Title: TBA

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Abstract: TBA

From quantum field theory to quantum gravity

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AdS/CFT correspondence

[Maldacena]
[Gubser, Klebanov, Polyakov; Witten]

$$Z[J(x)] = \int D\phi(x) e^{-S_{field\ theory}[\phi]}$$ D-dim QFT

$$= \int D \text{ ``}J(x,z)\text{''}\, e^{-S'[J(x,z)]}\Big|_{J(x,0)=J(x)}$$ (D+1)-dim gravity

- D-dim QFT is dual to (D+1)-dim gravitational theory
  - N=4 SU(N) gauge theory in 4D = IIB superstring theory in $AdS_5 x S^5$
  - Weak coupling description for strongly coupled QFT for a large N
  - Non-perturbative definition of quantum gravity
- Believed to be a general framework for a large class of QFT's

[Akhmedov; Das, Jevicki; Gopakumar; Heemskerk, Penedones, Polchinski; Lee; Faulkner, Liu, Rangamani; Douglas, Mazzucato, Razamat,...]

- No first-principle derivation yet

Holographic RG :
Classical RG = Classical GR

Beta functionals of boundary field theory

= Saddle point equations in the bulk [Verlinde]

Low energy

$$J(x,z) \;=\; J(x,0)$$

z

$$S_0 + \int dx J(x) O(x)$$

High energy

x

D –dim flat space

The identification of bulk EOM with beta function is meaningful only in the large N limit.

What is the general framework ?

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This talk :

- Can one explicitly construct a quantum theory of gravity in the bulk from a boundary QFT ?
  - To identify gravitational dual for general QFT's that arise in nature, e.g., condensed matter systems
  - To give microscopic justification for the AdS/CFT conjecture
- Goal : QFT ➔ bulk theory
- Quantum RG = Quantum GR

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- Can one explicitly construct a quantum theory of gravity in the bulk from a boundary QFT ?
  - To identify gravitational dual for general QFT's that arise in nature, e.g., condensed matter systems
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- Quantum RG = Quantum GR

Step 0 : Partition function is a functional of spacetime dependent sources

$$Z[J(x)] = \int_{|k|<\Lambda} D\phi \; e^{-S[\phi; J_n(x)]}$$

$$S = \int d^D x \left[J^{\mu\nu}(x)\partial_\mu\phi\partial_\nu\phi + J_2(x)\phi^2 + J_4(x)\phi^4 + ... \right]$$

Sources : background metric, spacetime dependent mass, etc

Step 1 : Separate low energy mode from high energy mode

$$Z[J(x)] = \int D\phi_< D\phi_> \; e^{-S[\phi_< + \phi_>; J_n(x)]}$$

$$\phi_< : |k| < \Lambda e^{-dz}, \quad \phi_> : \Lambda e^{-dz} < |k| < \Lambda$$

Cf. In conventional RG, one integrates out the high energy mode to obtain an effective action for the low energy mode with renormalized couplings

$$Z[J(x)] = \int D\phi_< \; e^{-S[\phi_<; J_n(x) + \delta J_n(x)]}$$

Classical Beta functional : $\beta_n(x) = \dfrac{dJ_n(x)}{dz}$

Here we do something else....

Step 2 : Interpret high energy mode as dynamical sources for low energy mode

$$Z[J(x)] = \int D\phi_< D\phi_> \; e^{-S[\phi_< + \phi_>; J_n]}$$

$$= \int D\phi_< D\phi_> \; e^{-S[\phi_<; J_n + f_n[\phi_>; J_m]]}$$

$$\sum_n J_n(x)(\phi_< + \phi_>)^n = [J_1 + 2J_2\phi_> + 3J_3\phi_>^2 + ..]\phi_<$$

$$+ [J_2 + 3J_3\phi_> + ...]\phi_<^2 + ...$$

$$f_1[\phi_>; J_m(x)] = 2J_2\phi_> + 3J_3\phi_>^2 + ..$$

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$$f_1[\phi_>; J_m(x)] = 2J_2\phi_> + 3J_3\phi_>^2 + ..$$

$$f_2[\phi_>; J_m(x)] = 3J_3\phi_> + ..$$

Step 3 : Decouple the high energy and low energy modes through auxiliary fields

$$Z[J(x)] = \int Dj_n^{(1)} Dp_n^{(1)} D\phi_< D\phi_> \, e^{-S'}$$

$$S' = \int dx \;\; i(j_n^{(1)} - J_n - f_n[\phi_>; J_m])p_n^{(1)} - S[\phi_<; j_n^{(1)}]$$

j_n : dynamical source $\quad p_n$: Lagrangian multiplier

At this stage, the auxiliary fields do not have any dynamics

Step 4 : integrate out high energy mode to generate dynamical action for the auxiliary fields

$$Z[J(x)] = \int Dj_n^{(1)} Dp_n^{(1)} D\phi_< \; e^{-S''}$$

$$S'' = \int dx \;\; i(j_n^{(1)} - J_n)p_n^{(1)} + S[\phi_<; j_n^{(1)}] + \delta S(J_n, p_n^{(1)})$$

We obtain a theory for low energy mode coupled with fluctuating sources whose dynamics is controlled by an action generated out of the high energy mode

Step 3 : Decouple the high energy and low energy modes through auxiliary fields

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Step 5 : repeat 1-4 again and again

$$Z[J(x)] = \int \prod_{l=1}^{\infty} Dj_n^{(l)} Dp_n^{(l)} \; e^{-S_{D+1}}$$

$$S_{D+1} = \sum_{l=1}^{\infty} \int dx \quad \left[i(j_n^{(l)} - j_n^{(l-1)}) p_n^{(l)} + H(j_n^{(l-1)}, p_n^{(l)}) dz \right]$$

- A set of dynamical sources are introduced at each step of RG at the expense of decimating high energy mode bit by bit
- The length scale becomes an extra coordinate

$$j_n^{(l)}(x) \rightarrow j_n(x, z) \qquad\qquad z = l dz$$

$$S_{D+1} = \int dx dz \quad [i(\partial_z j_n) p_n + H(j_n, p_n)]$$

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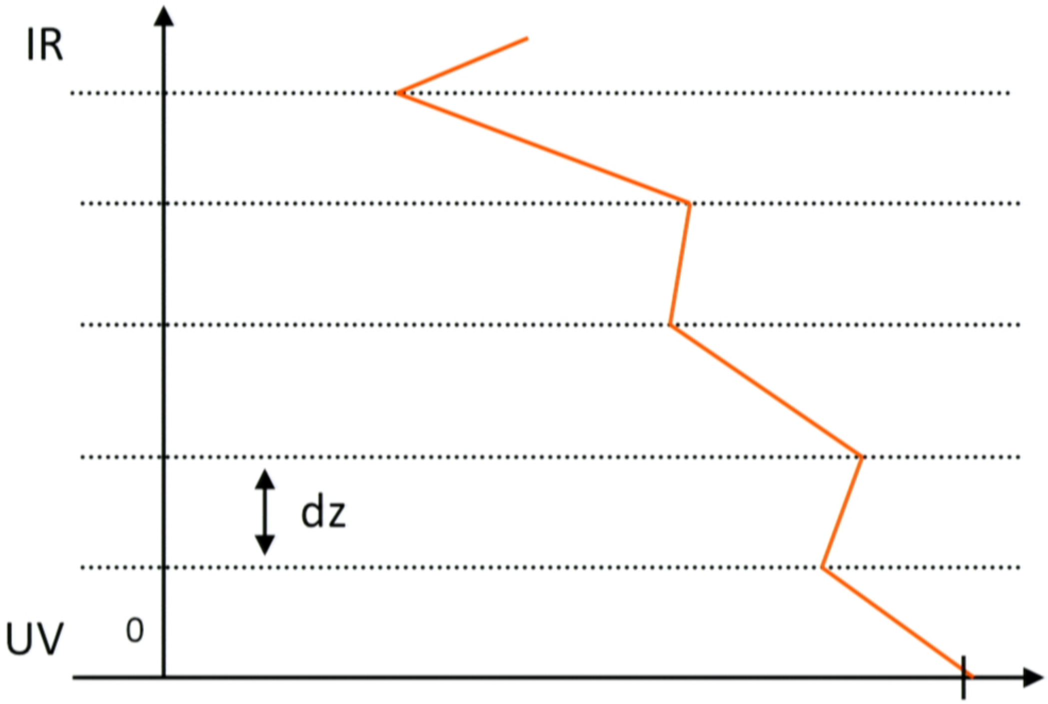
Extra dimension as a length scale



$j_n^{(l)},\ p_n^{(l)}$

Boundary condition $j_n^{(0)}(x) = J_n(x)$

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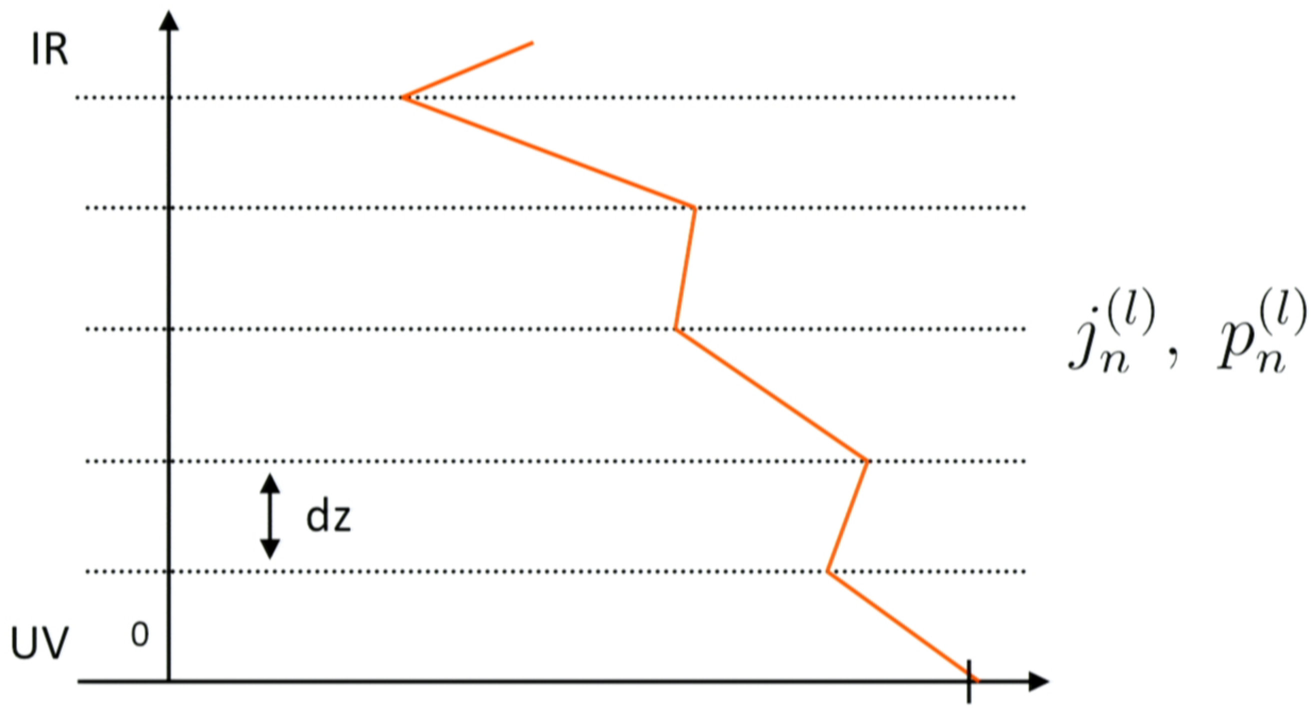

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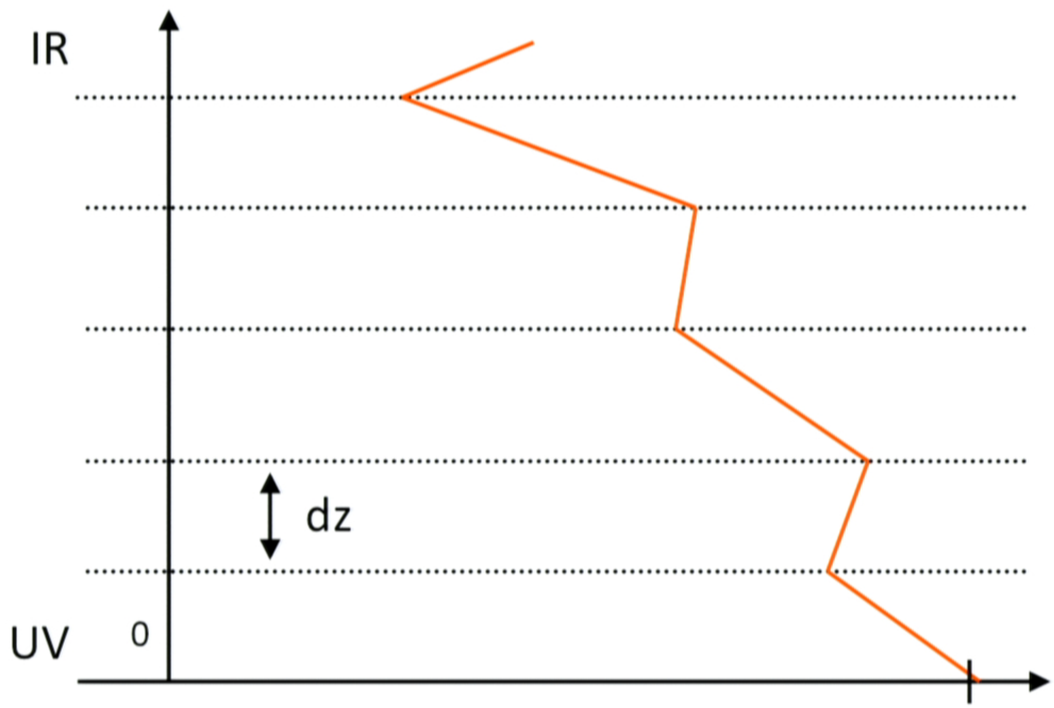

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Key features

- An exact change of variables
- D-dimensional partition function can be written as (D+1)-dimensional functional integration for dynamical sources and their conjugate fields
- General scheme : can be applied to any field theory
- Degrees of freedom are dynamical sources and dynamical operators

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Quantum beta function

$$Z = \lim_{\beta \to \infty} < \Psi_f | e^{-\beta H} | \Psi_i >$$

Wavefunction for D - dimensional spacetime dependent sources

- Partition function is written as a transition amplitude of D-dimensional quantum wavefunction of coupling constants if one interprets z (scale) as `time'
- The Hamiltonian generates scale transformation for dynamical couplings
- The Heisenberg equation : quantum beta function

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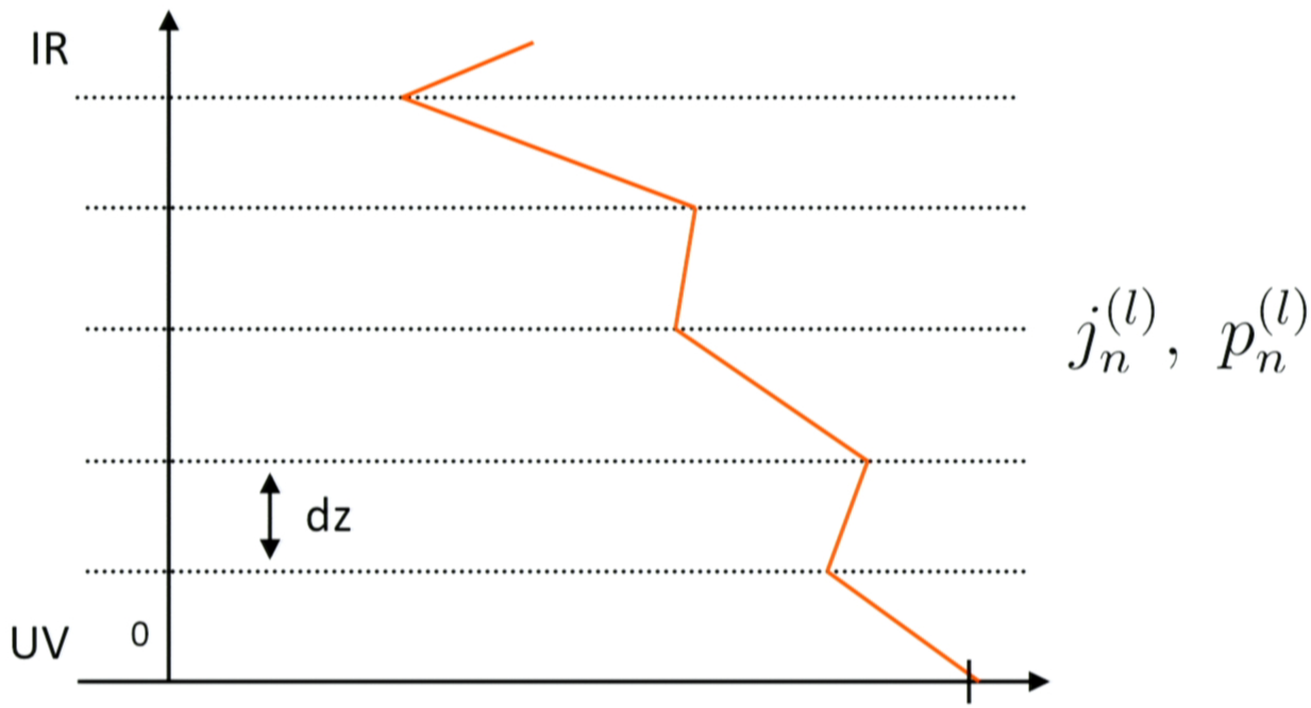

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Is the holographic theory a gravitational theory ?

- Energy momentum tensor ➔spin-2 mode in the bulk
- What is the origin of the (D+1)-dimensional diffeomorphism invariance ?

Local RG prescription

- Spacetime dependent coarse graining

$$-\phi\left(\nabla^2+\frac{\nabla^4}{\Lambda^2}+\ldots\right)\phi \to -\phi\left(\nabla^2+\frac{\nabla^4}{\Lambda(x)'^2}+\ldots\right)\phi$$

$$\Lambda(x)' = \Lambda e^{-\alpha(x)dz}$$

Speed of coarse graining

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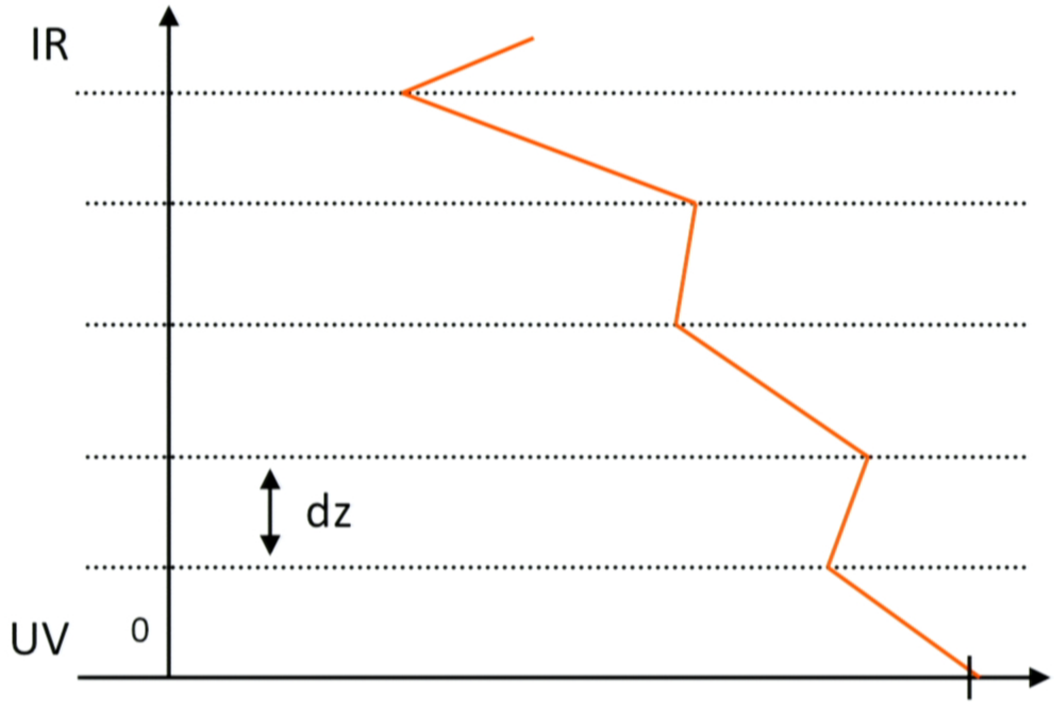

$$j_n^{(l)},\ p_n^{(l)}$$

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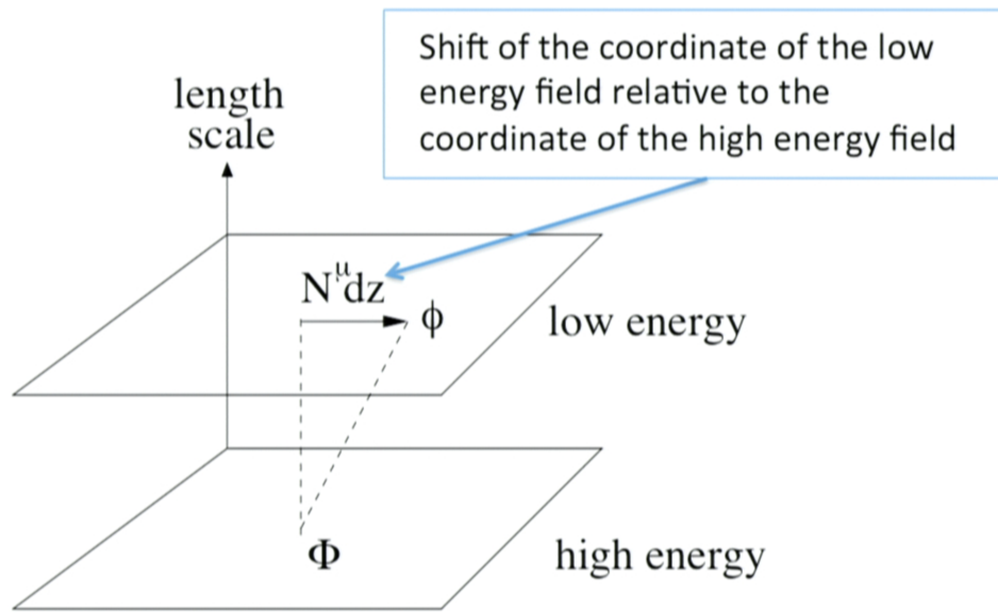
Shift

- One does not have to choose the coordinate of the low energy field as the coordinate of the high energy mode



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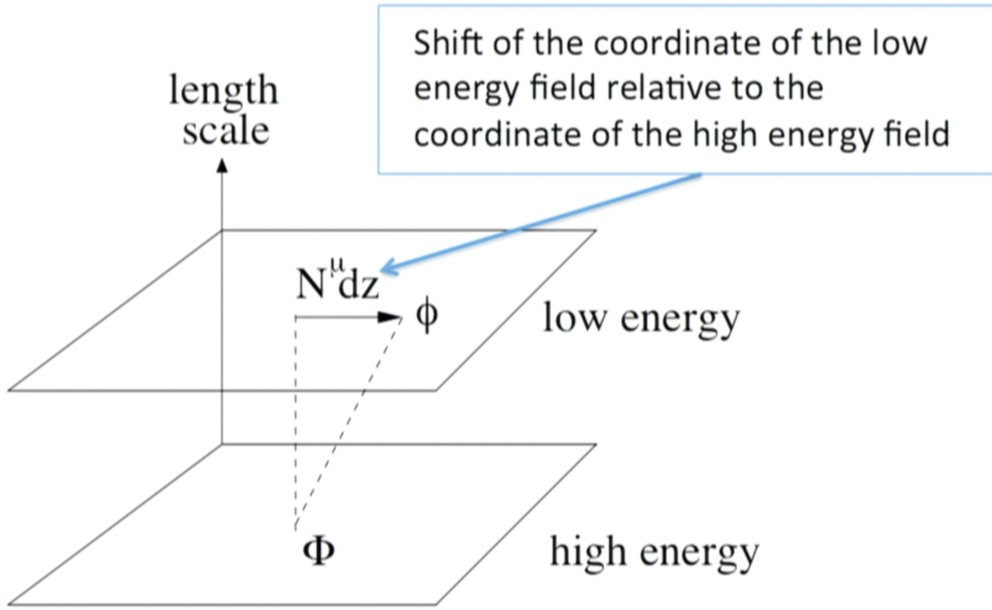
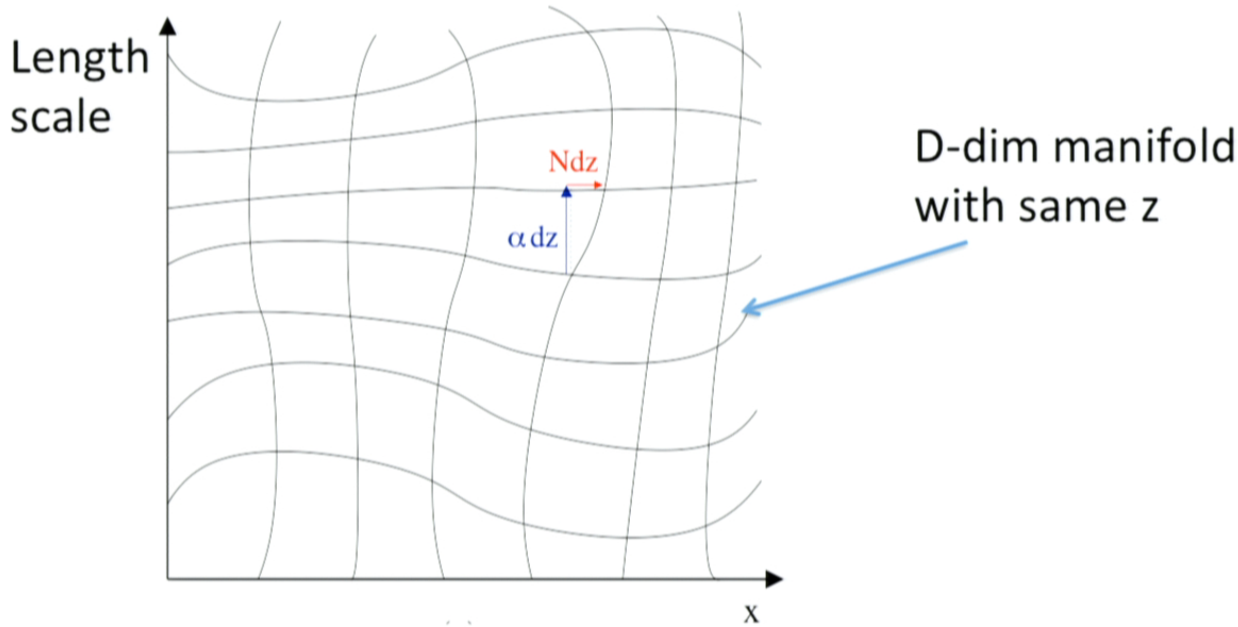


Diffeomorphism = Freedom to choose different local RG schemes



D-dimensional matrix field theory

single-trace operators

$$O_{[q+1;\{\mu_j^i\}]} = \frac{1}{N}\mathrm{tr}\left[\Phi\left(\partial_{\mu_1^1}\partial_{\mu_2^1}..\partial_{\mu_{p_1}^1}\Phi\right)\left(\partial_{\mu_1^2}\partial_{\mu_2^2}..\partial_{\mu_{p_2}^2}\Phi\right)\dots\left(\partial_{\mu_1^q}\partial_{\mu_2^q}..\partial_{\mu_{p_q}^q}\Phi\right)\right]$$

Spacetime dependent sources

Φ : N x N tracelss symmetric real matrix field

$$Z[\mathcal{J}] = \int D\Phi \;\exp\left[iN^2\int d^Dx\left(-\mathcal{J}^m O_m + V[O_m;\mathcal{J}^{\{m_i\},\{\nu_j^i\}}]\right)\right]$$

multi-trace deformation

$$V[O_m;\mathcal{J}^{\{m_i\},\{\nu_j^i\}}] = \sum_{q=1}^{\infty}\mathcal{J}^{\{m_i\},\{\nu_j^i\}}O_{m_1}\left(\partial_{\nu_1^1}..\partial_{\nu_{p_1}^1}O_{m_2}\right)\left(\partial_{\nu_1^2}..\partial_{\nu_{p_2}^2}O_{m_3}\right)\dots\left(\partial_{\nu_1^q}..\partial_{\nu_{p_q}^q}O_{m_{q+1}}\right)$$

Introduce auxiliary fields to remove multi-trace operator

$$\mathcal{L} = N^2\Big\{-\mathcal{J}^{(0)m}O^{(0)}_m + V[O^{(0)}_m;\mathcal{J}^{(0);\{m_i\},\{\nu^i_j\}}]\Big\}$$

$$Z \;=\; \int Dj^{(1)n}Dp^{(1)}_n D\Phi \;\; e^{i\int d^Dx\;\mathcal{L}_1}$$

$$\mathcal{L}_1 \;=\; N^2\Big\{j^{(1)m}(p^{(1)}_m - O^g_m) - \mathcal{J}^{(0)m}f_m{}^n(G^{(0)},g)p^{(1)}_n + V[f_m{}^n(G^{(0)},g)p^{(1)}_n;\mathcal{J}^{(0);\{m_i\},\{\nu^i_j\}}]\Big\}$$

- $j^{(1)m}$: dynamical source, $p^{(1)}{}_m$: dynamical operator
- Only single-trace operators
- Due to multi-trace operators, the sources for the single-trace operators become dynamical

Local RG

- Integrate out the high energy field

$$\mathcal{L}_2 = N^2\Big\{V[f_m{}^n(0,1)P_n^{(1)};\mathcal{J}^{(0);\{m_i\},\{\nu_j^i\}}]$$

Casimir energy [Sakharov (68)]

$$+\left(J^{(1)n}-\mathcal{J}^{(0)m}f_m{}^n(0,1)\right)P_n^{(1)}+\delta_{\alpha^{(1)}}\mathcal{L}[J^{(1)m}]$$

$$-(J^{(1)m}+\delta_{\alpha^{(1)}}J^{(1)m\{\mu\}}\nabla^{(1)}_{\{\mu\}})O_m^{(1)}+\frac{\delta_{\alpha^{(1)}}J^{(1)mn\{\mu\}\{\nu\}}}{\sqrt{|G^{(1)}|}}(\nabla^{(1)}_{\{\mu\}}O_m^{(1)})(\nabla^{(1)}_{\{\nu\}}O_n^{(1)})\Big\}$$

Quantum corrections to single-trace operators

Double-trace operators

Wilsonian beta function

$$\delta_{\alpha^{(1)}}\mathcal{L}[J^{(1)m}] = dz\,\alpha^{(1)}(x)\sqrt{|G^{(1)}|}\,\{C_0[J^{(1)}]+C_1[J^{(1)}]\mathcal{R}+\ldots\},$$

$$\delta_{\alpha^{(1)}}J^{(1)m\{\mu\}} = dz\,\alpha^{(1)}(x)A^{m\{\mu\}}[J^{(1)}],$$

$$\delta_{\alpha^{(1)}}J^{(1)mn\{\mu\}\{\nu\}} = dz\,\alpha^{(1)}(x)B^{mn\{\mu\}\{\nu\}}[J^{(1)}].$$

Introduce auxiliary fields to remove multi-trace operator

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(D+1)-dimensional gravity

$$Z[\mathcal{J}] = \int \mathcal{D}J(x,z)\mathcal{D}\pi(x,z)\ e^{i\left(S_{UV}[\pi(x,0)]+S[J(x,z),\pi(x,z)]+S_{IR}[J(x,\infty)]\right)}\Bigg|_{J(x,0)=\mathcal{J}(x)}$$

Bulk action : $$S = N^2 \int d^D x dz \left[(\partial_z J^n)\pi_n - \alpha(x,z)\mathcal{H} - N^\mu(x,z)\mathcal{H}_\mu \right]$$

Hamiltonian constraint : $$\mathcal{H} = \tilde{A}^{\mu\nu}[J(x)]\pi_{[2,\mu\nu]} - \frac{\tilde{B}^{\mu\nu\lambda\sigma}[J(x)]}{\sqrt{|G|}}\pi_{[2,\mu\nu]}\pi_{[2,\lambda\sigma]} - \sqrt{|G|}\left\{ C_0[J(x)] + C_1[J(x)]\mathcal{R} \right\} + \ldots,$$

Momentum constraint :

$$\mathcal{H}_\mu = -2\nabla^\nu \pi_{[2,\mu\nu]} - \sum_{[q,\{\mu^i_j\}]\neq[2,\mu\nu]} \left[\sum_{a,b} \nabla_\nu \left(J^{[q,\{\mu^1_1\mu^1_2\ldots\mu^a_{b-1}\nu\mu^a_{b+1}\ldots\}]}\pi_{[q,\{\mu^1_1\mu^1_2\ldots\mu^a_{b-1}\mu\mu^a_{b+1}\ldots\}]} \right) + (\nabla_\mu J^{[q,\{\mu^i_j\}]})\pi_{[q,\{\mu^i_j\}]} \right].$$

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Bulk equation of motion

- In the large N limit, the bulk fields become classical

E.O.M. $$\partial_z J^n = \{J^n, \mathbf{H}\}, \quad \partial_z \pi_n = \{\pi_n, \mathbf{H}\}$$

Hamiltonian $$\mathbf{H} = \int d^D x [\alpha \mathcal{H} + N^\mu \mathcal{H}_\mu]$$

Poisson bracket $$\{A, B\} = \int d^D x \left[\frac{\delta A}{\delta J^n} \frac{\delta B}{\delta \pi_n} - \frac{\delta A}{\delta \pi_n} \frac{\delta B}{\delta J^n} \right]$$

Bulk equation of motion

- In the large N limit, the bulk fields become classical

E.O.M. $$\partial_z J^n = \{J^n, \mathbf{H}\}, \quad \partial_z \pi_n = \{\pi_n, \mathbf{H}\}$$

Hamiltonian $$\mathbf{H} = \int d^D x[\alpha\mathcal{H} + N^\mu \mathcal{H}_\mu]$$

Poisson bracket $$\{A, B\} \;=\; \int d^D x \left[\frac{\delta A}{\delta J^n}\frac{\delta B}{\delta \pi_n} - \frac{\delta A}{\delta \pi_n}\frac{\delta B}{\delta J^n}\right]$$

Summary

- A D-dimensional QFT is explicitly mapped into a (D+1)-dimensional quantum theory of gravity based on a local RG
  – Length scale can be changed in a spacetime dependent way
  – Coordinate of low energy field can be shifted relative to the coordinate of high energy field

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Maldacena's AdS/CFT = Wilson's RG + Sakharov's Induced gravity