Title: Quantum Communication Between Localized, Non-Inertial Observers

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Abstract: An unsolved problem in relativistic quantum information research is how to model efficient, directional quantum communication between localised parties in a fully quantum field theoretical framework. We propose a tractable approach to this problem based on calculating expectation values of localized field observables in the Heisenberg Picture. We illustrate our approach by analysing, and obtaining approximate analytical solutions to, the problem of communicating quantum states between an inertial sender, Alice and an accelerated homodyne receiver, Rob. We discuss the effect on quantum protocols carried out over such a communication channel.

Quantum Communication between Localized Non-inertial Observers

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T.G.Downes, T.C.Ralph, N.Walk,
arXiv:1203.2716 (2012)

The University of Queensland
AUSTRALIA

Quantum Optics Quantum Information Theory Group

Combining Quantum Optics with Relativistic Quantum Field Theory

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The University of Queensland
Australia

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Relativistic Quantum Information 6

Workshop

Customs House, Brisbane

Australia

late November 2012

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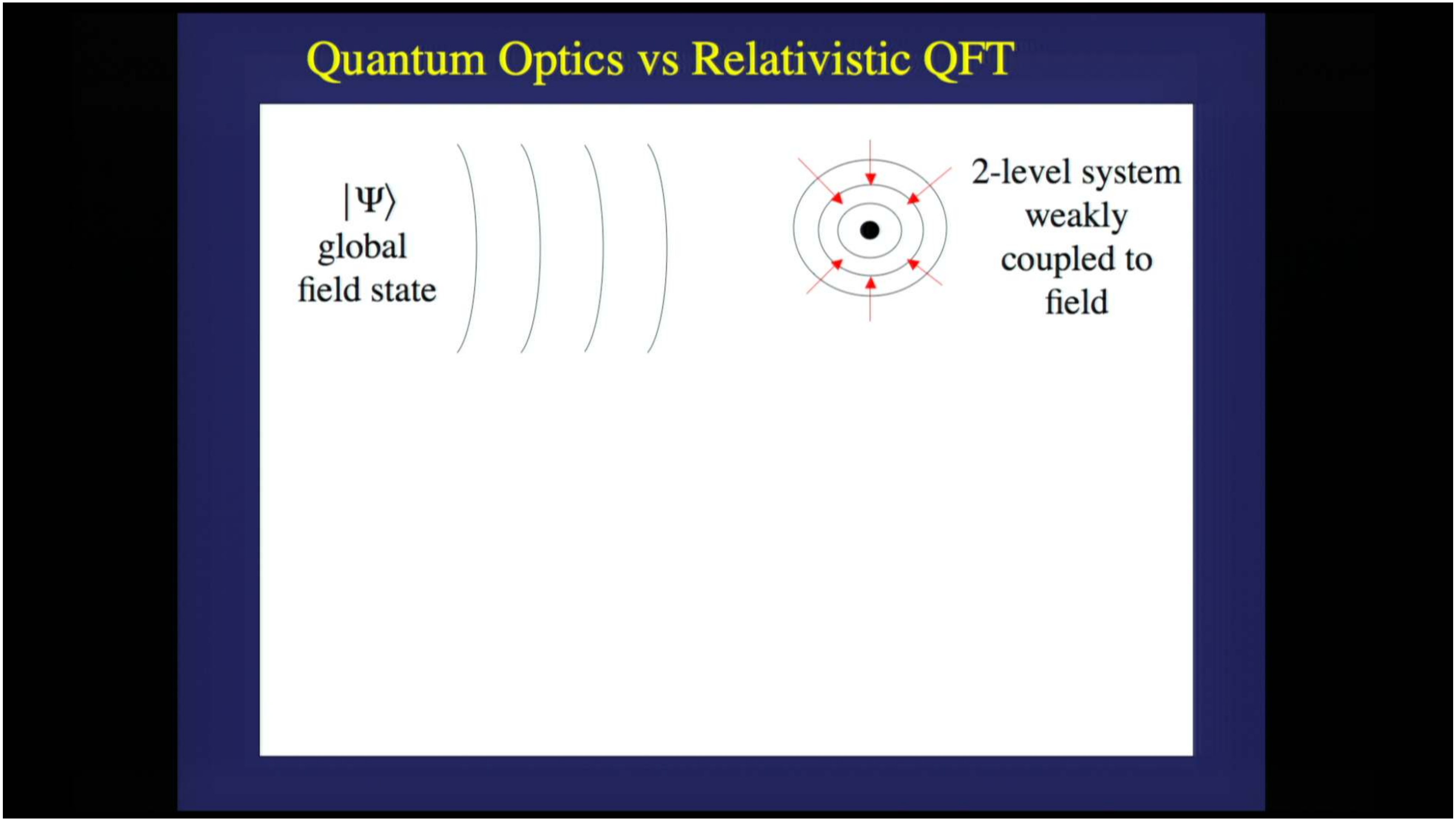
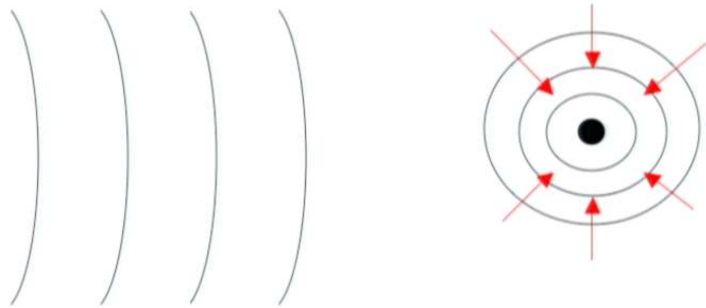
RQI2

2008


Quantum Optics vs Relativistic QFT
|Ψ⟩
global
field state
2-level system
weakly
coupled to
field

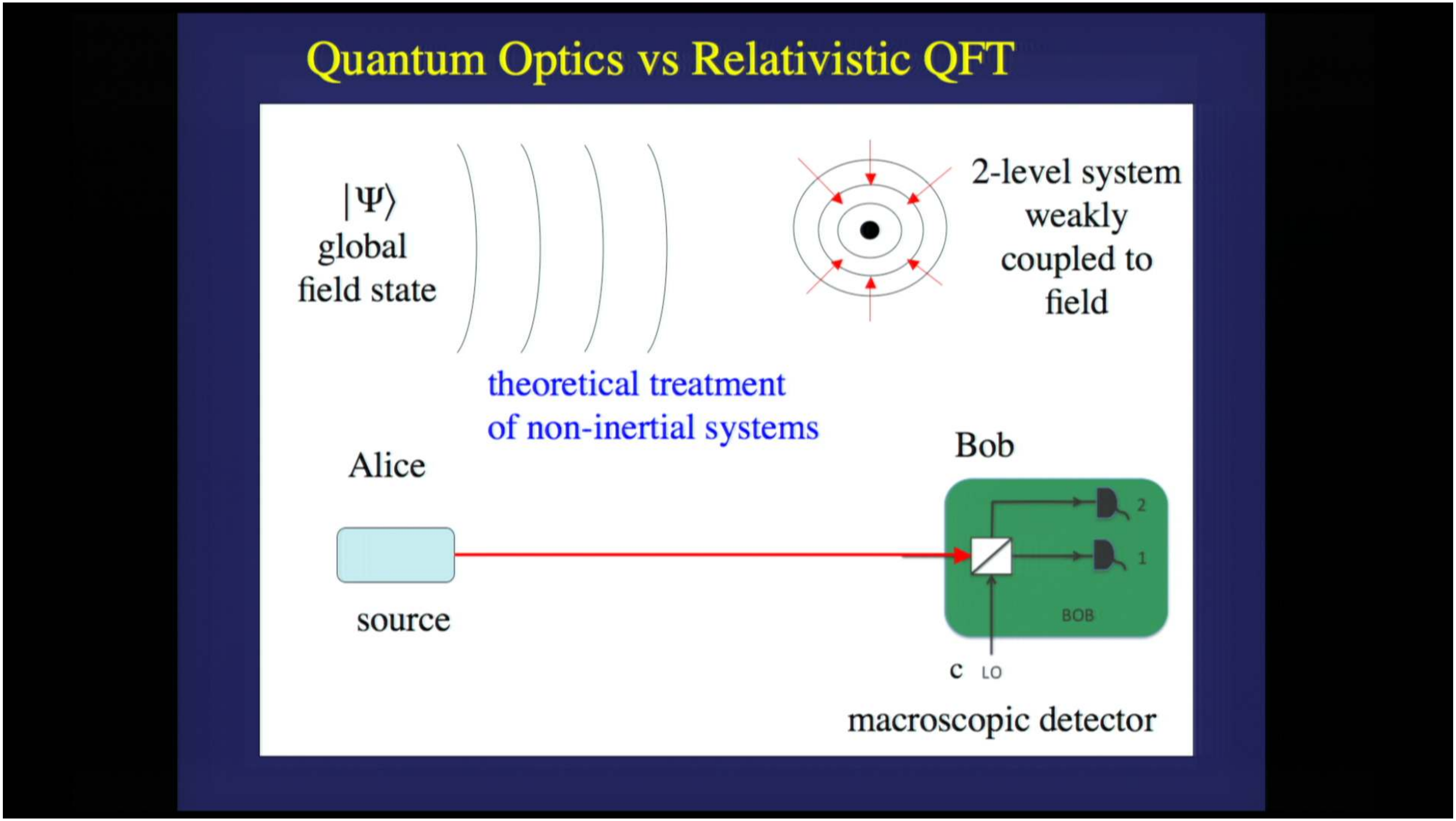
Quantum Optics vs Relativistic QFT

$|\Psi\rangle$
global
field state



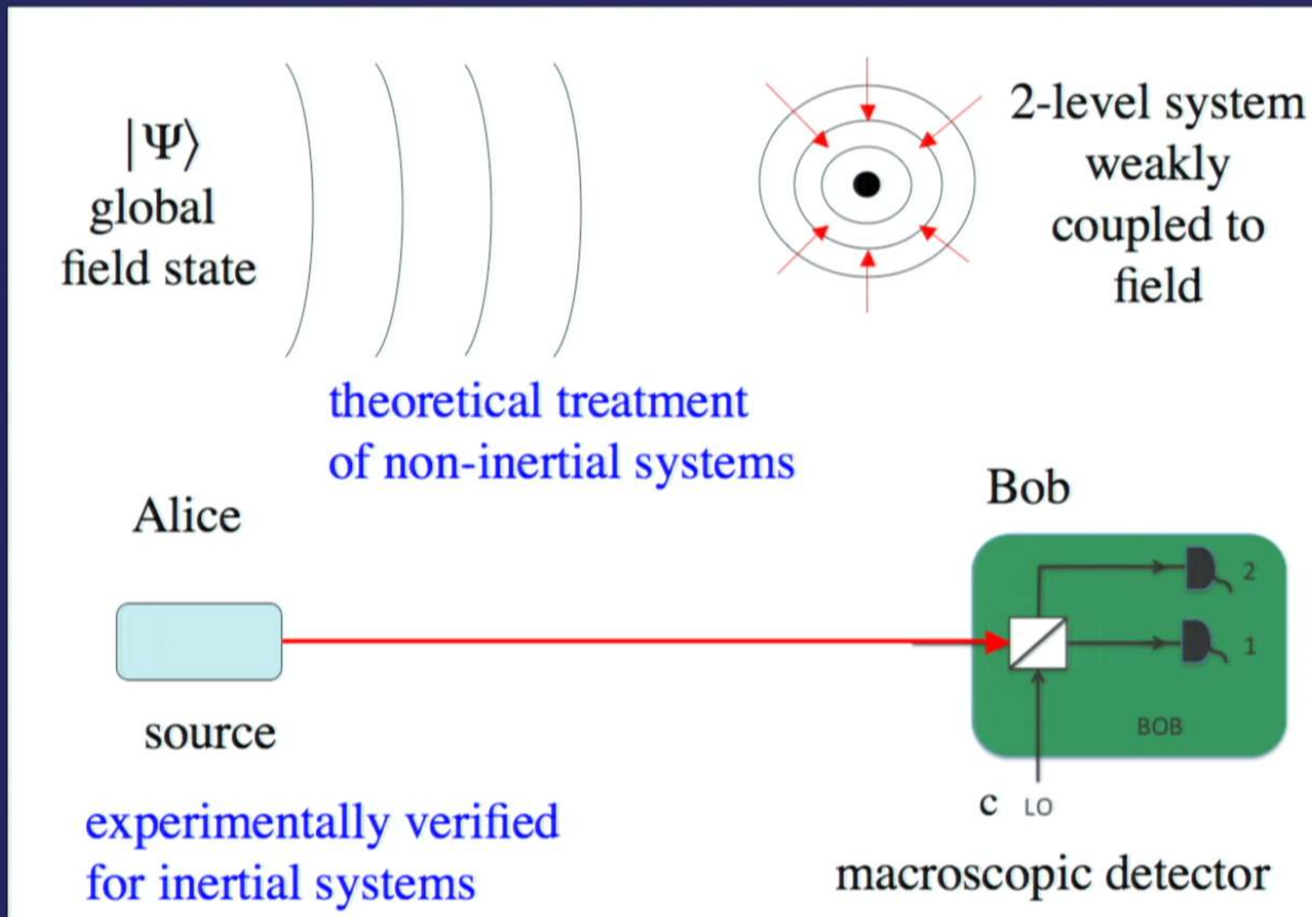
2-level system
weakly
coupled to
field

theoretical treatment
of non-inertial systems

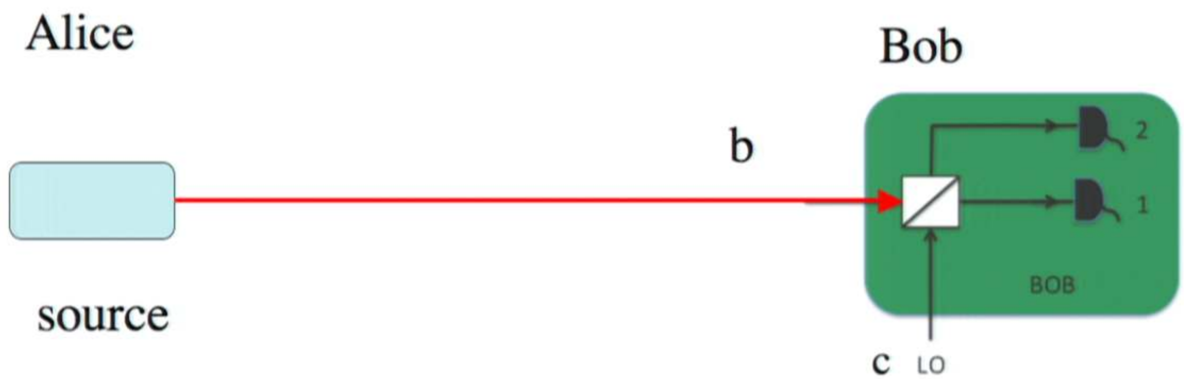


Quantum Optics vs Relativistic QFT
|Ψ⟩
global
field state
2-level system
weakly
coupled to
field
theoretical treatment
of non-inertial systems
Alice
Bob
source
2
1
BOB
c LO
macroscopic detector

Quantum Optics vs Relativistic QFT



Quantum Communication



$$|\psi\rangle = U(a)\,|0\rangle$$

$$\int d\tau\, (b^\dagger_\tau c_\tau + c^\dagger_\tau b_\tau)$$

$$\langle b^\dagger b\rangle = \int d\tau\, \langle 0|\, U^\dagger(a)\, (b^\dagger_\tau c_\tau + c^\dagger_\tau b_\tau)\, U(a)\,|0\rangle$$

a, b, c ≡ mode annihilation operators

Quantum Communication

Alice

Bob

source

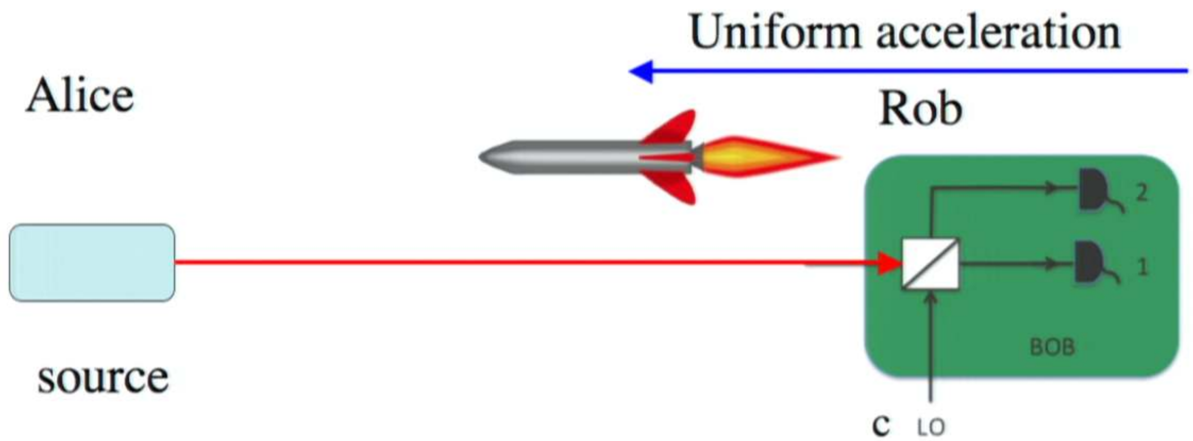
$$|\psi\rangle = U(a)\,|0\rangle$$

$$\int d\tau\,(b^{\dagger}_{\tau}\,c_{\tau} + c^{\dagger}_{\tau}\,b_{\tau})$$

$$\langle b^{\dagger}\, b\rangle = \int d\tau\,\langle 0\,|\,U^{\dagger}(a)\,(b^{\dagger}_{\tau}\,c_{\tau} + c^{\dagger}_{\tau}\,b_{\tau})\,U(a)\,|\,0\rangle$$

a, b, c ≡ mode annihilation operators

Quantum Communication with acceleration

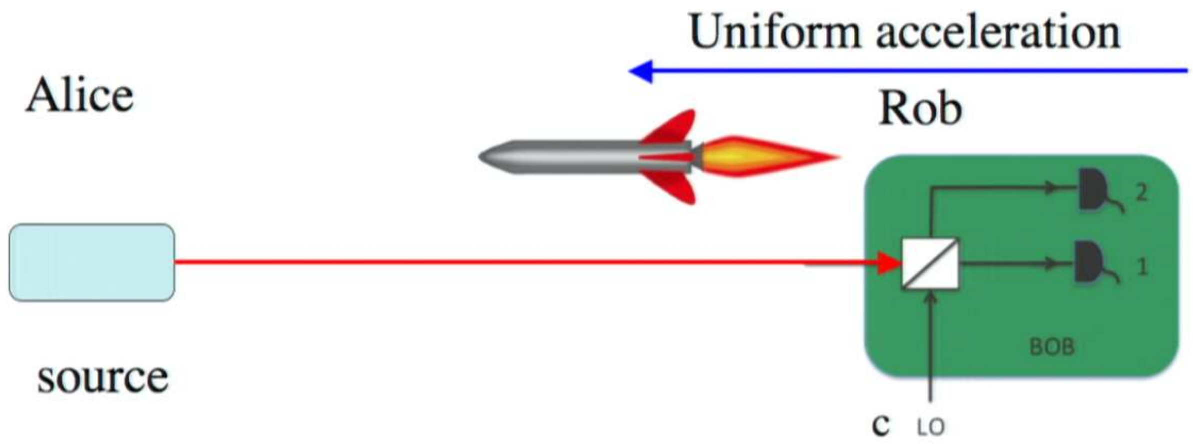


$$|\psi\rangle = U(a)\,|0\rangle \qquad \int d\tau\,(b^\dagger_\tau\, c_\tau + c^\dagger_\tau\, b_\tau)$$

$$\langle b^\dagger b\rangle = \int d\tau\,\langle 0|\,U^\dagger(a)\,B^\dagger\,(b^\dagger_\tau\, c_\tau + c^\dagger_\tau\, b_\tau)\,B\,U(a)\,|0\rangle$$

a, b, c ≡ mode annihilation operators

Quantum Communication with acceleration



$|\psi\rangle = U(a)\,|0\rangle$

$\int d\tau\,(b^\dagger_\tau\, c_\tau + c^\dagger_\tau\, b_\tau)$

$$\langle b^\dagger b\rangle = \int d\tau\,\langle 0|\,U^\dagger(a)\,B^\dagger\,(b^\dagger_\tau\, c_\tau + c^\dagger_\tau\, b_\tau)\,B\,U(a)\,|0\rangle$$

a, b, c ≡ mode annihilation operators

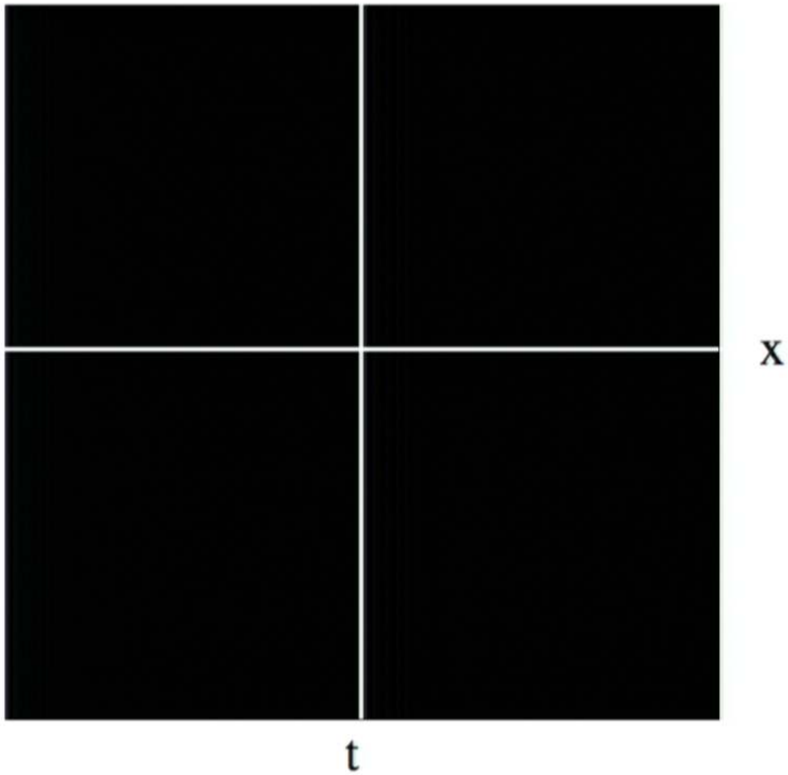
Overview

* Problems with analysing localized non-inertial sources and/or detectors

* Efficient, localized detection of Unruh radiation.

* Quantum Communication between Alice and Rob

Vacuum



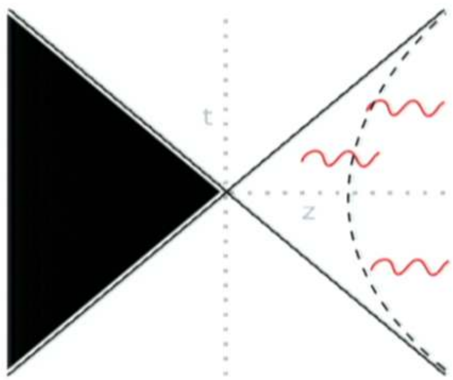
Vacuum Entanglement

The Minkowski vacuum, $|0_M\rangle$, is an entangled state of $\phi(x_L)$ and $\phi(x_R)$.

$$|0_M\rangle = \prod_i C_i \sum_{n_i=0}^{\infty} \frac{e^{-\pi n_i \omega_i / a}}{n_i!} (\hat{a}_{\omega_i}^{L\dagger} \hat{a}_{\omega_i}^{R\dagger})^{n_i} |0_R\rangle$$

$$\hat{a}_{\omega_i}^{L} |0_R\rangle = \hat{a}_{\omega_i}^{R} |0_R\rangle = 0$$

Entanglement → Thermal Bath → Unruh effect



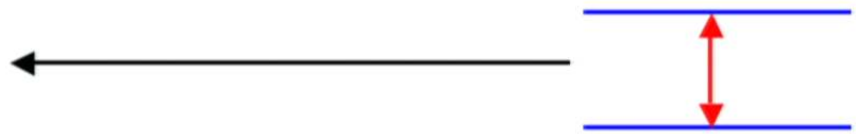
$$\hat{\rho}_R = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i / a} |n_i^R\rangle\langle n_i^R| \right]$$

$$T_U = \frac{\hbar a}{2\pi c k_B} \qquad i\frac{\partial}{\partial \tau}\Psi = H_0 \Psi$$

One degree corresponds to a = 10^{20} m/s^2.

Entanglement → Thermal Bath → Unruh effect

Field theorists detector



2-level detector

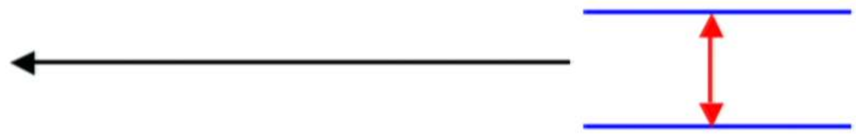
Uniform acceleration

$$F(E) = \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' e^{-iE(\eta-\eta')} D^{+}(\eta, \eta')$$

W.G. Unruh, Phys. Rev. D14, 870 (1976).

Entanglement → Thermal Bath → Unruh effect

Field theorists detector



2-level
detector

Uniform acceleration

$$F(E) = \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' e^{-iE(\eta-\eta')} D^{+}(\eta,\eta')$$

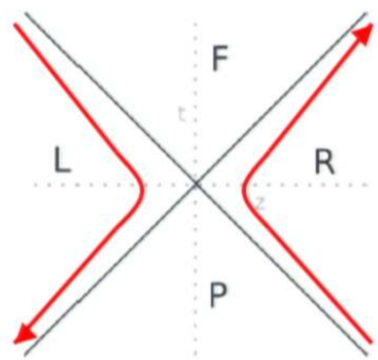
W.G. Unruh, Phys. Rev. D14, 870 (1976).

Vacuum Entanglement

The Unruh vacuum, $|0_M\rangle$, is an entangled state of $\phi(x_L)$ and $\phi(x_R)$.

$$|0_M\rangle = \sum_{n=0}^{\infty} \frac{e^{-\pi n \omega/a}}{n!} (\hat{a}_\omega^{L\dagger} \hat{a}_\omega^{R\dagger})^n \, |0_R\rangle$$

$$\hat{a}_{\omega_i}^{L} |0_R\rangle = \hat{a}_{\omega_i}^{R} |0_R\rangle = 0$$

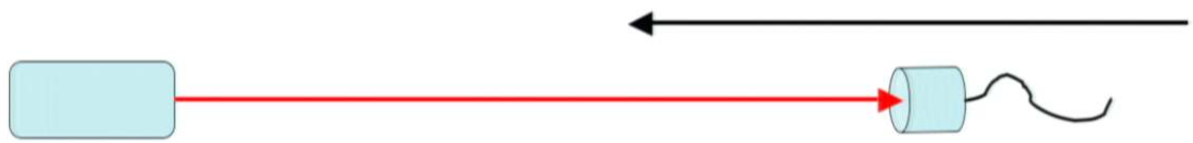


Quantum Communication

Uniform acceleration

Alice

Rob



source

detector

$$|\psi\rangle = U(a)|0\rangle \qquad \int d\tau\, b^\dagger_\tau\, b_\tau$$

Bogolyubov Transformation

$$\langle b^\dagger b\rangle = \int d\tau\, (\langle 0|\, U^\dagger(a)\, B^\dagger)\, b^\dagger_\tau\, b_\tau\, (B\, U(a)\, |0\rangle)$$

Schrödinger Picture

Bobolyubov Transformation

$$b_i = \int dk\, f_i(k) b_k$$

Rob's mode in Rindler co-ordinates

Bobolyubov Transformation

$$b_i = \int dk\, f_i(k) b_k$$

Rob's mode in
Rindler co-ordinates

$$b_k = \int dk' \left[A_{kk'} a_{k'} + B_{kk'} a^\dagger_{k'} \right]$$

Bobolyubov transform
from
Rindler to Minkowski

Bobolyubov Transformation

$$b_i = \int dk\, f_i(k) b_k$$

Rob's mode in Rindler co-ordinates

$$b_k = \int dk' \left[A_{kk'} a_{k'} + B_{kk'} a^\dagger_{k'} \right]$$

Bobolyubov transform from Rindler to Minkowski

$$b_i = \int \int dk dk' f_i(k) \left[A_{kk'} a_{k'} + B_{kk'} a^\dagger_{k'} \right] \equiv (\mathbf{B}^\dagger\, \mathbf{b}_\tau\, \mathbf{B})$$

Bobolyubov Transformation

$$b_i = \int dk\, f_i(k) b_k$$

Rob's mode in
Rindler co-ordinates

$$b_k = \int dk' \left[A_{kk'} a_{k'} + B_{kk'} a^\dagger_{k'} \right]$$

Bobolyubov transform
from
Rindler to Minkowski

$$b_i = \int \int dk dk' f_i(k) \left[A_{kk'} a_{k'} + B_{kk'} a^\dagger_{k'} \right] \equiv (\mathrm{B}^\dagger\, \mathrm{b}_\tau\, \mathrm{B})$$

$$A_{k_d k_s} = \delta(\vec{k}_d - \vec{k}_s) \frac{1}{\sqrt{2\pi\omega_s(1 - e^{-2\pi k_{d1}})}} \left(\frac{\omega_s + k_{s1}}{\omega_s - k_{s1}} \right)^{i\frac{1}{2}k_{d1}}$$

$$B_{k_d k_s} = \delta(\vec{k}_d + \vec{k}_s) \frac{1}{\sqrt{2\pi\omega_s(e^{2\pi k_{d1}} - 1)}} \left(\frac{\omega_s + k_{s1}}{\omega_s - k_{s1}} \right)^{i\frac{1}{2}k_{d1}}$$

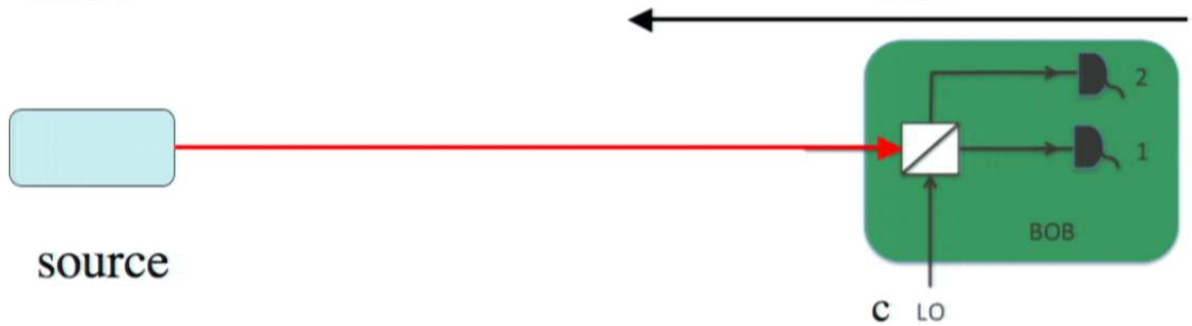
Takagi, Prog. Theo. Phys., **88**, 1, 1986

Quantum Communication

Uniform acceleration

Alice

Rob



source

$$|\psi\rangle = U(a)\,|0\rangle$$

$$\int d\tau\,(b^\dagger_\tau c_\tau + c^\dagger_\tau b_\tau)$$

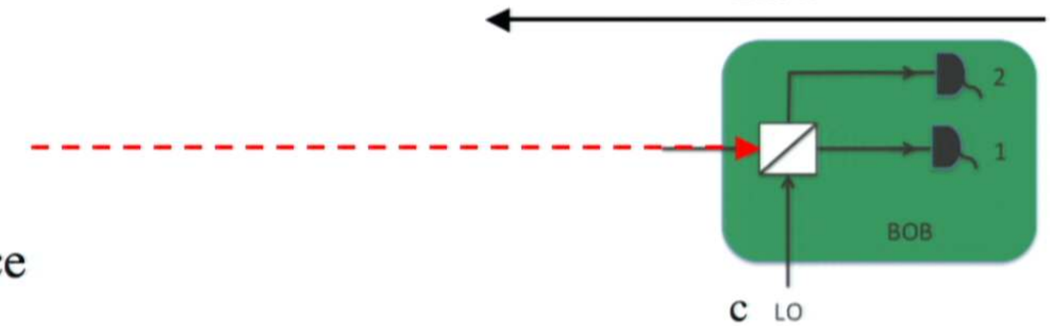
$$\langle b^\dagger b\rangle = \int d\tau\,\langle 0|\,U^\dagger(a)\,B^\dagger\,(b^\dagger_\tau c_\tau + c^\dagger_\tau b_\tau)\,B\,U(a)\,|0\rangle$$

Unruh radiation

Uniform acceleration

Alice

Rob



source

$|\psi\rangle = |0\rangle$

$\int d\tau\, (b^{\dagger}_{\tau}\, c_{\tau} + c^{\dagger}_{\tau}\, b_{\tau})$

$$\langle b^{\dagger} b \rangle = \int d\tau\, \langle 0 | B^{\dagger} (b^{\dagger}_{\tau}\, c_{\tau} + c^{\dagger}_{\tau}\, b_{\tau}) B | 0 \rangle$$

Unruh radiation

Uniform acceleration

Alice

Rob

source

$$|\psi\rangle = |0\rangle$$

$$\int d\tau\, (b^{\dagger}_{\tau}\, c_{\tau} + c^{\dagger}_{\tau}\, b_{\tau})$$

$$b_{\tau} = \int dk\, F\, e^{ik\tau}\, b_k$$

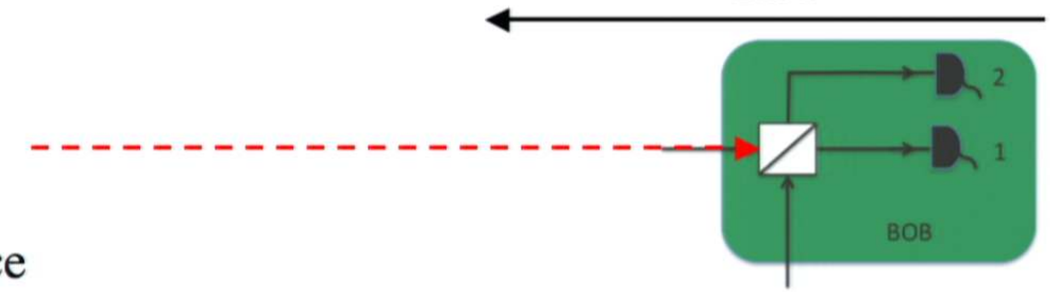
spatially and temporally strongly localized

Unruh radiation

Uniform acceleration

Alice

Rob



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c_{LO}

$|\psi\rangle = |0\rangle$

$|LO\rangle = U(c)\,|0\rangle$

U(c) = Displacement by β

$\int d\tau\, (b^{\dagger}_{\tau}\, c_{\tau} + c^{\dagger}_{\tau}\, b_{\tau})$

$b_{\tau} = \int dk\, F\, e^{ik\tau}\, b_k$

spatially and temporally strongly localized

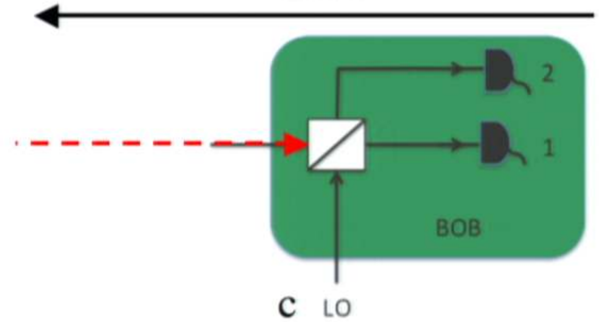
Unruh radiation

Uniform acceleration

Rob

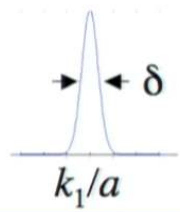
$$\hat{O}(\tau) = \hat{b}_i^S \hat{b}_i^{L\dagger} e^{i\phi} + \hat{b}_i^{S\dagger} \hat{b}_i^L e^{-i\phi}$$

$$V = \langle (\int d\tau \hat{O}(\tau))^2 \rangle - \langle \int d\tau \hat{O}(\tau) \rangle^2$$



$$\int d\tau \, (b^\dagger_\tau \, c_\tau + c^\dagger_\tau \, b_\tau)$$

$$b_\tau = \int dk \, F \, e^{ik\tau} b_k$$

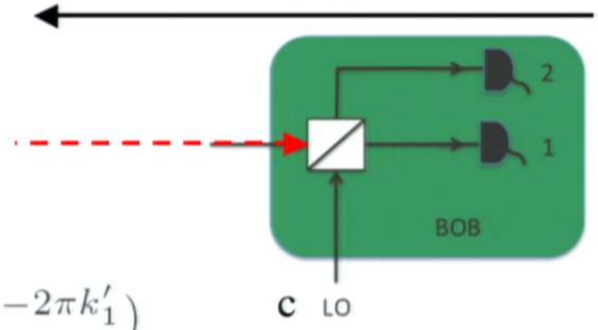


spatially and temporally strongly localized

Unruh radiation

Uniform acceleration

Rob



$$\hat{O}(\tau) = \hat{b}_i^S \hat{b}_i^{L\dagger} e^{i\phi} + \hat{b}_i^{S\dagger} \hat{b}_i^L e^{-i\phi}$$

$$V = \langle (\int d\tau \hat{O}(\tau))^2 \rangle - \langle \int d\tau \hat{O}(\tau) \rangle^2$$

$$\beta >> 1$$

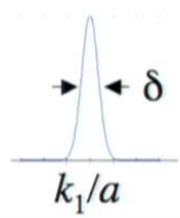
$$V = \beta^2 \int dk_1' |f(k_1')|^2 \frac{(1 + e^{-2\pi k_1'})}{(1 - e^{-2\pi k_1'})}$$

$$\approx \beta^2 \frac{(1 + e^{-2\pi k_1/a})}{(1 - e^{-2\pi k_1/a})}$$

$\int d\tau \, (b^\dagger_\tau \, c_\tau + c^\dagger_\tau \, b_\tau)$

$b_\tau = \int dk \, F \, e^{ik\tau} \, b_k$

$\delta << k_1/a$



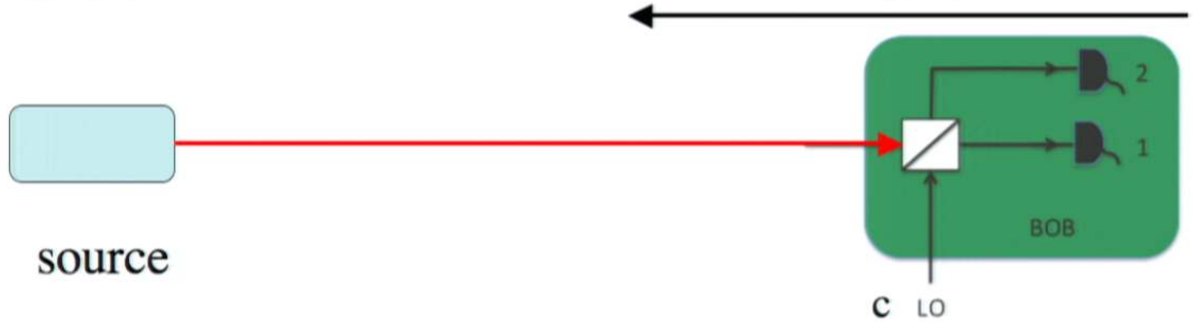
spatially and temporally strongly localized

Quantum Communication

Uniform acceleration

Alice

Rob



source

$|\psi\rangle = U(a)\,|0\rangle$

U(a) = Displacement by α

$\int d\tau\,(b^{\dagger}_{\tau}\, c_{\tau} + c^{\dagger}_{\tau}\, b_{\tau})$

$b_{\tau} = \int dk\, F\, e^{ik\tau}\, b_k$

spatially and temporally strongly localized

Quantum Communication

Uniform acceleration

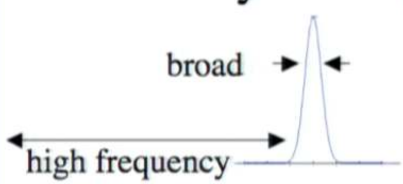
Alice　　　　Rob

$|\psi\rangle = U(a)|0\rangle$

$\int d\tau\, (b^\dagger_\tau\, c_\tau + c^\dagger_\tau\, b_\tau)$

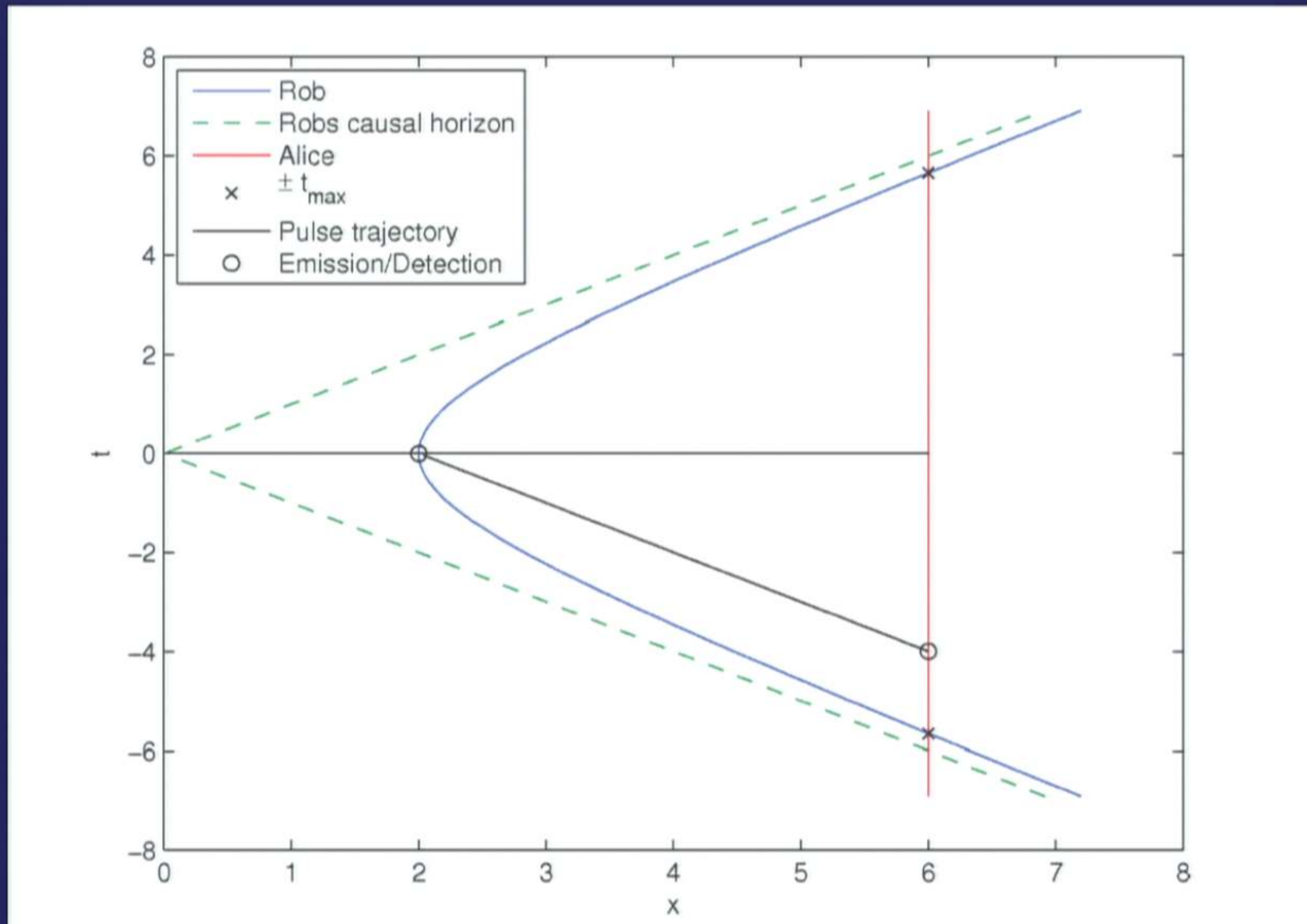
U(a) = Displacement
by α and β

$b_\tau = \int dk\, F\, e^{ik\tau}\, b_k$



spatially and temporally
strongly localized

Quantum Communication



Quantum Communication

approximate analytical solution

$$\langle X_B(\theta)\rangle_A = X/\bar{\beta} \;=\; \frac{\alpha e^{i\phi} + \alpha^* e^{-i\phi}}{\sqrt{(1 - e^{-2\pi|k_{so}|(x+t)})}}$$

$$\langle \Delta X_B(\theta)^2\rangle_A = V/\bar{\beta}^2 \;=\; \frac{(1 + e^{-2\pi|k_{so}|(x+t)})}{(1 - e^{-2\pi|k_{so}|(x+t)})}$$

linear amplifier with gain…..

$$G = 1/(1 - e^{-2\pi|k_{so}|(x+t)})$$

T.G.Downes, T.C.Ralph, N.Walk, arXiv:1203.2716 (2012)

Quantum Communication

approximate analytical solution

$$\langle X_B(\theta)\rangle_A = X/\bar{\beta} \; = \; \frac{\alpha e^{i\phi} + \alpha^* e^{-i\phi}}{\sqrt{(1 - e^{-2\pi |k_{so}|(x+t)})}}$$

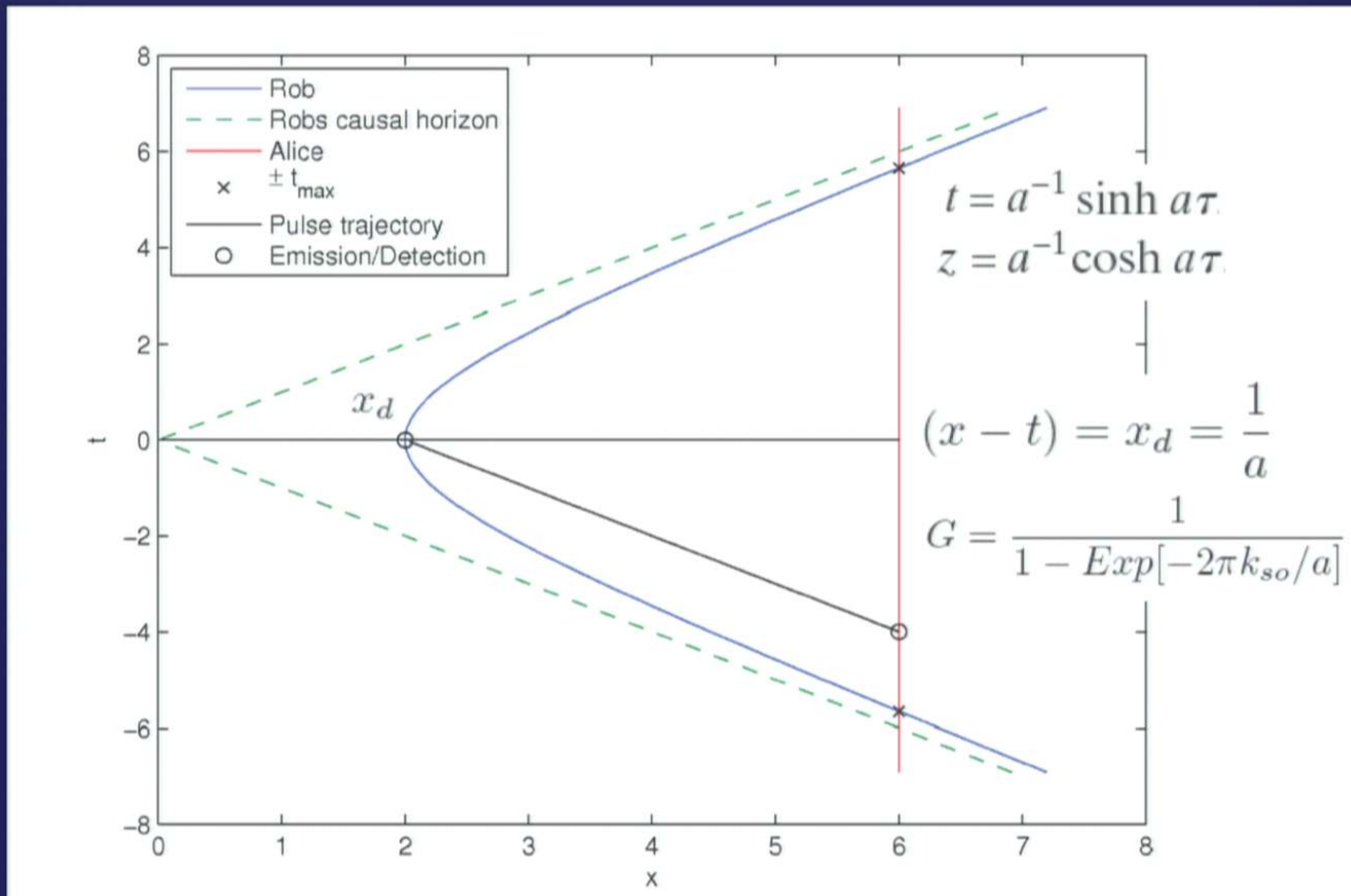
$$G = 1/(1 - e^{-2\pi |k_{so}|(x+t)})$$

$$Exp[2\pi k_{so}(x - t)] - 1$$

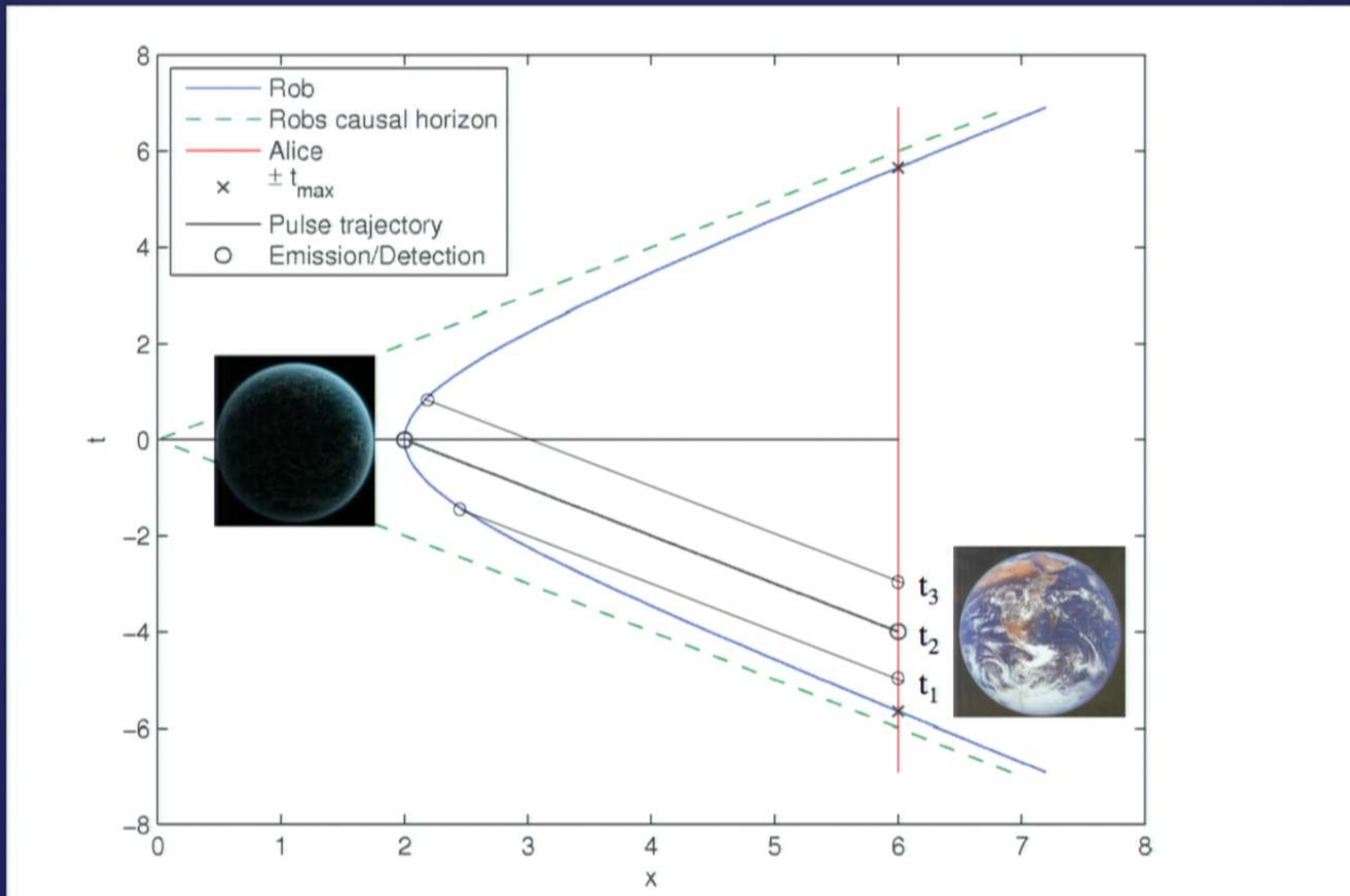
linear amplifier with gain….. $G = 1/(1 - e^{-2\pi |k_{so}|(x+t)})$

T.G.Downes, T.C.Ralph, N.Walk, arXiv:1203.2716 (2012)

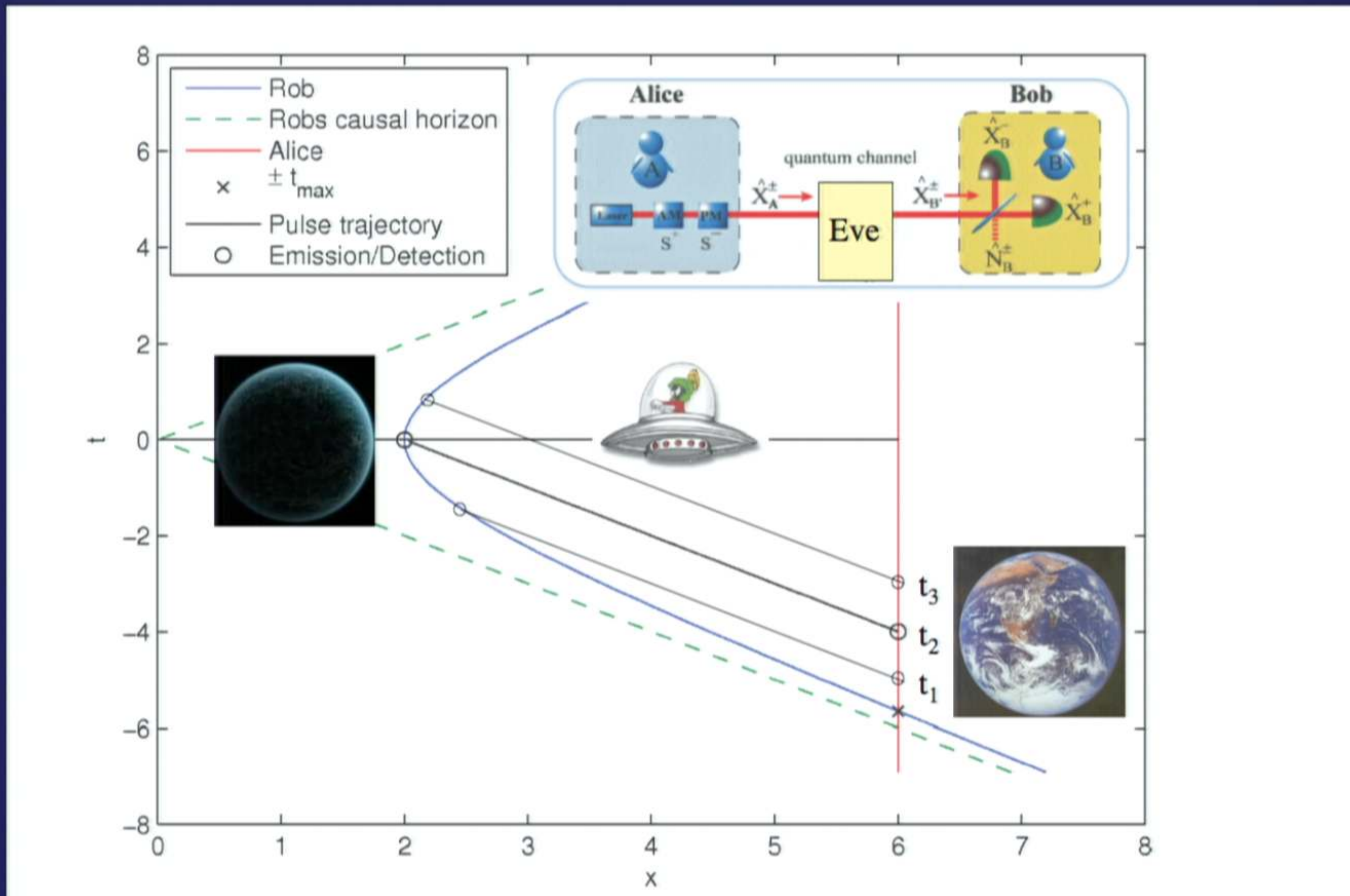
Quantum Communication



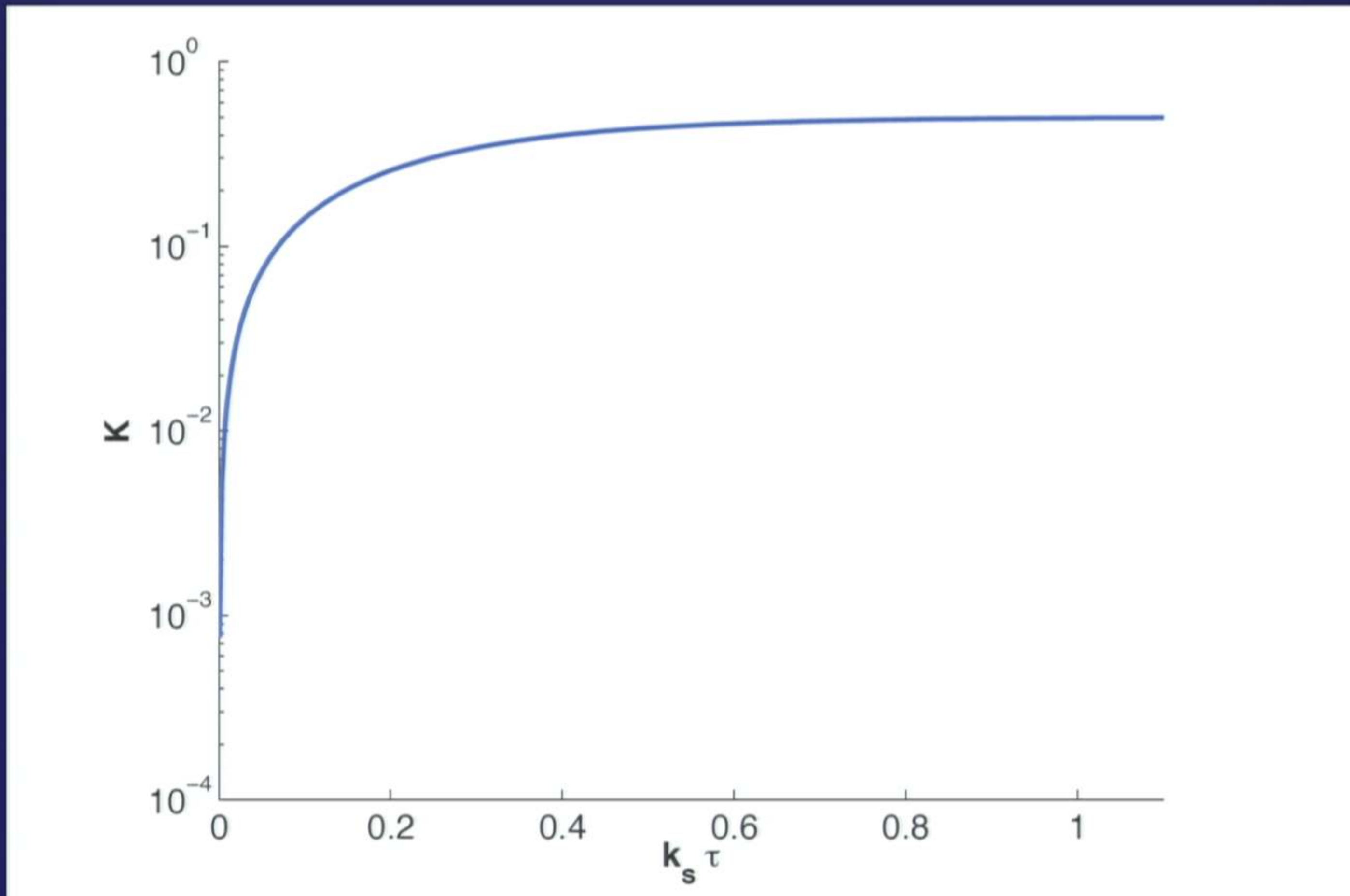
E.g Quantum Key Distribution between Alice and Rob



E.g Quantum Key Distribution between Alice and Rob



E.g Quantum Key Distribution between Alice and Rob



Summary

* Presented general approach for treating **localized**, **directional** quantum optical detection by an accelerated observer.

* Obtained approximate analytical results for the case of quantum limited homodyne detection of a signal sent from an inertial source

* Applied the technique to analysing a continuous variable QKD protocol between Alice and Rob.