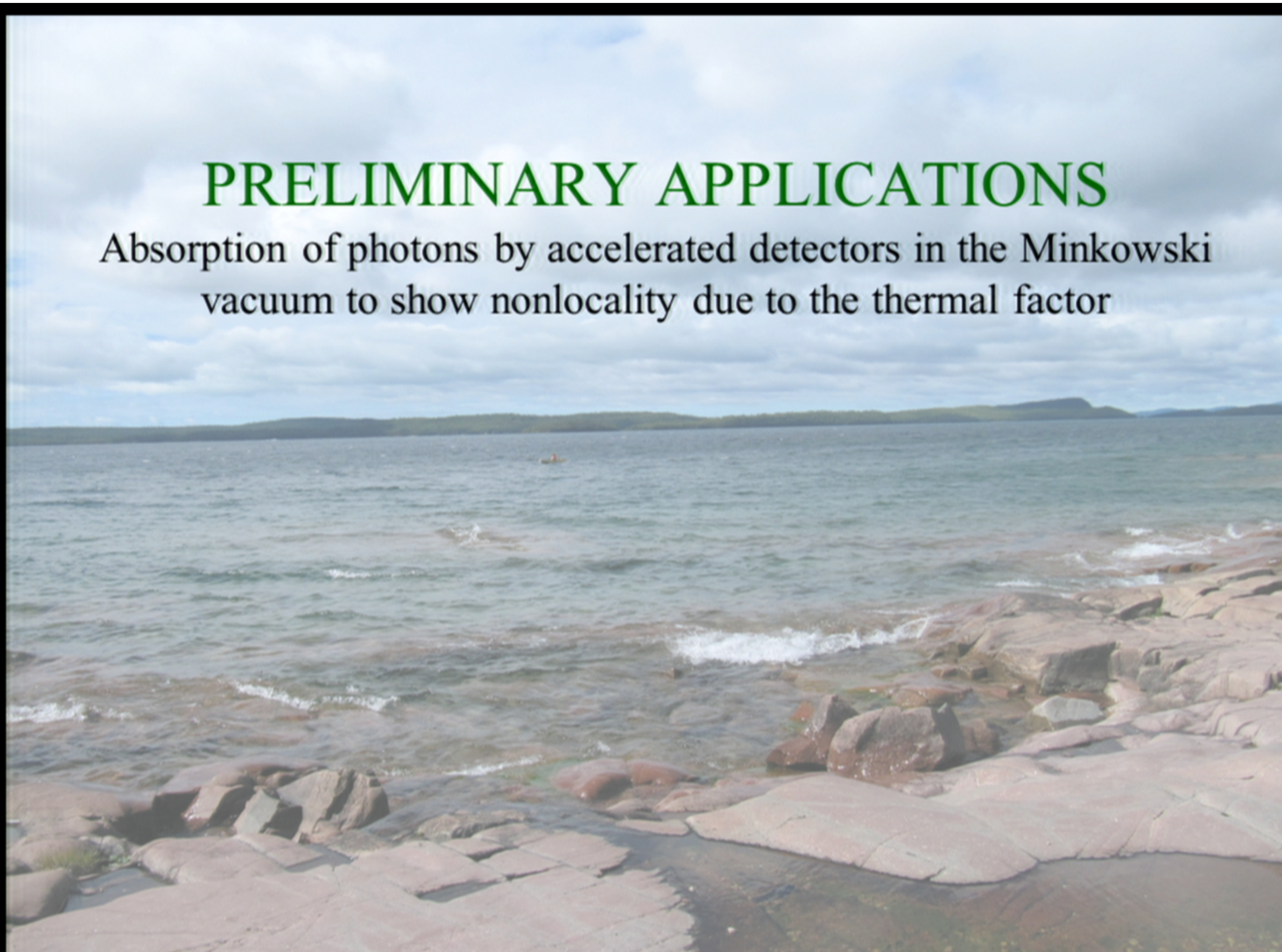
$$u_{\eta',\xi',II}\,(\Omega,K) = (2\Omega)^{1/2}\,\frac{\exp\left(-i\Omega\eta' - iK\xi'\right)}{(2\pi)^{1/2}}$$

$$\left(u_{\eta',\xi',I}, u_{\eta',\xi'',I}\right) = \delta\left(\xi' - \xi''\right)\ .\ .\ .$$

The spacelike hypersurface η'=const is a Cauchy surface with Killing vector $\partial_{\eta'}$ in I and $\partial_{-\eta'}$ in II. The localized basis separates into +*ve* and –*ve* ω parts and annihilation and creation operators can be defined.

The ξ'=const hypersurface (the path of the Rindler detector) is not a Cauchy surface so +*ve* and –*ve* frequencies are not separated in the basis.




PRELIMINARY APPLICATIONS
Absorption of photons by accelerated detectors in the Minkowski vacuum to show nonlocality due to the thermal factor

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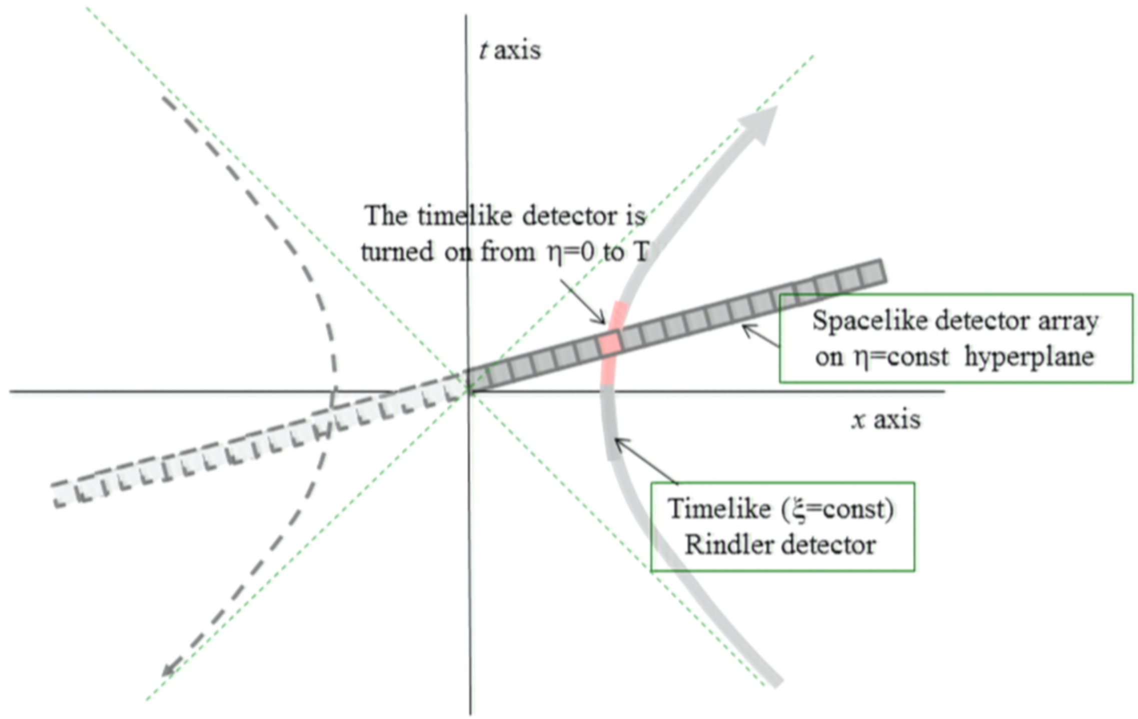
ABSORPTION OF A PHOTON BY A LOCALIZED RINDLER DETECTOR



I'll work with a spacelike basis as is usual in QFT, but the Rindler detector actually lives on the timelike hypersurface and I'll discuss it briefly.

AS IN UNRUH AND WALD EXCEPT IN x-BASIS

$$\widehat{a}_{\eta',\xi',I} = \int_{-\infty}^{\infty} dK \frac{\exp\left(-i\Omega\eta' + iK\xi'\right)}{(4\pi\Omega)^{1/2}} \widehat{a}_{\Omega,K,I}$$

annihilates a Rindler photon in state localized at (η', ξ') in I. In the Unruh basis

$$\widehat{a}_{\Omega,K,I} = \frac{\widehat{A}_{\Omega,K,I} + \exp(-\pi\Omega/a)\widehat{A}^{\dagger}_{\Omega,-K,II}}{[1-\exp(-2\pi\Omega/a)]^{1/2}}$$

The Unruh vacuum is the same as the Minkowski vacuum. Photons cannot be annihilated when the RHS acts on $|0_M>$ so annihilation of a Rindler photon in I is seen as emission of a photon in the Unruh basis, primarily in II. This is the usual argument except that here a photon is absorbed locally so I integrate over K.

Working in the Unruh basis where $|1_{\Omega,K,II}\rangle = \widehat{A}^{\dagger}_{\Omega,K,II}\,|0_M\rangle$

$$|\psi\rangle = \widehat{a}_{\eta',\xi',I}\,|0_M\rangle$$

$$= \int_{-\infty}^{\infty} dK\,\frac{\exp\big(i\Omega\eta' - iK\xi'\big)}{(4\pi\Omega)^{1/2}}\,\frac{\exp(-\pi\Omega/a)}{[1-\exp(-2\pi\Omega/a)]^{1/2}}\,|1_{\Omega,K,II}\rangle$$

$$\equiv |1_{\eta',\xi',a,II}\rangle$$

$$\langle 1_{\eta',\xi,II}|\psi\rangle = \int_{-\infty}^{\infty} dK' \int_{-\infty}^{\infty} dK\,\frac{\exp\big(iK'\xi - i\Omega'\eta'\big)}{(4\pi\Omega)^{1/2}}\,\frac{\big(2\Omega'\big)^{1/2}\exp\big(i\Omega\eta' - iK\xi'\big)}{(4\pi\Omega')^{1/2}}$$

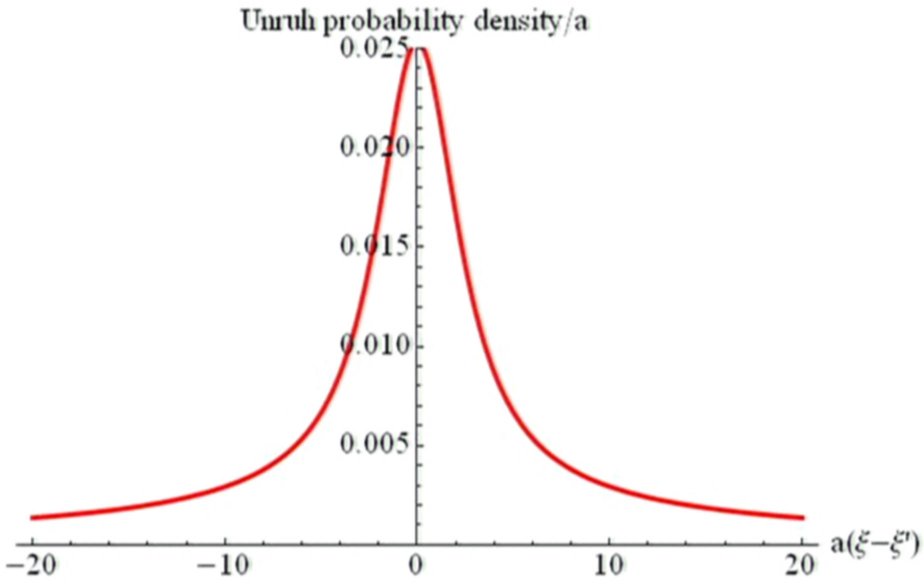
$$\times\frac{\exp(-\pi\Omega/a)}{[1-\exp(-2\pi\Omega/a)]^{1/2}}\left\langle 0_M \left| \widehat{A}^{\dagger}_{\Omega,K,I}\widehat{A}_{\Omega',K',I} \right| 0_M \right\rangle$$

$$= \int_{-\infty}^{\infty} dK\,\frac{\exp\big[iK\big(\xi-\xi'\big)\big]}{2\pi}\,\frac{\exp(-\pi\Omega/a)}{[1-\exp(-2\pi\Omega/a)]^{1/2}}$$

$$\langle 1_{\eta',\xi,I}|\psi\rangle = 0$$

This describes the spatial extent of the emitted photon density (primarily in wedge II) as seen by a Minkowski observer using the bi-localized Unruh basis.

ONE PHOTON UNRUH STATE II CREATED BY CLICK OF A LOCALIZED RINDLER DETECTOR IN I



This would be a δ-function without the thermal factor ($T=a/2\pi$).

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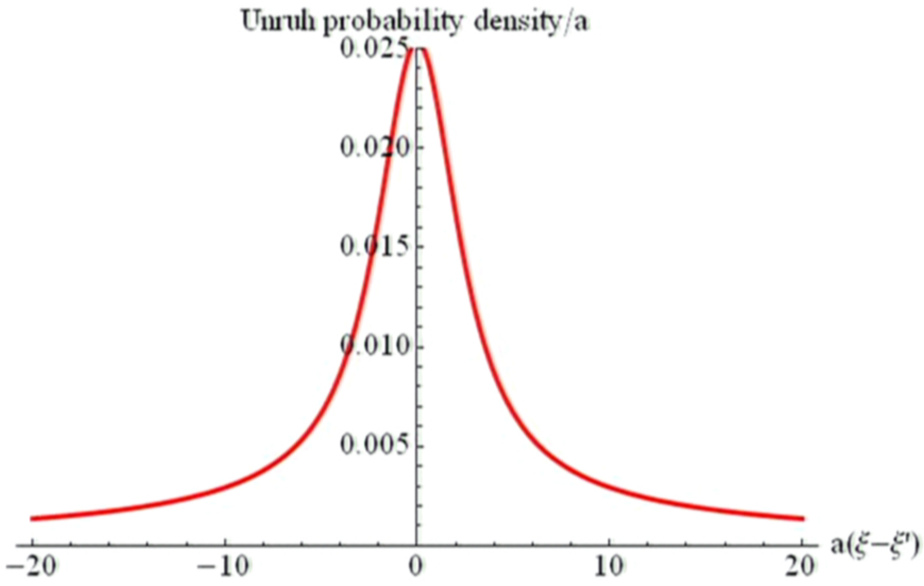

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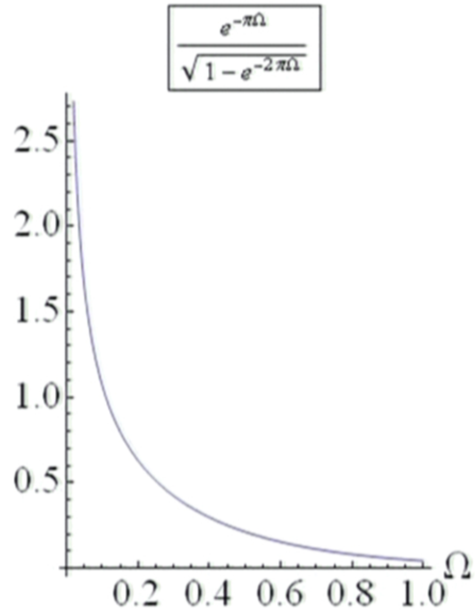

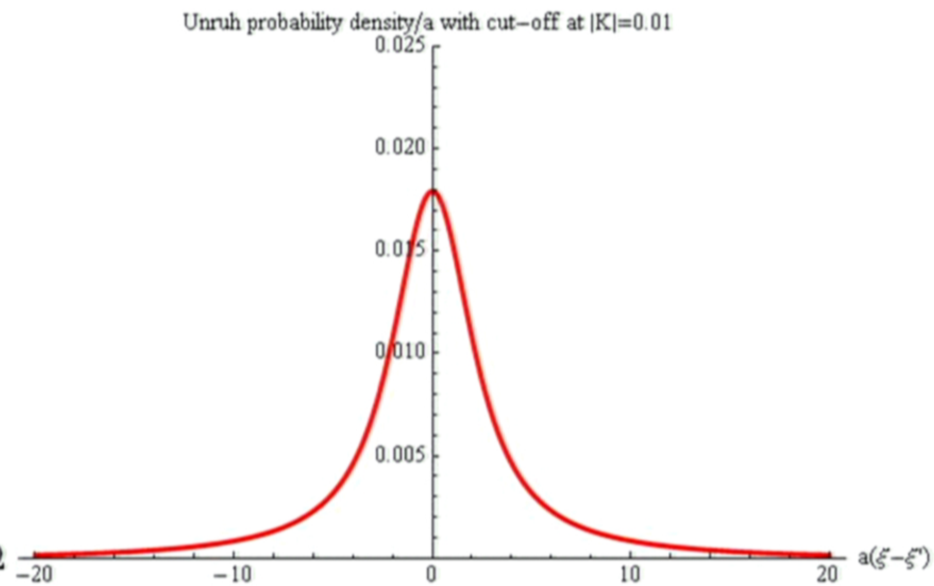

The slow decline of the wave function is due to divergence of the thermal factor as $\Omega \to 0$. In the graph above a cut-off at $|\Omega|=0.01$ was introduced.

CONCLUSION

Localized bases (POVMs) describing small photon counting hyperpixels were constructed in Rindler and in Unruh coordinates.

Here localized means that $(u_{\eta',\xi',J}, u_{\eta',\xi'',J}) = \delta(\xi' - \xi'')$ for ξ' and ξ'' on η' hypersurface Σ in wedge J (or on ξ' =const, but this basis is timelike).

The Unruh state created when a photon is absorbed by a localized Rindler detector is broaden due to the thermal factor $\exp(-\pi\Omega/a) / [1 - \exp(-2\pi\Omega/a)]^{1/2}$.