

Title: Numerical Holographic Striped Phases

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Abstract: TBA



a place of mind



# Numerical Holographic Striped Phases

Exploring AdS-CFT Dualities in Dynamical Settings

Jared Stang

June 8<sup>th</sup>, 2012

with M. Rozali, D. Smyth, and E. Sorkin

# Motivation and goal I

1. AdS/CMT: stripes in high- $T_c$  superconductors
2. Understanding strongly coupled physics

Goal:

Gravity dual of a 2+1 system with stripes

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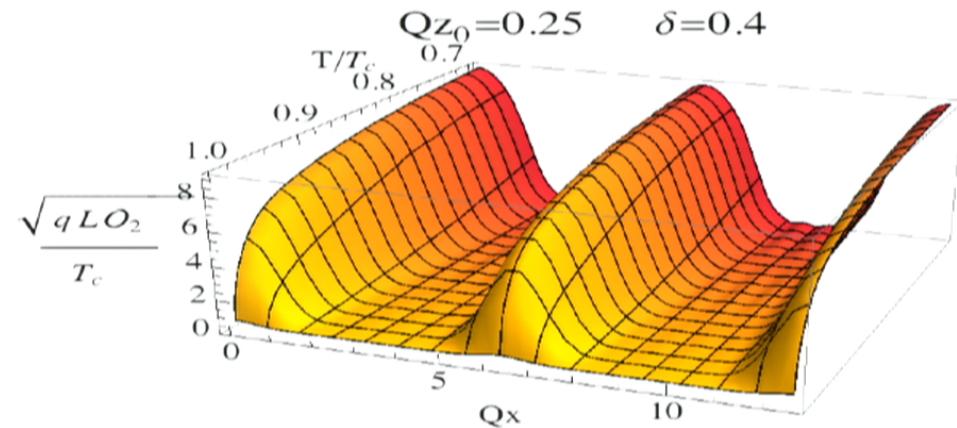
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# Background I

- One method: source stripes in the Hartnoll-Herzog-Horowitz superconductor

Flauber, Pajer and Papanikolaou, 1010.1775



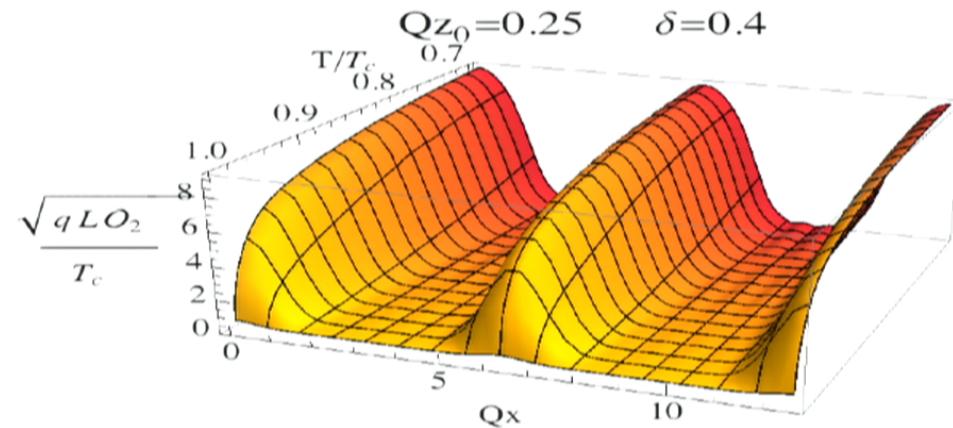
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# Background II

- Instabilities with Chern-Simons terms

- Holographic QCD

Domokos and Harvey, 0704.1604

- Maxwell-Chern-Simons in 4+1

Nakamura, Ooguri and Park, 0911.0679

- D7 probe brane in D3 background

Bergman *et al.*, 1106.3883

- Einstein-Maxwell-axion in 3+1

Donos and Gauntlett, 1106.2004

# Motivation and goal II

1. AdS/CMT: stripes
2. Understanding strongly coupled physics
3. How do the stripes back-react on the geometry?

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Goal:

Back-reacted gravity dual of 2+1 system with spontaneous stripes

# The model

- AdS gravity, neutral pseudo-scalar, and gauge field:  
Donos and Gauntlett, 1106.2004

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}(R + \Lambda) - \frac{1}{2}\sqrt{-g}(\partial^\mu\psi\partial_\mu\psi + m^2\psi^2) - \frac{1}{4}\sqrt{-g}F^{\mu\nu}F_{\mu\nu} - \frac{c_1}{8}\epsilon^{\mu\nu\rho\sigma}\psi F_{\mu\nu}F_{\rho\sigma}$$

- Homogeneous solution is AdS-Reissner-Nordstrom:

$$ds_{RN}^2 = -2r^2f_{RN}dt^2 + \frac{dr^2}{2r^2f_{RN}} + 2r^2(dx^2 + dy^2)$$

$$A = \left(1 - \frac{r_+}{r}\right)dt, \quad \psi = 0 \quad f_{RN} = 1 - \left(1 + \frac{1}{4r_+^2}\right)\left(\frac{r_+}{r}\right)^3 + \frac{1}{4r_+^2}\left(\frac{r_+}{r}\right)^4$$

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# The linearised system

Donos and Gauntlett, 1106.2004

- Consider this fluctuation and look for normalisable modes:

$$\delta g_{ty} = \lambda r(r - r_+) w(r) \sin(kx)$$

$$\delta A_y = \lambda a(r) \sin(kx)$$

$$\delta \psi = \lambda \phi(r) \cos(kx)$$

- Asymptotic expansions:

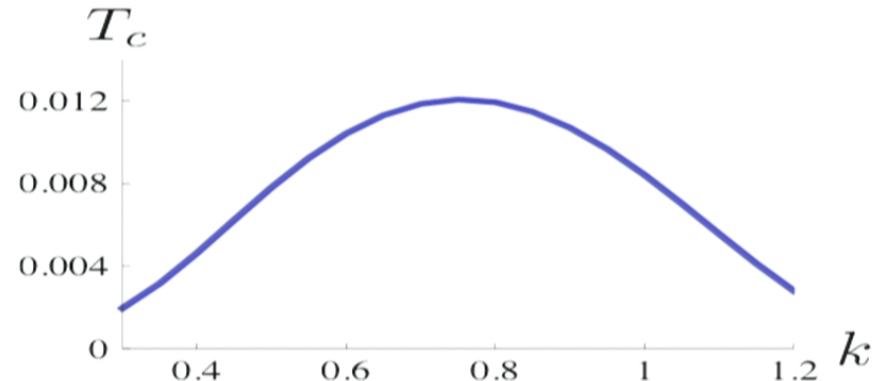
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- At the next order:

$$\delta Z = \lambda^2 (Z^{(0)}(r) + Z^{(1)}(r) \cos(2kx)), \quad Z = \{g_{tt}, g_{xx}, g_{yy}, A_t\}$$



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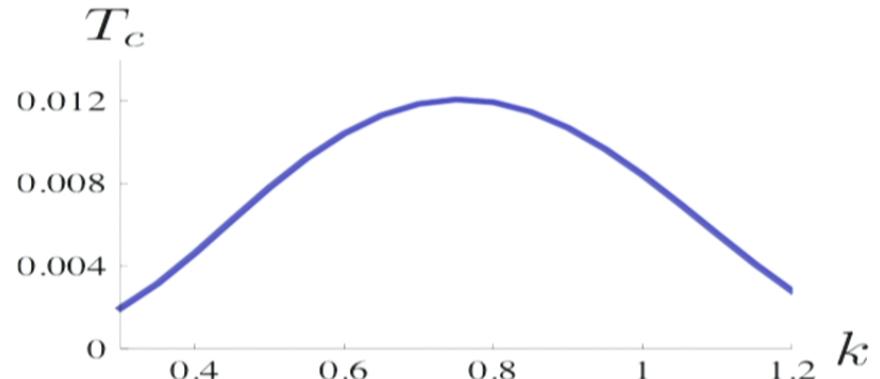
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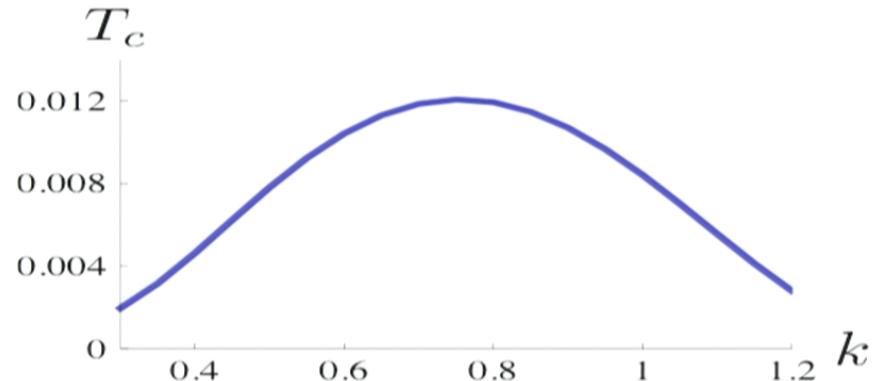
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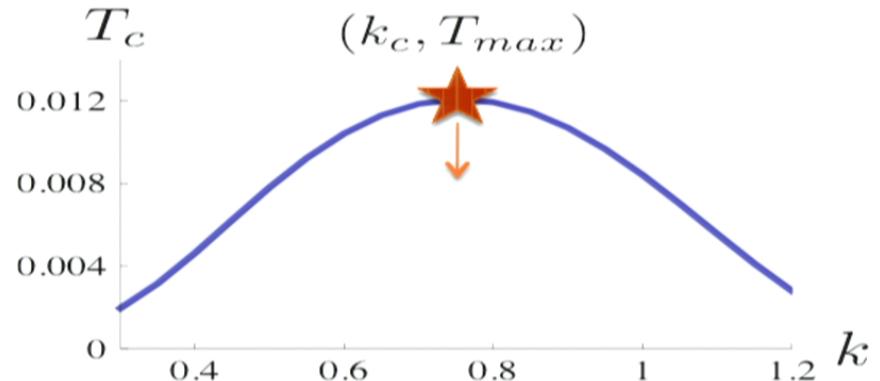
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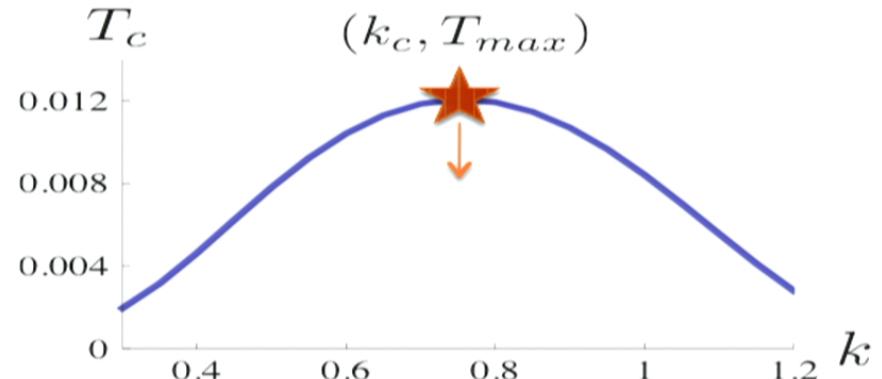
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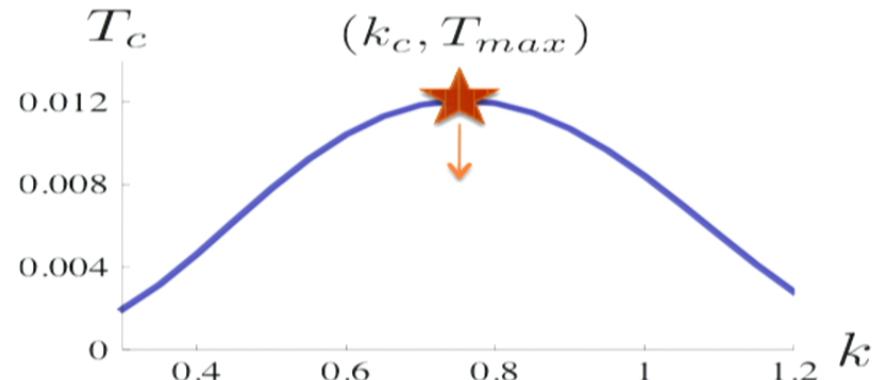
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## Non-linear system: Ansatz

$$ds^2 = -2r^2 f_{RN} e^{2A(r,x)} dt^2 + e^{2B(r,x)} \left( \frac{dr^2}{2r^2 f_{RN}} + 2r^2 dx^2 \right) + 2r^2 e^{2C(r,x)} (dy - W(r,x)dt)^2$$
$$A = A_t(r, x)dt + A_y(r, x)dy, \quad \psi = \psi(r, x)$$

$$T = T_{max} e^{A-B}|_{r=r_+}$$

$$x = \pi/k_c$$

7 fields:

$$A, B, C, A_t, W, A_y, \psi$$

$$x = 0$$

$$r = r_+$$

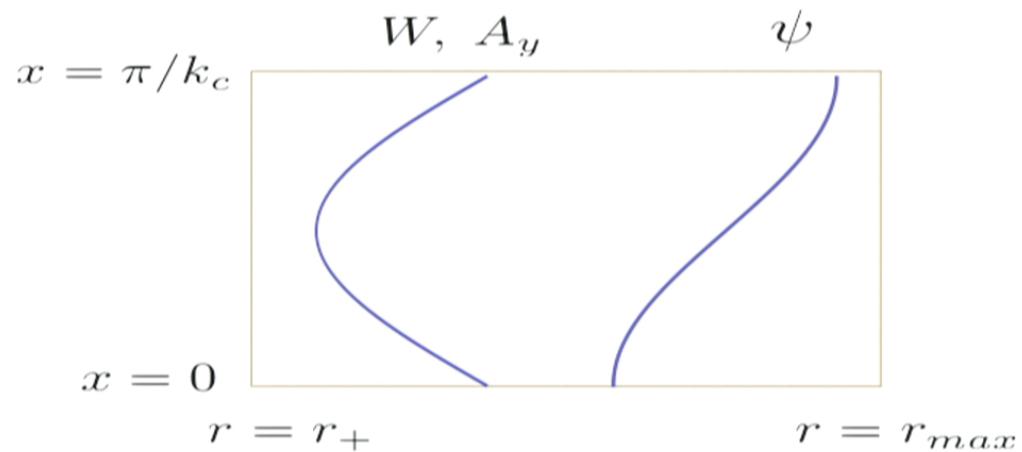
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# Non-linear system: Equations of motion

$$\begin{array}{lcl} G_t^t & = & T_t^t \\ G_y^t & = & T_y^t \\ G_y^x & = & T_y^x \\ G_r^r + G_x^x & = & T_r^r + T_x^x \end{array} \quad \left. \right\} \quad \begin{array}{l} \text{2nd order, elliptic E.O.M. for} \\ A, B, C, W \end{array}$$

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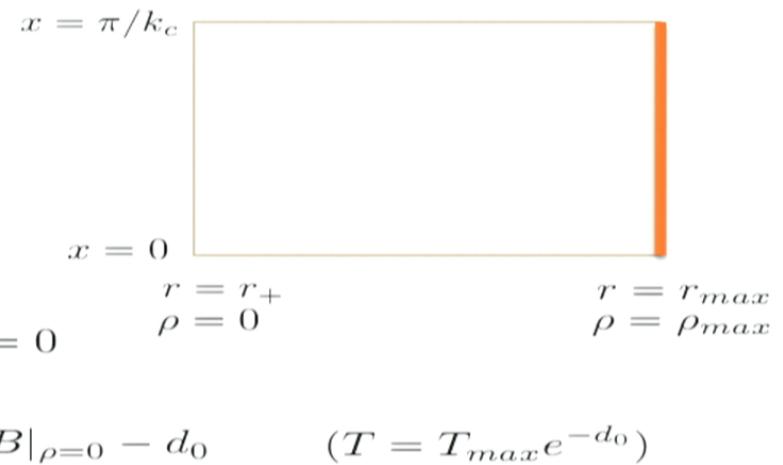
E.O.M. + boundary conditions  $\Rightarrow$  constraints are satisfied

Wiseman, hep-th/0209051

# Non-linear system: Boundary conditions

1.  $r = r_{max}$  : falloff conditions

$$A = \cancel{A_0}(x) + \dots + \frac{A_3(x)}{r^3} + \dots$$



2.  $r = r_+$  :

- regularity conditions

With  $\rho = \sqrt{r^2 - r_+^2}$ ,  $\partial_\rho C|_{\rho=0} = 0$

- constraints

$$\partial_x(A - B)|_{\rho=0} = 0 \Rightarrow A|_{\rho=0} = B|_{\rho=0} - d_0 \quad (T = T_{max}e^{-d_0})$$

3.  $x = 0$ ,  $x = \pi/k_c$  : Neumann or Dirichlet conditions

Solve: Finite difference, use Gauss-Siedel relaxation

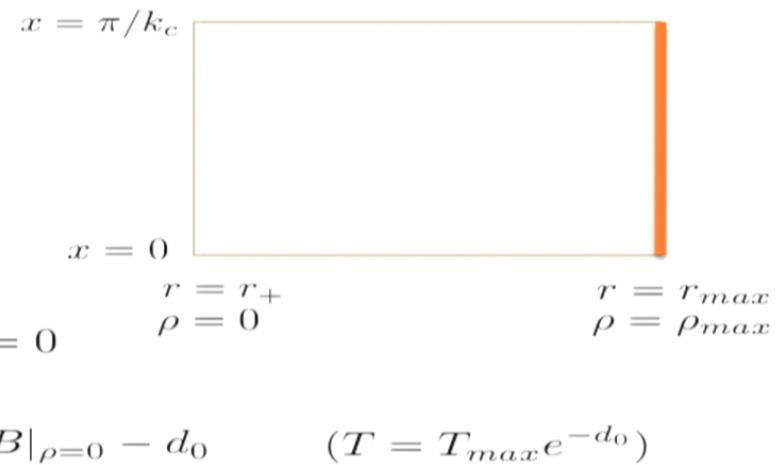
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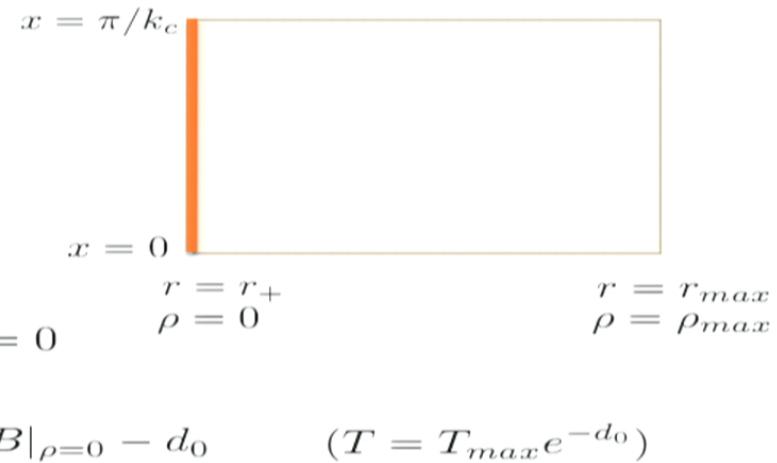
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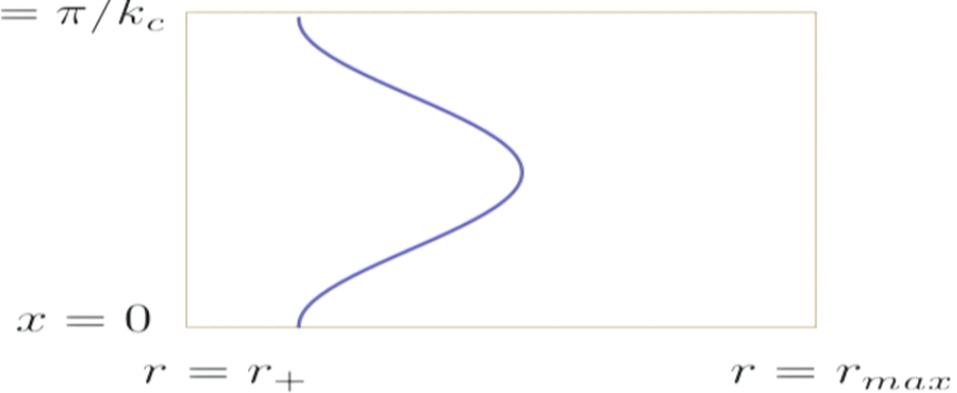
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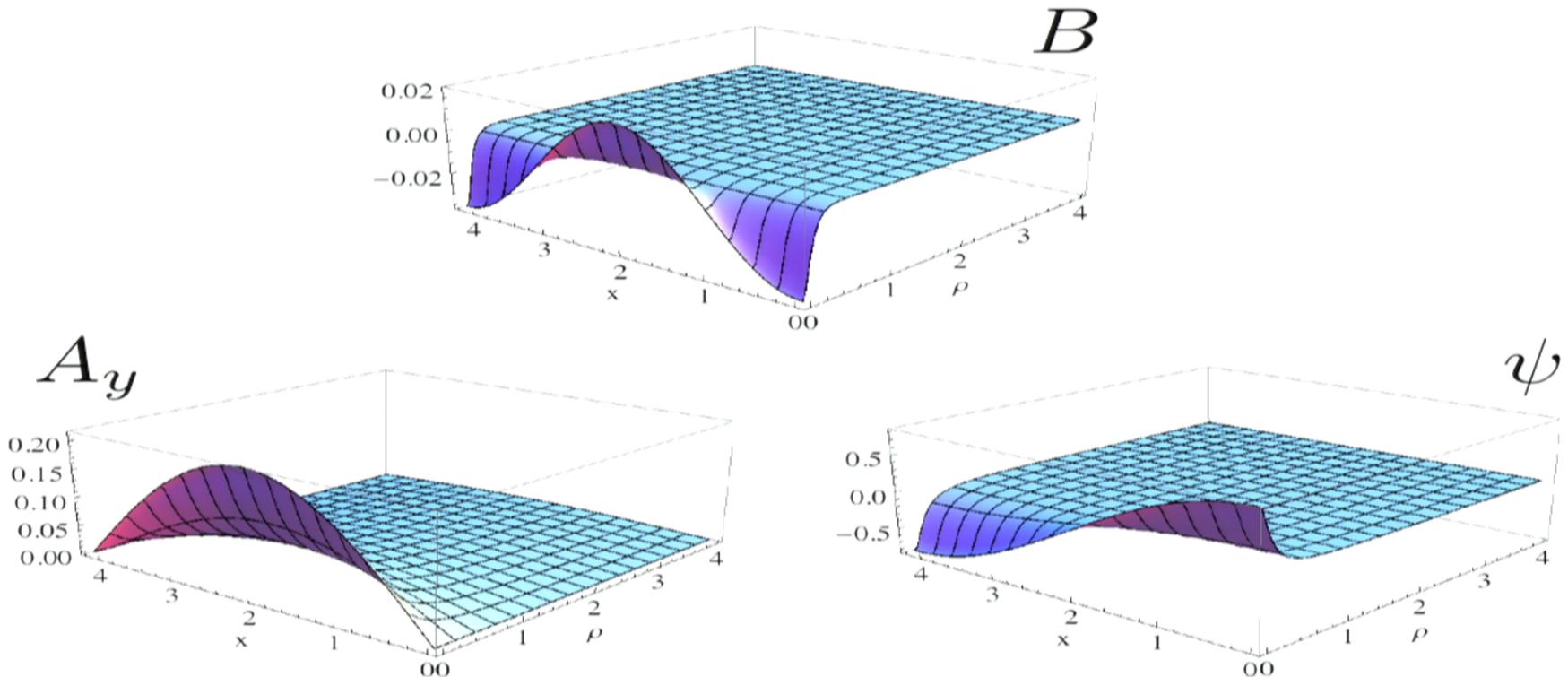
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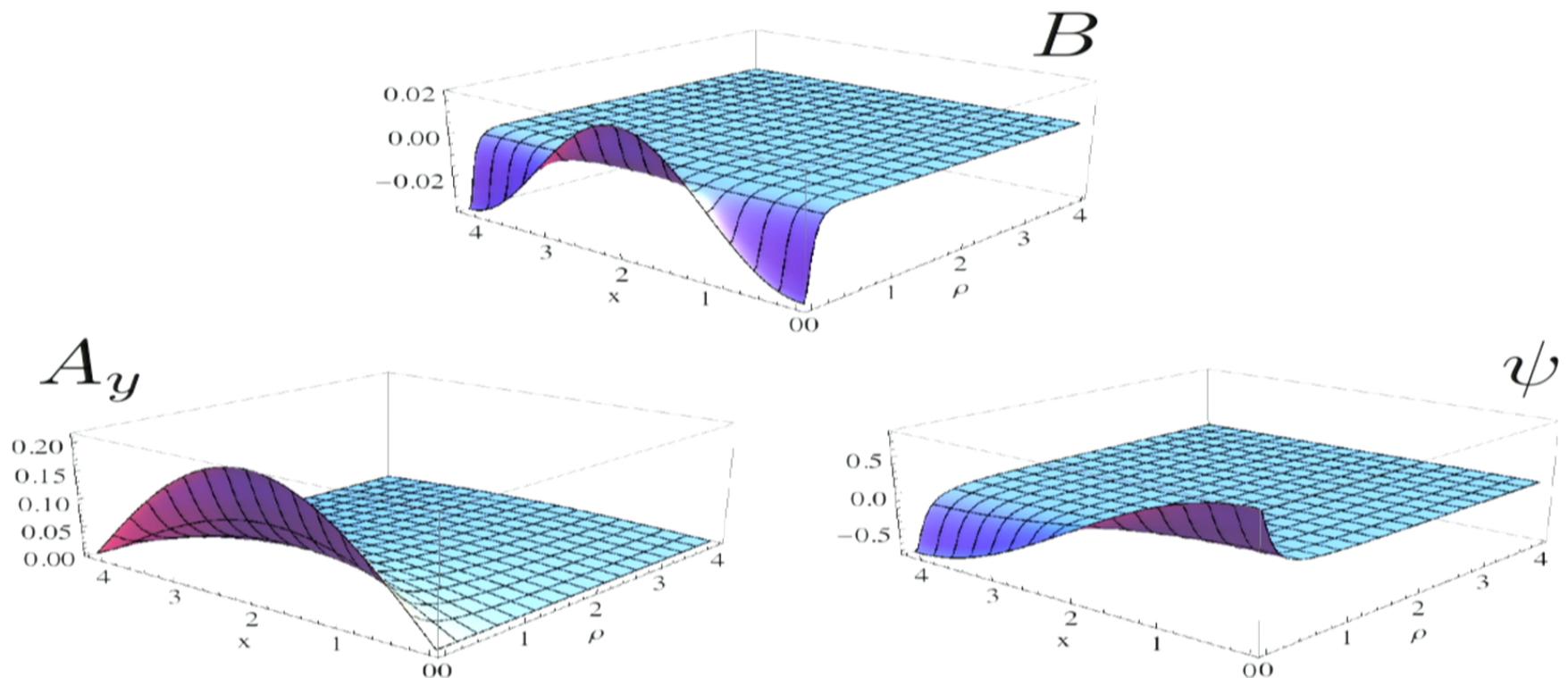
# Preliminary results



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## Further questions

1. What are the properties of the field theory?
2. Is this the dominant phase?
3. Zero temperature limit of this phase?