Title: Numerical Holographic Striped Phases

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Abstract: TBA

a place of mind

NSERC CRSNG

Numerical Holographic Striped Phases

Exploring AdS-CFT Dualities in Dynamical Settings

Jared Stang

June 8th, 2012

with M. Rozali, D. Smyth, and E. Sorkin

Motivation and goal I

1. AdS/CMT: stripes in high-T_c superconductors
2. Understanding strongly coupled physics

Goal:

Gravity dual of a 2+1 system with stripes

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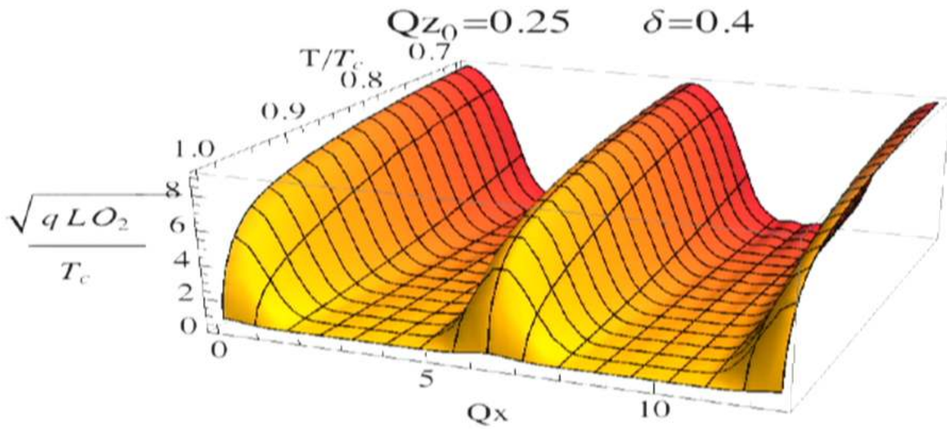
Background I

- One method: source stripes in the Hartnoll-Herzog-Horowitz superconductor

Flauger, Pajer and Papanikolaou, 1010.1775



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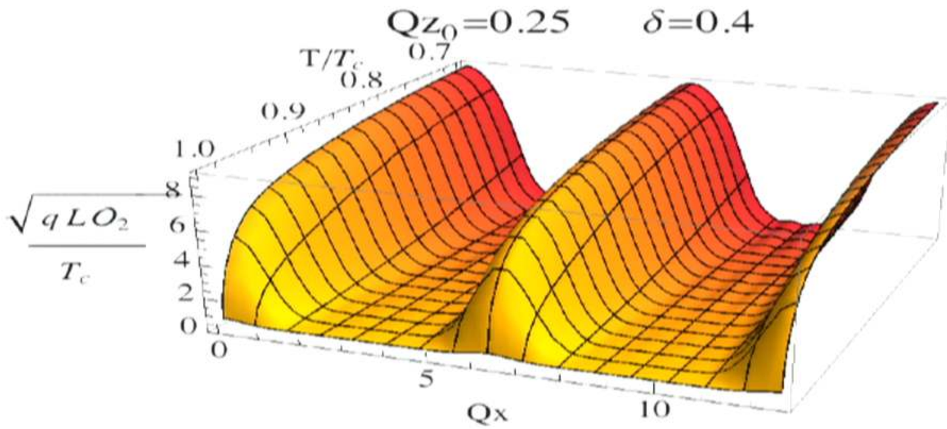

Background II

- Instabilities with Chern-Simons terms
  - Holographic QCD
    Domokos and Harvey, 0704.1604
  - Maxwell-Chern-Simons in 4+1
    Nakamura, Ooguri and Park, 0911.0679
  - D7 probe brane in D3 background
    Bergman *et al.*, 1106.3883
  - Einstein-Maxwell-axion in 3+1
    Donos and Gauntlett, 1106.2004

Motivation and goal II

1. AdS/CMT: stripes
2. Understanding strongly coupled physics
3. How do the stripes back-react on the geometry?

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Gravity dual of a 2+1 system with stripes

Motivation and goal II

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Goal:

Back-reacted gravity dual of 2+1 system with spontaneous stripes

The model

- AdS gravity, neutral pseudo-scalar, and gauge field:

Donos and Gauntlett, 1106.2004

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\,(R+\Lambda) - \frac{1}{2}\sqrt{-g}\,(\partial^\mu\psi\partial_\mu\psi + m^2\psi^2) - \frac{1}{4}\sqrt{-g}\,F^{\mu\nu}F_{\mu\nu} - \frac{c_1}{8}\epsilon^{\mu\nu\rho\sigma}\psi F_{\mu\nu}F_{\rho\sigma}$$

- Homogeneous solution is AdS-Reissner-Nordstrom:

$$ds^2_{RN} = -2r^2 f_{RN} dt^2 + \frac{dr^2}{2r^2 f_{RN}} + 2r^2(dx^2 + dy^2)$$

$$A = \left(1 - \frac{r_+}{r}\right) dt, \;\; \psi = 0 \qquad f_{RN} = 1 - \left(1 + \frac{1}{4r_+^2}\right)\left(\frac{r_+}{r}\right)^3 + \frac{1}{4r_+^2}\left(\frac{r_+}{r}\right)^4$$

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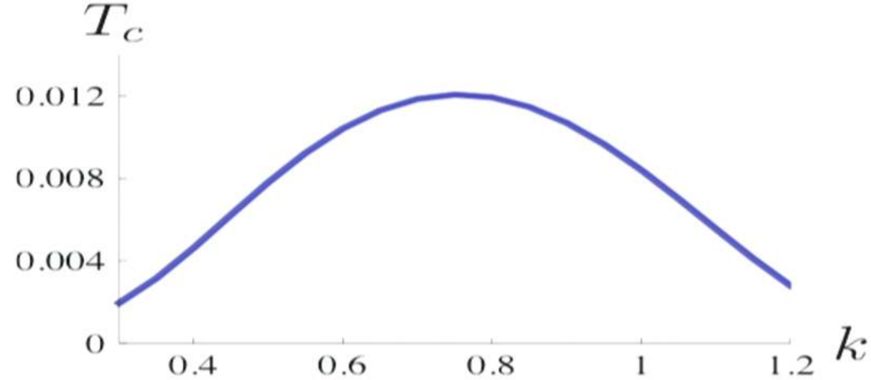
The linearised system

Donos and Gauntlett, 1106.2004

- Consider this fluctuation and look for normalisable modes:

$$\delta g_{ty} = \lambda r(r-r_+)w(r)\sin(kx)$$
$$\delta A_y = \lambda a(r)\sin(kx)$$
$$\delta\psi = \lambda\phi(r)\cos(kx)$$



- Asymptotic expansions:

$$w = w_0 + \cdots + \frac{w_3}{r^3} + \ldots$$
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- At the next order:

$$\delta Z = \lambda^2(Z^{(0)}(r) + Z^{(1)}(r)\cos(2kx)),\ Z = \{g_{tt}, g_{xx}, g_{yy}, A_t\}$$

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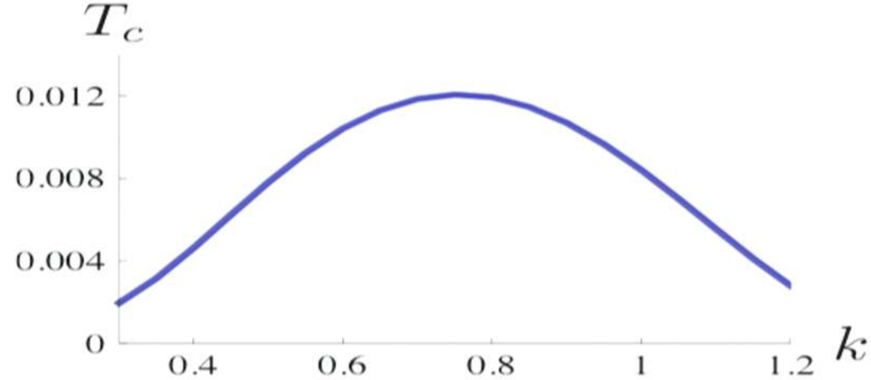

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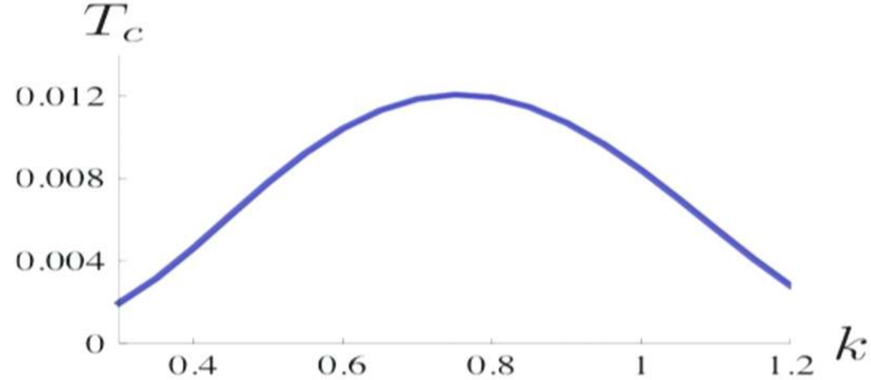

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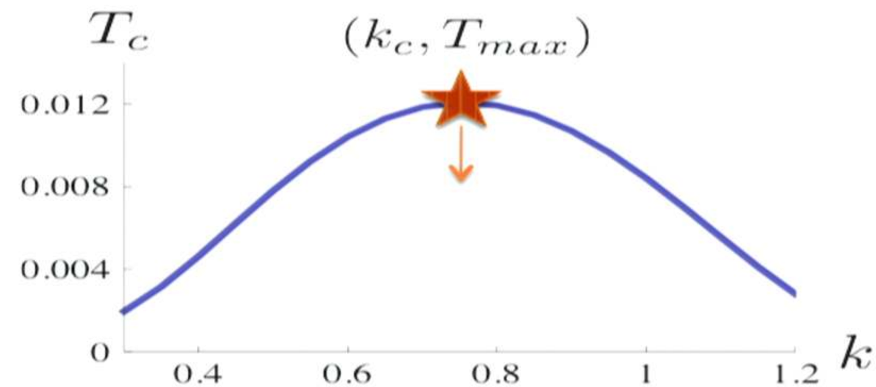

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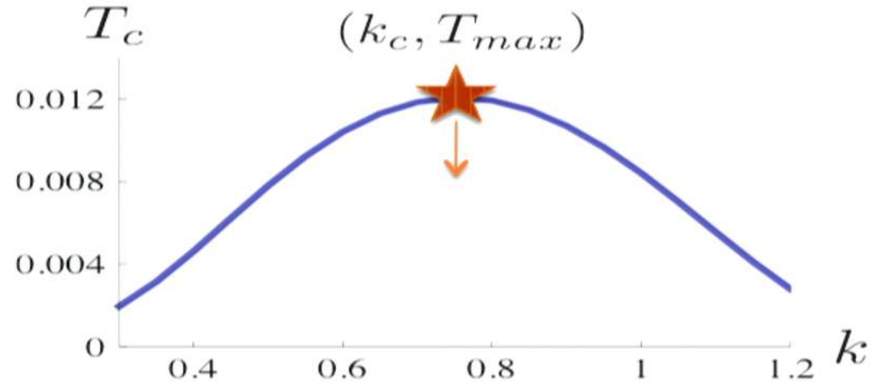

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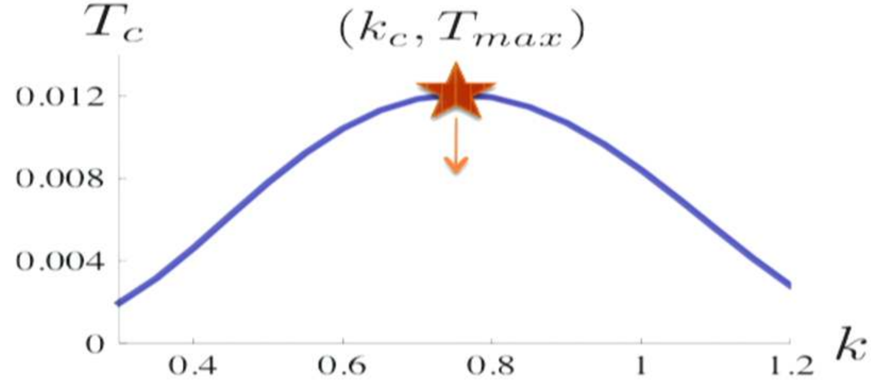

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Non-linear system: Ansatz

$$ds^2 = -2r^2 f_{RN} e^{2A(r,x)} dt^2 + e^{2B(r,x)} \left(\frac{dr^2}{2r^2 f_{RN}} + 2r^2 dx^2 \right) + 2r^2 e^{2C(r,x)} (dy - W(r,x)dt)^2$$

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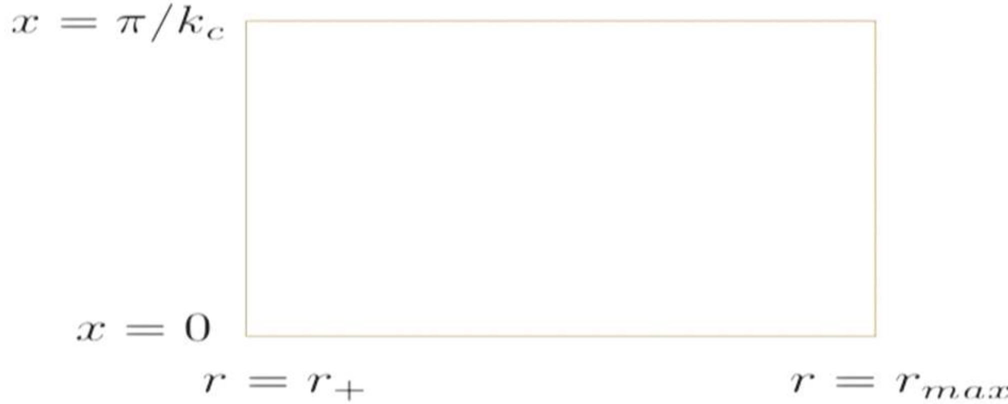

7 fields:
$A, B, C, A_t, W, A_y, \psi$

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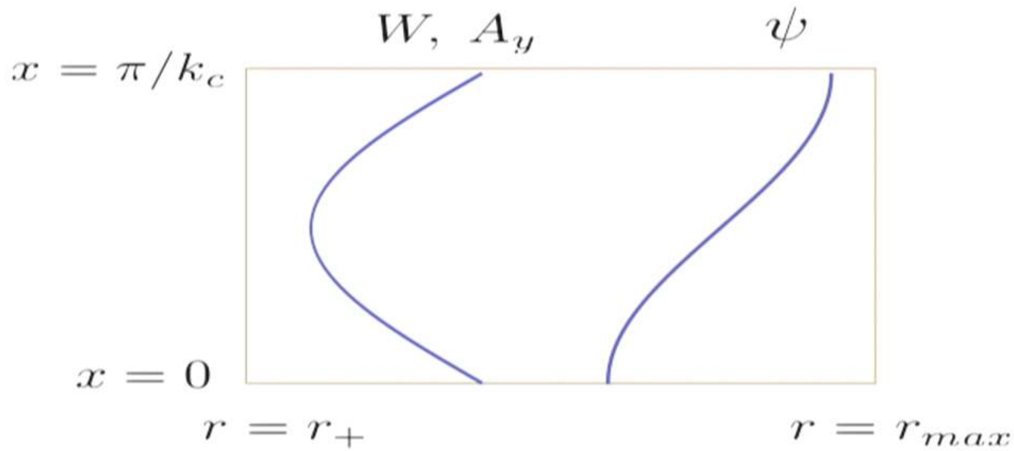


Non-linear system: Equations of motion

$$\begin{aligned} G^t{}_t &= T^t{}_t \\ G^t{}_y &= T^t{}_y \\ G^t{}_y &= T^t{}_y \\ G^r{}_r + G^x{}_x &= T^r{}_r + T^x{}_x \end{aligned}$$

2^{nd} order, elliptic E.O.M. for A, B, C, W

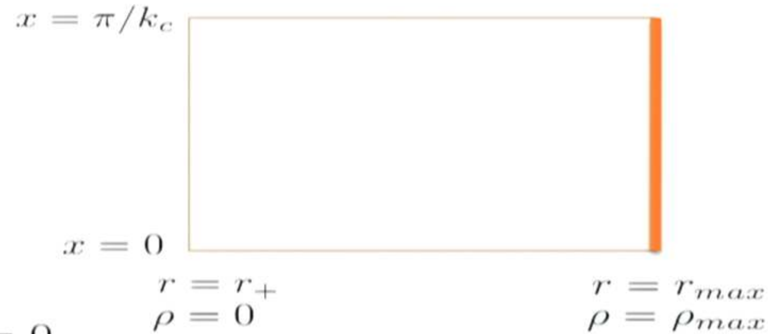
$$\begin{aligned} G^r{}_x &= T^r{}_x \\ G^r{}_r - G^x{}_x &= T^r{}_r - T^x{}_x \end{aligned}$$

constraint equations

E.O.M. + boundary conditions $\Longrightarrow$ constraints are satisfied

Wiseman, hep-th/0209051

Non-linear system: Boundary conditions



1. $r = r_{max}$: falloff conditions

$$A = \cancel{A_0}(x) + \cdots + \frac{A_3(x)}{r^3} + \ldots$$

2. $r = r_+$:
   - regularity conditions
     With $\rho = \sqrt{r^2 - r_+^2}$, $\partial_\rho C|_{\rho=0} = 0$
   - constraints
     $$\partial_x (A - B)|_{\rho=0} = 0 \Rightarrow A|_{\rho=0} = B|_{\rho=0} - d_0 \qquad (T = T_{max} e^{-d_0})$$

3. $x = 0, \ x = \pi/k_c$: Neumann or Dirichlet conditions

Solve: Finite difference, use Gauss-Siedel relaxation

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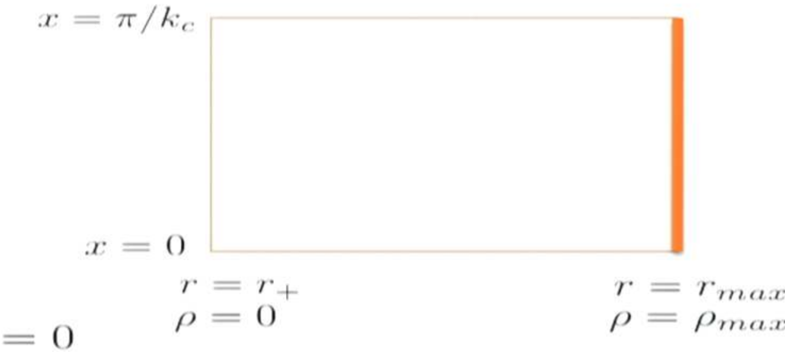

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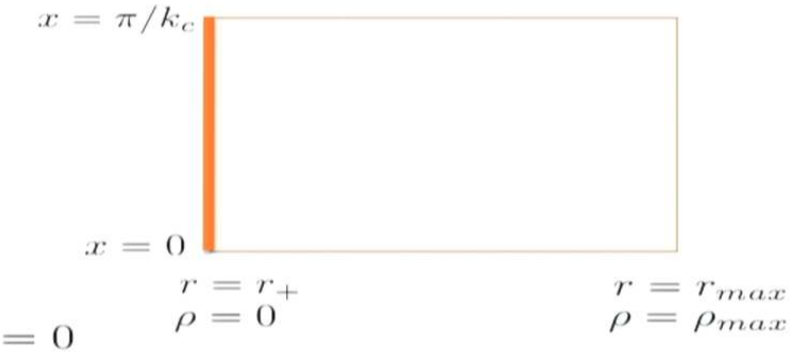

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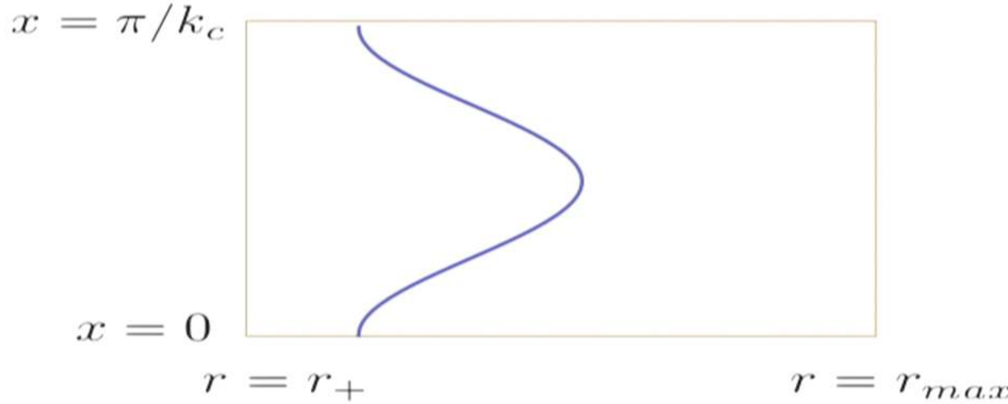

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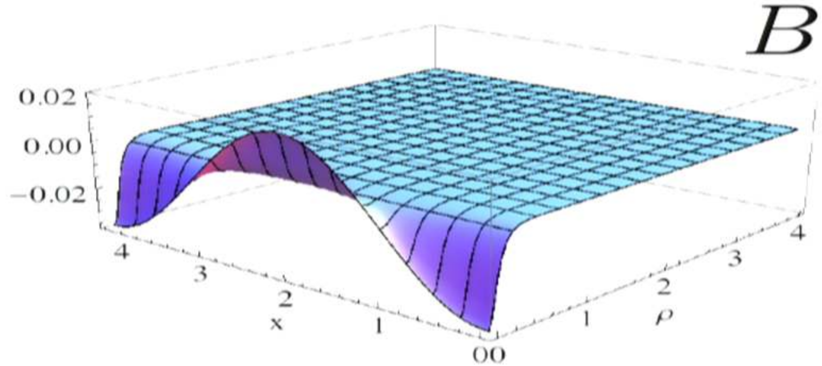
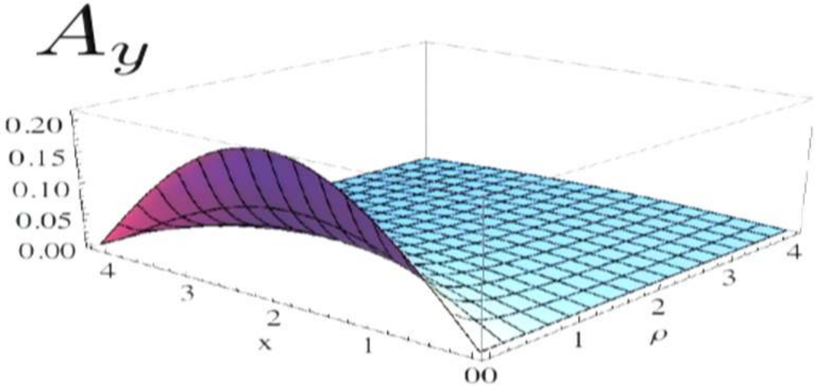
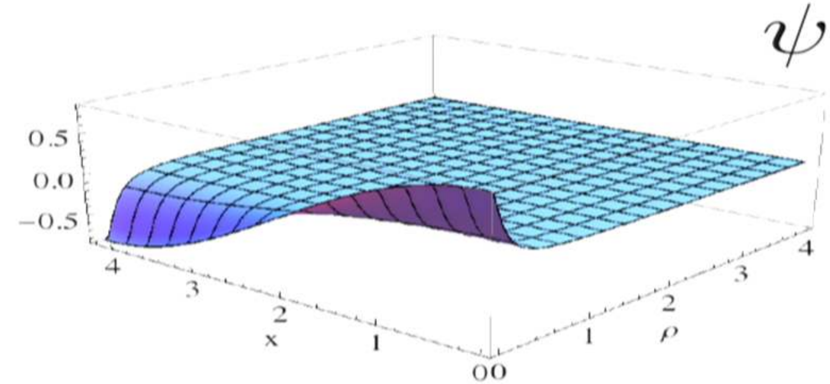
Preliminary results







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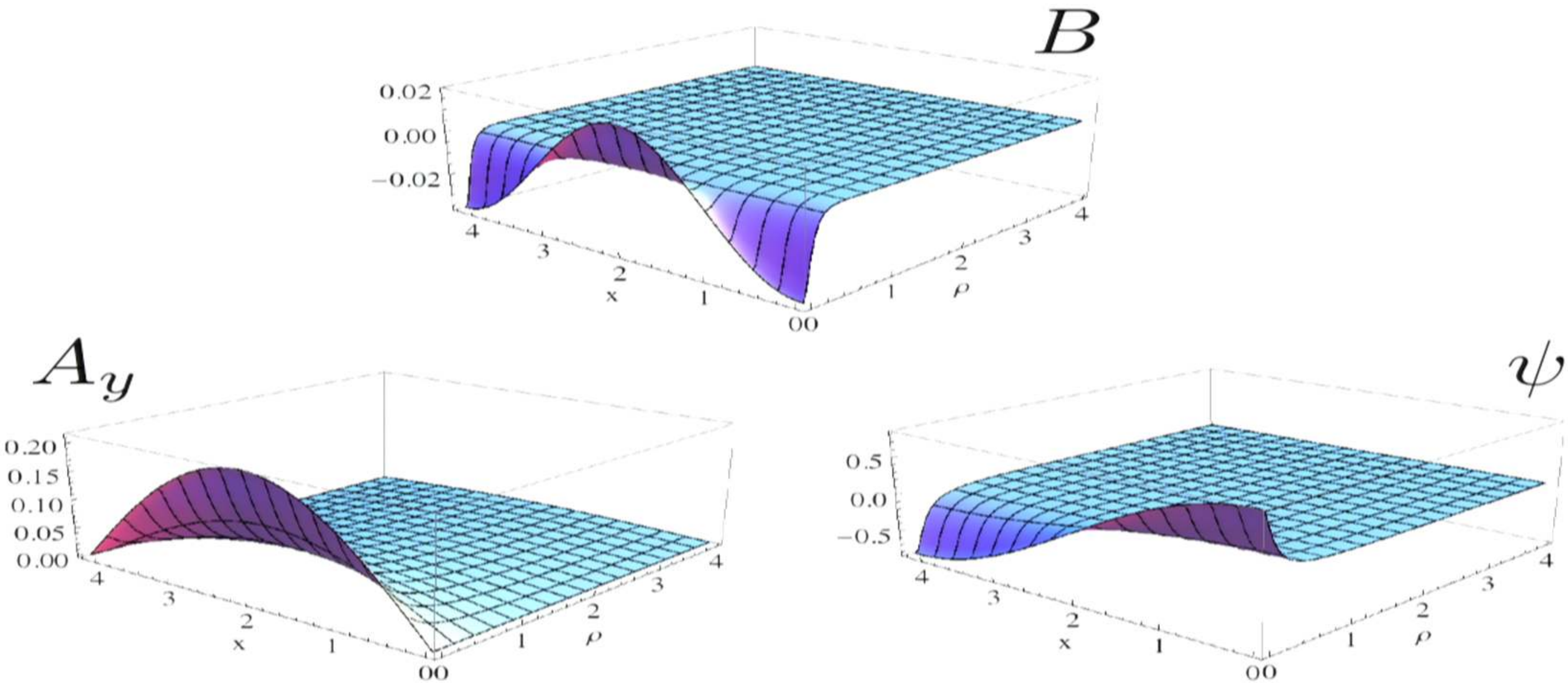

Further questions

1. What are the properties of the field theory?
2. Is this the dominant phase?
3. Zero temperature limit of this phase?