

Title: TBA

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Abstract: TBA

# Another Mass Gap in the BTZ Geometry? Hairy Black Holes in 2+1 Gravity

Sean Stotyn

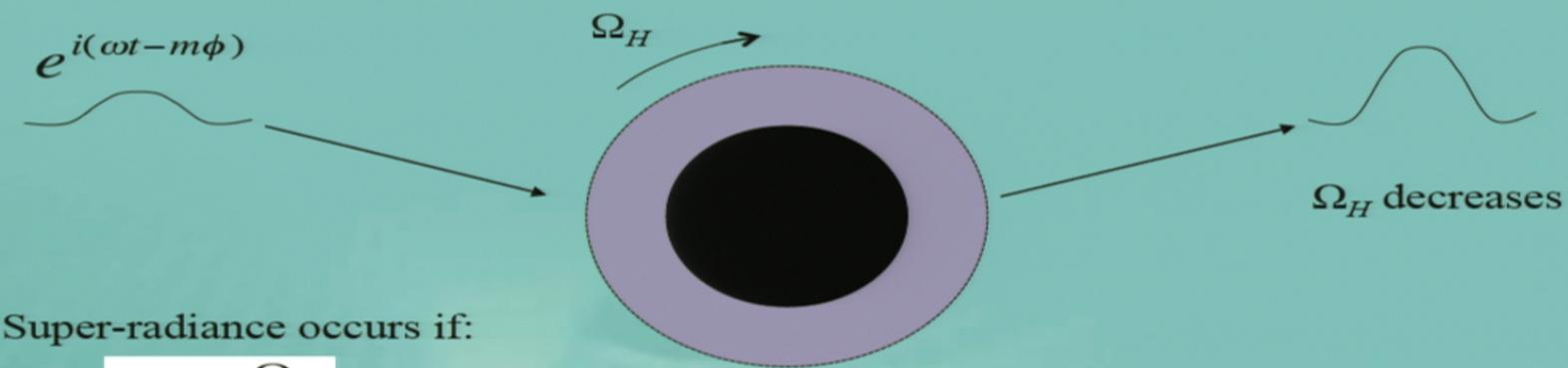
AdS/CFT in Dynamical Settings  
Perimeter Institute, June 8, 2012

based on Stotyn-Mann arXiv:1203.0214  
extends Dias-Horowitz-Santos ( $D=5$ ) and Stotyn-Park-McGrath-Mann ( $D=5,7,9,11$ )

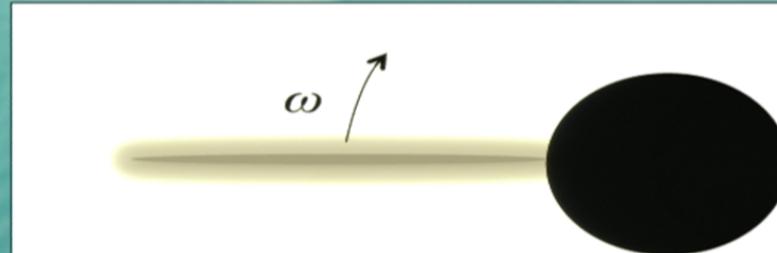
## Outline

1. Motivation and review of the BTZ black hole
2. Perturbative boson stars around  $\text{AdS}_3$  and conical singularities
3. No perturbative hairy black holes with one Killing field
4. Analytic evidence of finite size hairy black holes
5. Future prospects

## Motivation: Super-radiance

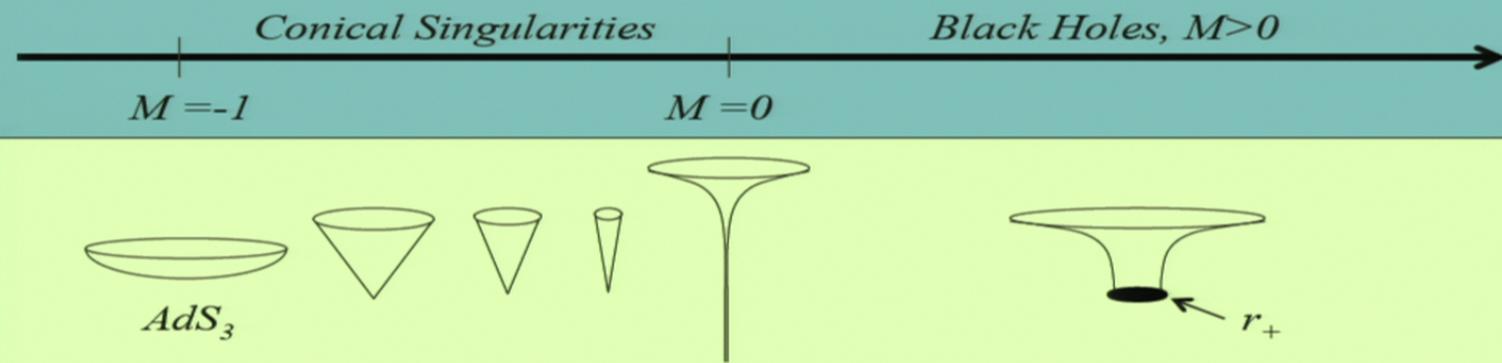


$m = 1$ :



$$\Omega_H = \frac{\omega}{m}$$

## BTZ Black Hole



$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\phi - \Omega(r)dt)^2$$

$$f(r) = \frac{r^2}{\ell^2} - M + \frac{J^2}{4r^2}$$

$$\Omega(r) = \frac{J}{2r^2}$$

## Ansatz

$$S = \frac{1}{16\pi} \int d^3x \sqrt{-g} \left( R + \frac{2}{\ell^2} - 2|\nabla\Pi|^2 \right)$$

Metric ansatz:  $ds^2 = -f(r)g(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\phi - \Omega(r)dt)^2$

Scalar field ansatz:

$$\Pi = \Pi(r)e^{-i(\omega t - m\phi)}$$

$$T_{ab} = 2\partial_{(a}\Pi^{*}\partial_{b)}\Pi - g_{ab}(\partial_c\Pi^{*}\partial^c\Pi)$$

The stress tensor shares the symmetries of the metric but the scalar field itself is only invariant under the particular combination

$$K = \partial_t + \frac{\omega}{m}\partial_\phi$$

Solutions with this matter are invariant under a single Killing vector field,  $K$ .

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## Equations of Motion

The ansatz leads to a consistent set of coupled, 2nd order ODEs:

$$f'' + f' \left( \frac{3}{r} - \frac{g'}{2g} \right) + \frac{fg'}{g} \left( \frac{1}{r} - \frac{g'}{2g} \right) + \frac{8\Pi'\Pi}{r} + \frac{4\Pi^2(\omega - \Omega)^2}{fg} + \frac{4\Pi^2}{r^2} - \frac{8\Pi^2\Omega'r}{fg}(\omega - \Omega) - \frac{8}{\ell^2} = 0$$

$$g'' + g' \left( \frac{2f'}{f} + \frac{1}{r} \right) - \frac{8\Pi^2}{f^2}(\omega - \Omega)^2 + \frac{8r\Pi^2\Omega'}{f^2}(\omega - \Omega) - \frac{8g\Pi'\Pi}{rf} = 0$$

$$\Omega'' + \frac{4\Pi^2}{fr^2}(\omega - \Omega) + \Omega' \left( \frac{3}{r} - \frac{g'}{2g} \right) = 0$$

$$\Pi'' + \frac{\Pi'(f^2gr^2)'}{2f^2gr^2} + \frac{\Pi}{f^2g}(\omega - \Omega)^2 - \frac{\Pi}{fr^2} = 0$$

$$G_{ab} - \frac{1}{\ell^2}g_{ab} = T_{ab}$$

$$\nabla^2\Pi = 0$$

There are also a further 2 constraint equations:

$$C_1 = \frac{r}{f}(f^2g)' + 4g\Xi + r^4\Omega'^2 = 0$$

$$\Xi = \Pi^2 - \frac{r^2}{\ell^2}$$

$$C_2 = \frac{\Pi^2(\omega - \Omega)^2}{f^2g} + \Pi'^2 + \frac{r^2\Omega'^2}{4fg} + \frac{f'}{2fr} + \frac{\Pi^2}{fr^2} - \frac{1}{f\ell^2} = 0$$

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## Constructing Solutions Perturbatively

Perform a double expansion of all fields in powers of  $\varepsilon$  and  $r_+$

$$F(r, \varepsilon, r_+) = \sum_{i=0}^m \sum_{j=0}^n F_{2j,2i}(r) \varepsilon^{2j} \left(\frac{r_+}{\ell}\right)^{2i}$$

$$\Pi(r, \varepsilon, r_+) = \sum_{i=0}^m \sum_{j=0}^n \Pi_{2j+1,2i}(r) \varepsilon^{2j+1} \left(\frac{r_+}{\ell}\right)^{2i}$$

Bootstrapping procedure: expand EOM in powers of  $\varepsilon$  and  $r_+$

$$\sum_{i=0}^m F_{0,2i}(r) \left(\frac{r_+}{\ell}\right)^{2i} \xrightarrow{\text{put non-trivial scalar field in gravitational background}} \varepsilon \sum_{i=0}^m \Pi_{1,2i}(r) \left(\frac{r_+}{\ell}\right)^{2i}$$

$\nwarrow$

$r_+ / \ell$  expansion of BTZ or AdS/conical sing.

## No Perturbative Hairy Black Holes

- Split up space-time into asymptotic and near-horizon regions.
- Solve EOM perturbatively in each region
- Match the solutions in a patch in between
- Problem! For small black holes ( $r_+ \ll l$ ) the matching condition requires the scalar field to vanish everywhere.

$$\Pi_{1,2}^{out} = C_1 \frac{\ell^2}{r^2}$$

*small-r  
limit*

$$\Pi_{1,2}^{in} = \frac{C_2}{\sqrt{z^2 - \ell^2}} K_1 \left( \sqrt{\frac{2\ell^3 \omega_{0,2}}{z^2 - \ell^2}} \right)$$

$$z = \frac{\ell r}{r_+}$$

$$C_1 \frac{\ell^2}{r^2} = \frac{C_2}{\sqrt{2\ell^3 \omega_{0,2}}}$$

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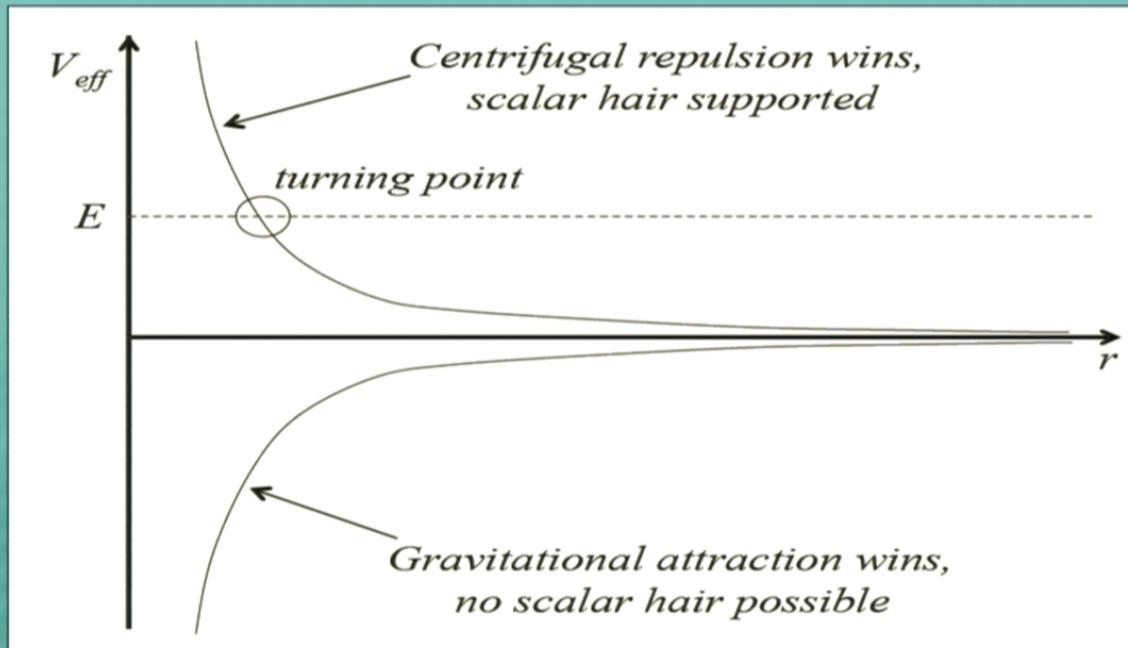
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## Effective Potential

$$V_{\text{eff}} = \frac{L^2}{\ell^2} - E^2 - \frac{L}{r^2}(ML - JE)$$



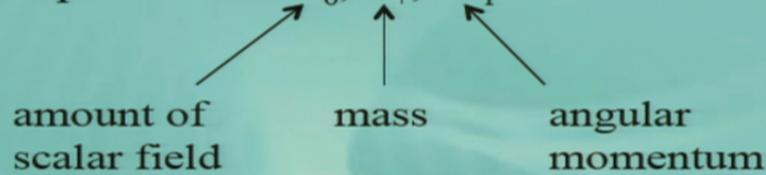
## Another Perspective

Perform a near-horizon expansion of the fields

$$F(r) = \sum_{i=0}^{\infty} F_i (r - r_+)^i$$

$$\Pi(r) = \sum_{i=0}^{\infty} \Pi_i (r - r_+)^i$$

Independent parameters:  $\Pi_0$ ,  $r_+$ ,  $\Omega_1$



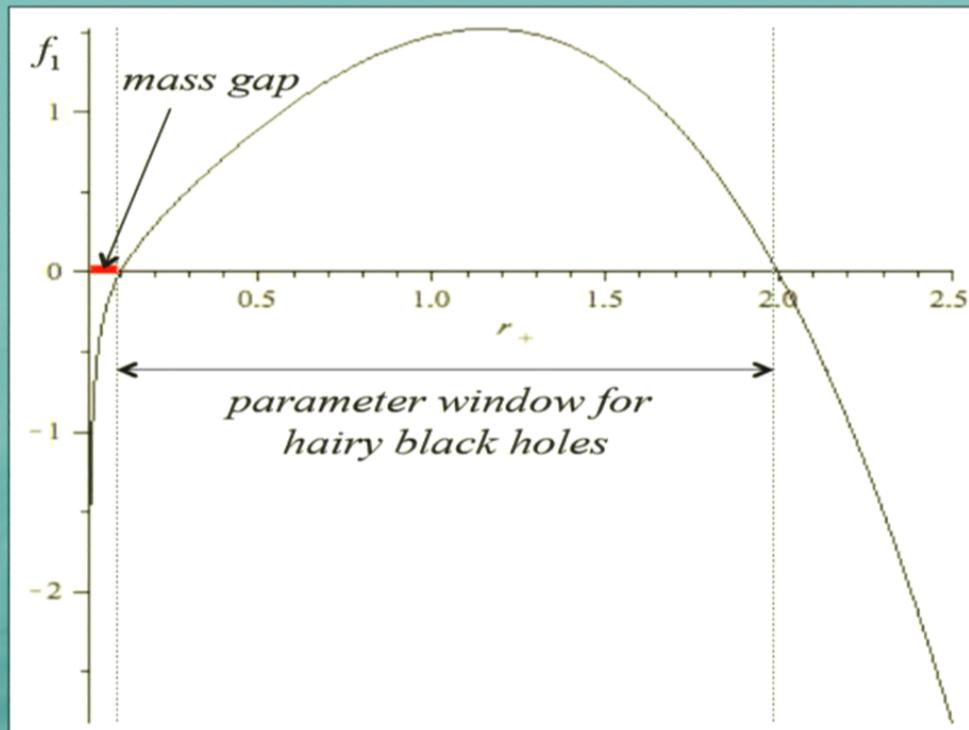
What we find from near-horizon expansion:

- Scalar field identically vanishes if  $\Pi_0 = 0$
- Existence of black hole horizon is constrained by sign of  $f_1$

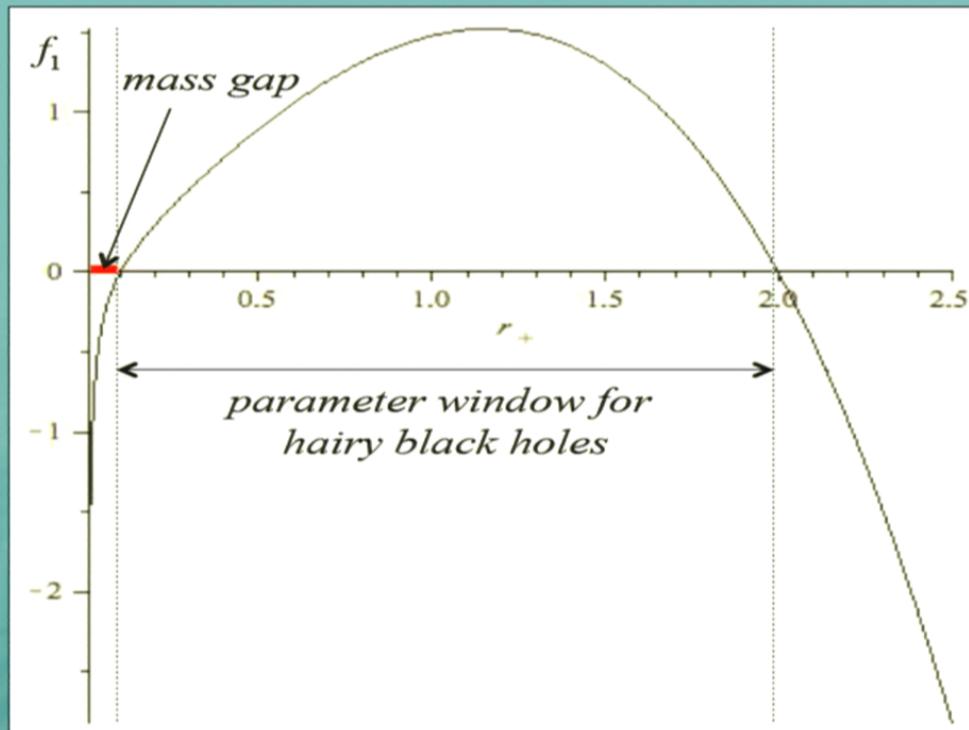
$$f_1 = -\frac{4\ell^2 g_0 \Pi_0^2 + \ell^2 r_+^4 \Omega_1^2 - 4g_0 r_+^2}{2\ell^2 g_0 r_+}$$

$$g_0 > 0$$

## Existence window



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## In Progress and Future Directions

- Searching for boson stars numerically
  - Expect very similar results to 5D results (Dias-Horowitz-Santos)
- Searching for hairy black holes numerically
  - Expect very different results from higher dimensions (Dias-Horowitz-Santos & Stotyn-Park-McGrath-Mann)
- Effect of the mode number,  $m$ , on the black hole existence window
  - Do these settle down at finite  $m$ ?
- Future: is the same condition found when looking for super-radiant scattering in 2+1? If yes, good corroborating evidence
- Ultimately: what do these states correspond to in the 1+1 CFT and can they tell us anything useful?

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