

Title: The Quest for Localization

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Abstract: This talk aims to review the obstacles met in QFT to reach an appropriate definition for such a basic concept as localization. The anti-local character of the square root of the "- Laplace-Beltrami + mass²" operator prevents the existence of localized states with a finite number of quanta. (Bosonic) quantum fields describe elementary excitations of an extended system whose ground state is the vacuum. No wonder, there is a complicated relationship between the cardinal

(quantal) and continuous (spatial) sides of the theory.

We will also analyze the roles (if any) played in RQI by the localized excitations of the vacuum.



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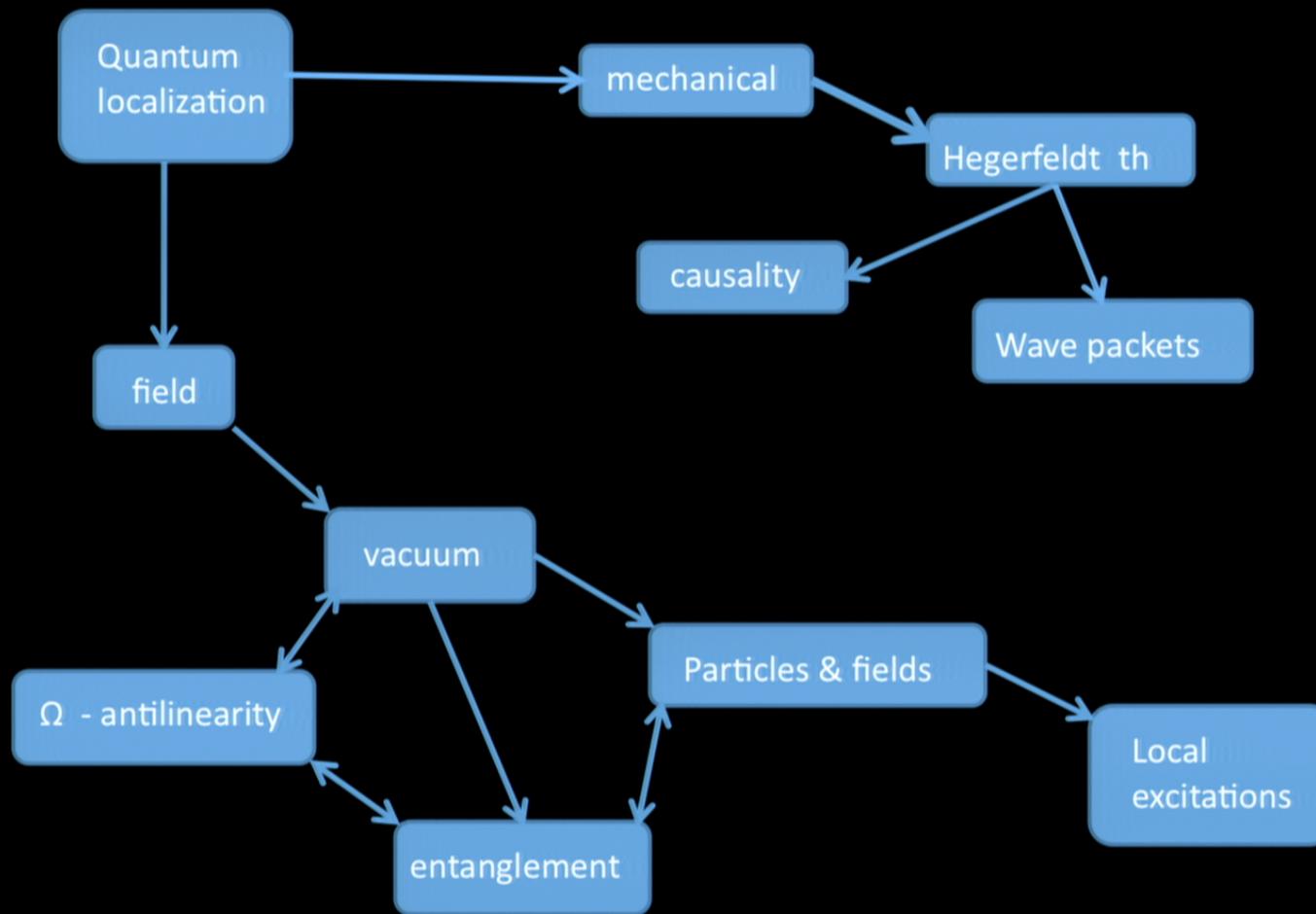
Quantum Information and Foundations Group



Acabas de llegar a la página del grupo de Información Cuántica y Fundamentos de Teoría Cuántica, ubicado en el Instituto de Física Fundamental, CSIC. Juntos formamos el grupo Quinfog, creado en 2006, y cuyos miembros encontrarás en la [página de personal](#).

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mechanical localization

if it is not there.... it is zero

Hegerfeldt Theorem

$$\begin{array}{l}
 \psi_t = e^{-iHt}\psi_0 \quad \hat{H} \geq c \\
 \mathcal{P}_A(t) = \langle \psi_t | A | \psi_t \rangle \quad \hat{A} \geq 0
 \end{array}
 \rightarrow
 \begin{cases}
 \text{Either } \mathcal{P}_A(t) \neq 0 \forall t \in \mathbf{R} \\
 \text{or } \mathcal{P}_A(t) = 0 \forall t \in \mathbf{R}.
 \end{cases}$$

R. Paley and N. Wiener Theorem XII

Take $A = \int_V |x\rangle\langle x|$ V Borel set in \mathbf{R}^3 , $|x\rangle$ position eigenstate

Either ψ is in V forever ($P_V(t) \neq 0 \forall t \in \mathbf{R}$)

or ψ is never in V ($P_V(t) = 0 \forall t \in \mathbf{R}$)

Strict localization of a system with H bounded from below incompatible with causality

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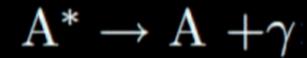
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Causality

Detection of spontaneous emission



Excited atom $|\psi\rangle$

No photons

Detector $|D_0\rangle$



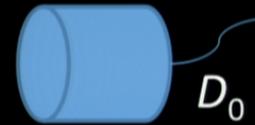
Final state atom $|\psi_t\rangle$

n photons

Excited detector $|D_e\rangle$

When the detector clicks?

$t < R/c$

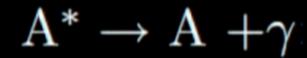


$$\mathcal{P}_{\text{click}}(t) = \langle \psi_t | \mathcal{O}_{\text{click}} | \psi_t \rangle$$

$$\mathcal{O}_{\text{click}} = \sum_e |D_e\rangle \langle D_e|$$

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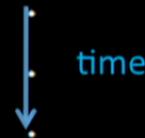
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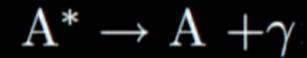


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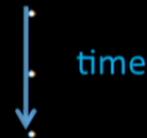
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Causality problem posed by Fermi in RMP 1931

Solved perturbatively by Power, Thirunamachandran PRA 1997

Solved non-perturbatively by Sabin et al PRL 2011
(circuit QED exp proposal included)

Finally,

$$P_{\text{click}}(t) = \text{for } t < R/c$$

“iff we subtract the vacuum contribution”
(detector self excitations)

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What if the particle is strictly localized at $t=0$?

Prigogine et al PRA 2000

i. at $t = 0$ $\Phi_{x_0,b}(0) = \frac{1}{2b} \Theta(b - |x - x_0|)$



ii. is a particle, $w > 0$ $\Phi_+(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \phi_+(k) e^{-i\omega_k t} e^{ikx}$

iii. be it a massless boson $\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} \right) \Phi(x, t) = 0$

If this describe a particle , one would expect

$$\Phi = 0 \text{ for } |x - x_0| > b + c t$$

This is not the case

N .B. **no well posed** "initial" conditions, $d_t \Phi(x,0) \neq 0$ "everywhere"

Best localized photon wave packets in the market

- i. Define the one photon mode $\tilde{f}(\mathbf{k}, \lambda)$

$$|1_f\rangle = \sum_{\lambda} \int d^3k \tilde{f}(\mathbf{k}, \lambda) a^{\dagger}(\mathbf{k}, \lambda) |0\rangle$$

- ii. Take into account the probabilistic nature of the wave function

$$\sum_{\lambda} \int d^3k |\tilde{f}(\mathbf{k}, \lambda)|^2 = 1$$

- iii. The fact that to represent a photon $\tilde{f}(\mathbf{k}, \lambda) = 0$ for $\omega_k < 0$

$$f(x, t) = Z(t + x/c) + Z(t - x/c)$$

$$\lim_{|\tau| \rightarrow \infty} Z(\tau) \gtrsim e^{-A\tau^{\gamma}}, \quad A > 0, \gamma < 1$$

Bialynicki-Birula PRL 97, **don't miss their 2012 minimal uncertainty wave packets**

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~~mechanical localization if it is not there.... it is zero~~

field localization:

If it is not there..... there is the vacuum!

State Ψ localized outside $G \rightarrow (\Psi, A \Psi) = (\Omega, A \Omega)$ for all A in $R(G)$

W partial isometry in $R'(G) \mid W^*W=1$

$\Psi = W\Omega$ strictly localized outside G

In general $WW^*=P \neq 1$

W strictly localized operator $\rightarrow (W\Phi_1, A W\Phi_2) = (\Phi_1, A \Phi_2)$

Iff $W^*W=1$ and W in $R(G')$

W strictly localized operator $\rightarrow W\Omega$ strictly localized state

All strictly localized states are of the form $\Psi = W \Omega$

W is unique in $R'(G)$

but other operators can create Ψ from the vacuum

example: P_Ω projector on the vacuum $\rightarrow WP_\Omega\Omega = \Psi$

but if $WP_\Omega = W \rightarrow P_\Omega = W^*W = 1 !!!$

For any P in $R(G)$ and any Ψ in H
there is an eigenstate of P equivalent to ψ in G'

Localized states do not form a vector space

$$\Phi_1 = W_1 \Omega \quad \Phi_2 = W_2 \Omega$$

$$\Psi = \alpha \Phi + \beta \Phi \text{ localized}$$

$$\text{iff } W_2^* W_1 = (\Omega, W_2^* W_1 \Omega) = r$$

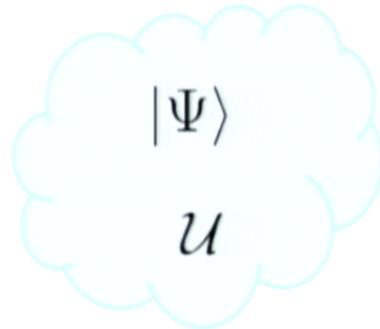
$$W_2 = r^* W_1 + (1 - |r|^2)^{1/2} U$$

$$U^* W = 0, U^* U = 1 \dots P_1 = W_1 W_1^*, P_U = U U^* \rightarrow P_1 P_U = 0$$

U strictly localized outside G and orthogonal to W_1

A state $|\Psi\rangle$ of a field $\phi(x)$ is localized in the region \mathcal{U} if

$$\langle \Psi | \prod_{i=1}^n \phi(x_i) | \Psi \rangle = \langle 0 | \prod_{i=1}^n \phi(x_i) | 0 \rangle, \quad n = 1, 2, \dots, \quad x_i \notin \mathcal{U}$$



Knight JMP 1961
Licht JMP 1963

Generic solution

$$|\Psi\rangle = \exp(iR)|0\rangle$$

$$R = q_0 + \int_{\mathcal{U}} dx q(x)\phi(x) + \int \int_{\mathcal{U}} dx_1 dx_2 q(x_1, x_2)\phi(x_1)\phi(x_2) + \dots$$

Localized in the diamond of \mathcal{U}

Ree-Schlieder theorem N Cim 1961

Ω is strongly non-local

$$\Omega^2 u(x) = - \sum_1^n \partial/\partial x_i (\gamma_{ij}(x) (\partial f(x)/\partial x_j)) + a(x) f(x)$$

If f and Ωf vanish in a set $\mathcal{U} \subset \mathcal{D}$ then

$f = 0$ almost everywhere in \mathcal{D}

This is the version after Masuda and Murata,

Arbitrary space dimensions,

Static spacetime

1. The vacuum is cyclic vector
2. The vacuum is separating vector

The following are equivalent:

- i. Ω is strongly non-local on B
- ii. No states with finite number of quanta are strictly local excitations of the vacuum with support in B

$$|1\text{ph}\rangle = \int d^3k f_+(\mathbf{k}) a^\dagger(\mathbf{k}) |0\rangle + \int d^3k f_-(\mathbf{k}) b^\dagger(\mathbf{k}) |0\rangle$$

$$\mathcal{H}_q(\mathbf{r}, t) = \langle 1\text{ph} | : \hat{\mathbf{F}}^\dagger(\mathbf{r}, t) \cdot \hat{\mathbf{F}}(\mathbf{r}, t) : | 1\text{ph} \rangle = |\mathbf{F}_+(\mathbf{r}, t)|^2 + |\mathbf{F}_-(\mathbf{r}, t)|^2$$

Incoherent sum \rightarrow Hegerfeldt (Paley-Wiener) prevent localization

$$|\text{coh}\rangle = N \exp \left[\int d^3k f_+(\mathbf{k}) a^\dagger(\mathbf{k}) + \int d^3k f_-(\mathbf{k}) b^\dagger(\mathbf{k}) \right] |0\rangle$$

$$\mathcal{H}_{\text{coh}}(\mathbf{r}, t) = \langle \text{coh} | : \hat{\mathbf{F}}^\dagger(\mathbf{r}, t) \cdot \hat{\mathbf{F}}(\mathbf{r}, t) : | \text{coh} \rangle = |\mathbf{F}_+(\mathbf{r}, t) + \mathbf{F}_-(\mathbf{r}, t)|^2$$

Coherent sum \rightarrow interferences defeat Hegerfeldt

No obstacle to the localization of coherent states