

Title: Quantum Communication Between Localized, Non-Inertial Observers

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URL: <http://pirsa.org/12060065>

Abstract: An unsolved problem in relativistic quantum information research is how to model efficient, directional quantum communication between localised parties in a fully quantum field theoretical framework. We propose a tractable approach to this problem based on calculating expectation values of localized field observables in the Heisenberg Picture. We illustrate our approach by analysing, and obtaining approximate analytical solutions to, the problem of communicating quantum states between an inertial sender, Alice and an accelerated homodyne receiver, Rob. We discuss the effect on quantum protocols carried out over such a communication channel.

QUANTUM COMMUNICATION BETWEEN LOCALIZED NON-INERTIAL OBSERVERS

T.C.Ralph, Tony Downes
and Nathan Walk
School of Maths & Physics
University of Queensland

T.G.Downes, T.C.Ralph, N.Walk,
arXiv:1203.2716 (2012)



COMBINING QUANTUM OPTICS WITH RELATIVISTIC QUANTUM FIELD THEORY

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School of Maths & Physics
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$Q|O\rangle Q|I\rangle$
Quantum Optics Quantum Information Theory Group





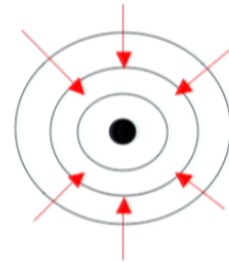
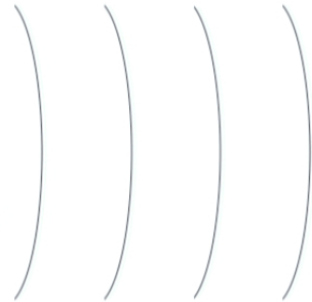
Relativistic Quantum Information 6
Workshop
Customs House, Brisbane
Australia
late November 2012



RQI2
2008

Quantum Optics vs Relativistic QFT

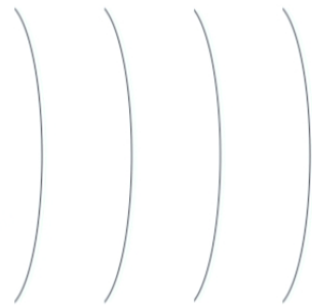
$|\Psi\rangle$
global
field state



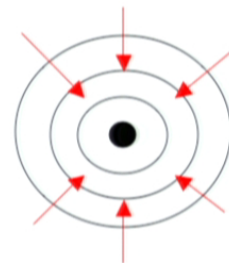
2-level system
weakly
coupled to
field

Quantum Optics vs Relativistic QFT

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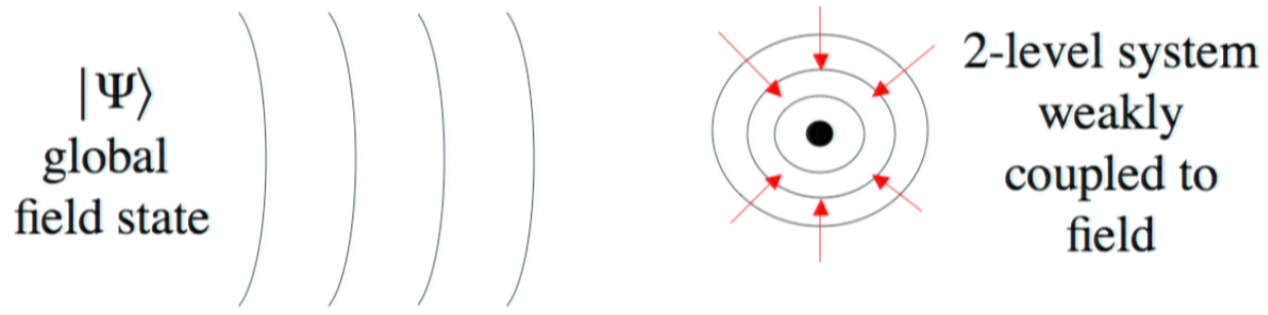


theoretical treatment
of non-inertial systems

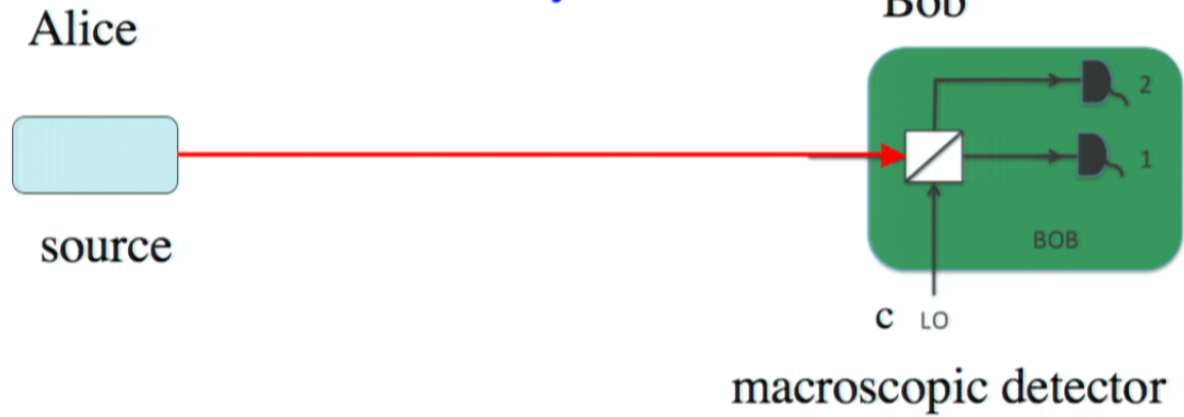


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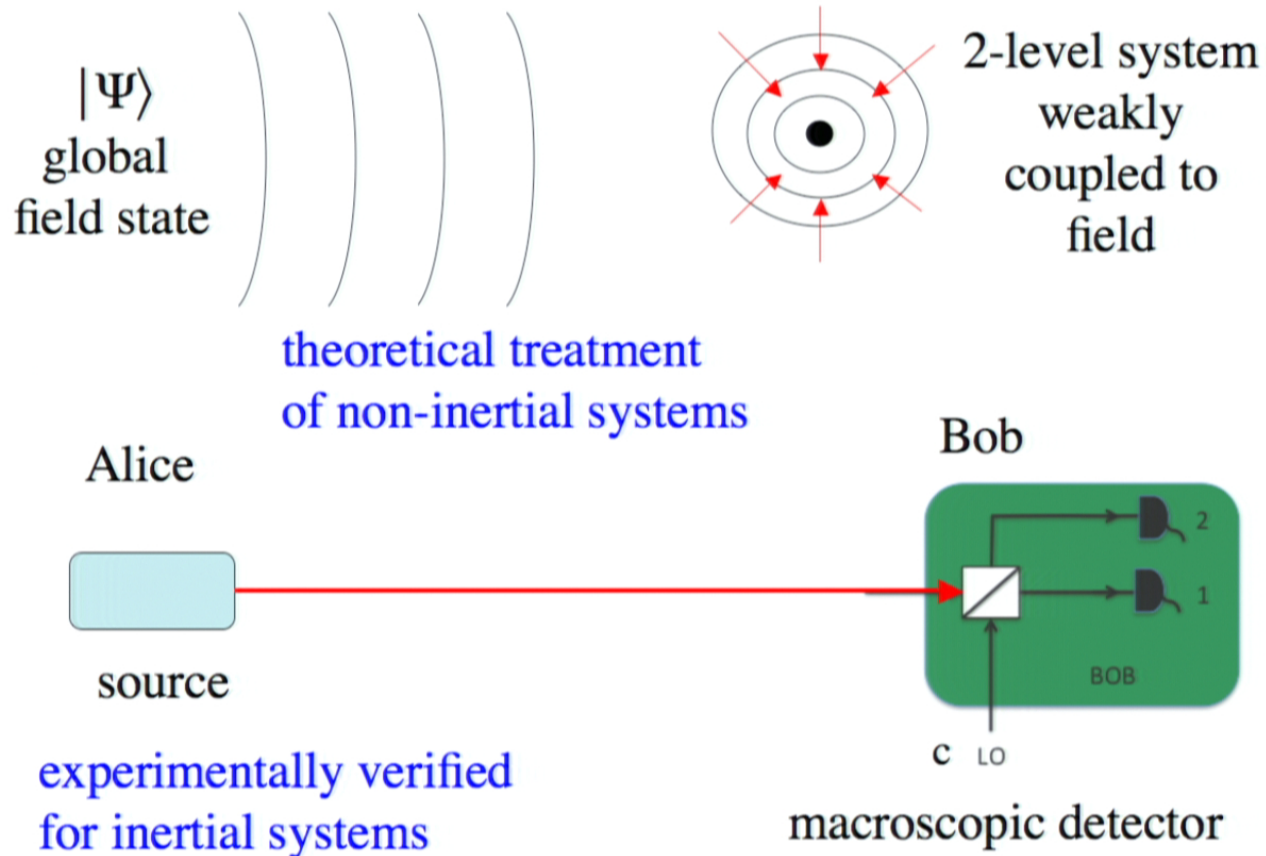
Quantum Optics vs Relativistic QFT



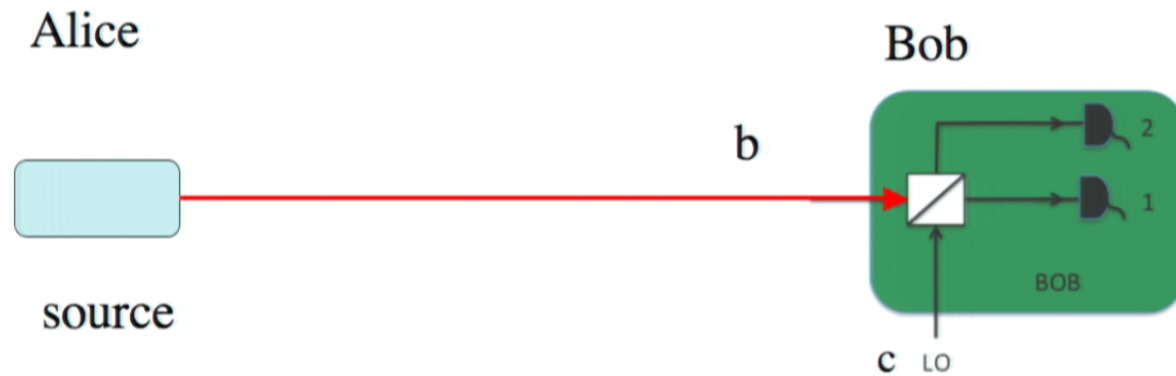
theoretical treatment
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Quantum Optics vs Relativistic QFT



Quantum Communication



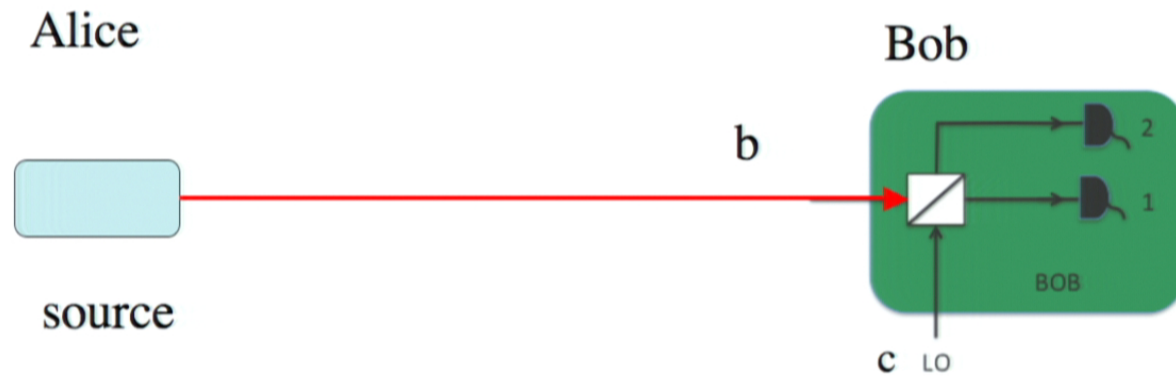
$$|\psi\rangle = U(a) |0\rangle$$

$$\int d\tau (b^\dagger_\tau c_\tau + c^\dagger_\tau b_\tau)$$

$$\langle b^\dagger b \rangle = \int d\tau \langle 0 | U^\dagger(a) (b^\dagger_\tau c_\tau + c^\dagger_\tau b_\tau) U(a) | 0 \rangle$$

$a, b, c \equiv$ mode annihilation operators

Quantum Communication



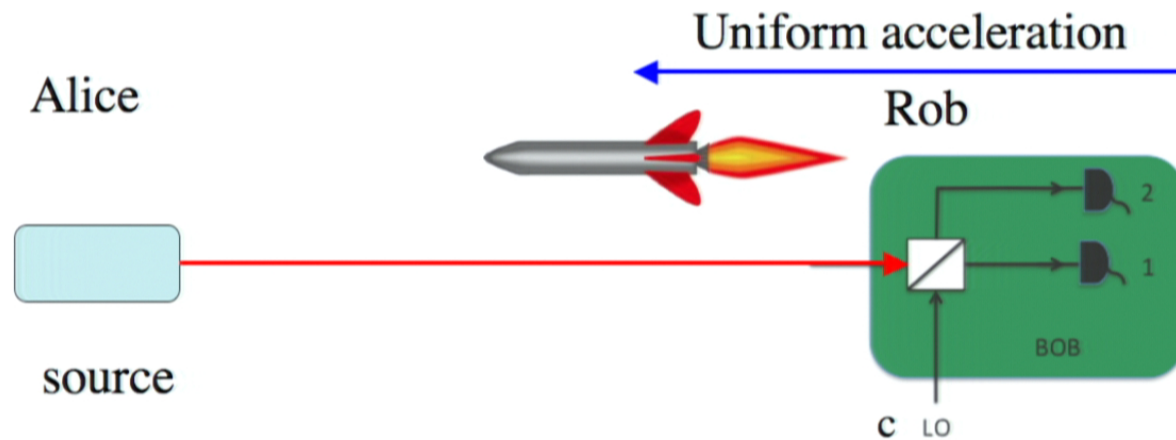
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Quantum Communication with acceleration



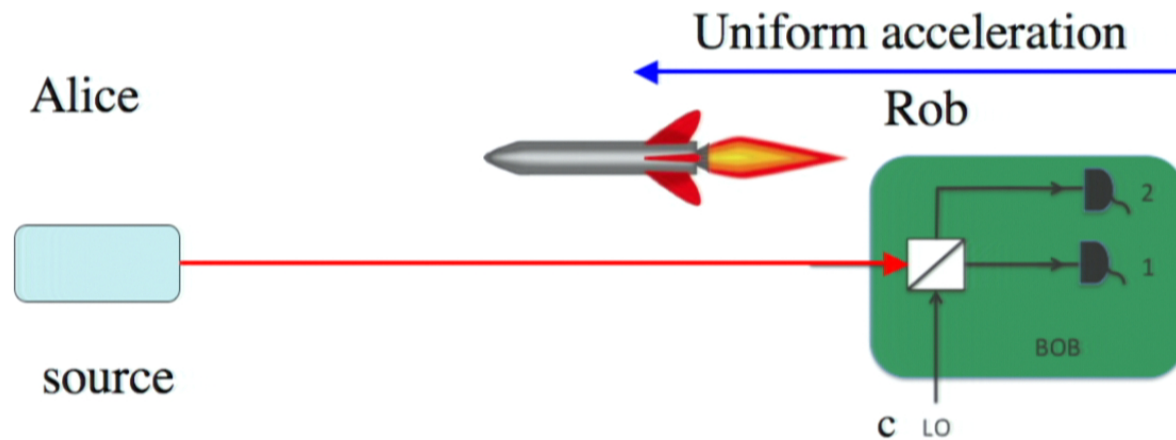
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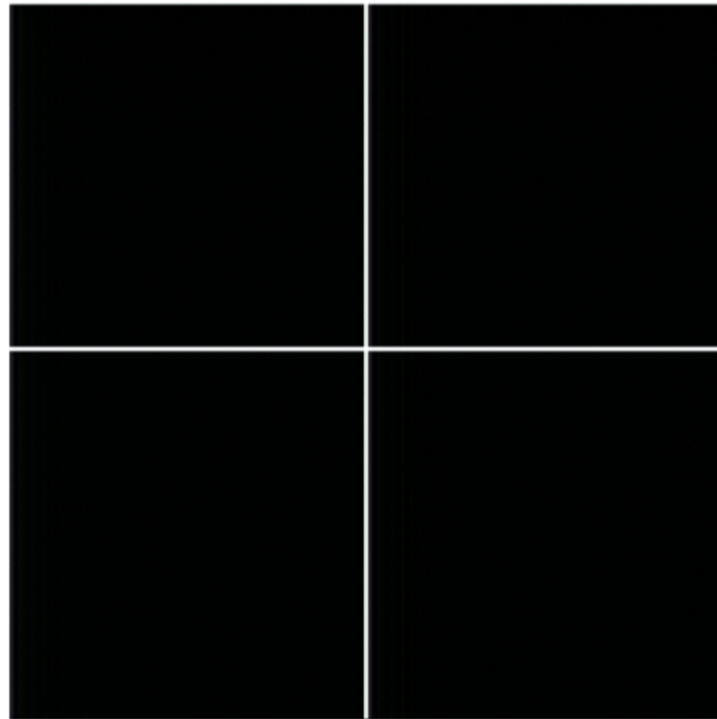
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Overview

- * Problems with analysing localized non-inertial sources and/or detectors
- * Efficient, localized detection of Unruh radiation.
- * Quantum Communication between Alice and Rob

Vacuum



X

t

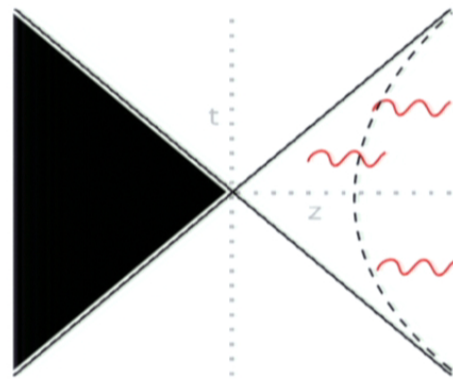
Vacuum Entanglement

The Minkowski vacuum, $|0_M\rangle$, is an entangled state of $\phi(x_L)$ and $\phi(x_R)$.

$$|0_M\rangle = \prod_i C_i \sum_{n_i=0}^{\infty} \frac{e^{-\pi n_i \omega_i / a}}{n_i!} (\hat{a}_{\omega_i}^{L\dagger} \hat{a}_{\omega_i}^{R\dagger})^{n_i} |0_R\rangle$$

$$\hat{a}_{\omega_i}^L |0_R\rangle = \hat{a}_{\omega_i}^R |0_R\rangle = 0$$

Entanglement \rightarrow Thermal Bath \rightarrow Unruh effect



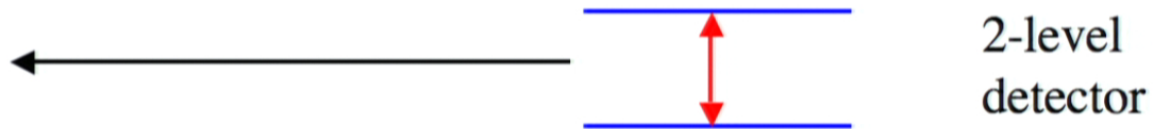
$$\hat{\rho}_R = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i / a} |n_i^R\rangle \langle n_i^R| \right]$$

$$T_U = \frac{\hbar a}{2\pi c k_B} \quad i \frac{\partial}{\partial \tau} \Psi = H_0 \Psi$$

One degree
corresponds to $a = 10^{20}$
 m/s^2 .

Entanglement \rightarrow Thermal Bath \rightarrow Unruh effect

Field theorists detector



2-level
detector

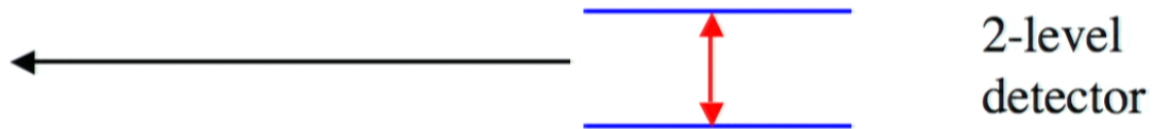
Uniform acceleration

$$F(E) = \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' e^{-iE(\eta-\eta')} D^+(\eta, \eta')$$

W.G. Unruh, Phys. Rev. D14, 870 (1976).

Entanglement \rightarrow Thermal Bath \rightarrow Unruh effect

Field theorists detector



Uniform acceleration

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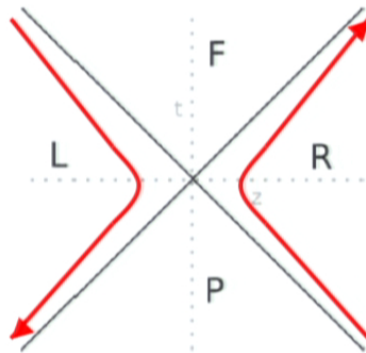
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$$|0_M\rangle = \sum_{n=0}^{\infty} \frac{e^{-\pi n \omega_i / a}}{n!} (\hat{a}_{\omega}^{L\dagger} \hat{a}_{\omega}^{R\dagger})^n |0_R\rangle$$

$$\hat{a}_{\omega_i}^L |0_R\rangle = \hat{a}_{\omega_i}^R |0_R\rangle = 0$$



Quantum Communication



$$|\psi\rangle = U(a) |0\rangle$$

$$\int d\tau b_{\tau}^{\dagger} b_{\tau}$$

Bogolyubov Transformation

$$\langle b^{\dagger} b \rangle = \int d\tau (\langle 0 | U^{\dagger}(a) B^{\dagger}) b_{\tau}^{\dagger} b_{\tau} (B U(a) | 0 \rangle)$$

Schrödinger Picture

Bobolyubov Transformation

$$b_i = \int dk f_i(k) b_k$$

Rob's mode in
Rindler co-ordinates

Bobolyubov Transformation

$$b_i = \int dk f_i(k) b_k$$

Rob's mode in
Rindler co-ordinates

$$b_k = \int dk' \left[A_{kk'} a_{k'} + B_{kk'} a_{k'}^\dagger \right]$$

Bobolyubov transform
from
Rindler to Minkowski

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Bobolyubov transform
from
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$$b_i = \int \int dk dk' f_i(k) \left[A_{kk'} a_{k'} + B_{kk'} a_{k'}^\dagger \right] \equiv (\mathbf{B}^\dagger \mathbf{b}_\tau \mathbf{B})$$

Bobolyubov Transformation

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Bobolyubov transform
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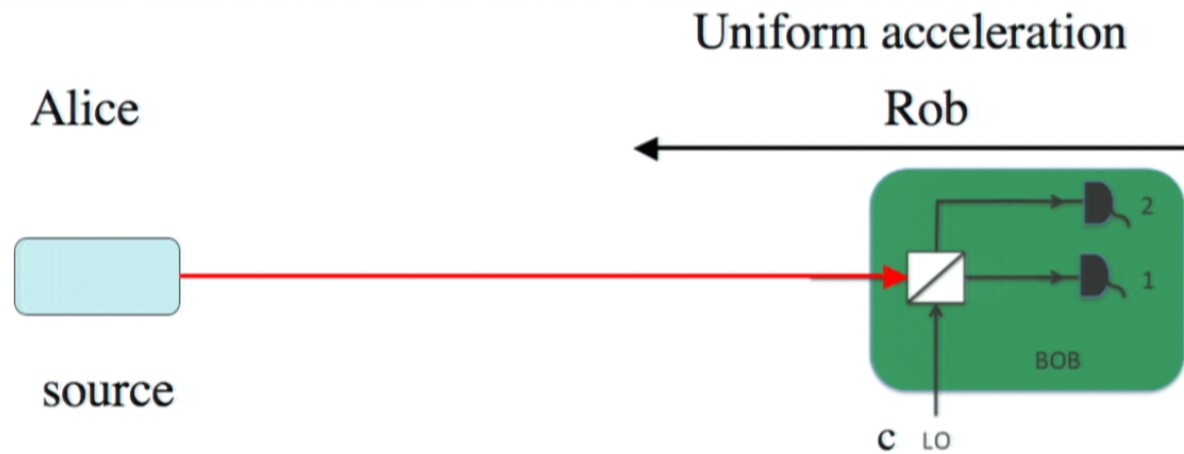
$$b_i = \int \int dk dk' f_i(k) \left[A_{kk'} a_{k'} + B_{kk'} a_{k'}^\dagger \right] \equiv (\mathbf{B}^\dagger \mathbf{b}_\tau \mathbf{B})$$

$$A_{k_d k_s} = \delta(\vec{k}_d - \vec{k}_s) \frac{1}{\sqrt{2\pi\omega_s(1 - e^{-2\pi k_{d1}})}} \left(\frac{\omega_s + k_{s1}}{\omega_s - k_{s1}} \right)^{i\frac{1}{2}k_{d1}}$$

$$B_{k_d k_s} = \delta(\vec{k}_d + \vec{k}_s) \frac{1}{\sqrt{2\pi\omega_s(e^{2\pi k_{d1}} - 1)}} \left(\frac{\omega_s + k_{s1}}{\omega_s - k_{s1}} \right)^{i\frac{1}{2}k_{d1}}$$

Takagi, Prog. Theo. Phys., **88**, 1, 1986

Quantum Communication

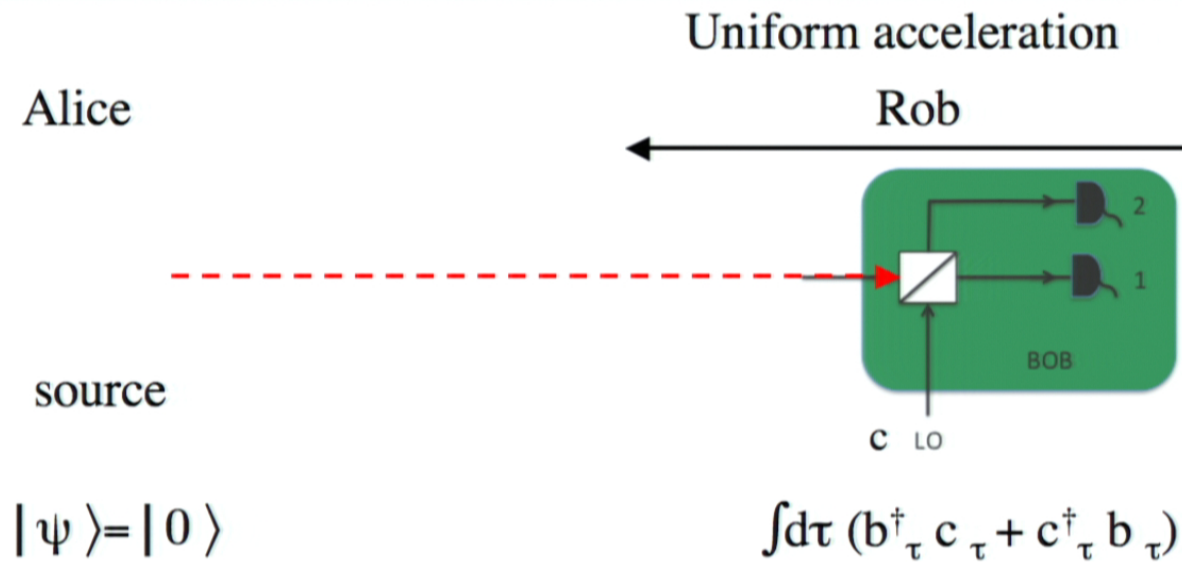


$$|\psi\rangle = U(a) |0\rangle$$

$$\int d\tau (b^\dagger_\tau c_\tau + c^\dagger_\tau b_\tau)$$

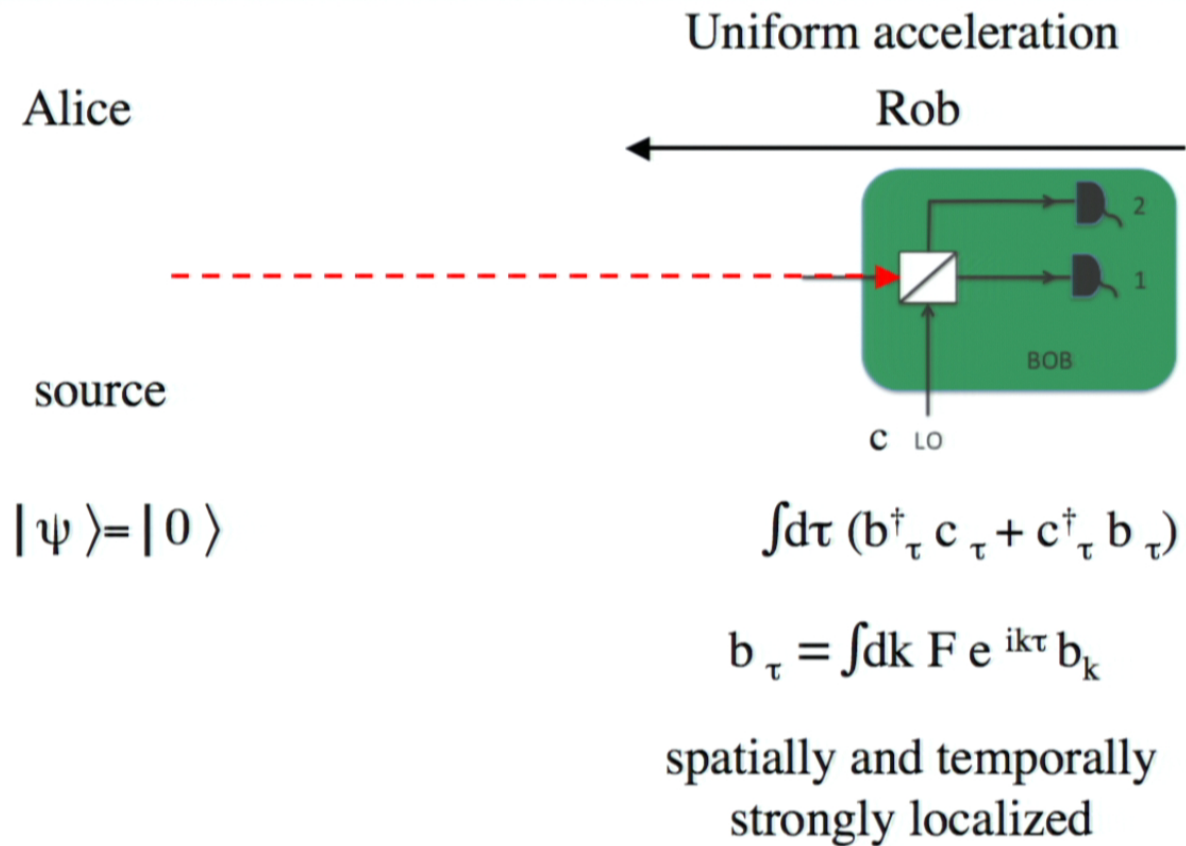
$$\langle b^\dagger b \rangle = \int d\tau \langle 0 | U^\dagger(a) B^\dagger (b^\dagger_\tau c_\tau + c^\dagger_\tau b_\tau) B U(a) | 0 \rangle$$

Unruh radiation

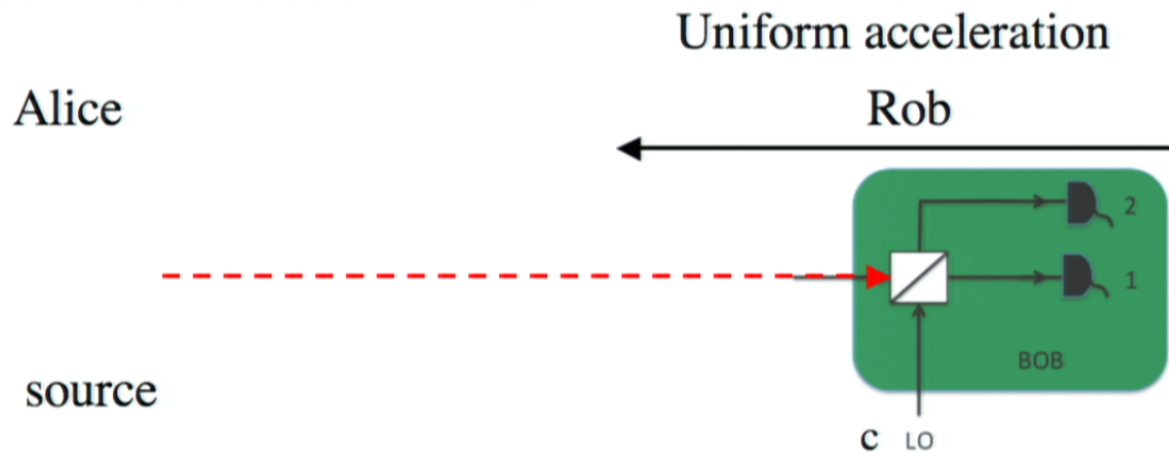


$$\langle b^\dagger b \rangle = \int d\tau \langle 0 | B^\dagger (b^\dagger_\tau c_\tau + c^\dagger_\tau b_\tau) B | 0 \rangle$$

Unruh radiation



Unruh radiation



$$|\psi\rangle = |0\rangle \quad |LO\rangle = U(c) |0\rangle \quad \int d\tau (b_\tau^\dagger c_\tau + c_\tau^\dagger b_\tau)$$

$U(c) = \text{Displacement}$
by β

$$b_\tau = \int dk F e^{ik\tau} b_k$$

spatially and temporally
strongly localized

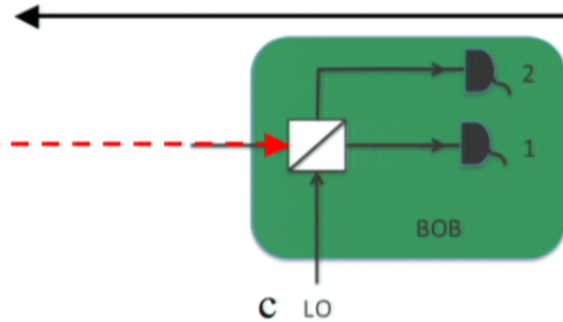
Unruh radiation

$$\hat{O}(\tau) = \hat{b}_i^S \hat{b}_i^{L\dagger} e^{i\phi} + \hat{b}_i^{S\dagger} \hat{b}_i^L e^{-i\phi}$$

$$V = \langle (\int d\tau \hat{O}(\tau))^2 \rangle - \langle \int d\tau \hat{O}(\tau) \rangle^2$$

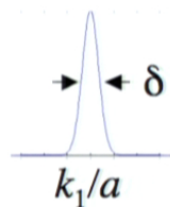
Uniform acceleration

Rob



$$\int d\tau (b_\tau^\dagger c_\tau + c_\tau^\dagger b_\tau)$$

$$b_\tau = \int dk F e^{ik\tau} b_k$$



spatially and temporally
strongly localized

Unruh radiation

$$\hat{O}(\tau) = \hat{b}_i^S \hat{b}_i^{L\dagger} e^{i\phi} + \hat{b}_i^{S\dagger} \hat{b}_i^L e^{-i\phi}$$

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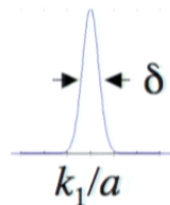
$$\beta \gg 1$$

$$V = \beta^2 \int dk'_1 |f(k'_1)|^2 \frac{(1 + e^{-2\pi k'_1})}{(1 - e^{-2\pi k'_1})} \int d\tau (b_\tau^\dagger c_\tau + c_\tau^\dagger b_\tau)$$

$$\approx \beta^2 \frac{(1 + e^{-2\pi k_1/a})}{(1 - e^{-2\pi k_1/a})}$$

$$b_\tau = \int dk F e^{ik\tau} b_k$$

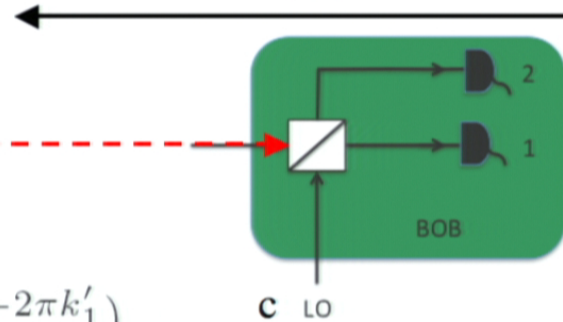
$$\delta \ll k_1/a$$



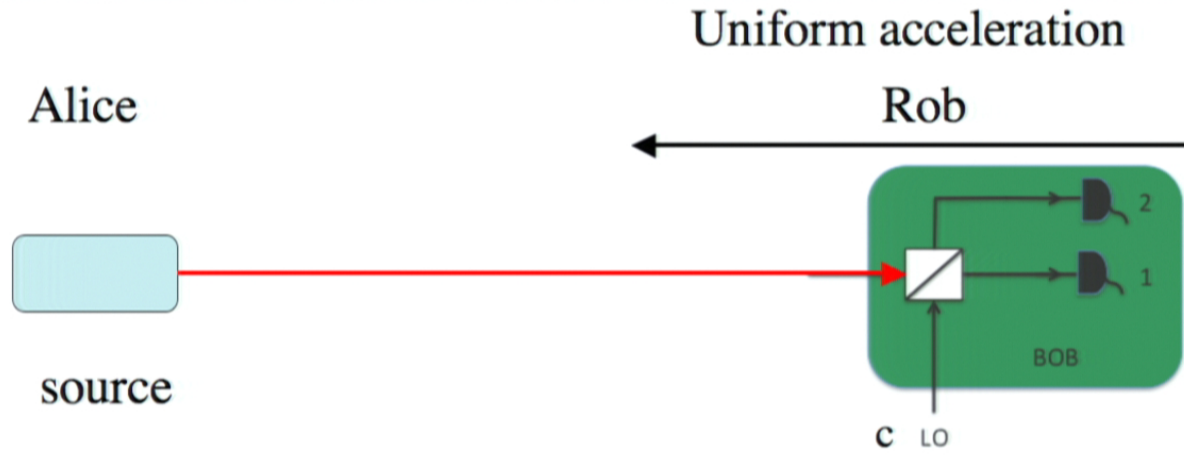
spatially and temporally strongly localized

Uniform acceleration

Rob



Quantum Communication



$$|\psi\rangle = U(a) |0\rangle$$

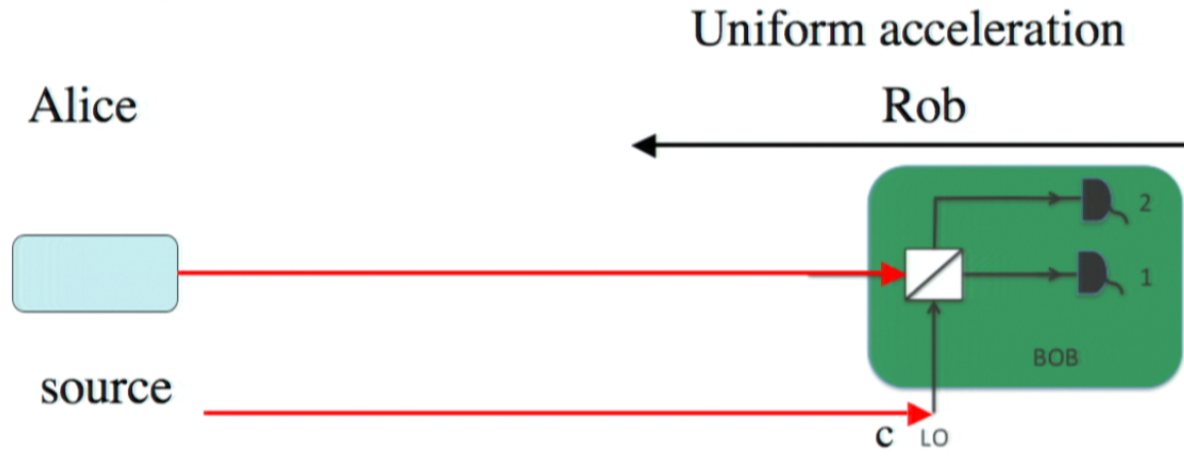
$U(a)$ = Displacement
by α

$$\int d\tau (b_{\tau}^{\dagger} c_{\tau} + c_{\tau}^{\dagger} b_{\tau})$$

$$b_{\tau} = \int dk F e^{ik\tau} b_k$$

spatially and temporally
strongly localized

Quantum Communication



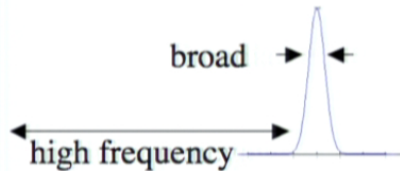
$$|\psi\rangle = U(a) |0\rangle$$

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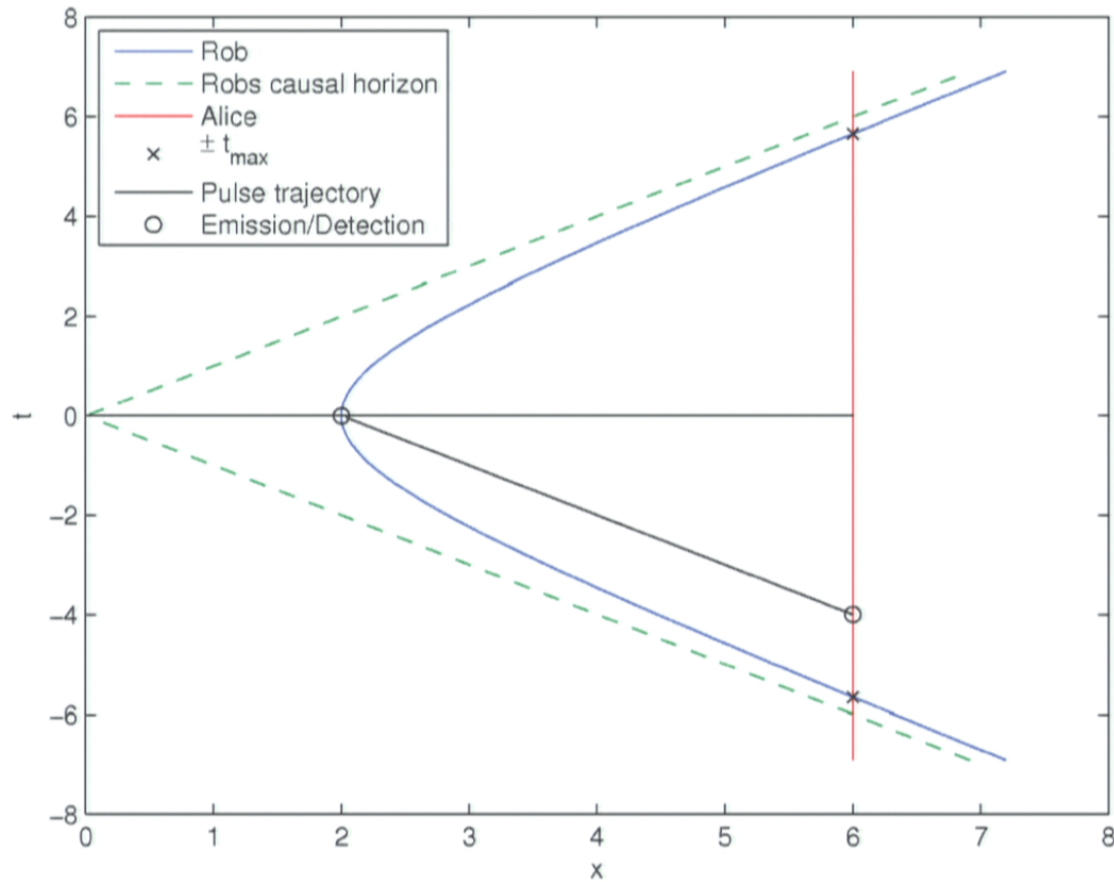
$U(a)$ = Displacement
by α and β

$$b_{\tau} = \int dk F e^{ik\tau} b_k$$

spatially and temporally
strongly localized



Quantum Communication



Quantum Communication

approximate analytical solution

$$\langle X_B(\theta) \rangle_A = X/\bar{\beta} = \frac{\alpha e^{i\phi} + \alpha^* e^{-i\phi}}{\sqrt{(1 - e^{-2\pi|k_{so}|(x+t)})}}$$

$$\langle \Delta X_B(\theta)^2 \rangle_A = V/\bar{\beta}^2 = \frac{(1 + e^{-2\pi|k_{so}|(x+t)})}{(1 - e^{-2\pi|k_{so}|(x+t)})}$$

linear amplifier
with gain.....

$$G = 1/(1 - e^{-2\pi|k_{so}|(x+t)})$$

T.G.Downes, T.C.Ralph, N.Walk, arXiv:1203.2716 (2012)

Quantum Communication

approximate analytical solution

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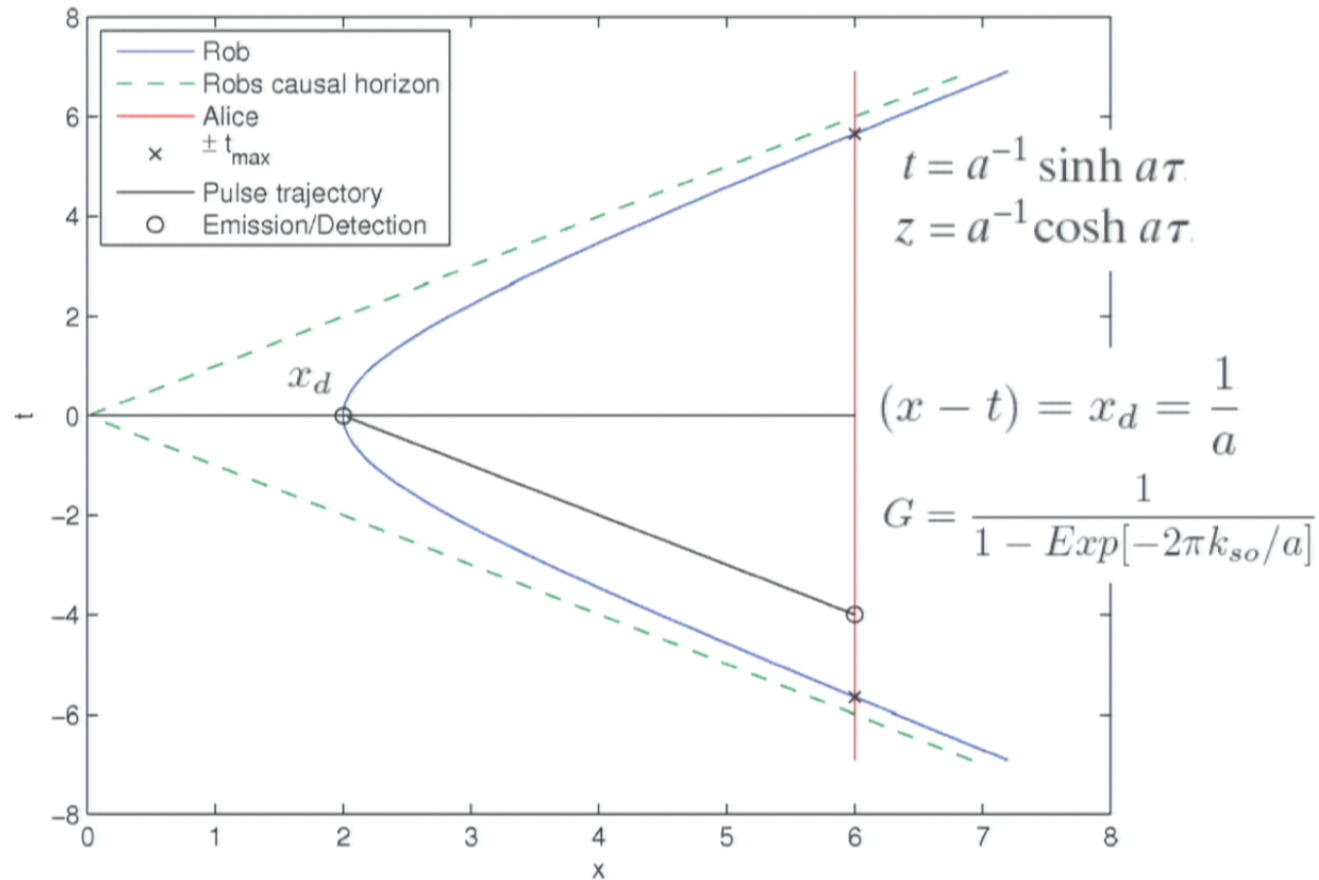
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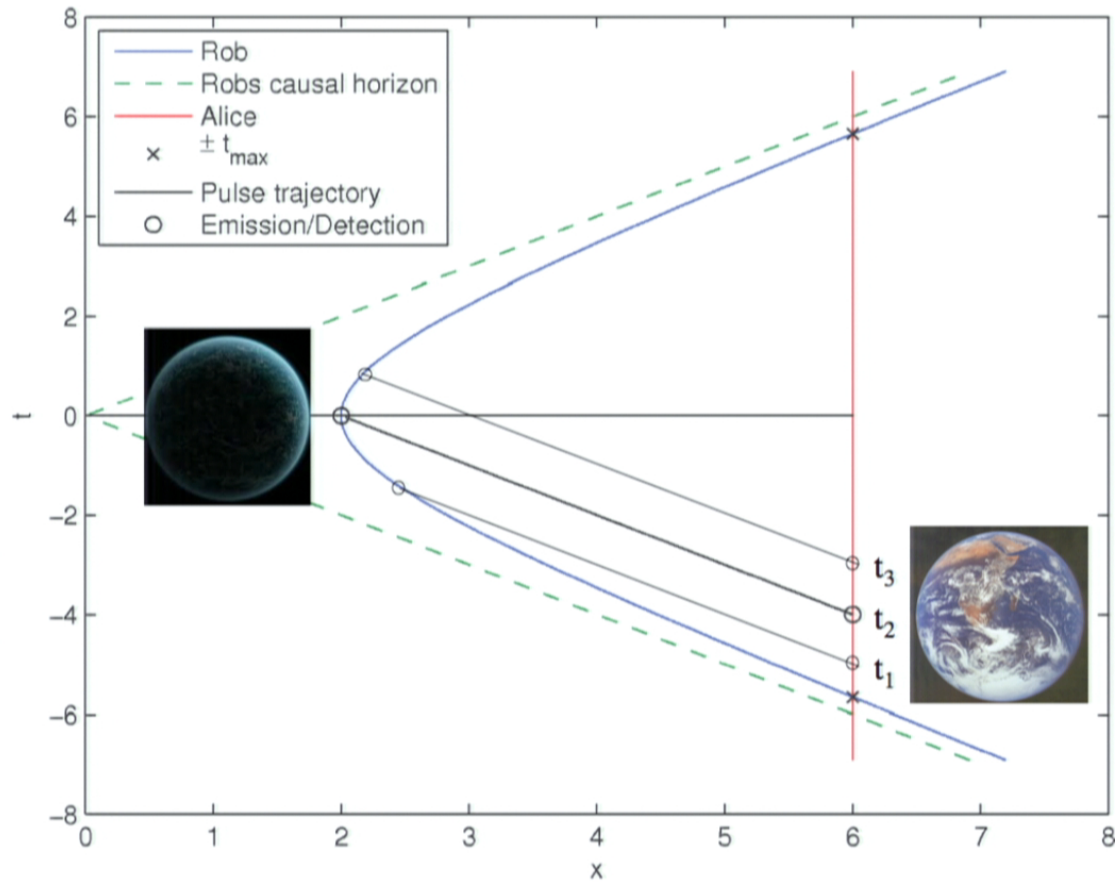
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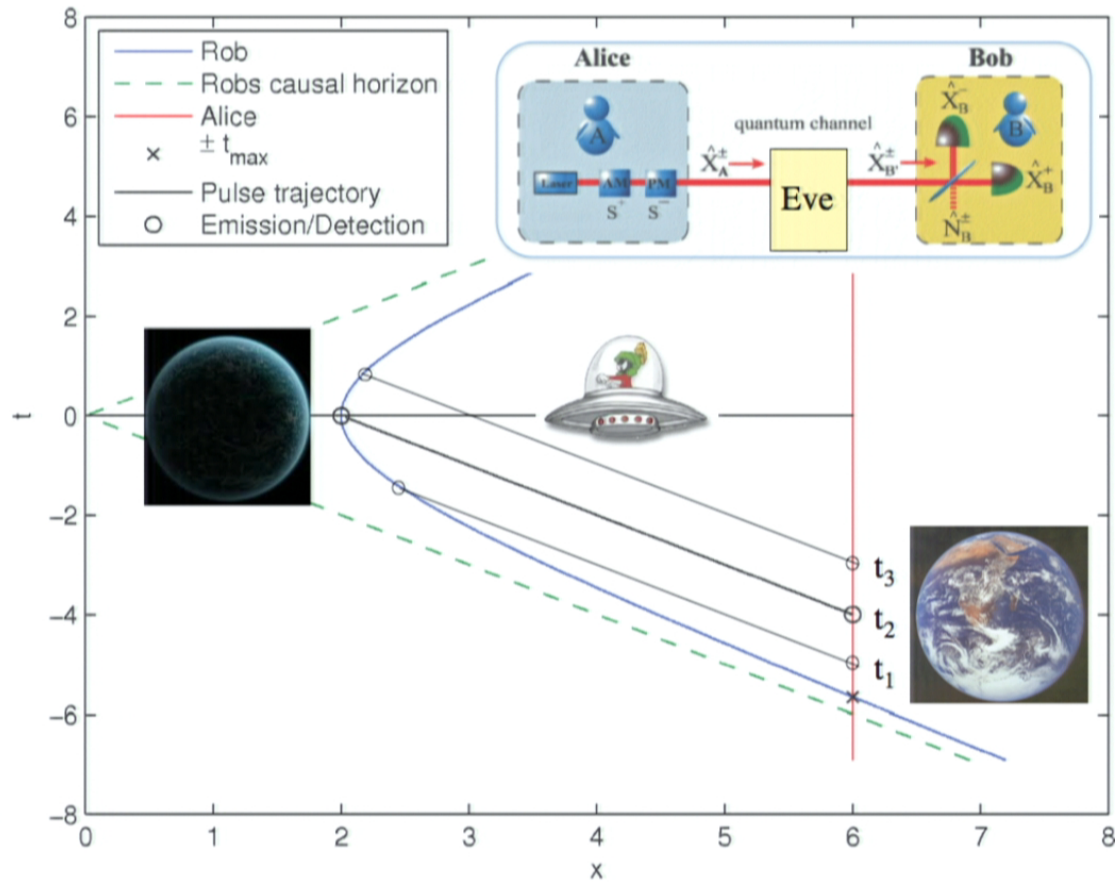
Quantum Communication



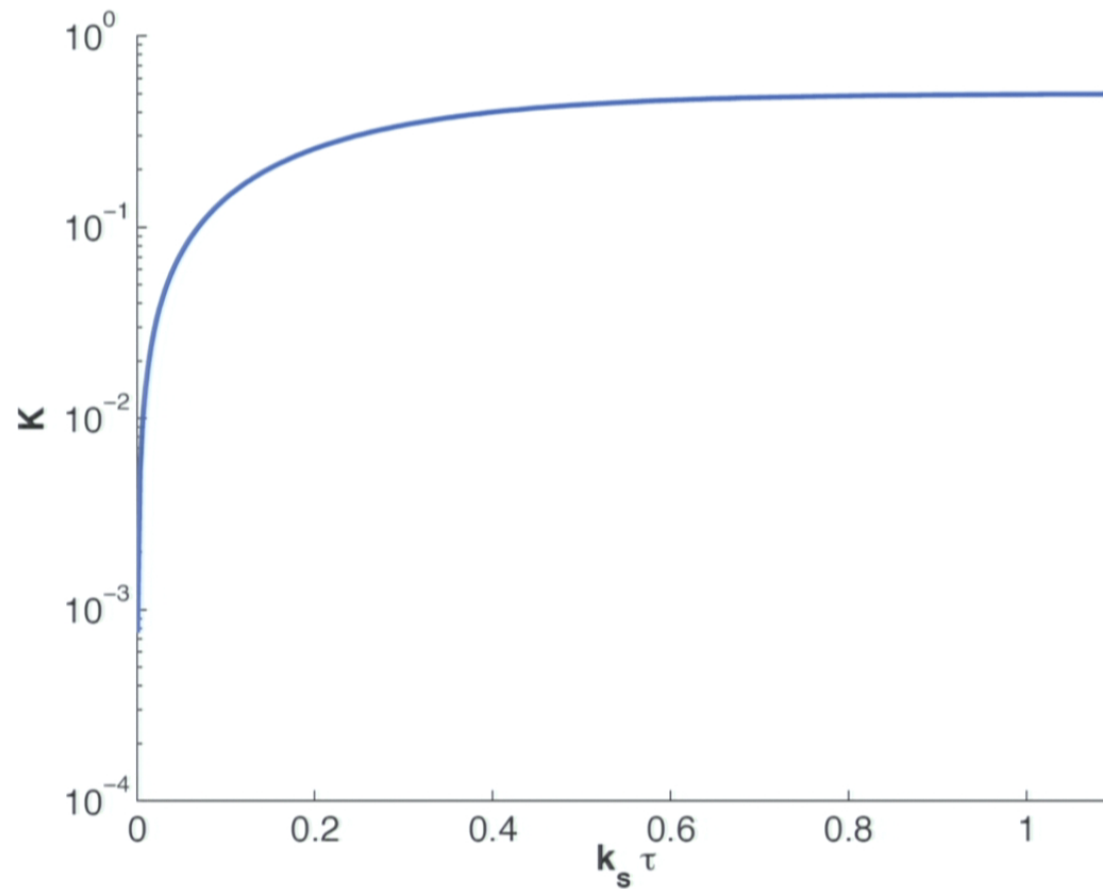
E.g Quantum Key Distribution between Alice and Rob



E.g Quantum Key Distribution between Alice and Rob



E.g Quantum Key Distribution between Alice and Rob



Summary

- * Presented general approach for treating **localized, directional** quantum optical detection by an accelerated observer.
- * Obtained approximate analytical results for the case of quantum limited homodyne detection of a signal sent from an inertial source
- * Applied the technique to analysing a continuous variable QKD protocol between Alice and Rob.