Title: Instabilities of MP Black Holes

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Abstract: TBA

Instabilities of Myers-Perry black holes:

Numerical relativity study

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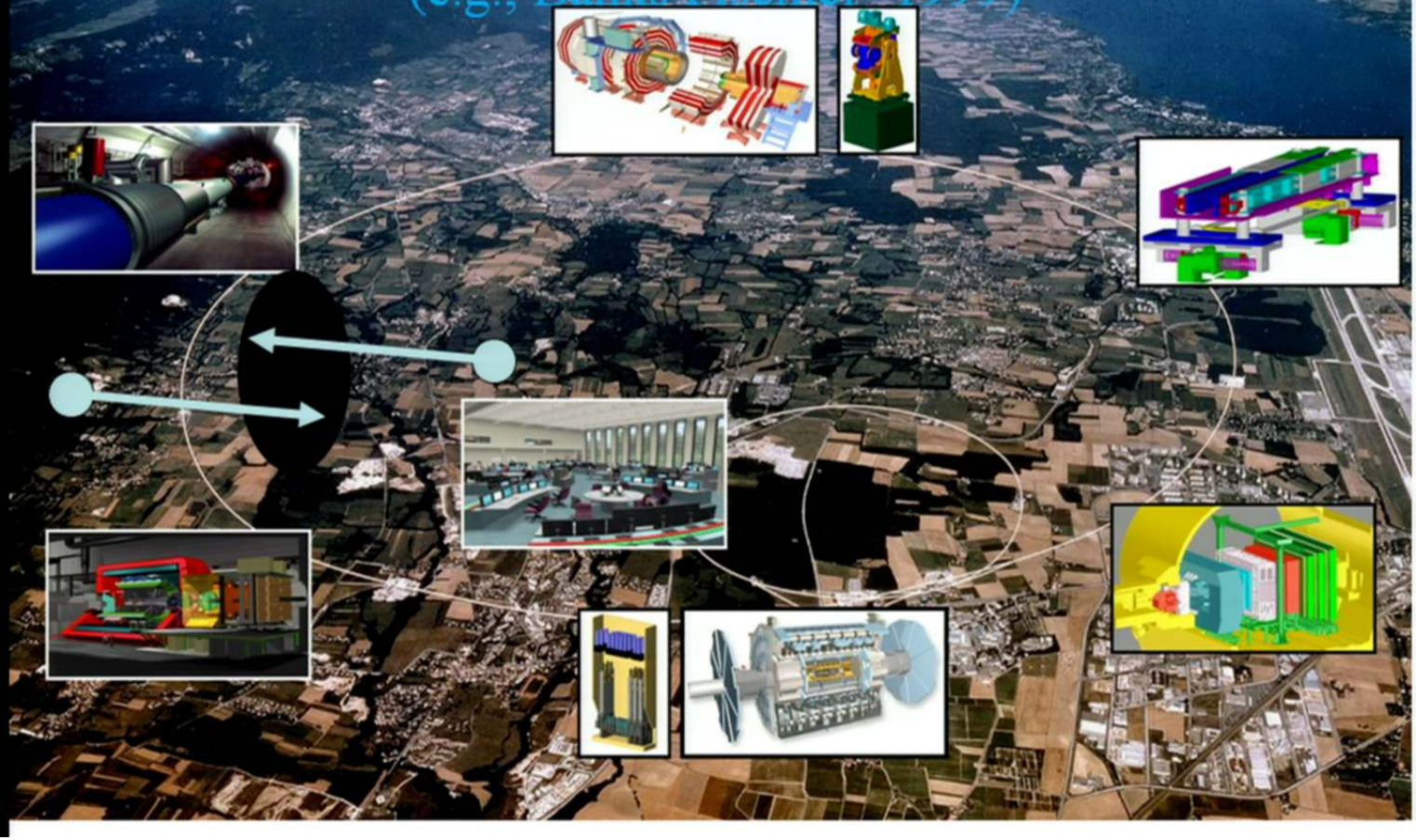
With H. Yoshino (KEK)

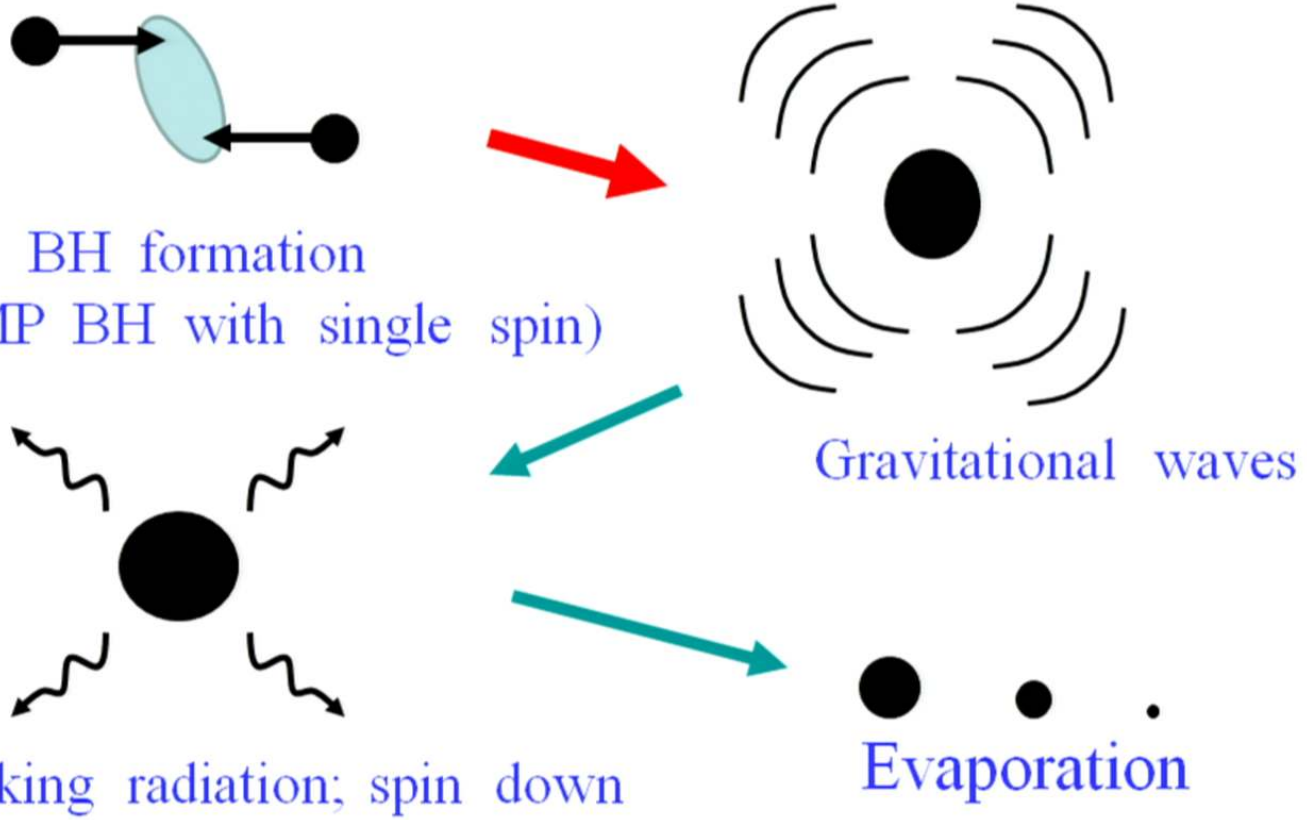
I Introduction

Particle collision → BH, if high dimension

(e.g., Banks-Fischler; 1999)



Often referred high-D & TeV-grav scenario



Classical regime is studied in numerical relativity

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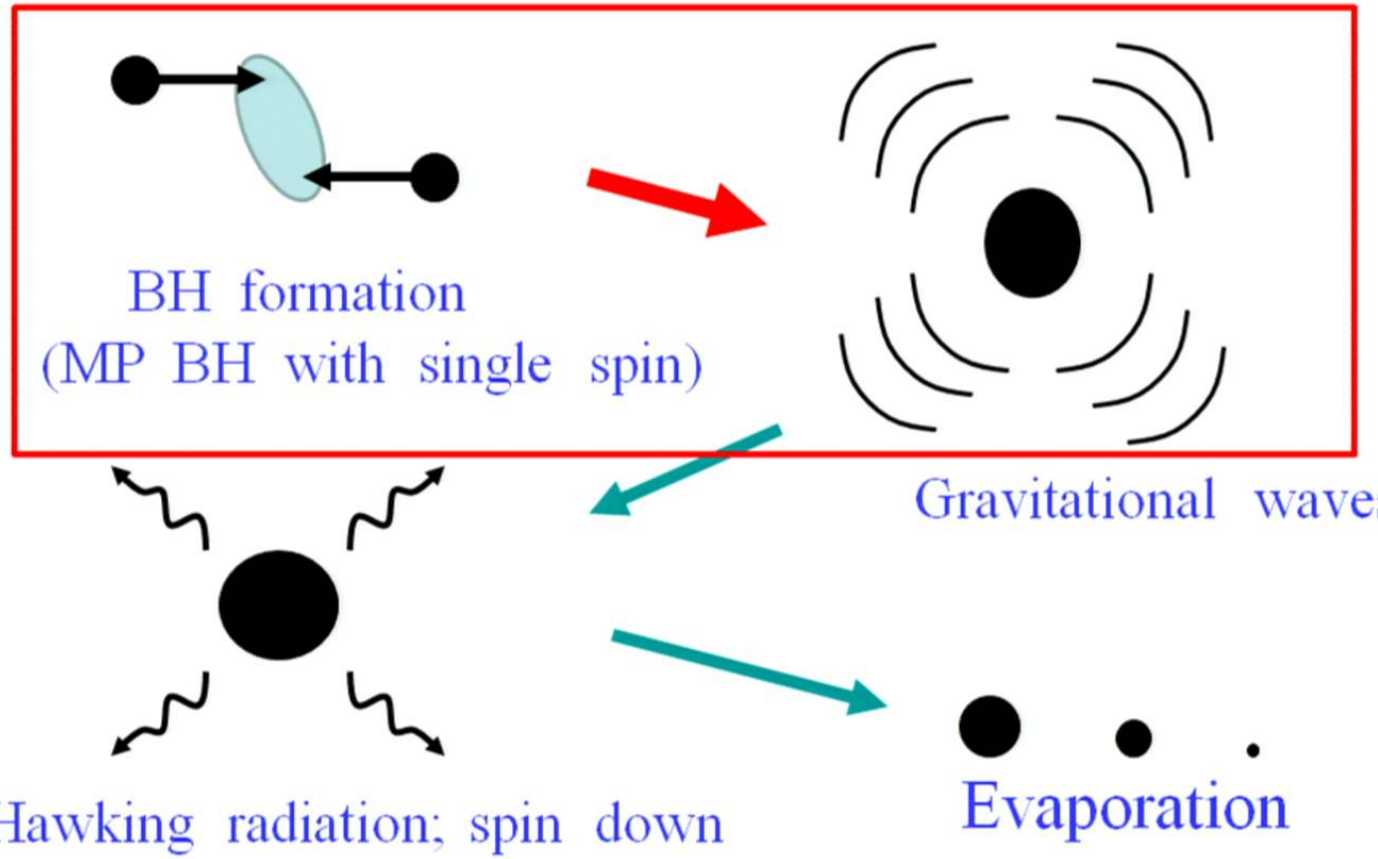


Classical regime is studied in numerical relativity

This talk focuses on a Myers-Perry black hole with single spin

$$ds^2 = -dt^2 + \frac{G_d \mu}{r^{d-5}\Sigma}\left(dt + a\sin^2\theta d\varphi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$
$$+\left(r^2 + a^2\right)\sin^2\theta d\varphi^2 + r^2\cos^2\theta d\Omega_{d-4}^2$$

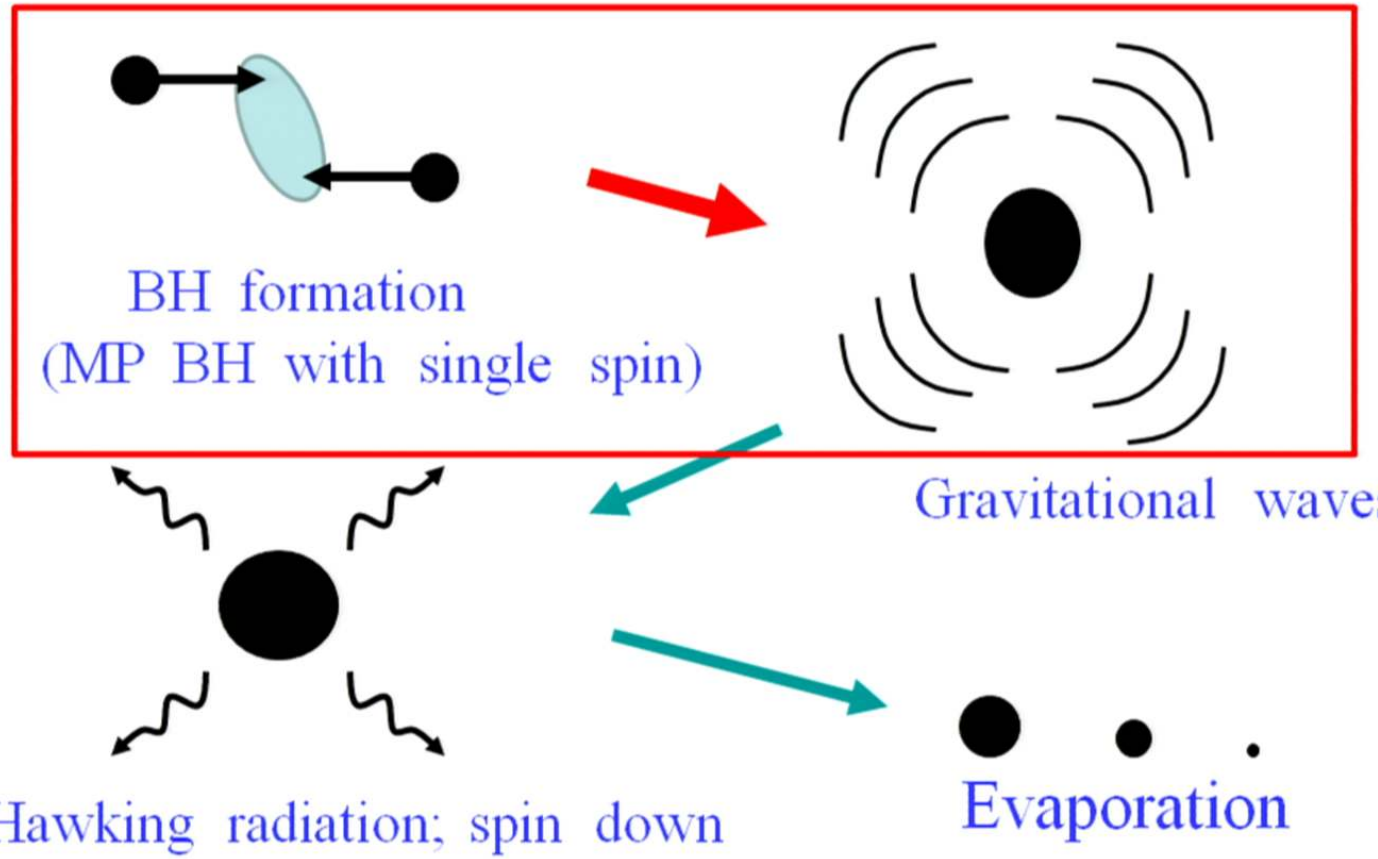
$$\Sigma = r^2 + a^2\cos^2\theta; \quad \Delta = r^2 + a^2 - \frac{G_d\mu}{r^{d-5}}$$

μ : mass parameter, a : spin parameter 2 parameter (a, μ)

$$\Rightarrow \quad M = \frac{(d-2)\Omega_{d-2}}{16\pi G_d}\mu, \quad J = \frac{2}{d-2}Ma$$

Length: $\left(G_d\mu\right)^{1/(d-3)}$, $q = \dfrac{a}{\left(G_d\mu\right)^{1/(d-3)}}$: nondim. spin

Often referred high-D & TeV-grav scenario



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Stability analysis

- For 4D, Kerr BH is stable; perturbation equation is separable (Teukolsky 1973), and the analysis can be done by solving ODE
- For higher-dimensions, no proof; perturbation equation has not been shown to be separable in general (except for special cases), and we need to solve PDEs numerically

Study for the stability of MP BH (1)

➢ **1st numerical stability analysis** (Dias et al. 09)

a. Restrict attention to axisymmetric mode
→ 2D problem (2D elliptic equations)

b. Solve numerically the eigen-value problem and try to find "zero" modes

c. For $D > 5$ (except for D=5), MP BH always has an unstable branch for the ultra-spinning case (against GL-type instability):
$q = a/\mu^{1/D-3} > \sim 1.6$ *with* D=6—9

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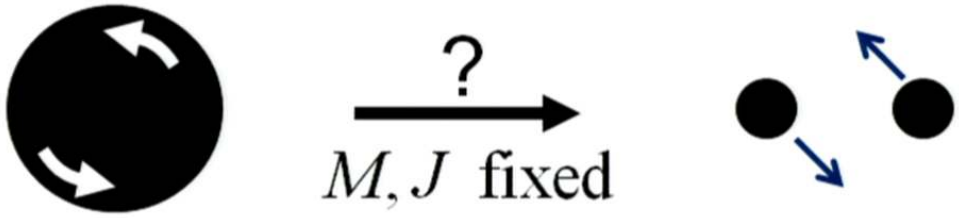
Study for the stability of MP BH (2)

- Emparan & Myers gave a conjecture based on *Thermodynamical argument* (2003)



$A = \Omega_{D-2} r_h \mu \rightarrow 0$
for high-spin, $D \geqq 5$

$A = 2A_o > 0$
Likely to be more stable

- EM suggests that ultra-spinning **BHs would be unstable for** $q = a/\mu^{1/D-3} > \sim 1$ **irrespective of** $D \geqq 5$; ***could set in for lower spin***

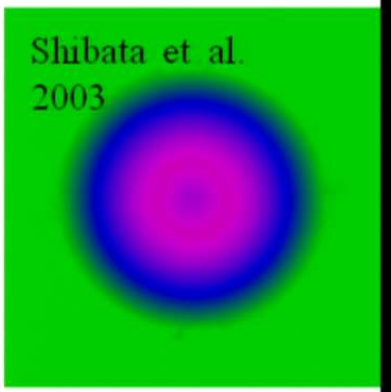
Analogy: Rotating star in 4D

- Rapidly rotating stars are unstable against *nonaxisymmetric* deformation (many works done, Chandra 69, Eriguchi, Tohline,…)
- Often found criterion, $T/W \sim 0.27$;

  T = rotational kinetic energy

  W = gravitational potential energy;

  or strongly differential rotation case



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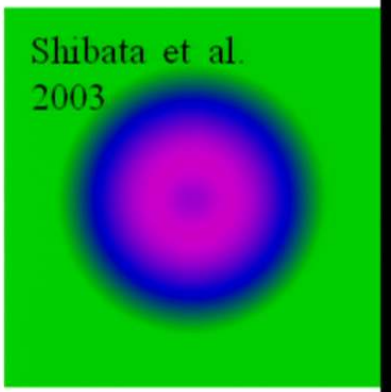

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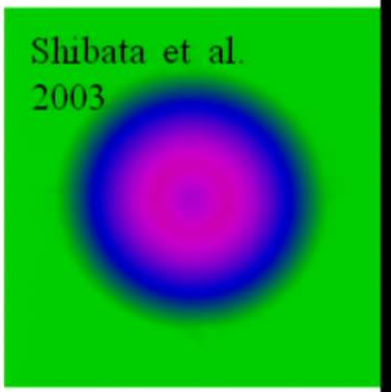

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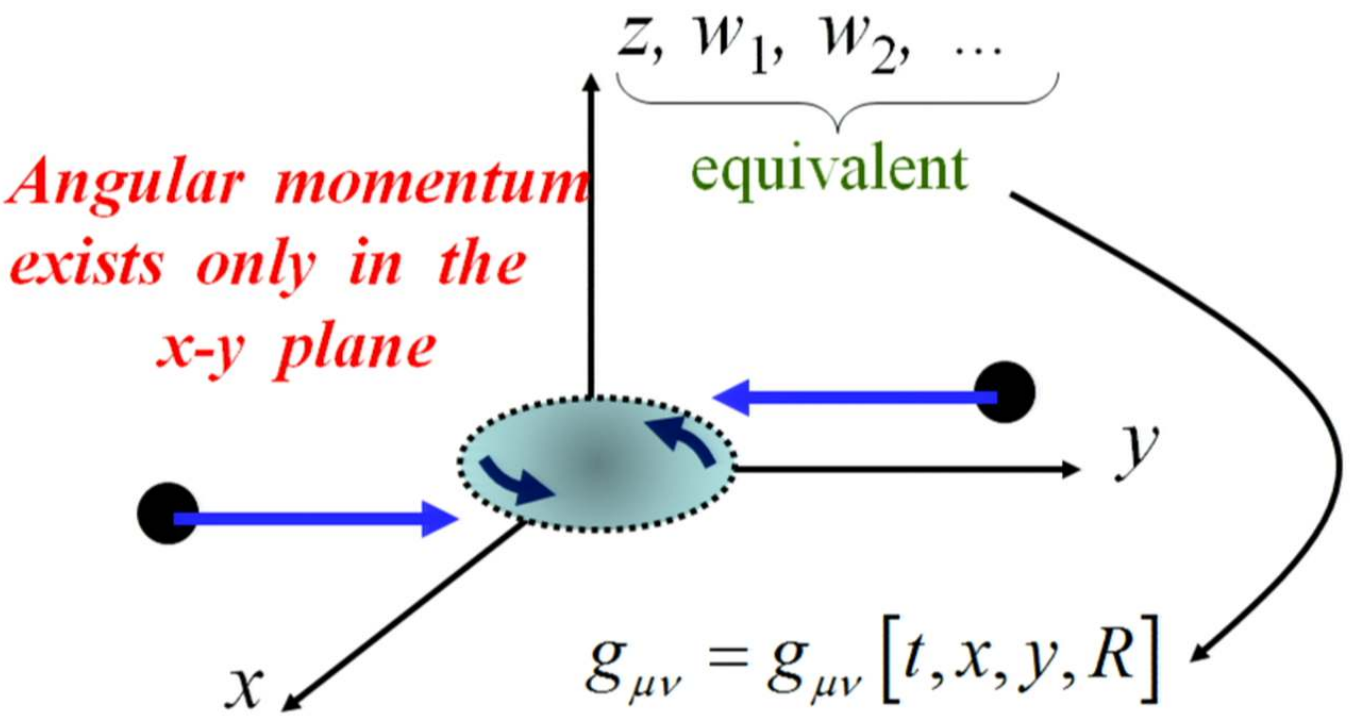
II High-dim numerical relativity: our approach

- Solve D-dim Einstein's equation $G_{\mu\nu}=0$ in $(N+1)$ formalism $(N=D-1)$

$$ds^2 = -\left(\alpha^2 - \beta_k \beta^k\right) dt^2 + 2\beta_k dx^k dt + \chi^{-2}\tilde{\gamma}_{ij} dx^i dx^j$$

- Specifically, BSSN + puncture formalism is employed as in the 3+1 case; works well
  (Shibata-Nakamura 95, Baumgarate-Shapiro 98, Campanelli, et al. 05, Yoshino-Shibata 09)
- Assume symmetry in the extra-dimensional directions; **SO(D–3) symmetry**
  → In computation, the number of dimension is "3" + 1 (time)

SO(*D*-3) Symmetry

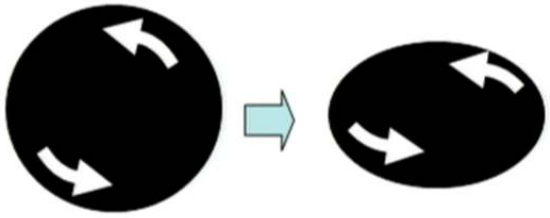


III Simulation

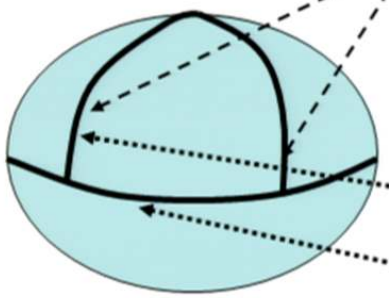
- Prepare a spinning Myers-Perry BH in the ***quasi-isotropic coordinates***, and follow time evolution using *puncture approach* (as in 4D)
- Initially, a small perturbation is given
- Method: 4^{th} order finite difference in time and space & puncture-gauge with BSSN (as in 4D)
- Fixed mesh refinement is used: A high grid resolution is necessary for high spin case

Method of analysis

- Analyze apparent horizon during simulation
- Calculate proper length of circumferential radius, and area
- Define deformation parameter



View from z-axis



Calculate the deformation, e.g.

$C_p(0)/\ C_p(\pi/2)$

$C_p(\phi)$

C_e

- Also, extract gravitational waves in the wave zone (along z axis)

5D case (SY, PRD **81**, 021501, 2010)

- Spin parameter $q = a/\mu^{1/2} = [0,1)$
  # For $a = \mu^{1/2}$, a naked singularity appears ($r_h = 0$, and hence, $A = 0$)
- Ellipticity increases with q:
  For $a \to \mu^{1/2} \to$ pancake-like black hole

$A = \Omega_{D-2}\, r_h \mu \to 0$

$C_p / C_e \to 0$

Rotating star is unstable against bar-mode deformation for such deformed case

Any 5D MP BH (Kerr type) is stable against axisymmetric deformation (cf. Dias et al. 09)

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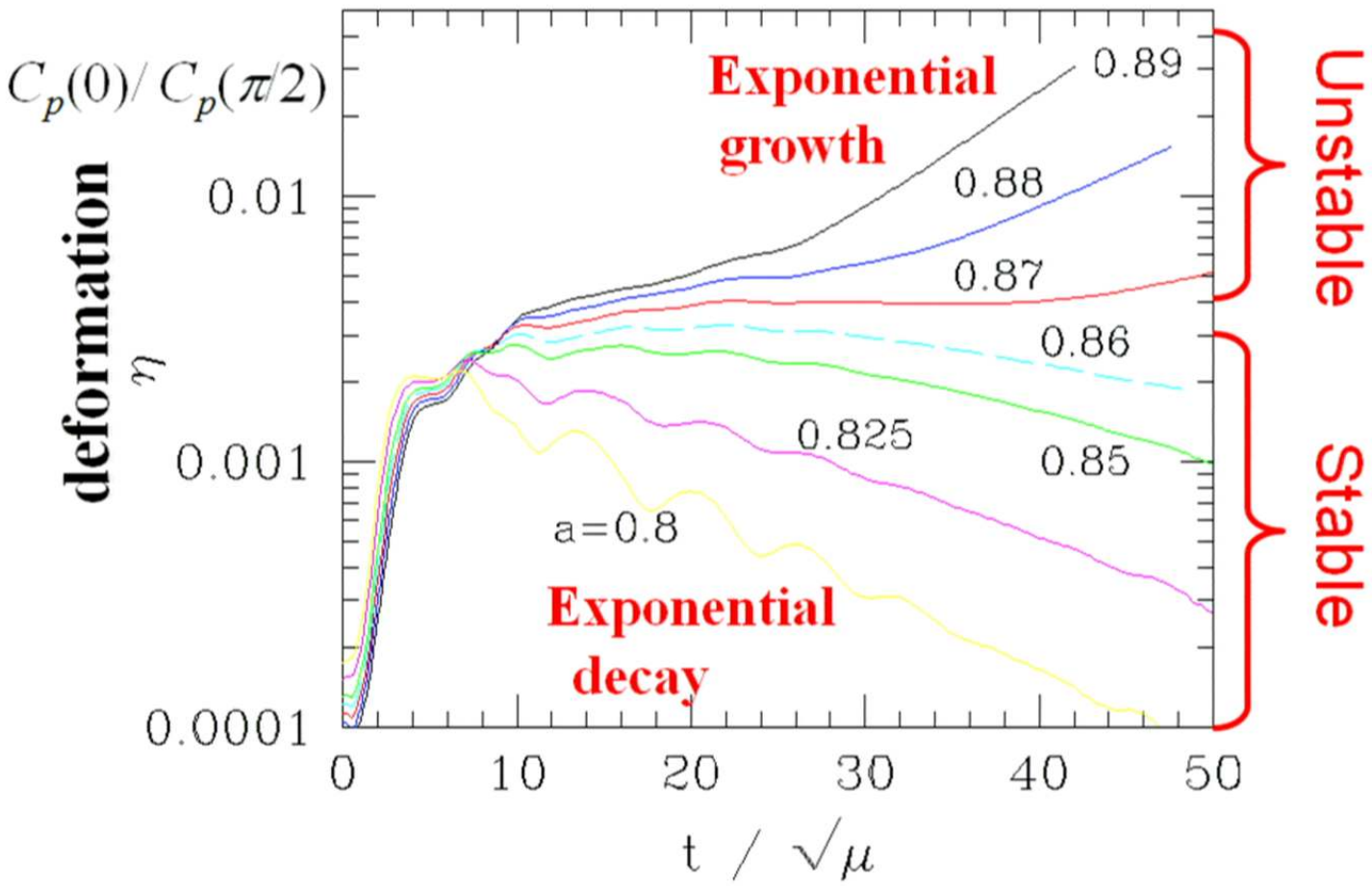
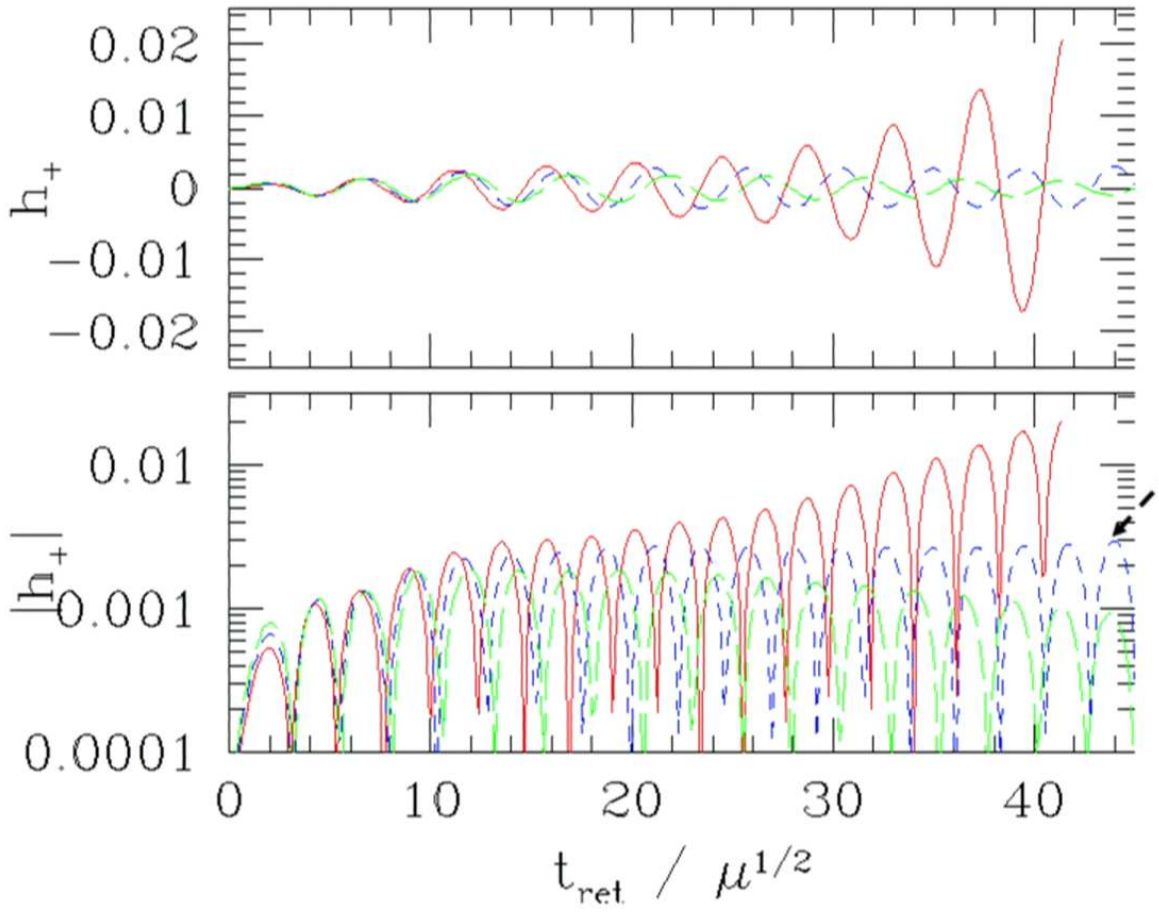
Evolution of deformation of AH



Gravitational waves



5D Results

- **Rapidly spinning BHs are unstable against bar-mode deformation**
- The threshold value of spin is $q \fallingdotseq 0.87$
- Final fate ??

  → *Simulation for 5D rapidly spinning BH is very difficult, unfortunately*

  → We cannot follow it for a very long time to determine the final fate

  → **But, this is not the case in 6D & 7D**

D-dim case ($D \geqq 6$)

- Spin parameter $q = a/\mu^{1/(D-3)} = [0,\infty)$

$$A = \Omega_{D-2}\, r_{\rm h} \mu \to 0 \quad \text{for} \quad q \to \infty$$

- Ellipticity increases with q, but increases slowly with q
- BH with $q > 1.6$ is unstable for ***axisymmetric*** deformation (Dias et al., 2009)
- ***Nonaxisymmetric*** instability sets in for much smaller spin (SY, PRD 81, 104035, 2010)

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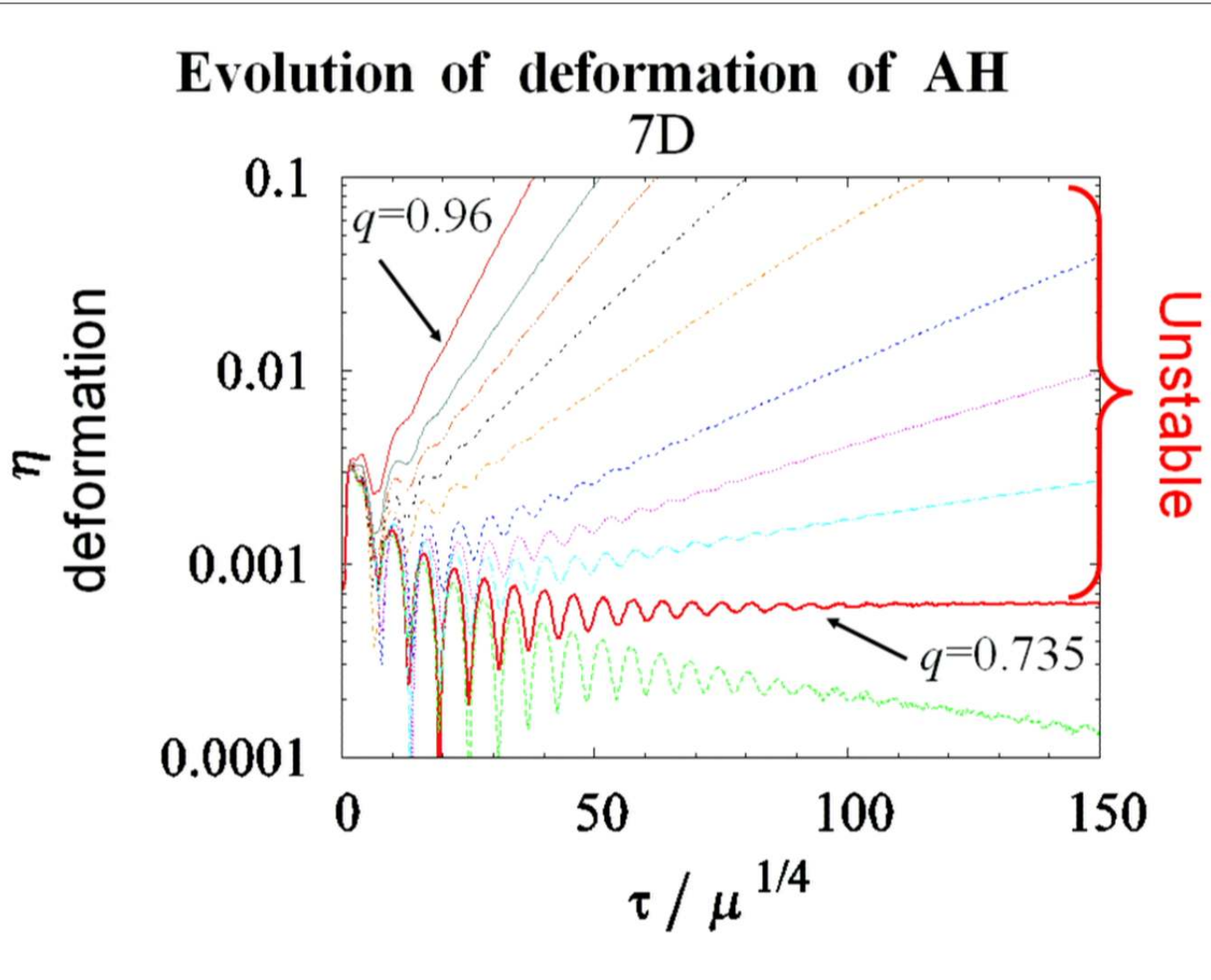
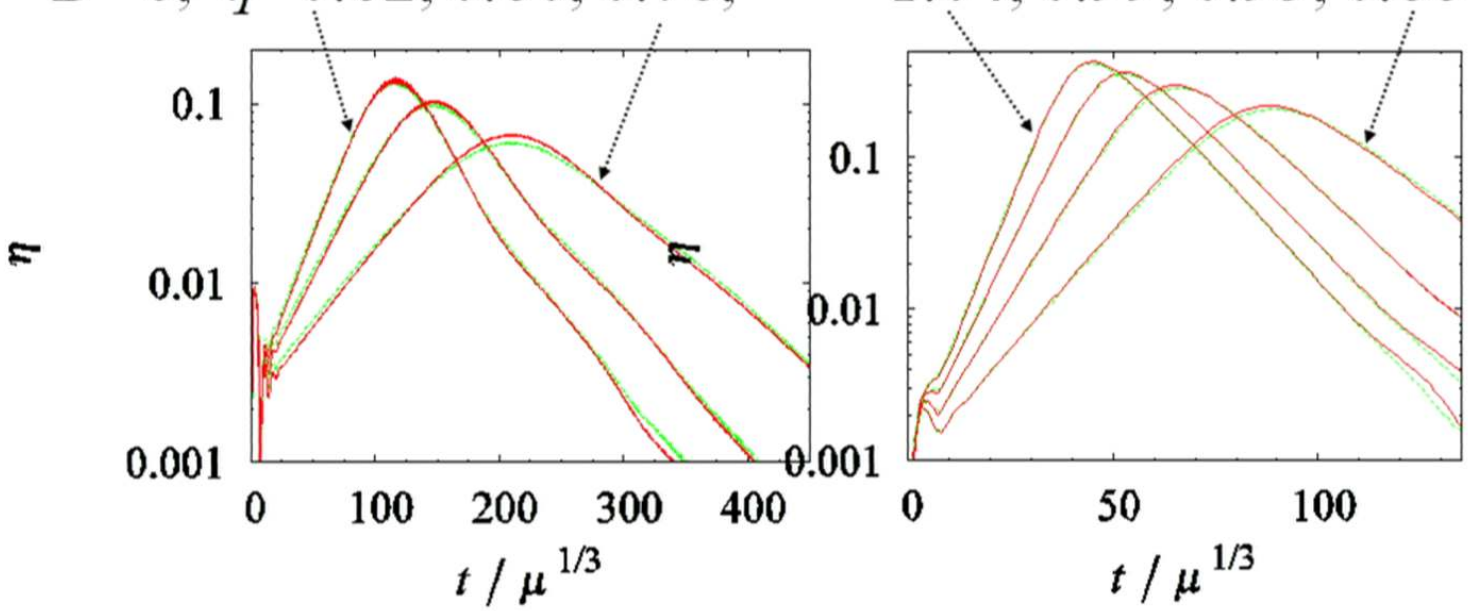

Evolution of deformation of AH
7D
0.1
0.01
0.001
0.0001
η
deformation
q=0.96
q=0.735
Unstable
0
50
100
150
τ / μ 1/4

Longterm evolution of deformation of AH

D=6, q=0.82, 0.80, 0.78, 1.04, 0.99, 0.93, 0.88



Growth → **Saturation by GW emission**

→ New stable BH of smaller spin

The same process as in 4D fast rotating star

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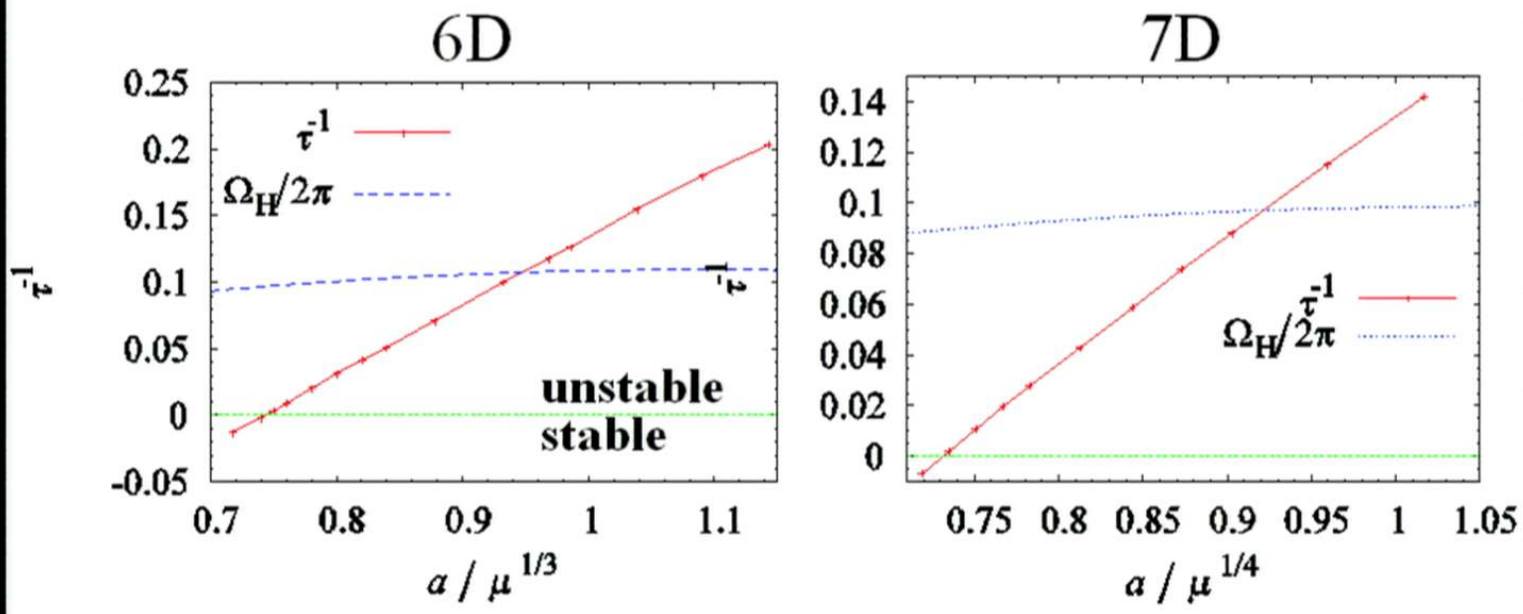
Growth rate



For $q > \sim 1$, growth time is shorter than rotational period of BH

→ **Very fast growth**

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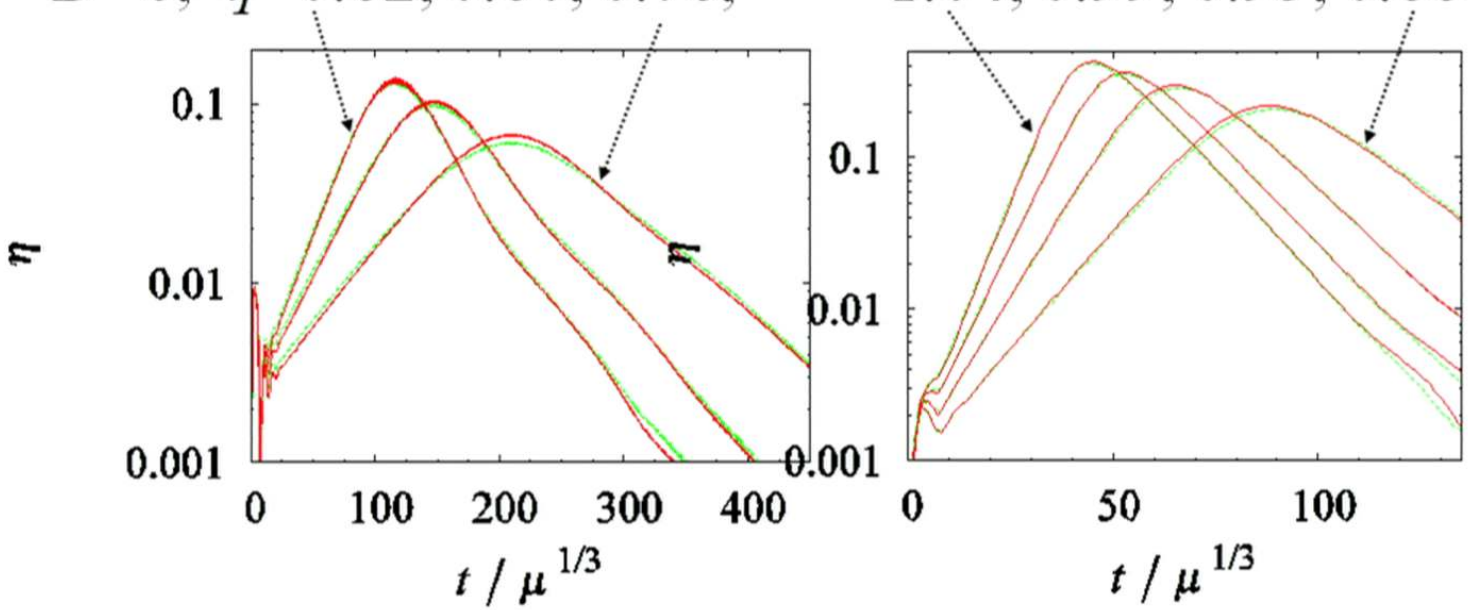

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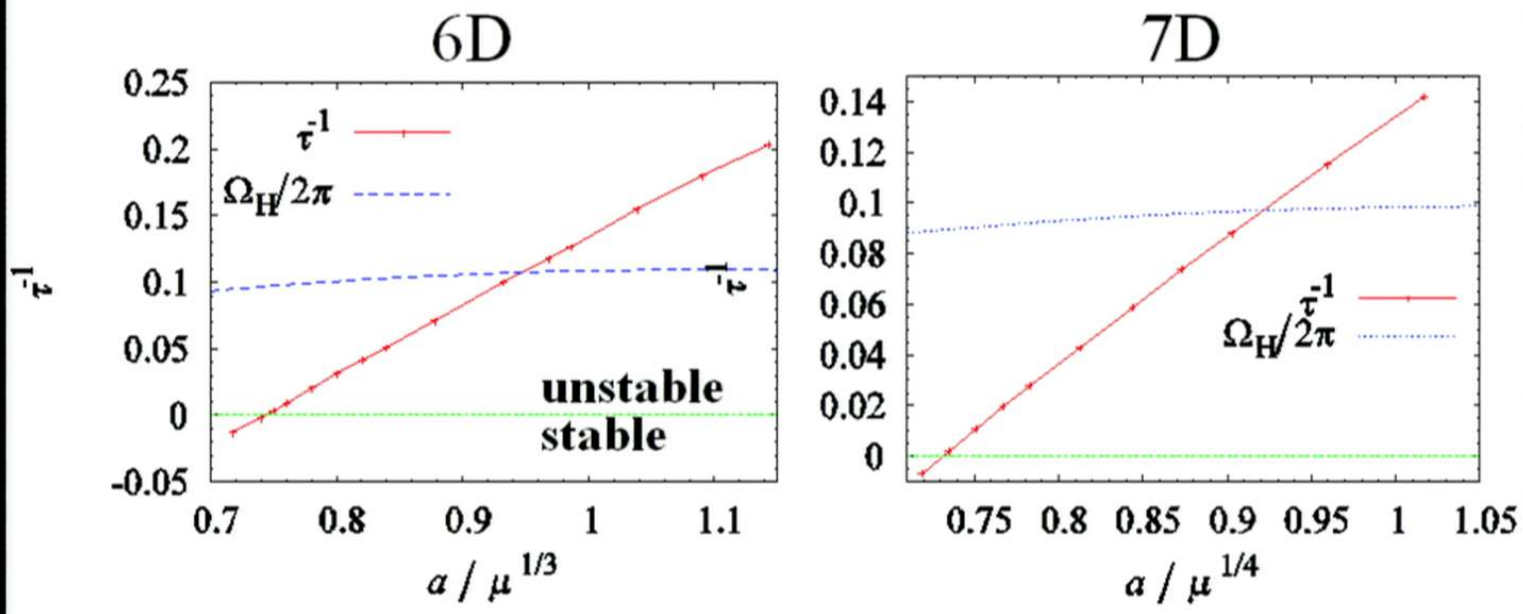

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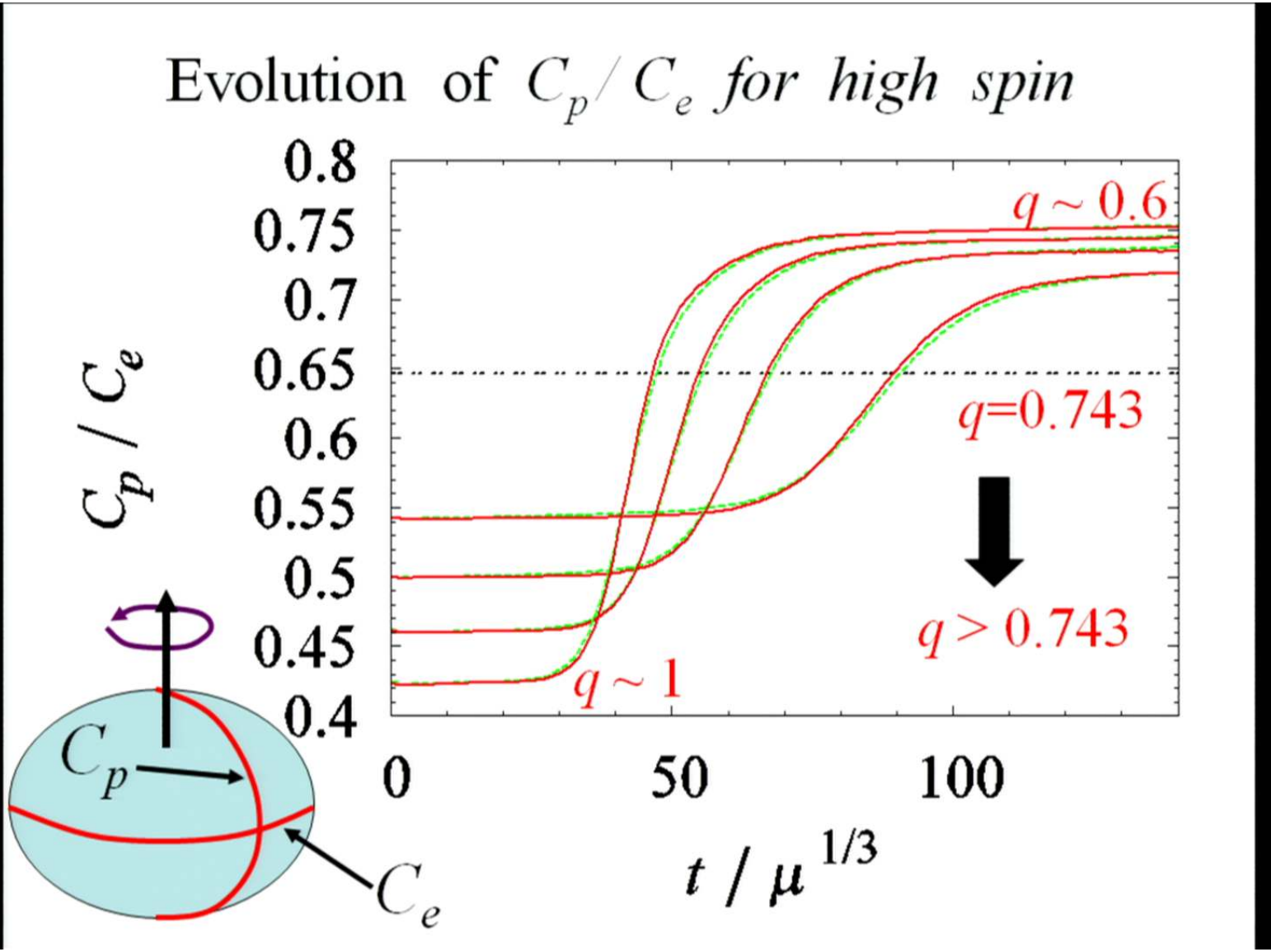

Evolution of C_p / C_e for high spin
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
C_p / C_e
$q \sim 0.6$
q=0.743
$q > 0.743$
$q \sim 1$
0
50
100
$t / \mu^{1/3}$
C_p
C_e

Check: 1st law of BH thermodynamics

$$\kappa \delta A = 8\pi \left(\delta M - \Omega_{\rm H} \delta J \right) > 0$$

For monochromatic gravitational waves:

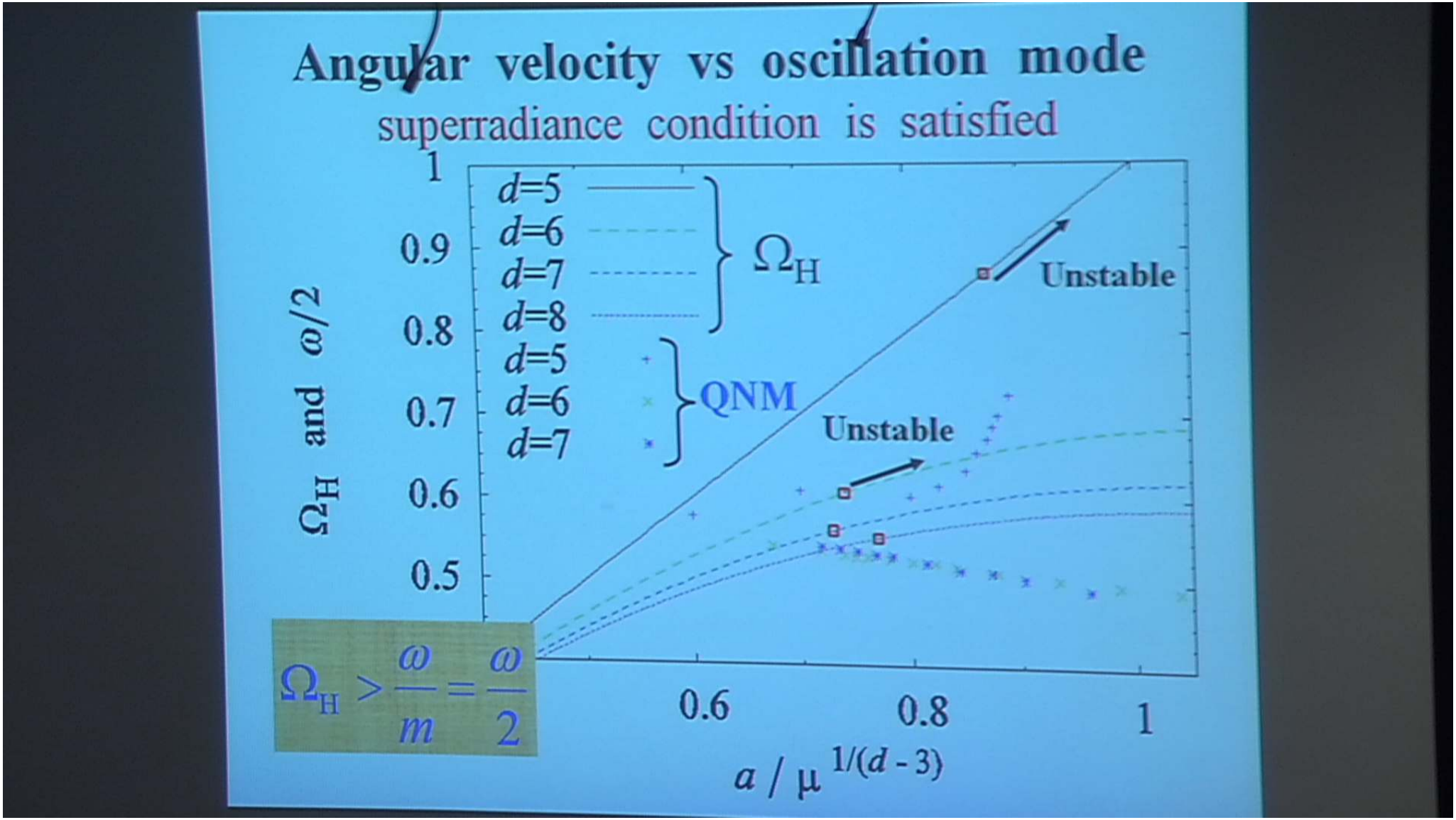
$$\delta M = -\delta E < 0, \quad \delta J < 0, \quad \delta E = \frac{\omega}{m} \delta J$$

$$\Rightarrow \kappa \delta A = 8\pi \, | \, \delta J \, | \left(\Omega_{\rm H} - \frac{\omega}{m} \right) > 0$$

For the unstable QNM mode, we find $\boxed{\Omega_{\rm H} > \frac{\omega}{m}}$

(Superradiance condition, e.g., Press & Teukolsky)

Evolution by GW emission is possible


Angular velocity vs oscillation mode
superradiance condition is satisfied
1
0.9
0.8
0.7
0.6
0.5
Ω_H and $\omega/2$
d=5
d=6
d=7
d=8
Ω_H
d=5
d=6
d=7
QNM
Unstable
Unstable
$\Omega_H > \frac{\omega}{m} = \frac{\omega}{2}$
0.6
0.8
1
$a / \mu^{1/(d-3)}$

Summary

- Rapidly spinning BHs are primarily unstable against bar-mode deformation
- The threshold value of spin is fairly small $q \sim$ **0.74 (D=6), 0.73 (D=7), and 0.77 (D=8);** cf. $q \sim 0.87$ (D=5),
- **Unstable BH radiates GWs and after the spin-down by sufficient radiation, the BH settles down to a new stable state**

Note: Ultra-spinning BH may have different fate; but it is not easy to simulate …

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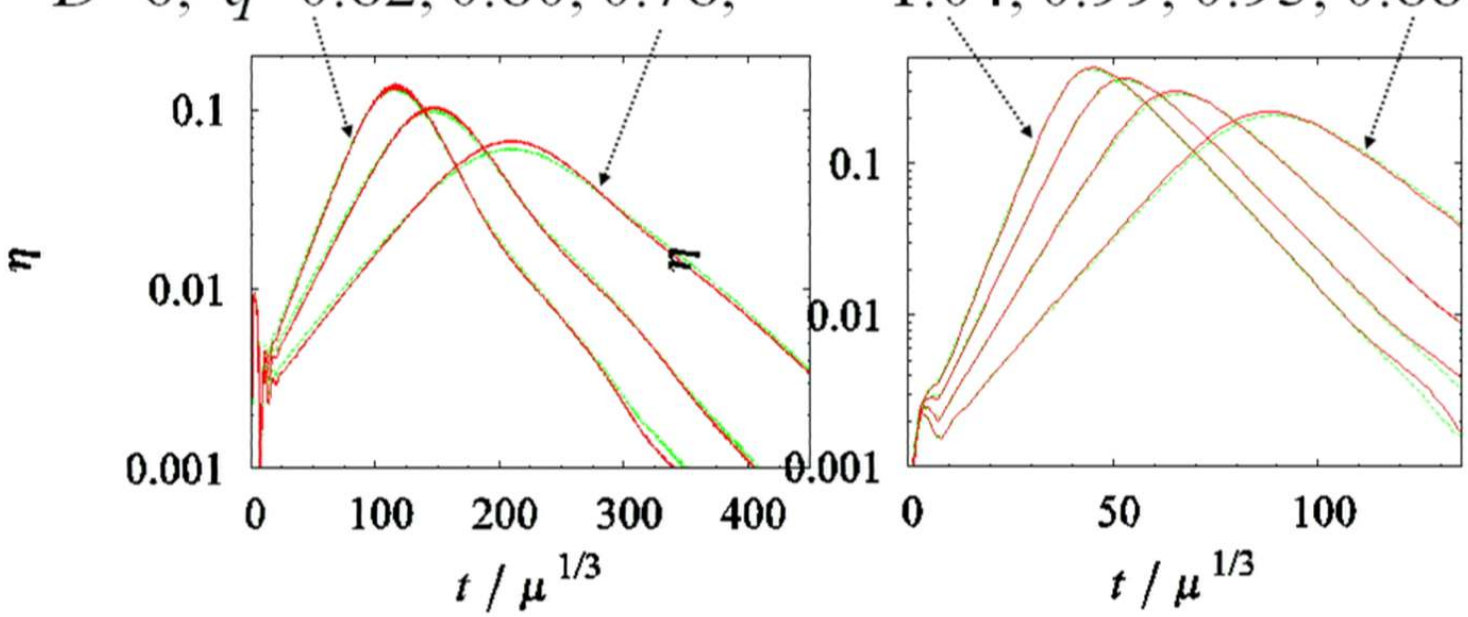

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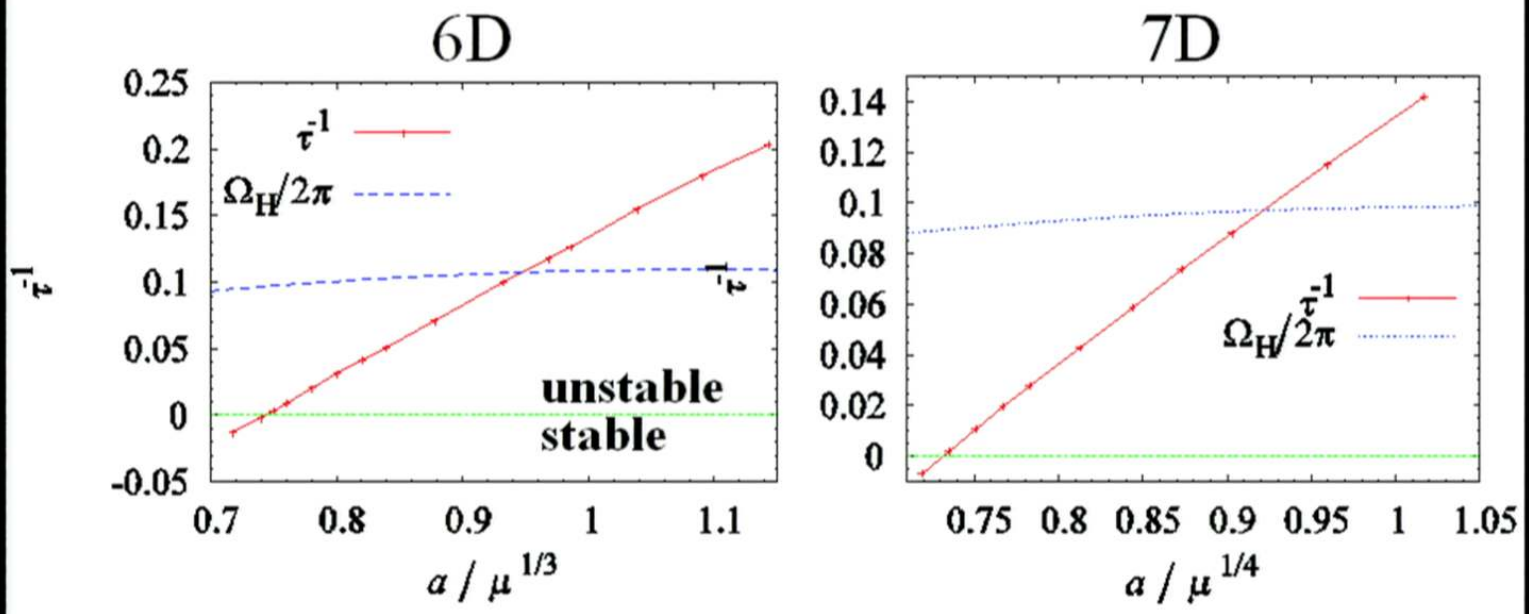

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