Title: Dynamics of AdS-CMT Quenches

Date: Jun 08, 2012 09:00 AM

URL: http://pirsa.org/12060027

Abstract: I will describe numerical simulations of quenches in AdS-CMT superconductors where we are able to construct a dynamical phase diagram for the system. I will describe how the late time behaviour is
understood in terms of the quasinormal modes of the system, and how a rather generic behaviour of the pole structure there leads to interesting physical consequences that have an analog in condensed matter calculations using integrable models.

Quenches in `holographic superconductors'

Toby Wiseman (Imperial)

with Joe Bhaseen, Ben Simons (Cambridge), Sonner (DAMTP), Gauntlett (Imperial)

- **arXiv:1206.xxxx**

Quenches in `holographic superconductors'

Toby Wiseman (Imperial)

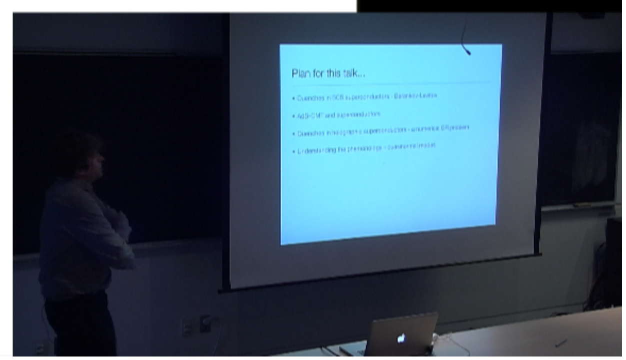
with Joe Bhaseen, Ben Simons (Cambridge), Julian Sonner (DAMTP), Jerome Gauntlett (Imperial)

- **arXiv:1206.xxxx**

Condensed matter theorists!

Plan for this talk...

- Quenches in BCS superconductors - Barankov-Levitov
- AdS-CMT and superconductors
- Quenches in holographic superconductors - a numerical GR problem
- Understanding the phemonology - quasinormal modes



AdS-CFT

- AdS-CFT states that a certain class of (rather special) CFTs are equivalent to gravitational theories (possibly higher spin or string theories) in spacetimes that asymptote to AdS.
- In particular this is made concrete in the case of *N*=4 susy SU(N) YM which is a CFT whose dual is understood to be a closed string theory. The vacuum geometry is $AdS_5 \times S^5$ - radius ℓ. The parameters are related as;

$$g_{YM}^2 = g_s \qquad\qquad \lambda = N^2 g_{YM} = \left(\frac{l}{l_s}\right)^4$$

- In the 't Hooft limit, $N \to \infty$ and finite $\lambda = N^2 g_{YM}$ the string coupling becomes small.
- In the large λ limit the target space becomes weakly curved, and stringy corrections can be ignored, reducing the dual to supergravity, which can be truncated to simply to 5-d gravity and a negative cosmological const.

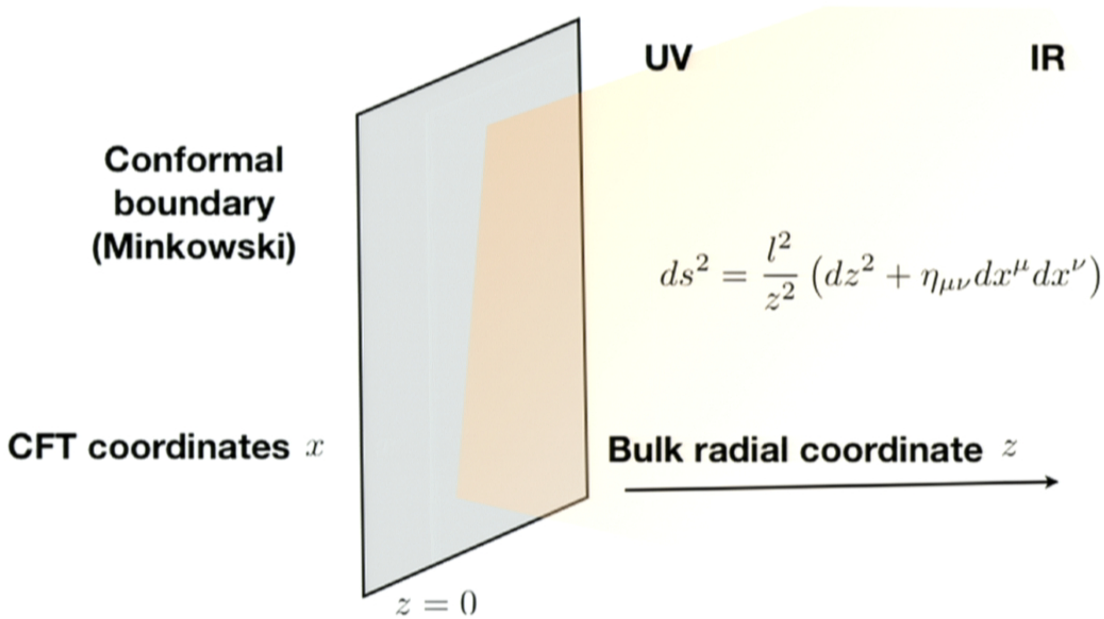
AdS-CFT

AdS-CFT

AdS-CFT

- The vacuum geometry is AdS - the CFT 'lives' on the boundary

$$\Lambda = -\frac{4}{l^2}$$

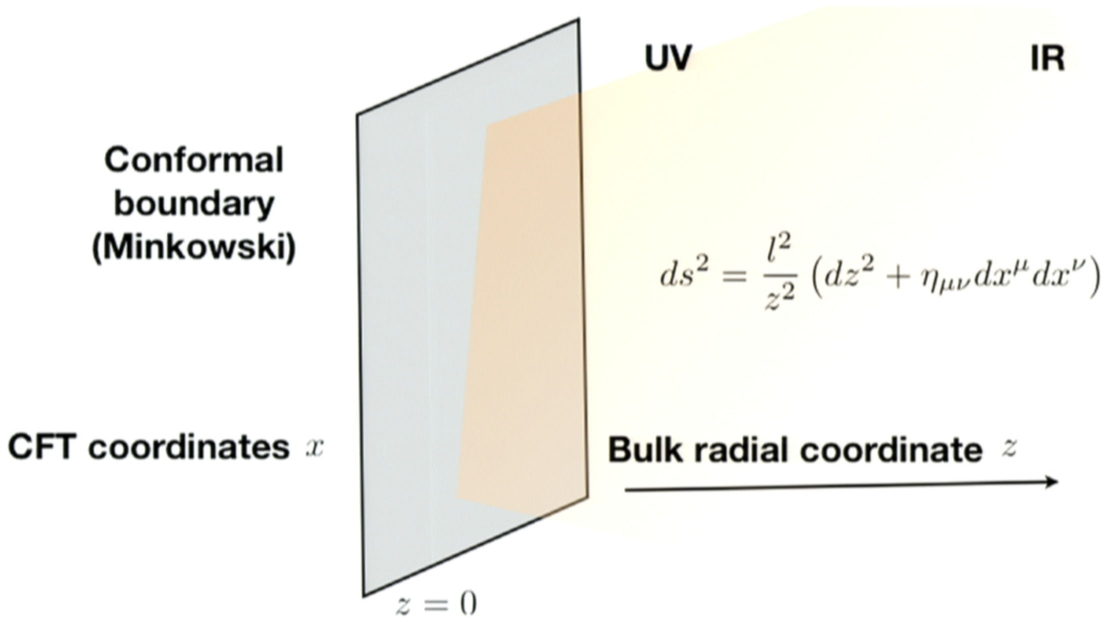


$$ds^2 = \frac{l^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

AdS-CFT

- The vacuum geometry is AdS - the CFT ‘lives’ on the boundary

$$\Lambda = -\frac{4}{l^2}$$

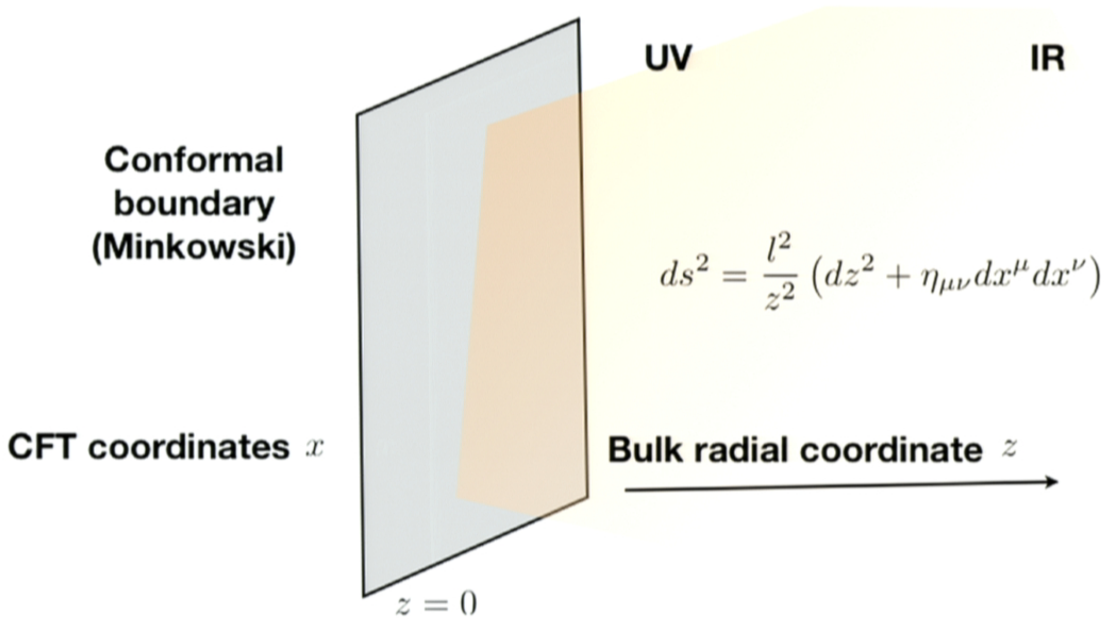


$$ds^2 = \frac{l^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

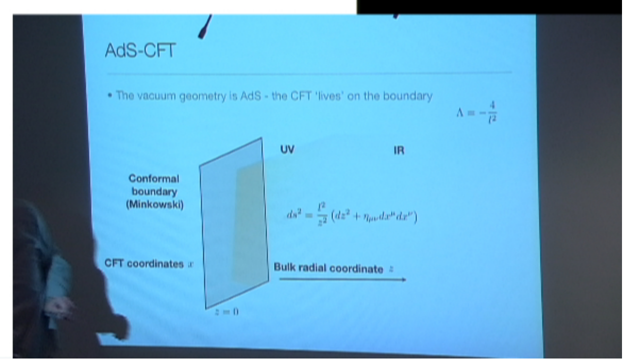
AdS-CFT

- The vacuum geometry is AdS - the CFT ‘lives’ on the boundary

$$\Lambda = -\frac{4}{l^2}$$



$$ds^2 = \frac{l^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$



AdS-CFT

- Every field in the gravity (string theory) $\phi(x,z)$ corresponds to an operator $O(x)$ in the CFT. The boundary requires boundary conditions - fix $J(x)$
- Solving the gravity gives CFT initially in vacuum with source $J(x)$

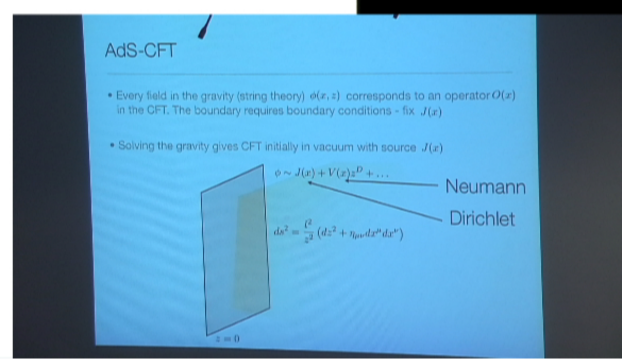
$$\phi \sim J(x) + V(x) z^D + \dots$$

Neumann

Dirichlet

$$ds^2 = \frac{l^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$

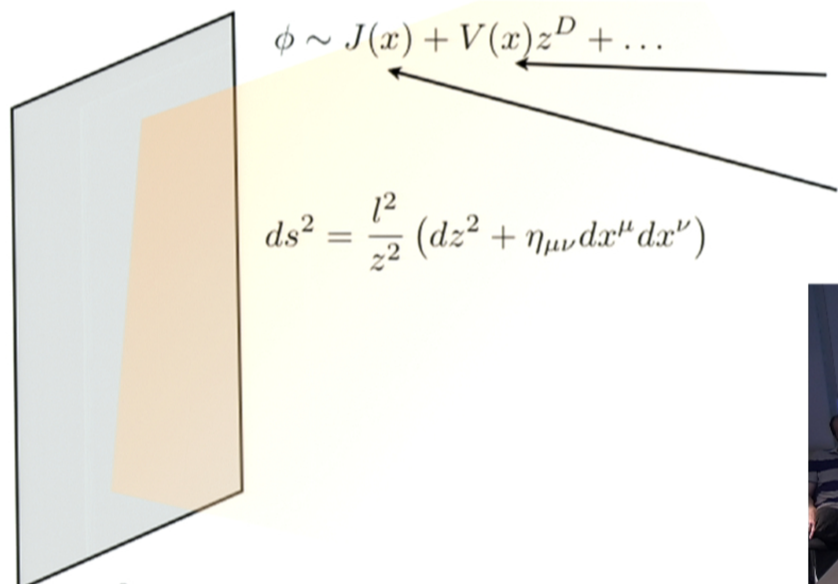
$z = 0$



AdS-CFT

- Every field in the gravity (string theory) $\phi(x,z)$ corresponds to an operator $O(x)$ in the CFT. The boundary requires boundary conditions - fix $J(x)$
- Solving the gravity gives CFT initially in vacuum with source $J(x)$

$$\phi \sim J(x) + V(x) z^D + \dots$$



$$ds^2 = \frac{l^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

$z = 0$

AdS-CFT

- The vev $< O(x) >= V(x)$
- Also correlation functions $< O(x_1)O(x_2)\dots >$, Wilson loops, entanglement S....

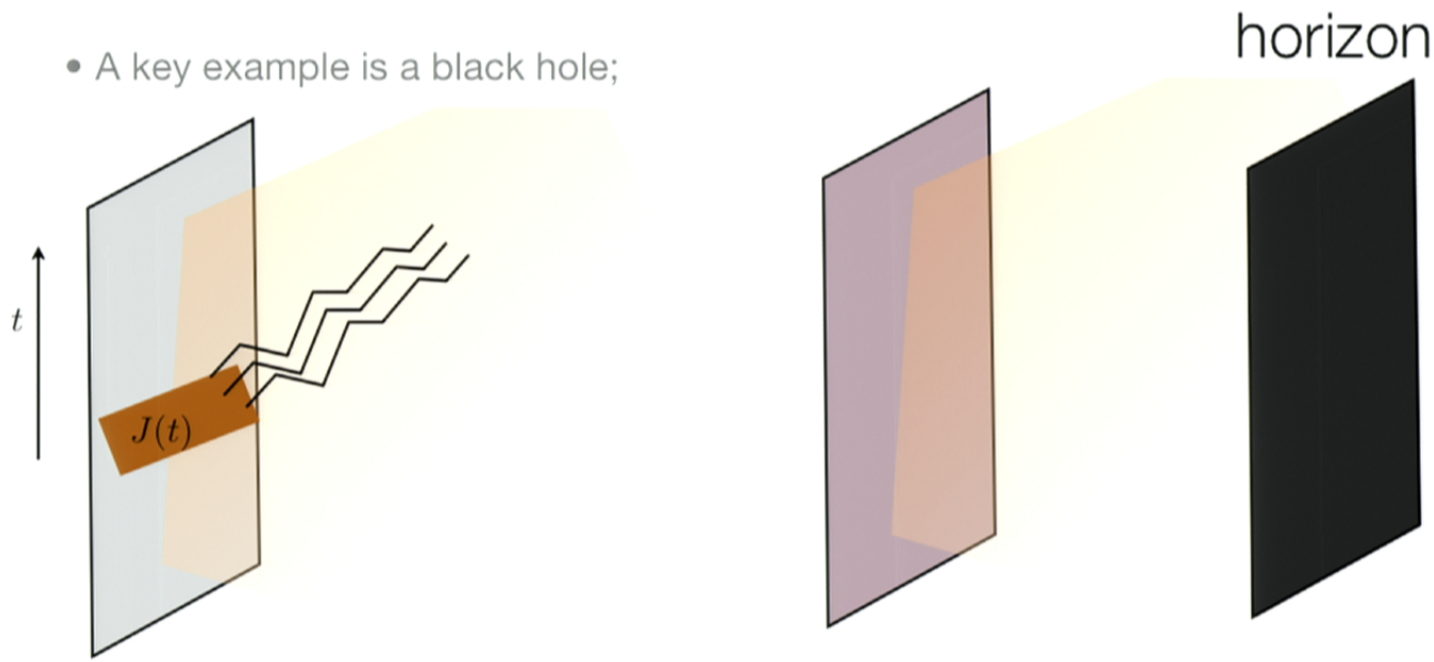
$$\phi \sim J(x) + V(x)z^D + \dots$$

$$ds^2 = \frac{l^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

AdS-CFT

- One perturbs the theory by turning on a source, or starting in a non-vacuum state.

- A key example is a black hole;



AdS-CMT

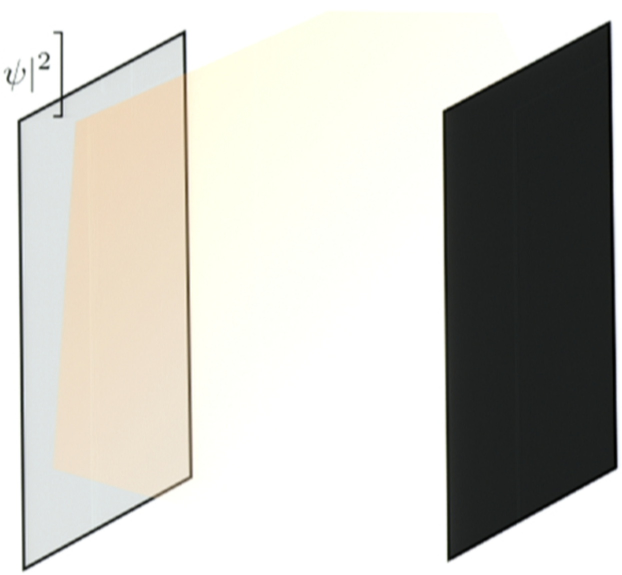
- Much interest has arisen also in rather exotic black holes where the gravity has a complex scalar charged under a vector.

$$S = \int d^4x\sqrt{-g}\left[R + \frac{6}{l^2} - \frac{1}{4}F^2 - |D\psi|^2 - m^2|\psi|^2\right]$$

$$F = dA \qquad D\psi = (d - 2iA)\psi$$

$$q = 2$$

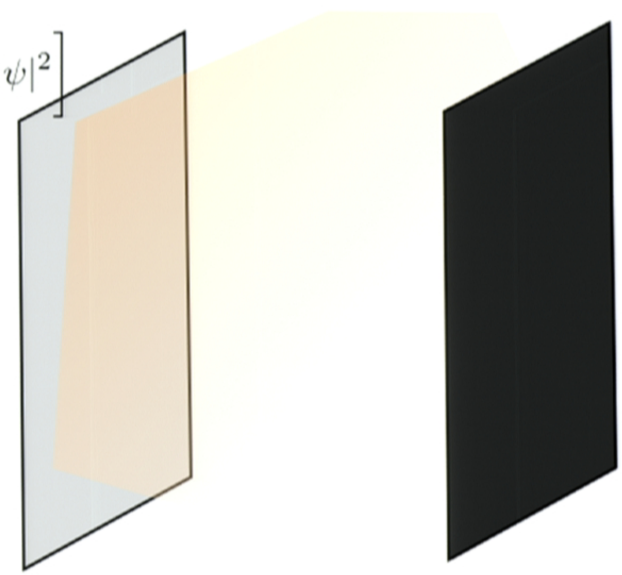
$$m = -\frac{2}{L^2}$$



AdS-CMT

- Much interest has arisen also in rather exotic black holes where the gravity has a complex scalar charged under a vector. The vector is dual to a conserved current in the boundary. We may then turn on a chemical potential source for this current.

$$S = \int d^4x\sqrt{-g}\left[R + \frac{6}{l^2} - \frac{1}{4}F^2 - |D\psi|^2 - m^2|\psi|^2\right]$$



$j \leftrightarrow A$

$$A(x,z) = A_0(x) + zA_1(x) + \ldots$$

$A_0 = \mu dt$ $\quad$ $A_1 = q\, dt$

chemical potential for q $\quad$ $< j >$

$O \leftrightarrow \psi$

$$\psi(x,z) = z\psi_1(x) + z^2\psi_2(x) + \ldots$$

$J_O(x)$ $\quad$ $< O >$

AdS-CMT

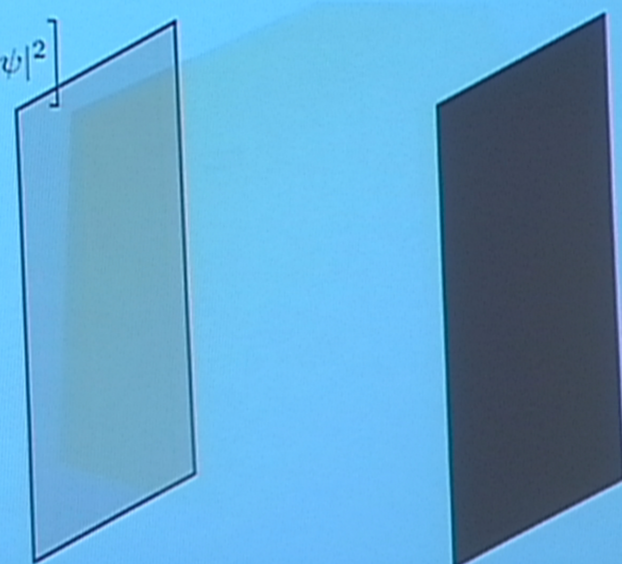
- Much interest has arisen also in rather exotic black holes where the gravity has a complex scalar charged under a vector.

$$S = \int d^4x\sqrt{-g}\left[R + \frac{6}{l^2} - \frac{1}{4}F^2 - |D\psi|^2 - m^2|\psi|^2\right]$$

$$F = dA \qquad D\psi = (d - 2iA)\psi$$

$$q = 2$$

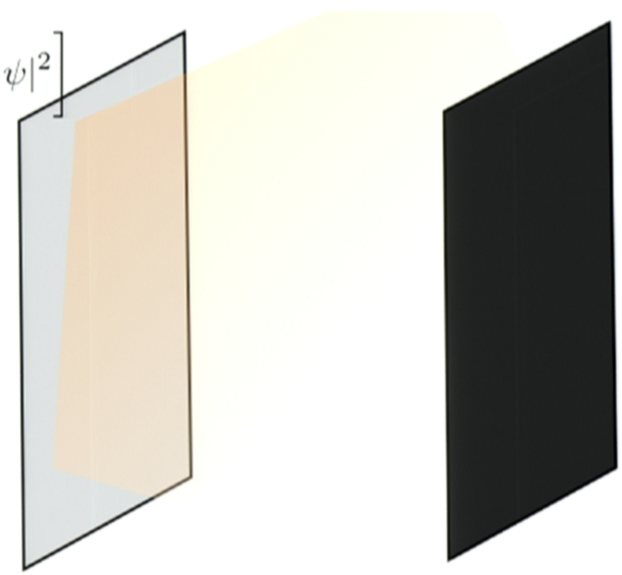
$$m = -\frac{2}{L^2}$$



AdS-CMT

- Much interest has arisen also in rather exotic black holes where the gravity has a complex scalar charged under a vector. The vector is dual to a conserved current in the boundary. We may then turn on a chemical potential source for this current.

$$S = \int d^4x\sqrt{-g}\left[R + \frac{6}{l^2} - \frac{1}{4}F^2 - |D\psi|^2 - m^2|\psi|^2\right]$$



$j \leftrightarrow A$

$$A(x,z) = A_0(x) + zA_1(x) + \dots$$

$A_0 = \mu dt$ $\qquad$ $A_1 = q\, dt$

chemical potential for q $\qquad$ $< j >$

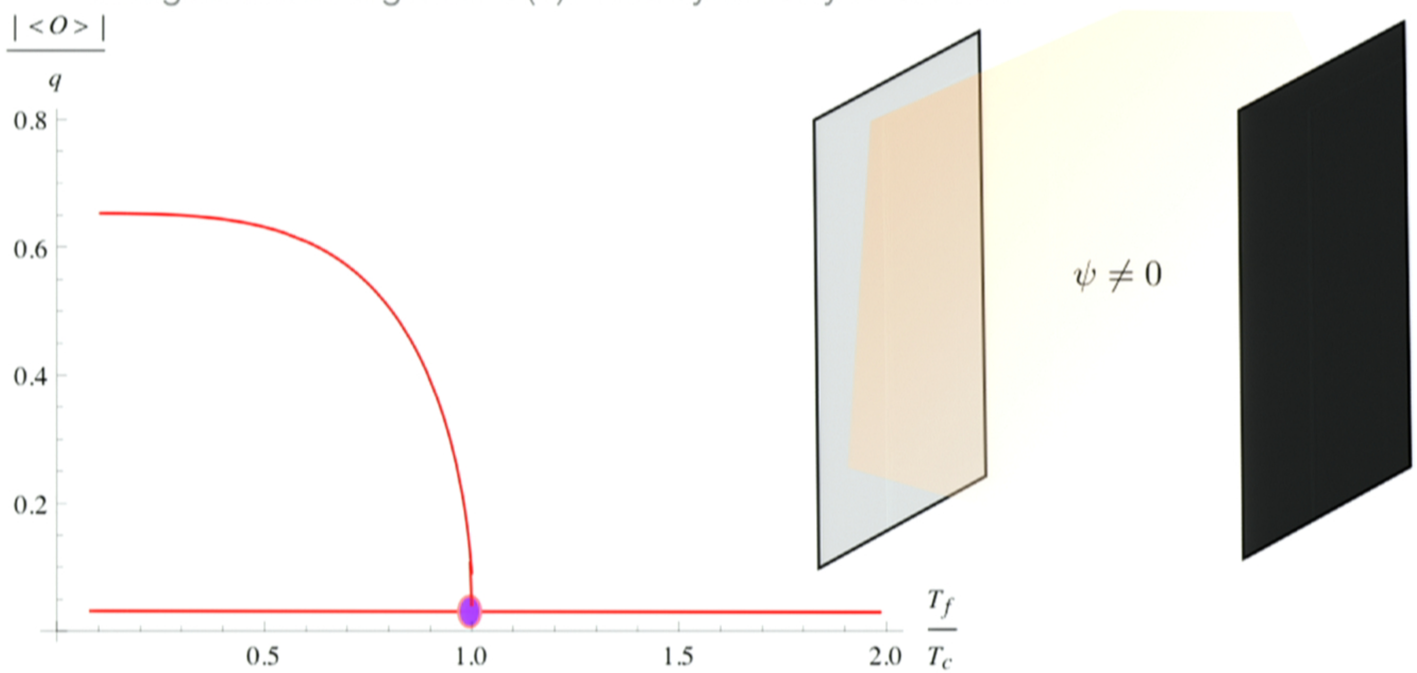
$O \leftrightarrow \psi$

$$\psi(x,z) = z\psi_1(x) + z^2\psi_2(x) + \dots$$

$J_O(x)$ $\qquad$ $< O >$

AdS-CMT

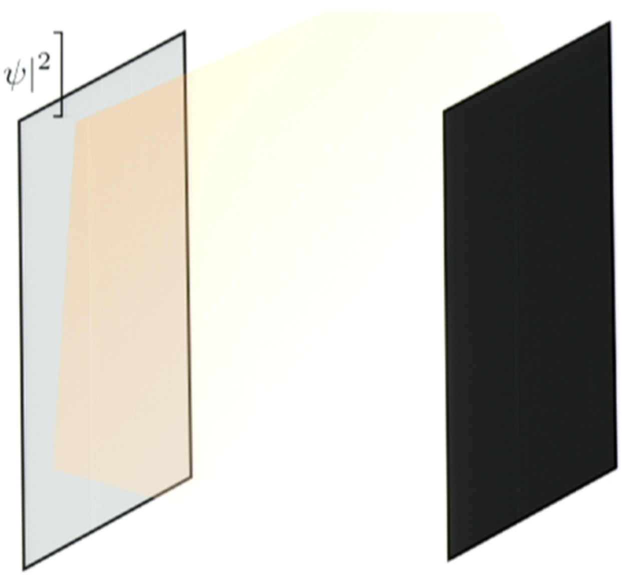
- [Gubser ; Hartnoll, Herzog, Horowitz] There is an exotic phase transition to black holes that superconduct - seen via the CFT current-current correlator. The complex scalar is ‘turned on’ and is dual to a complex scalar operator, O, charged under a global U(1). This symmetry is broken.



AdS-CMT

- Much interest has arisen also in rather exotic black holes where the gravity has a complex scalar charged under a vector. The vector is dual to a conserved current in the boundary. We may then turn on a chemical potential source for this current.

$$S = \int d^4x\sqrt{-g}\left[R + \frac{6}{l^2} - \frac{1}{4}F^2 - |D\psi|^2 - m^2|\psi|^2\right]$$



$j \leftrightarrow A$

$$A(x,z) = A_0(x) + zA_1(x) + \ldots$$

$A_0 = \mu dt$ $\qquad$ $A_1 = q\, dt$

chemical potential for q $\qquad$ $< j >$

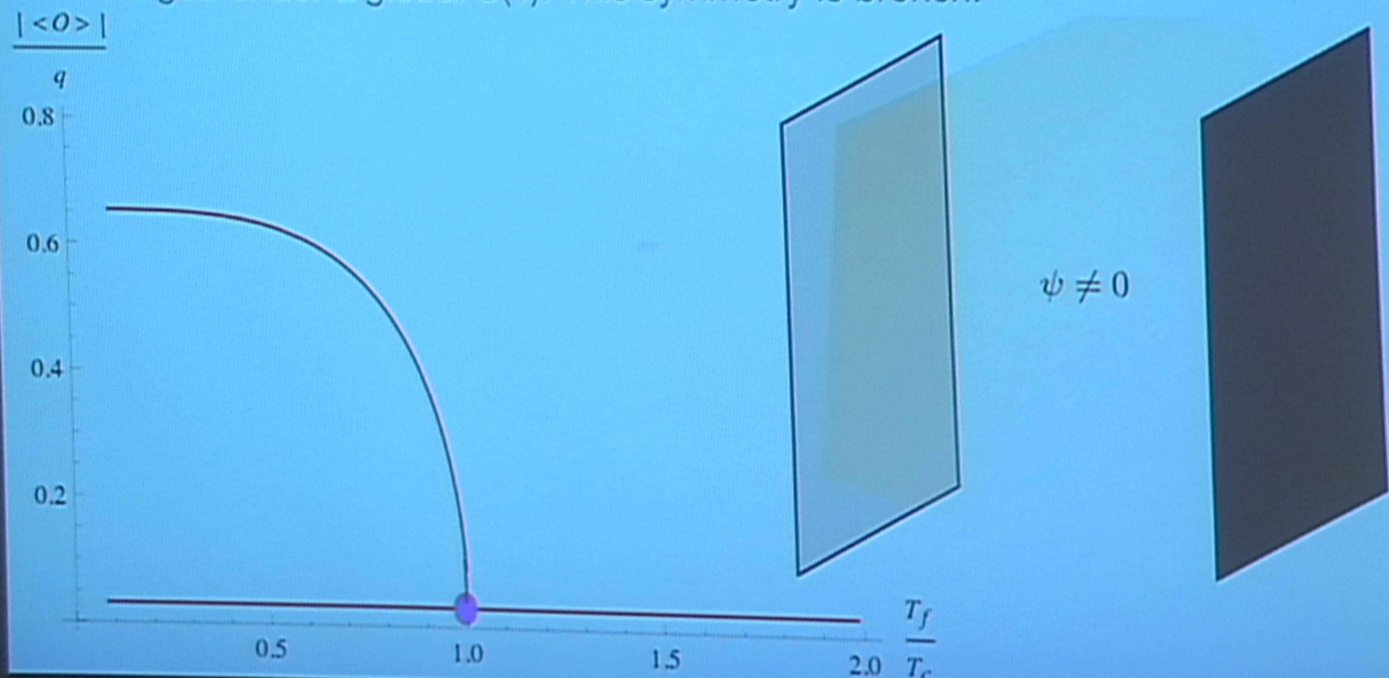
$O \leftrightarrow \psi$

$$\psi(x,z) = z\psi_1(x) + z^2\psi_2(x) + \ldots$$

$J_O(x)$ $\qquad$ $< O >$

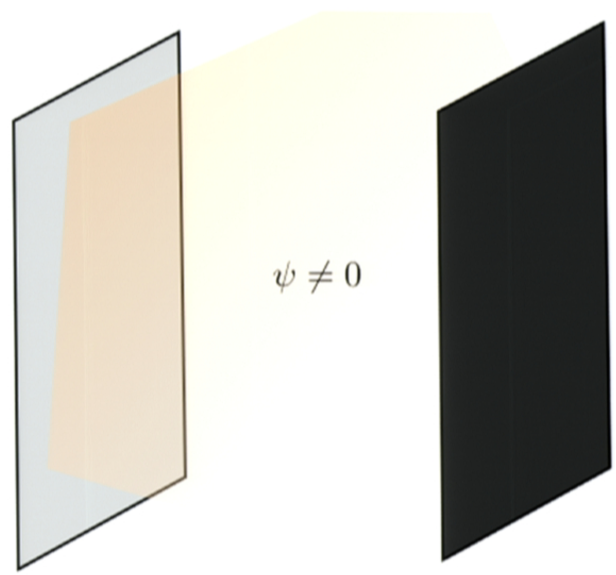
AdS-CMT

- [Gubser ; Hartnoll, Herzog, Horowitz] There is an exotic phase transition to black holes that superconduct - seen via the CFT current-current correlator. The complex scalar is 'turned on' and is dual to a complex scalar operator, O, charged under a global U(1). This symmetry is broken.



AdS-CMT

- Superconductivity;
  - Broken U(1) in boundary - cf. BCS
  - Infinite DC conductivity in `broken phase' - computed from $< j\, j >$



AdS-CMT

- Many many questions;
  - Do these describe real strongly coupled superconductors?
  - Do `holographic' materials describe other CMT physics - fermi surfaces, strange metals etc...?
  - Can this be superconductor be embedded in string theory? [Gauntlett, Sonner, TW ; Gubser, Herzog, Pufu, Tesileanu]
- Here we will focus on dynamics and in particular strongly non-adiabatic dynamics - so called `quenches'

AdS-CMT

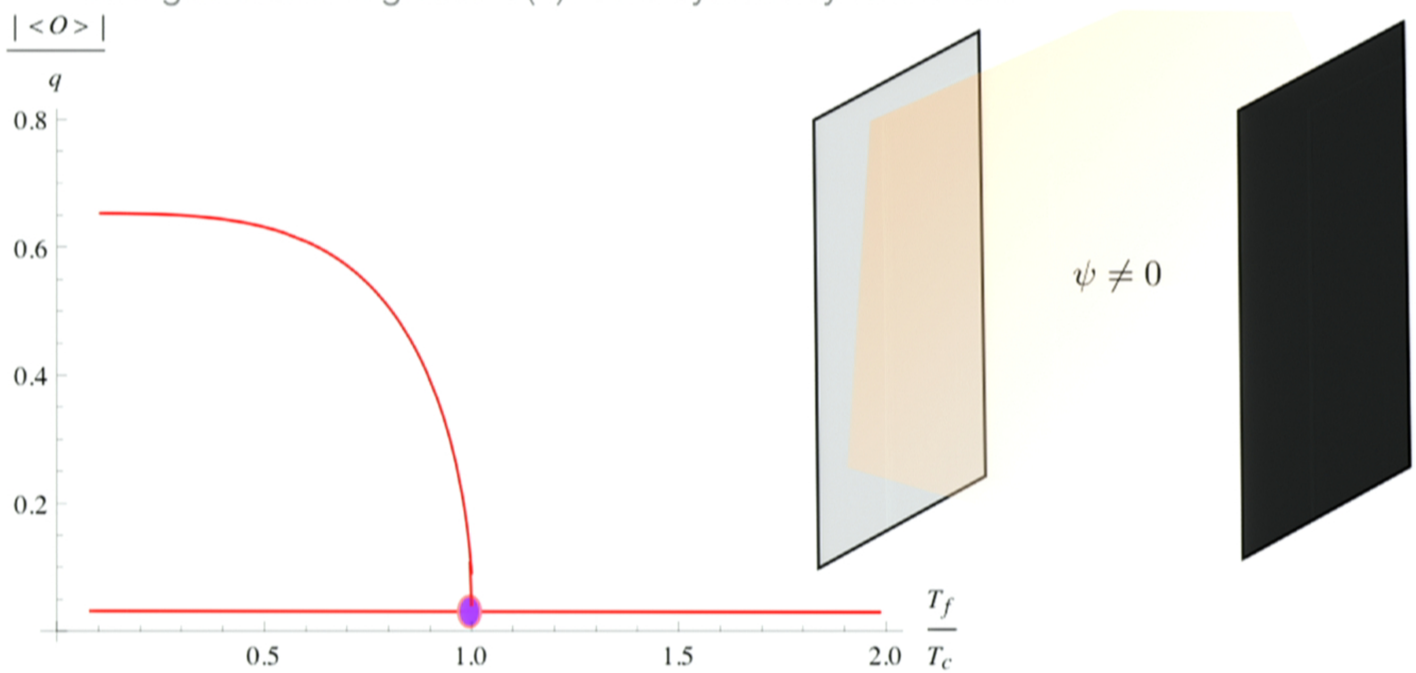
- The dynamics of superconductors is an area of interest in CMT. An important question is what happens if one starts with a superconductor and injects energy. This is a notoriously tough problem - the state of the art is work of Barankov-Levitov where approximations in BCS theory lead to a dynamical phase diagram.

[1] Barankov and Levitov, *"Collective Rabi Oscillations and Solitons in a Time-Dependent BCS Pairing Hamiltonian"*, PRL **93**, 160401 (2004)

[2] Barankov, Levitov and Spivak, *"Synchronization in the BCS Pairing Dynamics as a critical phenomenon"*, PRL **96**, 020403 (2006)

AdS-CMT

- [Gubser ; Hartnoll, Herzog, Horowitz] There is an exotic phase transition to black holes that superconduct - seen via the CFT current-current correlator. The complex scalar is ‘turned on’ and is dual to a complex scalar operator, O, charged under a global U(1). This symmetry is broken.



AdS-CMT

- The dynamics of superconductors is an area of interest in CMT. An important question is what happens if one starts with a superconductor and injects energy. This is a notoriously tough problem - the state of the art is work of Barankov-Levitov where approximations in BCS theory lead to a dynamical phase diagram.

[1] Barankov and Levitov, *"Collective Rabi Oscillations and Solitons in a Time-Dependent BCS Pairing Hamiltonian"*, PRL **93**, 160401 (2004)

[2] Barankov, Levitov and Spivak, *"Synchronization in the BCS Pairing Dynamics as a critical phenomenon"*, PRL **96**, 020403 (2006)

AdS-CMT

- Solve time dependent BCS theory
  - Several approximations; zero dimensional (mean field), collisionless

$$H = -\sum_{p,\sigma} \epsilon_p a^\dagger_{p,\sigma} a_{p,\sigma} - \frac{\lambda(t)}{2} \sum_{p,q} a^\dagger_{p,+\frac{1}{2}} a^\dagger_{-p,-\frac{1}{2}} a_{-q,-\frac{1}{2}} a_{q,+\frac{1}{2}}$$

$$\lambda(t) = \begin{cases} \lambda_s & t < t_\star \\ \lambda & t \geq t_\star \end{cases}$$

$$|\Psi(t) >= \Pi_p \left[u_p(t) + v_p(t) a^\dagger_{p,+\frac{1}{2}} a^\dagger_{-p,-\frac{1}{2}} \right] |0>$$

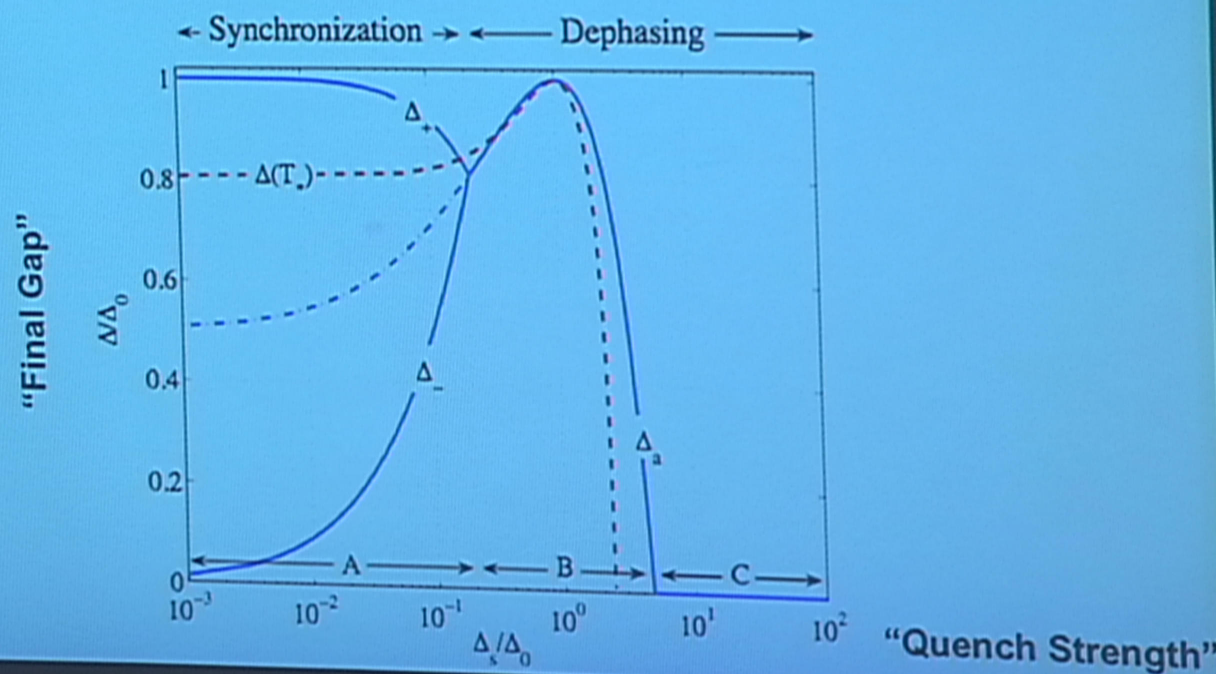
$$\Delta(t) = \lambda(t) \sum_p u_p(t) v^\star_p(t)$$

Initial pairing Δ_s

Eqm final pairing Δ_0

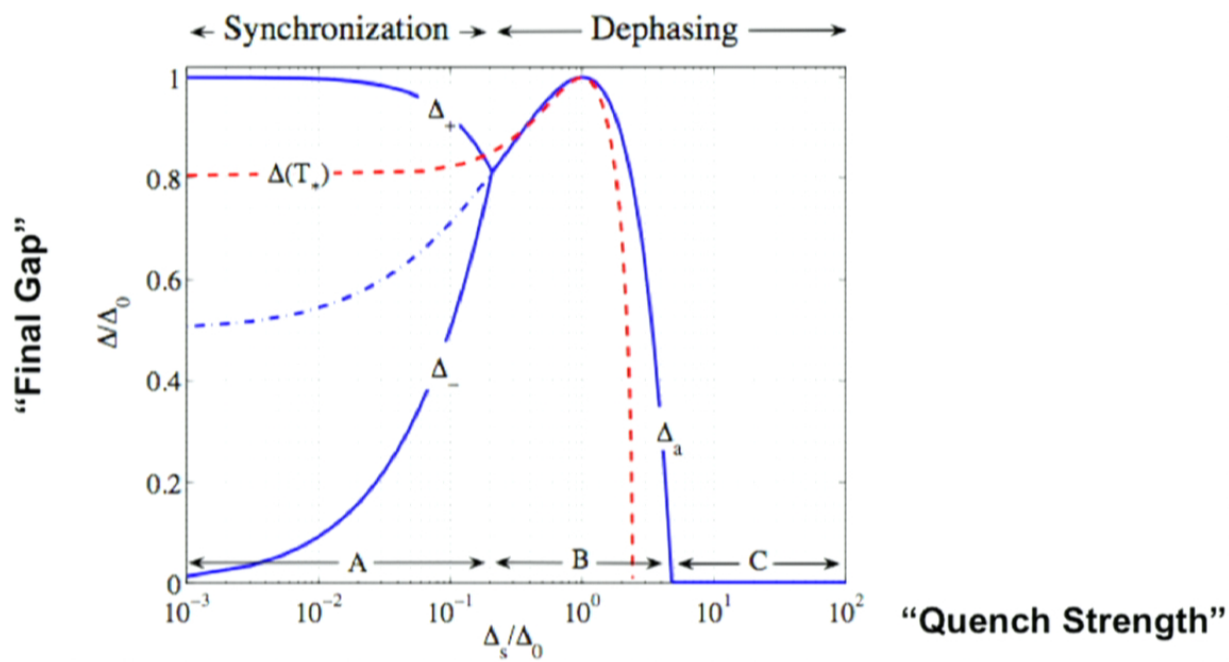
AdS-CMT

- Obtain `**dynamical phase diagram'** - 3 regimes;
    - oscillation of pairing, decay to non-zero pairing, decay to zero pairing



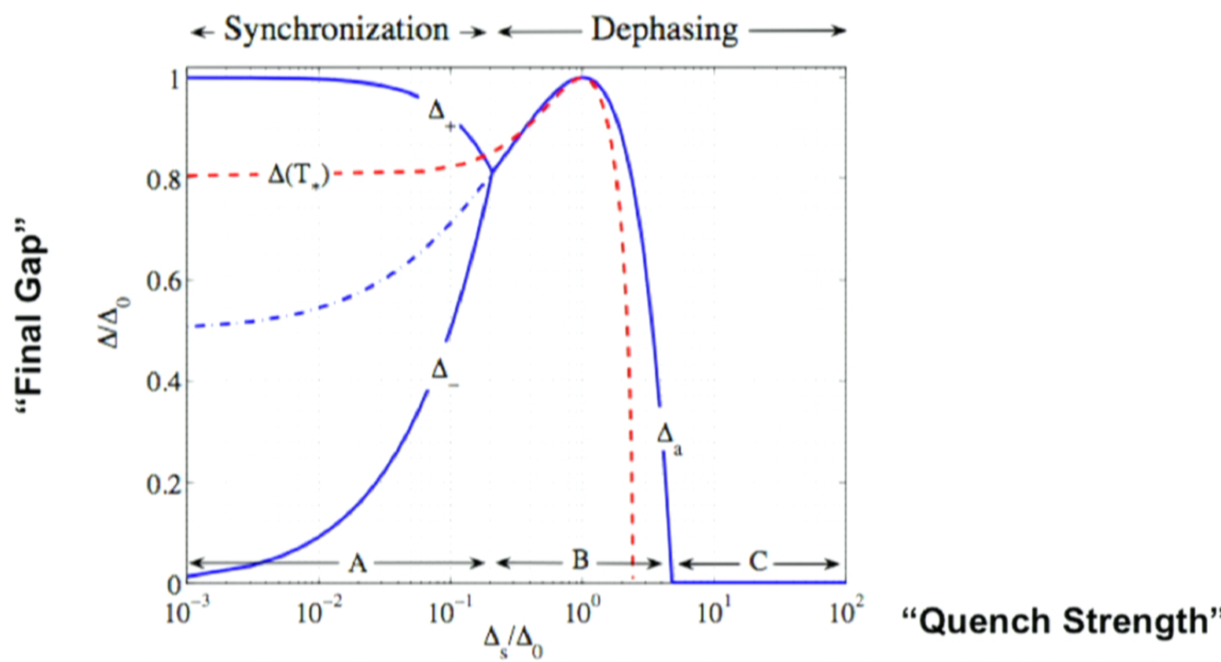
AdS-CMT

- Obtain `**dynamical phase diagram'** - 3 regimes;
  - oscillation of pairing, decay to non-zero pairing, decay to zero pairing



AdS-CMT

- Obtain `**dynamical phase diagram' -** 3 regimes;
    - oscillation of pairing, decay to non-zero pairing, decay to zero pairing

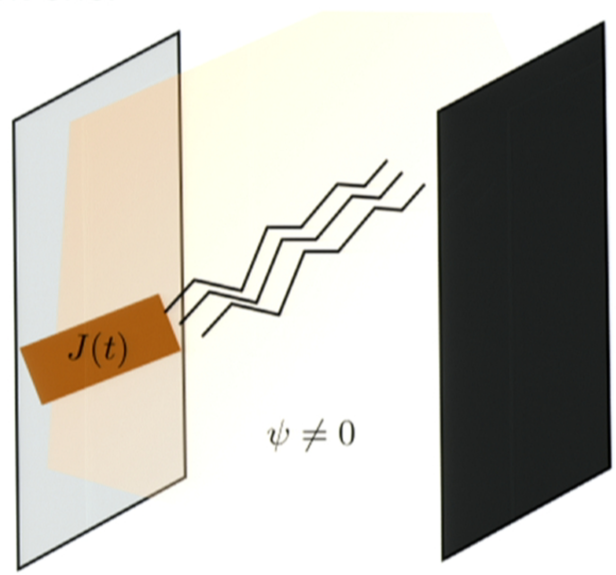


AdS-CMT

- Obtain `**dynamical phase diagram' -** 3 regimes;
    - oscillation of pairing, decay to non-zero pairing, decay to zero pairing
- Unclear what the status of these 3 regimes is if one adds collisions, thermal damping and non-mean field.

B-L in AdS/CMT

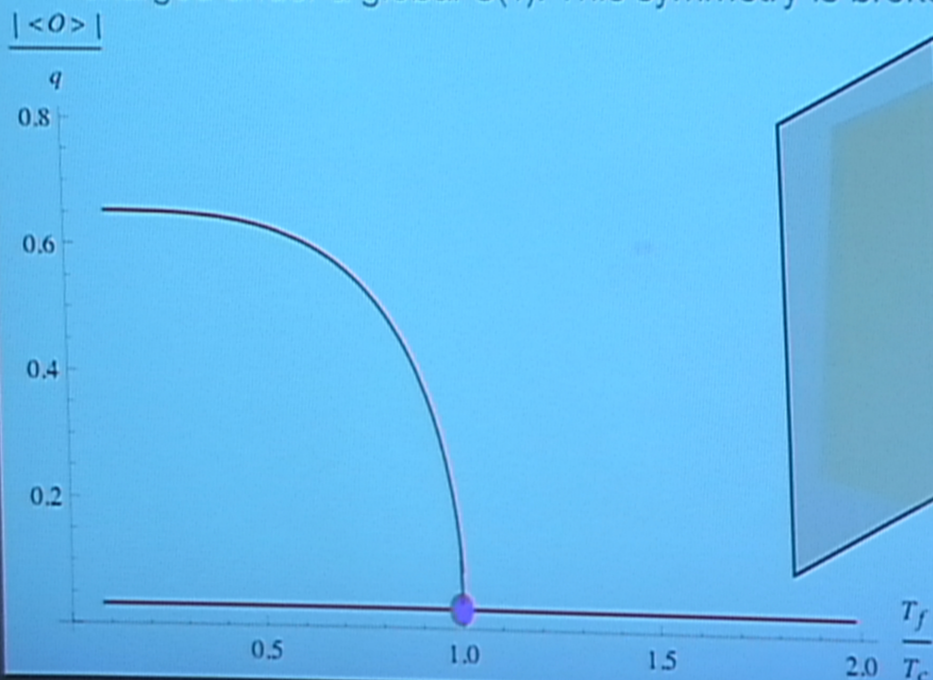
- In B-L various approximations are made. By performing a simple 1+1 dynamical simulation we may ask whether the same is observed in a strongly coupled superconductor. We quickly turn on and off a source to `quench' the system from one pairing state to a different one.

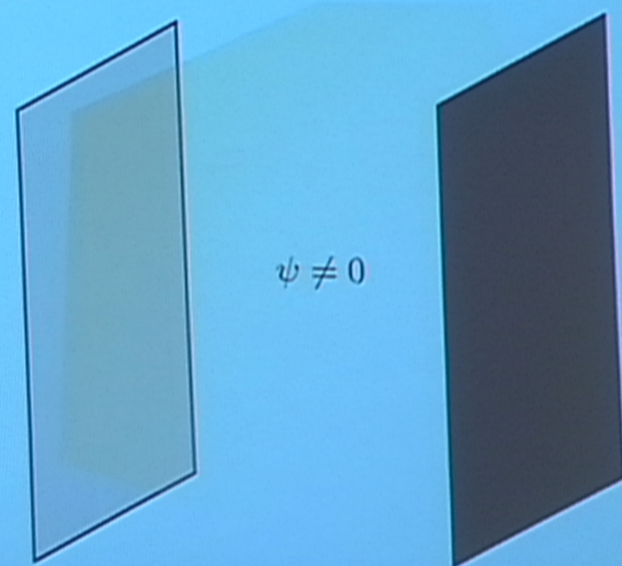


cf. Murata et al

AdS-CMT

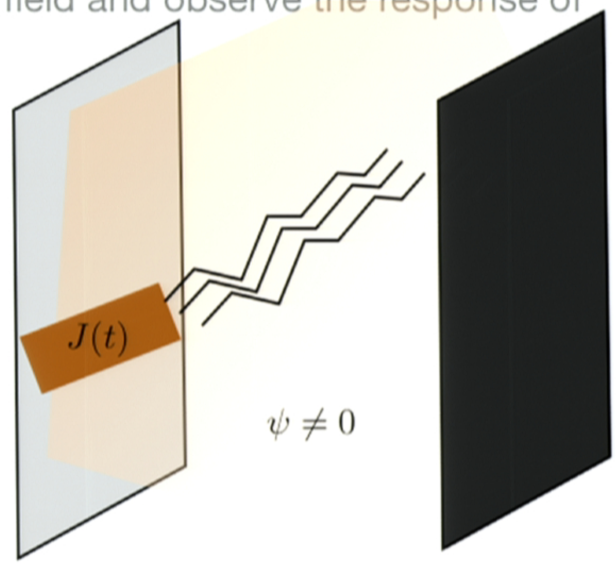
- [Gubser ; Hartnoll, Herzog, Horowitz] There is an exotic phase transition to black holes that superconduct - seen via the CFT current-current correlator. The complex scalar is ‘turned on’ and is dual to a complex scalar operator, O, charged under a global U(1). This symmetry is broken.





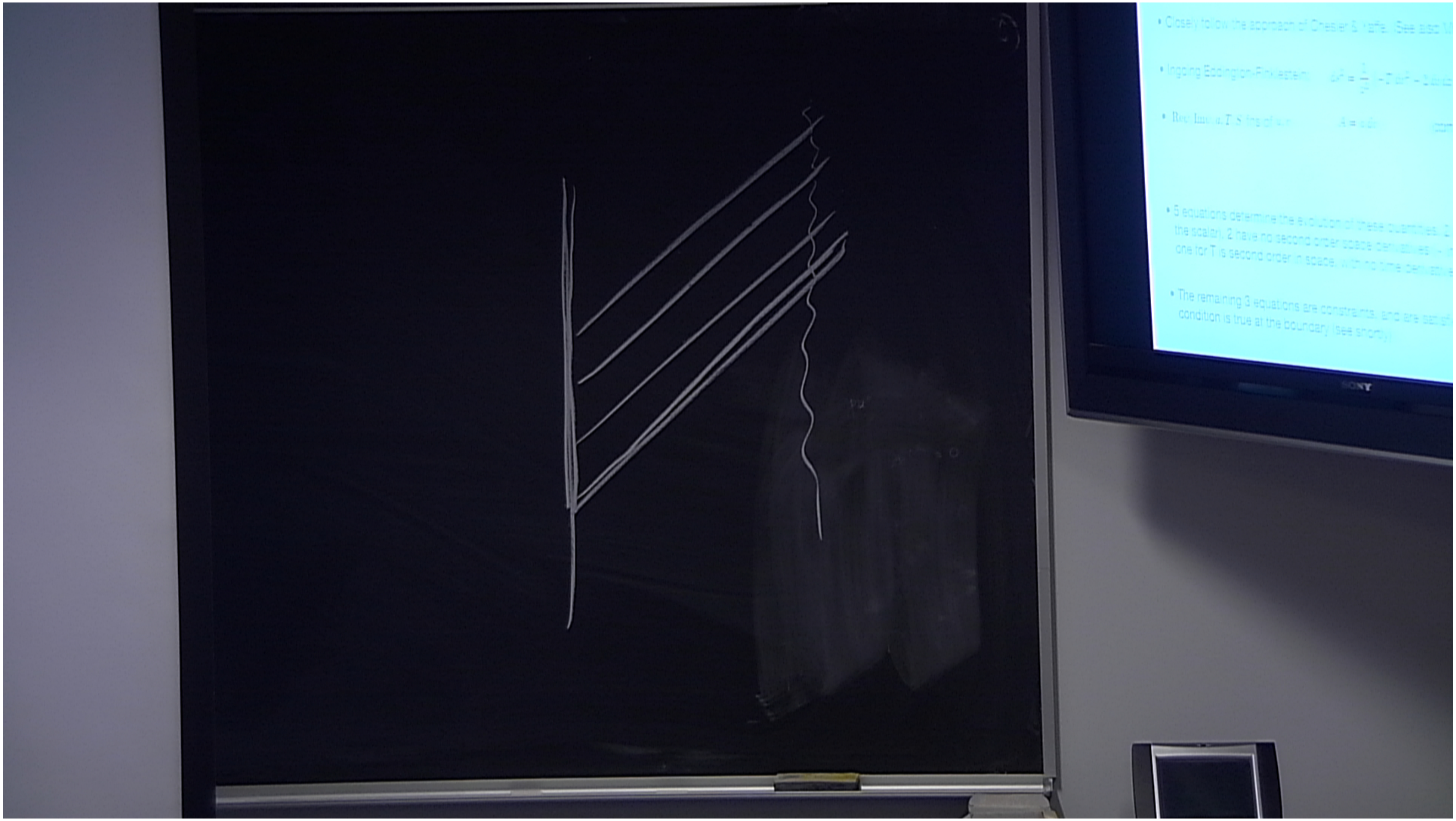
B-L in AdS/CMT

- Birkhoff's theorem tells one that in fact the only source one can turn on is for the scalar itself.
- cf. one may quench an applied magnetic field and observe the response of the magnetisation
- Sachdev - 'injecting condensate'
- Note: vortices will **not** be excited!



The numerical GR calculation

- Closely follow the approach of Chesler & Yaffe. (See also Murata)
- Ingoing Eddington-Finklestein; $ds^2 = \frac{1}{z^2}\left(-T\,dv^2 - 2\,dvdz + S^2 dx_i^2\right)$
- $\mathrm{Re}\psi, \mathrm{Im}\psi, a, T, S$ fns of u, v $A = a\,dv$ (complex) ψ
- 5 equations determine the evolution of these quantities. 2 are wavelike (for the scalar), 2 have no second order space derivatives (~ integrals in z) and one for T is second order in space, with no time derivatives.
- The remaining 3 equations are constraints, and are satisfied provided one condition is true at the boundary (see shortly)


Closely follow the approach of Chesler & Yaffe
Ingoing Eddington-Finkelstein
5 equations determine the evolution of these quantities
the scalar, 2 have no second order space derivatives
one for T is second order in space
The remaining 3 equations are constraints, and are
condition is true at the boundary (see shortly)
SONY

The numerical GR calculation

- Closely follow the approach of Chesler & Yaffe. (See also Murata)
- Ingoing Eddington-Finklestein; $ds^2 = \frac{1}{z^2}\left(-T\,dv^2 - 2\,dvdz + S^2 dx_i^2\right)$
- $\mathrm{Re}\psi, \mathrm{Im}\psi, a, T, S$ fns of u, v $A = a\,dv$ (complex) ψ
- 5 equations determine the evolution of these quantities. 2 are wavelike (for the scalar), 2 have no second order space derivatives (~ integrals in z) and one for T is second order in space, with no time derivatives.
- The remaining 3 equations are constraints, and are satisfied provided one condition is true at the boundary (see shortly)

The numerical GR calculation

- Decompose fields as; $\psi = z\left(\psi_0 + \hat{\psi}\right)$

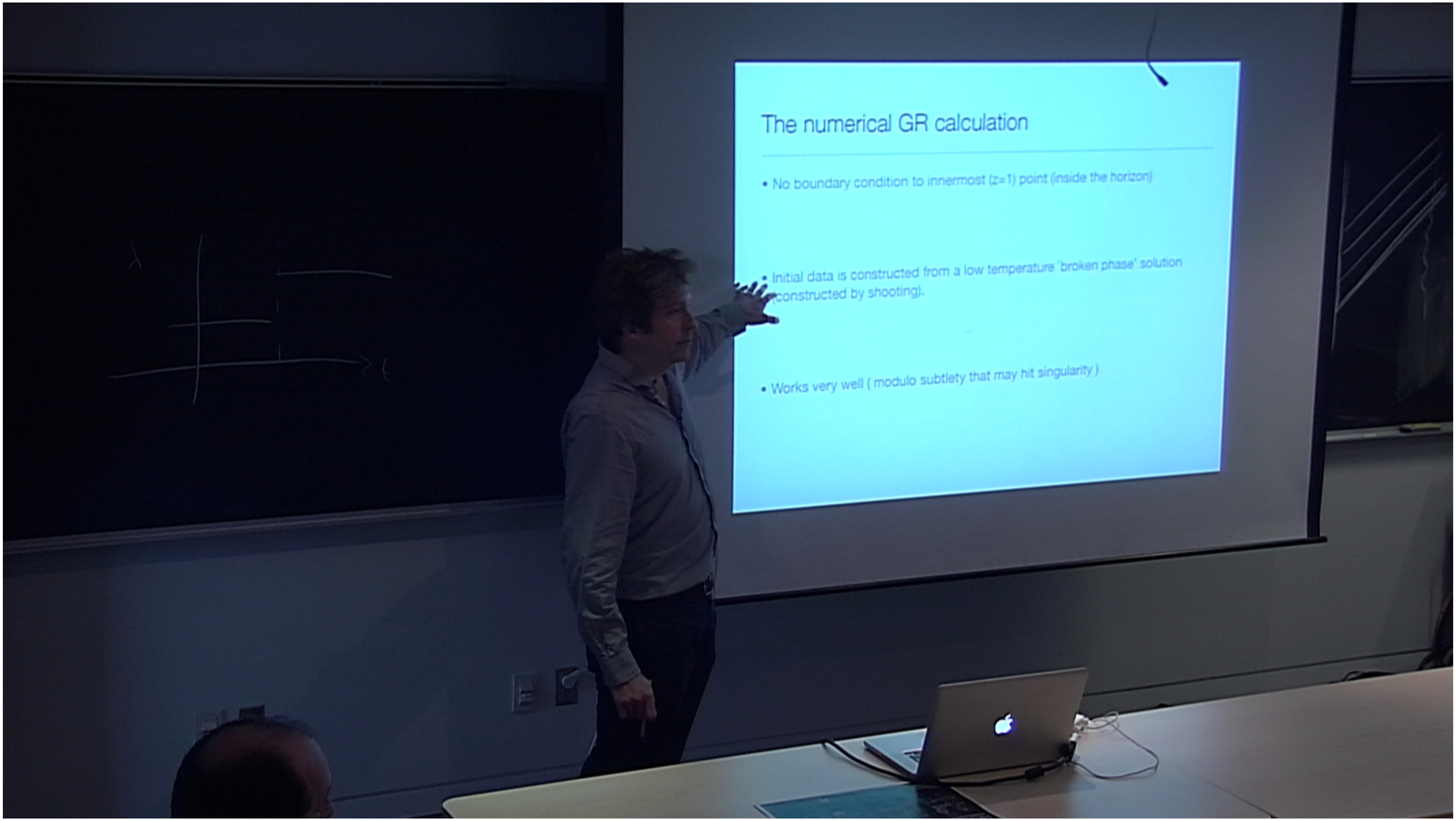
$$a = \mu + \hat{a}$$

$$T = 1 + z^2 \hat{T} \qquad S = 1 + z^2 \hat{S}$$

- Chebychev collocation method
- Dirichlet zero b.c. at z=0 for $\hat{\psi}, \hat{a}, \hat{T}, \hat{S}$ with sources μ, ψ_0 specified
- May choose μ s.t. $\mathrm{Im}\hat{\psi} = 0$
- Determine remaining data for $\hat{T}$ (from boundary expansion) as;

$$\mathrm{Re}\left[\partial_{v,z}\hat{T} + \psi_0^\star\left(\partial_{v,z}\hat{\psi} - \partial_v^2\psi_0 - 2\,i\,a\left(\partial_z\hat{\psi} - 2\partial_v\psi_0\right) + 4\,a^2\,\psi_0\right)\right]\Big|_{z=0} = 0$$

(stress energy conservation)


The numerical GR calculation
• No boundary condition to innermost (z=1) point (inside the horizon)
• Initial data is constructed from a low temperature 'broken phase' solution (constructed by shooting).
• Works very well (modulo subtlety that may hit singularity)

The numerical GR calculation

- No boundary condition to innermost (z=1) point (inside the horizon)

- Initial data is constructed from a low temperature `broken phase' solution (constructed by shooting).

- Works very well (modulo subtlety that may hit singularity)

The numerical GR calculation

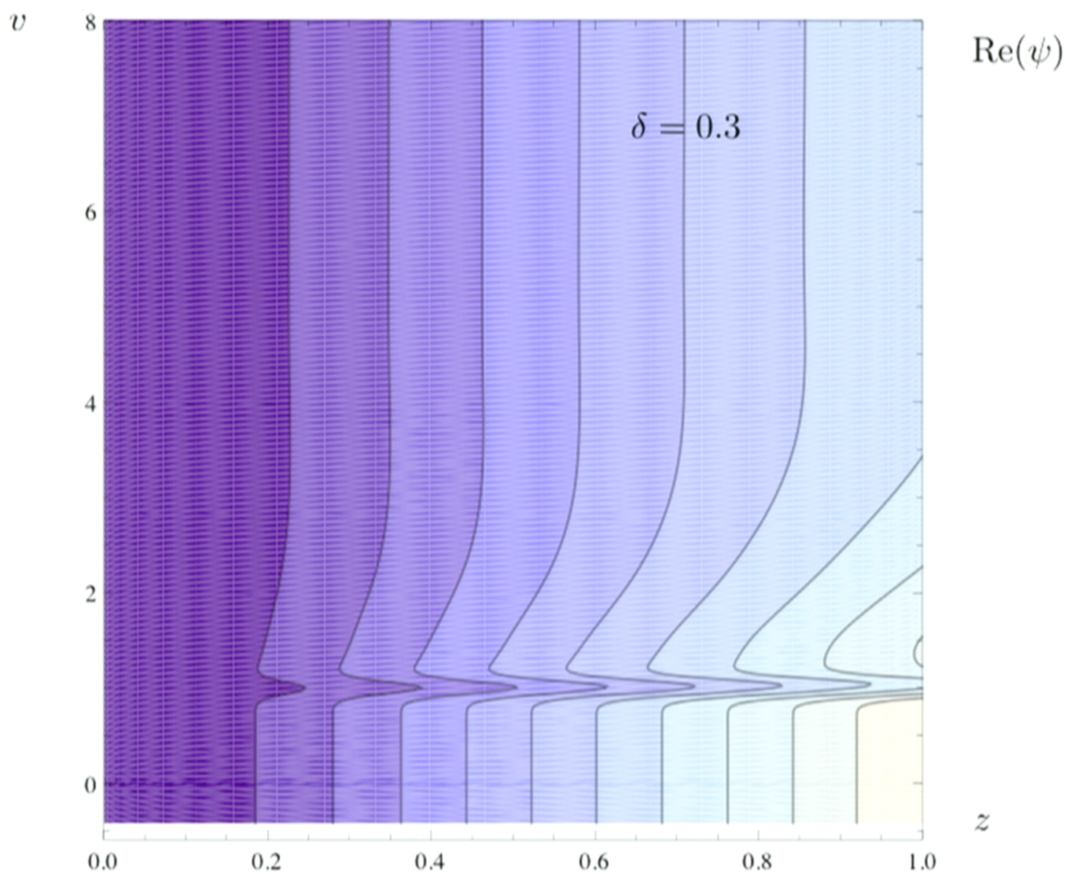
- No boundary condition to innermost (z=1) point (inside the horizon)

- Initial data is constructed from a low temperature `broken phase' solution (constructed by shooting).

- Works very well (modulo subtlety that may hit singularity)

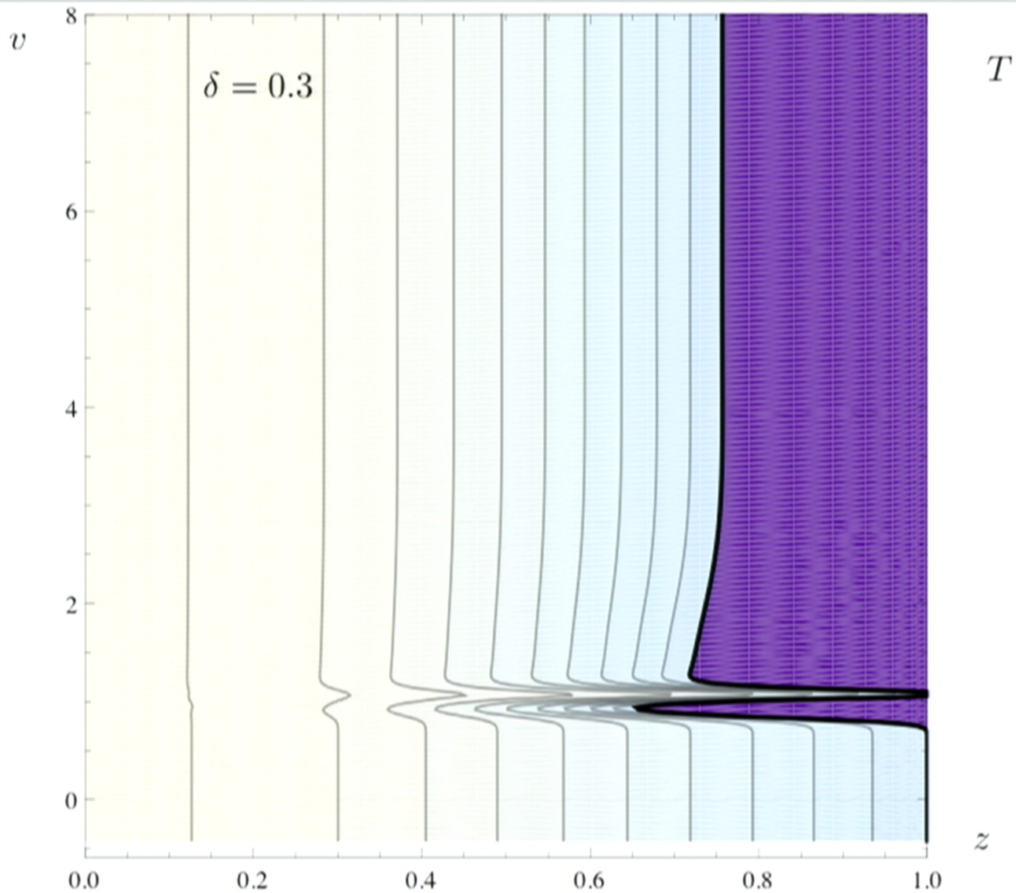
Example solution

$$\psi_0 = \delta\, i\, e^{-100(v-1)^2}$$



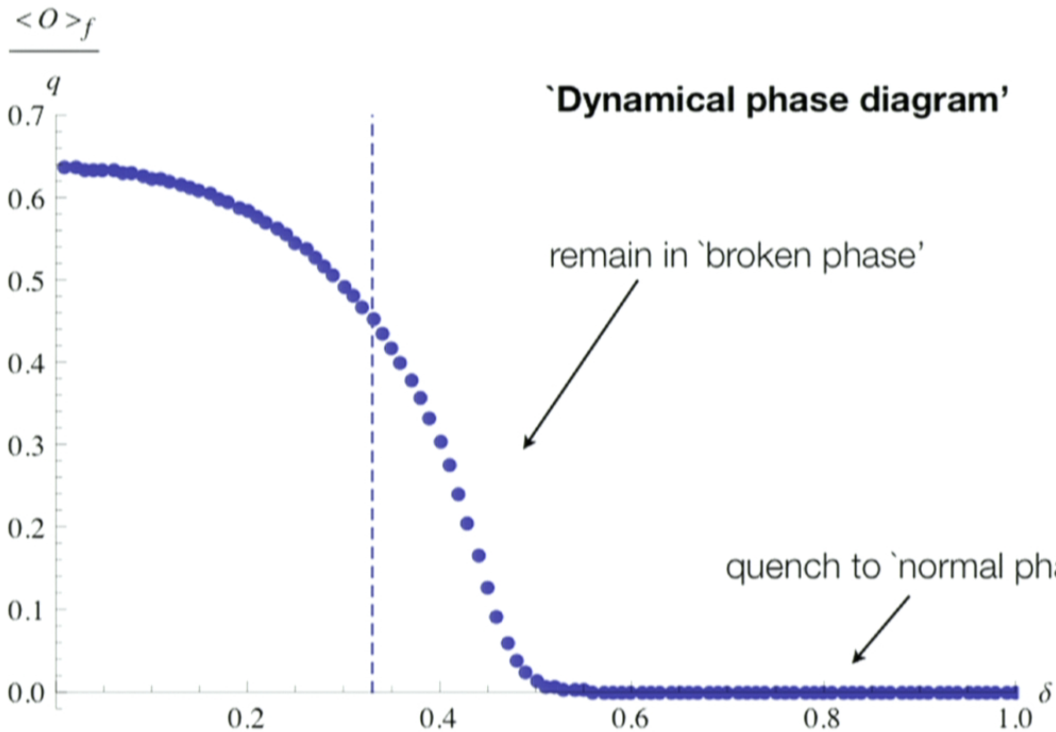
Example solution

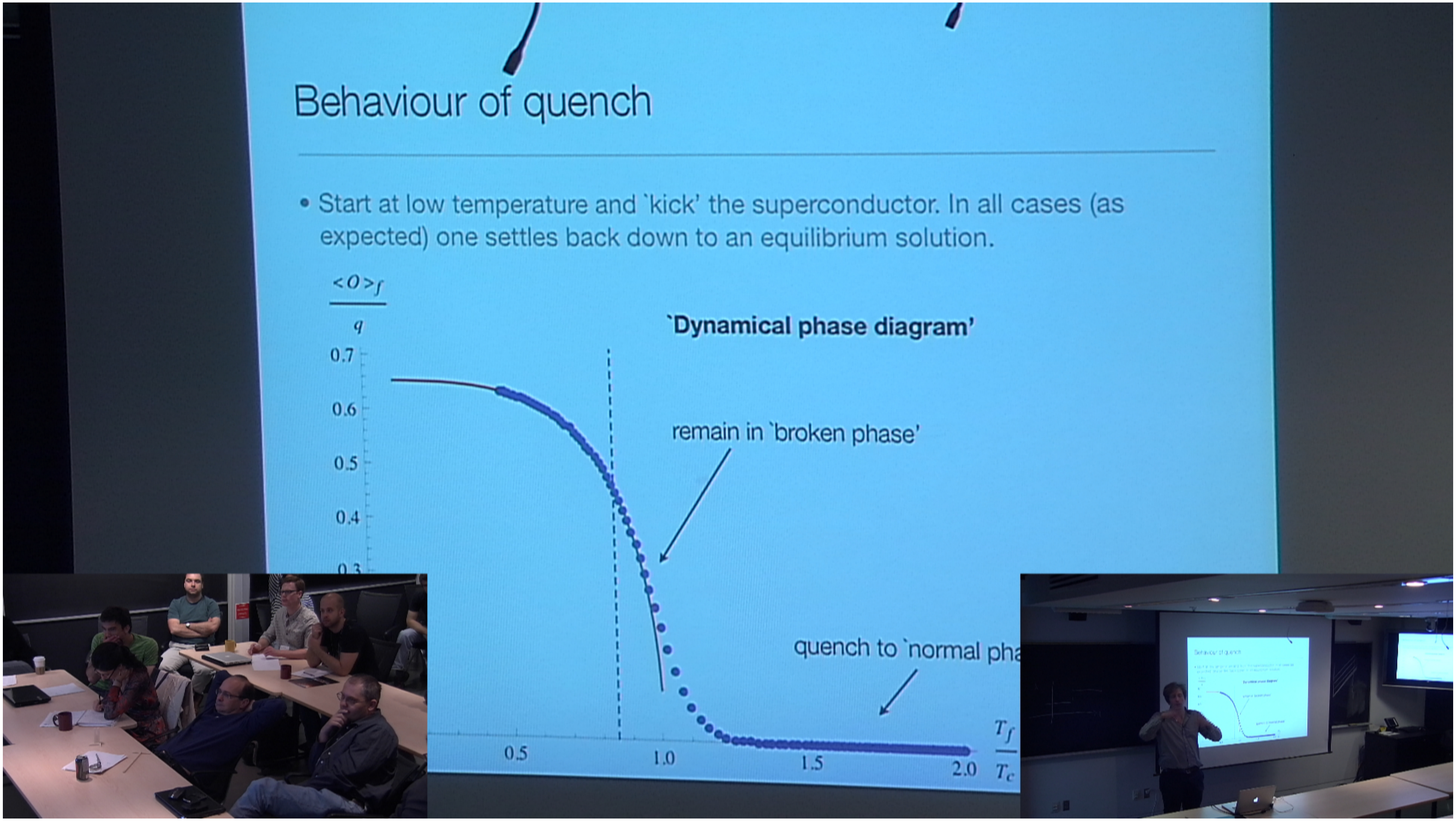
$$\psi_0 = \delta\, i\, e^{-100(v-1)^2}$$



Behaviour of quench

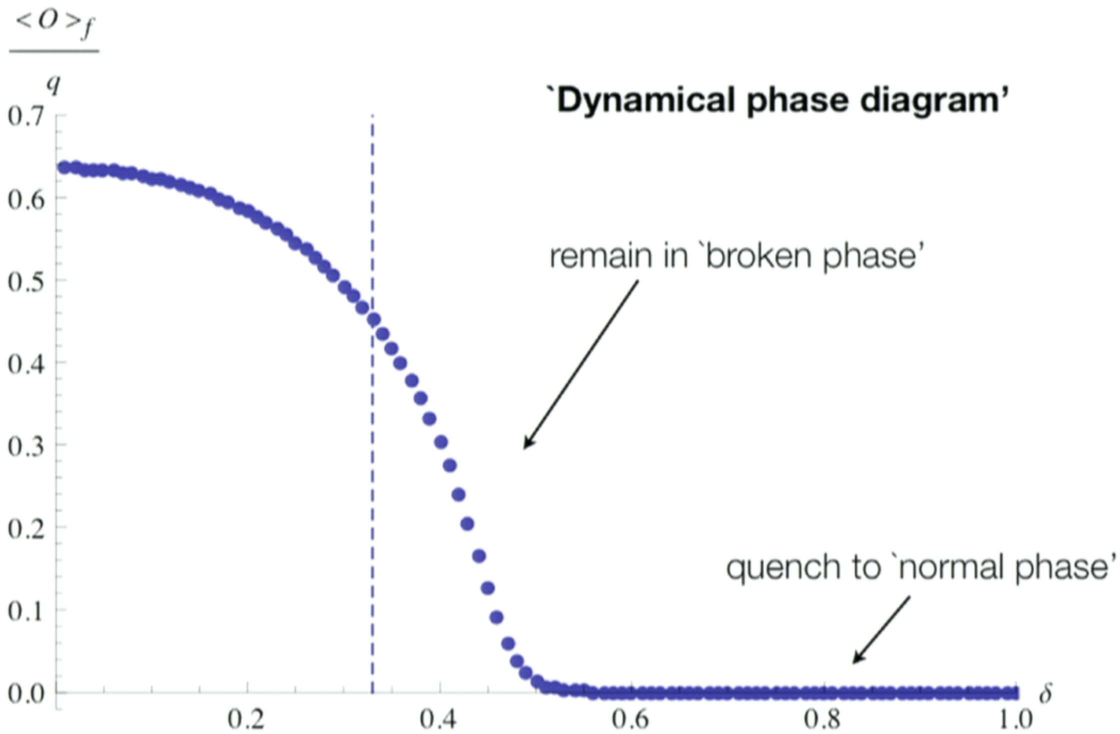
- Start at low temperature and `kick' the superconductor. In all cases (as expected) one settles back down to an equilibrium solution.




Behaviour of quench
• Start at low temperature and `kick' the superconductor. In all cases (as expected) one settles back down to an equilibrium solution.
`Dynamical phase diagram'
<O>_f / q
0.7
0.6
0.5
0.4
remain in `broken phase'
quench to `normal pha
0.5
1.0
1.5
2.0
T_f / T_c

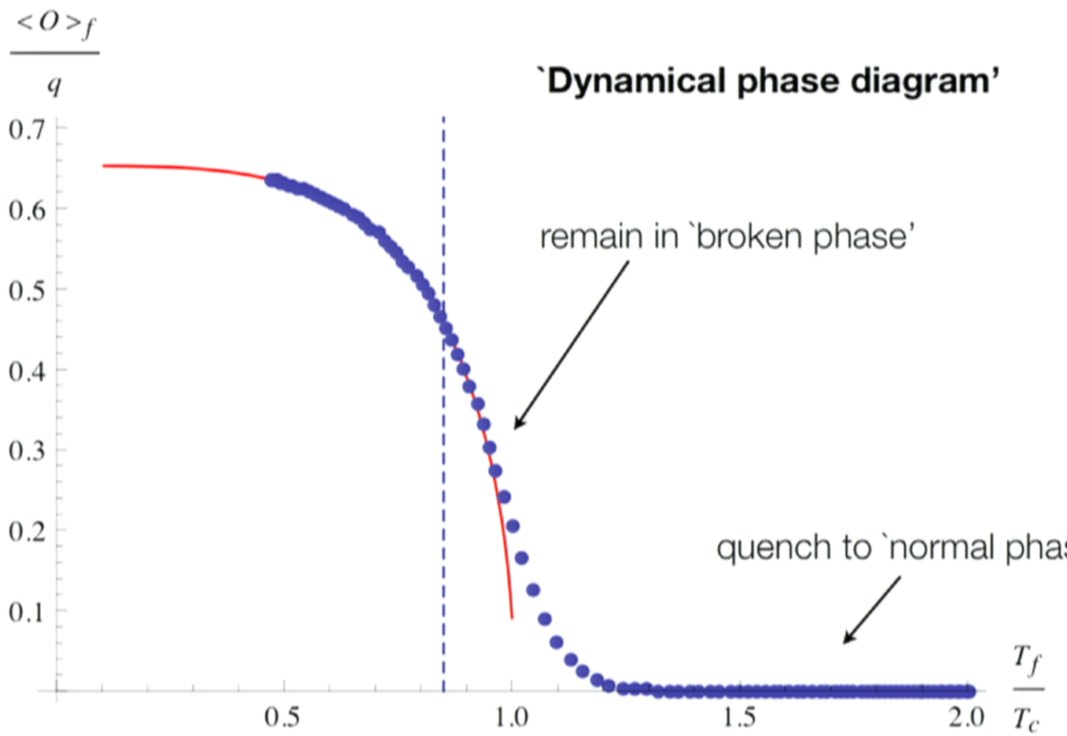
Behaviour of quench

- Start at low temperature and `kick' the superconductor. In all cases (as expected) one settles back down to an equilibrium solution.



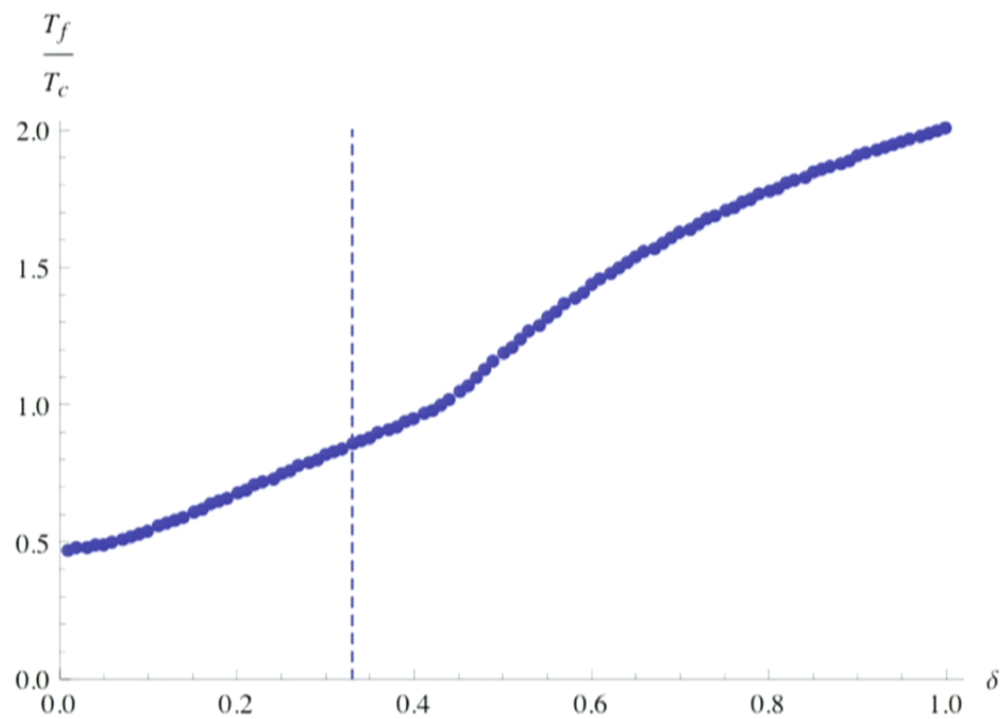
Behaviour of quench

- Start at low temperature and `kick' the superconductor. In all cases (as expected) one settles back down to an equilibrium solution.



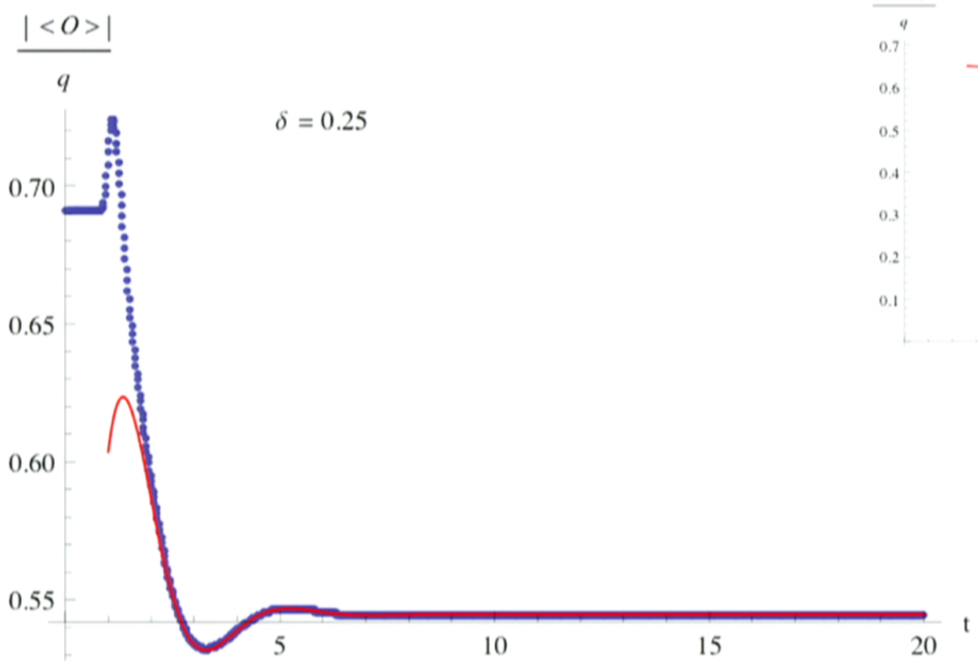
Behaviour of quench

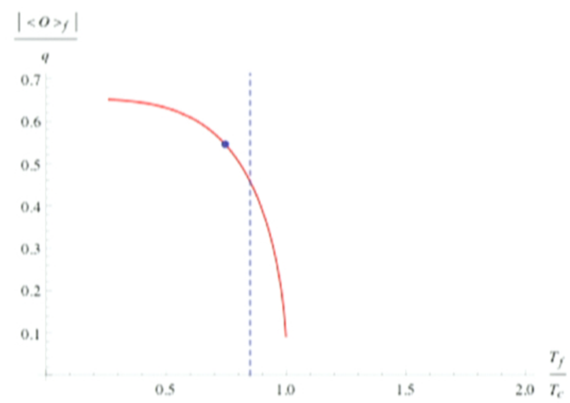
- Start at low temperature and `kick' the superconductor. In all cases (as expected) one settles back down to an equilibrium solution.



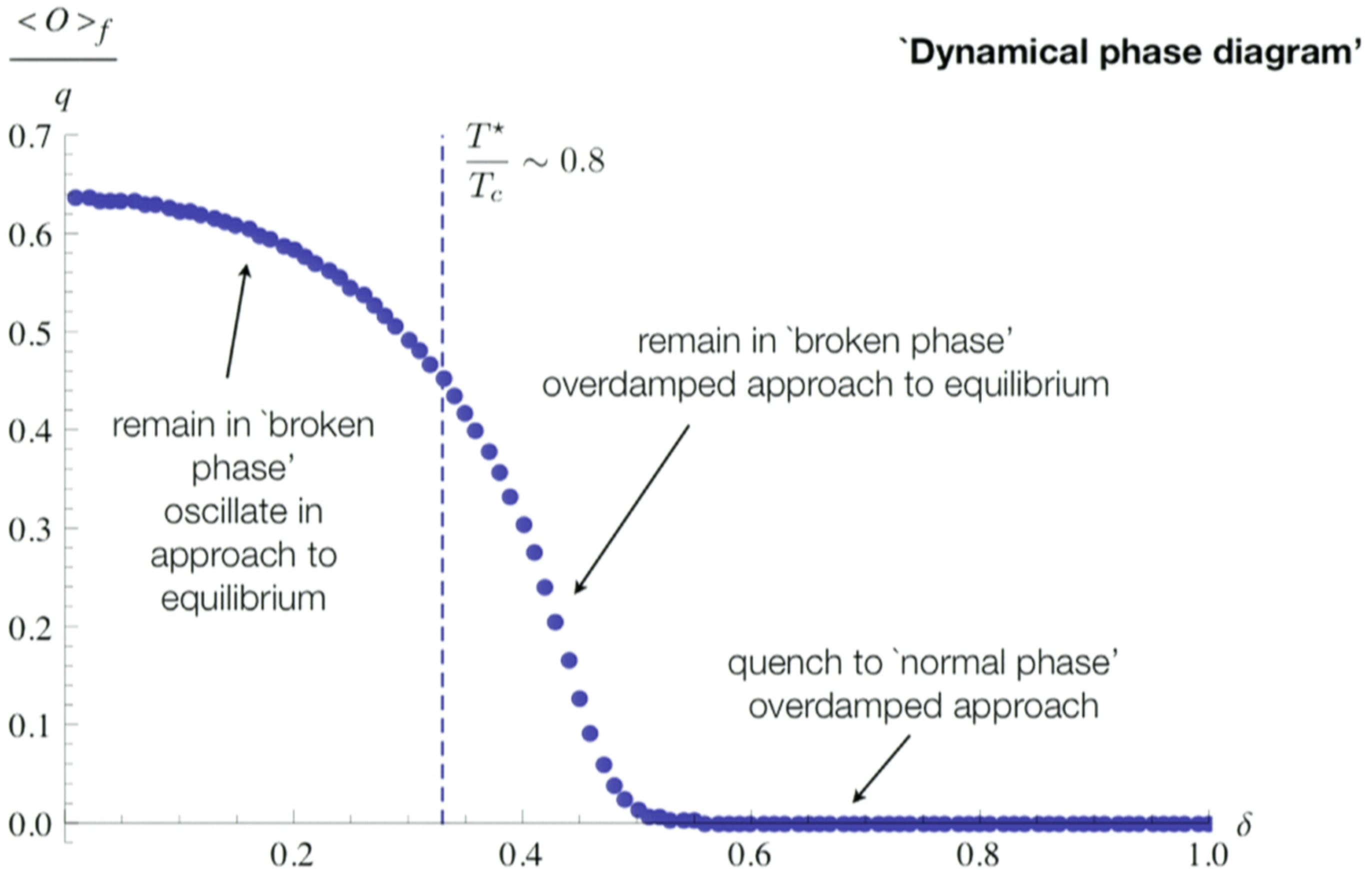
`B-L' like behaviour

- We see analogous behaviour - for a small δ we find oscilating relaxation to the broken phase



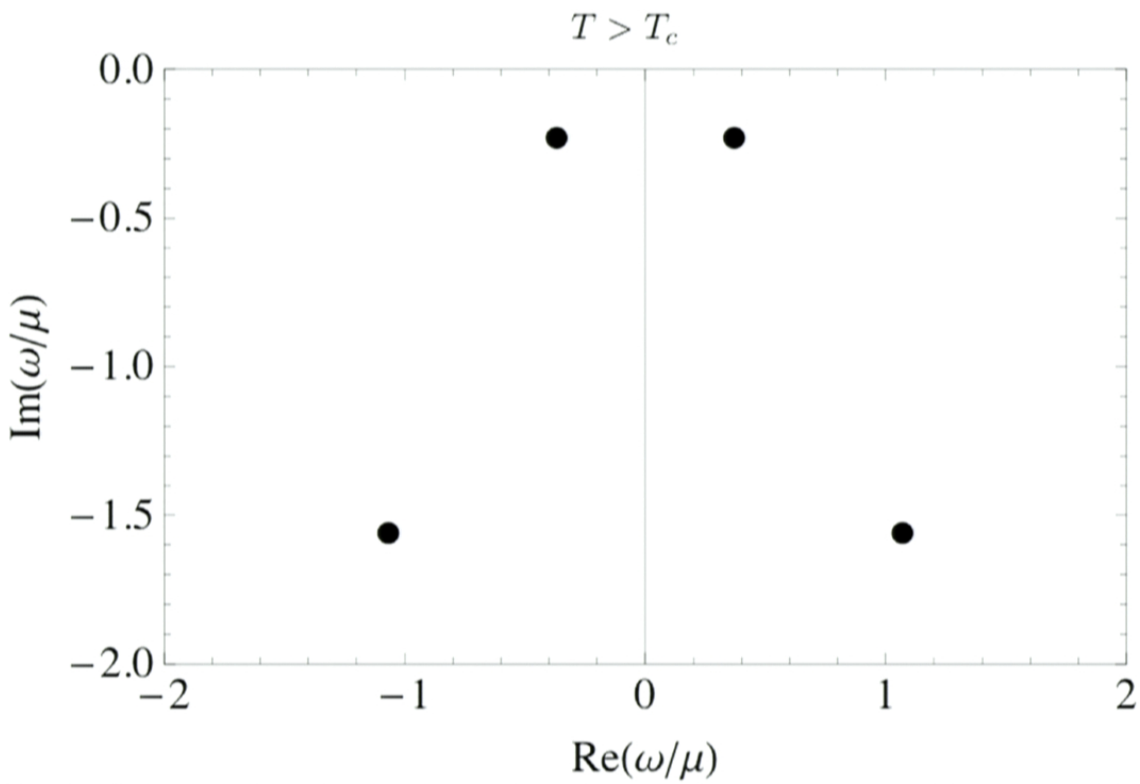


B-L like behaviour



Quasinormal modes

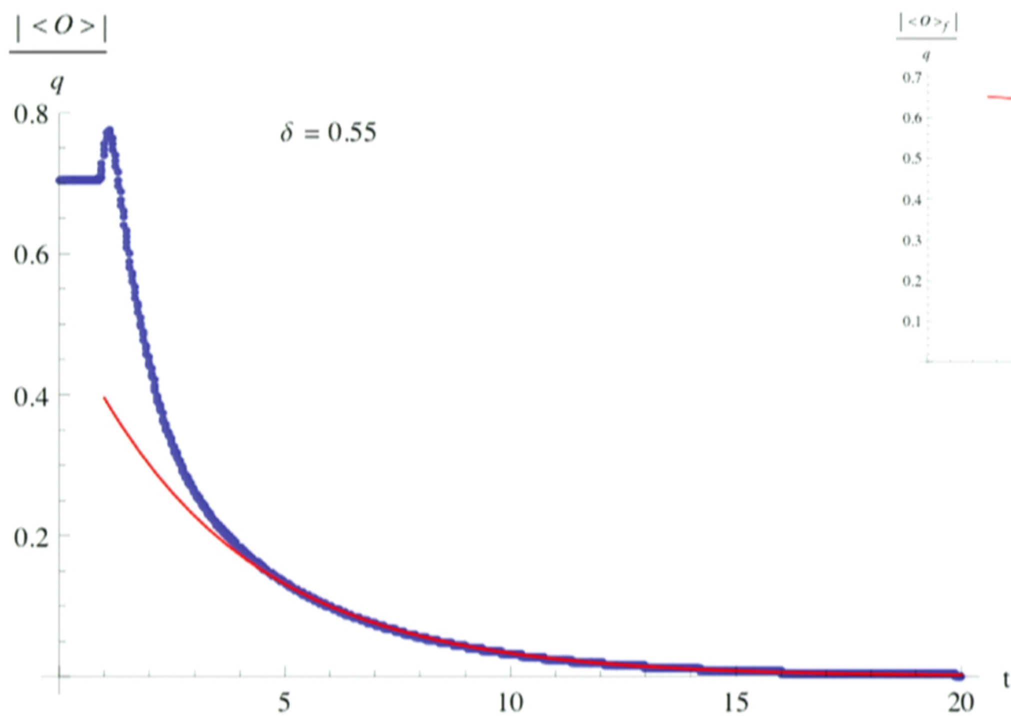
- In the normal phase we have no oscillations; $\psi(v,z) \sim e^{-i\omega v} f(z)$

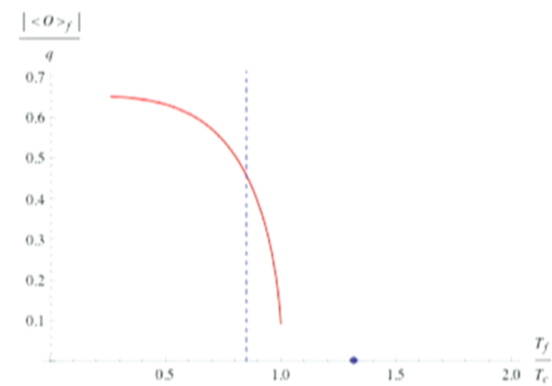


$|\psi| \sim e^{\mathrm{Im}(\omega) v} f(z)$

`B-L' like behaviour

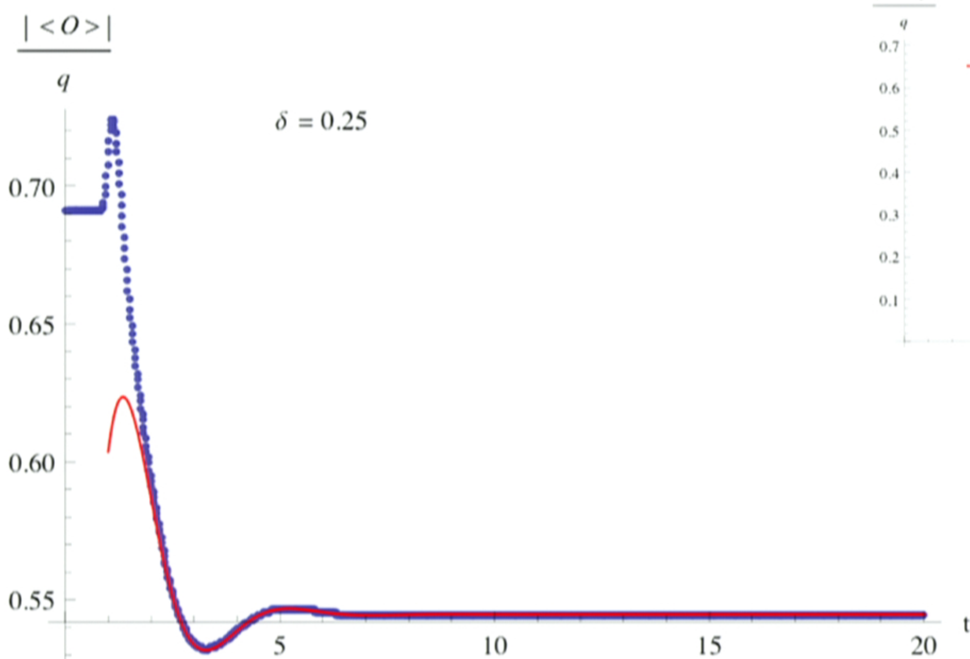
- We see analogous behaviour - for a small δ we find oscilating relaxation to the broken phase

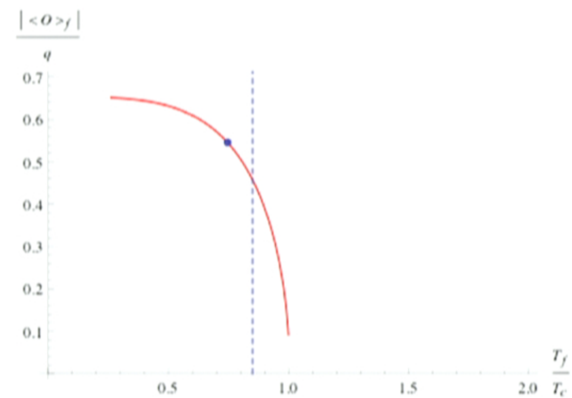




`B-L' like behaviour

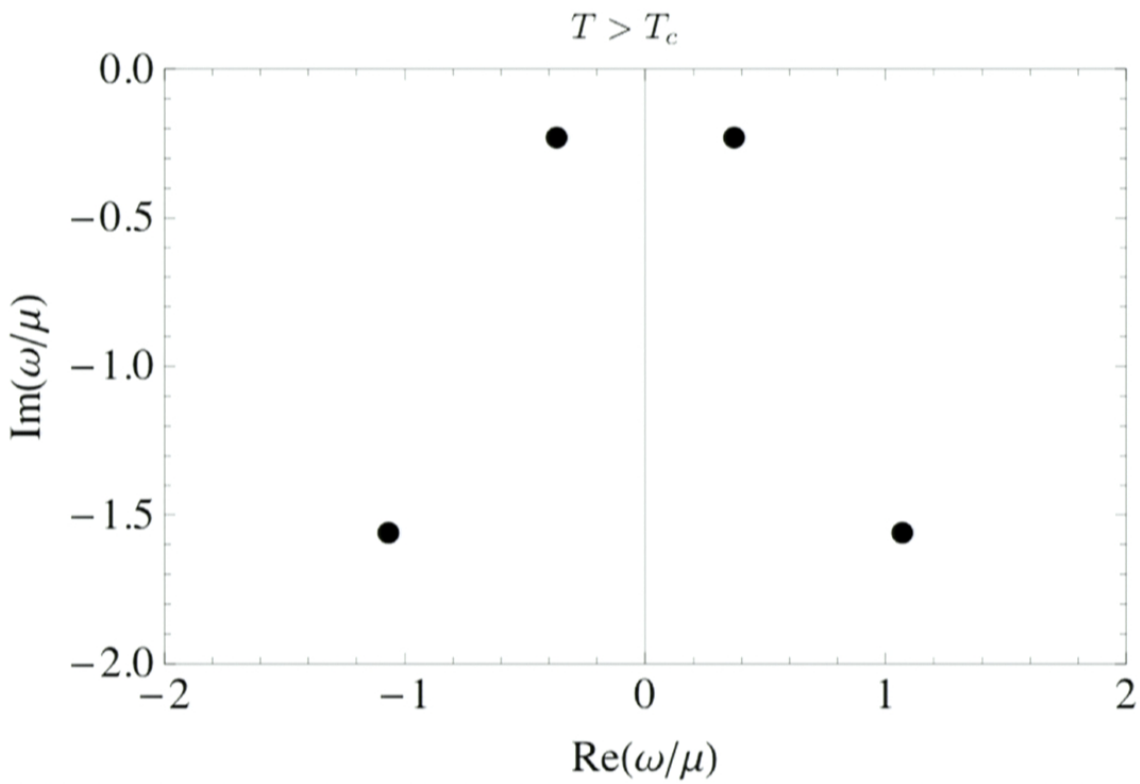
- We see analogous behaviour - for a small δ we find oscilating relaxation to the broken phase





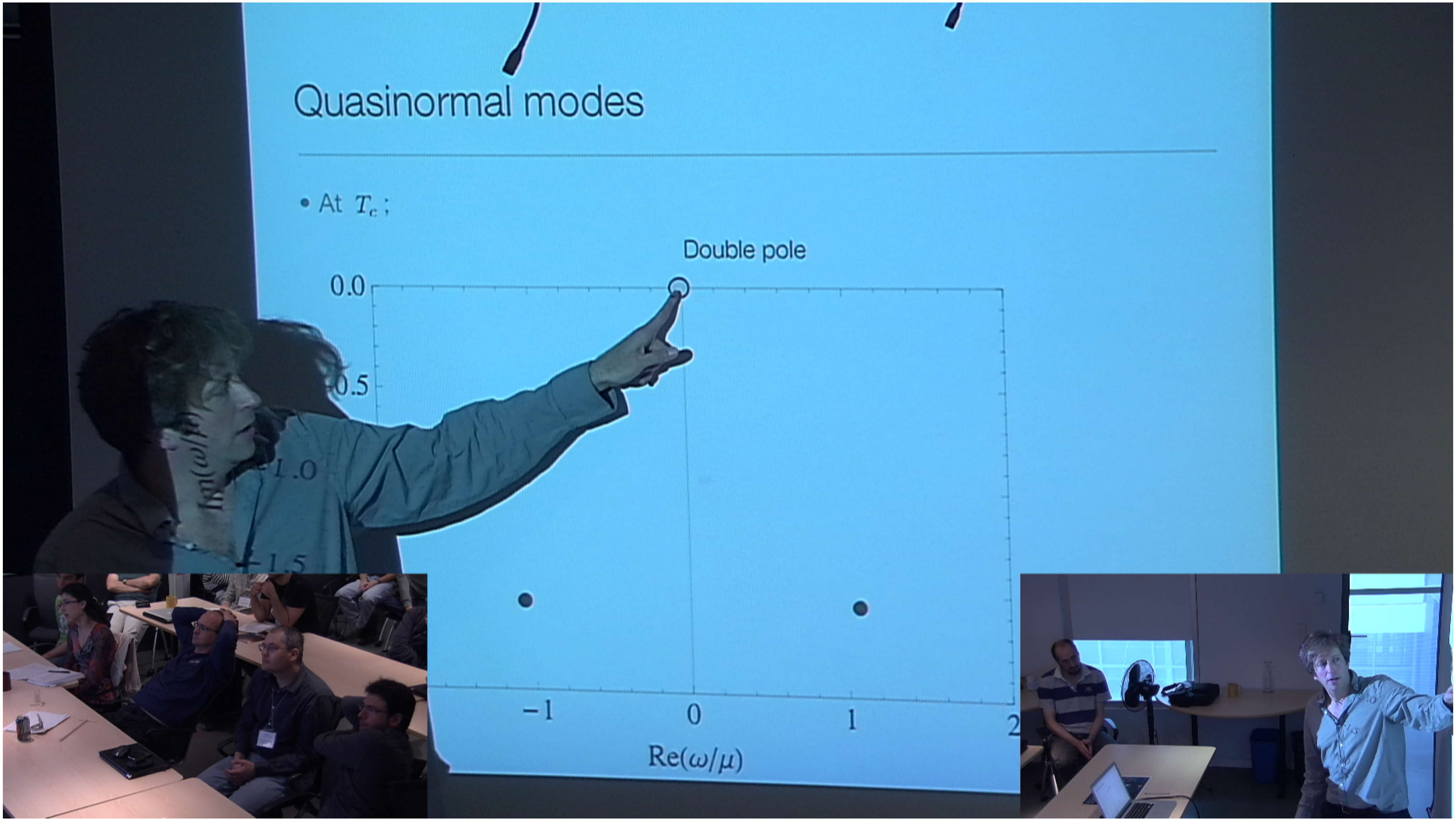
Quasinormal modes

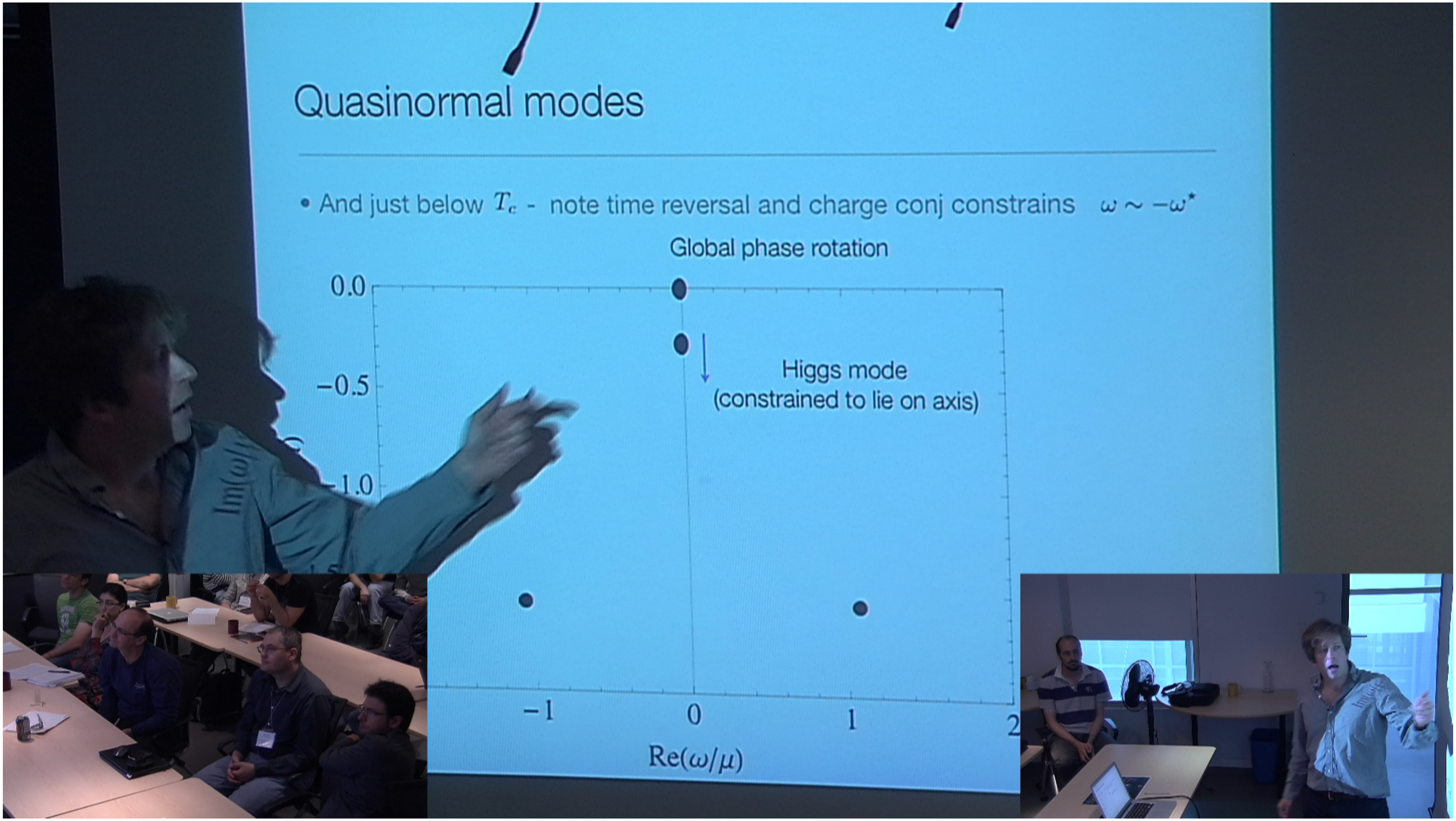
- In the normal phase we have no oscillations; $\psi(v,z) \sim e^{-i\omega v} f(z)$



$|\psi| \sim e^{\mathrm{Im}(\omega) v} f(z)$

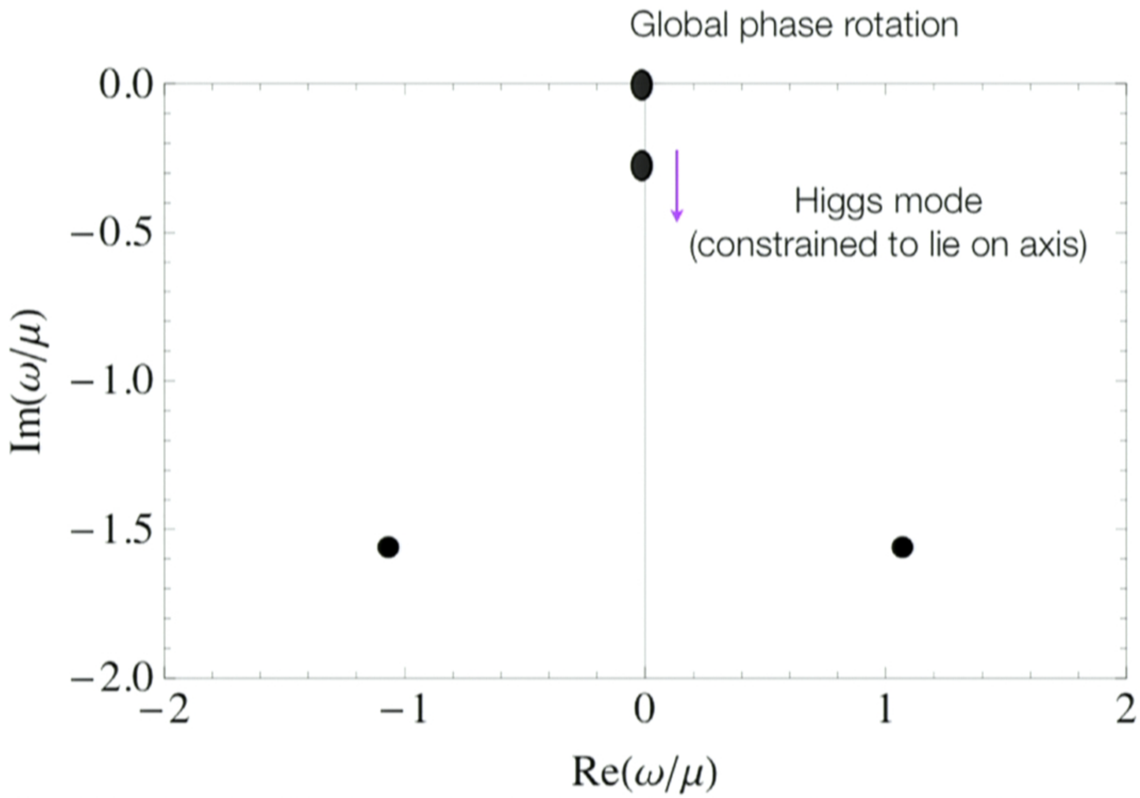

Quasinormal modes
• Approaching T_c ;
$T > T_c$
0.0
−1
0
1
2
Re(ω/μ)


Quasinormal modes
• At T_c ;
Double pole
0.0
0.5
1.0
1.5
−1
0
1
2
Re(ω/μ)


Quasinormal modes
• And just below T_c - note time reversal and charge conj constrains $\omega \sim -\omega^*$
Global phase rotation
0.0
−0.5
−1.0
Higgs mode
(constrained to lie on axis)
−1
0
1
2
Re(ω/μ)

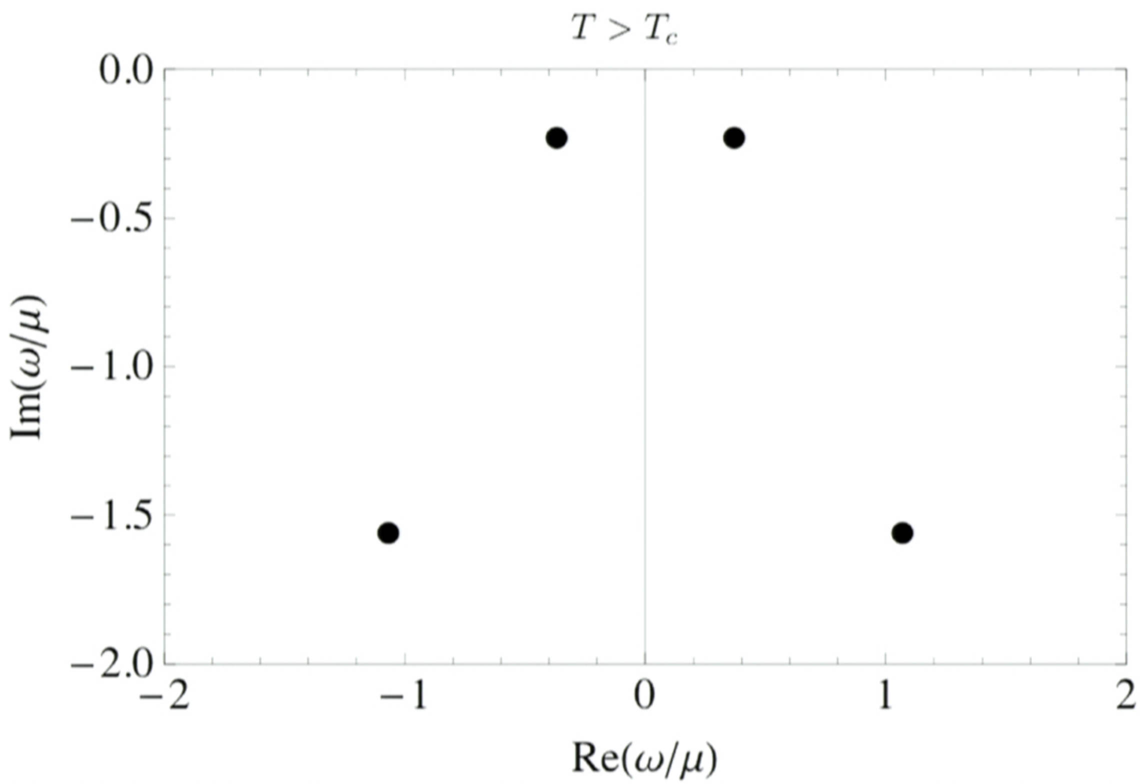
Quasinormal modes

- And just below T_c - note time reversal and charge conj constrains $\omega \sim -\omega^\star$



Quasinormal modes

- In the normal phase we have no oscillations; $\psi(v,z) \sim e^{-i\omega v} f(z)$

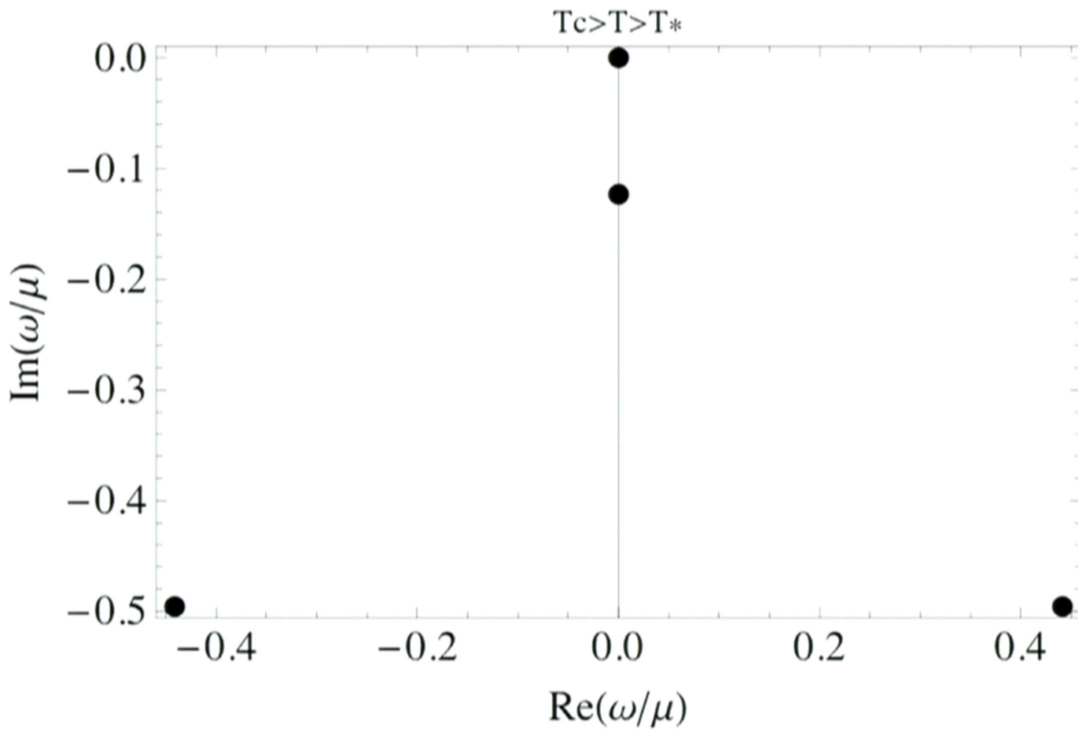


$|\psi| \sim e^{\mathrm{Im}(\omega)v} f(z)$

Quasinormal modes

- Between $T_c > T > T_\star$ see overdamped decay;

$$\psi(v,z) = g(z) + e^{-i\omega v} f(z)$$

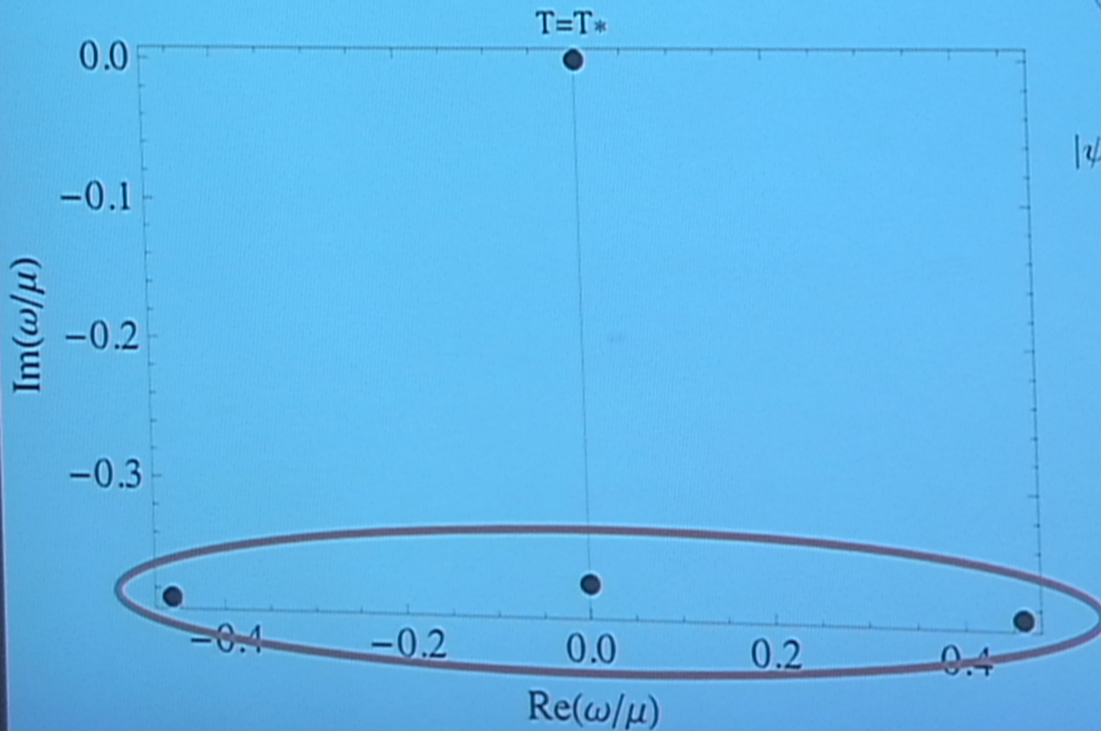


$$|\psi| \sim |g + e^{-i\omega v} f|$$

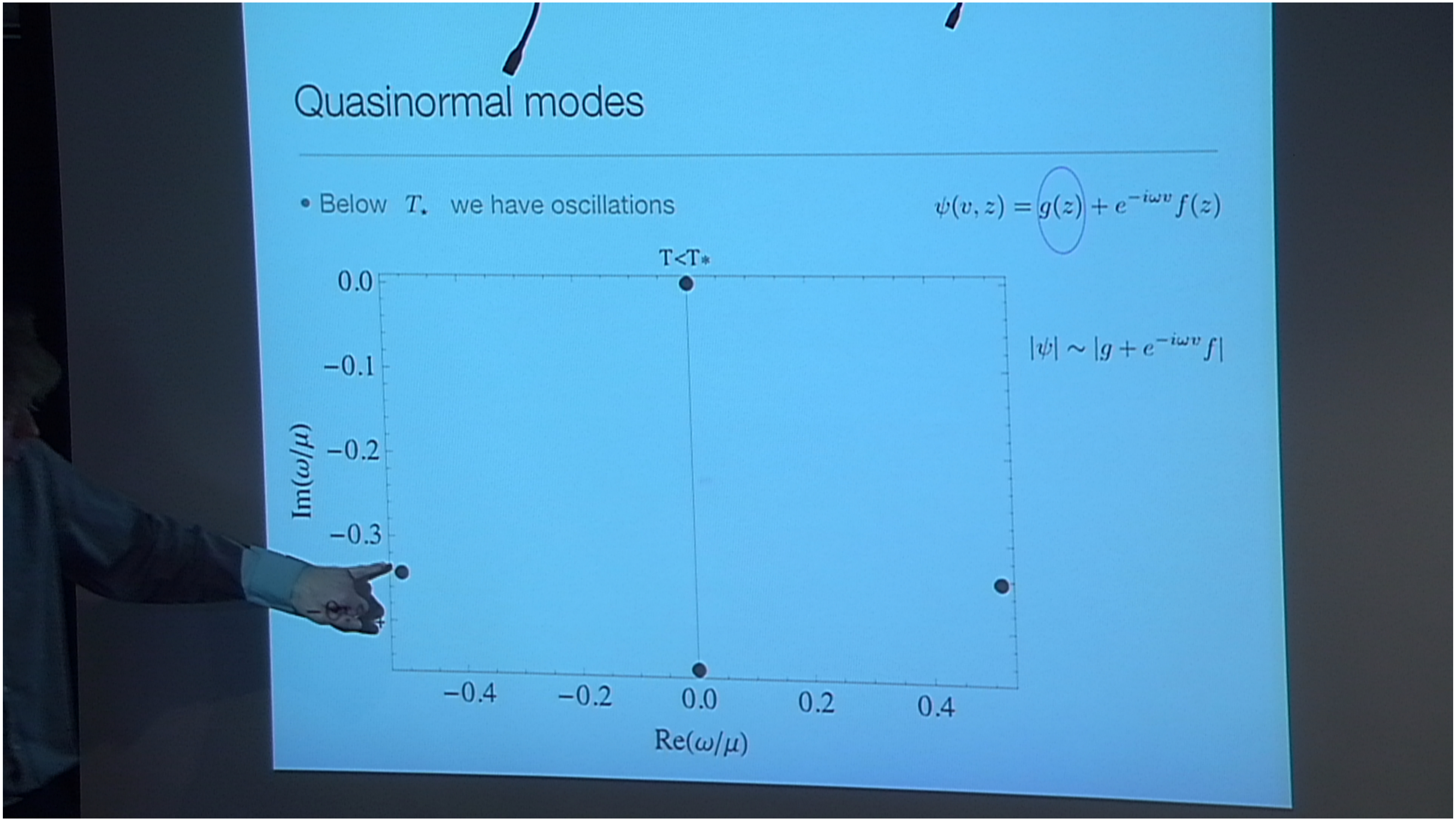
Quasinormal modes

- At T_*

$$\psi(v,z) = g(z) + e^{-i\omega v} f(z)$$

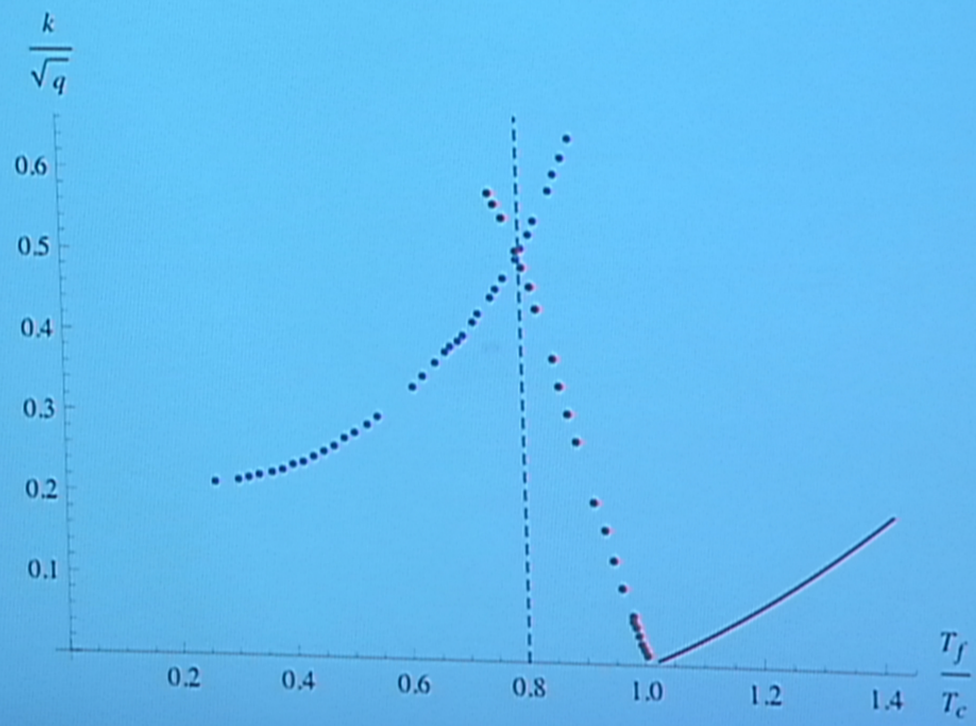


$$|\psi| \sim |g + e^{-i\omega v} f|$$


Quasinormal modes
• Below T_* we have oscillations
$\psi(v,z) = g(z) + e^{-i\omega v} f(z)$
$|\psi| \sim |g + e^{-i\omega v} f|$
T<T*
0.0
−0.1
−0.2
−0.3
Im(ω/μ)
−0.4
−0.2
0.0
0.2
0.4
Re(ω/μ)

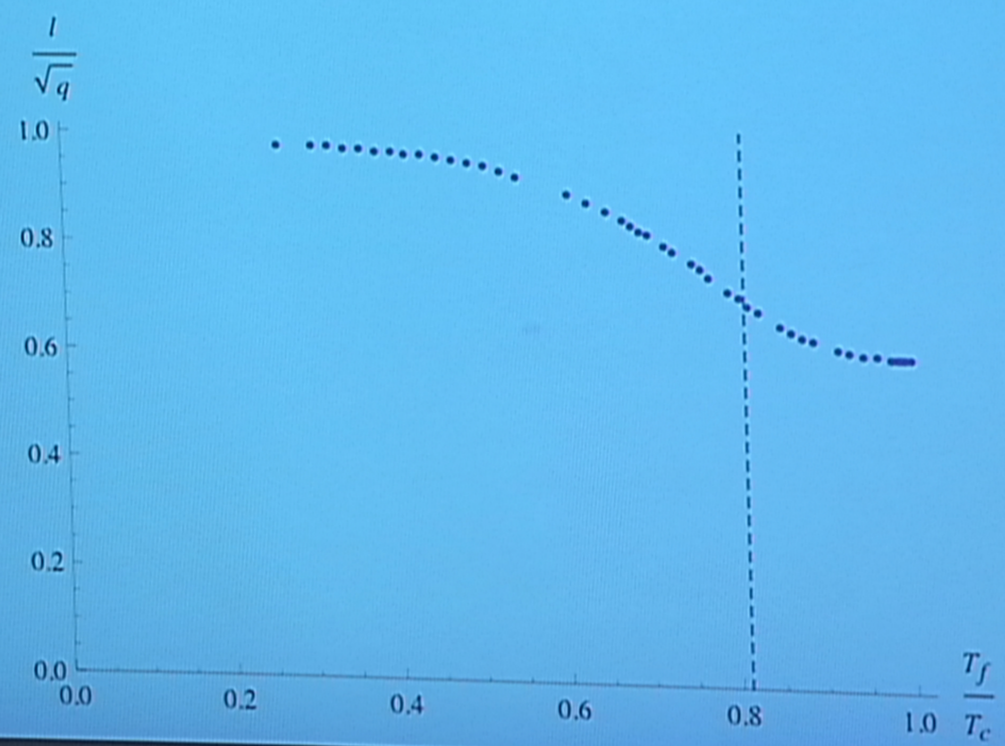
Quasinormal modes

- Imaginary part;



Quasinormal modes

- Real part;



Quasinormal modes

- At low temperatures it seems reasonable to expect oscillations as it is the temperature that gives dissipation.
- Then we see that a rather general structure simply due to symmetry breaking leads to the various behaviours.
- One might even expect such a result is more general than holographic superconductors, and in fact arguments might be made in linear response.

Summary

- AdS-CFT provides a possibility to describe certain strongly coupled superconductors
- Whilst it is unclear if these can describe 'real world' materials they do provide a computational setting to study questions that are very hard from a CMT approach.
- We see the 3 regimes of B-L are precisely seen in quenches of our superconductor, and the reason behind this is due to the structure of symmetry breaking and QNMs.
- Adds weight to the idea that these 3 regimes in quenches of superconductors are rather general and survive thermal effects and are beyond mean field.