Title: A Fully-Relativistic Bandlimit on Quantum Fields' Two-Point Correlation Functions

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Abstract: The bridge between continuous information and discrete information is provided by sampling theory. In this talk, I will discuss an application of covariant sampling theory to cosmology (see the previous talk by Dr. R. Martin). In cosmology, the two-point correlation function of a quantum field is of central importance because it is a measure of the size of the fluctuations of the quantum field and of the entanglement of the vacuum in a given spacetime. Furthermore, the two-point function is experimentally accessible through the cosmic microwave background. Using covariant sampling theory, I will show how an information-theoretic bandlimit imposed at the Planck scale manifests itself in the two-point function. We will examine this bandlimit in Minkowski space and in de Sitter space.

A Fully-Relativistic Bandlimit on Quantum Fields' Two-Point Correlation Functions

Aidan Chatwin-Davies [1] Achim Kempf [1] Robert Martin [2]

[1]University of Waterloo

[2]University of Cape Town

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Introduction

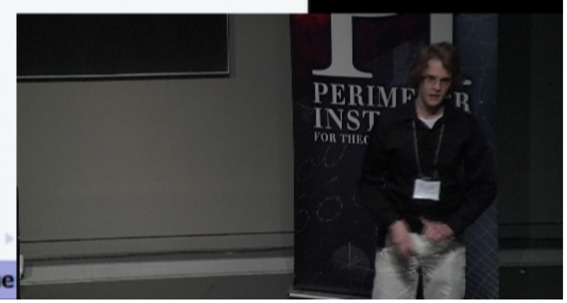
Q: Is there a covariant notion of a minimum length?

A: Yes - formulate it using ideas from information theory.

- Idea: minimum length scale $\rightarrow$ bandlimited fields in nature
- Recall,

$$f(x) \text{ bandlimited} \iff \hat{F}(k) \in \mathcal{L}^2[-\Omega^2, \Omega^2] \tag{1}$$

- What is a covariant bandlimit?



A Bandlimit on Spacetime

$f(x)$ bandlimited $\iff$ $f(x)$ spanned by eigenfunctions of $-\frac{\partial^2}{\partial x^2}$ with eigenvalues $\in [0, \Omega^2]$.

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\Omega}^{\Omega} e^{ikx}\, \hat{F}(k)\, dk \qquad (2)$$

- Idea: require $\mathrm{spec}\{\Box\} \in [-\Omega^2, \Omega^2]$
- Q: What does this mean physically, and how do we implement it?

Application to Cosmology

Model: massive scalar field $\hat{\phi}(x)$ on curved spacetime background.

- Consider the *Two-Point Function*

$$G_F(x, x') := \frac{\int \phi(x)\phi(x') e^{iS[\phi]}\, \mathcal{D}[\phi]}{\int e^{iS[\phi]}\, \mathcal{D}[\phi]} \tag{3}$$

- Importance: measures quantum fluctuation spectrum of $\hat{\phi}$

$$G_F(t = t', \vec{k}) \sim \delta\phi_k(t) \tag{4}$$

- Can measure in the CMB.

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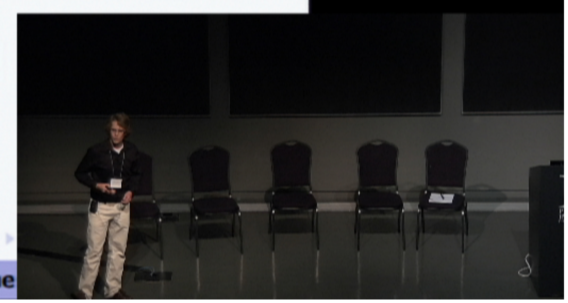
$$G_F(t=t',\vec{k}) \sim \delta\phi_k(t) \qquad (4)$$

- Can measure in the CMB.
- Cutoff: restriction on fields integrated over in the path integral

$$G_F{}^{c}(x,x') := \frac{\int \phi(x)\phi(x')e^{iS[\phi]}\,\mathcal{D}^c[\phi]}{\int e^{iS[\phi]}\,\mathcal{D}^c[\phi]}$$



Minkowski Space (No Bandlimit)

$G_F(x, x')$ obeys

$$(\Box_x + m^2)G_F(x - x') = -i\delta^4(x - x') \tag{6}$$

- Fourier transform w.r.t. $x - x'$:

$$G_F(p) = \frac{i}{(2\pi)^2} \frac{1}{p_0^2 \underbrace{-|\vec{p}\,|^2 - m^2}_{:= -\omega^2} + i\epsilon} \tag{7}$$

Then,

$$\boxed{G_F(t - t', \vec{p}\,) = \frac{i}{(2\pi)^{5/2}} \int_{-\infty}^{\infty} dp_0 \, \frac{e^{ip_0(t-t')}}{p_0^2 - \omega^2 + i\epsilon}}$$

The Covariant Bandlimit

- Assuming a covariant bandlimit, how does the calculation change?
- Eigenfunctions of $\Box$:

$$\Box\, e^{ip\cdot x} = \left(\frac{\partial^2}{\partial t^2} - \triangle\right) e^{ip_0 t - i\vec{p}\cdot\vec{x}} = (-p_0^2 + |\vec{p}\,|^2)e^{ip\cdot x} \qquad (9)$$

- So, a bandlimit necessitates

$$\left|p_0^2 - |\vec{p}\,|^2\right| \leq \Omega^2 \qquad (10)$$

$$G_F \quad \rightarrow \quad G_F^c(t-t',\vec{p}\,) = \frac{i}{(2\pi)^{5/2}} \int_{\mathcal{I}} dp_0 \, \frac{e^{ip_0(t-t')}}{p_0^2 - \omega^2 + i\epsilon} \qquad (11)$$

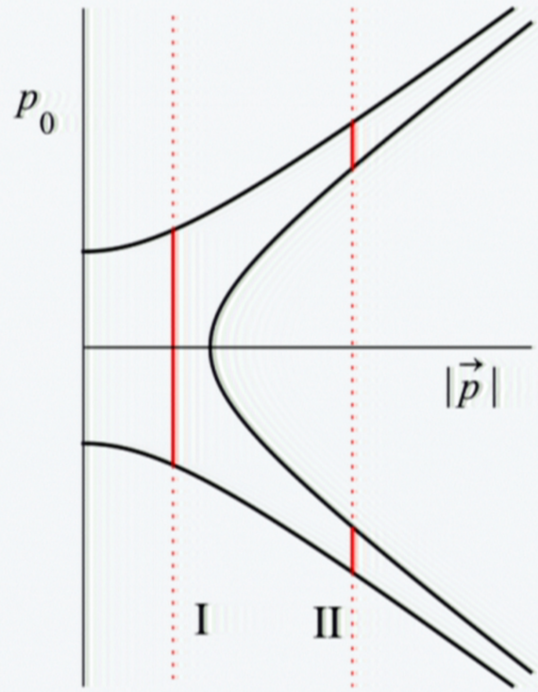
Minkowski Space (With Bandlimit)

$$G_F^c(t-t',\vec{p}) = \frac{i}{(2\pi)^{5/2}}\int_{\mathcal{I}} dp_0\, \frac{e^{ip_0(t-t')}}{p_0^2 - |\vec{p}\,|^2 - m^2 + i\epsilon}$$

Bandlimit

$\Downarrow$

$$\left|p_0^2 - |\vec{p}\,|^2\right| \leq \Omega^2$$



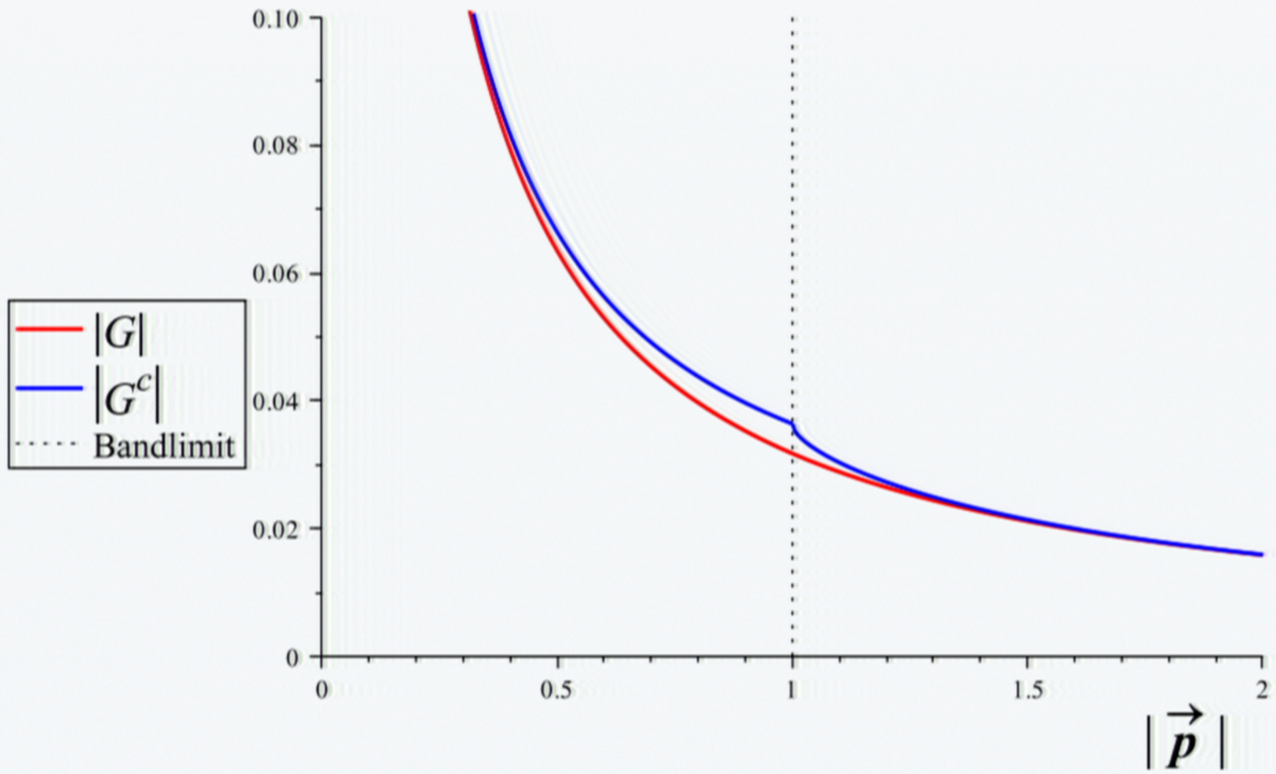
Comparison

$$G_F(t=t',\vec{p}\,) = \frac{1}{(2\pi)^{3/2}}\frac{1}{2\omega}$$

$$G_F^c(t=t',\vec{p}\,) = \begin{cases} \frac{1}{(2\pi)^{3/2}}\frac{1}{2\omega} - \frac{i}{(2\pi)^{5/2}}\frac{1}{\omega}\ln\left|\frac{B+\omega}{B-\omega}\right| & |\vec{p}\,| \le \Omega \\ \frac{1}{(2\pi)^{3/2}}\frac{1}{2\omega} - \frac{i}{(2\pi)^{5/2}}\frac{1}{\omega}\left(\ln\left|\frac{B+\omega}{B-\omega}\right| - \ln\left|\frac{\omega+b}{\omega-b}\right|\right) & |\vec{p}\,| > \Omega \end{cases}$$

where $\quad \omega = \sqrt{|\vec{p}\,|^2 + m^2} \qquad B = \sqrt{|\vec{p}\,|^2 + \Omega^2} \qquad b = \sqrt{|\vec{p}\,|^2 - \Omega^2}$

Comparison



FLRW Cosmology

- Current (& future) work: expanding spacetime
- E.g. de Sitter space, flat slicing, conformal time:

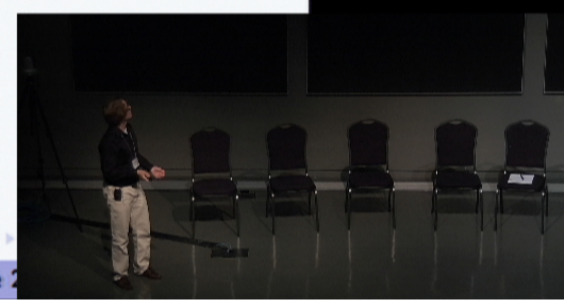
$$ds^2 = a^2(\eta)[dt^2 - d\vec{x}^2] \qquad ; \qquad a(\eta) = \frac{1}{H\eta} \tag{12}$$

- Equation of Motion:

$$(\Box_k + m^2)G_F(\eta, \eta', \vec{k}) = -\frac{i}{(2\pi)^{3/2}}\delta(\eta - \eta') \tag{13}$$

where

$$\Box_k = H^2\eta^2\frac{\partial^2}{\partial\eta^2} - 2H^2\eta\frac{\partial}{\partial\eta} + H^2\eta^2k^2$$



Two-Point Function in De Sitter Space

$$(\Box_k + m^2) G_F(\eta, \eta', \vec{k}) = -\frac{i}{(2\pi)^{3/2}} \delta(\eta - \eta')$$

- Idea: diagonalize $(\Box_k + m^2)$, i.e. find $(\Box_k + m^2)|\lambda\rangle = \lambda|\lambda\rangle$
- Then $(\Box_k + m^2)^{-1} = \sum \lambda^{-1} |\lambda\rangle\langle\lambda|$
- Choosing a basis $\langle\eta|\lambda\rangle := \phi_\lambda(\eta)$,

$$G_F(\eta, \eta', \vec{k}) = \sum_\lambda \frac{1}{\lambda} \phi_\lambda(\eta) \int_0^\infty [\phi_\lambda(\eta'')]^* f(\eta'' - \eta') a^4(\eta'')\, d\eta'' \qquad (15)$$

Approach to Implementing the Bandlimit

$$G_F(\eta, \eta', \vec{k}) = \sum_{\lambda} \frac{1}{\lambda} \phi_\lambda(\eta) \int_0^\infty [\phi_\lambda(\eta'')]^* f(\eta'' - \eta') a^4(\eta'')\, d\eta''$$

- To implement the cutoff, restrict the sum to just those λ such that

$$\lambda - m^2 \in [-\Omega^2, \Omega^2] \tag{16}$$

Issue:

- $\Box_k$ is only symmetric, not self-adjoint
  - need to fix a particular self-adjoint extension

Summary and Outlook

Main Ideas

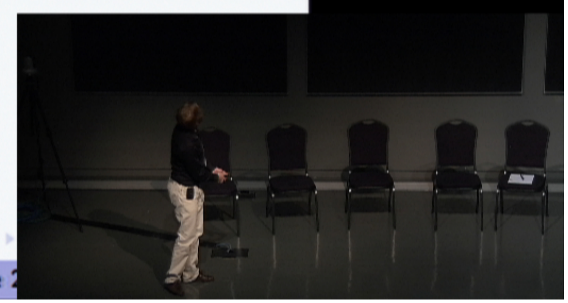
- Minimum length $\rightarrow$ bandlimit $\rightarrow \mathrm{spec}\{\Box\} \in [-\Omega^2, \Omega^2]$
- Interpretation: constrains fields in path integral
- Alters the two-point function in Minkowski space
  - suggests expanding spacetimes altered as well
  - interesting for cosmology

Short-term Outlook

- de Sitter and Power-law expansion

Long-term Outlook

- Backreacting spacetime
- Compare to CMB data



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