Title: Gravitational Turbulent Instability of Anti-de Sitter Space

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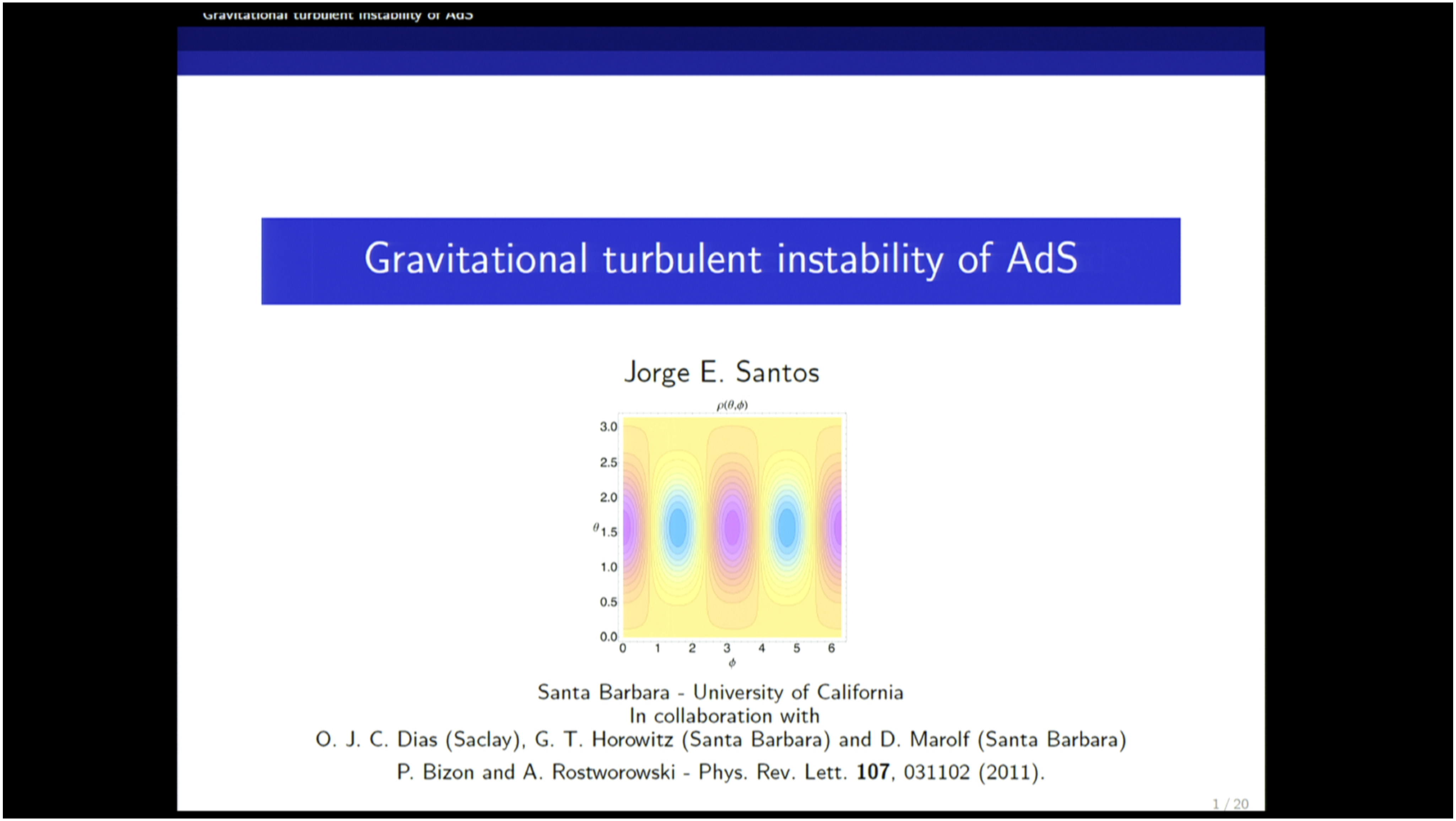
Abstract: TBA


Gravitational turbulent instability of AdS
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Jorge E. Santos
ρ(θ,ϕ)
3.0
2.5
2.0
θ 1.5
1.0
0.5
0.0
0 1 2 3 4 5 6
ϕ
Santa Barbara - University of California
In collaboration with
O. J. C. Dias (Saclay), G. T. Horowitz (Santa Barbara) and D. Marolf (Santa Barbara)
P. Bizon and A. Rostworowski - Phys. Rev. Lett. 107, 031102 (2011).
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Anti-de Sitter spacetime - 1/2

Anti-de Sitter space is a maximally symmetric solution to

$$S = \frac{1}{16\pi G}\int \mathrm{d}^d x\sqrt{-g}\left[R + \frac{(d-1)(d-2)}{L^2}\right],$$

which in global coordinates can be expressed as

$$\mathrm{d}s^2 \equiv \bar{g}_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(\frac{r^2}{L^2}+1\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\frac{r^2}{L^2}+1} + r^2\mathrm{d}\Omega^2_{d-2}.$$

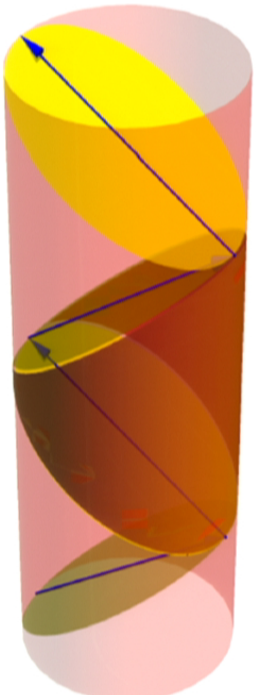
The Poincaré coordinates

$$\mathrm{d}s^2 = R^2(-\mathrm{d}\tau^2 + \mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}) + \frac{L^2\mathrm{d}R^2}{R^2}$$

do not cover the entire spacetime.

Anti-de Sitter spacetime - 2/2

Conformally, AdS looks like the interior of a cylinder



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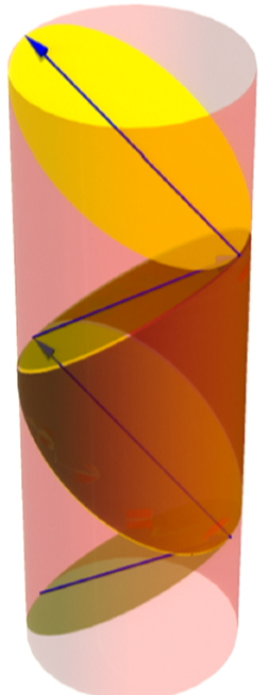


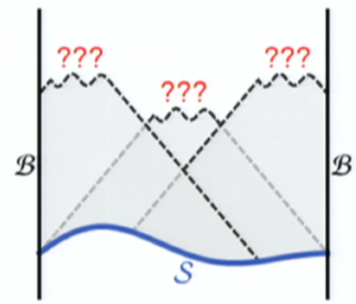
- Poincaré coordinates cover the brown-shaded region.
- The instability described in this talk will occur in global AdS only.

The stability problem for spacetimes in general relativity

The setup:

Consider a spacetime $(\mathcal{M}, g)$, together with prescribed boundary conditions $\mathcal{B}$ if timelike boundary exists.

- Take small perturbations (in a suitable sense) on a Cauchy surface $\mathcal{S}$.
- Does the solution spacetime $(\mathcal{M}, g')$ that arises still has the same asymptotic causal structure as $(\mathcal{M}, g)$?
- If so, can we bound the "difference" between the asymptotic form of g and g' in terms of initial data defined on $\mathcal{S}$?



In particular, if a geodesically complete spacetime is perturbed, does it remain "complete"?

Minkowski, dS and AdS spacetimes

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Minkowski, dS and AdS spacetimes

- At the linear level, Anti de-Sitter space-time appears just as stable as the Minkowski or de-Sitter spacetimes.
- For the Minkowski and de-Sitter spacetimes, it has been shown that small, but finite, perturbations remain small - D. Christodoulou and S. Klainerman '93 and Friedrich '86.
- Why has this not been shown for Anti de-Sitter?
  - It is just *not* true!

Claim:

Generic small (but finite) perturbations of AdS become large and eventually form black holes.

Folklore

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NO :

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Why is AdS unstable?

- AdS acts like a confining finite box. Any generic finite excitation which is added to this box might be expected to explore all configurations consistent with the conserved charges of AdS - including small black holes.
- Special (fine tuned) solutions need not lead to the formation of black holes.
  - We will see that for some linearized gravitational *mode* there will be a corresponding nonlinear solution - geon.

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Perturbative construction - 1/2

- Expand the metric as

$$g = \bar{g} + \sum_i \epsilon^i h^{(i)}.$$

- At each order in perturbation theory, the Einstein equations yield:

$$\tilde{\Delta}_L h_{ab}^{(i)} = T_{ab}^{(i)},$$

where $T^{(i)}$ depends on $\{h^{(j \leq i-1)}\}$ and their derivatives and

$$2\tilde{\Delta}_L h_{ab}^{(i)} \equiv -\bar{\nabla}^2 h_{ab}^{(i)} - 2\bar{R}_a{}^c{}_b{}^d h_{cd}^{(i)} - \bar{\nabla}_a \bar{\nabla}_b h^{(i)} + 2\bar{\nabla}_{(a}\bar{\nabla}^c h_{b)c}^{(i)}.$$

- Any smooth symmetric two-tensor can be expressed as a sum of fundamental building blocks, $\mathcal{T}_{ab}^{\ell m}$, that have definite transformation properties under the $SO(d-1)$ subgroup of AdS.

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Perturbative construction - 2/2

- For concreteness, set $d = 4$. Perturbations come in three classes:
  - Scalar-type perturbations: perturbations are constructed from spherical harmonics on S^2 - $\mathbb{Y}_{\ell m}$.
  - Vector-type perturbations: perturbations are constructed from vector harmonics on S^2 - these are $\star_{S^2}\nabla\mathbb{Y}_{\ell m}$.
  - Tensor-type perturbations: only exist in $d \geq 5$.

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  - Vector-type perturbations: perturbations are constructed from vector harmonics on S^2 - these are $\star_{S^2} \nabla \mathbb{Y}_{\ell m}$.
  - Tensor-type perturbations: only exist in $d \geq 5$.
- We go beyond linear order: need real representation for $\mathbb{Y}_{\ell m}$ - $\mathbb{Y}^c_{\ell m} = \cos\phi \, \mathrm{L}^m_\ell(\theta)$ and $\mathbb{Y}^s_{\ell m} = \sin\phi \, \mathrm{L}^m_\ell(\theta)$.

Linear Perturbations

- At the linear level ($i = 1$) we can further decompose our perturbations as

$$\Phi^{\alpha,(i)}_{\ell m}(t,r) = \Phi^{\alpha,(i),c}_{\ell m}(r)\cos(\omega_\ell t) + \Phi^{\alpha,(i),s}_{\ell m}(r)\sin(\omega_\ell t).$$

- Because AdS acts like a confining box, only certain frequencies are allowed to propagate

$$\omega_\ell^2 L^2 = (1 + \ell + 2p)^2,$$

  where p is the radial overtone. These are the so-called normal modes of AdS. The fact that $\omega^2 L^2 > 0$ means that AdS is linearly stable.
- For simplicity, we will take $p = 0$, in which case one finds

$$\Phi^{\alpha,(1),\kappa}(r) = A^{\alpha,(1),\kappa}\frac{r^{\ell+1}}{(r^2+L^2)^{\frac{\ell+1}{2}}},$$

  where $A^{\alpha,(1),\kappa}$ is a normalization constant.

General Structure

1. Start with a given perturbation $\Phi^{\alpha,(i),\kappa}_{\ell m}(r)$, and determine the corresponding $h^{(i)}_{\ell m}(t,r,\theta,\phi)$ through a linear differential map.
2. Compute $T^{(i+1)}_{ab}$ and decompose it as a sum of the building blocks $\mathcal{T}^{\ell m}_{ab}$.
3. Compute source term $\tilde{T}^{\alpha,(i+1)}_{\ell m}(t,r)$, and determine $\Phi^{\alpha,(i+1)}_{\ell m}(t,r)$.
4. If $\tilde{T}^{\alpha,(i+1)}_{\ell m}(t,r)$ has an harmonic time dependence $\cos(\omega\, t)$, then $\Phi^{\alpha,(i+1)}_{\ell m}(t,r)$ will exhibit the same dependence, EXCEPT when ω agrees with one of the normal frequencies of AdS:

$$\Phi^{\alpha,(i+1)}_{\ell m}(t,r) = \Phi^{\alpha,(i+1),c}_{\ell m}(r)\cos(\omega\, t) + \Phi^{\alpha,(i+1),s}_{\ell m}(r)\, t\, \sin(\omega\, t).$$

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5. If for a given perturbation one can construct $\Phi^{\alpha,(i)}_{\ell m}$ to any order, without ever introducing a term growing linearly in time, the solution is said to be stable and is unstable otherwise.

Geons

- Start with a single mode $\ell = m = 2$ initial data.
- At second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order there is a resonant mode, but one can set the amplitude of the growing mode to zero by changing the frequency slightly

$$\omega L = 3 - \frac{14703}{17920}\epsilon^2.$$

- The structure of the equations indicate that there is only one resonant term at each odd order, and that the amplitude of the growing mode can be set to zero by correcting the frequency.
- One can compute the asymptotic charges to fourth order, and they readily obey to the first order of thermodynamics:

$$E_g = \frac{3J_g}{2L}\left(1 - \frac{4901\, J_g}{7560\pi L^2}\right), \quad \omega_2 = \frac{3}{L}\left(1 - \frac{4901\, J_g}{3780\pi L^2}\right),$$

where we defined ϵ by $J_g = \frac{27}{128}\pi L^2 \epsilon^2$.

Colliding Geons - 1/2

- Start with a linear combination of $\ell = m = 2$ and $\ell = m = 4$.
- Alike the single mode initial data, at second order there are no resonant modes and the solution can be rendered regular everywhere.
- At third order, there are four resonant modes:
  - The amplitude of the growing modes in two of the resonant modes can be removed by adjusting the frequency of the initial data ($\omega_2 L = 3$ and $\omega_4 L = 5$) just like we did for the single mode initial data.

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AdS is nonlinearly unstable!

Colliding Geons - 2/2

- The frequency of the growing mode is higher than any of the frequencies we started with!
- The "energy" is transferred to modes of higher frequency.
- Expect this to continue. When the $\omega L = 7$, $\ell = m = 6$ mode grows, it will source even higher frequency modes with growing amplitude.

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Possibility I:

The endpoint is a rotating black hole.

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Possibility II:

Possibility I relies on the stability of the Kerr-AdS black hole! However, all black holes in gobal AdS might turn out to be nonlinearly unstable...

A tale of two timescales: 1/3

- Let us estimate the timescale for the breakdown of perturbation theory in the pure AdS setup:
    - This occurs when the third order term that grows linearly in time is of the same size as the first order terms: $t \sim \epsilon^{-2}$.

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- Let us estimate the timescale for the breakdown of perturbation theory in the pure AdS setup:
  - This occurs when the third order term that grows linearly in time is of the same size as the first order terms: $t \sim \epsilon^{-2}$.
  - This matches the scaling found by Bizon and Rostworowski for the time it takes to form a black hole.
- We have to compare this timescale, to the time it takes for a black hole to absorb a field of amplitude ϵ:
  - The standard lore is that this behavior is exponential in time: quasinormal modes.
  - However, this is a rather crude and incomplete answer.

A tale of two timescales: 2/3

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A tale of two timescales: 2/3

- Consider a minimally coupled scalar field ψ in the background of a Schwarzschild-AdS black hole.
- Due to $\mathbb{R}_t \times SO(3)$ symmetry of the background geometry we can decompose Φ as:

$$\psi(t,r,\theta,\phi) = \sum_{n,\ell,m} e^{-i\omega_{n\,\ell\,m}} \psi_{n,\ell,m}(r) \mathbb{Y}_{\ell,m}(\theta,\phi).$$

- Quasinormal modes calculations, in the large ℓ limit, find that

$$t^{-1} \equiv \mathrm{Im}(\omega_{n\,\ell\,m}) \propto \mathrm{Exp}(-\ell).$$

A tale of two timescales: 3/3

- We have confirmed these estimates numerically.
- Adding rotation only makes it worse: we studied generic perturbations around Kerr-AdS as well.
- It seems that the turbulent instability might have time to develop in the DOC of the Schwarzschild-AdS black hole!
- Furthermore, approximate resonances exist for large ℓ: in the WKB approximation $\mathrm{Re}(\omega_{n\,\ell\,m}) = \omega^{\mathrm{AdS}}_{n\,\ell\,m} + \mathcal{O}(\ell^{-\frac{d-3}{2}})$.

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What is the endpoint?:

1. Start with a generic perturbation about Schwarzschild-AdS.
2. This perturbation will likely form a new small black hole far away from the central black hole you started with.
3. The new black hole, coined black planet, will merge with the central black hole, and as it does so it radiates.
4. The radiation gets trapped at infinity: go to 2.

What happens in String Theory?

Consider IIB string theory on $\mathrm{AdS}_5 \times S^5$, with AdS length scale L.

There are two energy scales: the Planck energy and the string energy $E_s < E_p$.

Possibilities:

- If the initial energy is larger than $E > \frac{N^2}{L}$, one forms a 5D AdS black hole.
- If the initial energy is $E_{corr} < E < \frac{N^2}{L}$, one forms a 10D black hole. Here, E_{corr} is the energy of a black hole of the string scale size.
- If the initial energy is $E_s < E < E_{corr}$, one forms an excited string.
- If $E < E_s$, the cascades stops at frequencies $\omega = E$, and one gets a gas of particles in AdS.

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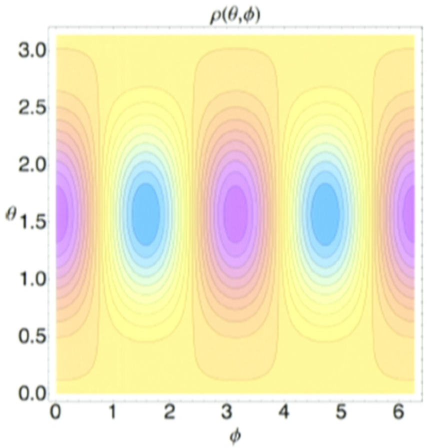

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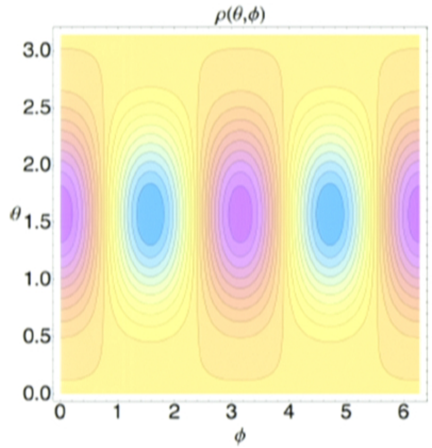

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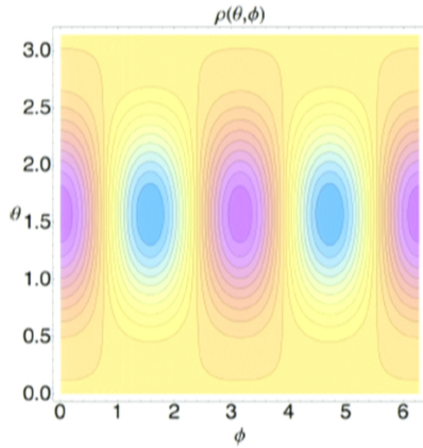
- Perhaps more intriguing, from the CFT perspective, is the existence of Geons: these high energy states do NOT thermalize.
- The boundary stress-tensor contains regions of negative and positive energy density around the equator:
  - It is invariant under

$$K = \frac{\partial}{\partial t} + \frac{\omega}{m}\frac{\partial}{\partial \phi},$$

which is timelike near the poles but spacelike near the equator.



Conclusions & Open questions

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- Understand the space of CFT states that do not thermalize.
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