Title: Geometric Discord in Non-Inertial Frames

Date: Jun 25, 2012 11:40 AM

URL: http://pirsa.org/12060058

Abstract: I review the recent work performed on computing the geometric discord in non-inertial frames. We consider the well-known case of an inertially maximally entangled state shared by inertial Alice and non-inertial Robb. It is found that for high accelerations the geometric discord decays to a negligible amount; this is in stark contrast to the entropic definition of quantum discord which asymptotes to a finite value in the same limit. Such a result has two different implications: the first being that usable quantum correlations are more limited in this regime than previously thought and the second being that geometric discord may not be a sufficient measure of quantum correlations. I will discuss both of these perspectives.

Geometric Discord in Non-Inertial Frames

Eric Brown

University of Waterloo
Kyle Cormier, Eduardo Martín-Martínez and Robert Mann

RQI 2012 - Perimeter Institute
June. 25. 2012

The Quantum and Geometric Discords

The Quantum and Geometric Discords

(Ollivier & Zurek, 01), (Henderson & Vedral, 01) and (Dakíc *et. al.*, 10)

The Quantum and Geometric Discords

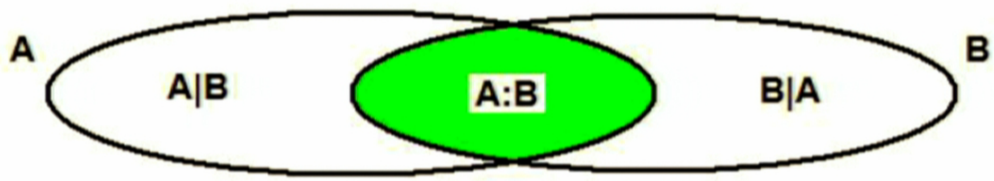
(Ollivier & Zurek, 01), (Henderson & Vedral, 01) and (Dakíc *et. al.*, 10)

Measures of quantum correlation beyond entanglement, for mixed states.

We find that mixed separable states can have nonzero discord.

Consider a bipartite state ρ_{AB} with reduced states ρ_A and ρ_B.

Recall the *mutual information*:



Classically Bayes' rule gives $I(A:B) = J(A:B)$.

In QM the conditional entropy depends on what measurement (PVM) $\{\Pi_i\}$ we perform on ρ_B,

$$S(A|B)_{\{\Pi_i\}} = \sum_i p_i S(\rho_{A|\Pi_i}).$$

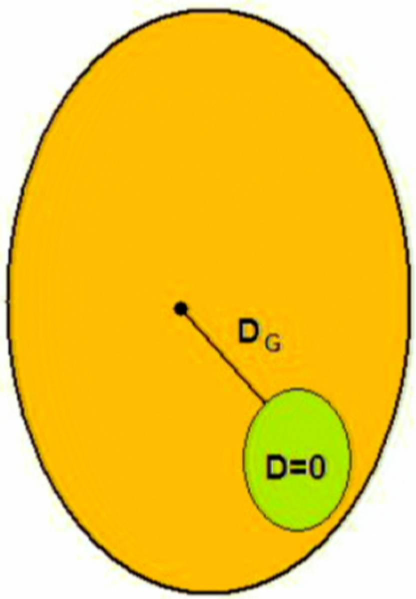
The geometric discord is

$$D_G(A,B) \equiv \min_{\chi\in C} ||\rho_{AB} - \chi||^2_{\mathrm{HS}} = \min_{\chi\in C} \mathrm{Tr}((\rho_{AB} - \chi)^2),$$

where C is the set of states with $D(A,B) = 0$.

This is equivalent to (Luo & Fu, 10)

$D_G(A,B) = \min_{\{\Pi_i\}} \mathrm{Tr}((\rho_{AB} - \rho'_{AB})^2)$.



The global state: $|\psi\rangle = (|0_s\rangle_M |0_k\rangle_M + |1_s\rangle_M |1_k\rangle_U)/\sqrt{2},$

The global state: $|\psi\rangle = (|0_s\rangle_M |0_k\rangle_M + |1_s\rangle_M |1_k\rangle_U)/\sqrt{2}$,

where $|1_k\rangle_U = c^{\dagger}_{k,R}|0\rangle_M$ is a one-particle Unruh mode ($q_R = 1$).

$$c_{k,R} = \cosh(r_k) a_{k,I} - \sinh(r_k) a^{\dagger}_{k,II}, \text{ where } \tanh(r_k) \equiv e^{-\pi|k|/a}.$$

Why use Unruh?

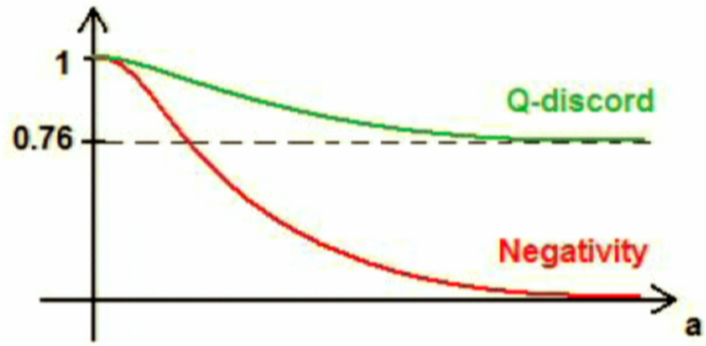
- Pure positive frequency in Minkowski basis $\implies |0\rangle_M = |0\rangle_U$,
- form complete orthonormal basis,
- they are single frequency in the Rindler basis.

Problems:

- highly oscillatory at horizon (not very physical),
- different accelerations correspond to different states.

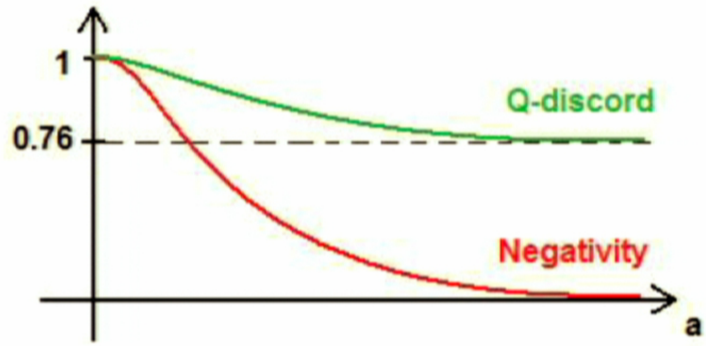
The Alice-Rob state: $\rho_{AR} = \mathrm{Tr}_{II}(|\psi\rangle\langle\psi|)$.

Previous findings (Alsing & Milburn, 03), (Fuentes-Schuller & Mann, 05) and (Datta, 09) :



The Alice-Rob state: $\rho_{AR} = \mathrm{Tr}_{II}(|\psi\rangle\langle\psi|)$.

Previous findings (Alsing & Milburn, 03), (Fuentes-Schuller & Mann, 05) and (Datta, 09) :



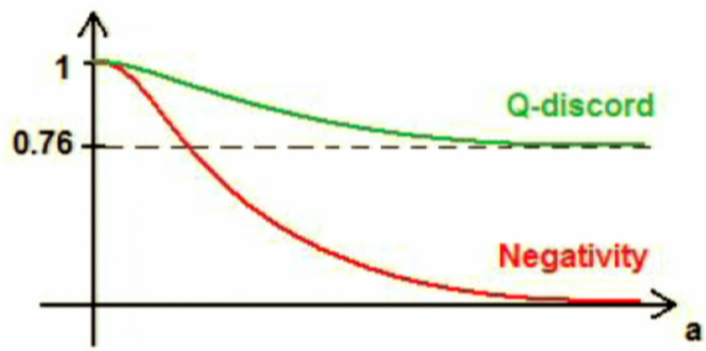
Q: What about the geometric discord? $D_G = \min_{\{\Pi_i\}} \mathrm{Tr}((\rho_{AB} - \rho'_{AB})^2)$

A:

$$D_G = \frac{(1-t^2)(2+t^2)}{4(1+t^2)^3}, \quad t \equiv e^{-\pi|k|/a}$$

The Alice-Rob state: $\rho_{AR} = \mathrm{Tr}_{II}(|\psi\rangle\langle\psi|)$.

Previous findings (Alsing & Milburn, 03), (Fuentes-Schuller & Mann, 05) and (Datta, 09) :



Q: What about the geometric discord? $D_G = \min_{\{\Pi_i\}} \mathrm{Tr}((\rho_{AB} - \rho'_{AB})^2)$

A:

$$D_G = \frac{(1-t^2)(2+t^2)}{4(1+t^2)^3}, \quad t \equiv e^{-\pi|k|/a}$$

(for measurements over Alice's system).

Decays to zero when $a \to \infty$!

In fact, we don't even need to minimize.

e.g. using $\Pi_n = |n\rangle\langle n| \implies \rho'_{AB} = \text{diag}(\rho_{AB})$.

$\implies D_G \to 0$ as $a \to \infty$ for measurements over both Alice and Rob.

In fact, we don't even need to minimize.

e.g. using $\Pi_n = |n\rangle\langle n| \implies \rho'_{AB} = \text{diag}(\rho_{AB})$.

$\implies D_G \to 0$ as $a \to \infty$ for measurements over both Alice and Rob.

This seems strange...

A result of infinite dimension and unbounded energy.

- Similar results seen in Gaussian geometric discord. (Adesso & Girolami, 11)
- In fact, same thing for geometric measure of entanglement! (Eisert, Simon & Plenio, 02)

Implication 1

The geometric discord is not a faithful measure of quantum correlations in continuous variable systems.

The Alice-Rob system appears to serve as an illustrative example for this.

- The trace (1-norm) distance $\mathrm{Tr}(\sqrt{(\rho_{AB} - \rho'_{AB})^2})$ does *not* vanish.
- Joint-correlators disagree. E.g. $\langle X_A X_R \rangle_{\rho'} = 0$ but $\langle X_A X_R \rangle_{\rho} \to \infty$.

Implication 2

Nevertheless, quantum communication procedures (among possibly others) are significantly limited for large accelerations.

Implication 2

Nevertheless, quantum communication procedures
(among possibly others) are significantly
limited for large accelerations.

In addition to other operational significance, the geometric discord...

- is the figure of merit for remote state preparation.

(Dakíc *et. al.*, 12)

- This is true even for $2 \times \infty$ dimensions!

(Tufarelli *et. al.*, 12)

No entanglement $\Longrightarrow$ no teleportation.

Conclusions

The geometric discord vanishes in the infinite acceleration limit, unlike the entropic definition of discord.

Conclusions

The geometric discord vanishes in the infinite acceleration limit, unlike the entropic definition of discord.

This result has two implications:

- The geometric discord is not a faithful quantifier of quantum correlations in infinite dimensions.

Conclusions

The geometric discord vanishes in the infinite acceleration limit, unlike the entropic definition of discord.

This result has two implications:

- The geometric discord is not a faithful quantifier of quantum correlations in infinite dimensions.
- Quantum communication for large accelerations is severely limited, despite the presence of quantum correlations.

Future Projects

- Scaling of discord in thermal bosonic field (R. Mann).
- Understanding discord in terms of entanglement on a dilated Hilbert space (A. Kempf, E. Webster and E. Martín-Martínez).
- Understand Unruh effect as viewed from inertial perspective. (A. Kempf, A. Chatwin-Davies, E. Martín-Martínez and R. Mann).