Title: Black Holes and Qubits

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Abstract: Two different branches of theoretical physics, string theory and quantum information theory (QIT), share many of the same features, allowing knowledge on one side to provide new insights on the other. In particular the matching of the classification of stringy black holes and the classification of four-qubit entanglement provides a falsifiable prediction in the field of QIT.

Black Holes and Qubits

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June 2012

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- Quantum entanglement lies at the heart of quantum information theory, with applications to quantum computing, teleportation, cryptography and communication. In the apparently separate world of quantum gravity, the Bekenstein-Hawking entropy of black holes has also occupied center stage.
- Here we describe a correspondence between the entanglement measures of qubits in quantum information theory and black hole entropy in string theory.

- Reviewed in Borsten, Dahanayake, Duff, Ebrahim, Rubens: "Black Holes, Qubits and Octonions" Phys. Rep. 471:113-219,2009 arXiv:0809.4685 [hep-th].
  Duff: "Black Holes and Qubits" CERN COURIER May 2010
  Borsten, Duff, Levay: "The Black Hole/Qubit Correspondence: an up-to-date review" arXiv:0809.4685 [hep-th].

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Repurposing string theory

- 1970s Strong nuclear interactions
- 1980s Quantum gravity; "theory of everything"
- 1990s AdS/CFT: QCD (revival of 1970s); quark-gluon plasmas
- 2000s AdS/CFT: superconductors
- 2000s Cosmic strings
- 2000s Fluid mechanics
- 2010s Black hole/qubit correspondence:
  entanglement in Quantum Information Theory
- Conclusion: May be right theory for some but not all

Entanglement measure

- The two qubit system Alice and Bob (where $A, B = 0, 1$) is described by the state

$$|\Psi\rangle = a_{AB}|AB\rangle$$
$$= a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle.$$

- The measure of the bipartite entanglement of Alice and Bob is given by the "two-tangle"

$$\tau_{AB} = 4|\det\, a_{AB}|^2 = 4|a_{00}a_{11} - a_{01}a_{10}|^2$$

- τ_{AB} is invariant under $SL(2)_A \times SL(2)_B$ with a_{AB} transforming as a $(2,2)$, and under a discrete duality that interchanges A and B.
- For normalized states

$$0 \leq \tau_{AB} \leq 1$$

Examples

- Example, separable state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle$$

$$\tau_{AB} = 0$$

No entanglement.

- Example, Bell state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$\tau_{AB} = 1$$

Maximal entanglement.

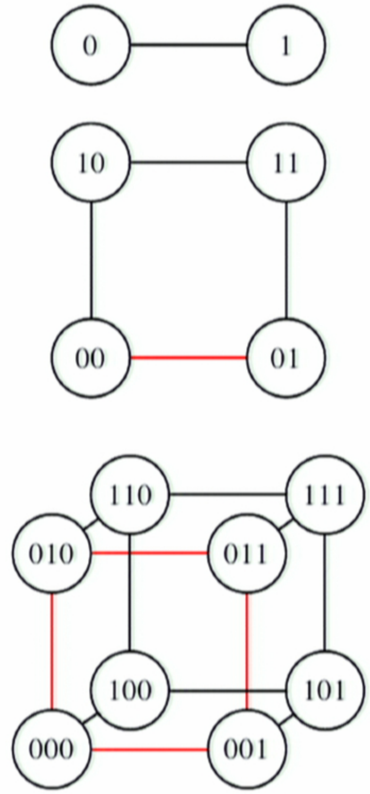
Three qubits

The three qubit system Alice, Bob and Charlie (where $A, B, C = 0, 1$) is described by the state

$$
\begin{aligned}
|\Psi\rangle &= a_{ABC}|ABC\rangle \\
&= a_{000}|000\rangle + a_{001}|001\rangle + a_{010}|010\rangle + a_{011}|011\rangle \\
&+ a_{100}|100\rangle + a_{101}|101\rangle + a_{110}|110\rangle + a_{111}|111\rangle.
\end{aligned}
$$

Hypermatrix

The 3-index quantity a_{ABC} is an example of what Cayley termed a *hypermatrix* in 1845. Its elements may be represented by the cube



Entanglement classification

- Null class: 0
- Separable class A-B-C (product states): $q_0|111\rangle$
- Biseparable class (bipartite entanglement):

$$A\text{-}BC : q_0|111\rangle - p^1|100\rangle$$
$$B\text{-}CA : q_0|111\rangle - p^2|010\rangle$$
$$C\text{-}AB : q_0|111\rangle - p^3|001\rangle$$

- Class W (tripartite entanglement):

$$-p^1|100\rangle - p^2|010\rangle - p^3|001\rangle$$

- Class GHZ (tripartite entanglement):

$$q_0|111\rangle - p^1|100\rangle - p^2|010\rangle - p^3|001\rangle$$

CAYLEY'S HYPERDETERMINANT (1845)

- The GHZ tripartite entanglement of Alice, Bob and Charlie is given by the three-tangle

$$\tau_{ABC} = 4|\mathrm{Det}\ a_{ABC}|,$$

Coffman et al: arXiv:quant-ph/9907047

- Det a_{ABC} is Cayley's hyperdeterminant

$$\mathrm{Det}\ a_{ABC} = -\frac{1}{2}\ \varepsilon^{A_1A_2}\varepsilon^{B_1B_2}\varepsilon^{C_1C_4}\varepsilon^{C_2C_3}\varepsilon^{A_3A_4}\varepsilon^{B_3B_4} \cdot a_{A_1B_1C_1}a_{A_2B_2C_2}a_{A_3B_3C_3}a_{A_4B_4C_4}$$

Miyake, Wadati: arXiv:quant-ph/0212146

- It is invariant under $SL(2)_A \times SL(2)_B \times SL(2)_C$, with a_{ABC} transforming as a $(\mathbf{2}, \mathbf{2}, \mathbf{2})$, and under a discrete triality that interchanges A, B and C.

CAYLEY

- Explicitly

$$
\begin{aligned}
&\text{Det } a_{ABC} = \\
&a_{000}^2 a_{111}^2 + a_{001}^2 a_{110}^2 + a_{010}^2 a_{101}^2 + a_{100}^2 a_{011}^2 \\
&- 2(a_{000}a_{001}a_{110}a_{111} + a_{000}a_{010}a_{101}a_{111} \\
&+ a_{000}a_{100}a_{011}a_{111} + a_{001}a_{010}a_{101}a_{110} \\
&+ a_{001}a_{100}a_{011}a_{110} + a_{010}a_{100}a_{011}a_{101}) \\
&+ 4(a_{000}a_{011}a_{101}a_{110} + a_{001}a_{010}a_{100}a_{111}).
\end{aligned}
$$

Charged black holes

The most general static spherically symmetric black hole solution of Einstein-Maxwell theory is given in spherical polar coordinates (t, r, θ, ϕ) by the Reissner-Nordström line-element

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2$$

$$+r^2(d\theta^2 + sin^2\theta d\phi^2) \qquad A_t = \frac{Q}{r}$$

where M and Q are the mass and electric charge of the black hole in units $G = \hbar = c = 1$.

Surface gravity and the area

- The R-N solution has two horizons determined by

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2}.$$

- Two important quantities are surface gravity and the area given by

$$\kappa_S = \frac{\sqrt{M^2 - Q^2}}{2M(M + \sqrt{M^2 - Q^2}) - Q^2}, \quad A = 4\pi(M + \sqrt{M^2 - Q^2})^2.$$

- *Extremal* black holes have $M = |Q|$. Two horizons, r_+ and r_- coincide.

Quantum black holes

- Hawking temperature

$$T_{\mathsf{H}} = \frac{\kappa_S}{2\pi}$$

- Bekenstein-Hawking entropy

$$S_{\mathsf{BH}} = \frac{A}{4}$$

- In the extremal case, the entropy is completely determined in terms of the charges

$$S_{BH} = \frac{A}{4} = \pi Q^2$$

and there is no Hawking radiation since

$$T_{\mathsf{H}} = 0$$

Magnetic charge

If magnetic monopoles are included into the theory,

$$A_\phi = P cos\theta$$

then a generalization to include magnetic charge P is obtained by replacing Q^2 by $Q^2 + P^2$ in the metric and other formulae.

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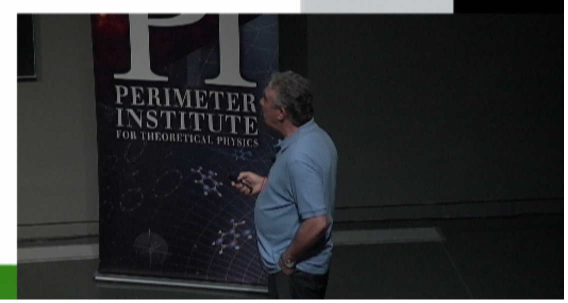
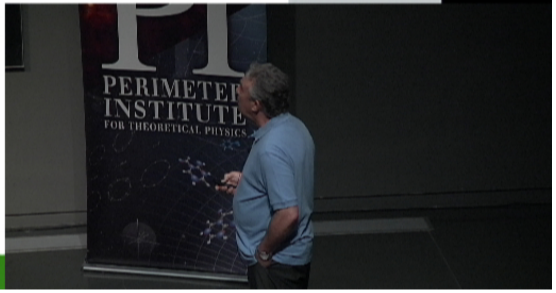


Extremal black holes

- Cosmic censorship: If $M \geq |Q|$, the singularity at $r = 0$ is hidden behind the event horizon; otherwise there is a naked singularity.
- *Extremal* black holes have $M = |Q|$. Two horizons, r_+ and r_- coincide.
- This also allows classically stable multi-centered black hole solutions obeying a *no-force condition*: the gravitational attraction is exactly cancelled by the Coulomb repulsion.



Black holes in supergravity

- Supergravity incorporates bose-fermi symmetry: the spin-2 graviton can have $1 \leq \mathcal{N} \leq 8$ spin 3/2 gravitino partners.
- The supergravity theories we shall consider have more than the one photon of Einstein-Maxwell theory. The $\mathcal{N} = 2$ STU model has 4; the $\mathcal{N} = 8$ model has 28, so the black holes will carry 8 or 56 electric and magnetic charges, respectively.
- Both also involve scalar fields.

BPS black holes

- A black hole that preserves some unbroken supersymmetry (admitting one or more Killing spinors) is said to be *BPS* (after Bogomol'nyi-Prasad-Sommerfield) and non-BPS otherwise.
- All BPS black holes are extremal but extremal black holes can be BPS or non-BPS.

STU model

- The STU supergravity model arises in string theory. Its bosonic sector consists of gravity coupled to 4 photons and three complex scalars, denoted S, T and U.
- The equations of motion display the symmetry $SL(2)_S \times SL(2)_T \times SL(2)_U$ and a discrete triality that interchanges S, T and U.
- Duff, Liu, Rahmfeld: arXiv:hep-th/9508094

Black hole/qubit correspondence

Duff: arXiv:hep-th/0601134 Identify STU with ABC and the 8 black hole charges with the 8 components of the three-qubit hypermatrix a_{ABC},

$$\begin{bmatrix} p^0 \\ p^1 \\ p^2 \\ p^3 \\ q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} a_{000} \\ -a_{001} \\ -a_{010} \\ -a_{100} \\ a_{111} \\ a_{110} \\ a_{101} \\ a_{011} \end{bmatrix}$$

Find that the black hole entropy is related to the 3-tangle as in

$$S = \pi\sqrt{|\text{Det } a_{ABC}|} = \frac{\pi}{2}\sqrt{\tau_{ABC}}$$

Turns out to be the tip of an iceberg.

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No force condition

- The 4-charge solution with just q_0, p^1, p^2, p^3 switched on obeys the no-force condition and may be regarded as a bound state of four individual black holes with charges q_0, p^1, p^2, p^3, with zero binding energy.
- This translates into the special GHZ (or Mermin) state

$$|\Psi\rangle = -p^3|001\rangle - p^2|010\rangle - p^1|100\rangle + q_0|111\rangle.$$

- Flipping the sign of q_0 flips the sign of $\mathrm{Det}\, a_{ABC}$ and corresponds to going from $1/8$ susy (BPS) to 0 susy (non-BPS) black hole.
- Similarly GHZ state

$$|\Psi\rangle = p^0|000\rangle + q_0|111\rangle.$$

corresponds to non-BPS black hole.

Microscopic analysis

String interpretation:

- Black holes are now 0-branes obtained by wrapping p-branes around p of the compactifying circles.
- Provides a *microscopic* explanation of Bekenstein-Hawking entropy A. Strominger and C. Vafa hep-th/9601029
- The 4-charge no-force solution is given by four D3-branes intersecting over a string, for example by wrapping the $(579), (568), (478), (469)$ cycles with wrapping numbers N_0, N_1, N_2, N_3. Klebanov and Tseytlin: arXiv:hep-th/9604166
- " Wrapped branes as qubits" Borsten, Dahanayake, Duff, Rubens, Ebrahim arXiv:0802.0840

dimension								$\vert ABC\rangle$
4	5	6	7	8	9	charges	branes	
x	o	x	o	x	o	p^0	0	$\vert 000\rangle$
o	x	o	x	x	o	q_1	0	$\vert 110\rangle$
o	x	x	o	o	x	q_2	0	$\vert 101\rangle$
x	o	o	x	o	x	q_3	0	$\vert 011\rangle$
o	x	o	x	o	x	q_0	N_0	$\vert 111\rangle$
x	o	x	o	o	x	$-p^1$	$-N_3$	$\vert 001\rangle$
x	o	o	x	x	o	$-p^2$	$-N_2$	$\vert 010\rangle$
o	x	x	o	x	o	$-p^3$	$-N_1$	$\vert 100\rangle$

Qubit interpretation

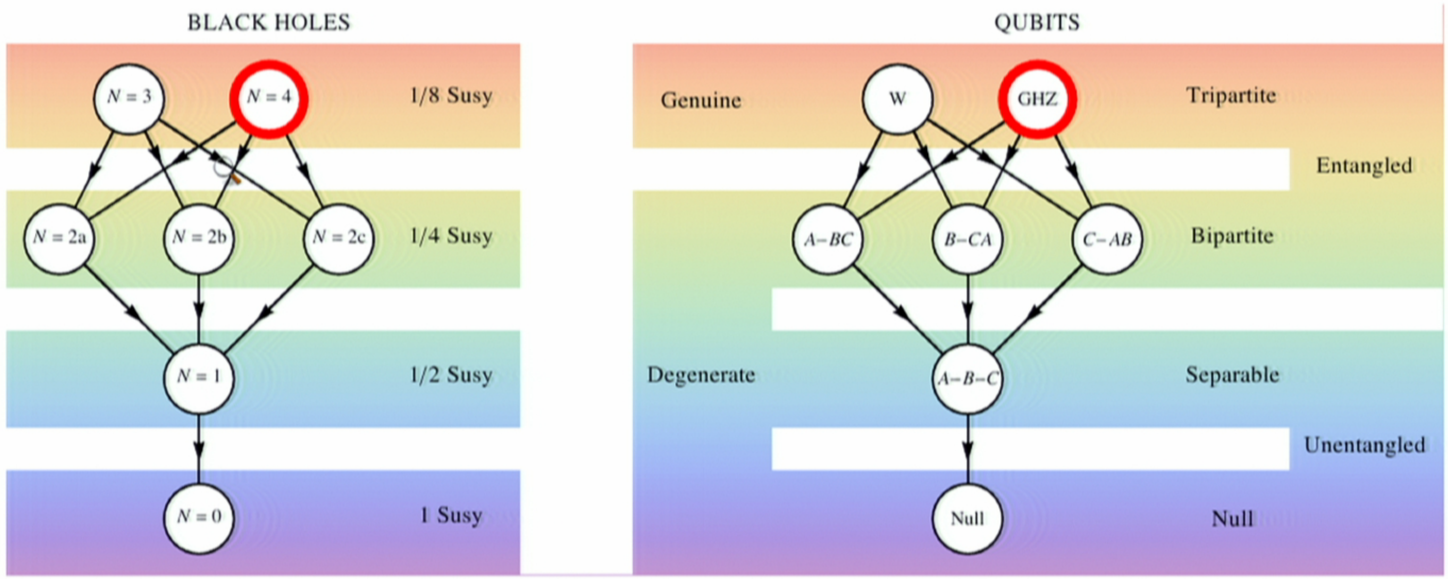
- We associate the three T^2 with the $SL(2)_A \times SL(2)_B \times SL(2)_C$ of the three qubits. The brane can wrap one circle or the other in each T^2.
- Provides an explanation for the appearance of the qubit two-valuedness (0 or 1): $|0\rangle$ corresponds to xo and $|1\rangle$ to ox. To wrap or not to wrap? That is the qubit.
- The number of qubits is three because of the number of extra dimensions is six.
- Performing a T-duality transformation, one obtains a Type IIA interpretation with N_0 D0-branes and N_1, N_2, N_3 D4-branes where $|0\rangle$ now corresponds to xx and $|1\rangle$ to oo.

5 PARAMETER STATE

- The most general real three-qubit state can be described by just five parameters. Acin et al: quant-ph/0009107
- It may conveniently be written

$$|\Psi\rangle = -N_3\cos^2\theta|001\rangle - N_2|010\rangle + N_3\sin\theta\cos\theta|011\rangle - N_1|100\rangle - N_3\sin\theta\cos\theta|101\rangle + (N_0 + N_3\sin^2\theta)|111\rangle.$$

$Inparticular, one$



- N= number of charges / number of kets

Embeddings

- The $\mathcal{N} = 2$ STU supergravity, where the 8 charges transform as a $(2, 2, 2)$ of $SL(2) \times SL(2) \times SL(2)$ can usefully be embedded in
- $\mathcal{N} = 4$ supergravity with symmetry $SL(2) \times SO(6, 6)$, where the 24 charges transform as a $(2, 12)$.
- $\mathcal{N} = 8$ supergravity with symmetry $E_{7(7)}$, where the 56 charges transform as an irreducible 56.
- Note the pattern $8(\mathcal{N} - 1) = 0, 8, 24, 56$ for $\mathcal{N} = 1, 2, 4, 8$
- Remarkably, the same five parameters suffice to describe all the black holes.

$E_{7(7)}$

- There is, in fact, a quantum information theoretic interpretation of the 56 charge $N = 8$ black hole in terms of a Hilbert space consisting of seven copies of the three-qubit Hilbert space. It relies on the decomposition $E_{7(7)} \supset [SL(2)]^7$

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Decomposition of the 56

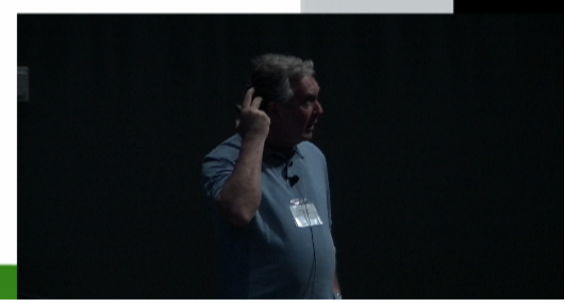
- Under

$$E_{7(7)} \supset$$

$$SL(2)_A \times SL(2)_B \times SL(2)_C \times SL(2)_D \times SL(2)_E \times SL(2)_F \times SL(2)_G$$

the 56 decomposes as

$$\begin{aligned} 56 \to \\ &(2,2,1,2,1,1,1) \\ &+(1,2,2,1,2,1,1) \\ &+(1,1,2,2,1,2,1) \\ &+(1,1,1,2,2,1,2) \\ &+(2,1,1,1,2,2,1) \\ &+(1,2,1,1,1,2,2) \\ &+(2,1,2,1,1,1,2) \end{aligned}$$



Seven qubits

- It admits the interpretation of a tripartite entanglement of seven qubits, Alice, Bob, Charlie, Daisy, Emma, Fred and George:

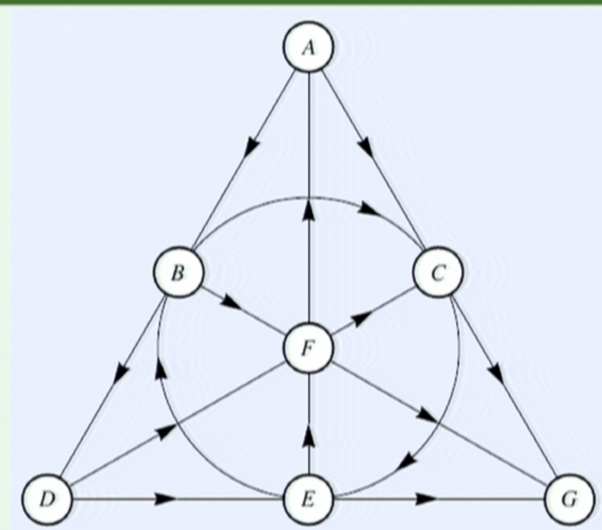
$$
|\psi\rangle = \begin{array}{l} a_{ABD}|ABD\rangle \\ + b_{BCE}|BCE\rangle \\ + c_{CDF}|CDF\rangle \\ + d_{DEG}|DEG\rangle \\ + e_{EFA}|EFA\rangle \\ + f_{FGB}|FGB\rangle \\ + g_{GAC}|GAC\rangle \end{array}
$$

Duff and Ferrara: quant-ph/0609227

Fano plane

A description of the entanglment is provided by the Fano plane which has seven points, representing the seven qubits, and seven lines (the circle counts as a line) with three points on every line, representing the tripartite entanglement, and three lines through every point.

Fano plane



Octonions

The Fano plane also provides the multiplication for the imaginary octonions:

	A	B	C	D	E	F	G
A		D	G	$-B$	F	$-E$	$-C$
B	$-D$		E	A	$-C$	G	$-F$
C	$-G$	$-E$		F	B	$-D$	A
D	B	$-A$	$-F$		G	C	$-E$
E	$-F$	C	$-B$	$-G$		A	D
F	E	$-G$	D	$-C$	$-A$		B
G	C	F	$-A$	E	$-D$	$-B$	

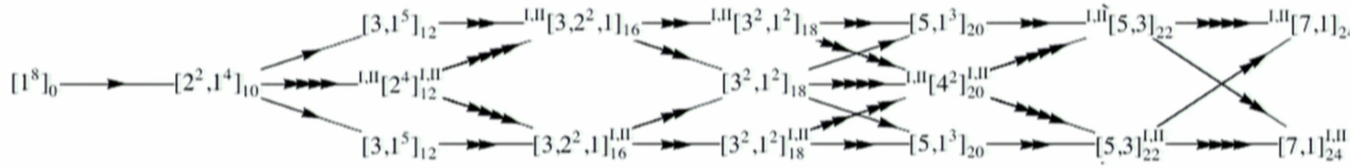
$\mathbf{R}, \mathbf{C}, \mathbf{H}, \mathbf{O}$

Lines	A	N	U	r	d	D
0	R	1	$SL(2)^7$	7	$16+16$	4
1	C	2	$SL(2)^3 \times SO(4,4)$	7	$32+32$	5
3	H	4	$SL(2) \times SO(6,6)$	7	$64+64$	7
7	O	8	$E_{7(-7)}$	7	$128+128$	11

- All admit membrane solutions
- Correspond to background geometry of membranes in D=4,5,7,11 (reduced to $D=4$).
- All are "self-mirror" with vanishing trace anomaly
  Duff and Ferrara arXiv:1010.3173

FAMILIES

- Under this finer classification there are 31 families of black hole



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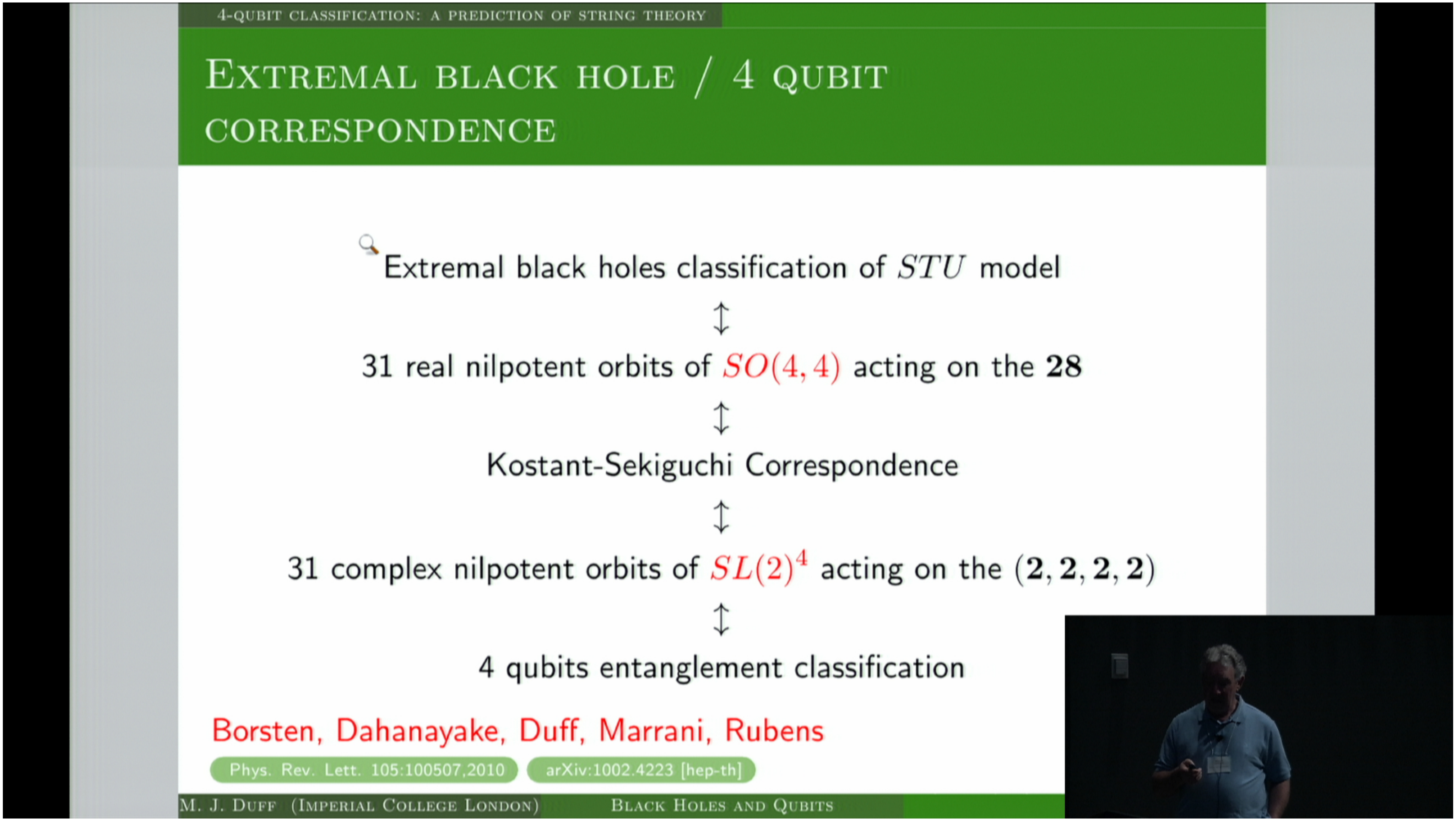
- Bergshoeff et al: arXiv:0806.2310v4 [hep-th]
  Bossard, Nicolai, Stelle: arXiv:0902.4438 [hep-th]
  Bossard, Michel, Pioline: arXiv:0908.1742v2 [hep-th]
- Suggests a way to classify 4 qubit entanglement
  Borsten, Dahanayake, Duff, Marrani, Rubens
  Phys. Rev. Lett. 105:100507,2010 arXiv:1002.4223 [hep-th]


4-QUBIT CLASSIFICATION: A PREDICTION OF STRING THEORY
EXTREMAL BLACK HOLE / 4 QUBIT CORRESPONDENCE
Extremal black holes classification of STU model
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31 real nilpotent orbits of $SO(4,4)$ acting on the $\mathbf{28}$
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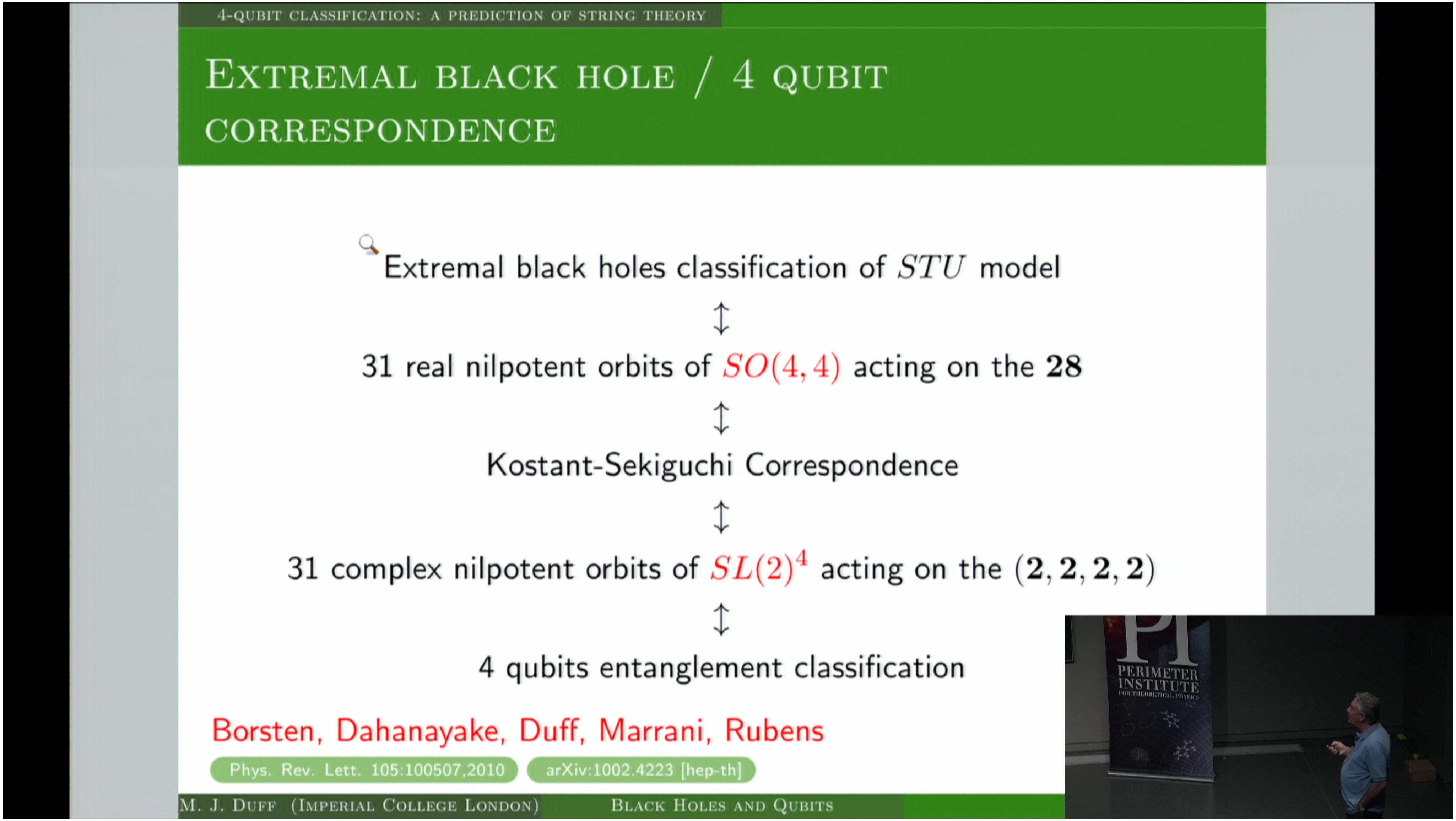
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STU black holes	perms	nilpotent rep		family
large $\frac{1}{2}$BPS and non-BPS $z_H = 0$	4	$\lvert 0000\rangle + \lvert 0111\rangle$	$\in$	$L_{0_{3\oplus\bar{1}}0_{3\oplus\bar{1}}}$
"extremal"	4	$\lvert 0000\rangle + \lvert 0101\rangle + \lvert 1000\rangle + \lvert 1110\rangle$	$\in$	$L_{0_{5\oplus\bar{3}}}$
"extremal"	4	$\lvert 0000\rangle + \lvert 1011\rangle + \lvert 1101\rangle + \lvert 1110\rangle$	$\in$	$L_{0_{7\oplus\bar{1}}}$

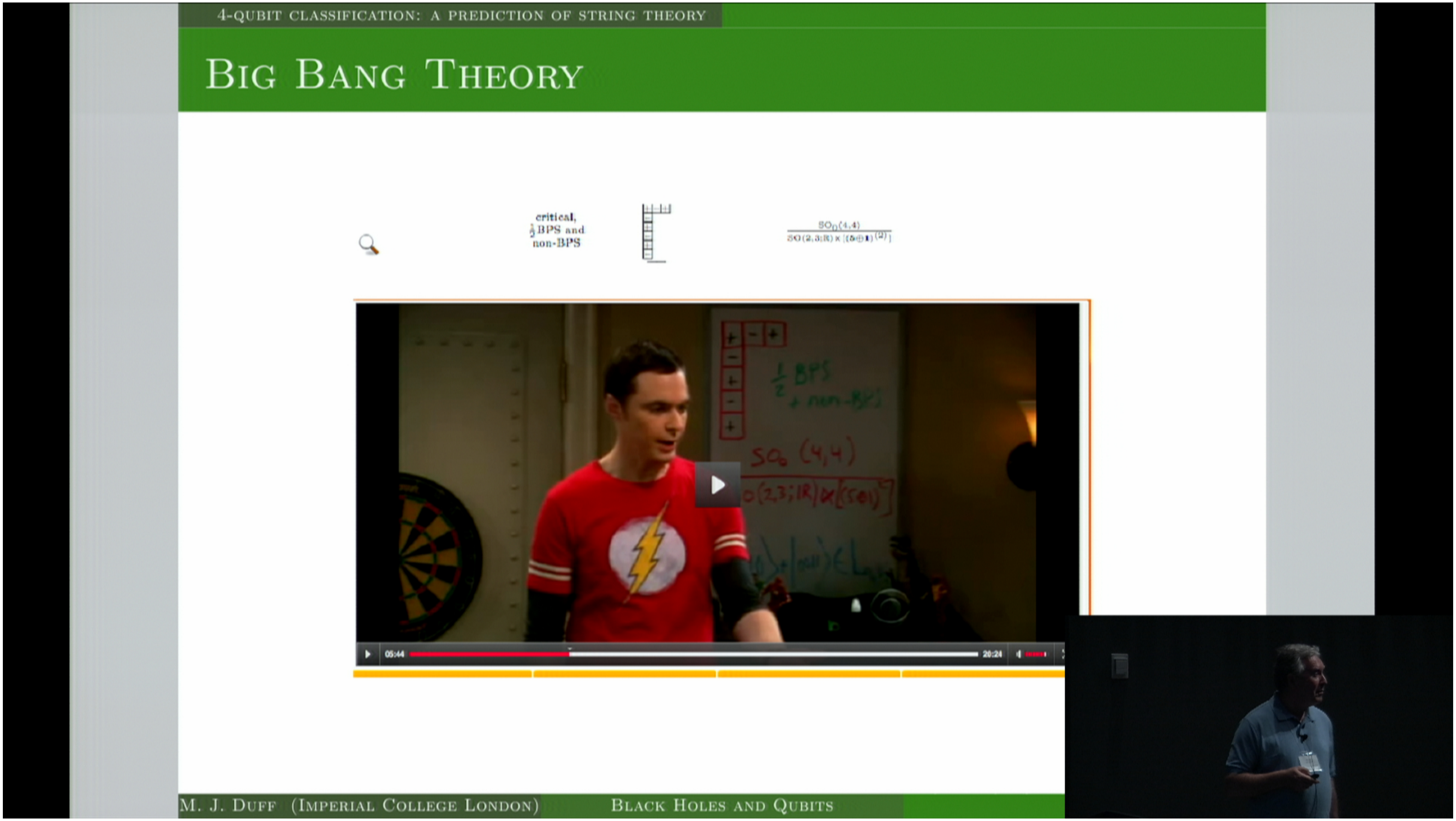
- Total number of families without permutations = 9 (agrees with Verstraete et al Phys.Rev. A65, 052112 (2002), quant-ph/0109033)
- Total number of families including permutations = 31 (new result)
- NB Trivially permuting the 9 yields many more than 31; (still need to check equivalence under SLOCC)

COMPARISON

Paradigm	Author	Year	result mod perms		result incl. perms	
classes	Lamata et al,	2006	8 genuine,	5 degenerate	16 genuine,	18 degenerate
	Cao et al	2007	8 genuine,	4 degenerate	8 genuine,	15 degenerate
	Li et al	2007	?		≥ 31 genuine,	18 degenerate
	Akhtarshenas et al	2010	?		11 genuine,	6 degenerate
families	Verstraete et al	2002	9		?	
	Chretrentahl et al	2007	9		?	
	String theory	2010	9		31	

PREDICTIONS

- Previous result 2006:
  STU black holes imply 5 ways to entangle three qubits
  Already known in QI; verified experimentally
- New result 2010:
  STU black holes imply 31 ways to entangle four qubits
  Not already known in QI: in principle testable in the laboratory


4-QUBIT CLASSIFICATION: A PREDICTION OF STRING THEORY
BIG BANG THEORY
critical,
$\frac{1}{2}$BPS and
non-BPS
$\frac{SO_0(4,4)}{SO(2,2;\mathbb{R})\times[(5\oplus 1)^{(2)}]}$
05:44
20:24
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