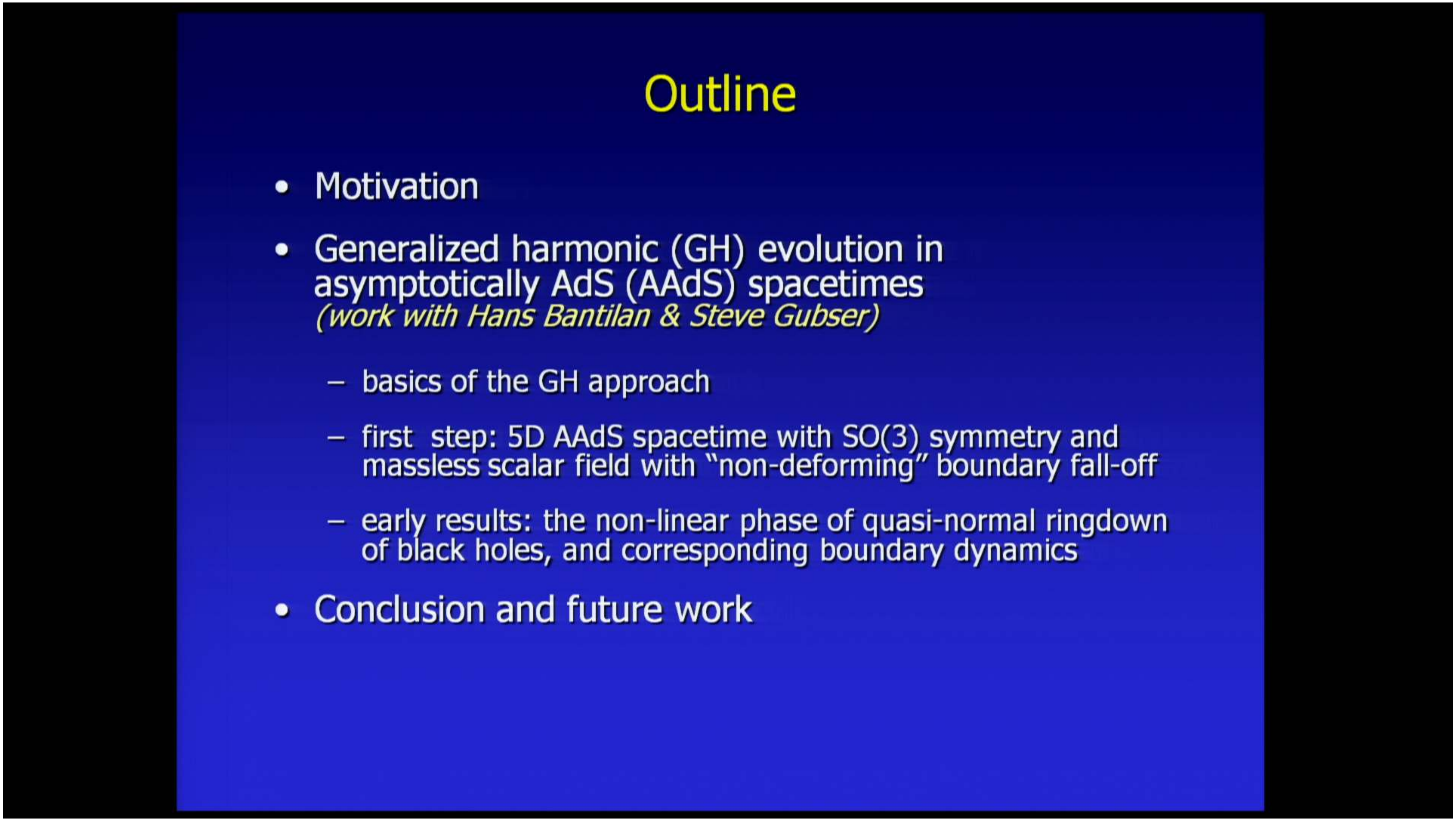
Title: Numerical Evolution of 5D Asymptotically AdS Spacetimes

Date: Jun 05, 2012 10:30 AM

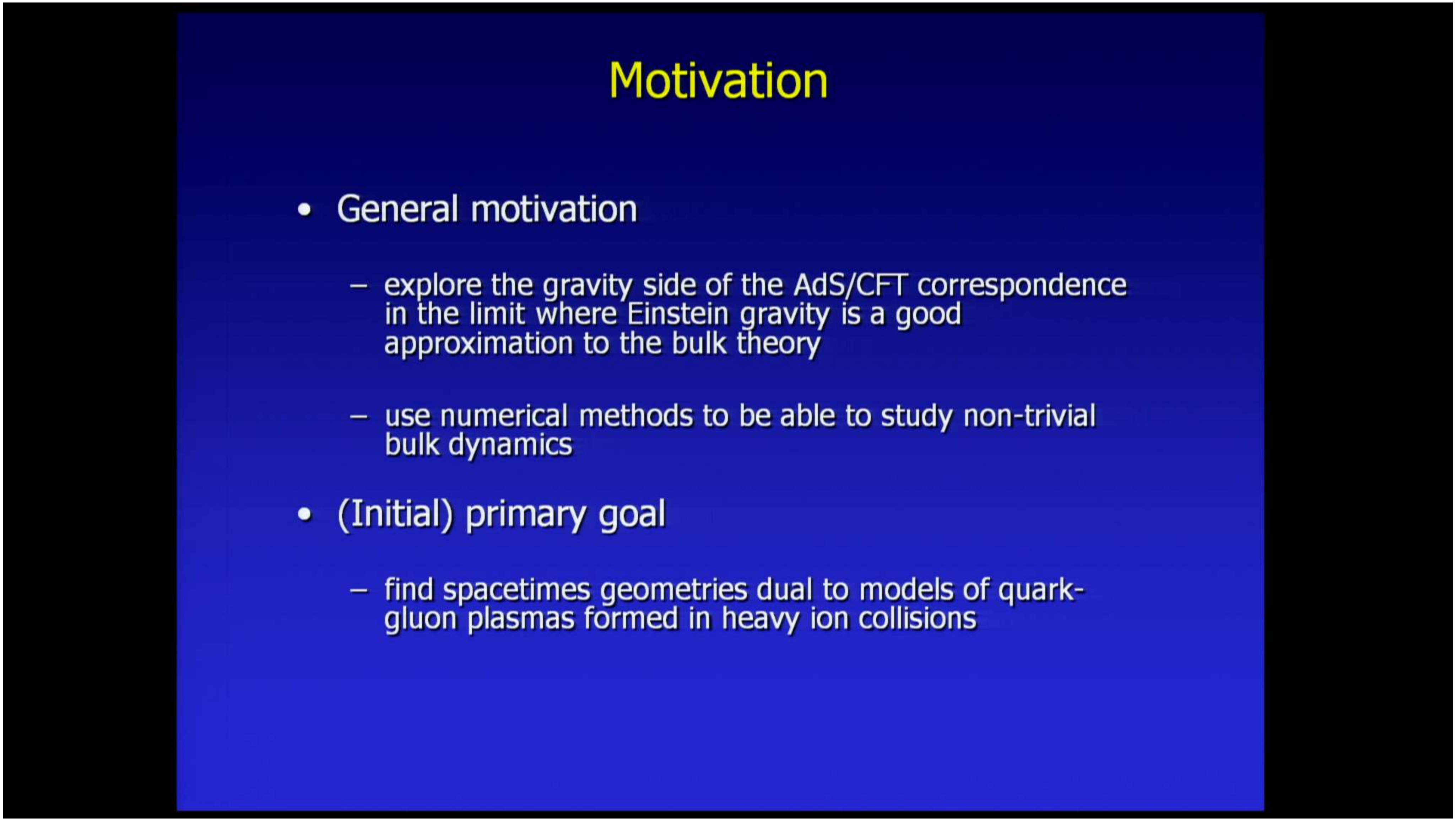
URL: http://pirsa.org/12060014

Abstract: I will describe a new numerical effort to solve Einstein gravity in 5-dimensional asymptotically Anti de Sitter spacetimes (AdS). The motivation is the gauge/gravity duality of string theory, with application to scenarios that on the gravity side are described by dynamical, strong-field solutions. For example, it has been argued that certain properties of the quark-gluon plasma formed in heavy-ion collisions can be modeled by a conformal field theory, with the dual description on the gravity side provided by the collision of black holes. As a first step towards modeling such more general phenomena, we initially focus on spacetimes with SO(3) symmetry in the bulk; i.e., axisymmetric gravity, dual to states with spherical or special conformal symmetry on the boundary. For a first application we study quasi-normal ringdown of highly deformed black holes in the bulk. Even though the initial states are far from equilibrium, the boundary state is remarkably well described as a hydrodynamic flow from early times. The code is based on the generalized harmonic formulation of the field equations, and though this method has been shown to work well in many asymptotically flat scenarios, there are unique challenges that arise in obtaining regular, stable solutions in asymptotically AdS spacetimes. I will describe these challenges, and the way we have addressed them.



Outline

- Motivation
- Generalized harmonic (GH) evolution in asymptotically AdS (AAdS) spacetimes
  (work with Hans Bantilan & Steve Gubser)
  - basics of the GH approach
  - first step: 5D AAdS spacetime with SO(3) symmetry and massless scalar field with "non-deforming" boundary fall-off
  - early results: the non-linear phase of quasi-normal ringdown of black holes, and corresponding boundary dynamics
- Conclusion and future work



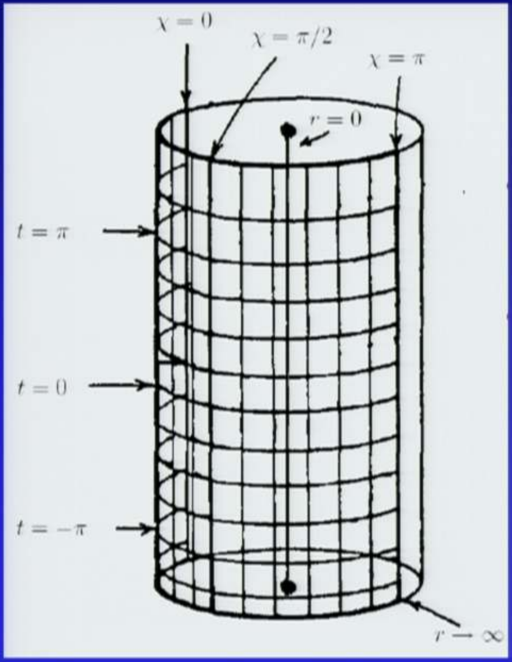
Motivation

- General motivation
  - explore the gravity side of the AdS/CFT correspondence in the limit where Einstein gravity is a good approximation to the bulk theory
  - use numerical methods to be able to study non-trivial bulk dynamics
- (Initial) primary goal
  - find spacetimes geometries dual to models of quark-gluon plasmas formed in heavy ion collisions

5D AdS spacetime

- Global AdS in spherical-polar type coordinates

$$ds^2 = -\left(1+\frac{r^2}{L^2}\right)dt^2 + \left(1+\frac{r^2}{L^2}\right)^{-1}dr^2 + r^2\left(d\chi^2 + \sin^2\chi\, d\Omega_2^2\right)$$

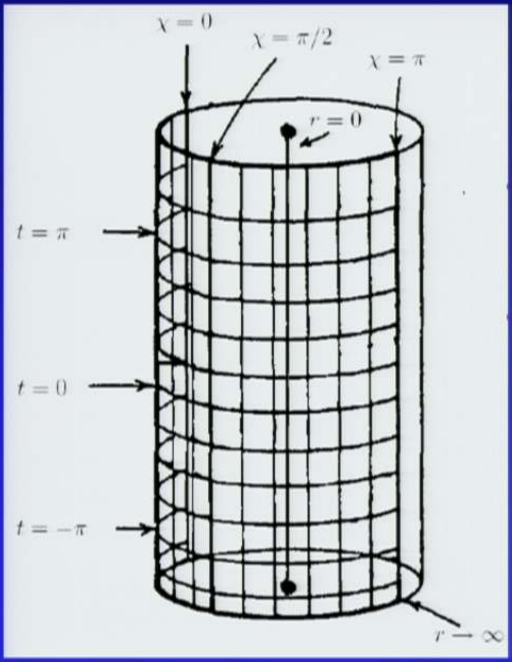


- we would like to solve the field equations in spacetimes with the same asymptotic structure, and in *global coordinates*
  - can tackle a more general class of problems than a code designed to evolve on a Poincare wedge, and for applications on the Poincare patch we can always map a desired segment of the global solution to it via a conformal transformation

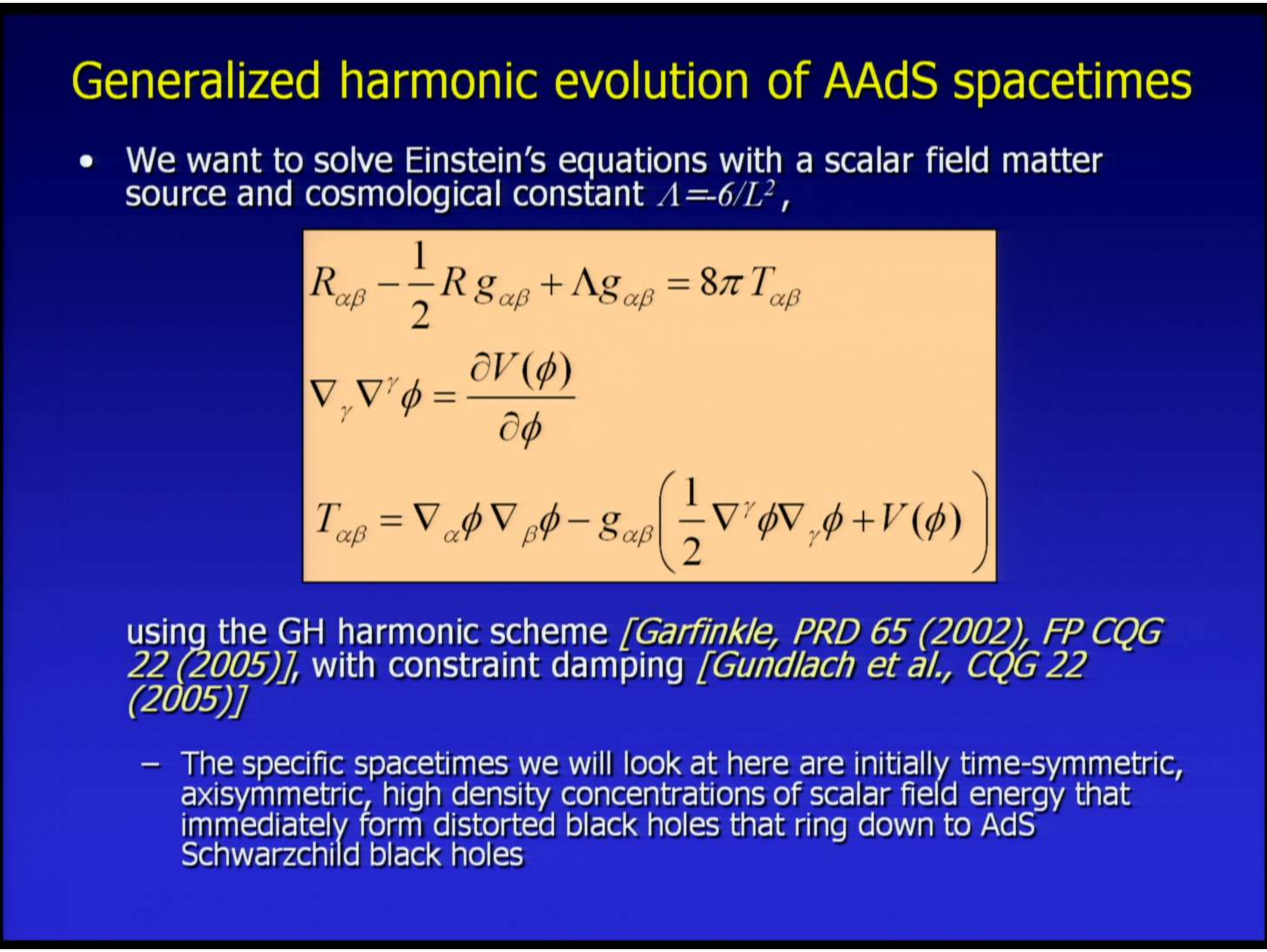
5D AdS spacetime

- Global AdS in spherical-polar type coordinates

$$ds^2 = -\left(1 + \frac{r^2}{L^2}\right)dt^2 + \left(1 + \frac{r^2}{L^2}\right)^{-1} dr^2 + r^2\left(d\chi^2 + \sin^2 \chi d\Omega_2^2\right)$$



- we would like to solve the field equations in spacetimes with the same asymptotic structure, and in *global coordinates*
  - can tackle a more general class of problems than a code designed to evolve on a Poincare wedge, and for applications on the Poincare patch we can always map a desired segment of the global solution to it via a conformal transformation



Generalized harmonic evolution of AAdS spacetimes

- We want to solve Einstein's equations with a scalar field matter source and cosmological constant $\Lambda = -6/L^2$,

$$R_{\alpha\beta} - \frac{1}{2} R\, g_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi\, T_{\alpha\beta}$$

$$\nabla_{\gamma} \nabla^{\gamma} \phi = \frac{\partial V(\phi)}{\partial \phi}$$

$$T_{\alpha\beta} = \nabla_{\alpha}\phi \nabla_{\beta}\phi - g_{\alpha\beta}\left(\frac{1}{2} \nabla^{\gamma}\phi \nabla_{\gamma}\phi + V(\phi) \right)$$

using the GH harmonic scheme *[Garfinkle, PRD 65 (2002), FP CQG 22 (2005)]*, with constraint damping *[Gundlach et al., CQG 22 (2005)]*

  - The specific spacetimes we will look at here are initially time-symmetric, axisymmetric, high density concentrations of scalar field energy that immediately form distorted black holes that ring down to AdS Schwarzchild black holes

Generalized harmonic evolution of AAdS spacetimes

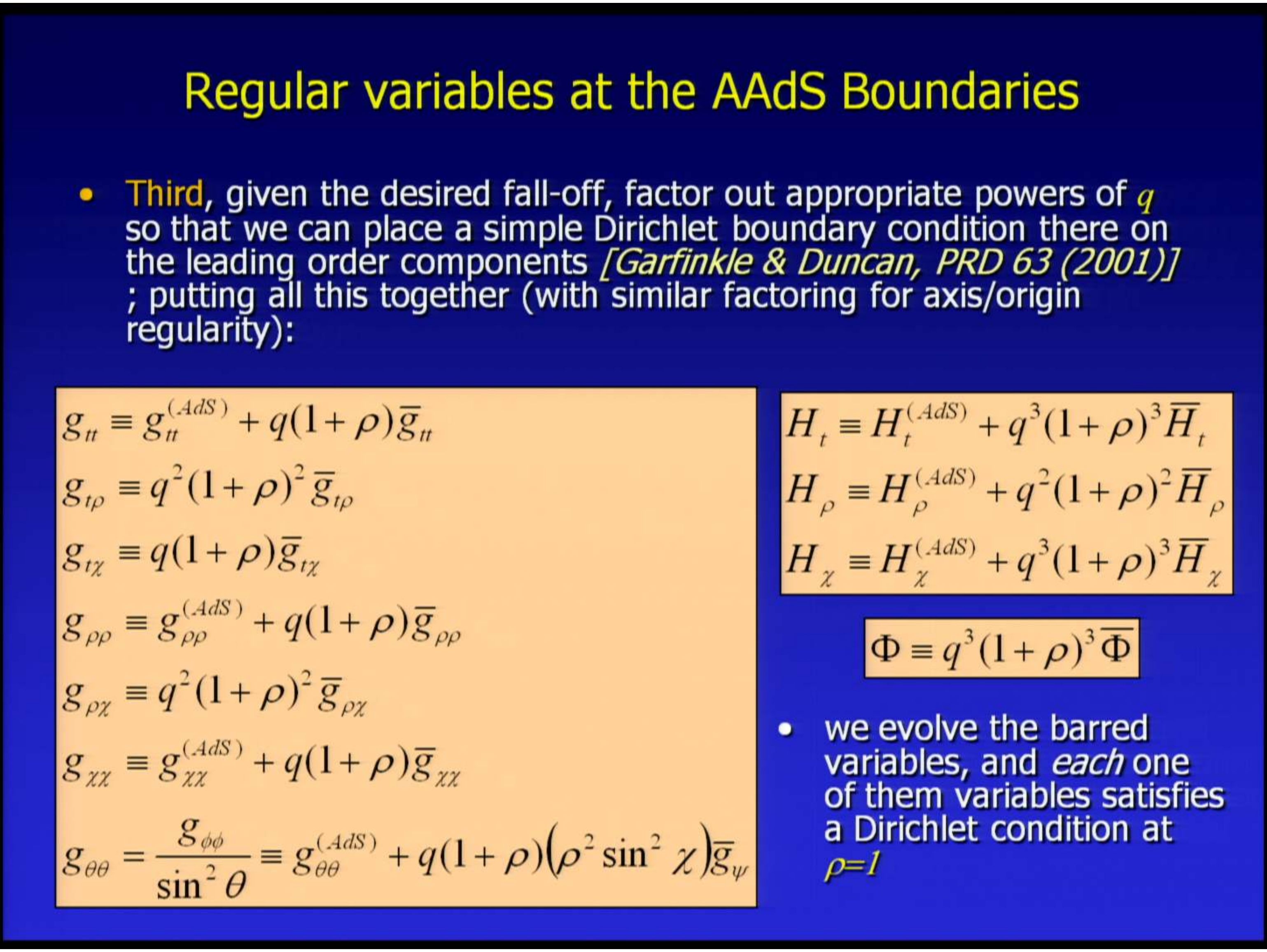
- Specifically, the Einstein equations in GH form are a set of coupled, qausi-linear hyperbolic PDEs, one for each metric element

$$\boxed{\begin{aligned}&-\frac{1}{2}g^{\gamma\delta}g_{\alpha\beta,\gamma\delta}-g^{\gamma\delta}{}_{,(\alpha}g_{\beta)\delta,\gamma}-\Gamma^{\gamma}_{\delta\beta}\Gamma^{\delta}_{\gamma\alpha}-H_{(\alpha,\beta)}+H_{\delta}\Gamma^{\delta}_{\alpha\beta}\\&-\kappa\left(2n_{(\alpha}C_{\beta)}-(1+P)g_{\alpha\beta}n^{\gamma}C_{\gamma}\right)=\frac{2}{3}\Lambda g_{\alpha\beta}+8\pi\left(T_{\alpha\beta}-\frac{1}{3}g_{\alpha\beta}T\right)\end{aligned}}$$

where $\boxed{C^{\mu}\equiv H^{\mu}-\nabla^{\alpha}\nabla_{\alpha}x^{\mu}=0;\ n_{\mu}=-\alpha\,\partial_{\mu}t}$

$\Gamma^{\alpha}{}_{\beta\sigma}$ is the metric connect, κ and P are constraint damping parameters, and α is the lapse function

- Because of the singular nature of the AAdS boundary, we cannot directly discretize these equations using the metric $g_{\mu\nu}$ and source functions H_{μ}
  - To describe the regularization, we first need to fix (in part) the gauge



Regular variables at the AAdS Boundaries

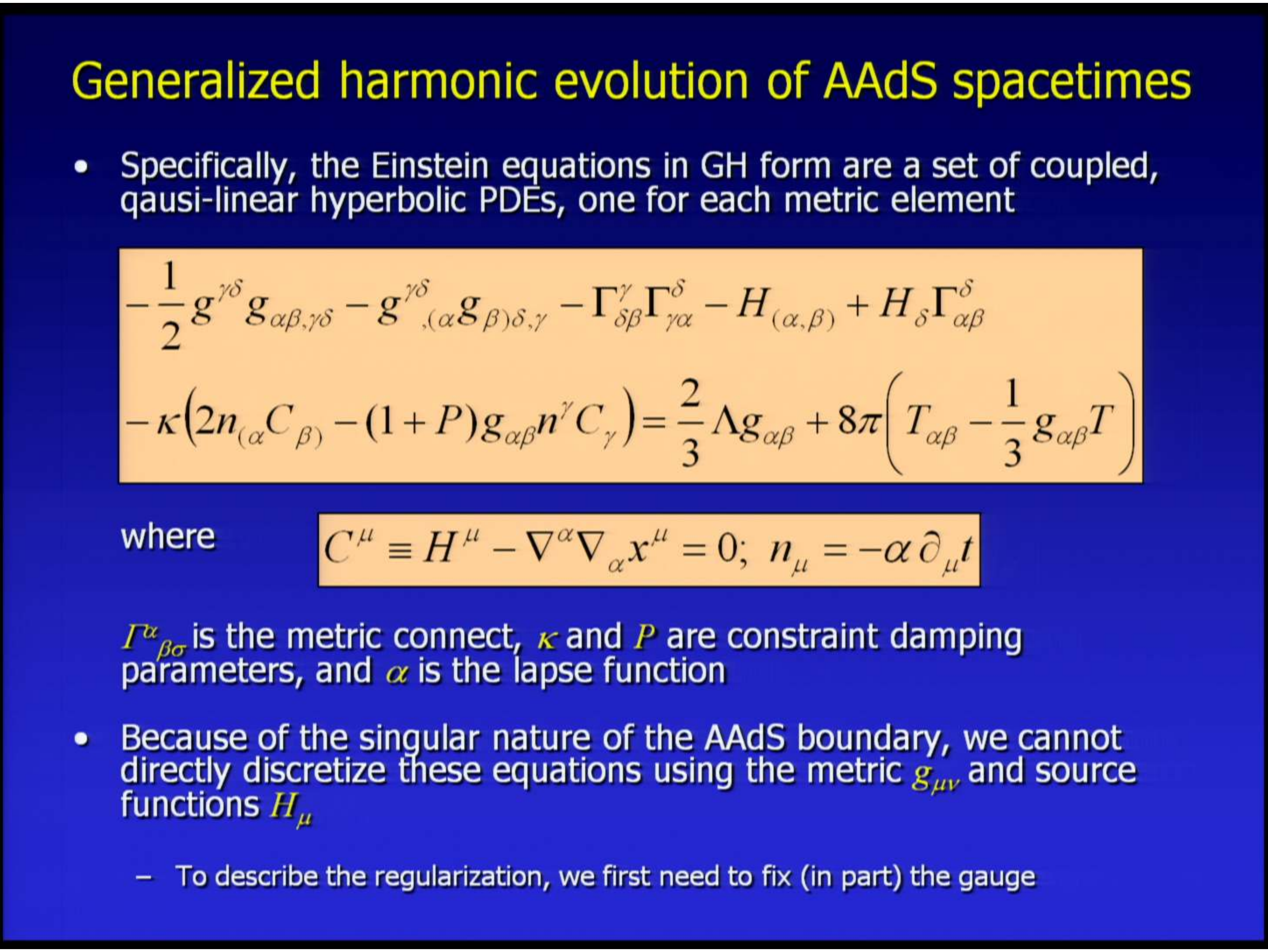
- Third, given the desired fall-off, factor out appropriate powers of q so that we can place a simple Dirichlet boundary condition there on the leading order components *[Garfinkle & Duncan, PRD 63 (2001)]* ; putting all this together (with similar factoring for axis/origin regularity):

$$
\begin{aligned}
g_{tt} &\equiv g_{tt}^{(AdS)} + q(1+\rho)\bar{g}_{tt} \\
g_{t\rho} &\equiv q^2(1+\rho)^2\bar{g}_{t\rho} \\
g_{t\chi} &\equiv q(1+\rho)\bar{g}_{t\chi} \\
g_{\rho\rho} &\equiv g_{\rho\rho}^{(AdS)} + q(1+\rho)\bar{g}_{\rho\rho} \\
g_{\rho\chi} &\equiv q^2(1+\rho)^2\bar{g}_{\rho\chi} \\
g_{\chi\chi} &\equiv g_{\chi\chi}^{(AdS)} + q(1+\rho)\bar{g}_{\chi\chi} \\
g_{\theta\theta} &= \frac{g_{\phi\phi}}{\sin^2\theta} \equiv g_{\theta\theta}^{(AdS)} + q(1+\rho)\left(\rho^2\sin^2\chi\right)\bar{g}_{\psi}
\end{aligned}
$$

$$
\begin{aligned}
H_t &\equiv H_t^{(AdS)} + q^3(1+\rho)^3\bar{H}_t \\
H_\rho &\equiv H_\rho^{(AdS)} + q^2(1+\rho)^2\bar{H}_\rho \\
H_\chi &\equiv H_\chi^{(AdS)} + q^3(1+\rho)^3\bar{H}_\chi
\end{aligned}
$$

$$\Phi \equiv q^3(1+\rho)^3\bar{\Phi}$$

- we evolve the barred variables, and *each* one of them variables satisfies a Dirichlet condition at $\rho=1$



Generalized harmonic evolution of AAdS spacetimes

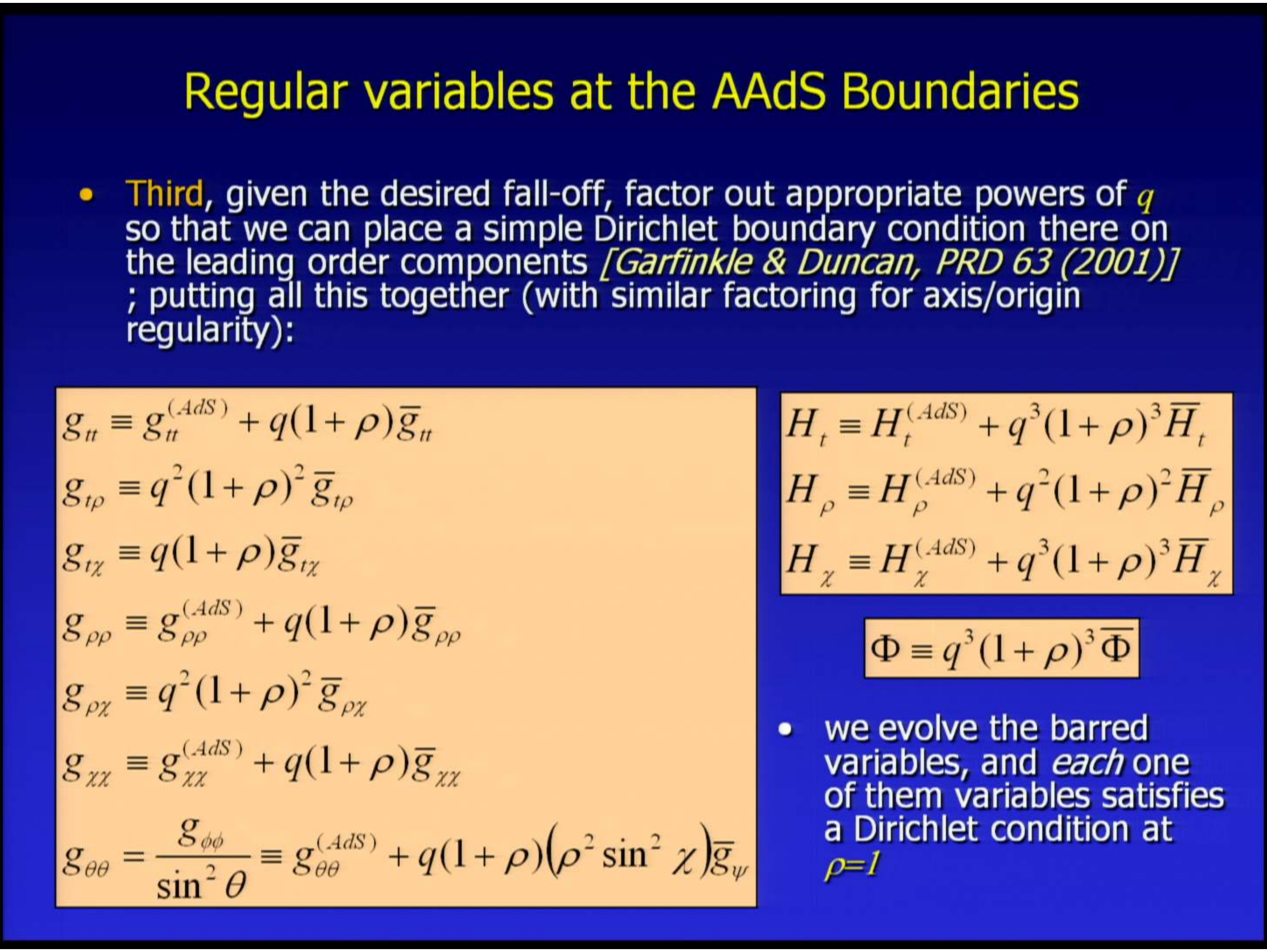
- Specifically, the Einstein equations in GH form are a set of coupled, qausi-linear hyperbolic PDEs, one for each metric element

$$\boxed{\begin{aligned}&-\frac{1}{2}g^{\gamma\delta}g_{\alpha\beta,\gamma\delta} - g^{\gamma\delta}{}_{,(\alpha}g_{\beta)\delta,\gamma} - \Gamma^{\gamma}_{\delta\beta}\Gamma^{\delta}_{\gamma\alpha} - H_{(\alpha,\beta)} + H_{\delta}\Gamma^{\delta}_{\alpha\beta}\\ &-\kappa\left(2n_{(\alpha}C_{\beta)} - (1+P)g_{\alpha\beta}n^{\gamma}C_{\gamma}\right) = \frac{2}{3}\Lambda g_{\alpha\beta} + 8\pi\left(T_{\alpha\beta} - \frac{1}{3}g_{\alpha\beta}T\right)\end{aligned}}$$

where $\boxed{C^{\mu} \equiv H^{\mu} - \nabla^{\alpha}\nabla_{\alpha}x^{\mu} = 0;\ n_{\mu} = -\alpha\,\partial_{\mu}t}$

$\Gamma^{\alpha}{}_{\beta\sigma}$ is the metric connect, κ and P are constraint damping parameters, and α is the lapse function

- Because of the singular nature of the AAdS boundary, we cannot directly discretize these equations using the metric $g_{\mu\nu}$ and source functions H_{μ}
  - To describe the regularization, we first need to fix (in part) the gauge



Regular variables at the AAdS Boundaries

- Third, given the desired fall-off, factor out appropriate powers of q so that we can place a simple Dirichlet boundary condition there on the leading order components *[Garfinkle & Duncan, PRD 63 (2001)]* ; putting all this together (with similar factoring for axis/origin regularity):

$$g_{tt} \equiv g_{tt}^{(AdS)} + q(1+\rho)\bar{g}_{tt}$$
$$g_{t\rho} \equiv q^2(1+\rho)^2\bar{g}_{t\rho}$$
$$g_{t\chi} \equiv q(1+\rho)\bar{g}_{t\chi}$$
$$g_{\rho\rho} \equiv g_{\rho\rho}^{(AdS)} + q(1+\rho)\bar{g}_{\rho\rho}$$
$$g_{\rho\chi} \equiv q^2(1+\rho)^2\bar{g}_{\rho\chi}$$
$$g_{\chi\chi} \equiv g_{\chi\chi}^{(AdS)} + q(1+\rho)\bar{g}_{\chi\chi}$$
$$g_{\theta\theta} = \frac{g_{\phi\phi}}{\sin^2\theta} \equiv g_{\theta\theta}^{(AdS)} + q(1+\rho)\left(\rho^2\sin^2\chi\right)\bar{g}_{\psi}$$

$$H_t \equiv H_t^{(AdS)} + q^3(1+\rho)^3\bar{H}_t$$
$$H_\rho \equiv H_\rho^{(AdS)} + q^2(1+\rho)^2\bar{H}_\rho$$
$$H_\chi \equiv H_\chi^{(AdS)} + q^3(1+\rho)^3\bar{H}_\chi$$

$$\Phi \equiv q^3(1+\rho)^3\bar{\Phi}$$

- we evolve the barred variables, and *each* one of them variables satisfies a Dirichlet condition at $\rho=1$

Regular variables at the AAdS Boundaries

- Third, given the desired fall-off, factor out appropriate powers of q so that we can place a simple Dirichlet boundary condition there on the leading order components *[Garfinkle & Duncan, PRD 63 (2001)]* ; putting all this together (with similar factoring for axis/origin regularity):

$$g_{tt} \equiv g_{tt}^{(AdS)} + q(1+\rho)\bar{g}_{tt}$$

$$g_{t\rho} \equiv q^2(1+\rho)^2\,\bar{g}_{t\rho}$$

$$g_{t\chi} \equiv q(1+\rho)\bar{g}_{t\chi}$$

$$g_{\rho\rho} \equiv g_{\rho\rho}^{(AdS)} + q(1+\rho)\bar{g}_{\rho\rho}$$

$$g_{\rho\chi} \equiv q^2(1+\rho)^2\,\bar{g}_{\rho\chi}$$

$$g_{\chi\chi} \equiv g_{\chi\chi}^{(AdS)} + q(1+\rho)\bar{g}_{\chi\chi}$$

$$g_{\theta\theta} = \frac{g_{\phi\phi}}{\sin^2\theta} \equiv g_{\theta\theta}^{(AdS)} + q(1+\rho)\left(\rho^2\sin^2\chi\right)\bar{g}_{\psi}$$

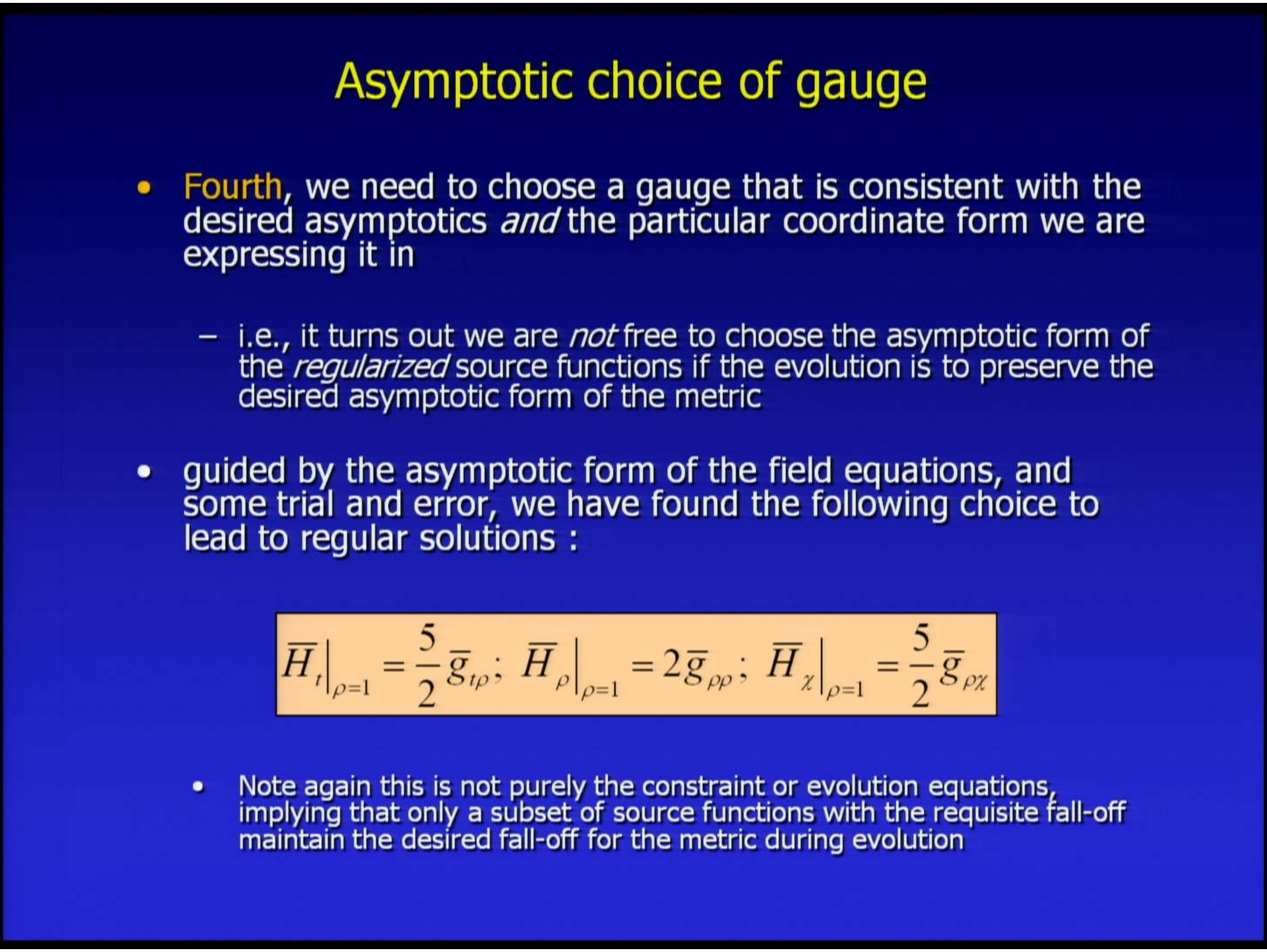
$$H_t \equiv H_t^{(AdS)} + q^3(1+\rho)^3\,\overline{H}_t$$

$$H_\rho \equiv H_\rho^{(AdS)} + q^2(1+\rho)^2\,\overline{H}_\rho$$

$$H_\chi \equiv H_\chi^{(AdS)} + q^3(1+\rho)^3\,\overline{H}_\chi$$

$$\Phi \equiv q^3(1+\rho)^3\,\overline{\Phi}$$

- we evolve the barred variables, and *each* one of them variables satisfies a Dirichlet condition at *ρ=1*

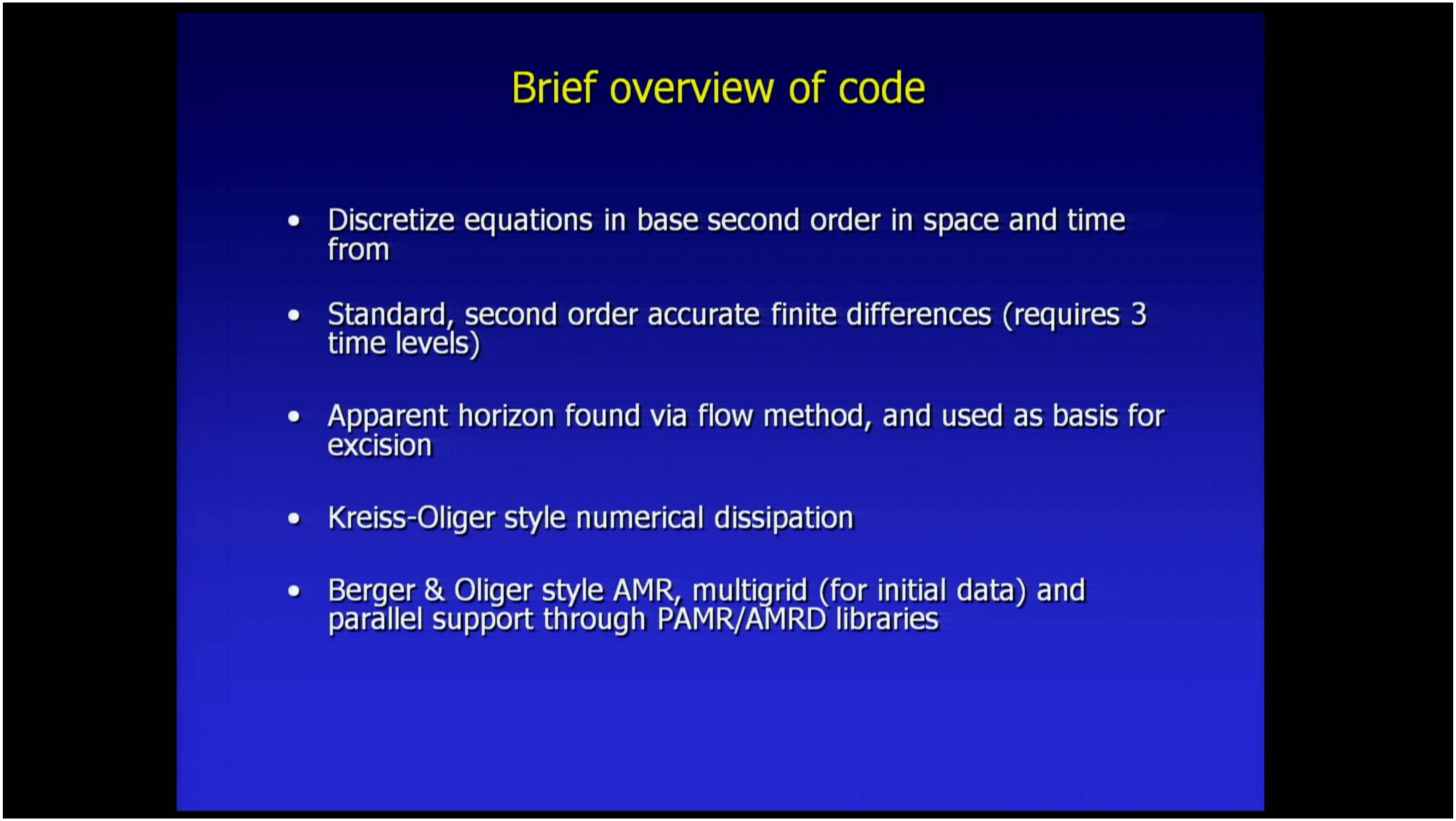


Asymptotic choice of gauge

- Fourth, we need to choose a gauge that is consistent with the desired asymptotics *and* the particular coordinate form we are expressing it in
  - i.e., it turns out we are *not* free to choose the asymptotic form of the *regularized* source functions if the evolution is to preserve the desired asymptotic form of the metric
- guided by the asymptotic form of the field equations, and some trial and error, we have found the following choice to lead to regular solutions :

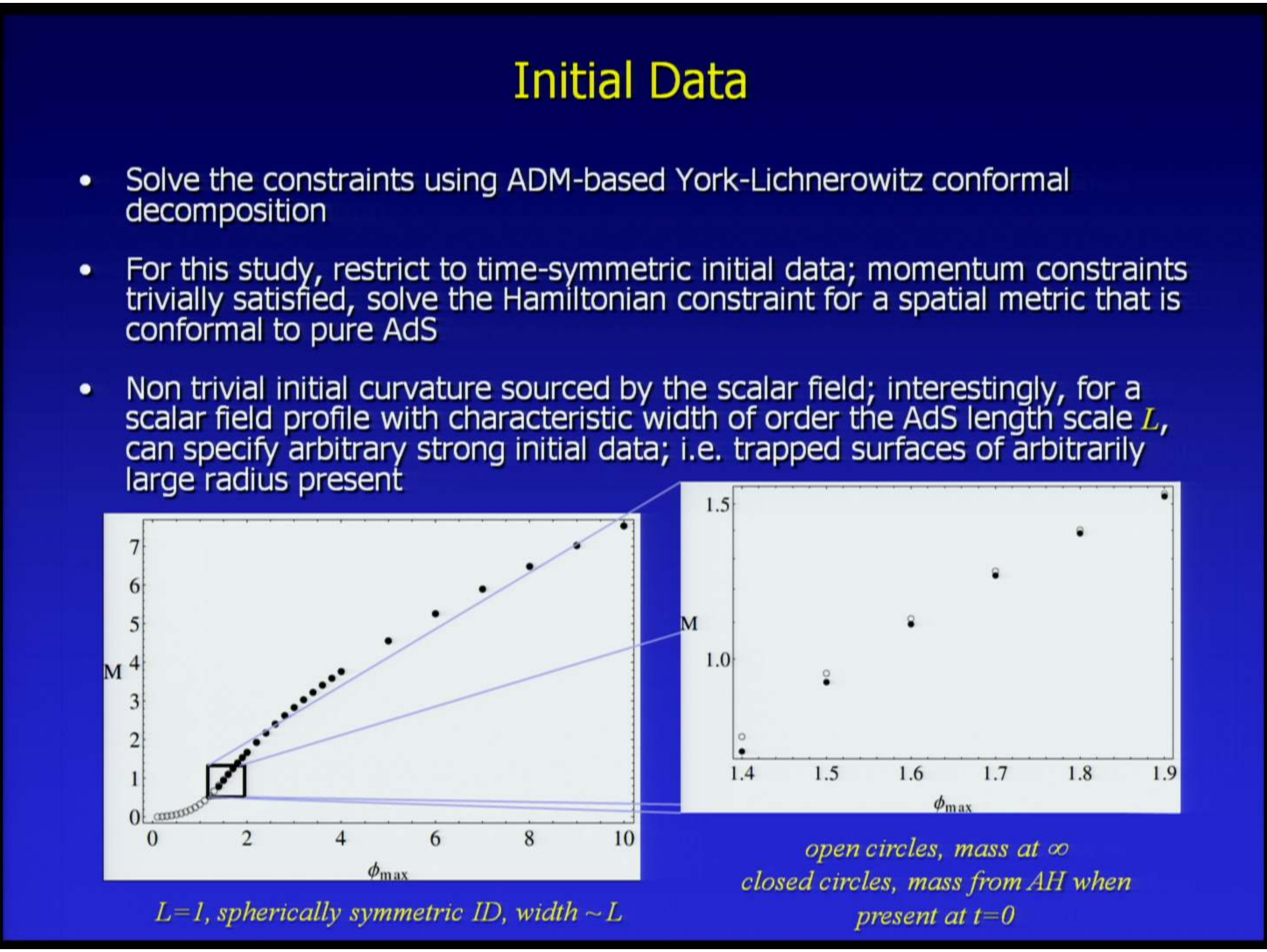
$$\boxed{\bar{H}_t\Big|_{\rho=1} = \frac{5}{2}\bar{g}_{t\rho};\ \ \bar{H}_\rho\Big|_{\rho=1} = 2\bar{g}_{\rho\rho};\ \ \bar{H}_\chi\Big|_{\rho=1} = \frac{5}{2}\bar{g}_{\rho\chi}}$$

  - Note again this is not purely the constraint or evolution equations, implying that only a subset of source functions with the requisite fall-off maintain the desired fall-off for the metric during evolution



Brief overview of code

- Discretize equations in base second order in space and time from
- Standard, second order accurate finite differences (requires 3 time levels)
- Apparent horizon found via flow method, and used as basis for excision
- Kreiss-Oliger style numerical dissipation
- Berger & Oliger style AMR, multigrid (for initial data) and parallel support through PAMR/AMRD libraries

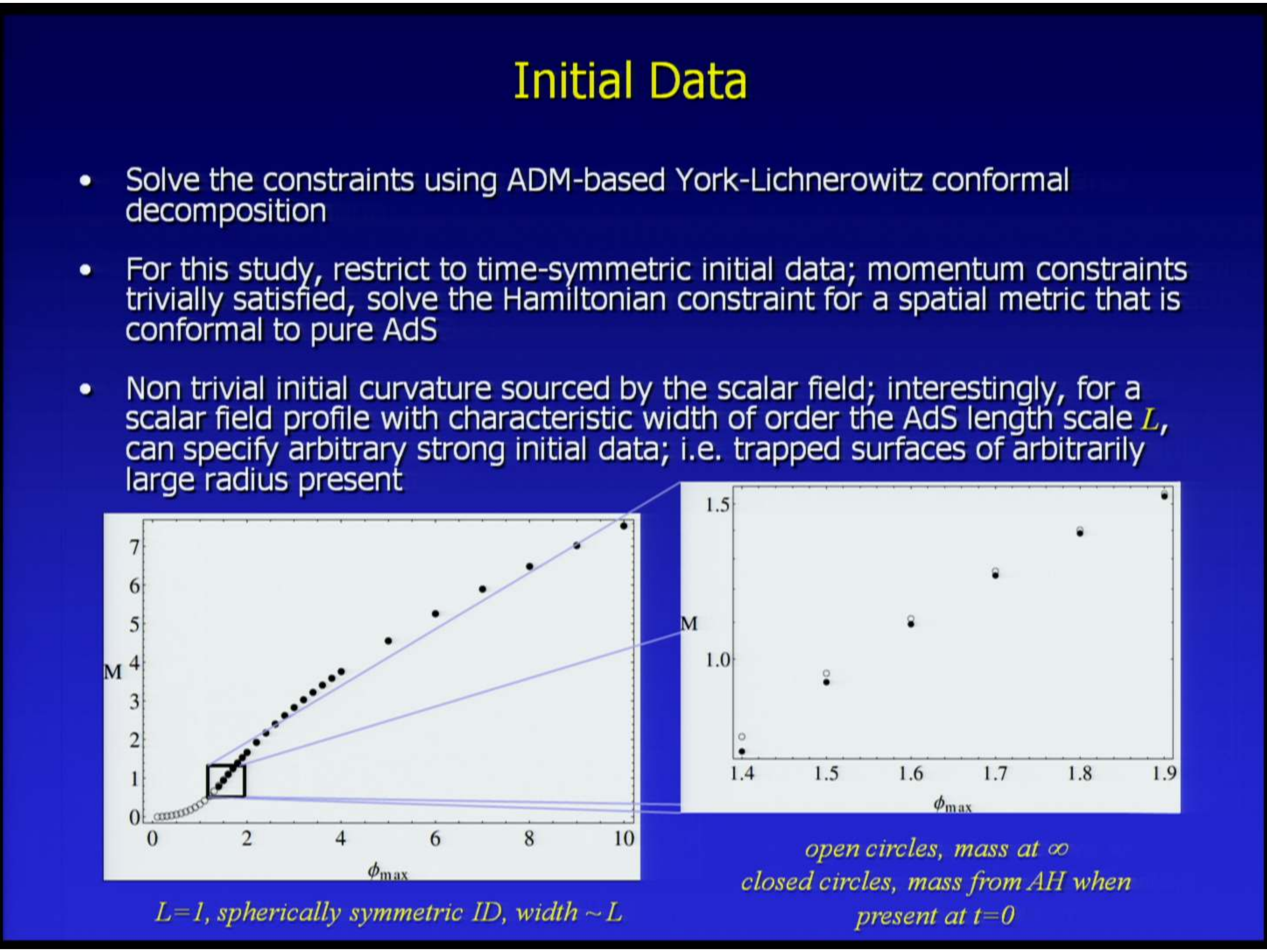


Initial Data

- Solve the constraints using ADM-based York-Lichnerowitz conformal decomposition
- For this study, restrict to time-symmetric initial data; momentum constraints trivially satisfied, solve the Hamiltonian constraint for a spatial metric that is conformal to pure AdS
- Non trivial initial curvature sourced by the scalar field; interestingly, for a scalar field profile with characteristic width of order the AdS length scale L, can specify arbitrary strong initial data; i.e. trapped surfaces of arbitrarily large radius present

$L=1$, spherically symmetric ID, width ~ L

open circles, mass at ∞
closed circles, mass from AH when present at $t=0$

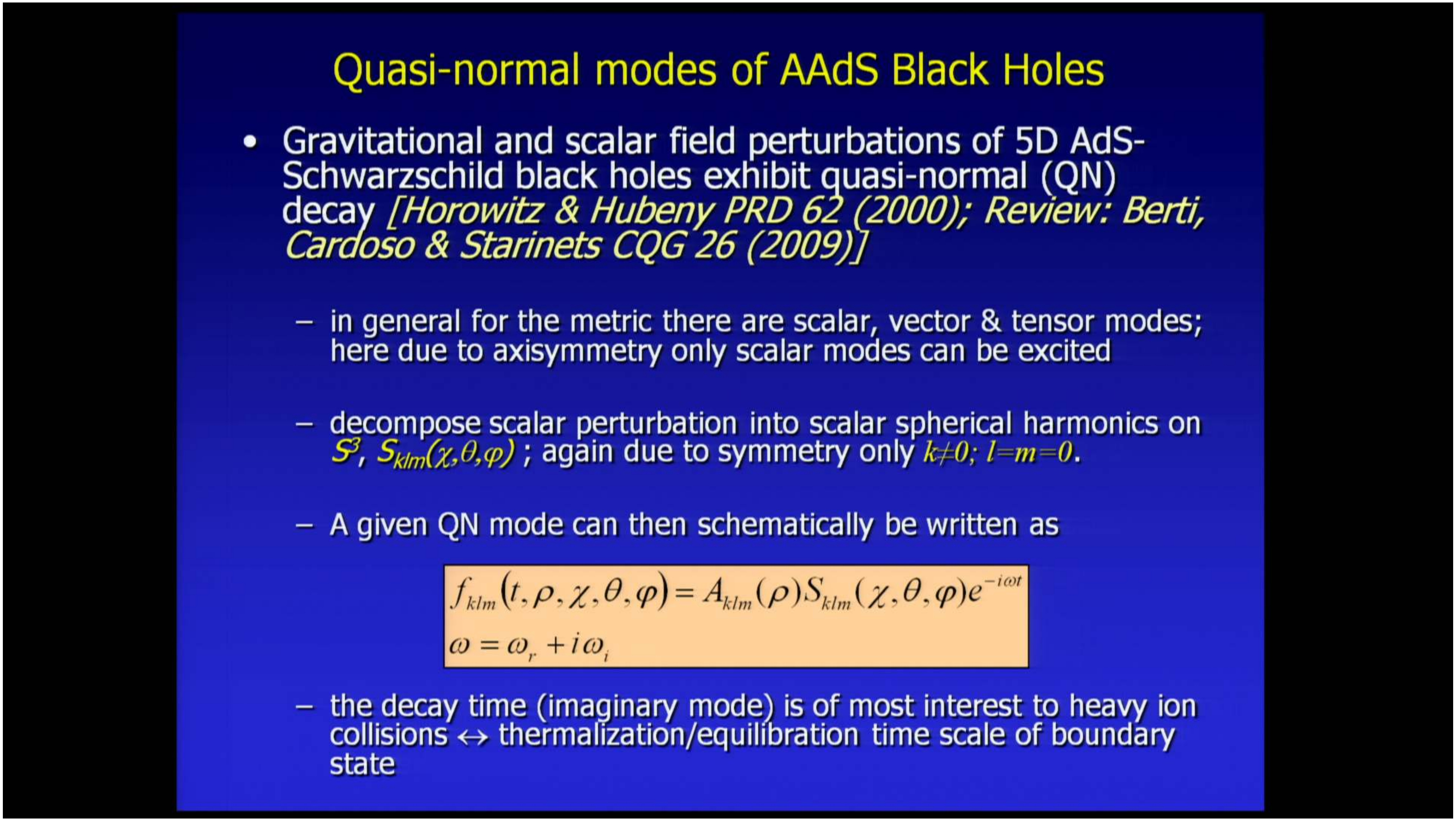


Initial Data

- Solve the constraints using ADM-based York-Lichnerowitz conformal decomposition
- For this study, restrict to time-symmetric initial data; momentum constraints trivially satisfied, solve the Hamiltonian constraint for a spatial metric that is conformal to pure AdS
- Non trivial initial curvature sourced by the scalar field; interestingly, for a scalar field profile with characteristic width of order the AdS length scale L, can specify arbitrary strong initial data; i.e. trapped surfaces of arbitrarily large radius present

L=1, spherically symmetric ID, width ~ L

open circles, mass at ∞
closed circles, mass from AH when present at t=0



Quasi-normal modes of AAdS Black Holes

- Gravitational and scalar field perturbations of 5D AdS-Schwarzschild black holes exhibit quasi-normal (QN) decay *[Horowitz & Hubeny PRD 62 (2000); Review: Berti, Cardoso & Starinets CQG 26 (2009)]*
  - in general for the metric there are scalar, vector & tensor modes; here due to axisymmetry only scalar modes can be excited
  - decompose scalar perturbation into scalar spherical harmonics on S^3, $S_{klm}(\chi,\theta,\varphi)$; again due to symmetry only $k \neq 0;\ l=m=0$.
  - A given QN mode can then schematically be written as

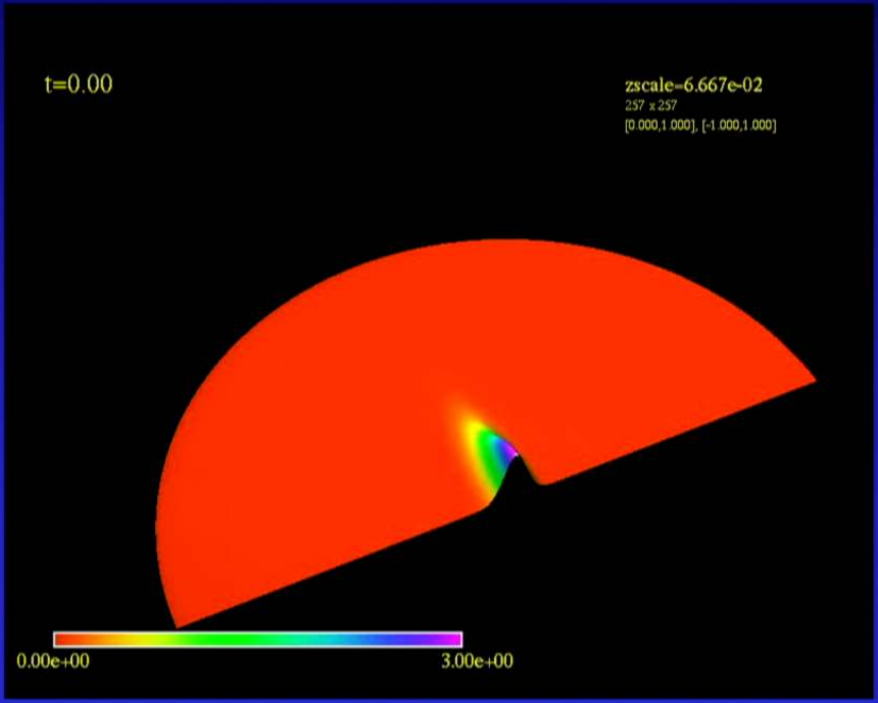
$$\boxed{\begin{aligned} & f_{klm}(t,\rho,\chi,\theta,\varphi) = A_{klm}(\rho) S_{klm}(\chi,\theta,\varphi) e^{-i\omega t} \\ & \omega = \omega_r + i\omega_i \end{aligned}}$$

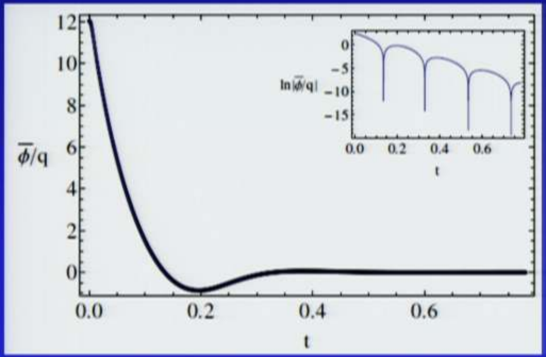
  - the decay time (imaginary mode) is of most interest to heavy ion collisions ↔ thermalization/equilibration time scale of boundary state

Quasi-normal modes of AAdS Black Holes

- form a distorted BH via asymmetric scalar field collapse

$$\overline{\Phi}(\rho,\chi,t=0) = Ae^{-\frac{\rho^2\cos^2\chi}{w_x^2}-\frac{\rho^2\sin^2\chi}{w_y^2}}$$





r_H=5.0; k=0

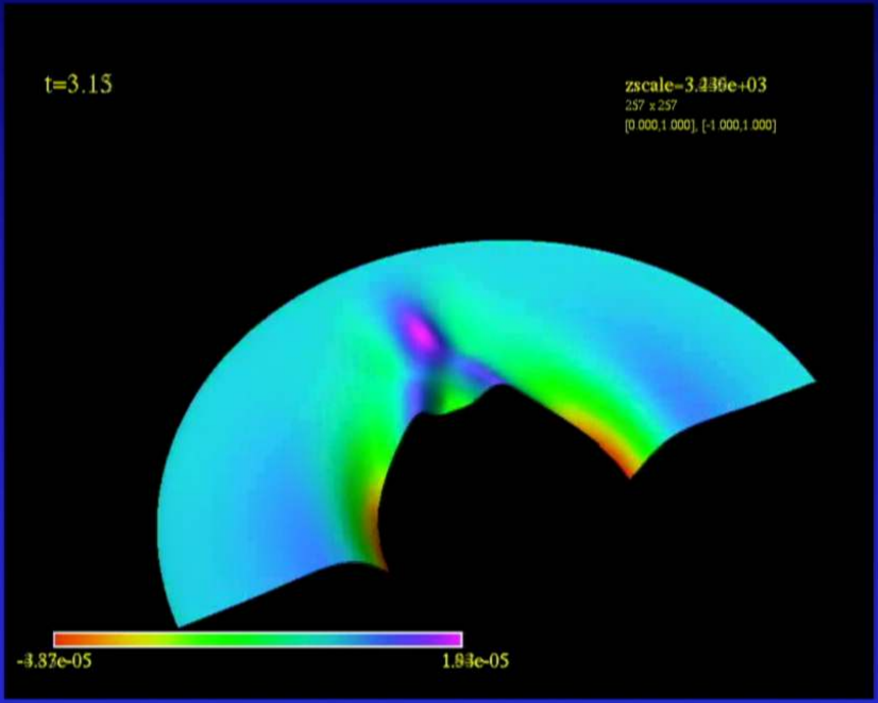
$$\overline{\Phi}(\rho,\chi,t)$$

r_H=12.2

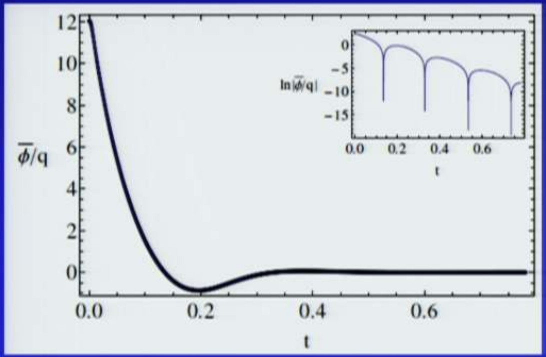
Quasi-normal modes of AAdS Black Holes

- form a distorted BH via asymmetric scalar field collapse

$$\overline{\Phi}(\rho,\chi,t=0)=Ae^{-\frac{\rho^2\cos^2\chi}{w_x^2}-\frac{\rho^2\sin^2\chi}{w_y^2}}$$



r_H=12.2



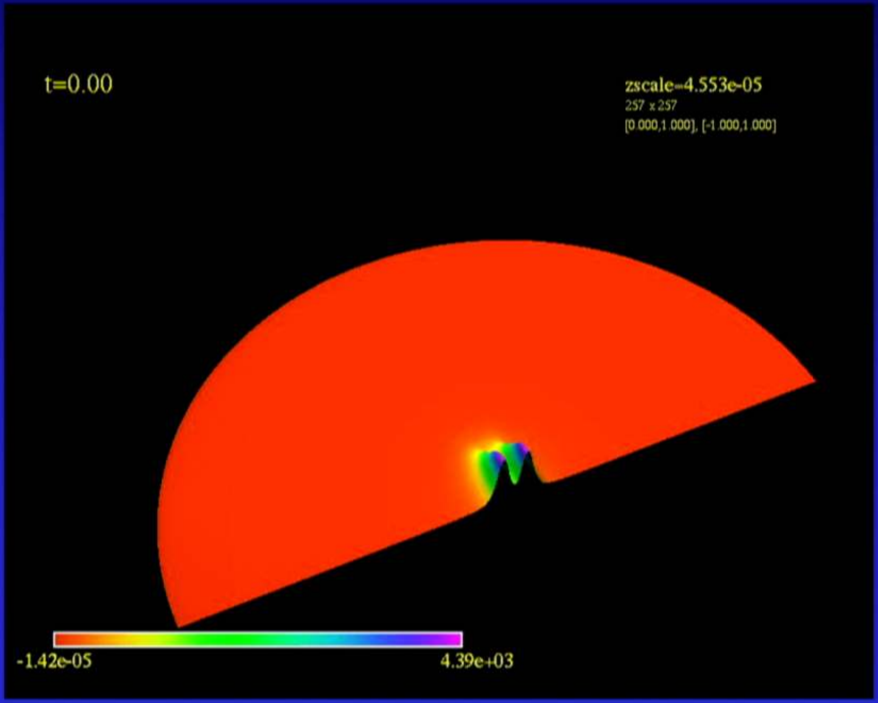
r_H=5.0; k=0

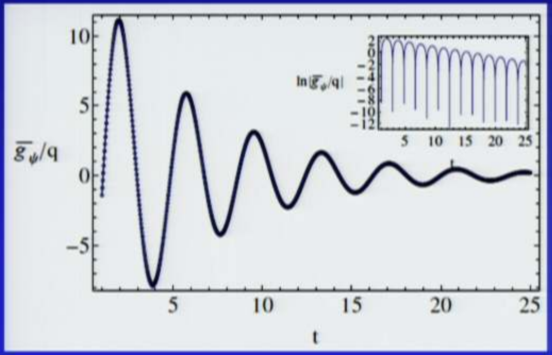
$\overline{\Phi}(\rho,\chi,t)$

Quasi-normal modes of AAdS Black Holes

- form a distorted BH via asymmetric scalar field collapse

$$\overline{\Phi}(\rho,\chi,t=0) = Ae^{-\frac{\rho^2\cos^2\chi}{w_x^2}-\frac{\rho^2\sin^2\chi}{w_y^2}}$$





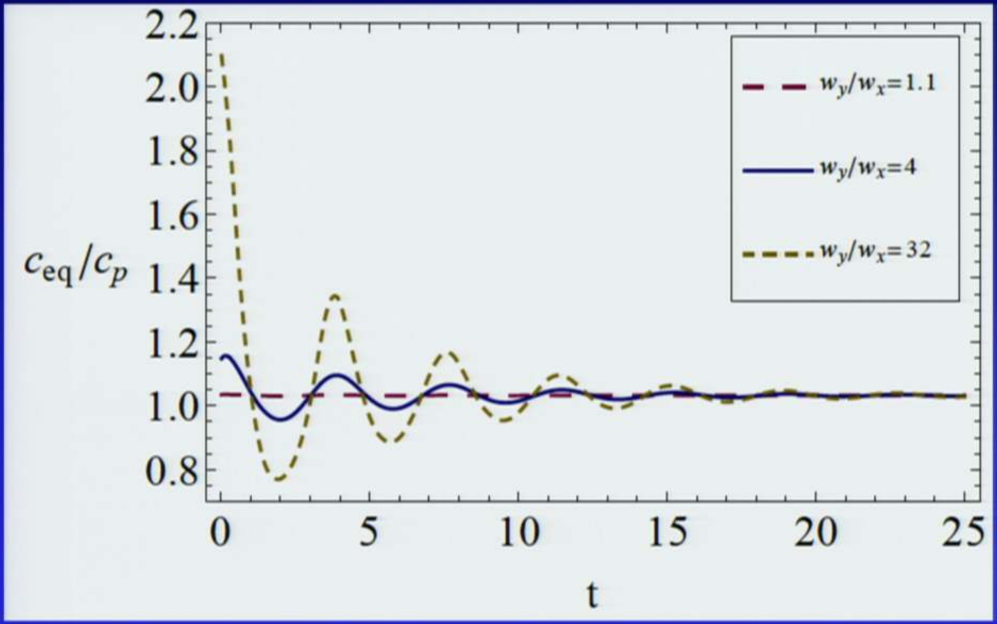
r_H=5.0; k=2

$$\boxed{\bar{g}_{\chi\chi}(\rho,\chi,t)}$$

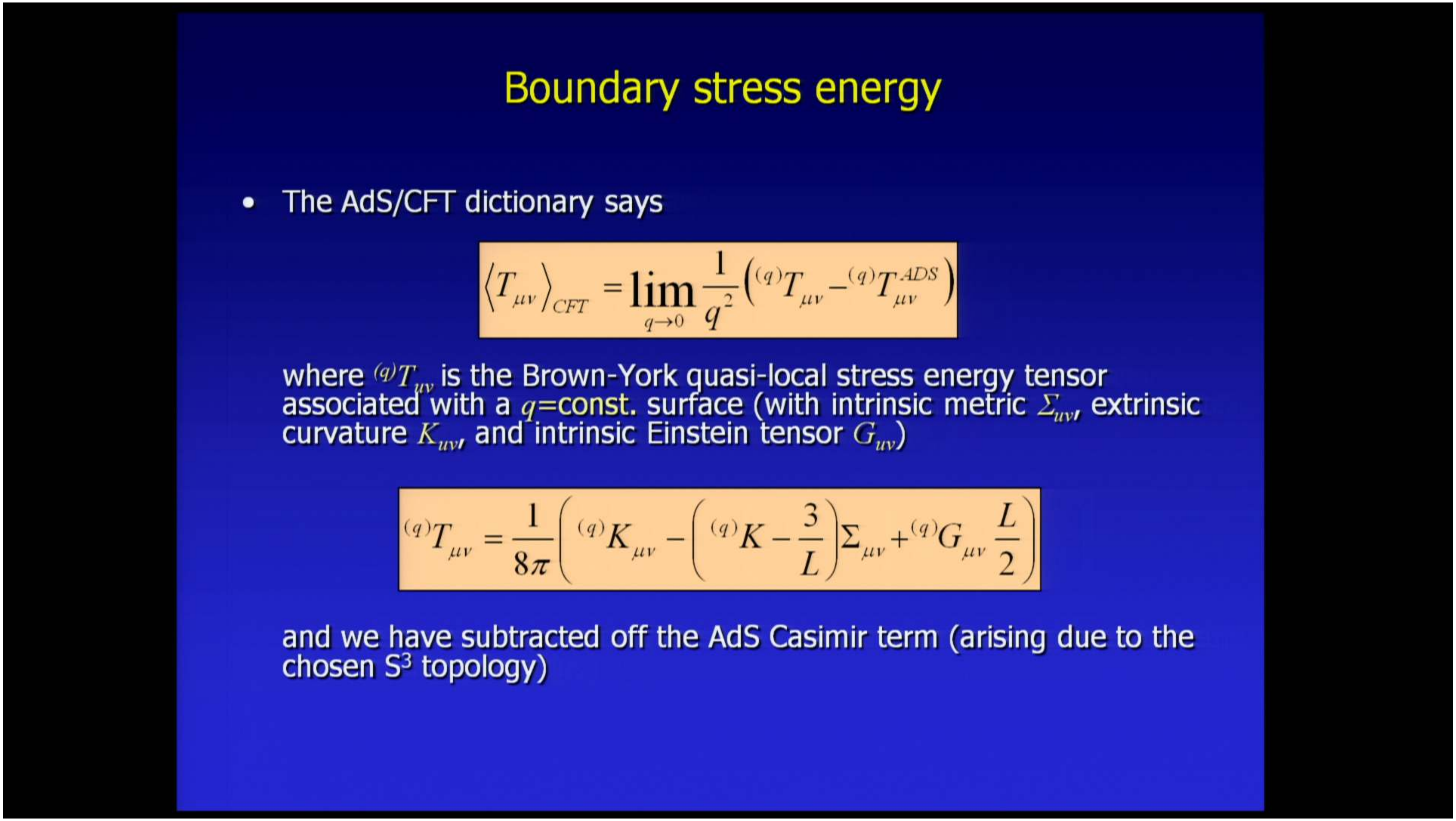
r_H=12.2

Quasi-normal modes of AAdS Black Holes

- To give some idea of how "non-linear" we are, below is the ratio of equatorial to polar circumference of AH for increasing asymmetric ID



- Can describe asymptotic behavior of fields as a superposition of linear QN modes, plus what appears to be a gauge mode (a purely decaying exponential); the non-linearity manifests in higher k-number modes through the appearance of harmonics of the lower k-modes



Boundary stress energy

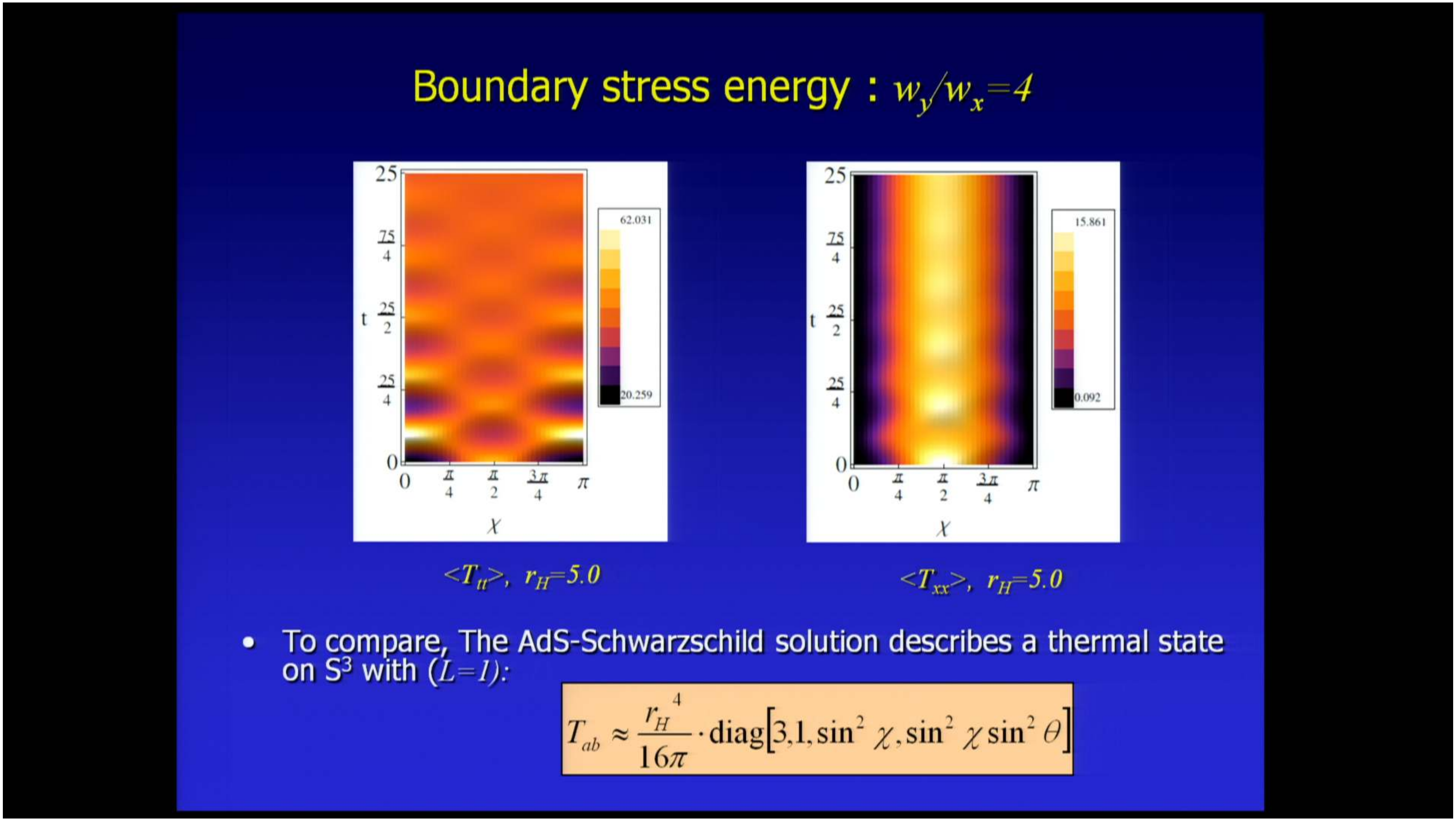
- The AdS/CFT dictionary says

$$\left\langle T_{\mu\nu} \right\rangle_{CFT} = \lim_{q\to 0} \frac{1}{q^2} \left({}^{(q)}T_{\mu\nu} - {}^{(q)}T^{ADS}_{\mu\nu} \right)$$

where ${}^{(q)}T_{uv}$ is the Brown-York quasi-local stress energy tensor associated with a q=const. surface (with intrinsic metric Σ_{uv}, extrinsic curvature K_{uv}, and intrinsic Einstein tensor G_{uv})

$${}^{(q)}T_{\mu\nu} = \frac{1}{8\pi} \left({}^{(q)}K_{\mu\nu} - \left({}^{(q)}K - \frac{3}{L} \right) \Sigma_{\mu\nu} + {}^{(q)}G_{\mu\nu} \frac{L}{2} \right)$$

and we have subtracted off the AdS Casimir term (arising due to the chosen S^3 topology)



Boundary stress energy : $w_y/w_x=4$

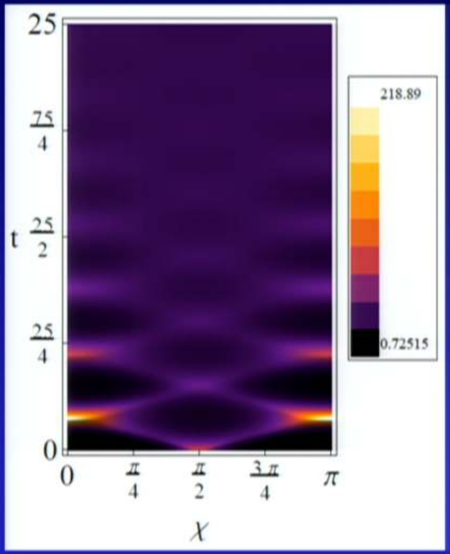
$<T_{tt}>$, $r_H=5.0$

$<T_{xx}>$, $r_H=5.0$

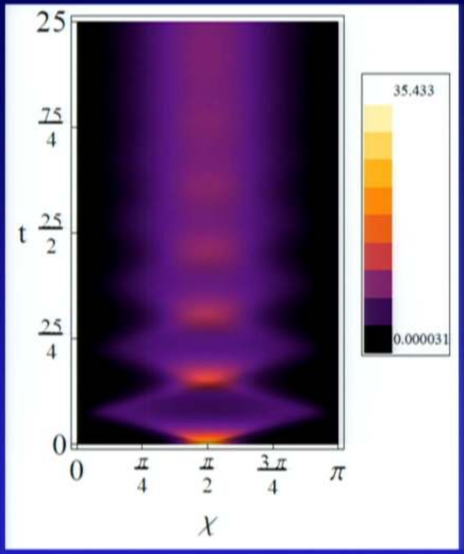
- To compare, The AdS-Schwarzschild solution describes a thermal state on S^3 with $(L=1)$:

$$T_{ab} \approx \frac{r_H^{\;4}}{16\pi} \cdot \mathrm{diag}\left[3, 1, \sin^2\chi, \sin^2\chi \sin^2\theta\right]$$

Boundary stress energy : $w_y/w_x=32$



$<T_{tt}>$, $r_H=5.0$



$<T_{xx}>$, $r_H=5.0$

- To compare, The AdS-Schwarzschild solution describes a thermal state on S^3 with $(L=1)$:

$$T_{ab} \approx \frac{r_H{}^4}{16\pi} \cdot \mathrm{diag}\left[3, 1, \sin^2\chi, \sin^2\chi \sin^2\theta\right]$$

Hydrodynamics of the boundary SET

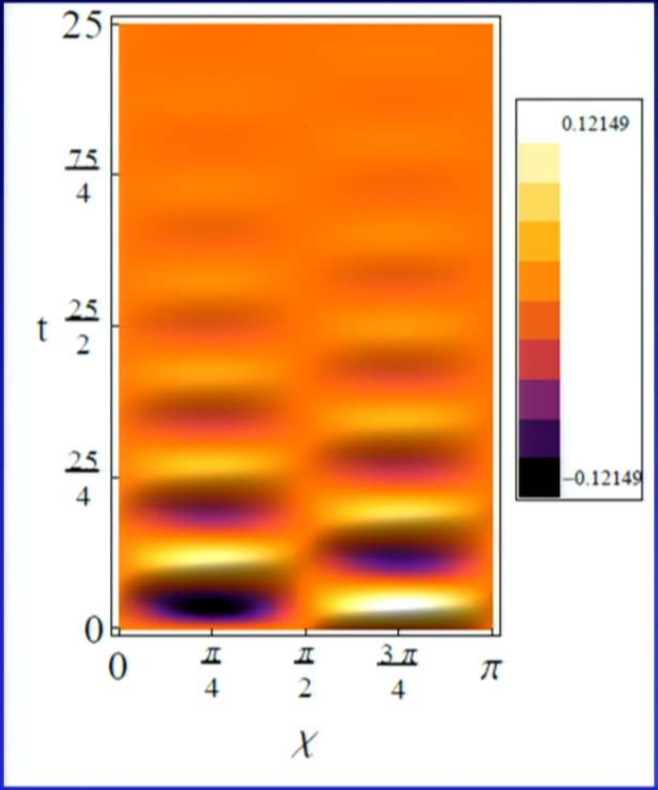
- Correspondence suggests if the bulk is dual to a thermal state on the boundary, the boundary SET should behave like a $\mathcal{N}=4$ *SYM* conformal fluid

$$T_{\mu\nu} = \sum_{i=0}^{\infty} T_{\mu\nu}^{(i)}$$

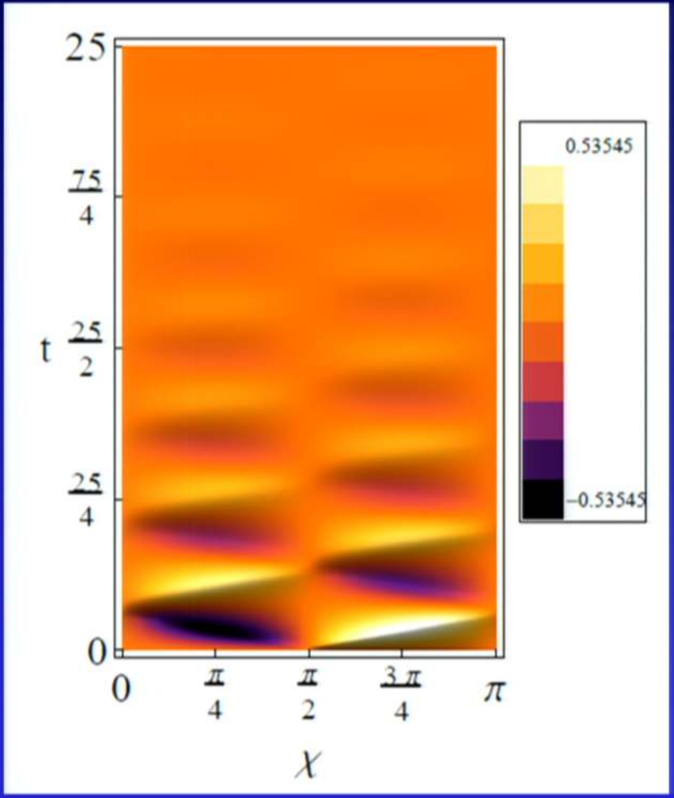
where up to 2nd order in a derivative expansion of the fluid velocity

$$\begin{aligned}
T_{\mu\nu}^{(0)} &= \epsilon u_\mu u_\nu + P \perp_{\mu\nu} \\
T_{\mu\nu}^{(1)} &= -2\eta\sigma_{\mu\nu} \\
T_{\mu\nu}^{(2)} &= -2\eta\left[-\tau_\pi u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \tau_\omega\left(\omega_\mu{}^\lambda \sigma_{\lambda\nu} + \omega_\nu{}^\lambda \sigma_{\lambda\mu}\right)\right] \\
&\quad + \xi_\sigma\left[\sigma_\mu{}^\lambda \sigma_{\lambda\nu} - \frac{\perp_{\mu\nu}}{3}\sigma^{\alpha\beta}\sigma_{\alpha\beta}\right] + \xi_C C_{\mu\alpha\nu\beta} u^\alpha u^\beta
\end{aligned}$$

Boundary Hydrodynamics : extracted velocities



$w_y/w_x=4$



$w_y/w_x=32$

Hydrodynamics of the boundary SET

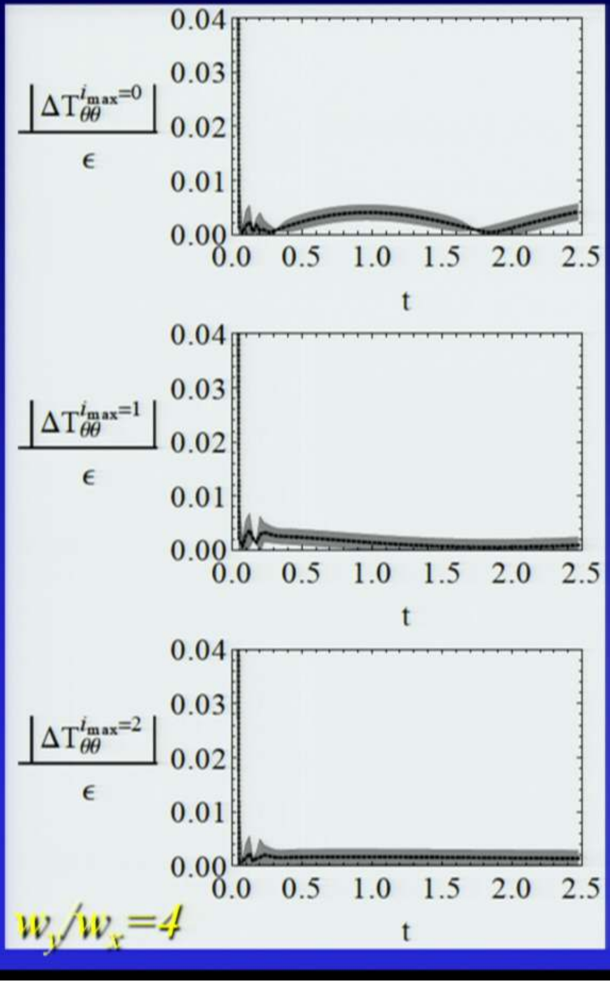
- Correspondence suggests if the bulk is dual to a thermal state on the boundary, the boundary SET should behave like a $\mathcal{N}=4$ *SYM* conformal fluid

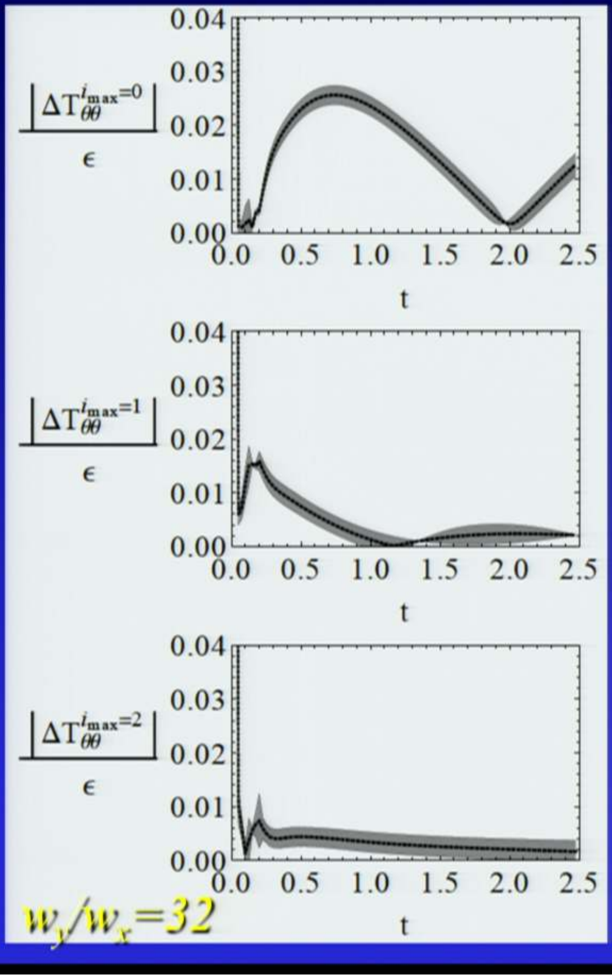
$$T_{\mu\nu} = \sum_{i=0}^{\infty} T_{\mu\nu}^{(i)}$$

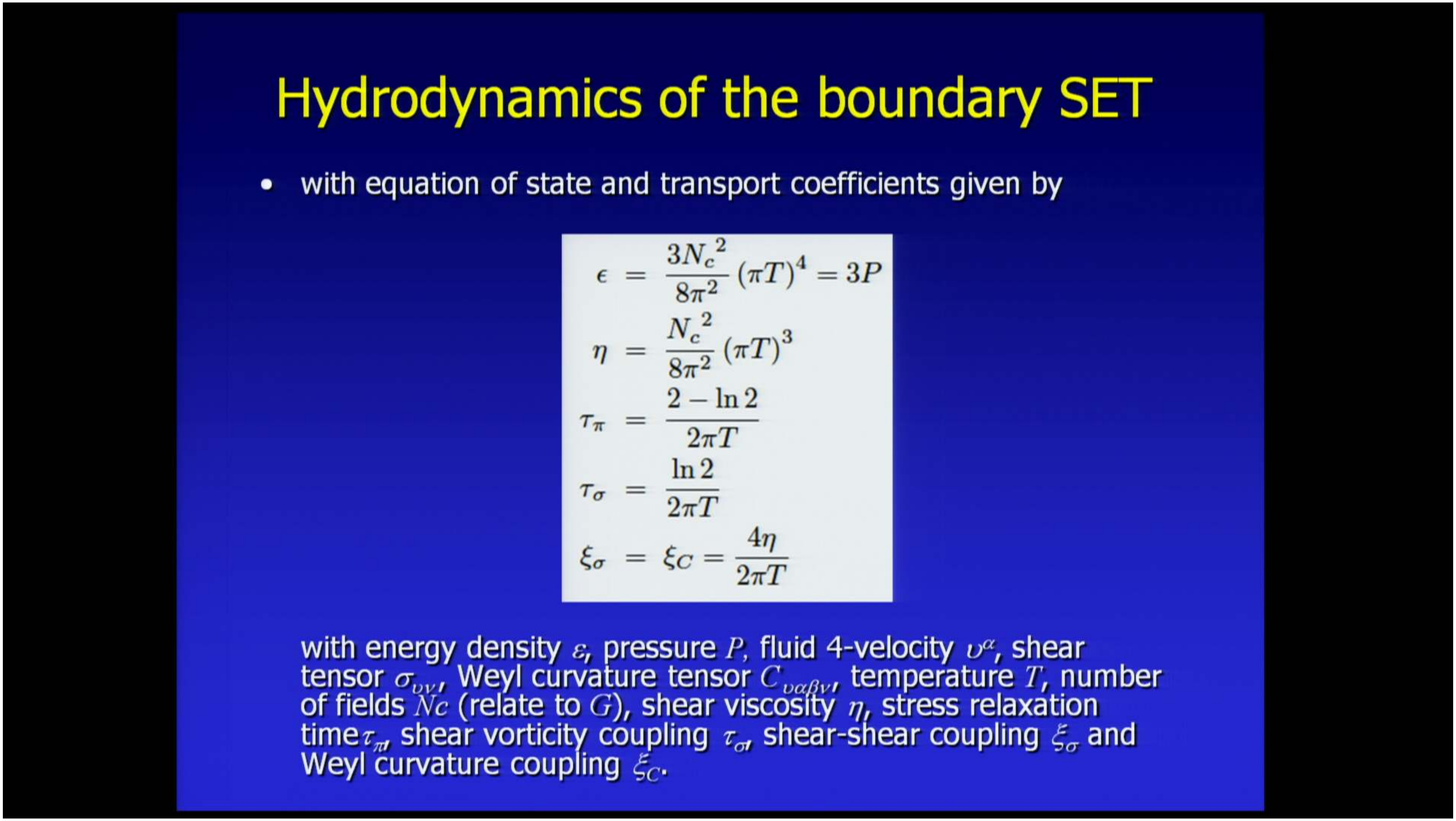
where up to 2nd order in a derivative expansion of the fluid velocity

$$\begin{aligned}
T_{\mu\nu}^{(0)} &= \epsilon u_\mu u_\nu + P \perp_{\mu\nu} \\
T_{\mu\nu}^{(1)} &= -2\eta\sigma_{\mu\nu} \\
T_{\mu\nu}^{(2)} &= -2\eta\left[-\tau_\pi u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \tau_\omega\left(\omega_\mu{}^\lambda \sigma_{\lambda\nu} + \omega_\nu{}^\lambda \sigma_{\lambda\mu}\right)\right] \\
&\quad + \xi_\sigma\left[\sigma_\mu{}^\lambda \sigma_{\lambda\nu} - \frac{\perp_{\mu\nu}}{3}\sigma^{\alpha\beta}\sigma_{\alpha\beta}\right] + \xi_C C_{\mu\alpha\nu\beta} u^\alpha u^\beta
\end{aligned}$$

Boundary Hydrodynamics : consistency with a SYM fluid





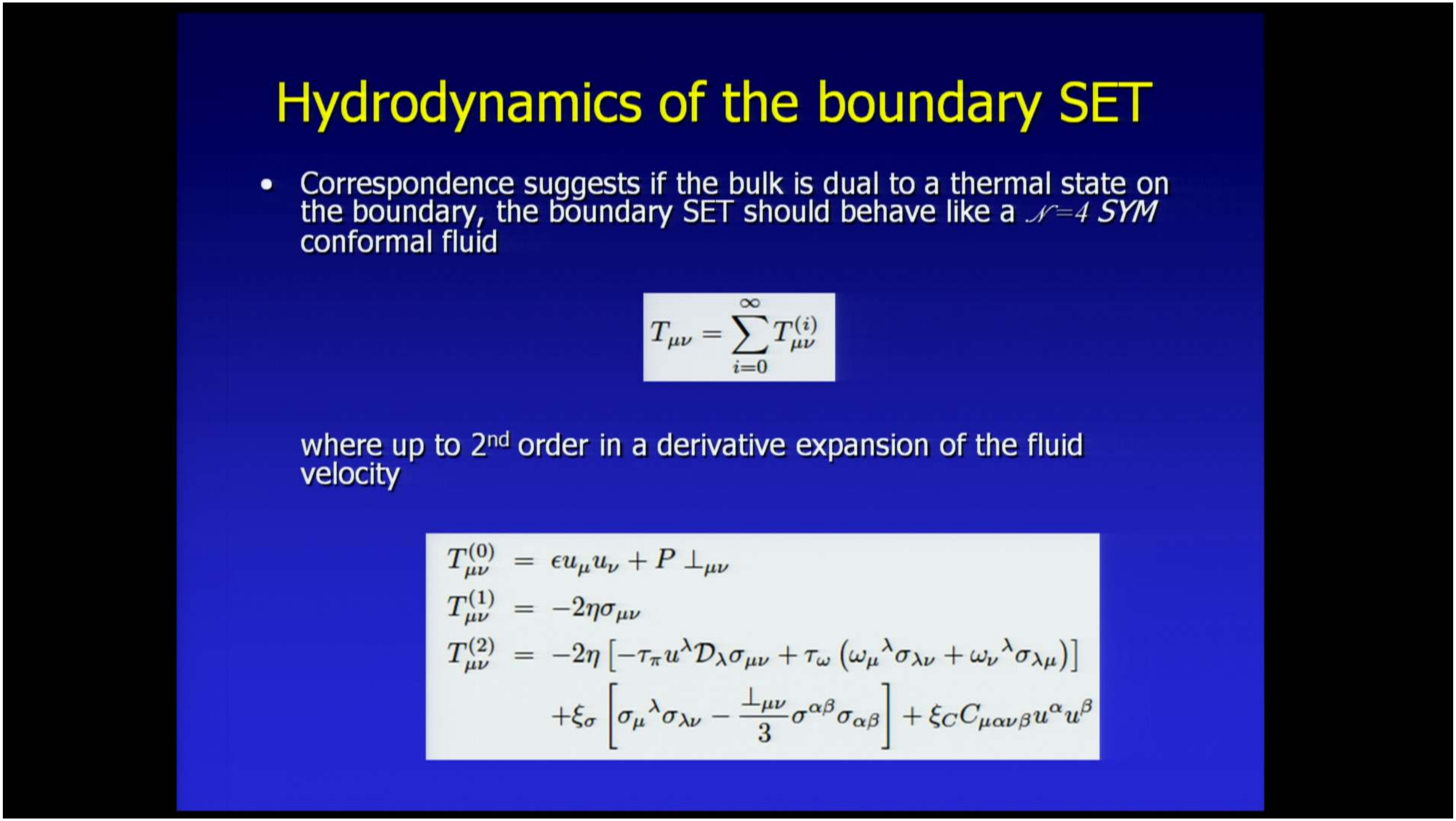


Hydrodynamics of the boundary SET

- with equation of state and transport coefficients given by

$$\begin{aligned}
\epsilon &= \frac{3{N_c}^2}{8\pi^2}(\pi T)^4 = 3P \\
\eta &= \frac{{N_c}^2}{8\pi^2}(\pi T)^3 \\
\tau_\pi &= \frac{2-\ln 2}{2\pi T} \\
\tau_\sigma &= \frac{\ln 2}{2\pi T} \\
\xi_\sigma &= \xi_C = \frac{4\eta}{2\pi T}
\end{aligned}$$

with energy density ε, pressure P, fluid 4-velocity υ^α, shear tensor $\sigma_{\upsilon\nu}$, Weyl curvature tensor $C_{\upsilon\alpha\beta\nu}$, temperature T, number of fields Nc (relate to G), shear viscosity η, stress relaxation timeτ_π, shear vorticity coupling τ_σ, shear-shear coupling ξ_σ and Weyl curvature coupling ξ_C.



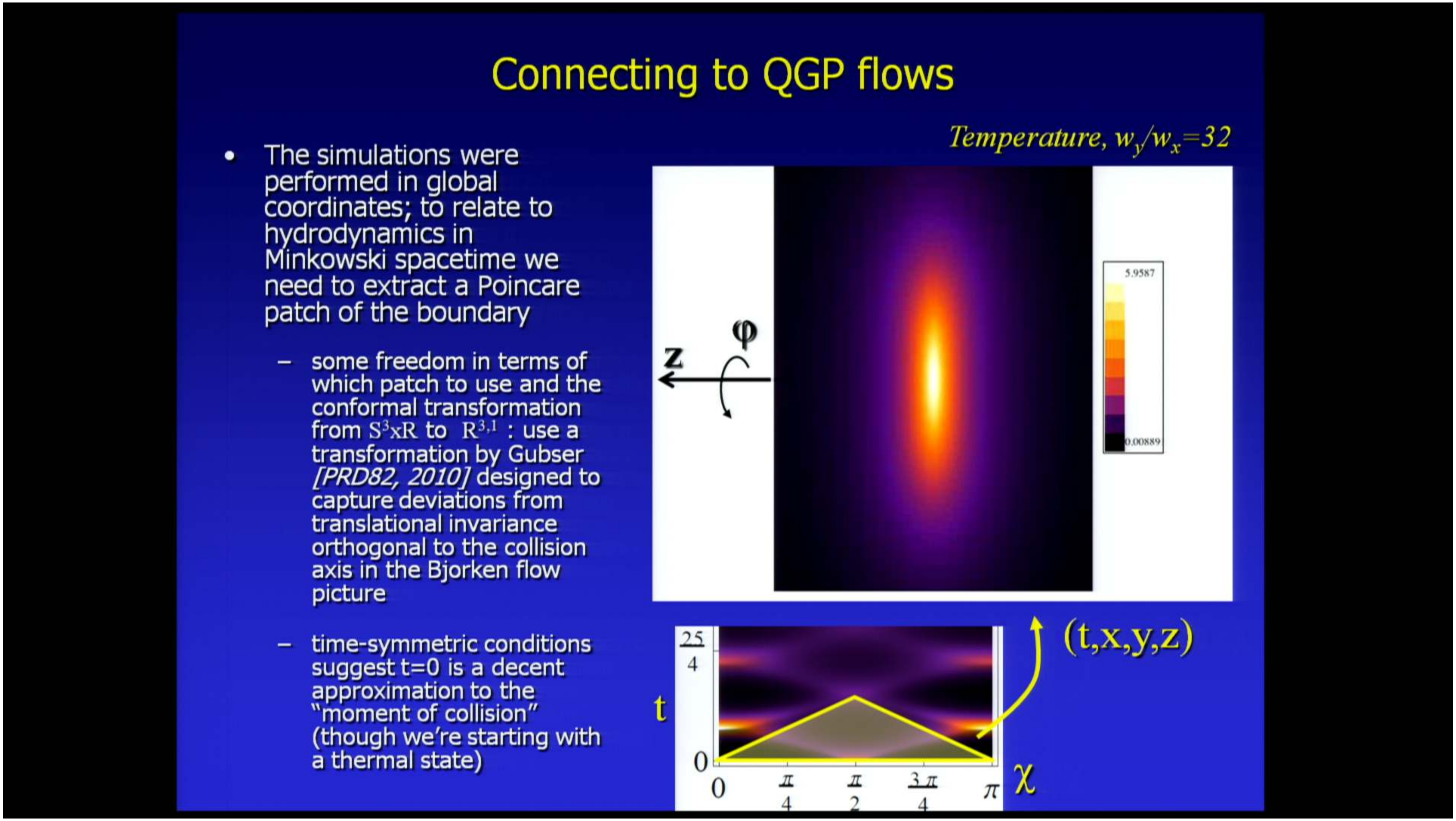
Hydrodynamics of the boundary SET

- Correspondence suggests if the bulk is dual to a thermal state on the boundary, the boundary SET should behave like a $\mathcal{N}=4$ *SYM* conformal fluid

$$T_{\mu\nu} = \sum_{i=0}^{\infty} T_{\mu\nu}^{(i)}$$

where up to 2nd order in a derivative expansion of the fluid velocity

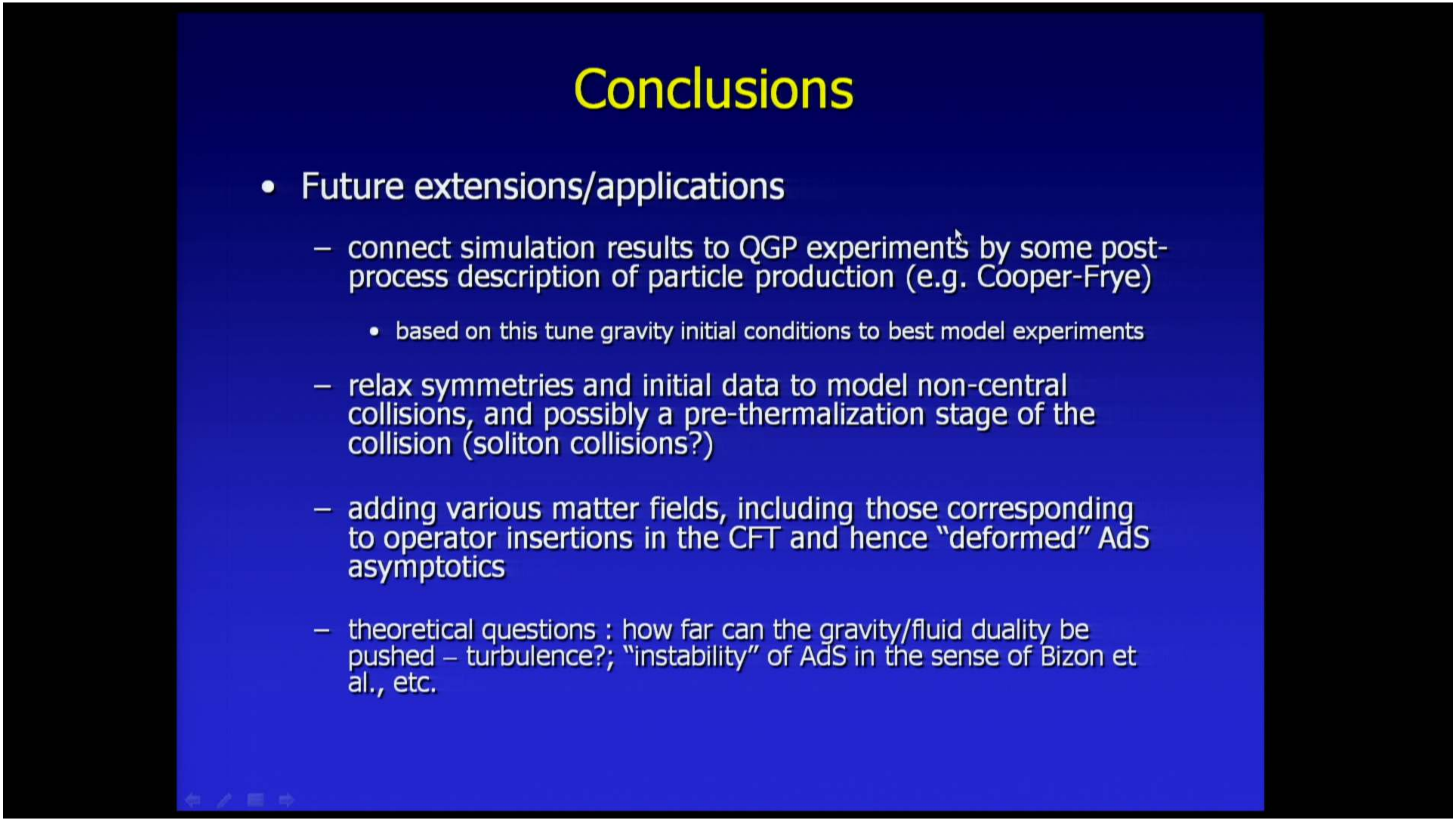
$$\begin{aligned}
T_{\mu\nu}^{(0)} &= \epsilon u_\mu u_\nu + P \perp_{\mu\nu} \\
T_{\mu\nu}^{(1)} &= -2\eta\sigma_{\mu\nu} \\
T_{\mu\nu}^{(2)} &= -2\eta\left[-\tau_\pi u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \tau_\omega\left(\omega_\mu{}^\lambda \sigma_{\lambda\nu} + \omega_\nu{}^\lambda \sigma_{\lambda\mu}\right)\right] \\
&\quad + \xi_\sigma\left[\sigma_\mu{}^\lambda \sigma_{\lambda\nu} - \frac{\perp_{\mu\nu}}{3}\sigma^{\alpha\beta}\sigma_{\alpha\beta}\right] + \xi_C C_{\mu\alpha\nu\beta}u^\alpha u^\beta
\end{aligned}$$



Connecting to QGP flows

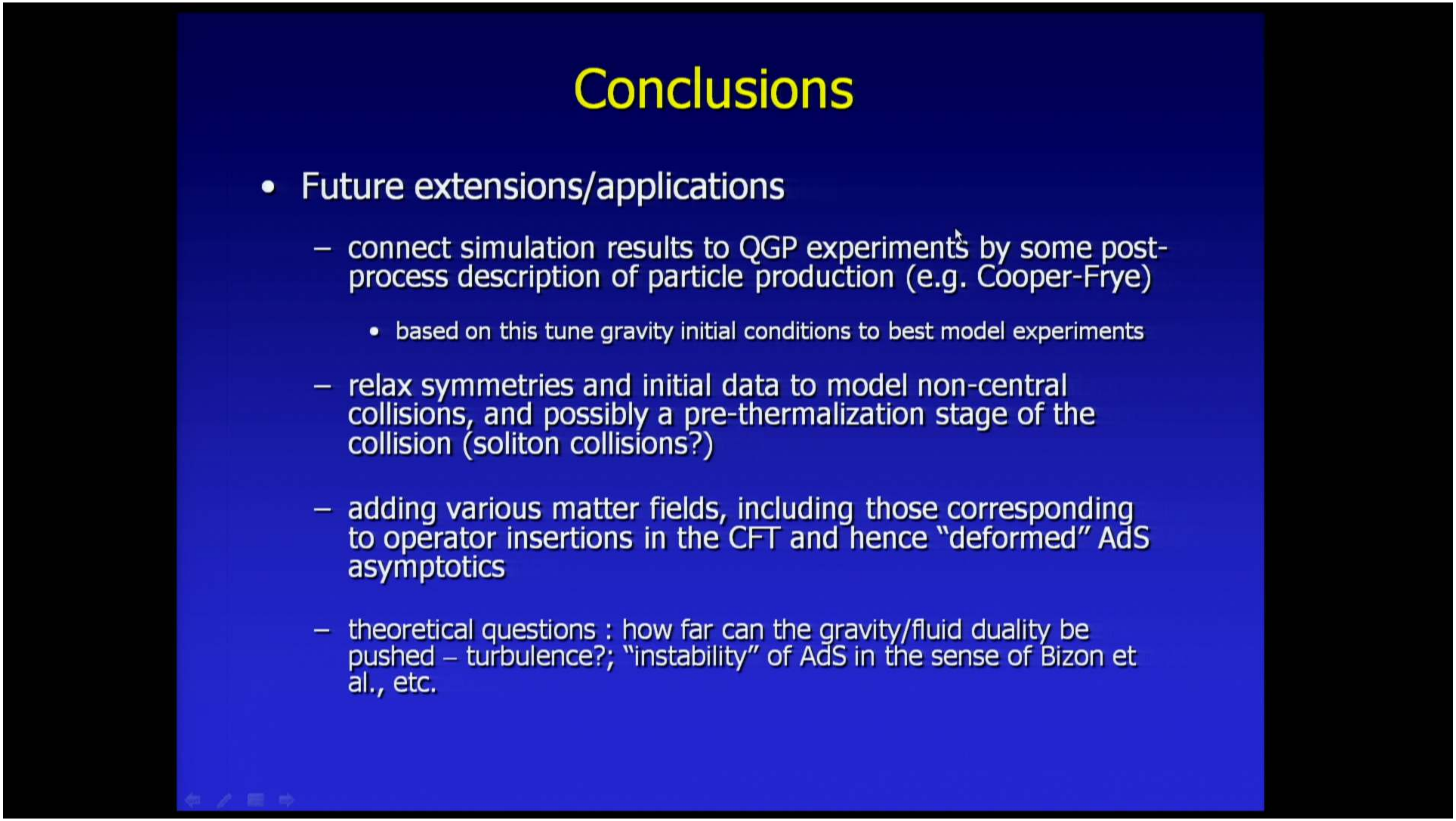
- The simulations were performed in global coordinates; to relate to hydrodynamics in Minkowski spacetime we need to extract a Poincare patch of the boundary
  - some freedom in terms of which patch to use and the conformal transformation from $S^3 \times R$ to $R^{3,1}$: use a transformation by Gubser *[PRD82, 2010]* designed to capture deviations from translational invariance orthogonal to the collision axis in the Bjorken flow picture
  - time-symmetric conditions suggest t=0 is a decent approximation to the "moment of collision" (though we're starting with a thermal state)

Temperature, $w_y/w_x=32$



Conclusions

- Future extensions/applications
  - connect simulation results to QGP experiments by some post-process description of particle production (e.g. Cooper-Frye)
    - based on this tune gravity initial conditions to best model experiments
  - relax symmetries and initial data to model non-central collisions, and possibly a pre-thermalization stage of the collision (soliton collisions?)
  - adding various matter fields, including those corresponding to operator insertions in the CFT and hence "deformed" AdS asymptotics
  - theoretical questions : how far can the gravity/fluid duality be pushed – turbulence?; "instability" of AdS in the sense of Bizon et al., etc.



Conclusions

- Future extensions/applications
  - connect simulation results to QGP experiments by some post-process description of particle production (e.g. Cooper-Frye)
    - based on this tune gravity initial conditions to best model experiments
  - relax symmetries and initial data to model non-central collisions, and possibly a pre-thermalization stage of the collision (soliton collisions?)
  - adding various matter fields, including those corresponding to operator insertions in the CFT and hence "deformed" AdS asymptotics
  - theoretical questions : how far can the gravity/fluid duality be pushed – turbulence?; "instability" of AdS in the sense of Bizon et al., etc.