Title: Tripartite Entanglement, Svetlichny Inequalities, and Non-inertial Observers

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Abstract: I discuss the behaviour of bipartite and tripartite non-locality between fermionic entangled states shared by observers, one of whom uniformly accelerates. Although fermionic entanglement persists for arbitrarily large acceleration, the Bell/CHSH inequalities cannot be violated for sufficiently large but finite acceleration. However the Svetlichny inequality, which is a measure of genuine tripartite non-locality, can be violated for any finite value of the acceleration.

Tripartite Entanglement, Svetlichny Inequalities, and Non-inertial Observers

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science.uwaterloo.ca

Robert Mann
Alexander Smith
Andreas Waldenburger
Paulina Corona-Ugalde
Eduarado Martin-Martinez

A. Smith and R.B. Mann, arXiv 1107.4633 (PRA to appear)

Introduction & Motivation

What happens to entanglement between quantum states if they are in relative motion?

Inertial Frames:

- Entanglement is constant between inertially moving observers
- Some degrees of freedom can be transferred to others

Peres,Scudo and Terno
Alsing and Milburn
Gingrich and Adami
Pachos and Solano

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Introduction & Motivation

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Pachos and Solano

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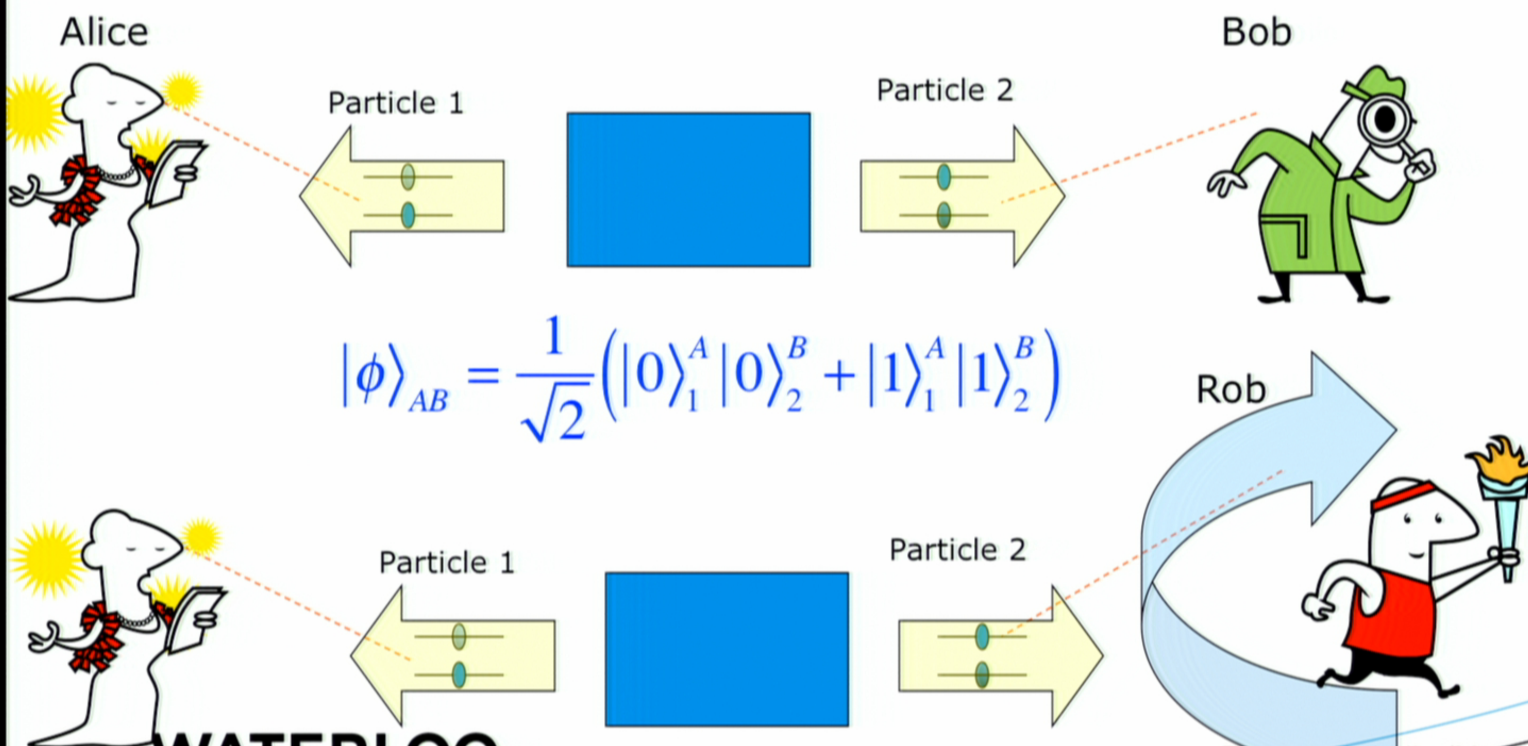
Entanglement for Uniformly Accelerated Observers



$$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_1^A|0\rangle_2^B + |1\rangle_1^A|1\rangle_2^B\right)$$

$$|\phi\rangle_{AB} = ?$$

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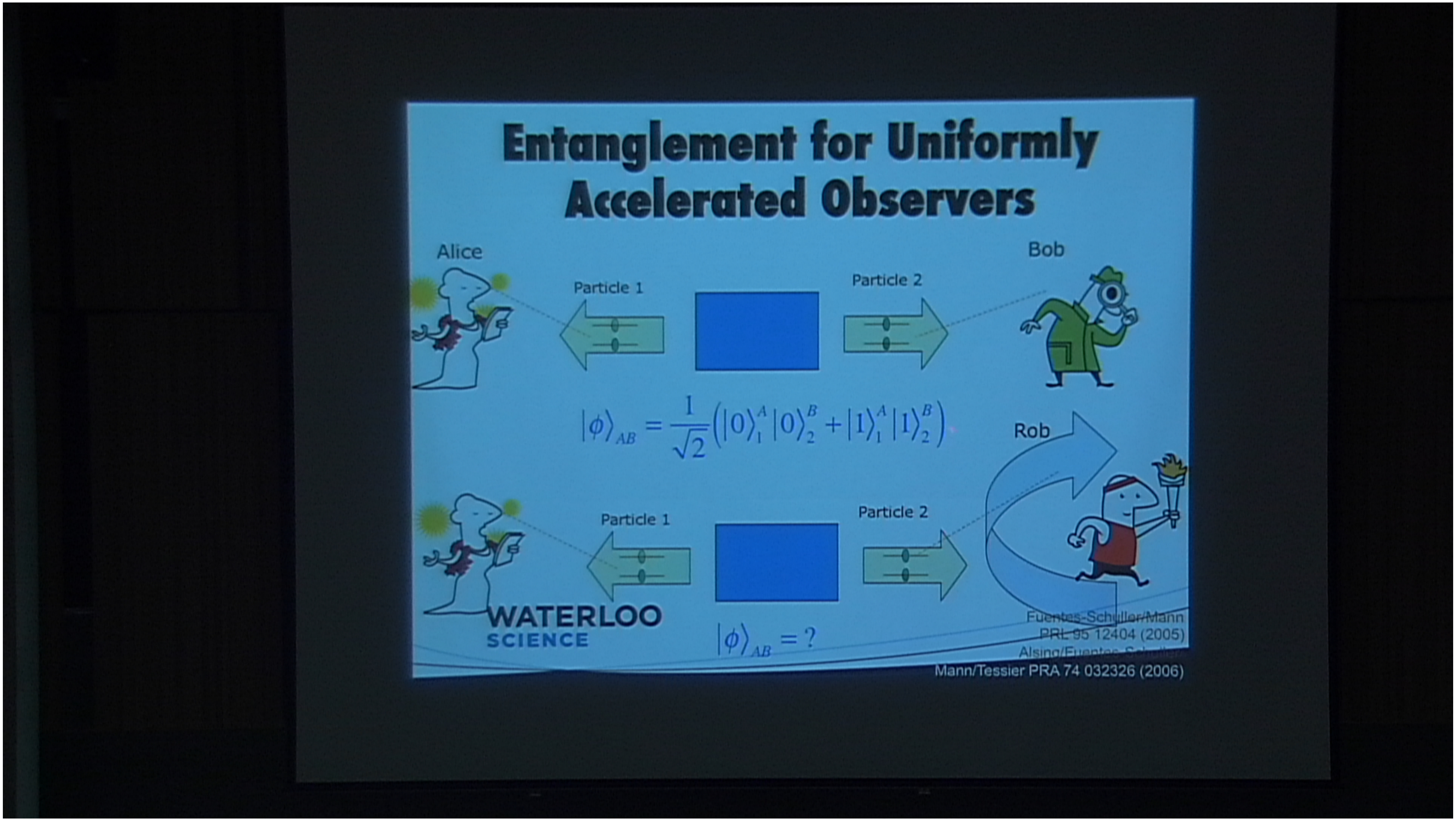
Fuentes-Schuller/Mann PRL 95 12404 (2005)
Alsing/Fuentes-Schuller/Mann/Tessier PRA 74 032326 (2006)


Entanglement for Uniformly Accelerated Observers
Alice
Bob
Particle 1
Particle 2
$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_1^A |0\rangle_2^B + |1\rangle_1^A |1\rangle_2^B\right)$
Rob
Particle 1
Particle 2
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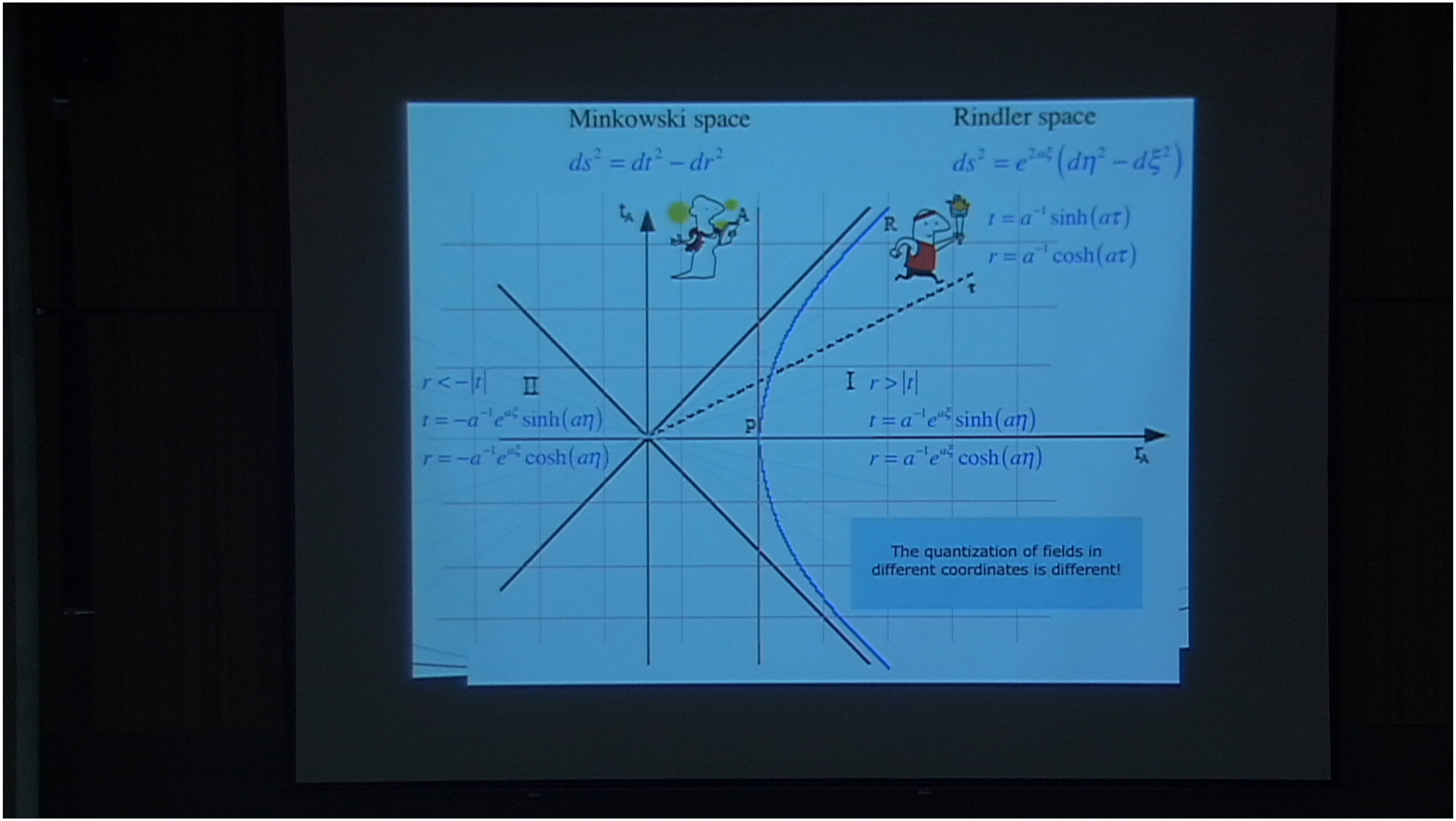
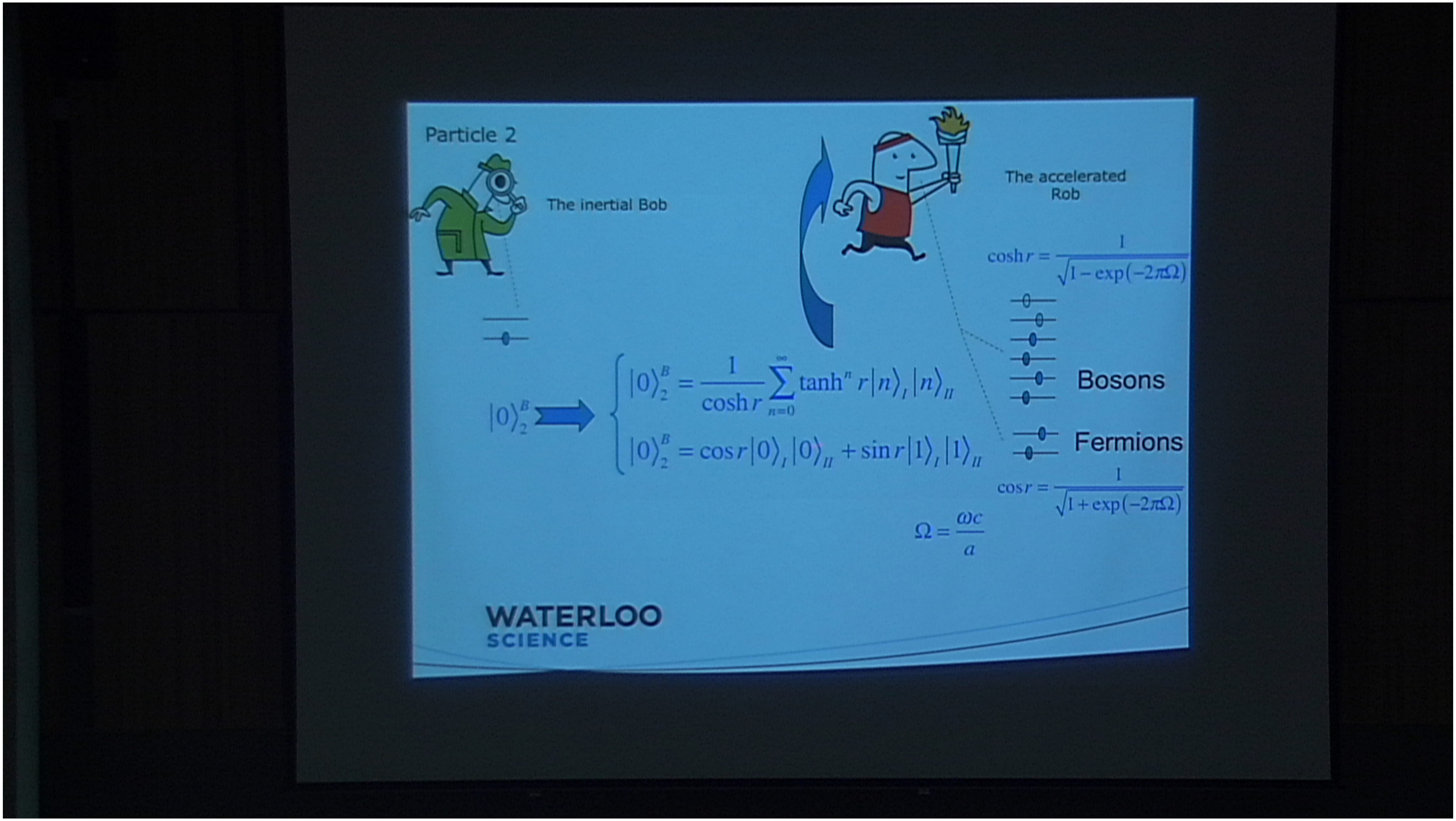
$|\phi\rangle_{AB} = ?$
Fuentes-Schuller/Mann
PRL 95 12404 (2005)
Alsing/Fuentes-Schuller
Mann/Tessier PRA 74 032326 (2006)



Minkowski space
$ds^2 = dt^2 - dr^2$
Rindler space
$ds^2 = e^{2a\xi}\left(d\eta^2 - d\xi^2\right)$
$t = a^{-1}\sinh(a\tau)$
$r = a^{-1}\cosh(a\tau)$
A
R
P
τ
II
$r < -|t|$
$t = -a^{-1}e^{a\xi}\sinh(a\eta)$
$r = -a^{-1}e^{a\xi}\cosh(a\eta)$
I
$r > |t|$
$t = a^{-1}e^{a\xi}\sinh(a\eta)$
$r = a^{-1}e^{a\xi}\cosh(a\eta)$
The quantization of fields in different coordinates is different!


Particle 2
The inertial Bob
The accelerated Rob
$\cosh r = \frac{1}{\sqrt{1-\exp(-2\pi\Omega)}}$
Bosons
Fermions
$\cos r = \frac{1}{\sqrt{1+\exp(-2\pi\Omega)}}$
$|0\rangle_2^B \Rightarrow \begin{cases} |0\rangle_2^B = \frac{1}{\cosh r}\sum_{n=0}^{\infty} \tanh^n r\, |n\rangle_I |n\rangle_{II} \\ |0\rangle_2^B = \cos r\, |0\rangle_I |0\rangle_{II} + \sin r\, |1\rangle_I |1\rangle_{II} \end{cases}$
$\Omega = \frac{\omega c}{a}$
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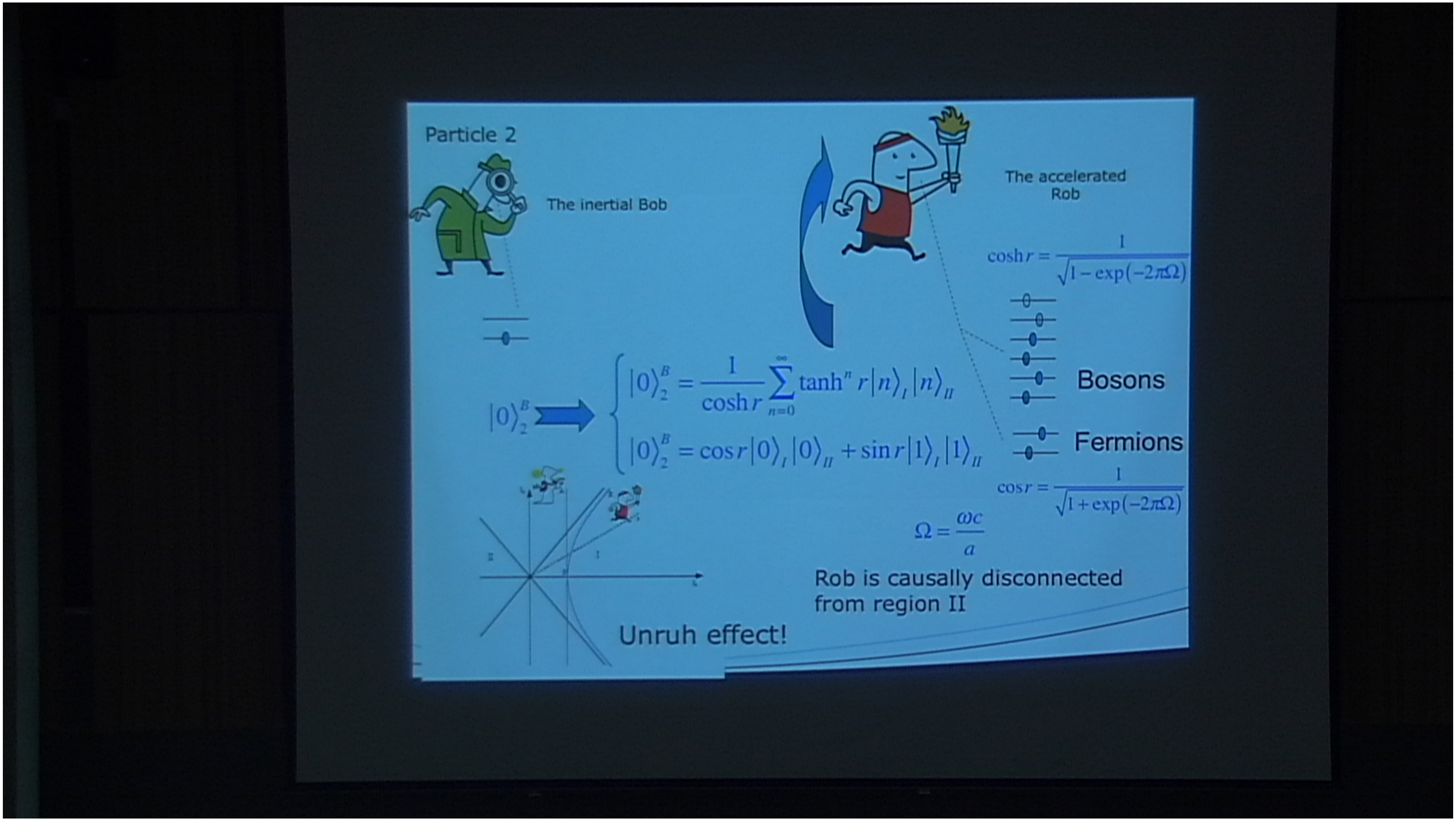
Particle 2
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$\cosh r = \frac{1}{\sqrt{1-\exp(-2\pi\Omega)}}$
$|0\rangle_2^B$
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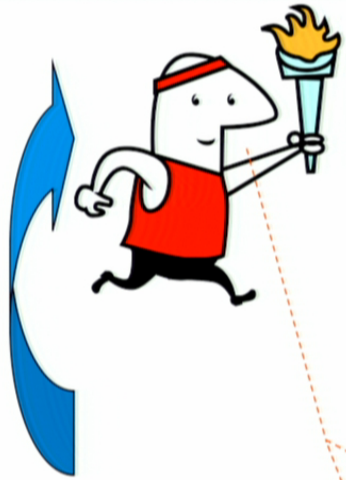
$|0\rangle_2^B = \cos r |0\rangle_I |0\rangle_{II} + \sin r |1\rangle_I |1\rangle_{II}$
Bosons
Fermions
$\cos r = \frac{1}{\sqrt{1+\exp(-2\pi\Omega)}}$
$\Omega = \frac{\omega c}{a}$
Rob is causally disconnected from region II
Unruh effect!

Particle 2

The inertial Bob



The accelerated Rob

$$\cosh r = \frac{1}{\sqrt{1-\exp(-2\pi\Omega)}}$$

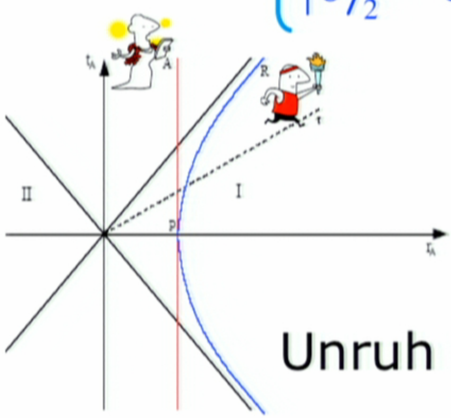
$$|0\rangle_2^B \Rightarrow \begin{cases} |0\rangle_2^B = \dfrac{1}{\cosh r}\displaystyle\sum_{n=0}^{\infty} \tanh^n r\,|n\rangle_I |n\rangle_{II} \\ |0\rangle_2^B = \cos r\,|0\rangle_I |0\rangle_{II} + \sin r\,|1\rangle_I |1\rangle_{II} \end{cases}$$

Bosons

Fermions

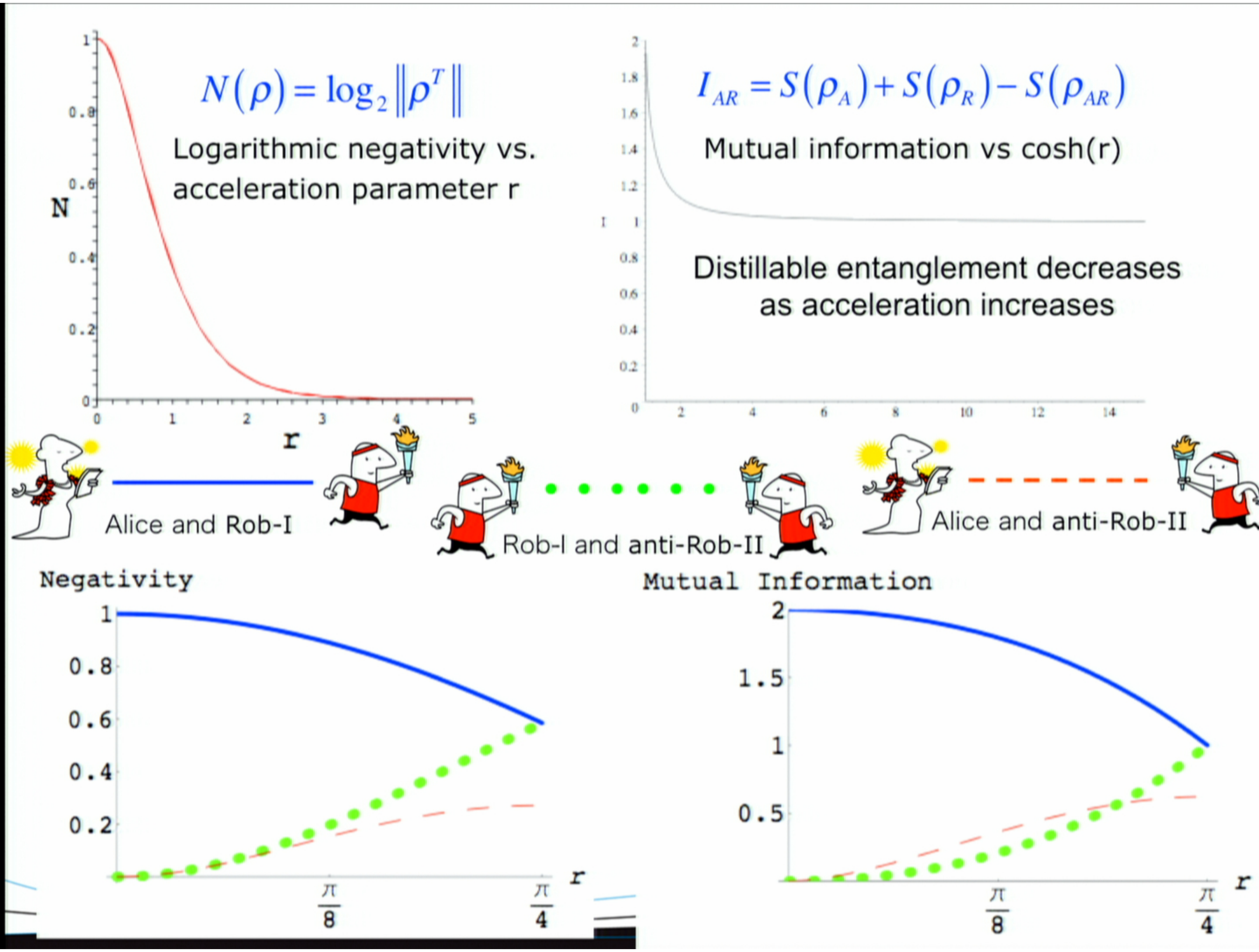
$$\cos r = \frac{1}{\sqrt{1+\exp(-2\pi\Omega)}}$$

$$\Omega = \frac{\omega c}{a}$$



Rob is causally disconnected from region II

Unruh effect!


$N(\rho) = \log_2 \|\rho^T\|$
Logarithmic negativity vs.
acceleration parameter r
N
r
$I_{AR} = S(\rho_A) + S(\rho_R) - S(\rho_{AR})$
Mutual information vs cosh(r)
Distillable entanglement decreases
as acceleration increases
Alice and Rob-I
Rob-I and anti-Rob-II
Alice and anti-Rob-II
Negativity
Mutual Information
r
$\frac{\pi}{8}$
$\frac{\pi}{4}$

How Robust is this Difference?

- Does the surviving entanglement depend on the choice of maximally entangled state?
  - No
- Does spin matter?
  - Yes, but only quantitatively -- entanglement degradation is greater but still does not vanish in the large acceleration limit
- Does the number of excited modes matter?
  - No for fermions -- entanglement degradation is still limited for fermions
  - Yes (weakly) for bosons -- entanglement loss depends on the number of modes occupied
- Does statistics matter?
  - Yes!

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Leon/Martin-Martinez
Phys. Rev. A 81, 052305 (2010)

Relativistic Tripartite Entanglement

- Bosonic
  - Tripartite Entanglement remains finite in the infinite acceleration limit
    - 1-tangles vanish in this limit
    - pi-tangle remains finite in this limit

Hwang/Park/Jung
PRA 82 012111 (2010)

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Relativistic Tripartite Entanglement

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Hwang/Park/Jung
PRA 82 012111 (2010)

- Fermionic
  - Tripartite Entanglement remains finite in the infinite acceleration limit
    - 1-tangles remain finite in this limit
    - pi-tangle remains finite in this limit
  - No bipartite entanglement generated from acceleration

Wang/Jing
PRA 83 022314 (2011)

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Measures of Tripartite Entanglement

3-tangle

Coffman/Kundu/ Wootters
PRA61 052396 (2000)
Wootters PRL**80** 2245 (1998)

$$\tau(\psi) = \mathcal{C}^2_{A(BC)} - \mathcal{C}^2_{A(B)} - \mathcal{C}^2_{B(C)}$$

$$\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

$$\lambda_i \geq \lambda_{i+1} \geq 0$$

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$$\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$ where λ's are the eigenvalues of $\sqrt{\tilde{\rho}\rho}$

$$\lambda_i \geq \lambda_{i+1} \geq 0 \qquad \tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

Problem: Only works for pure states

Ou/Fan PRA**75** 062308 (2007)

Pi-tangle

$$\pi = \frac{\pi_A + \pi_B + \pi_C}{3}$$

$$\mathcal{N}^2_{A(BC)} = \left\|\rho^{T_A}_{ABC}\right\| - 1 \qquad \text{1-tangle}$$

$$\pi_A(\psi) = \mathcal{N}^2_{A(BC)} - \mathcal{N}^2_{A(B)} - \mathcal{N}^2_{B(C)}$$

$$\mathcal{N}^2_{A(B)} = \left\|\left(\mathrm{Tr}_C[\rho_{ABC}]\right)^{T_A}\right\| - 1 \qquad \text{2-tangle}$$

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Measures of Tripartite Entanglement

3-tangle

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Ou/Fan PRA**75**
062308 (2007)

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$$\mathcal{N}^2_{A(BC)} = \left\|\rho^{T_A}_{ABC}\right\| - 1$$ 1-tangle

$$\mathcal{N}^2_{A(B)} = \left\|\left(\mathrm{Tr}_C[\rho_{ABC}]\right)^{T_A}\right\| - 1$$ 2-tangle

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Measures of Tripartite Entanglement

3-tangle

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PRA61 052396 (2000)
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$$\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \quad \text{where } \lambda\text{'s are the eigenvalues of } \sqrt{\tilde{\rho}\rho}$$

$$\lambda_i \geq \lambda_{i+1} \geq 0 \qquad \tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

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062308 (2007)

Pi-tangle

$$\pi = \frac{\pi_A + \pi_B + \pi_C}{3} \qquad \mathcal{N}^2_{A(BC)} = \left\|\rho^{T_A}_{ABC}\right\| - 1 \quad \text{1-tangle}$$

$$\pi_A(\psi) = \mathcal{N}^2_{A(BC)} - \mathcal{N}^2_{A(B)} - \mathcal{N}^2_{B(C)} \qquad \mathcal{N}^2_{A(B)} = \left\|\left(\mathrm{Tr}_C[\rho_{ABC}]\right)^{T_A}\right\| - 1 \quad \text{2-tangle}$$

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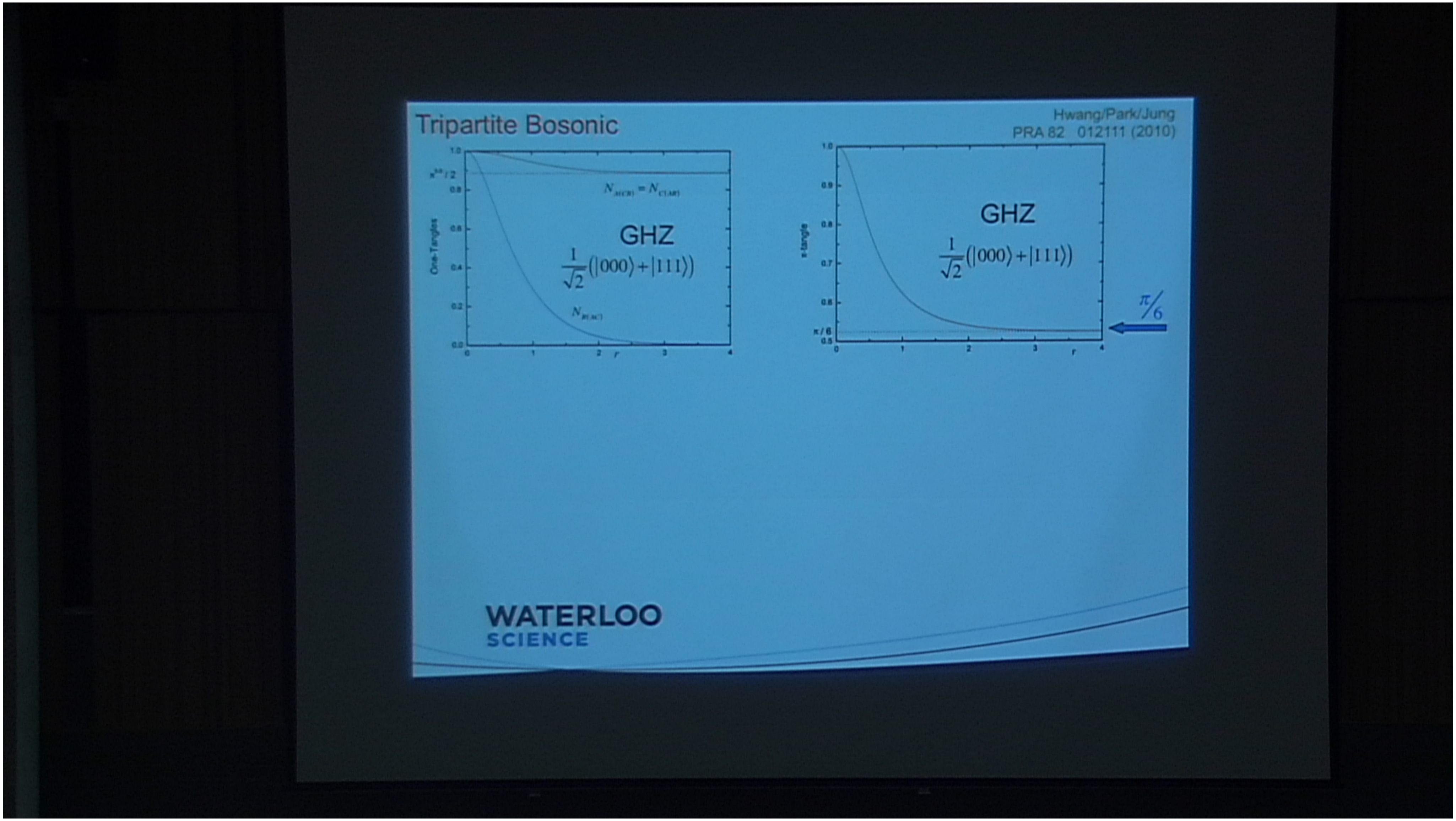
Tripartite Bosonic
Hwang/Park/Jung
PRA 82 012111 (2010)
GHZ
$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$
$N_{A(CB)} = N_{C(AB)}$
$N_{B(AC)}$
One-Tangles
GHZ
$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$
$\pi/6$
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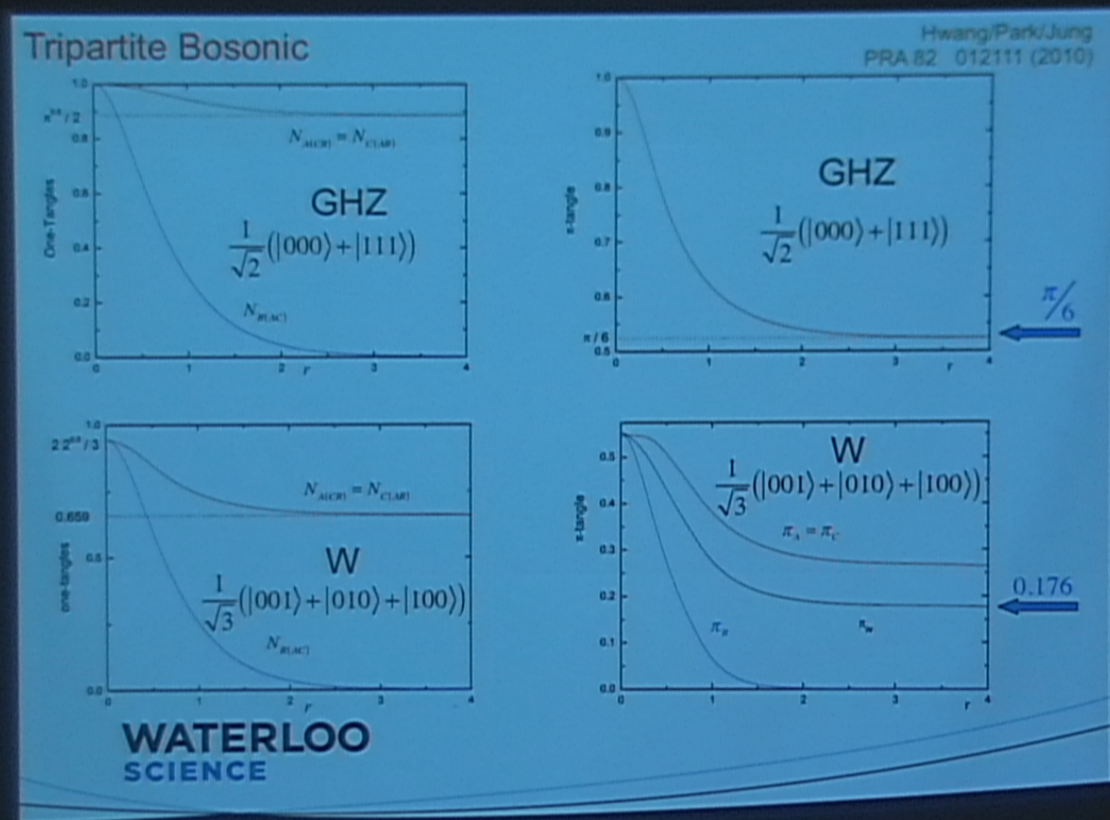

Tripartite Bosonic
Hwang/Park/Jung
PRA 82 012111 (2010)
GHZ
$\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)$
$N_{A(CB)}=N_{C(AB)}$
$N_{B(AC)}$
One-Tangles
$\pi/6$
W
$\frac{1}{\sqrt{3}}(|001\rangle+|010\rangle+|100\rangle)$
0.659
one-tangles
π-tangle
$\pi_A=\pi_C$
π_B
0.176
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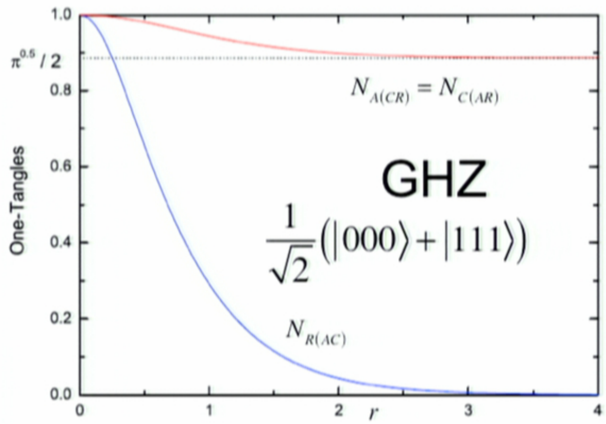
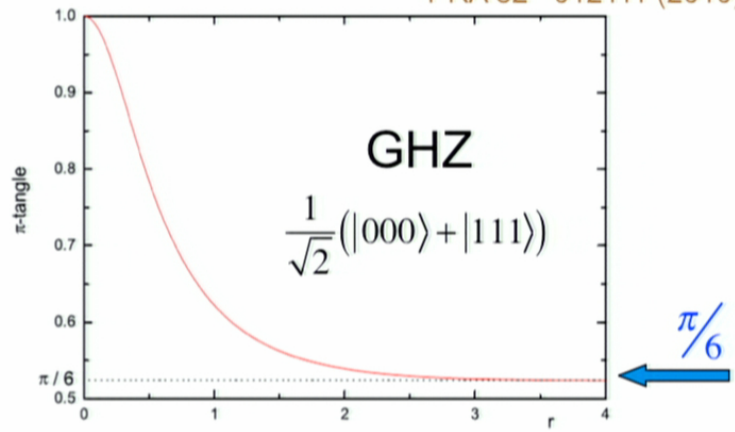
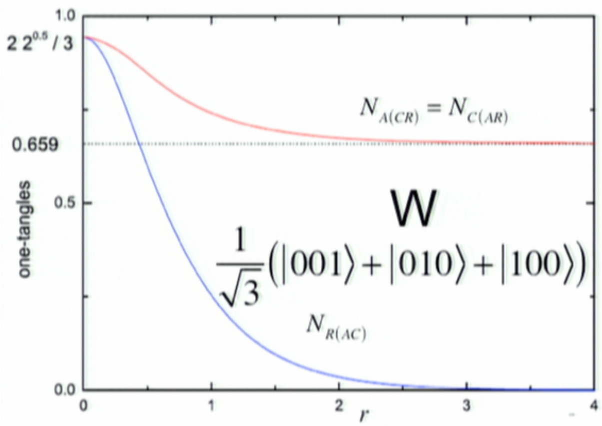
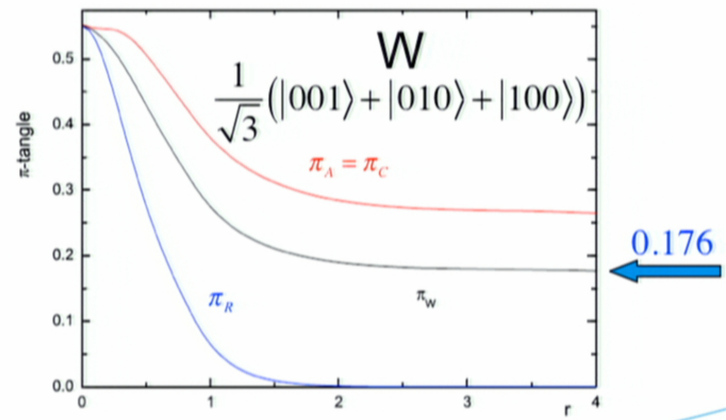
Tripartite Bosonic

Hwang/Park/Jung
PRA 82 012111 (2010)









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Generalized Tripartite States

GGHZ state: $|\psi_g\rangle = \cos\theta_1|000\rangle + \sin\theta_1|111\rangle$

3-tangle:

MS state: $|\psi_{MS}\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |11\rangle\left(\cos\theta_3|0\rangle + \sin\theta_3|1\rangle\right)\right)$

3-tangle:

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Generalized Tripartite States

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3-tangle:

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Bell and Svetlichny Inequalities

Bell Phys **1** 135 (1964)

Bell Inequality $\left|\left\langle A\left(B+B'\right)+A'\left(B-B'\right)\right\rangle\right|\leq 2$

- All pure bipartite entangled states violate Bell/CHSH
- Amount of violation increases with increasing entanglement

Svetlichny Inequality

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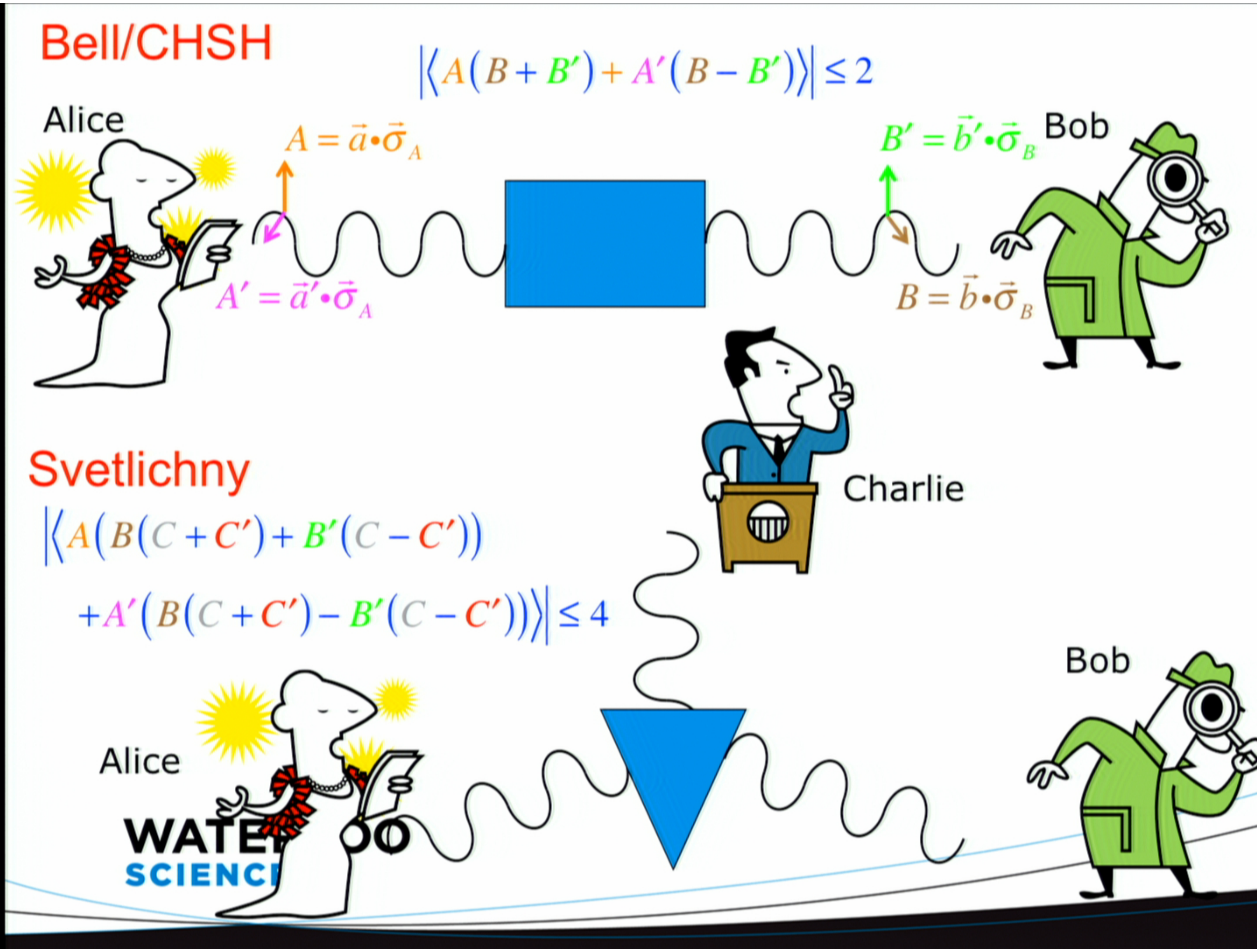
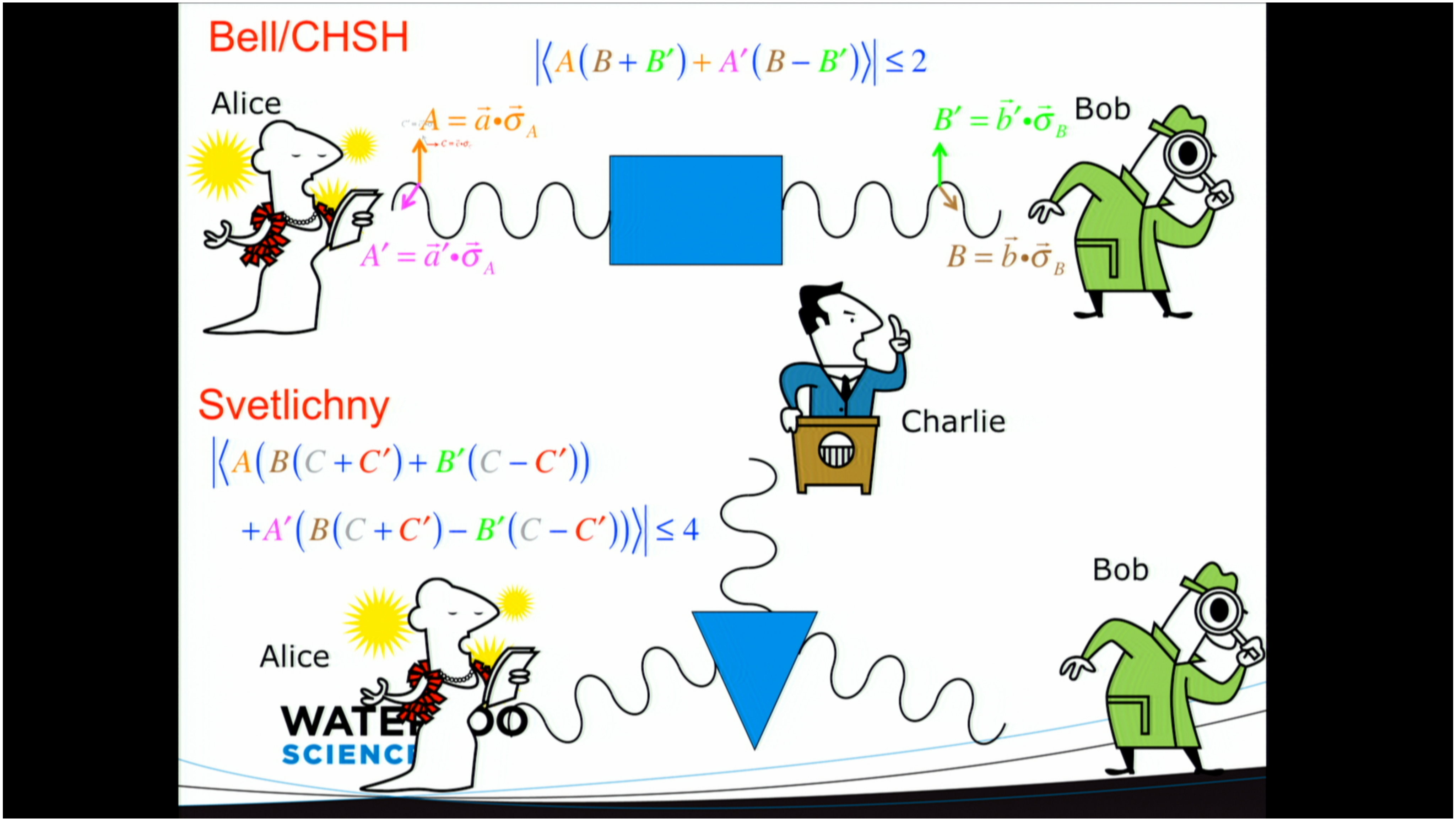
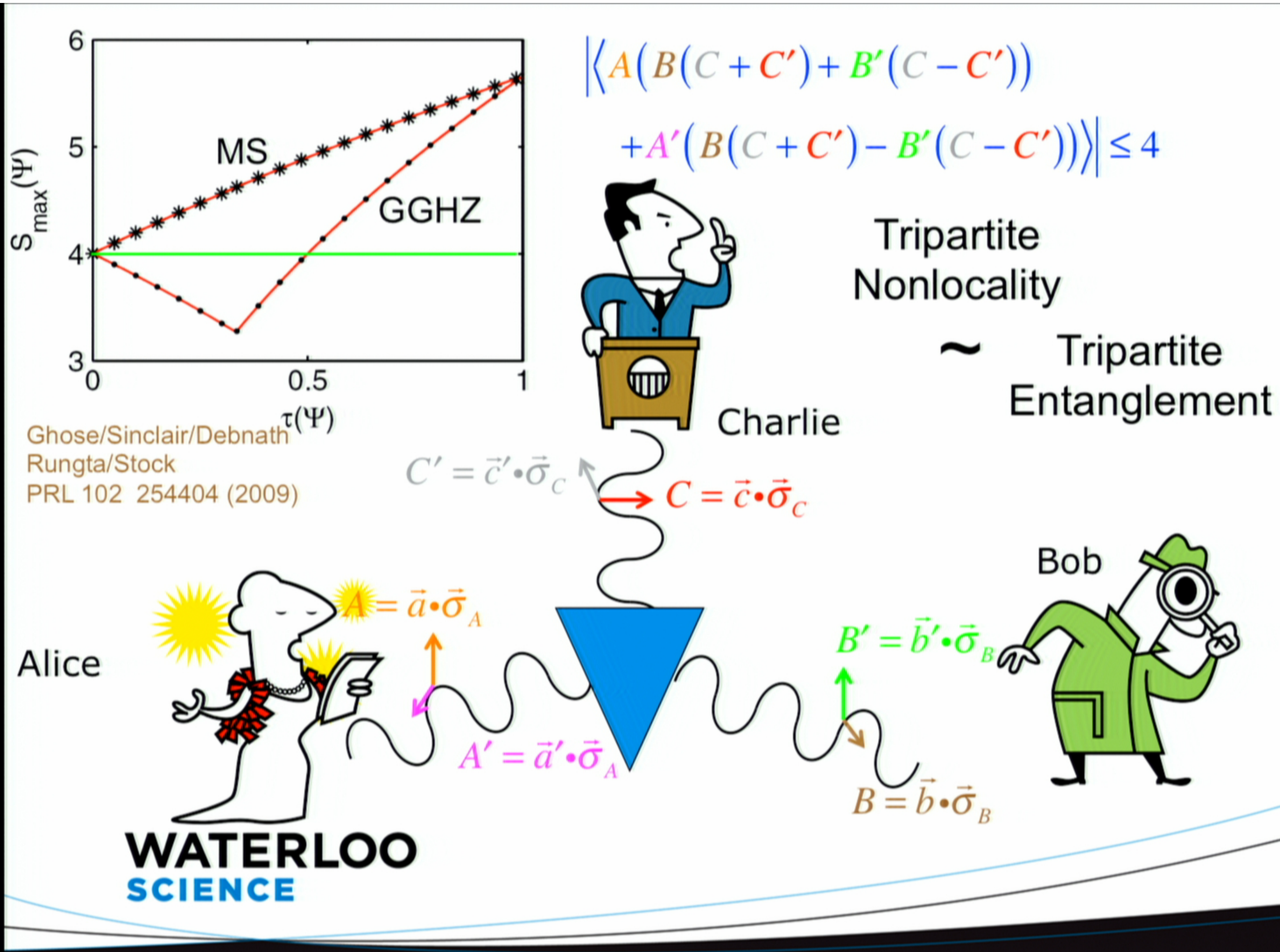

Bell/CHSH
$\left|\left\langle A(B+B')+A'(B-B')\right\rangle\right|\leq 2$
Alice
$A=\vec{a}\cdot\vec{\sigma}_A$
$A'=\vec{a}'\cdot\vec{\sigma}_A$
$B'=\vec{b}'\cdot\vec{\sigma}_B$
Bob
$B=\vec{b}\cdot\vec{\sigma}_B$
Charlie
Svetlichny
$\left|\left\langle A\left(B(C+C')+B'(C-C')\right)\right.\right.$
$\left.\left.+A'\left(B(C+C')-B'(C-C')\right)\right\rangle\right|\leq 4$
Bob
Alice


Bell/CHSH
$\left|\left\langle A(B+B')+A'(B-B')\right\rangle\right| \le 2$
Alice
$A=\vec{a}\bullet\vec{\sigma}_A$
$A'=\vec{a}'\bullet\vec{\sigma}_A$
Bob
$B'=\vec{b}'\bullet\vec{\sigma}_B$
$B=\vec{b}\bullet\vec{\sigma}_B$
Charlie
Svetlichny
$\left|\left\langle A\left(B(C+C')+B'(C-C')\right)\right.\right.$
$\left.\left.+A'\left(B(C+C')-B'(C-C')\right)\right\rangle\right| \le 4$
Alice
Bob



6
5
4
3
0
0.5
1
$S_{max}(\Psi)$
$\tau(\Psi)$
MS
GGHZ
Ghose/Sinclair/Debnath
Rungta/Stock
PRL 102 254404 (2009)
$\left|\left\langle A\left(B(C+C')+B'(C-C')\right)\right.\right.$
$\left.\left.+A'\left(B(C+C')-B'(C-C')\right)\right\rangle\right|\leq 4$
Tripartite
Nonlocality
~
Tripartite
Entanglement
Charlie
$C'=\vec{c}'\bullet\vec{\sigma}_C$
$C=\vec{c}\bullet\vec{\sigma}_C$
Bob
Alice
$A=\vec{a}\bullet\vec{\sigma}_A$
$A'=\vec{a}'\bullet\vec{\sigma}_A$
$B'=\vec{b}'\bullet\vec{\sigma}_B$
$B=\vec{b}\bullet\vec{\sigma}_B$
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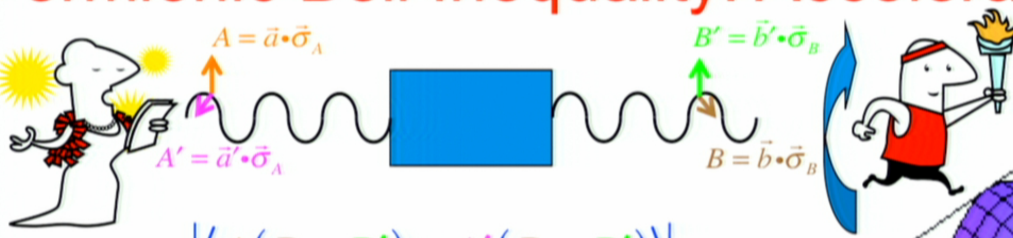
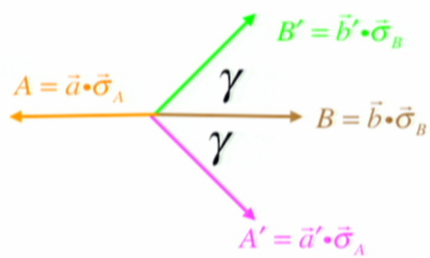
Fermionic Bell Inequality: Accelerated Observers



$$\left| \left\langle A(B+B') + A'(B-B') \right\rangle \right| :$$

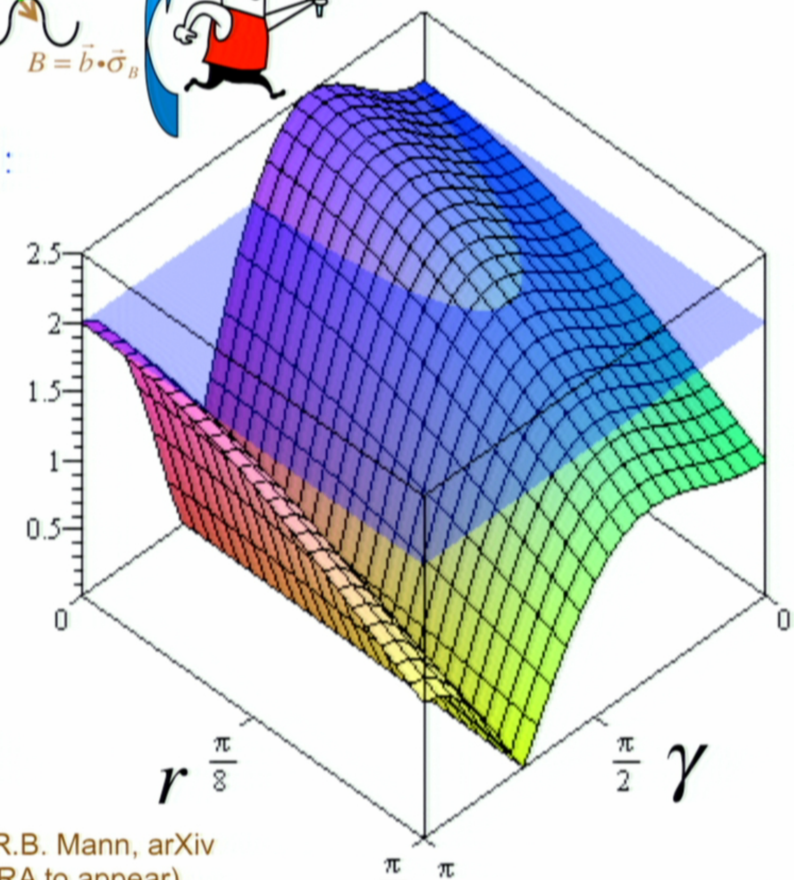
Singlet $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$

choose



$$\left| \left\langle A(B+B') + A'(B-B') \right\rangle \right|$$

$$= \frac{1}{2}(1+\cos 2r)\left|1 + 2\cos\gamma - \cos 2\gamma\right|$$



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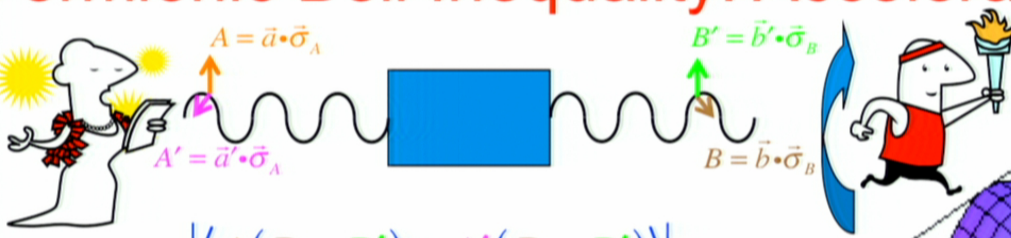
A. Smith and R.B. Mann, arXiv 1107.4633 (PRA to appear)

See also N. Friis, P. Kohler, E. Martin-Martinez, R Bertleman, Phys. Rev. A 84, 062111 (2011)

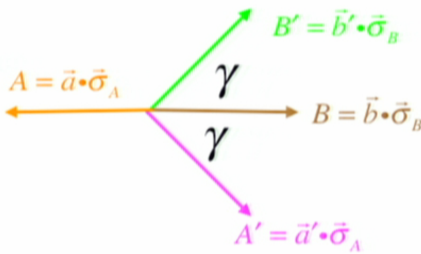
Fermionic Bell Inequality: Accelerated Observers

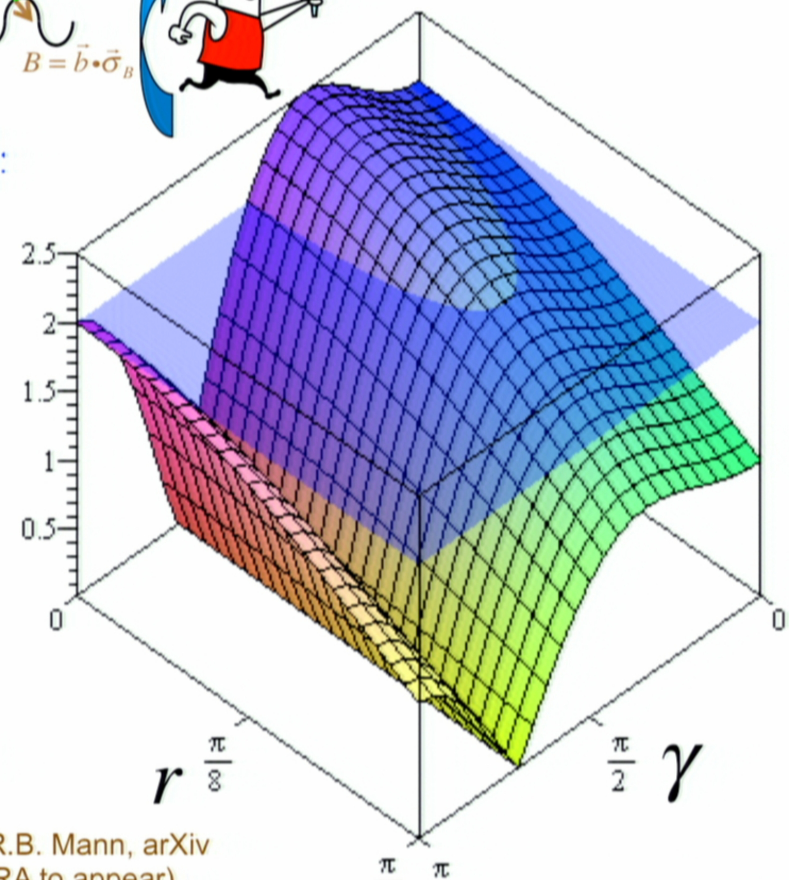


$$\left| \left\langle A(B+B') + A'(B-B') \right\rangle \right| :$$

Singlet $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$

choose





$$\left| \left\langle A(B+B') + A'(B-B') \right\rangle \right|$$

$$= \frac{1}{2}(1+\cos 2r)\left|1 + 2\cos\gamma - \cos 2\gamma\right|$$

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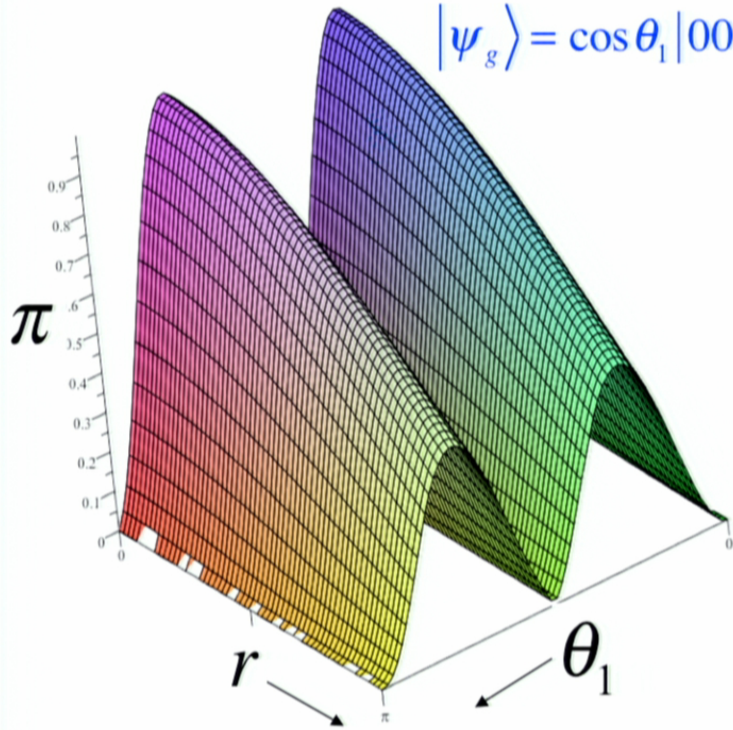
Fermionic GGHZ Entanglement and Nonlocality

$$|\psi_g\rangle = \cos\theta_1 |000\rangle + \sin\theta_1 |111\rangle$$



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A. Smith and R.B. Mann, arXiv 1107.4633 (PRA to appear)

Fermionic MS Entanglement

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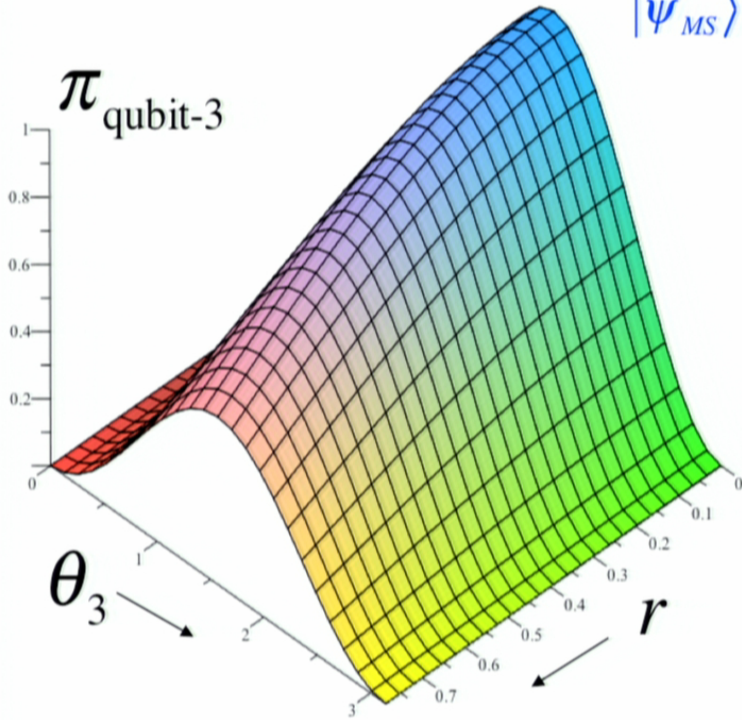
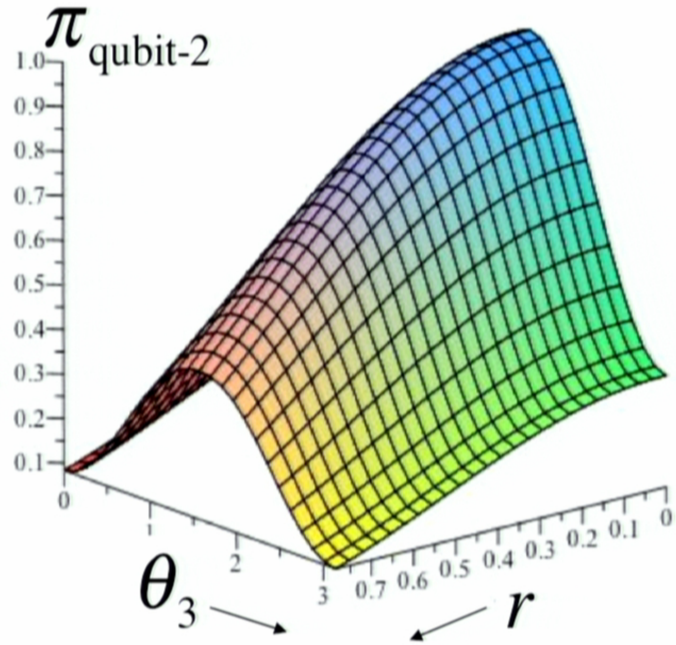


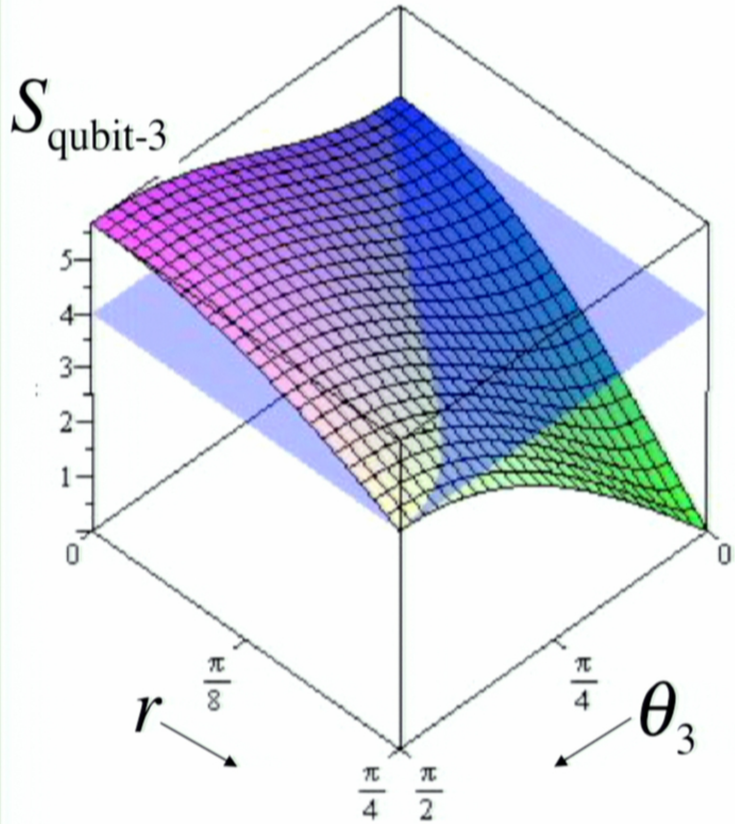
Always finite in the infinite acceleration limit

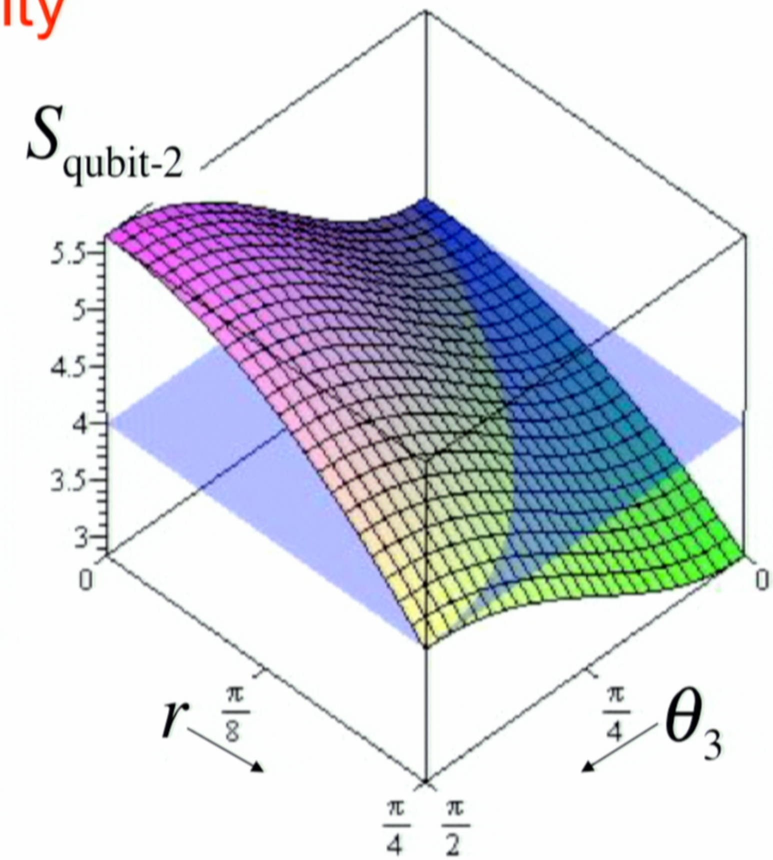


Always finite in the infinite acceleration limit

WATERLOO SCIENCE

Fermionic MS Non-locality





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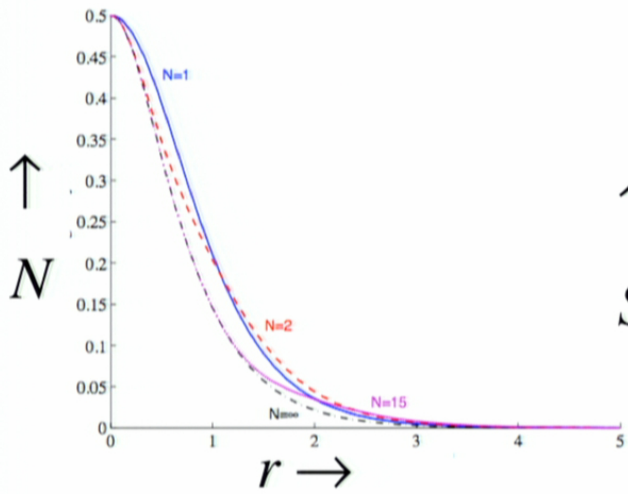
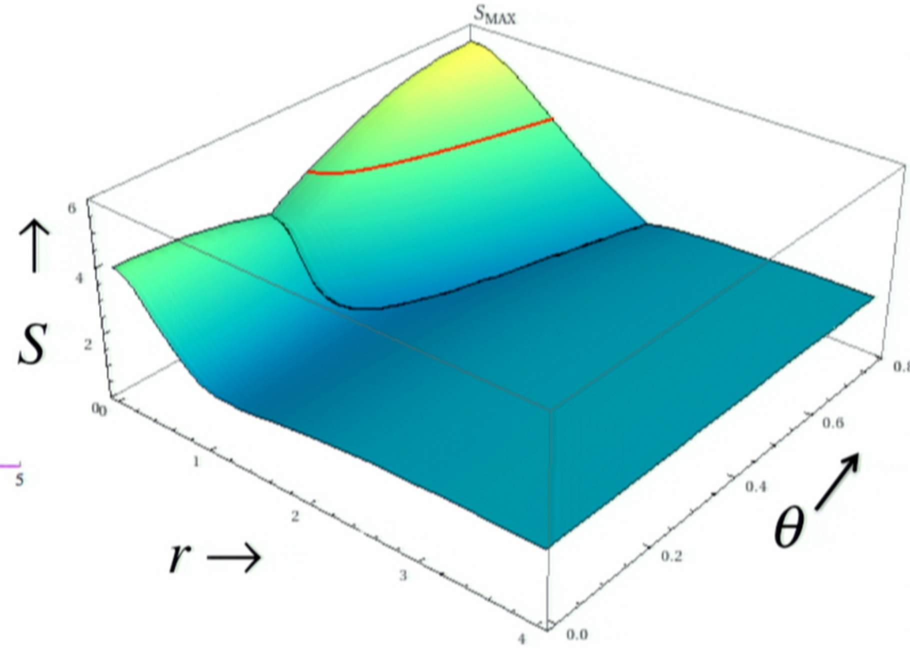
SCIENCE

Bosonic GGHZ Nonlocality

- Infinite number of mode excitations
- Truncate to a 2-level system → expect results to be qualitatively similar to actual case





J. Leon, E. Martin-Martinez
Phys.Rev.A81:032320,2010

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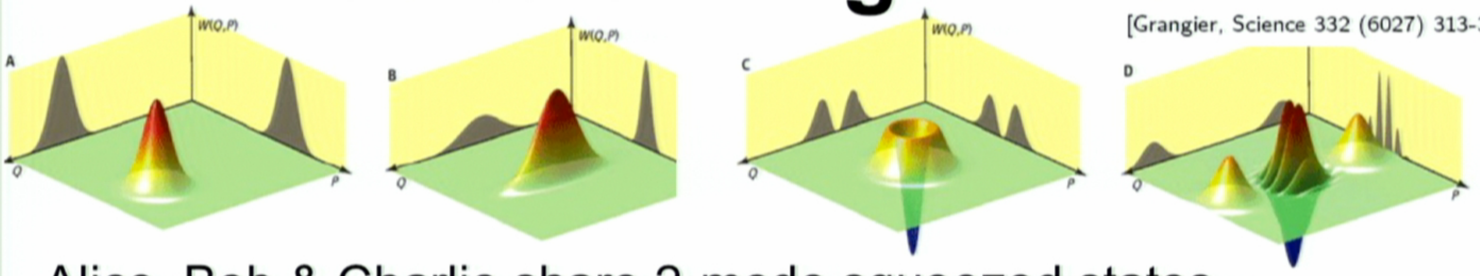
SCIENCE

P. Corona-Ugalde, E. Martin-Martinez, R. Mann (in progress)

Tripartite Bosonic Squeezed State Entanglement



[Grangier, Science 332 (6027) 313-314]

Alice, Bob & Charlie share 2-mode squeezed states

$$\left[a_i, a_j^\dagger\right] = \delta_{ij} \qquad \left|\psi_{sq}\right\rangle_{ij} = U(s)|0\rangle_i \otimes |0\rangle_j \qquad U(s) = \exp\left[-\frac{s}{2}\left(a_i^\dagger a_j^\dagger - a_i a_j\right)\right]$$

$$\Updownarrow$$

$$\left[\hat{X}_i, \hat{X}_j\right] = 2i\Omega_{ij}$$

$$\hat{X} = \left\{x_1, p_1, x_2, p_2, \ldots x_N, p_N\right\}$$

$$\sigma + i\Omega \geq 0$$

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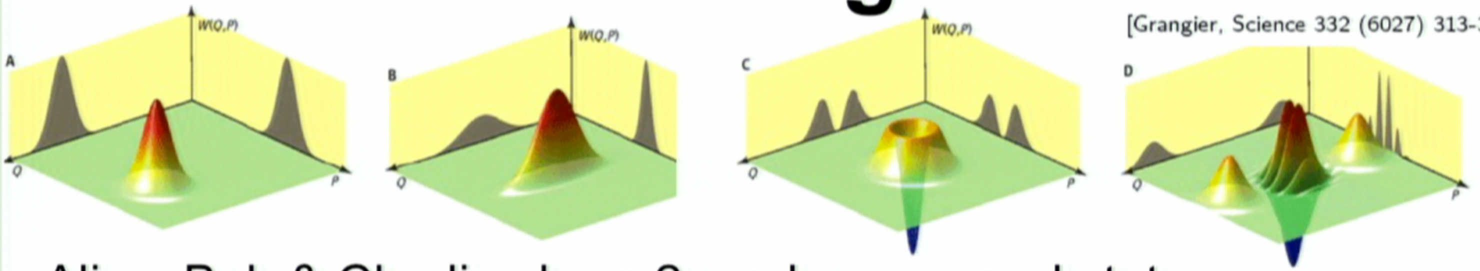
Tripartite Bosonic Squeezed State Entanglement

[Grangier, Science 332 (6027) 313-314]



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$$\Updownarrow \qquad \Updownarrow \qquad \Updownarrow$$

$$\left[\hat{X}_i, \hat{X}_j\right] = 2i\Omega_{ij} \qquad \sigma = S(s)\mathbb{I}S^T(s) \qquad S(s) = \begin{pmatrix} \cosh s & 0 & \sinh s & 0 \\ 0 & \cosh s & 0 & -\sinh s \\ \sinh s & 0 & \cosh s & 0 \\ 0 & -\sinh s & 0 & \cosh s \end{pmatrix}$$

$$\hat{X} = \{x_1, p_1, x_2, p_2, \ldots x_N, p_N\}$$

$$\sigma + i\Omega \geq 0$$

WATERLOO SCIENCE

Bipartite case

- Entanglement vanishes between lowest frequency modes

$$\sigma_{AB} = S_{AB}\mathbb{I}_4 S_{AB}^T$$

$$S_{AB} = S(B)S(A)$$

$$\sigma_{AB} = S_{R\bar{R}} S_{AR}\mathbb{I}_4 S_{R\bar{R}}^T$$

$$S_{AR} = S(R)S(A) \oplus \mathbb{I}_{\bar{R}}$$

$$S_{R\bar{R}} = \mathbb{I}_A \oplus S(\bar{R})S(R)$$

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SCIENCE

Bipartite case

$$\sigma_{AB} = S_{AB}\mathbb{I}_4 S_{AB}^T$$

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- Inertial entanglement redistributed into multipartite quantum correlations amongst accessible and inaccessible modes

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$$S_{R\bar{R}} = \mathbb{I}_A \oplus S(\bar{R})S(R)$$

- One accelerated observer
  - Entanglement redistributed in tripartite correlations
  - Classical correlations unaffected in tripartite correlations
- Two accelerated observers
  - Entanglement redistributed in quatrapartite correlations
  - Entanglement vanishes completely at finite acceleration
  - Classical correlations degraded

Adesso/Fuentes/Ericsson
PRA **76** 062112 (2007)

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SCIENCE

Bipartite case

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Adesso/Fuentes/Ericsson
PRA **76** 062112 (2007)

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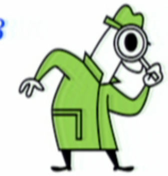
Tripartite case

Inertial

$$\sigma_{ACB} = S_{CB} S_{AC} \mathbb{I}_6 S_{AC}^T S_{CB}^T$$



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SCIENCE

Quantifying Entanglement

Residual Entanglement

CKW Inequality $$E\left(\sigma_{I|(\cdots)}\right)=\sum_{K\neq I}^{N}\sigma_{I|K} \Rightarrow E\left(\sigma_{I|(\cdots)}\right)-\sum_{K\neq I}^{N}E\left(\sigma_{I|K}\right)=\Upsilon\left(\sigma_{I}\right)$$

Adesso/Illuminati
PRA **72** 032334 (2005)
NJ Phys 8 15 (2006)

Contangle

$$\tau\left(\sigma_{I|J}\right)=g\left(m_{I|J}^{2}\right)$$

$$g(x)=\left(\sinh^{-1}\left(\sqrt{x-1}\right)\right)^{2}$$

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Quantifying Entanglement

Residual Entanglement

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Adesso/Illuminati
PRA **72** 032334 (2005)
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Contangle

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$$g(x)=\left(\sinh^{-1}\left(\sqrt{x-1}\right)\right)^{2}$$

Residual Contangle

$$\tau_r=\min_{I,\cdots}\left[\tau\left(\sigma_{I|(\cdots)}\right)-\sum_{K\neq I}^{N}\tau\left(\sigma_{I|K}\right)\right]$$

$$m_{I|J}=m\left(\sigma_{I|J}^{\mathrm{opt}}\right)=\sqrt{\det\sigma_{I}^{\mathrm{opt}}}=\sqrt{\det\sigma_{J}^{\mathrm{opt}}}$$

$$m\left(\sigma_{I|J}^{\mathrm{opt}}\right)=1 \Longleftarrow \text{separable}$$

$$\sigma_{I|J}^{\mathrm{opt}}=\min\left\{\sigma_{I|J}^{\mathrm{p}}\,\middle|\,\sigma_{I|J}^{\mathrm{p}}\leq\sigma_{I|J}\right\}$$

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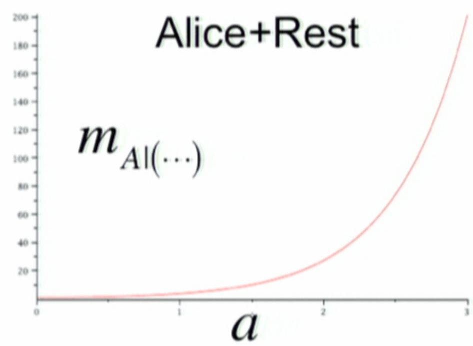
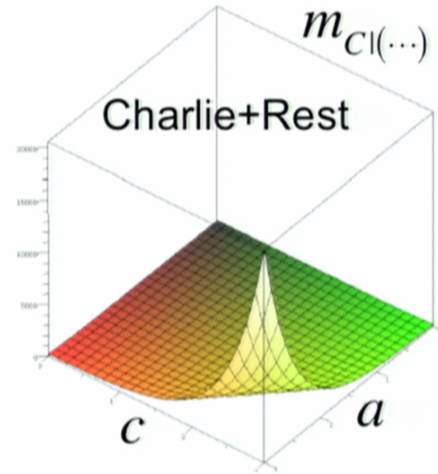
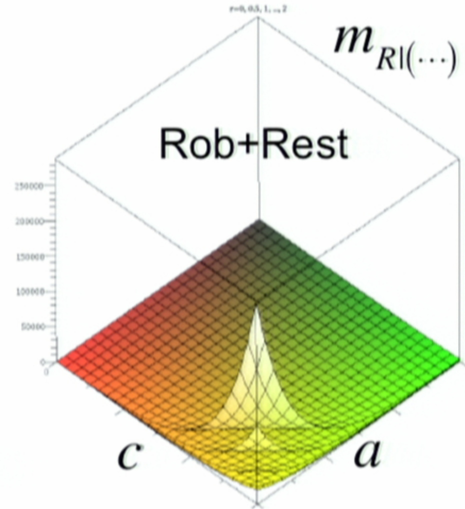
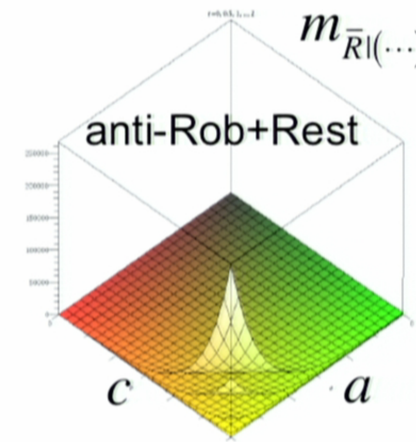
1×(Rest)









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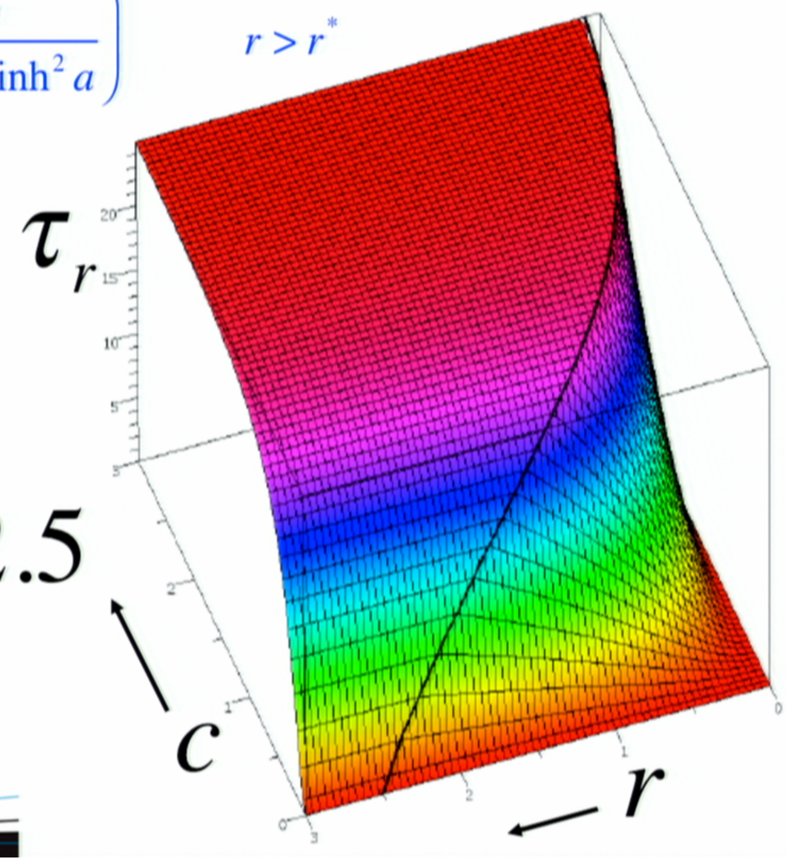
Residual Quatra-partite Entanglement

$$\tau_r = \begin{cases} \left[\sinh^{-1}\left(\sqrt{\left(2(\cosh a \sinh c \sinh r)^2 + \cosh 2r\right)^2 - 1}\right)\right]^2 - 4r^2 & r < r^* \\ 4a^2 - \sinh^{-1}\left(\dfrac{\cosh c \sinh 2a}{(\cosh c \cosh a)^2 - \sinh^2 a}\right) & r > r^* \end{cases}$$

$$\cosh r^* = \frac{\cosh c \cosh a}{(\sinh c \cosh a)^2 + 1}$$

Saturates at finite r

$$a = 2.5$$



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Summary

- Relativistic Tripartite Entanglement
  - degrades less than bipartite entanglement
  - Residual contangle saturates at finite acceleration
- Relativistic Tripartite Nonlocality
  - Correlated with Tripartite Entanglement
  - Persists to arbitrarily large accelerations for fermions, unlike its bipartite counterpart
  - (Truncated) bosonic non-locality vanishes for finite accelerations

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