Title: Black Holes Beyond Astrophysics

Date: Jun 06, 2012 02:00 PM

URL: http://pirsa.org/12060015

Abstract: In the context of the possible existence of large extra dimensions, and also the context of the AdS-CFT correspondence, there has been much interest in black holes solutions in theories of gravity and matter that are exotic - they might live in spacetime dimension other than 4, or have exotic matter and boundary conditions. I will review the types of physics that are accessible by studying such exotic black holes, ranging from LHC phenomenology to potential applications to condensed matter physics (via the AdS-CFT correspondence). One common theme is that traditional analytic methods
to find solutions tend not to work when confronted with these more exotic solutions and instead we are increasingly forced to use numerical techniques. I will discuss a numerical approach to finding static and stationary solutions, and give some example applications. I will also show how dynamical numerical simulations in these exotic contexts are playing an increasingly important role.

Novel black hole physics and numerics

Toby Wiseman (Imperial)

with Joe Bhaseen, Ben Simons (Cambridge), Sonner (DAMTP), Gauntlett (Imperial)

- to appear soon

and Pau Figueras (DAMPT), James Lucietti (Edinburgh)

- arXiv:1104.4489 , 1105.2558 and work to appear soon

and Matt Headrick (Brandeis), Sam Kitchen and Alex Adam (Imperial)

- arXiv: 0905.1822, 1105.6347 and for a review, arXiv: 1107.5513

Plan for this talk...

- Conventional 4d black holes; analytics and numerics
- Novel black hole physics;
  - Large compact extra dimensions and LHC;
    - Compact extra dimensions - largely numerical
    - Asymptotically flat extra dimensions - some analytic
  - Exotic gravity theories; black holes in the Randall-Sundrum model
  - AdS-CFT; dynamics of superconducting black holes

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  - AdS-CFT; dynamics of superconducting black holes

Conventional black hole physics

- Analytic control of equilibrium (ie. static/stationary) vacuum solutions
  - The static Schwarzschild bh has long been known.
  - The Kerr black hole solution is known explicitly and is the most general solution - given mass M and ang mom J there is a uniqueness thm.
- Describe the end state of stellar collapse or collapse of cold dark matter

Numerics in 4d

- Including matter or going out of equilibrium requires numerical methods to solve the PDEs.
- Stationary matter solutions; relativistic stars
  - Axisymmetric; suitable coordinates reduce this to '2d' elliptic problem
- Spherical dynamical collapse; '1 + 1' PDE
- Full dynamical GR; '1 + 3' PDE
  - Mergers of binary systems to create gravity wave templates for experiment

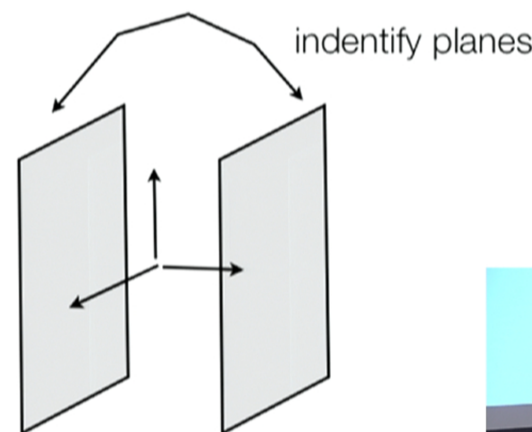
Part I

- Conventional 4d black holes; analytics and numerics
- Novel black hole physics;
  - **Large compact extra dimensions and LHC**
  - Exotic gravity theories
  - AdS-CFT

Large Compact Extra Dimensions

- Already in 1920's Kaluza & Klein proposed extra dimensions to unify gravity and electromagnetism.
- In modern context string theory requires of 10 (or 11) spacetime dimensions

geometry = $\mathbb{R}^{1,3} \times \mathcal{M}$



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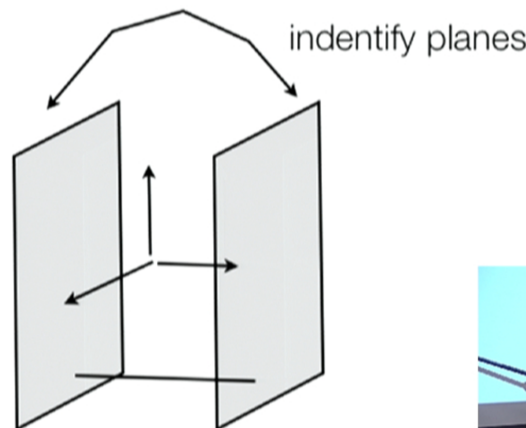
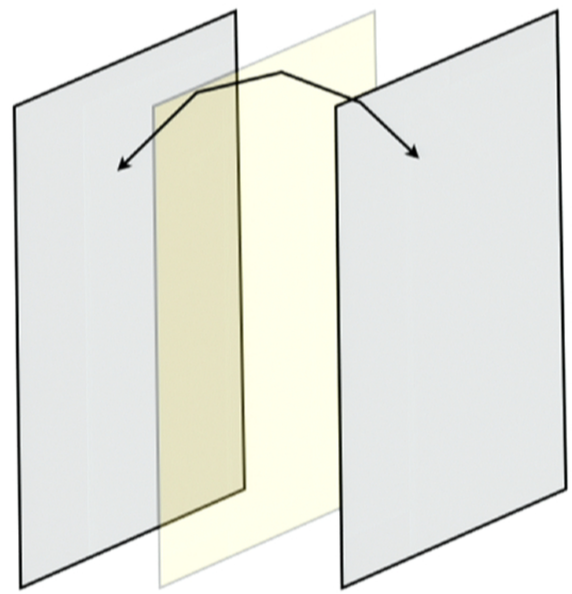


Large compact extra dimensions [ADD]

- Naively; $L < (TeV)^{-1} \sim 10^{-18} m$
- However if matter is confined to a 1+3 subsurface (a 'brane') the key experimental probe is the Cavendish experiment; L < 0.1 mm



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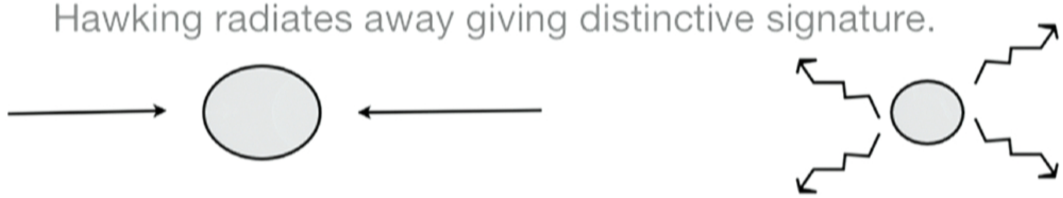
- However if matter is confined to a 1+3 subsurface (a 'brane') the key experimental probe is the Cavendish experiment; L < 0.1 mm
- Fundamental Planck scale M_D is reduced the larger the extra dimensions;

$$(M_D)^{D-2} = (M_4)^2 / L^{D-4} \qquad M_4 \sim 10^{19} GeV$$

- Choose $M_D \sim TeV$ to 'solve' hierarchy problem, then requires $D > 5$ and $L >> (TeV)^{-1}$

Collider physics with LXD

- If collide fundamental particles with center of mass > Planck mass, they will form a black hole. [Giddings, Thomas]
  - Compton wavelength of product $\lambda \sim 1/m$
  - Black hole radius for product $R^{D-3} \sim M_D^{2-D} m$
- In LXD however require only TeV -> possible at LHC if XD exist. Then Hawking radiates away giving distinctive signature.



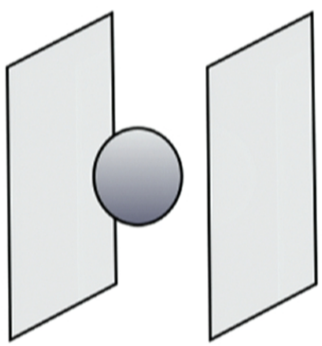
- If all XD are large, such a b.h. is tiny -> effectively 'sees' D dimensional spacetime - `asymptotically flat spacetime'
- If some dimensions are TeV size, some are large, then may also see some compact dimensions.

Large Extra Dimensions

- Compact extra dimensions - static black holes
- Asymptotically flat extra dimensions - rotating black holes

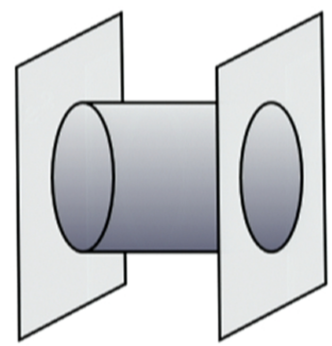
Compact XD

- Consider 1 extra compact dimension - Kaluza-Klein (KK) theory. We restrict to solutions asymptoting to $Mink^4 \times S^1$
- Consider static black holes to start.
- There are two obvious static black hole solutions in KK; the **homogeneous black string** and the **localized black hole**. Only homogeneous is analytic.



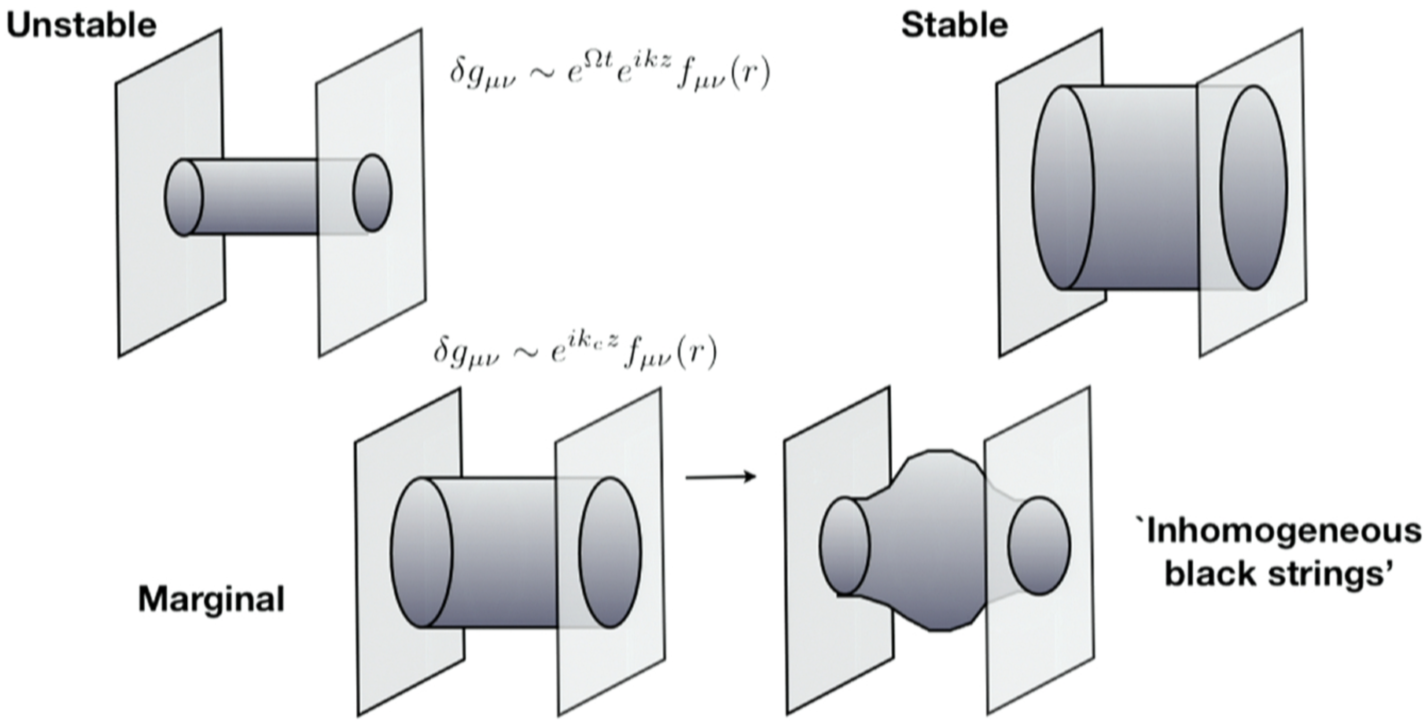
$$ds^2 \simeq ds^2_{5dSch}$$

$$ds^2 = ds^2_{4dSch} + dz^2$$



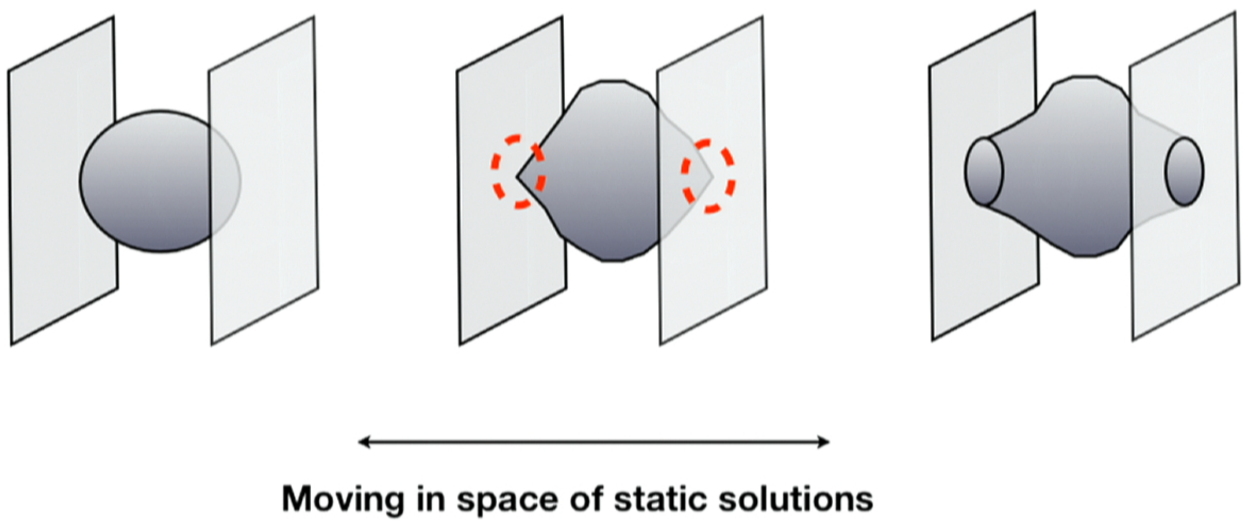
GL instability

- Space of static solutions is mediated by two effects;
- Thin black string is unstable to GL instability;



Topology change

- Kol proposed a localized black hole and inhomogeneous string may meet in the space of solutions at a topology changing solution.

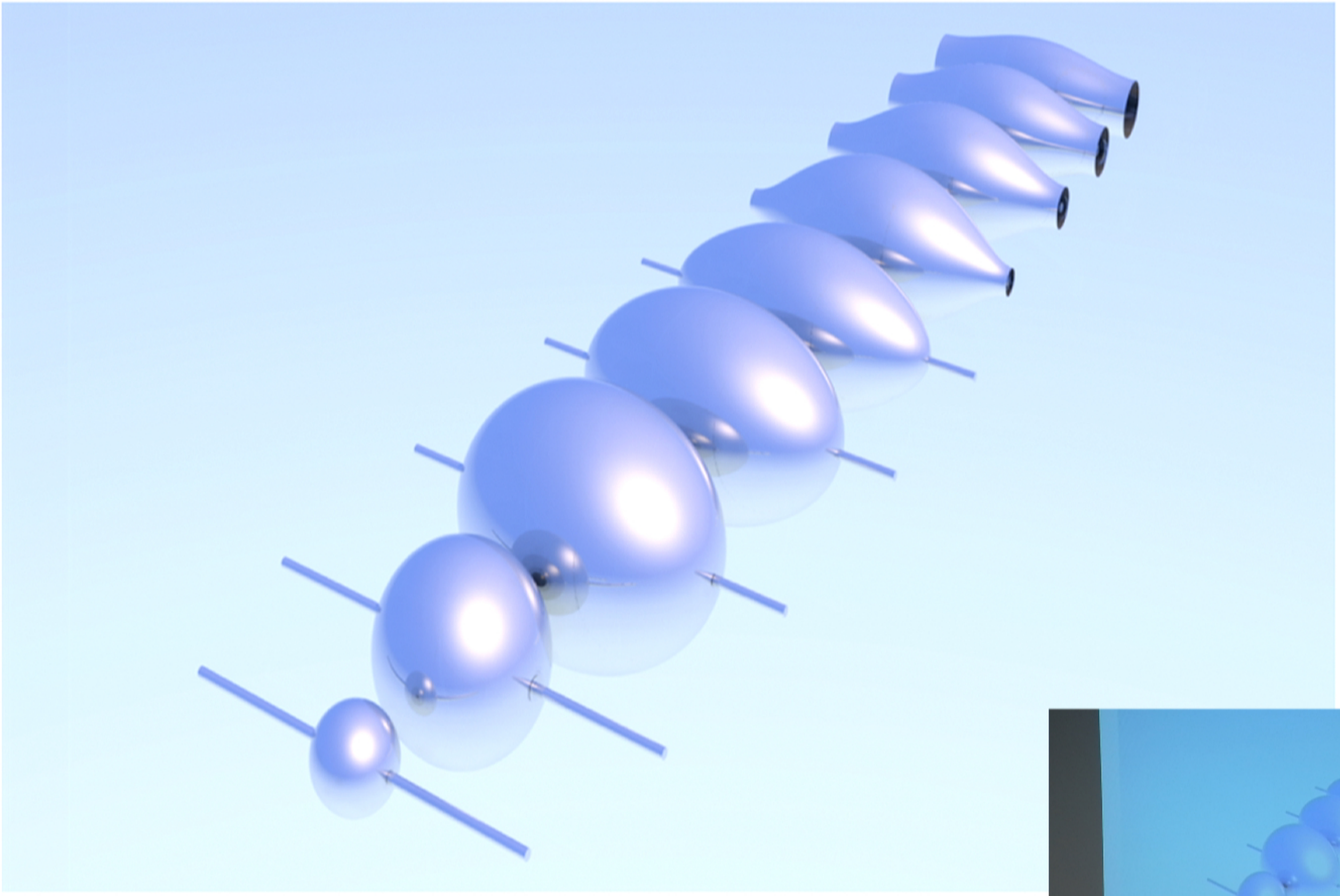


Static/stationary numerical problem

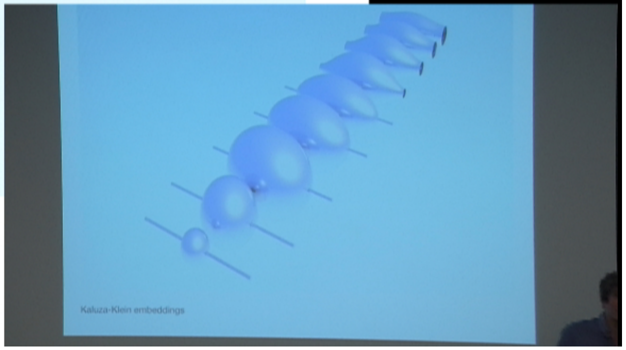
- Static/stationary problem should be elliptic - cf. the Poisson equation for electrostatics; fix asymptotics and horizon regularity

- In order to achieve this one must remove the gauge invariance. Either fix a gauge - which is messy. Or add 'gauge' fixing terms. Must then understand that solution to gauge fixed Einstein equations is indeed a solution.

- Use isometries to reduce the 'effective' dimension of the PDE problem. Here it is effectively `2d' - radial and extra dimension dependence.



Kaluza-Klein embeddings



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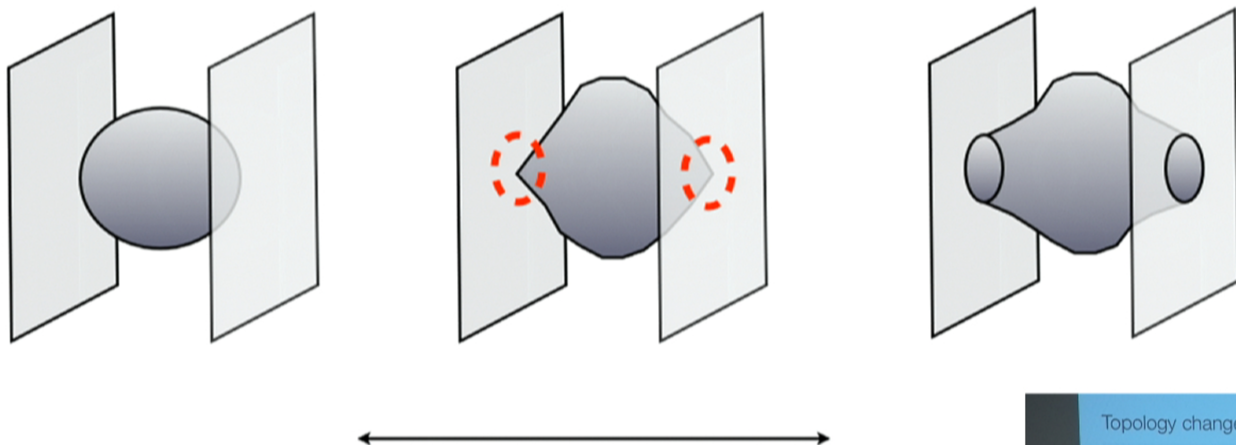
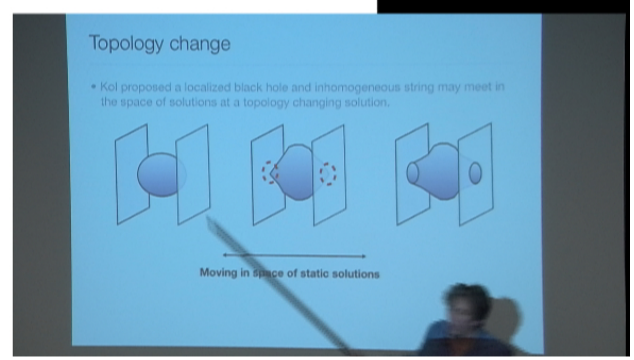
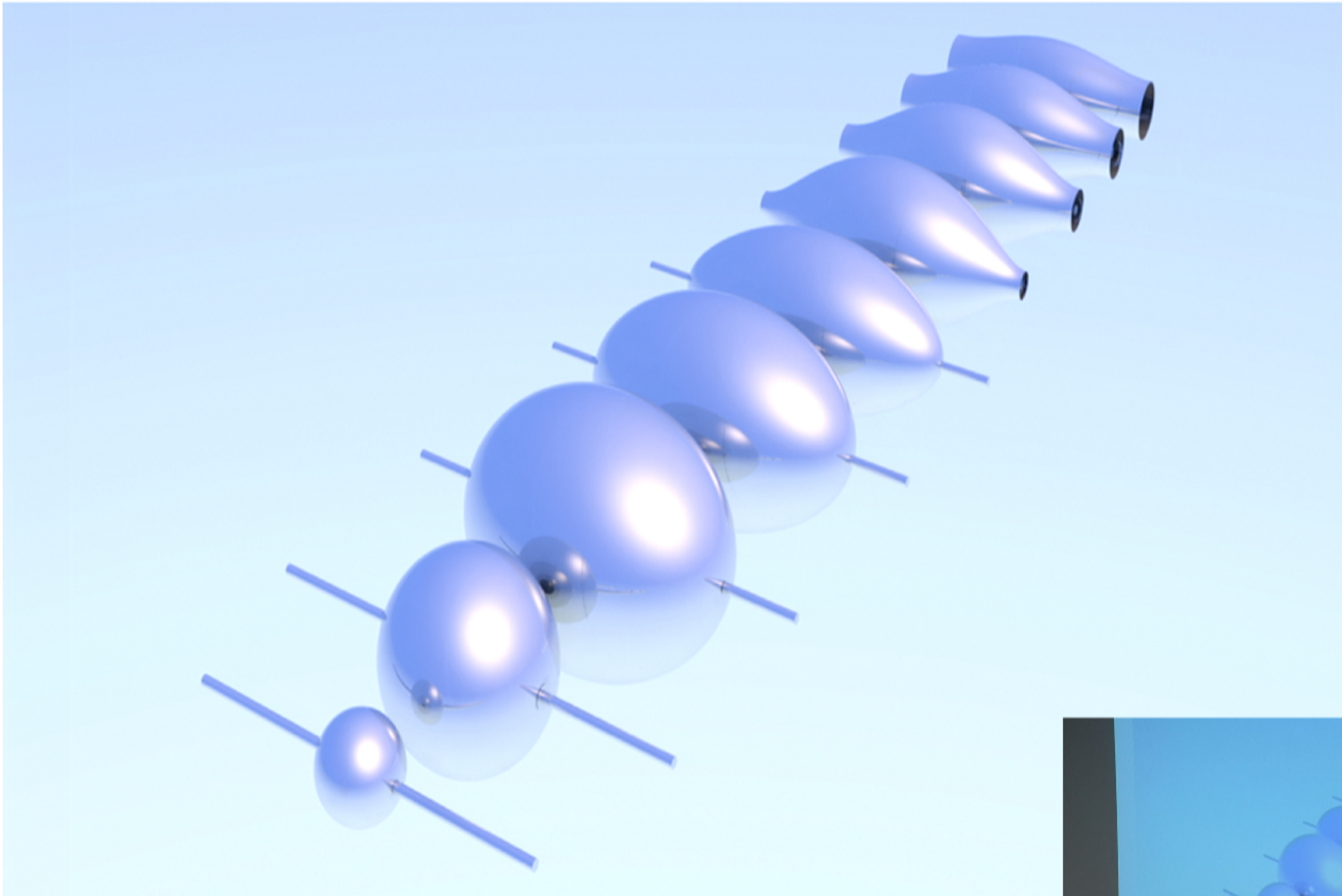


Moving in space of static solutions





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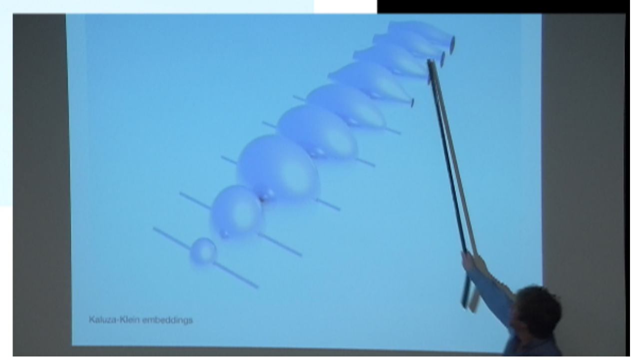
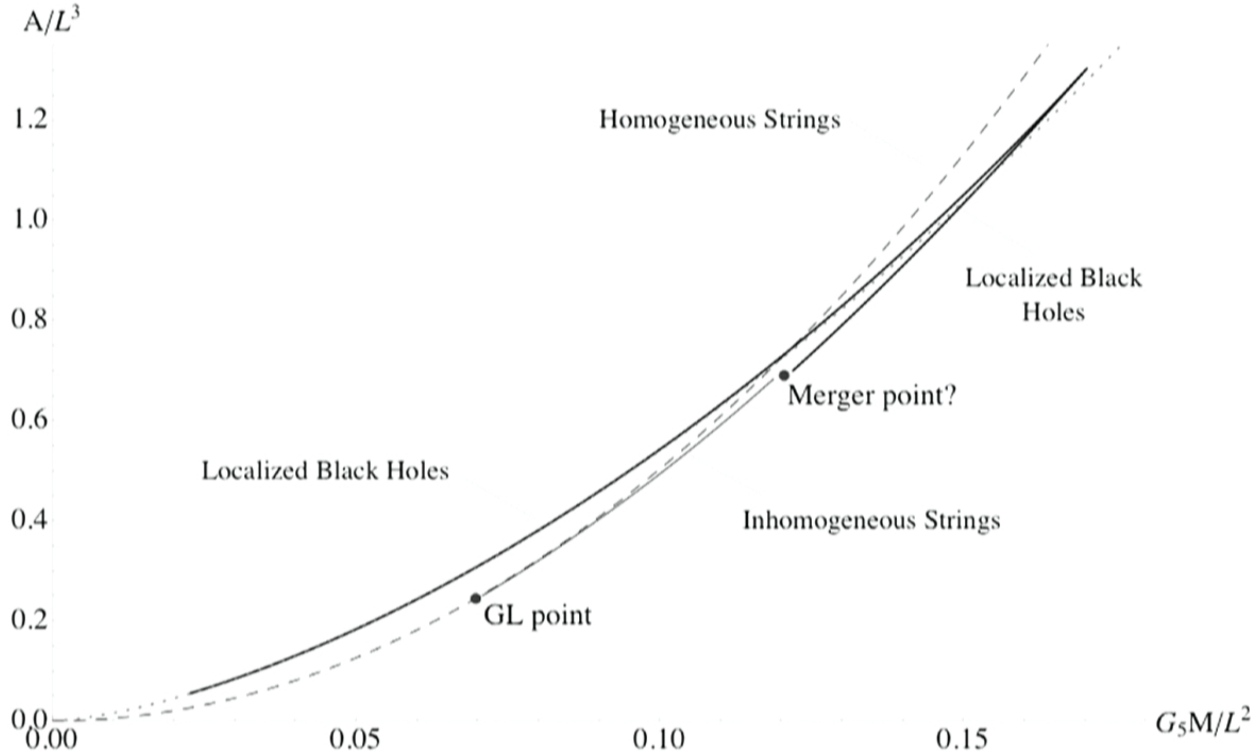
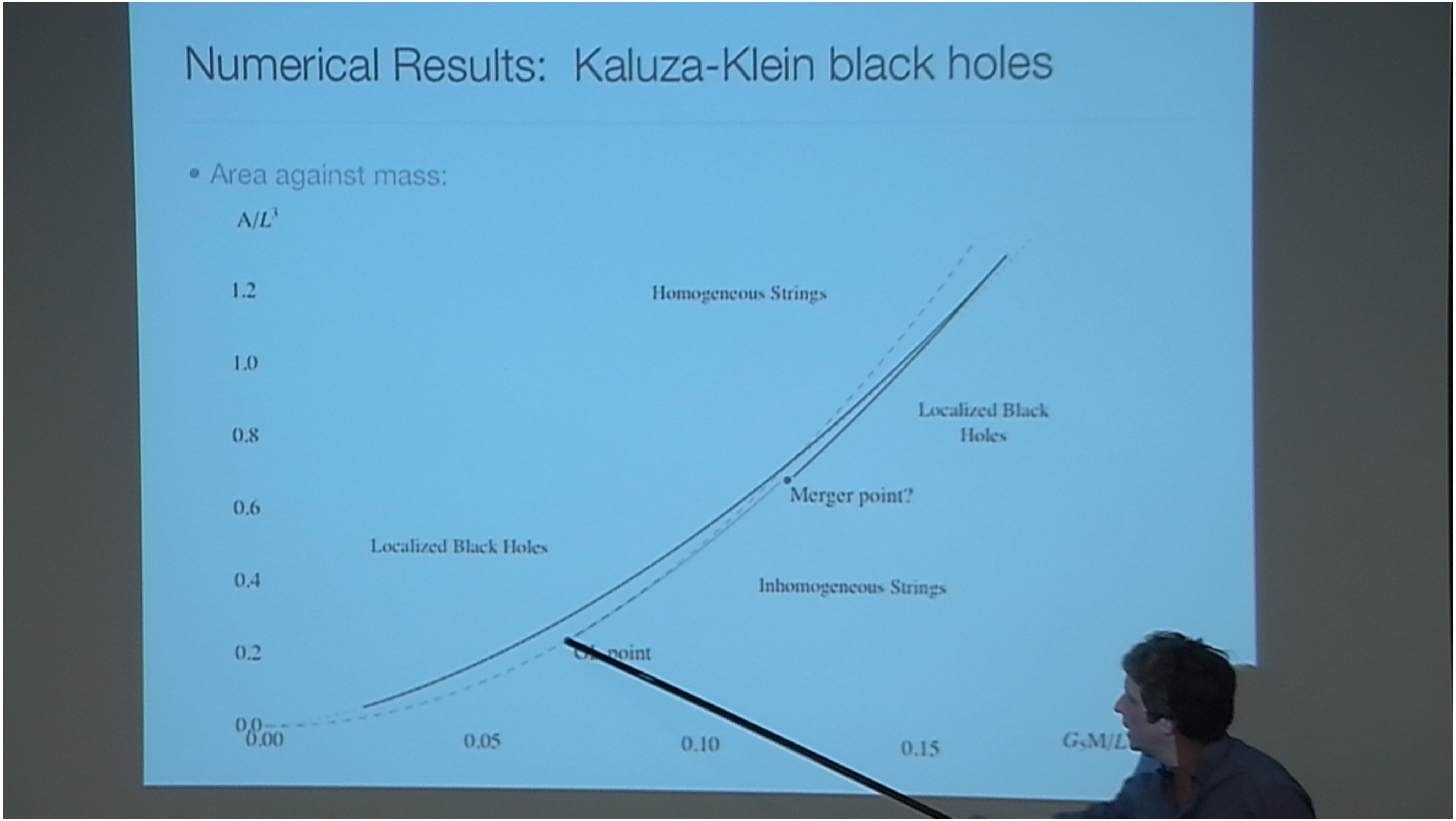


Numerical Results: Kaluza-Klein black holes

- Area against mass:

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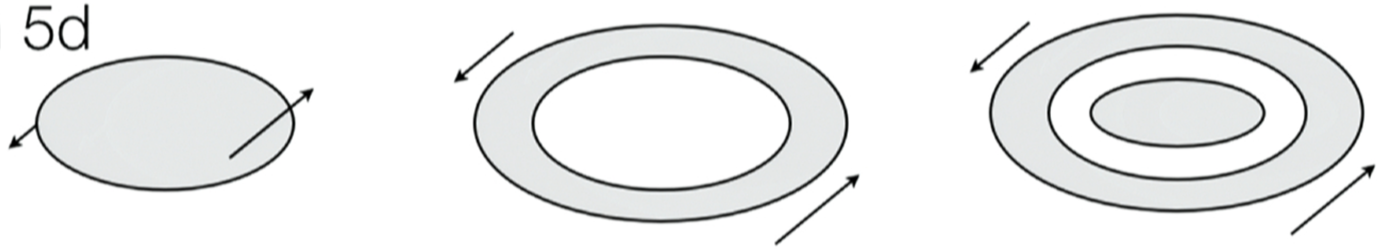
• Area against mass:
A/L³
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
Homogeneous Strings
Localized Black Holes
Merger point?
Localized Black Holes
Inhomogeneous Strings
point

Asymptotically flat XD

- Consider asymptotically flat spacetime. The static Schwarzschild b.h. has a simple generalization and is unique.
- The Kerr solution has 'simple' generalization - the Myers-Perry solution - modulo ang mom being more complicated.
- However for rotation there is no uniqueness theorem. Indeed in 5d Emparan and Reall made a remarkable discovery finding the `black ring'.
- Beyond M-P and the 5d black ring there is little analytic control - we will return to this later....

In 5d

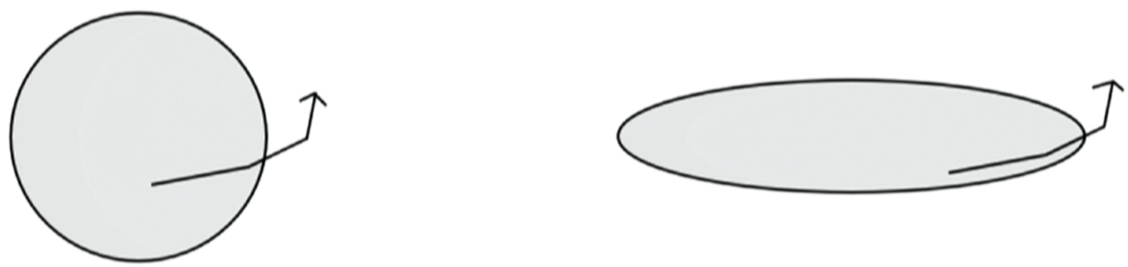
`Black saturn'



[Elvang, Figueras]

Asymptotically flat, D>5

- Beyond D=5 the MP solution has no maximum spin. But for high spins it becomes very ‘flat’. It presumably would want to ‘break up’ into little black holes (aka Gregory-Laflamme) [Myers, Emparan]



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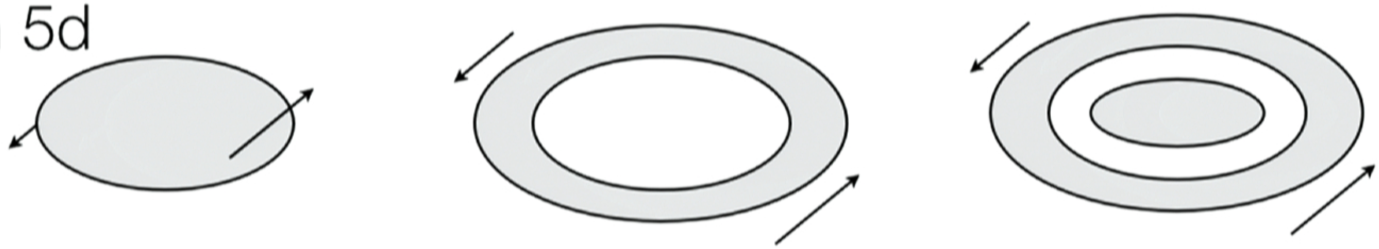
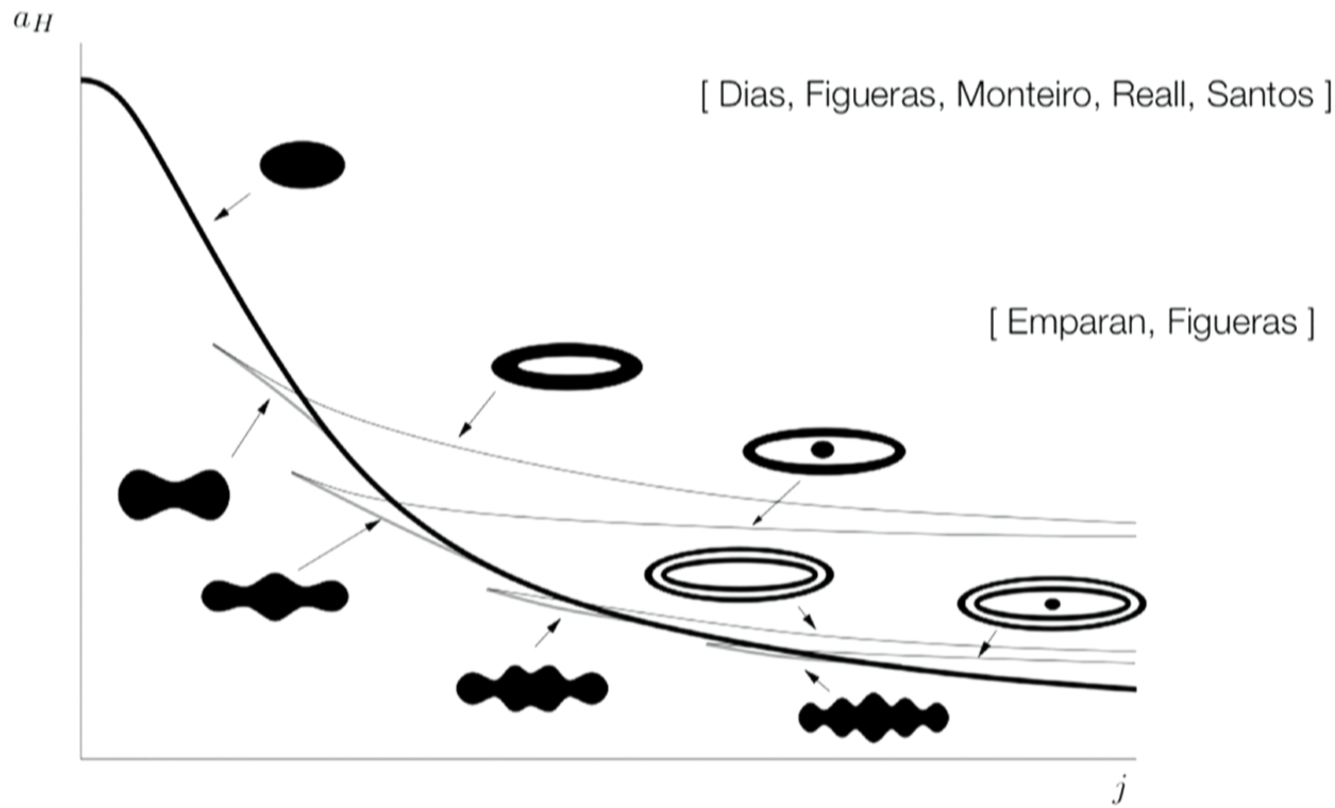


Asymptotically flat, D>5

- Beyond D=5 the MP solution has no maximum spin. The conjecture is;



Stability and dynamics

- The numerical problem of determining stability is only just starting.
  - Shibata and Yoshino have shown the MP solution with large angular momentum is unstable, but to a different instability.
  - Cardoso et al have simulated head on collision of black holes.
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  - **Exotic gravity theories**
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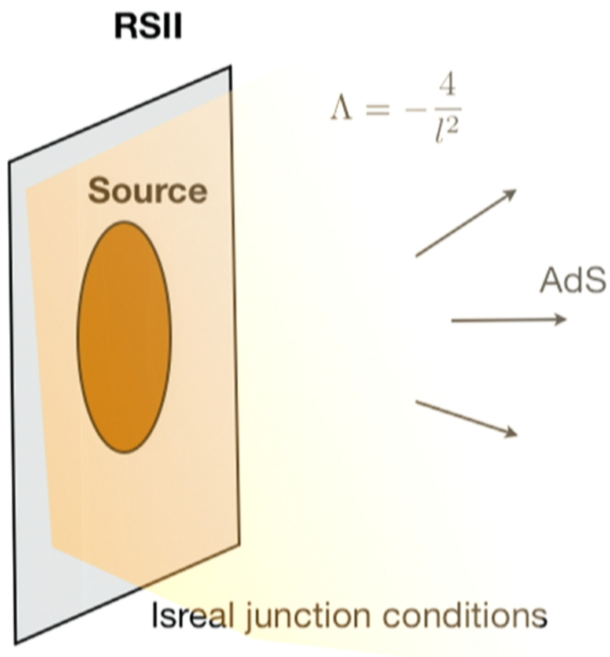
Exotic theories of gravity

- Fascinating situation where one might test the theory via black holes rather through the force law.
- 4-d but not Einstein; scalar-tensor theory, Einstein-Aether, TeVeS
- Theories with XD but not compact; Randall-Sundrum (RS), DGP

RS II

- The 5d Randall-Sundrum II model is a remarkable 'compactification'. 4d gravity is recovered on the brane, at scales larger than l;



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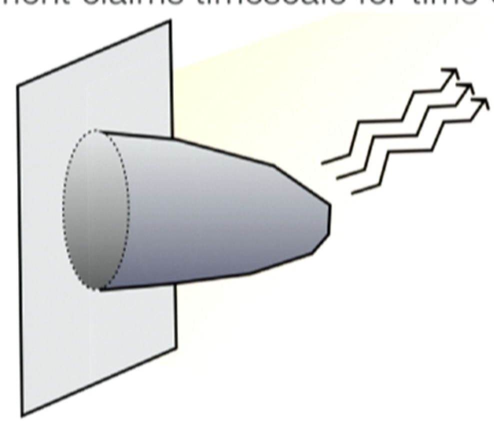
- 4d propagator goes as; $\sim \frac{1}{r} + \frac{l^2}{r^3} + \dots$ so there is no mass gap.
- Claim [Tanaka; Emparan, Kaloper, Fabbri '02]: For black holes, radius $R_4 \ll l$ there exist no static solutions. Counter argument [Fitzpatrick, Randall, TW '06]
- Reasonable argument claims timescale for time dependence; $\tau \sim \left(\frac{M}{M_\odot}\right)^3 \left(\frac{1\mathrm{mm}}{l}\right)^2 \mathrm{yr}$



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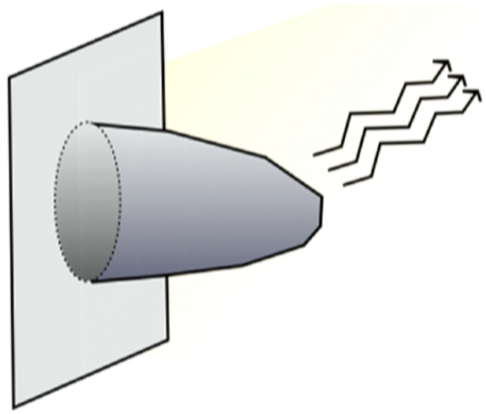
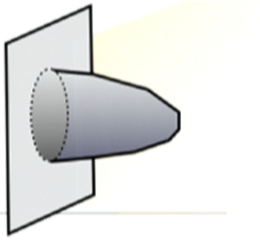
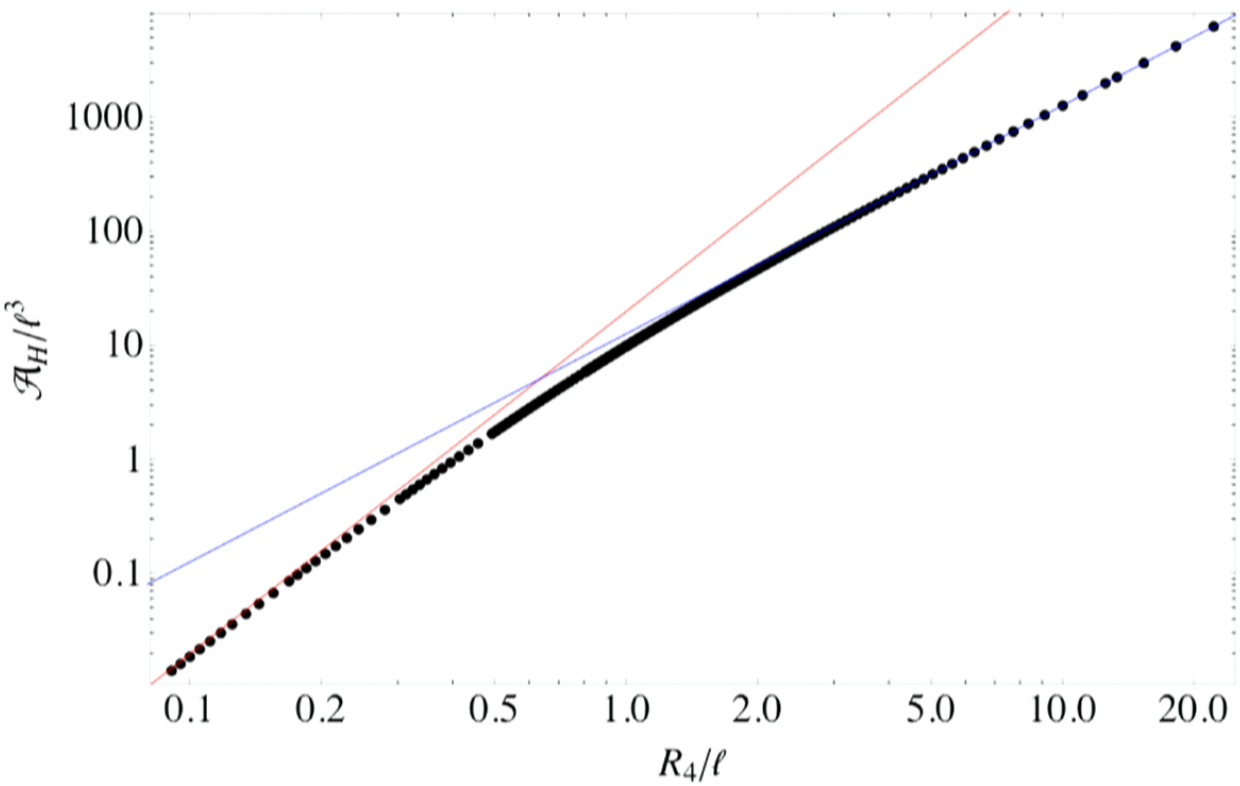


RSII results

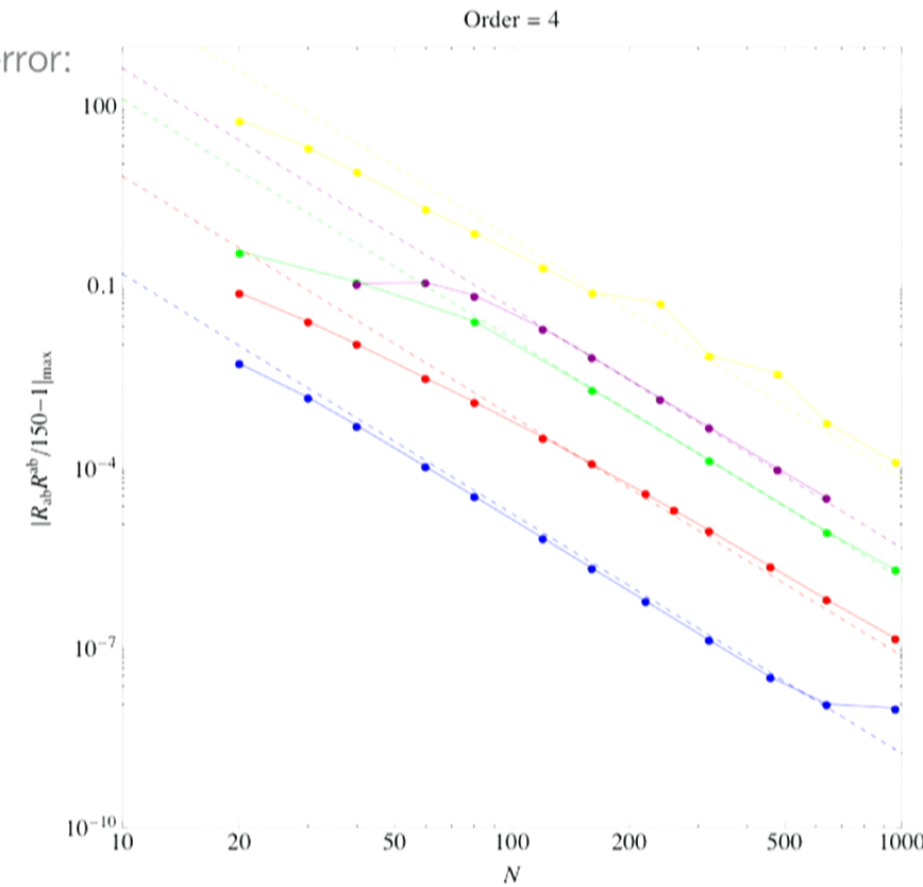


- Can find solutions from very small to very large black holes ($R_4/\ell \sim 100$).



RSII results

- Numerical error:



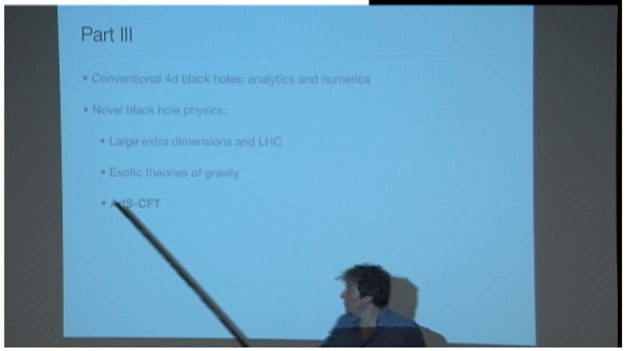
Another example...

- In Einstein-Aether theory there is a vector field that `falls into the black hole'. There is no analytic solution for the equivalent of Schwarzschild - solved as a `1-d' PDE.

- For the generalization of Kerr the solutions the previous methods would be required.

Part III

- Conventional 4d black holes; analytics and numerics
- Novel black hole physics;
  - Large extra dimensions and LHC
  - Exotic theories of gravity
  - **AdS-CFT**



AdS-CFT

- AdS-CFT states that a certain class of (rather special) CFTs are equivalent to gravitational theories (possibly higher spin or string theories) in spacetimes that asymptote to AdS.
- In particular this is made concrete in the case of *N*=4 susy SU(N) YM which is a CFT whose dual is understood to be a closed string theory. The vacuum geometry is $AdS_5 \times S^5$ - radius ℓ. The parameters are related as;

$$g_{YM}^2 = g_s \qquad \lambda = N^2 g_{YM} = \left(\frac{l}{l_s}\right)^4$$

- In the 't Hooft limit, $N \to \infty$ and finite $\lambda = N^2 g_{YM}$ the string coupling becomes small.
- In the large λ limit the target space becomes weakly curved, and stringy corrections can be ignored, reducing the dual to supergravity, which can be truncated to simply to 5-d gravity and a negative cosmological const.

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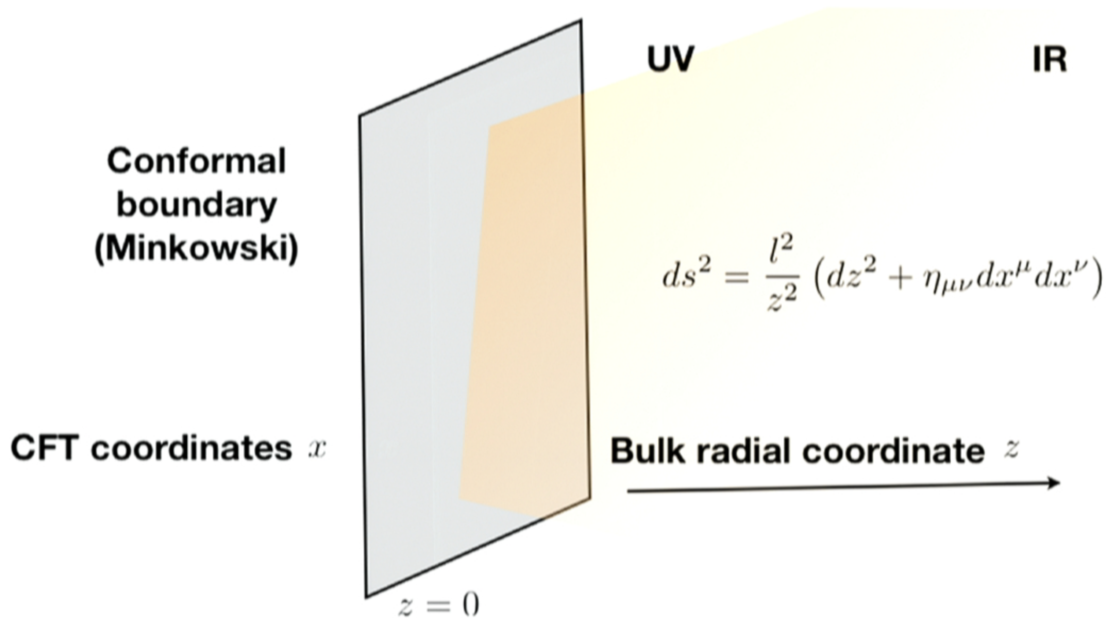
AdS-CFT

- One direction is to understand (quantum) gravity by studying the CFT.
- However here we will be interested in the reverse - understanding features of strongly coupled QFTs by thinking about gravity in asymptotically AdS spacetime.
- The key power in this approach is that the AdS-CFT gives a separation in the DOF in the problem. The complicated quantum stringy physics is separated from the gravity even though this is far from obvious in the QFT.

AdS-CFT

- The vacuum geometry is AdS - the CFT 'lives' on the boundary

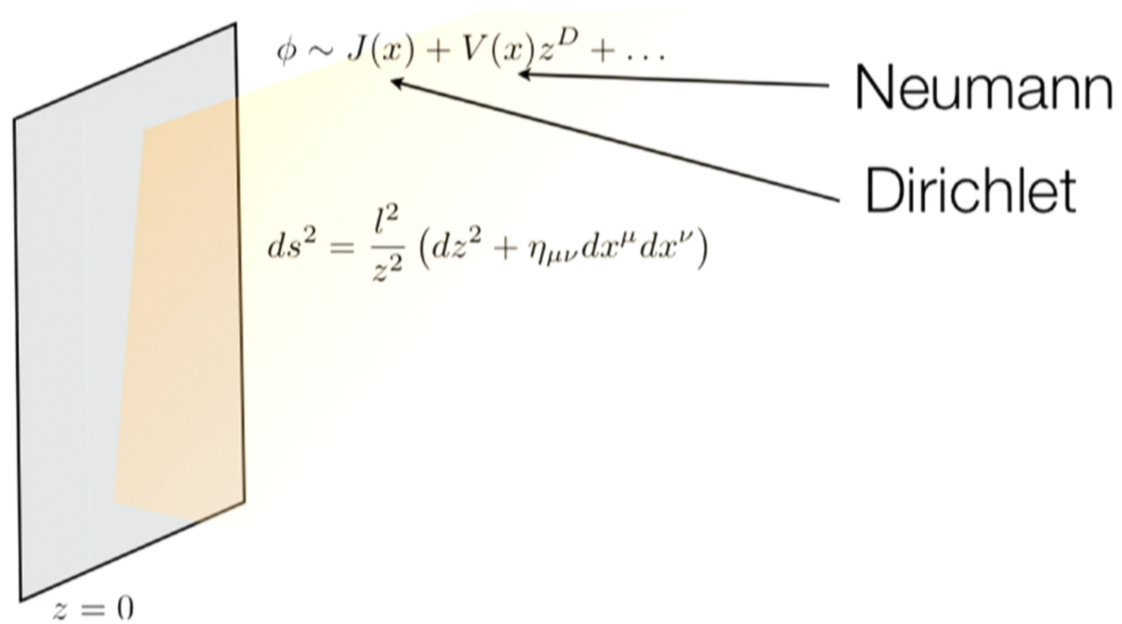
$$\Lambda = -\frac{4}{l^2}$$



AdS-CFT

- Every field in the gravity (string theory) $\phi(x, z)$ corresponds to an operator $O(x)$ in the CFT. The boundary requires boundary conditions - fix $J(x)$

- Solving the gravity gives CFT initially in vacuum with source $J(x)$



AdS-CFT

- The vev $< O(x) >= V(x)$
- Also correlation functions $< O(x_1)O(x_2)\ldots >$, Wilson loops, entanglement S....

$$\phi \sim J(x) + V(x)z^D + \ldots$$

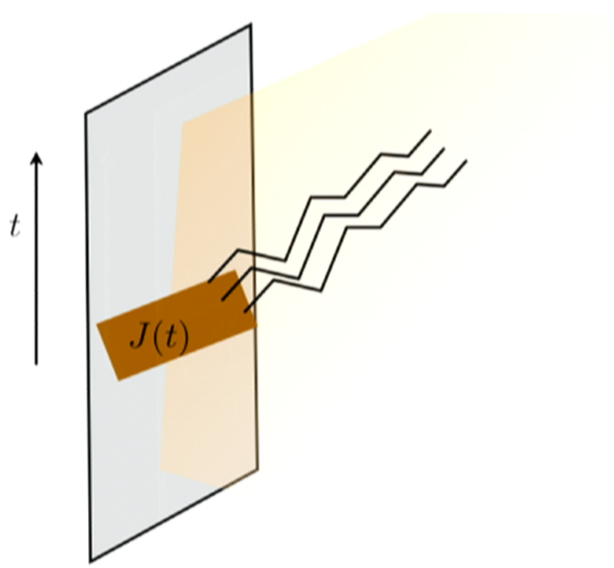
$$ds^2 = \frac{l^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

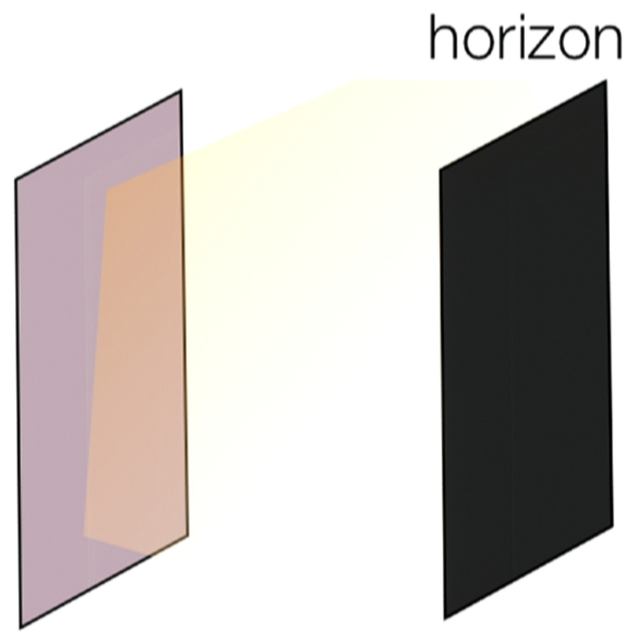
$z = 0$

AdS-CFT

- One perturbs the theory by turning on a source, or starting in a non-vacuum state.

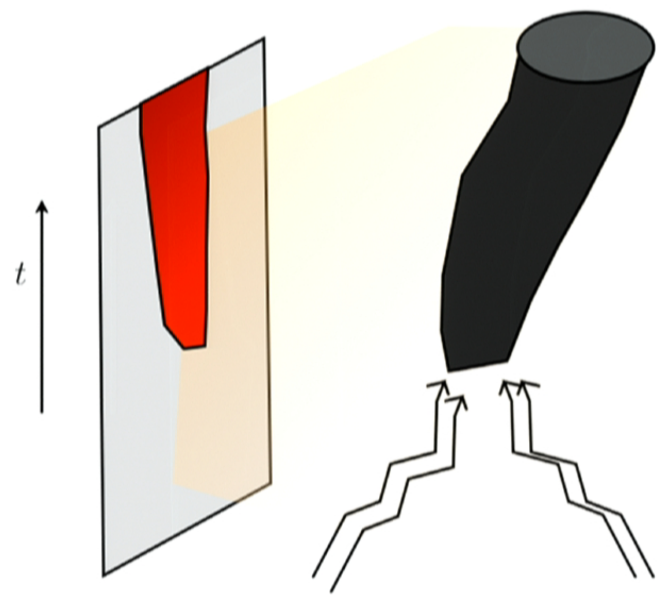
- A key example is a black hole;





AdS-CFT

- Black holes are fascinating objects in AdS-CFT as they are understood as being dual to regions of the CFT where there is local thermal equilibrium ie. it i a plasma - c.f. a fluid. The variety of black holes yields the various thermal phases of the CFT. The black hole dynamics then gives equilibration of this plasma.



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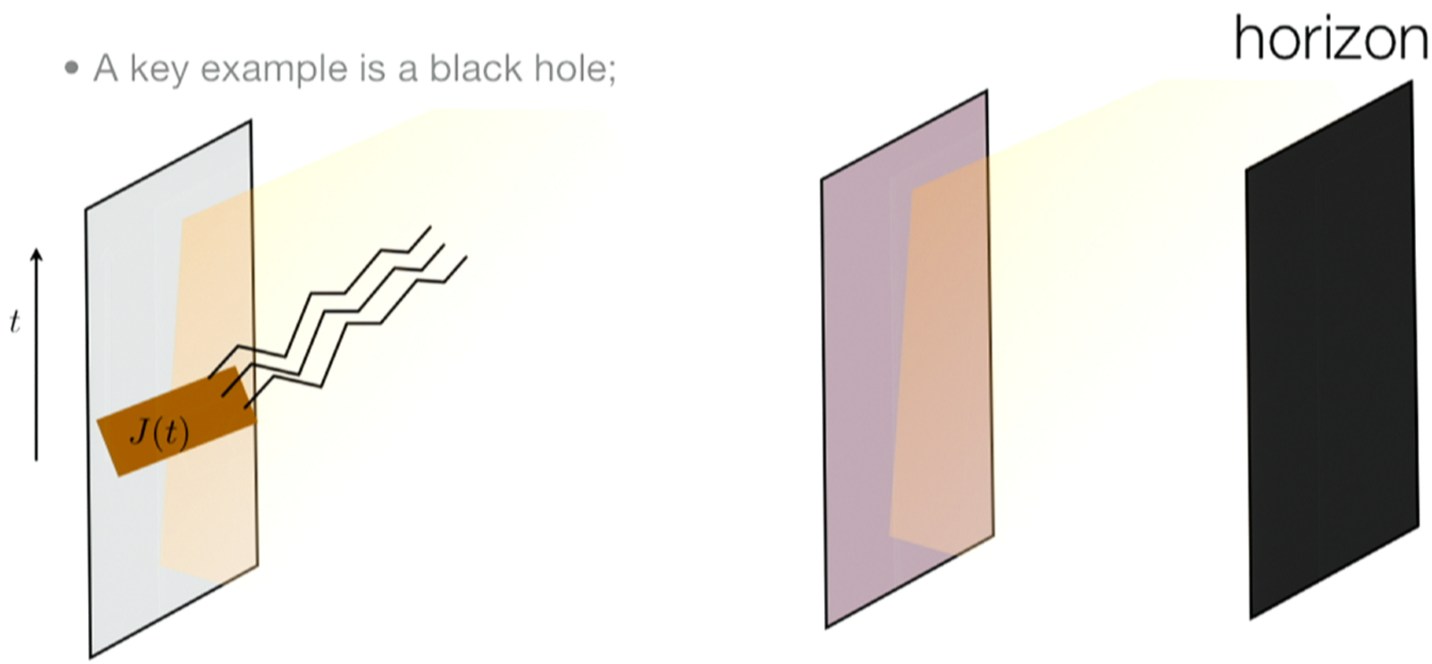

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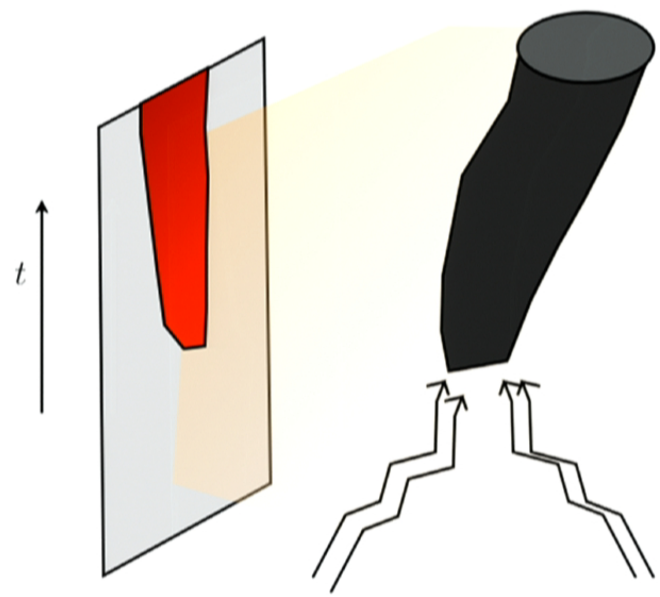
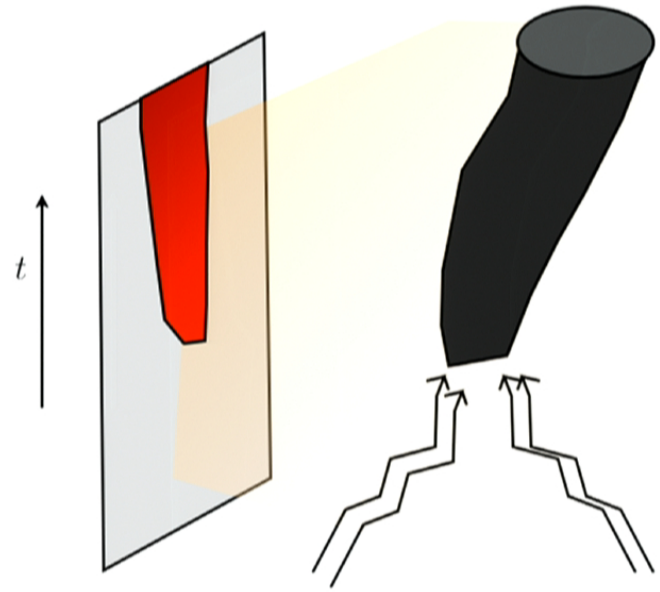


AdS-CFT

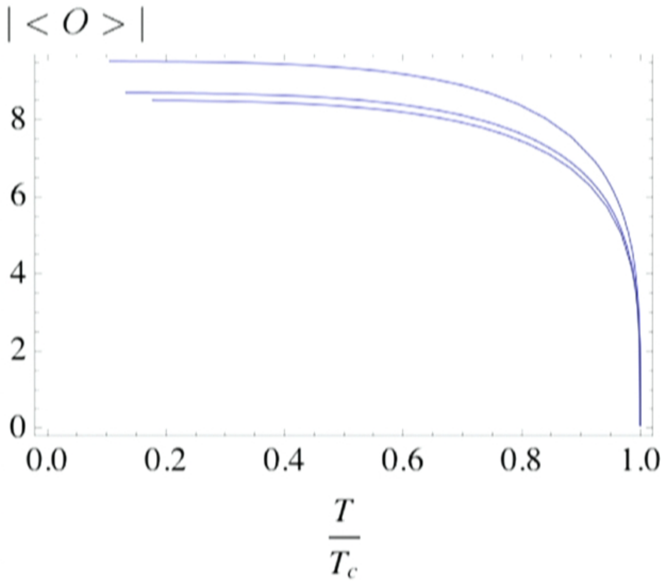
- Understanding strongly coupled plasma in QCD is at the heart of the heavy ion collision expts such as RHIC and ALICE. Much work has focussed on modelling QCD and understanding the gravity dual plasma.

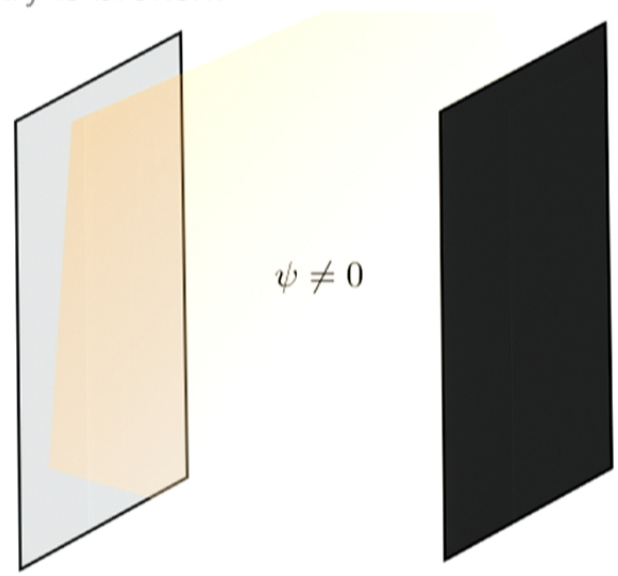


AdS-CMT

- [Gubser ; Hartnoll, Herzog, Horowitz] There is an exotic phase transition to black holes that superconduct - seen via the CFT current-current correlator. The complex scalar is 'turned on' and is dual to a complex scalar operator, O, charged under a global U(1). This symmetry is broken.

$\psi \to < O >$

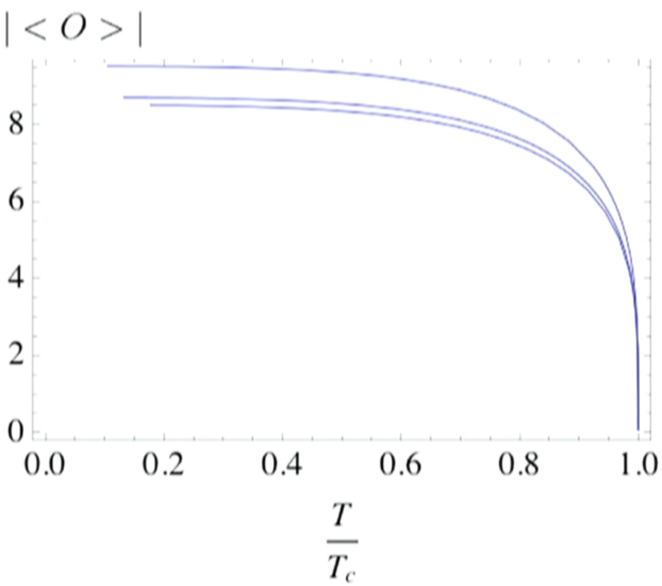


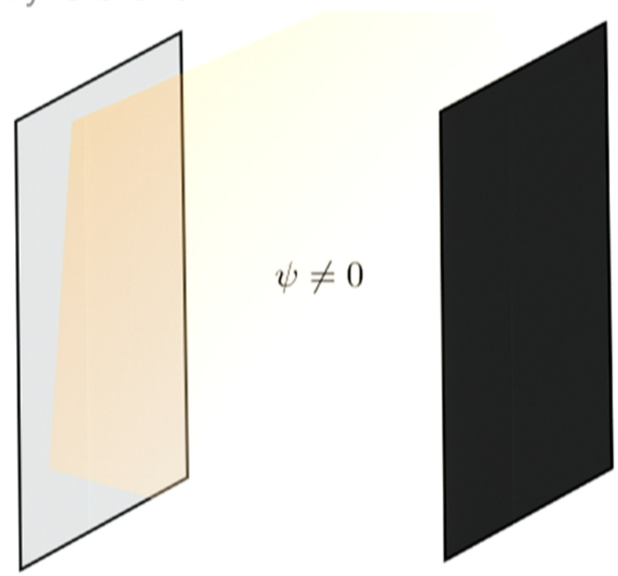


AdS-CMT

- [Gubser ; Hartnoll, Herzog, Horowitz] There is an exotic phase transition to black holes that superconduct - seen via the CFT current-current correlator. The complex scalar is ‘turned on’ and is dual to a complex scalar operator, O, charged under a global U(1). This symmetry is broken.

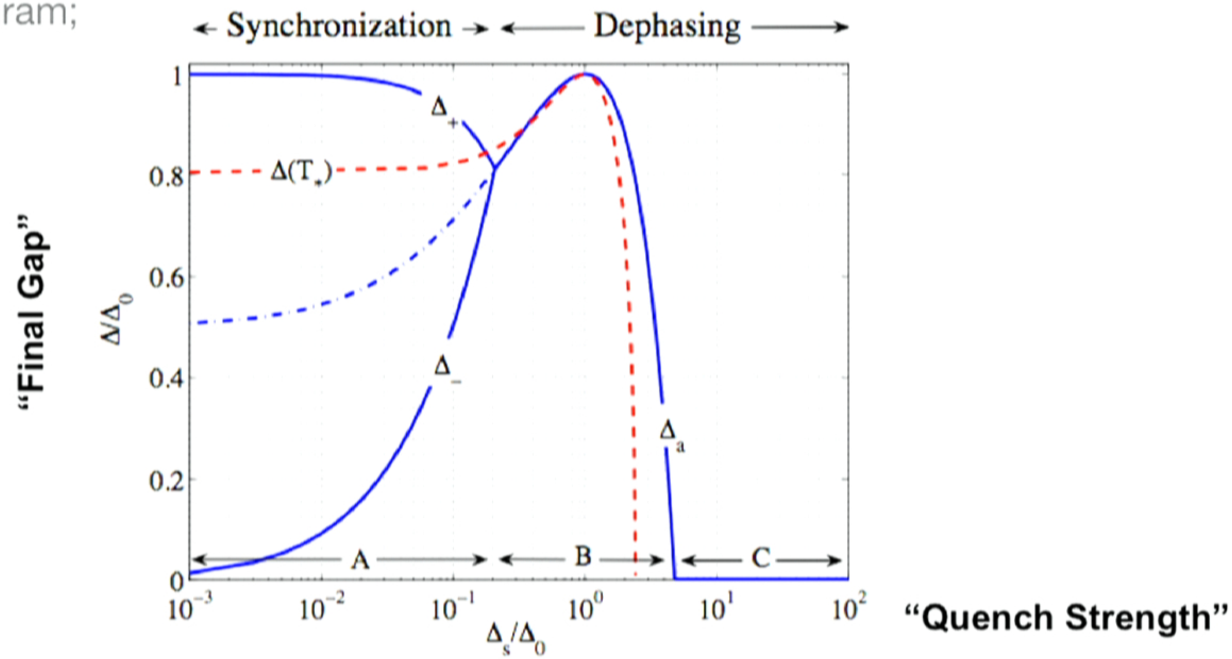
$\psi \rightarrow \langle O \rangle$





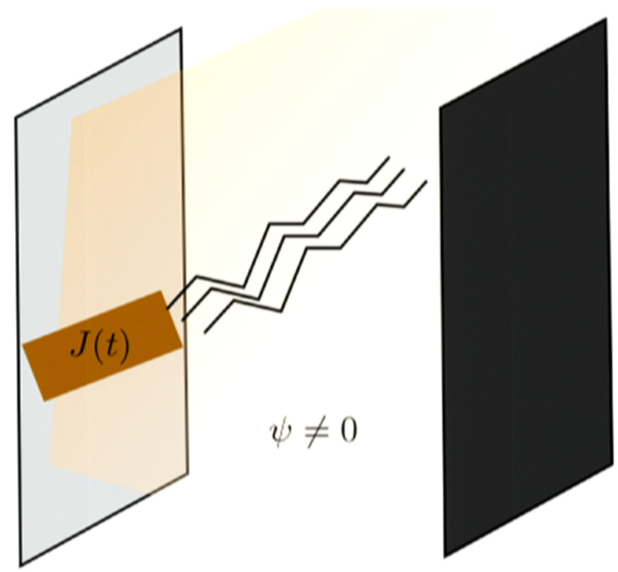
AdS-CMT

- The dynamics of superconductors is an area of interest in CMT. An important question is what happens if one starts with a superconductor and injects energy. This is a notoriously tough problem - the state of the art is work of Barankov-Levitov where approximations in BCS theory lead to a dynamical phase diagram;



B-L in AdS/CMT

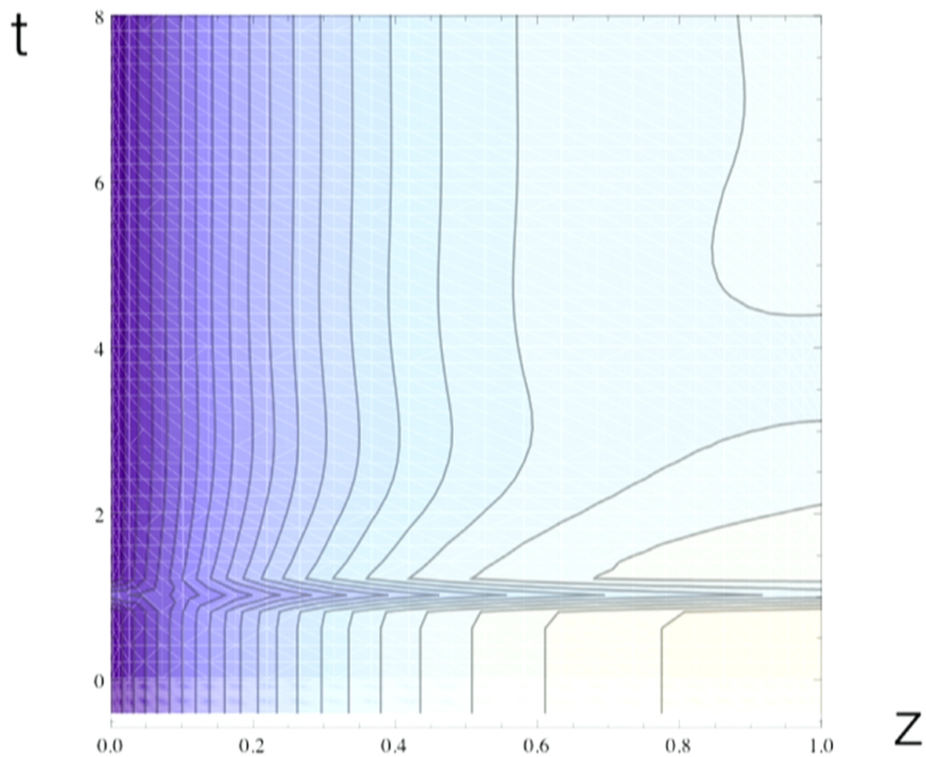
- In B-L various approximations are made. By performing a simple 1+1 dynamical simulation we may ask whether the same is observed in a strongly coupled superconductor. We quickly turn on and off a source for the order parameter, which injects energy.



cf. Murata et al

B-L in AdS-CFT

- We see analogous behaviour - for a source with strength δ we find;



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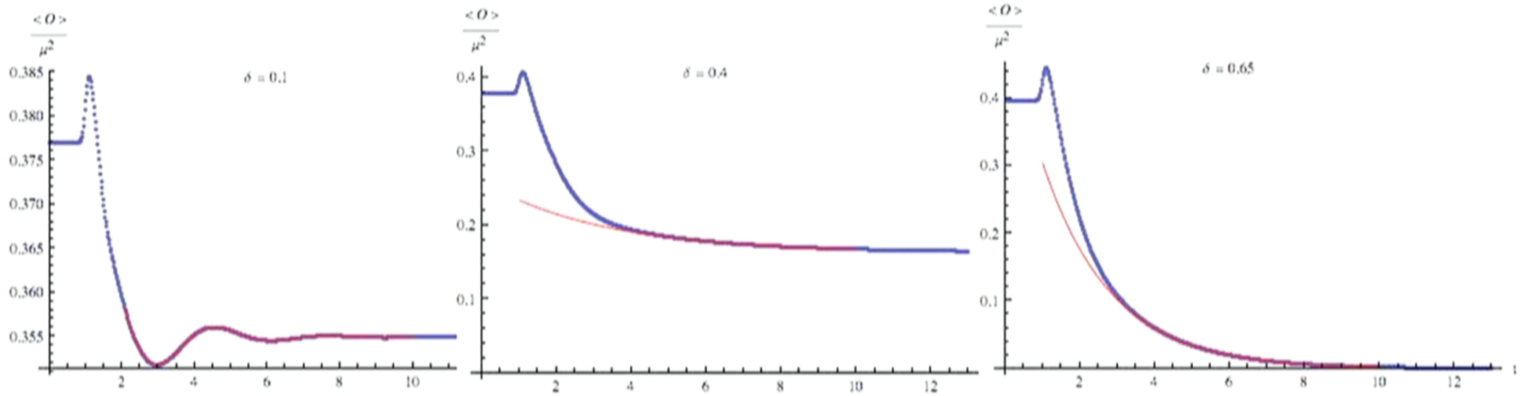

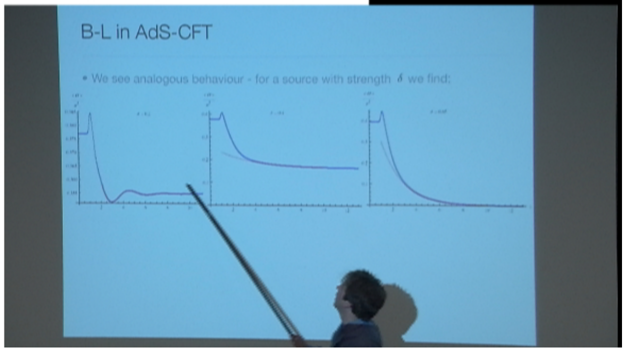

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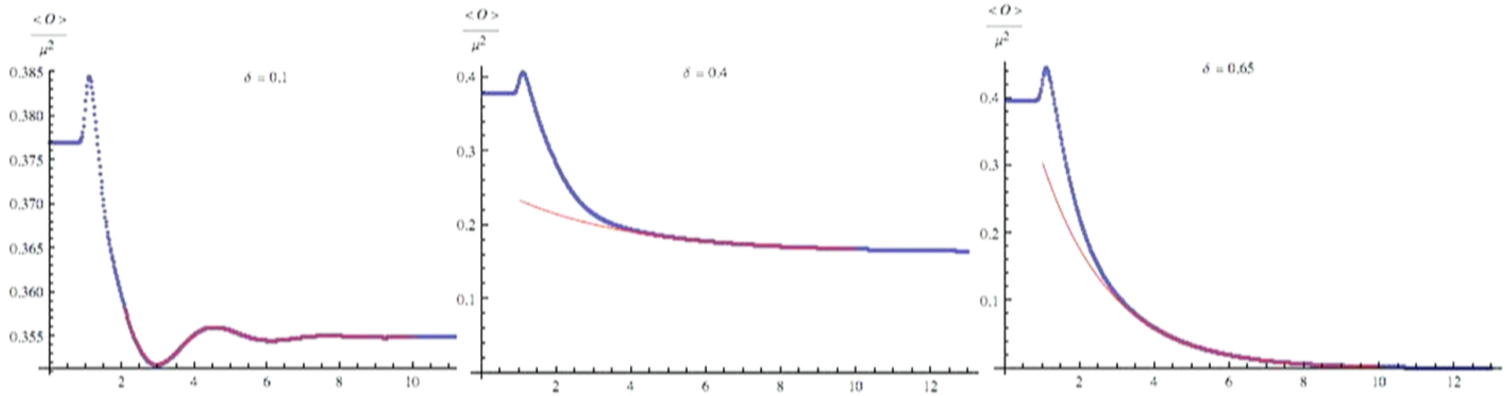

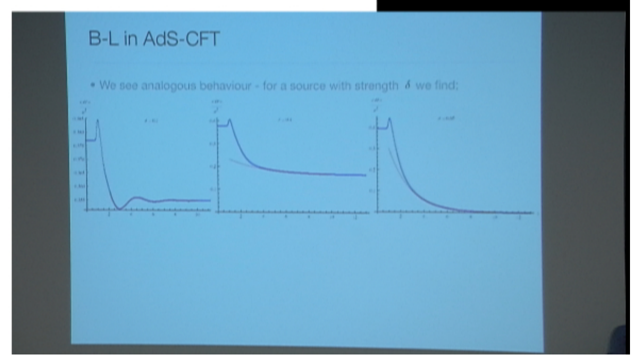

Summary

- The phenomenology of extra dimensions and modifications to gravity lead to a very rich arena of black hole physics
  - There is the possibility to create black holes in LHC
  - There is the possibility to constrain theories using observations of astrophysical black holes (A-E)
- Black holes in AdS-CFT play the role of thermalized plasma. Equilibrium bhs give thermal phases in CFT. Their dynamics describes out-of-equilibrium behaviour of plasma, interesting both for QCD and CMT.

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