Title: Physical Results with Fermions in RQI

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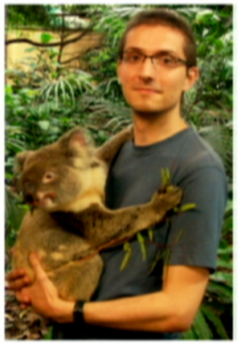
Abstract: A number of works in the field of relativistic quantum information have been devoted to the study of entanglement on certain simple families of Unruh-mode entangled states in non-inertial frames. In the fermionic case remarkable results such as the survival of entanglement at infinite acceleration have been obtained. In this talk we will present and analyze some issues related to the anticommuting character of fermionic field operators, which have been overlooked in the past, sometimes leading to unphysical results. We provide a simple way of obtaining physical results, yielding interesting consequences such as convergence of field entanglement for different families of Unruh mode-entangled states in the infinite acceleration limit.

- *Fermionic entanglement ambiguity in noninertial frames*, M. Montero and E. Martín-Martínez, Phys. Rev. A 83, 062323 (2011).
  Follow-up: K. Brádler, R. Jáuregui, Phys. Rev. A 85, 016301 (2012), M. Montero and E. Martín-Martínez, Phys. Rev. A 85, 016302 (2012).
- *Convergence of fermionic-field entanglement at infinite acceleration in relativistic quantum information*, M. Montero and E. Martín-Martínez, Phys. Rev. A 85, 024301 (2012).



The Unruh effect

- Acceleration/spacetime curvature have nontrivial effects on quantum fields.
- Unruh effect:

$$|0\rangle_{\mathrm{M}} \quad \rightarrow \quad \text{Thermal state at } T_{\mathrm{U}}$$

- Hawking effect:

$$|0\rangle_{\mathrm{in}} \quad \rightarrow \quad \text{Thermal state at } T_{\mathrm{H}}$$

Entanglement of field states

Alice

$$|\Psi\rangle$$

Rob

Quantization depends on the observer

- Quantization in Minkowski coordinates: Minkowski modes u_{M},

$$c_{\text{M}} \left|0\right\rangle_{\text{M}} = d_{\text{M}} \left|0\right\rangle_{\text{M}} = 0.$$

($c_{\text{M}} \rightarrow$ particle modes, $d_{\text{M}} \rightarrow$ antiparticle modes)

- Alice $\rightarrow$ Minkowski modes.
- Quantization in Rindler coordinates: Rindler modes u_{I}, u_{II},

$$c_{\text{I}} \left|0\right\rangle_{\text{Rindler}} = c_{\text{II}} \left|0\right\rangle_{\text{Rindler}} = d_{\text{I}} \left|0\right\rangle_{\text{Rindler}} = d_{\text{II}} \left|0\right\rangle_{\text{Rindler}} = 0.$$

- Rob $\rightarrow$ Rindler modes.

The Unruh modes

- They define the same vacuum as the Minkowski modes: $c_{\mathrm{U}}(r, q_{\mathrm{R}}) \left|0\right\rangle_{\mathrm{M}} = 0.$
- They have a simple expression in terms of Rindler modes.
- Parametrized by two parameters: q_{R} and r.

The parameters and the states

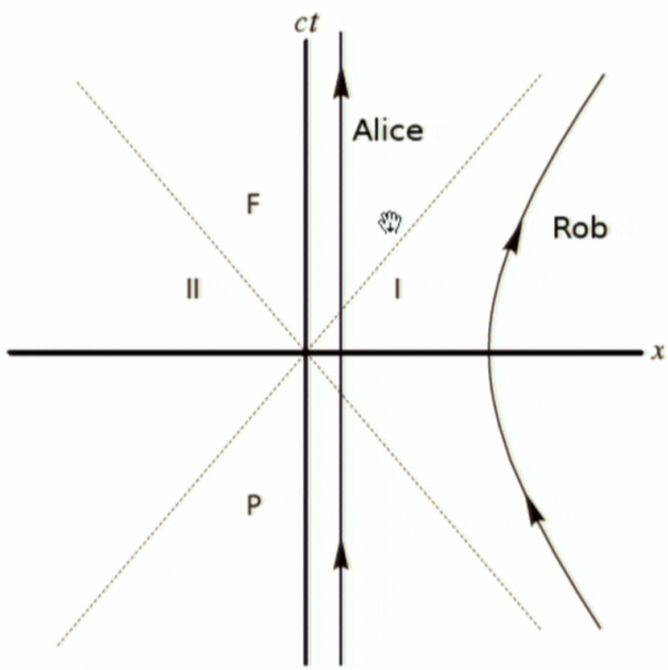
- The parameter r can be related to Rob's acceleration and probing frequency:

$$\tan r = e^{-\pi c\omega/a}, \quad r \in [0, \pi/4].$$

- q_{R} is related to the spatial localization of the Unruh modes at $r = 0$ (equivalently, $a = 0$).
- We consider linear combinations of $|0\rangle_{\mathsf{M}}$ and Unruh excitations, $c^{\dagger}_{\mathsf{U}}(r, q_{\mathsf{R}})\,|0\rangle_{\mathsf{M}}$, such as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\mathbf{I} + c^{\dagger}_{\text{Alice}}c^{\dagger}_{\text{U,Rob}}(r, q_{\mathsf{R}})\right]|0\rangle_{\mathsf{M}}\,.$$

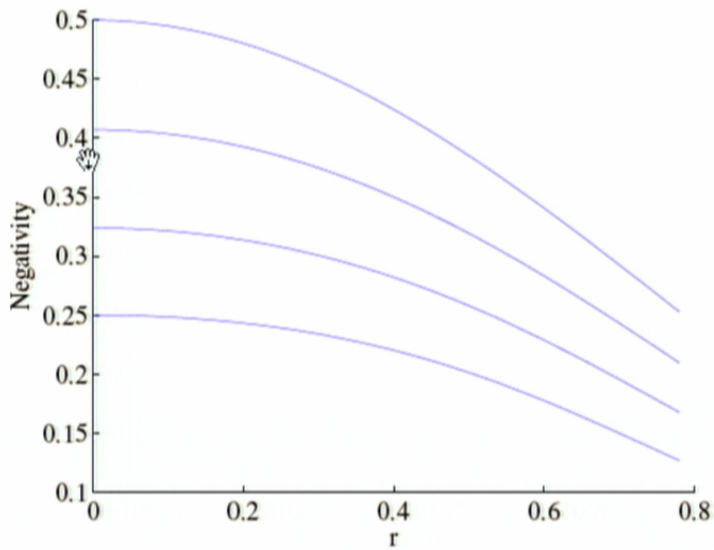
Computing entanglement measures



1. Express Rob's part of the state in the Rindler basis.
2. Compute the density matrix $\rho = |\Psi\rangle\langle\Psi|$ and trace out region II unobservable modes.
3. Compute entanglement measures.

Previous results

Negativity as a function of r, for different q_R:



We can show that at $r = \pi/4$ entanglement measures must be independent of q_R → Contradiction!

Convergence at infinite acceleration: Why

- At infinite acceleration, the Unruh excitations are

$$c_{\mathsf{U}}^\dagger = \frac{1}{\sqrt{2}}[(q_{\mathsf{R}} c_{\mathsf{I}}^\dagger - q_{\mathsf{L}} d_{\mathsf{I}}) + (q_{\mathsf{L}} c_{\mathsf{II}}^\dagger - q_{\mathsf{R}} d_{\mathsf{II}})] = a_{\mathsf{I}}^\dagger + a_{\mathsf{II}}^\dagger,$$

$$q_{\mathsf{L}} = \sqrt{1 - q_{\mathsf{R}}^2}, \quad a_{\mathsf{I}}^\dagger \left|0\right\rangle_{\mathsf{M}} = a_{\mathsf{II}}^\dagger \left|0\right\rangle_{\mathsf{M}},$$

so that the field state may be written as

$$\left|\Psi\right\rangle = A_{\mathsf{I}} \left|0\right\rangle_{\mathsf{M}}.$$

- If we compute the reduced state

$$\mathrm{Tr}_{\mathsf{II}}(\left|\Psi\right\rangle\left\langle\Psi\right|) = A_{\mathsf{I}} \mathrm{Tr}_{\mathsf{II}}(\left|0\right\rangle_{\mathsf{M}}\left\langle 0\right|_{\mathsf{M}}) A_{\mathsf{I}}^\dagger \propto A_{\mathsf{I}} A_{\mathsf{I}}^\dagger.$$

it does not depend on q_{R}.

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Fock space bases and orderings

- Toy model: Two fermionic modes a and b,

$$\{a, a^\dagger\} = \{b, b^\dagger\} = 1, \quad \{a, b\} = \{a^\dagger, b^\dagger\} = 0.$$

- Some possible Fock space bases:
    - a, b ordering,

$$|00\rangle = |0\rangle\,, |10\rangle = a^\dagger\,|0\rangle\,,\; |01\rangle = b^\dagger\,|0\rangle\,,\; |11\rangle = a^\dagger b^\dagger\,|0\rangle\,.$$

    - b, a ordering,

$$|00\rangle' = |0\rangle\,, |10\rangle' = a^\dagger\,|0\rangle\,,\; |01\rangle' = b^\dagger\,|0\rangle\,,\; |11\rangle' = b^\dagger a^\dagger\,|0\rangle\,.$$

- Each operator ordering defines a basis.
- The change of basis from one ordering to another is nonlocal.

Partial traces

- We want to perform a partial trace over a modes on a given state ρ.
- Naïve idea: Write down ρ in the a, b or the b, a basis
- Perform the trace as in a qubit system.
- This is what we call a fermion-qubit mapping.

The reduced state depends on the particular fermion-qubit mapping.$\rightarrow$ Which one, if any, is correct?

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The physical ordering

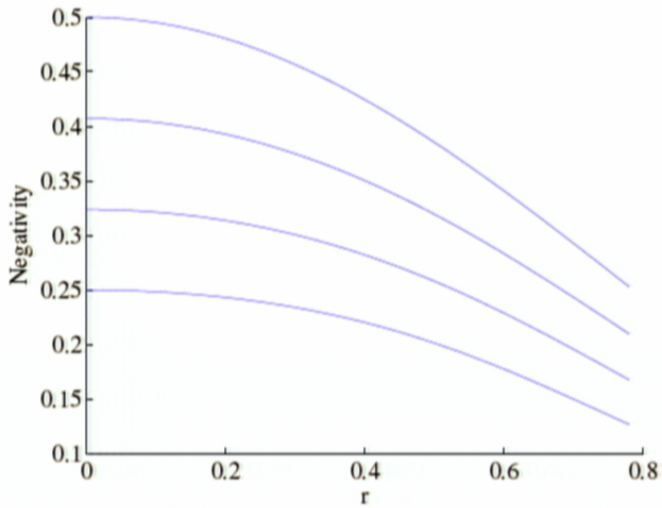
- A fermion-qubit mapping returns the correct reduced state of the fermionic system if in the corresponding operator ordering all traced modes appear to the right of all untraced modes.

$$|1\dots 1\rangle = \underbrace{b_1^\dagger, \dots b_k^\dagger}_{\text{Untraced modes}} \underbrace{a_1^\dagger, \dots a_l^\dagger}_{\text{Traced out modes}} |0\rangle \,.$$

- This way the a modes will give no spurious signs when computing matrix elements.
- Previous literature made use of fermion-qubit mappings but did not employ the physical operator ordering.

Physical ordering: RQI

Previous results:



Physical ordering: RQI

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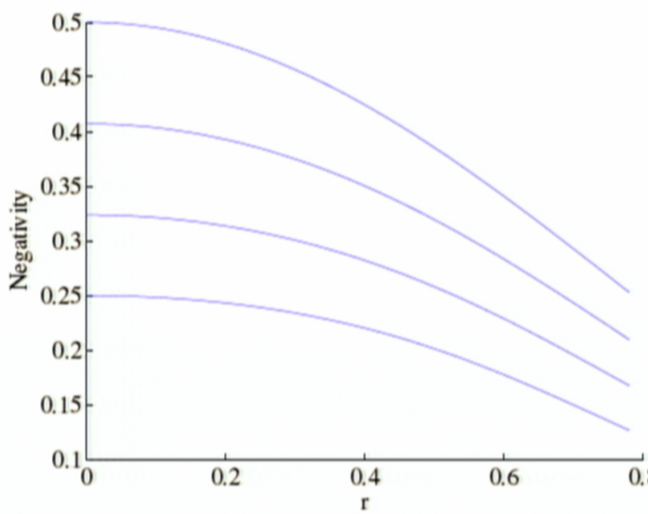
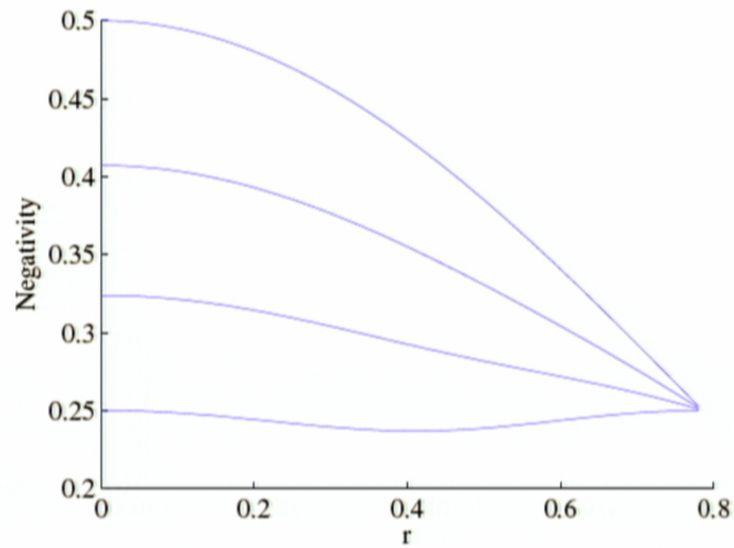
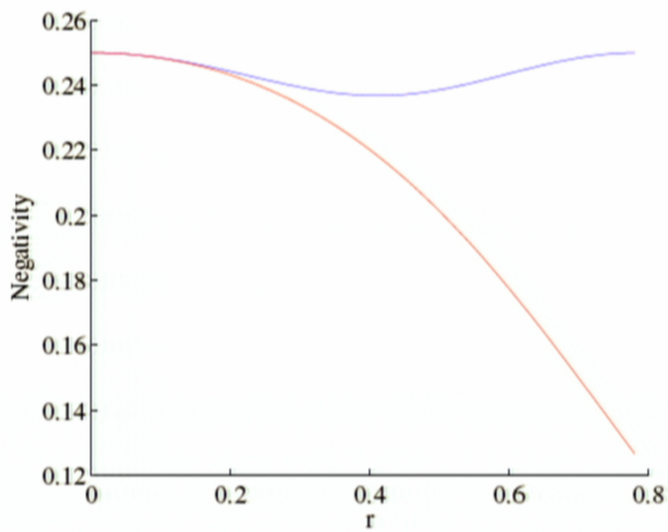


Physical ordering:



More examples: Chang and Kwon, Phys. Rev. A 85, 032302 (2012).

The physical ordering: Results



- $q_R = \frac{1}{\sqrt{2}}$.
- Blue: Physical ordering. Red: Ordering in previous works.
- Entanglement revival.

Summary

- At infinite acceleration, entanglement is independent of the choice of Unruh modes.
- The literature has made extensive use of fermion-qubit mappings.
- Only mappings associated with the physical ordering yield physical results.