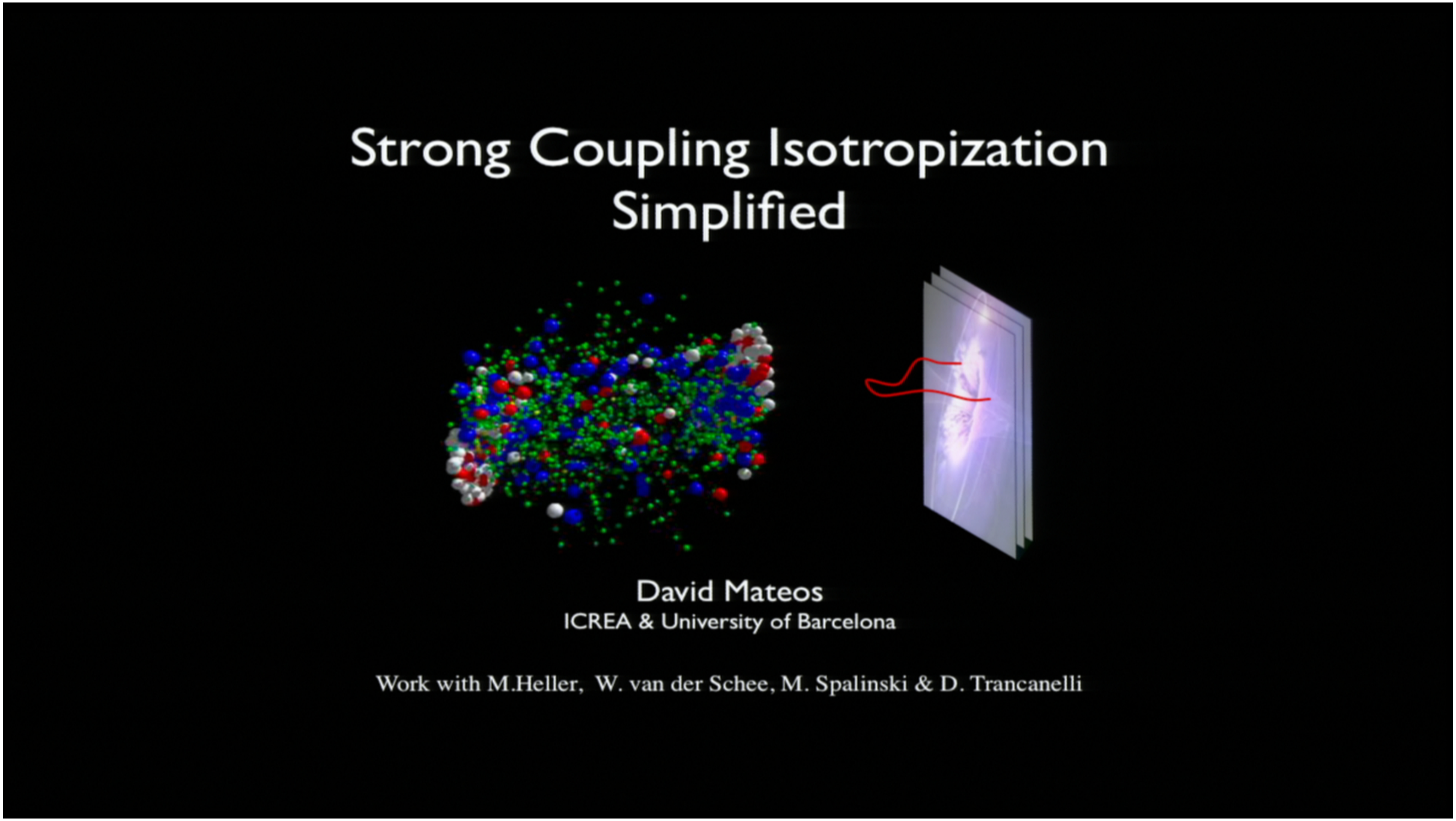
Title: Strong Coupling Isotropization Simplified

Date: Jun 04, 2012 09:00 AM

URL: http://pirsa.org/12060013

Abstract: We study the isotropization of a homogeneous, strongly coupled, non-Abelian plasma by means of its gravity dual. We compare the time evolution of a large number of initially anisotropic states as determined, on the one hand, by the full non-linear Einstein's equations and, on the other, by the Einstein's equations linearized around the final equilibrium state. The linear approximation works remarkably well even for states that exhibit large anisotropies. For example, it predicts with a 20% accuracy the isotropization time, which is of order 1/T, with T the final equilibrium temperature. We comment on possible extensions to less symmetric situations.

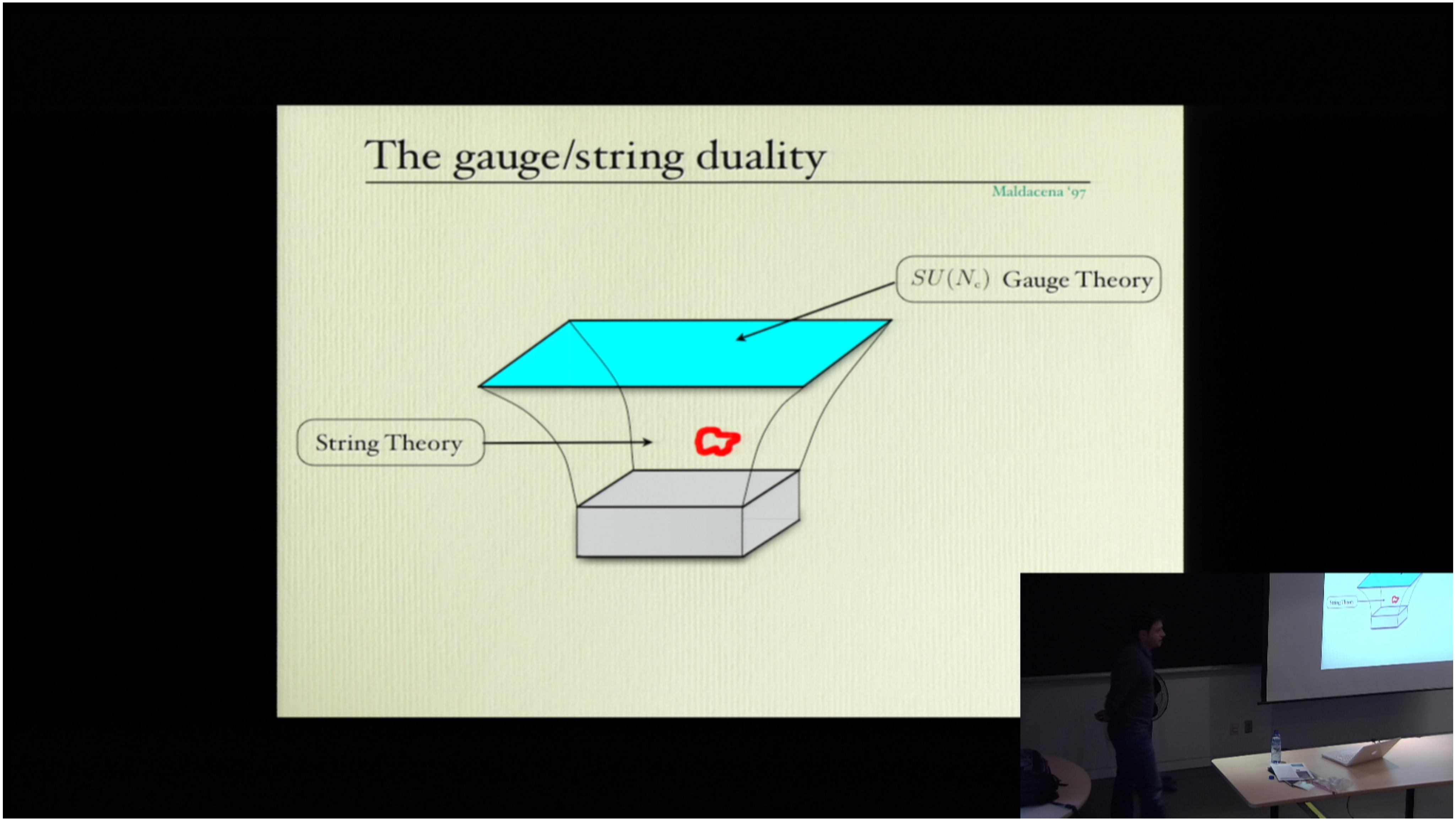


Strong Coupling Isotropization Simplified

David Mateos

ICREA & University of Barcelona

Work with M.Heller, W. van der Schee, M. Spalinski & D. Trancanelli



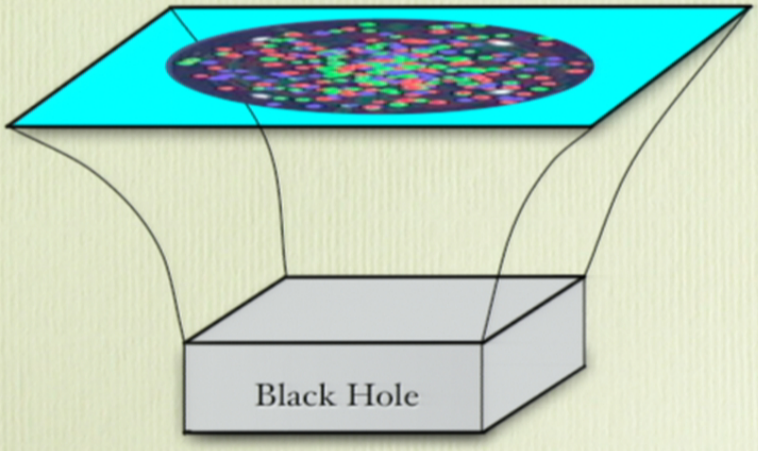
The gauge/string duality

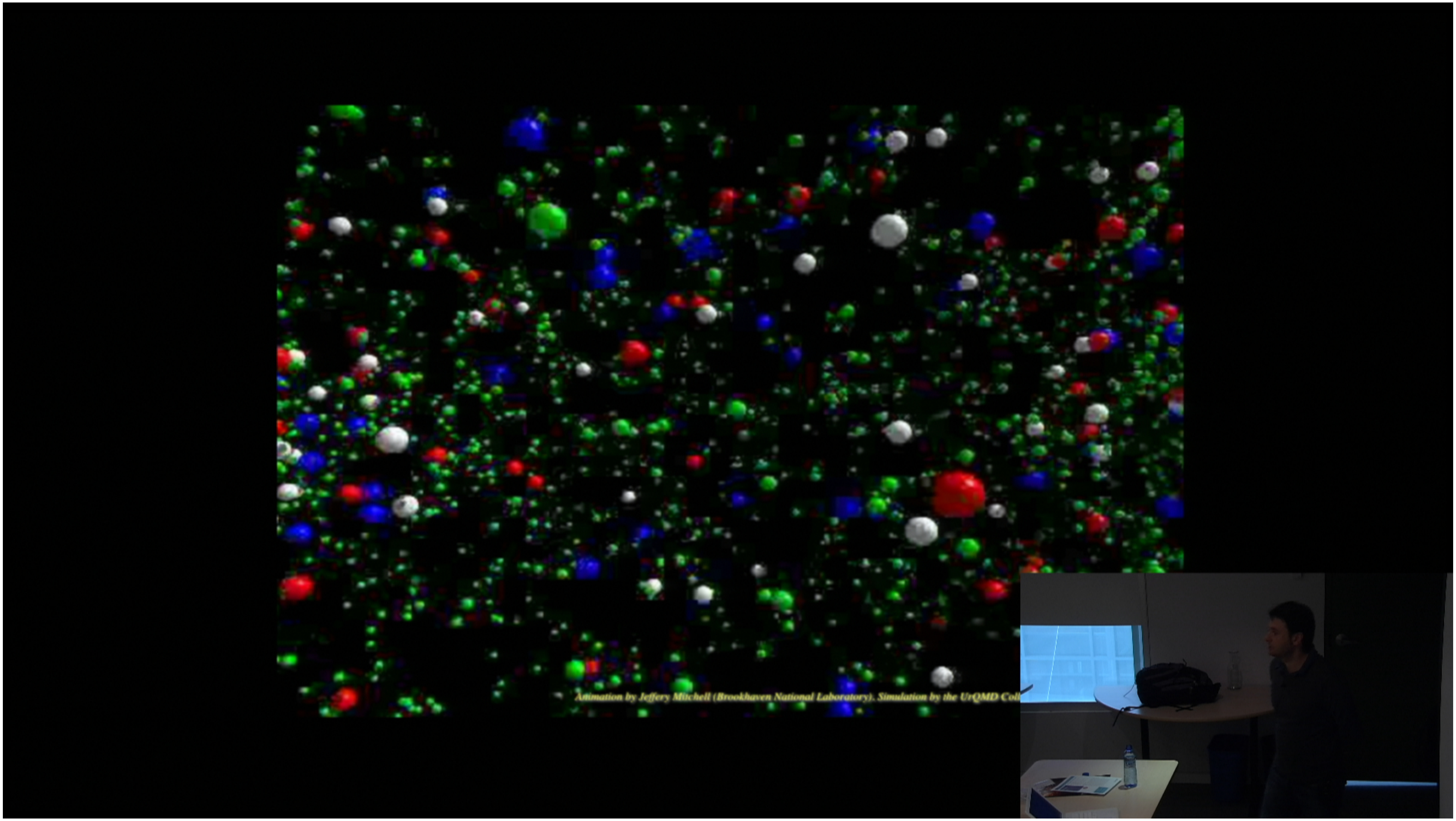
Maldacena '97

$SU(N_c)$ Gauge Theory

String Theory

Deconfinement (QGP) = Black Hole





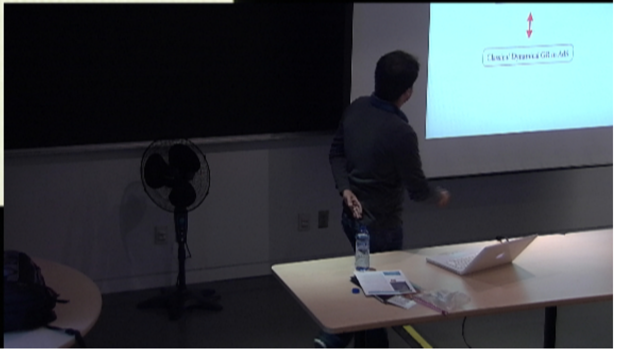
Animation by Jeffery Mitchell (Brookhaven National Laboratory). Simulation by the UrQMD Coll

Out of equilibrium

Out-of-equilibrium QFT

↕

Classical Dynamical GR in AdS



Remember

- QCD dual is beyond supergravity.

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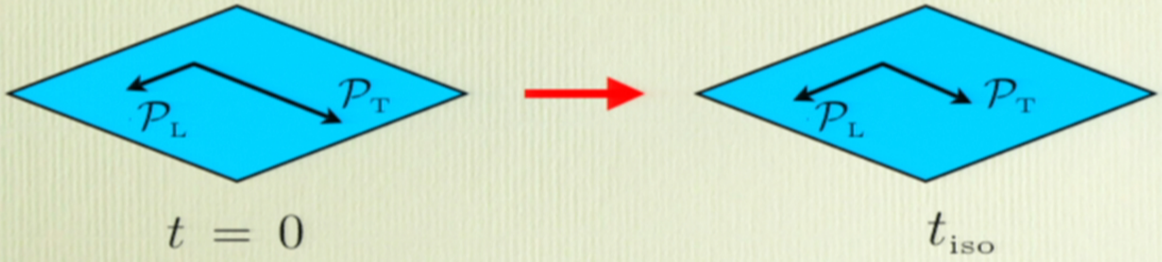
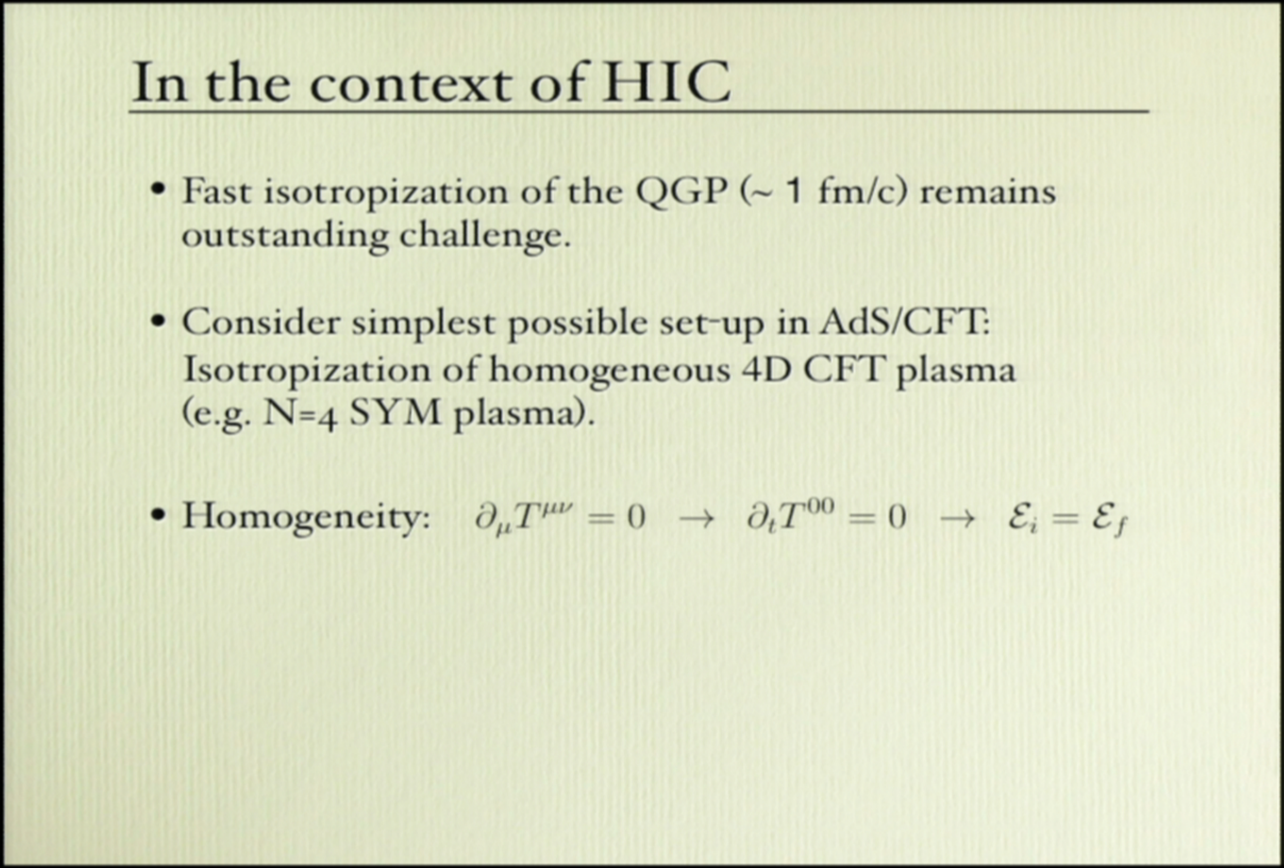
- QCD dual is beyond supergravity.
- Do not try to do *precision* QCD physics.
- Search for physical insights.

In the context of HIC

- Fast isotropization of the QGP (~ 1 fm/c) remains outstanding challenge.
- Consider simplest possible set-up in AdS/CFT: Isotropization of homogeneous 4D CFT plasma (e.g. N=4 SYM plasma).





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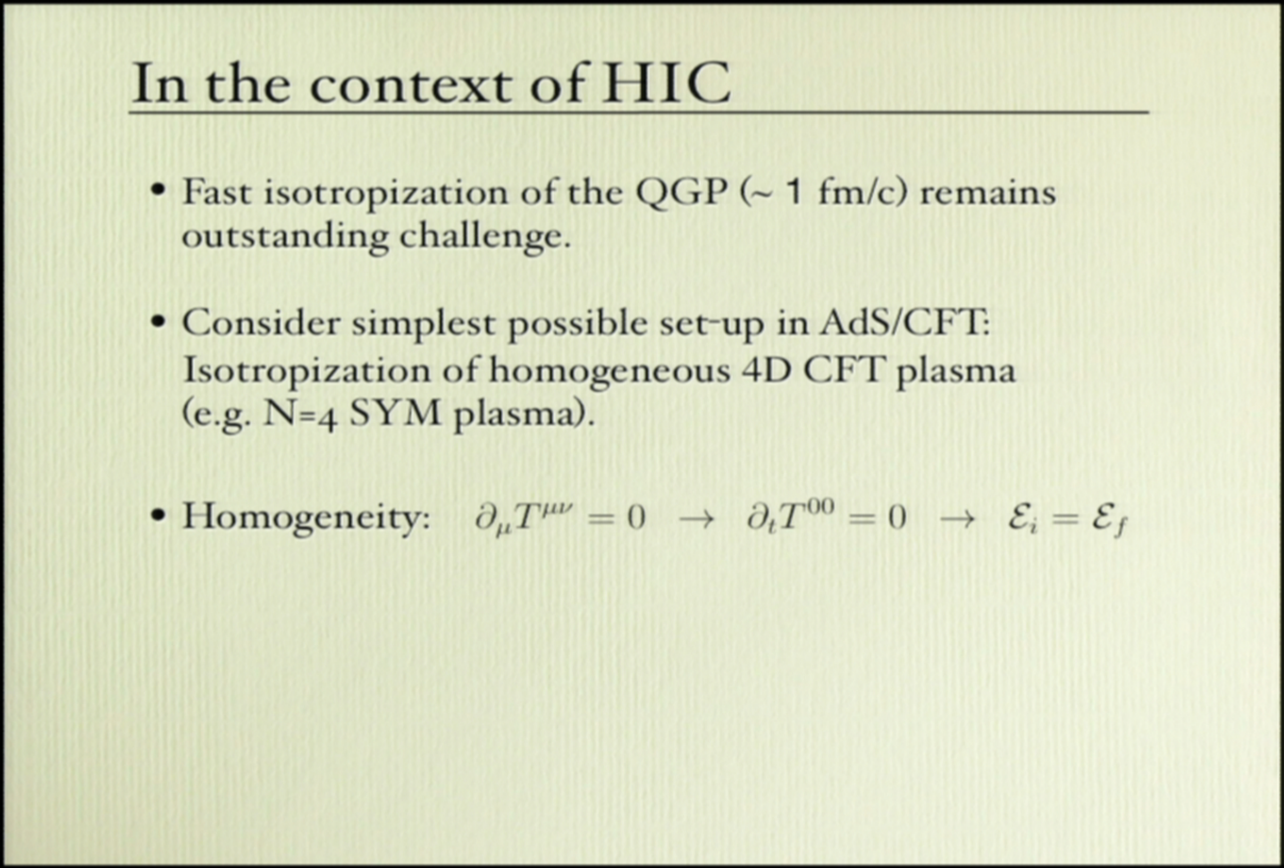

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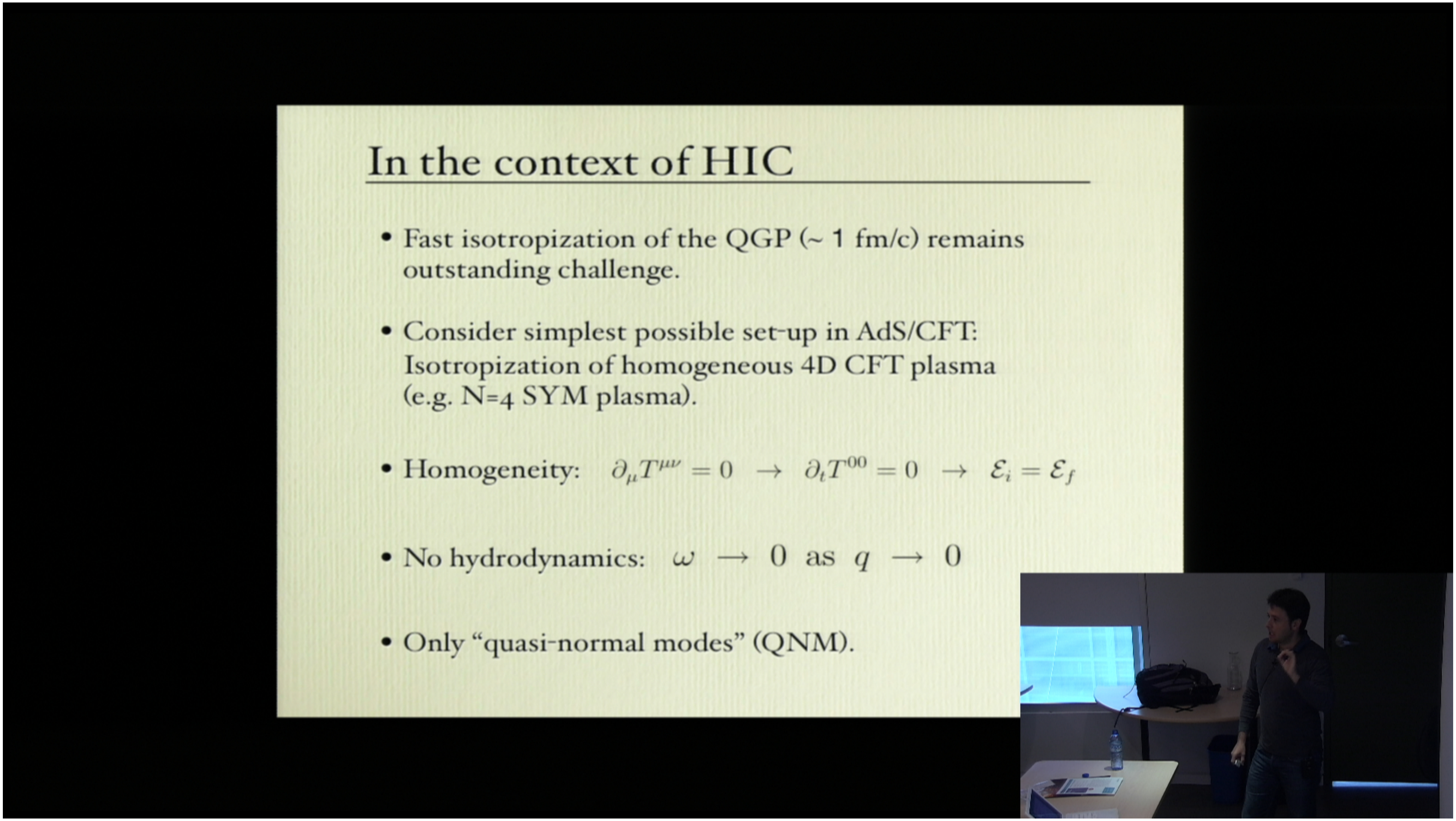

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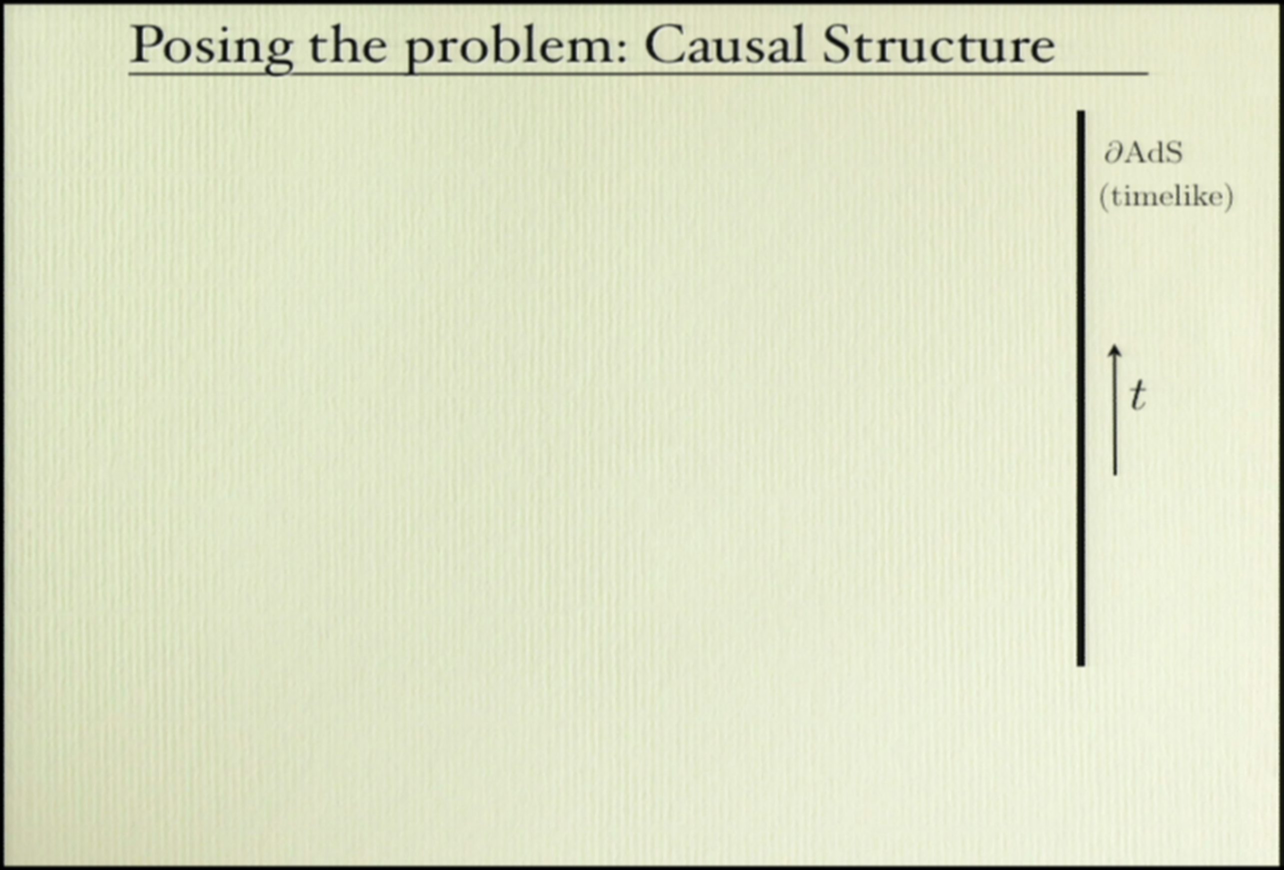
- Homogeneity: $\partial_\mu T^{\mu\nu} = 0 \;\rightarrow\; \partial_t T^{00} = 0 \;\rightarrow\; \mathcal{E}_i = \mathcal{E}_f$
- No hydrodynamics: $\omega \;\longrightarrow\; 0$ as $q \;\longrightarrow\; 0$
- Only "quasi-normal modes" (QNM).



Posing the problem: Causal Structure

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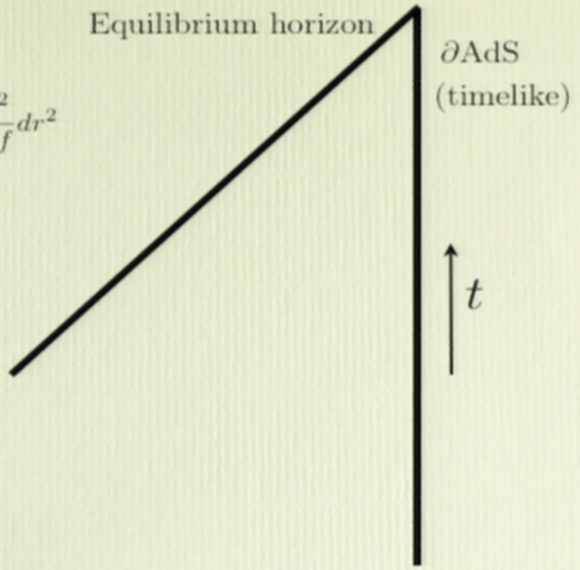
$$ds^2 = \frac{r^2}{R^2}\left(-f dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{R^2}{r^2 f} dr^2$$

$$f(r) = 1 - \frac{r_0^4}{r^4}$$



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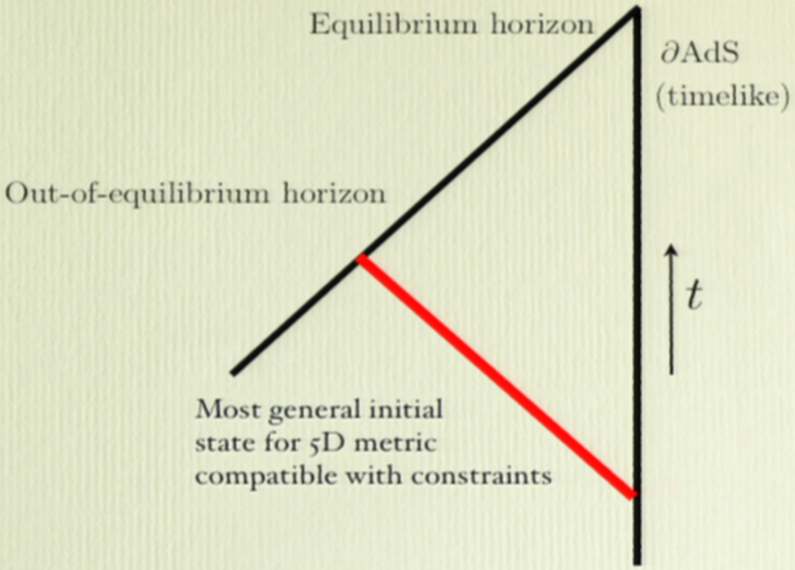


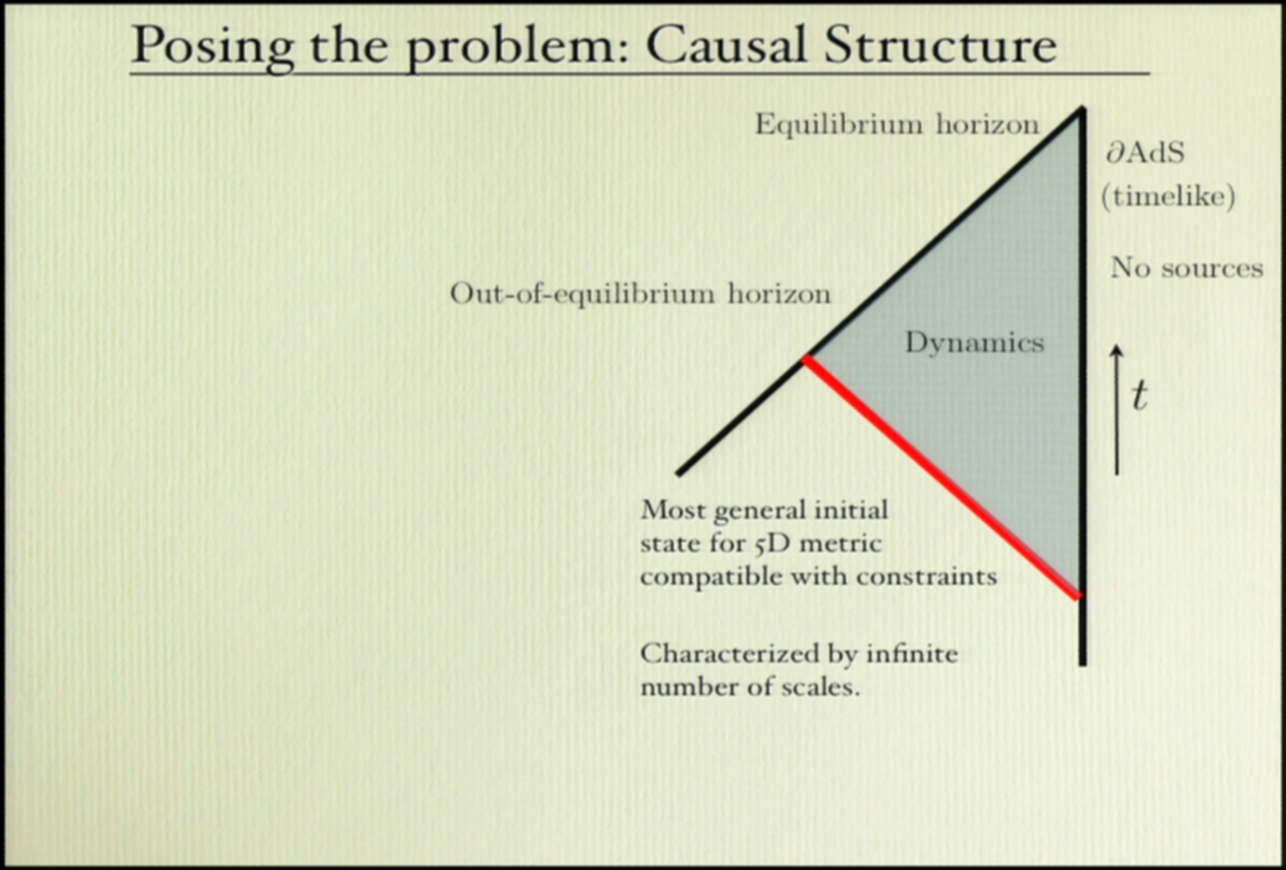
Equilibrium horizon

∂AdS
(timelike)

Out-of-equilibrium horizon

t

Most general initial
state for 5D metric
compatible with constraints



Posing the problem: Causal Structure

Equilibrium horizon

∂AdS
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No sources

Out-of-equilibrium horizon

Dynamics

t

Most general initial
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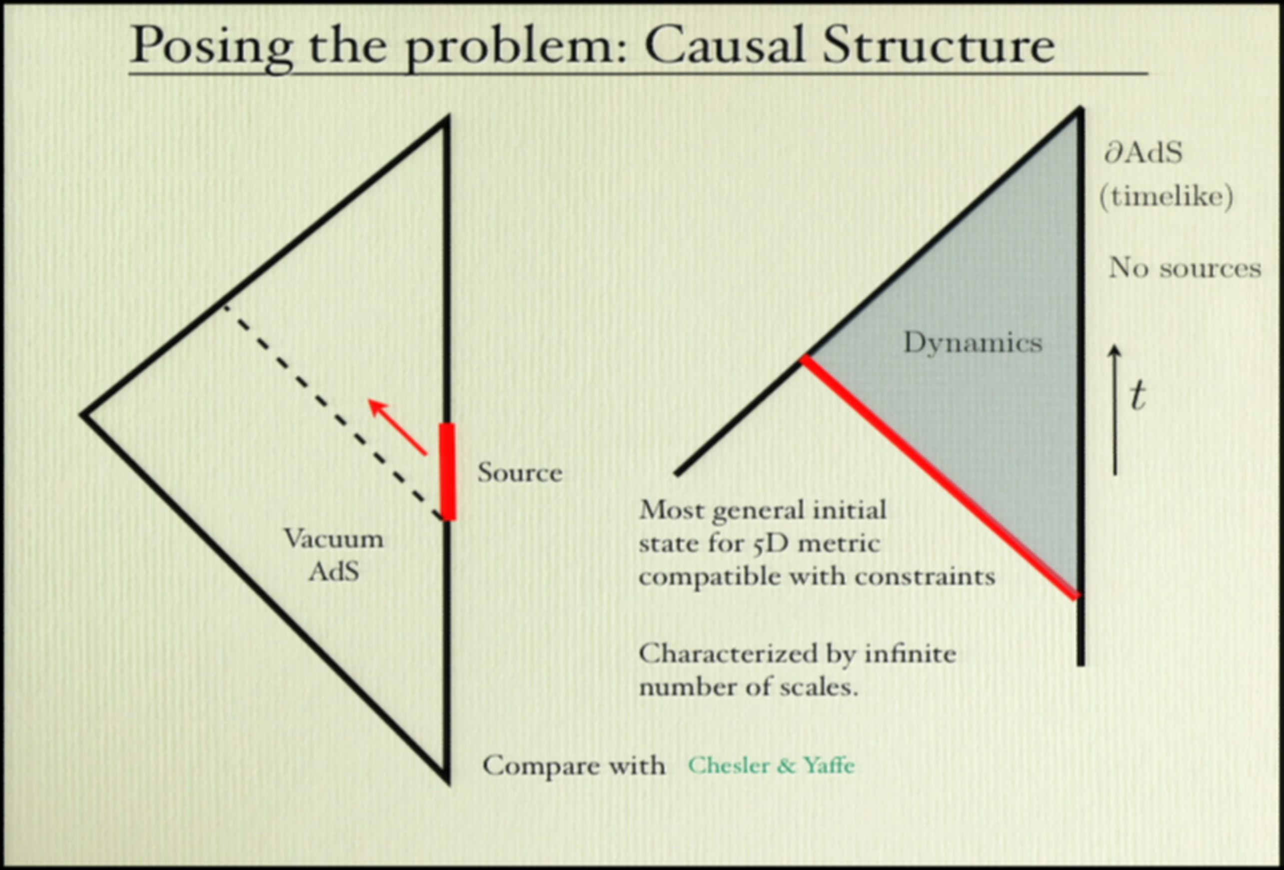
Characterized by infinite
number of scales.



Posing the problem: Causal Structure

Source

Vacuum
AdS

Compare with Chesler & Yaffe

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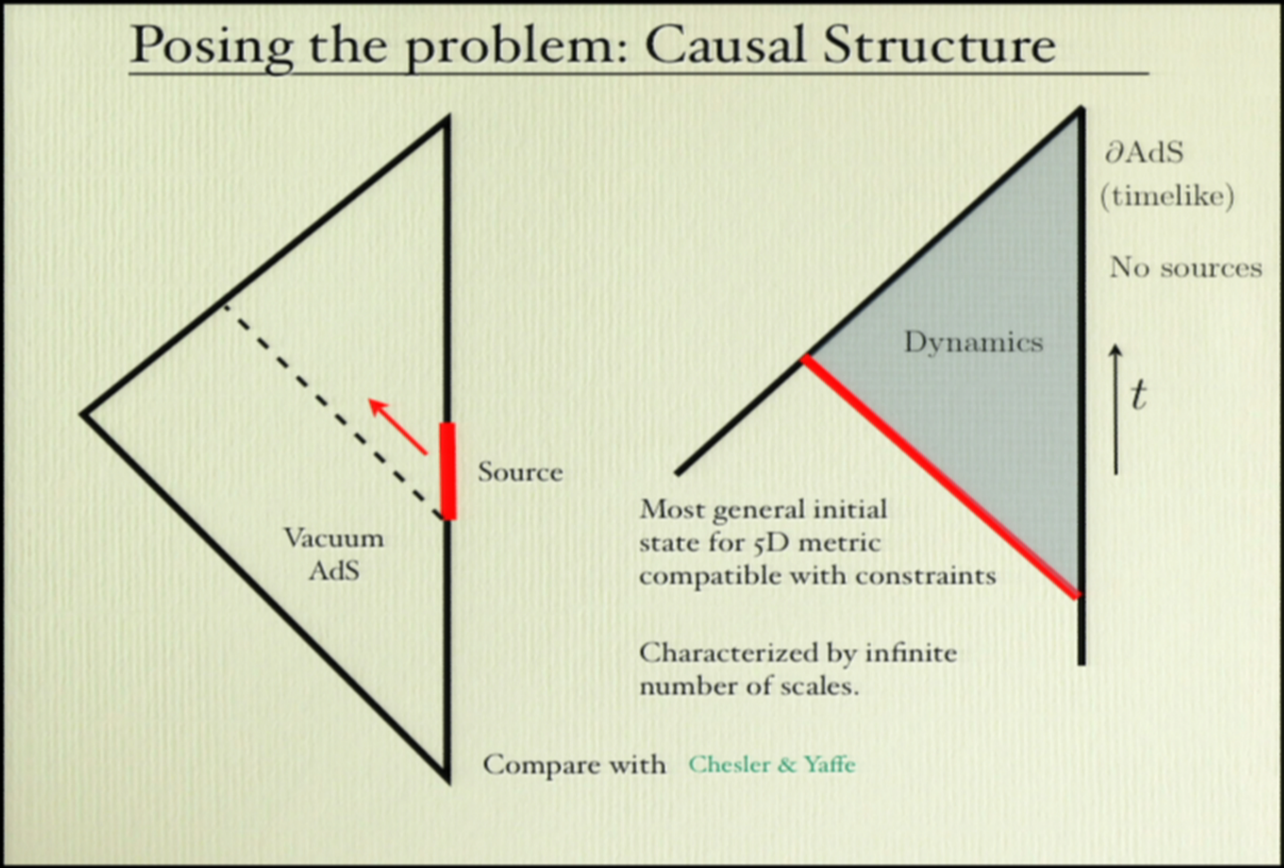

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Most general initial state for 5D metric compatible with constraints

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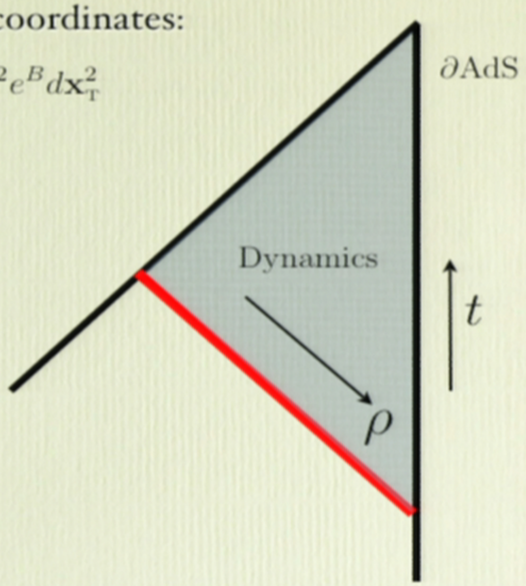
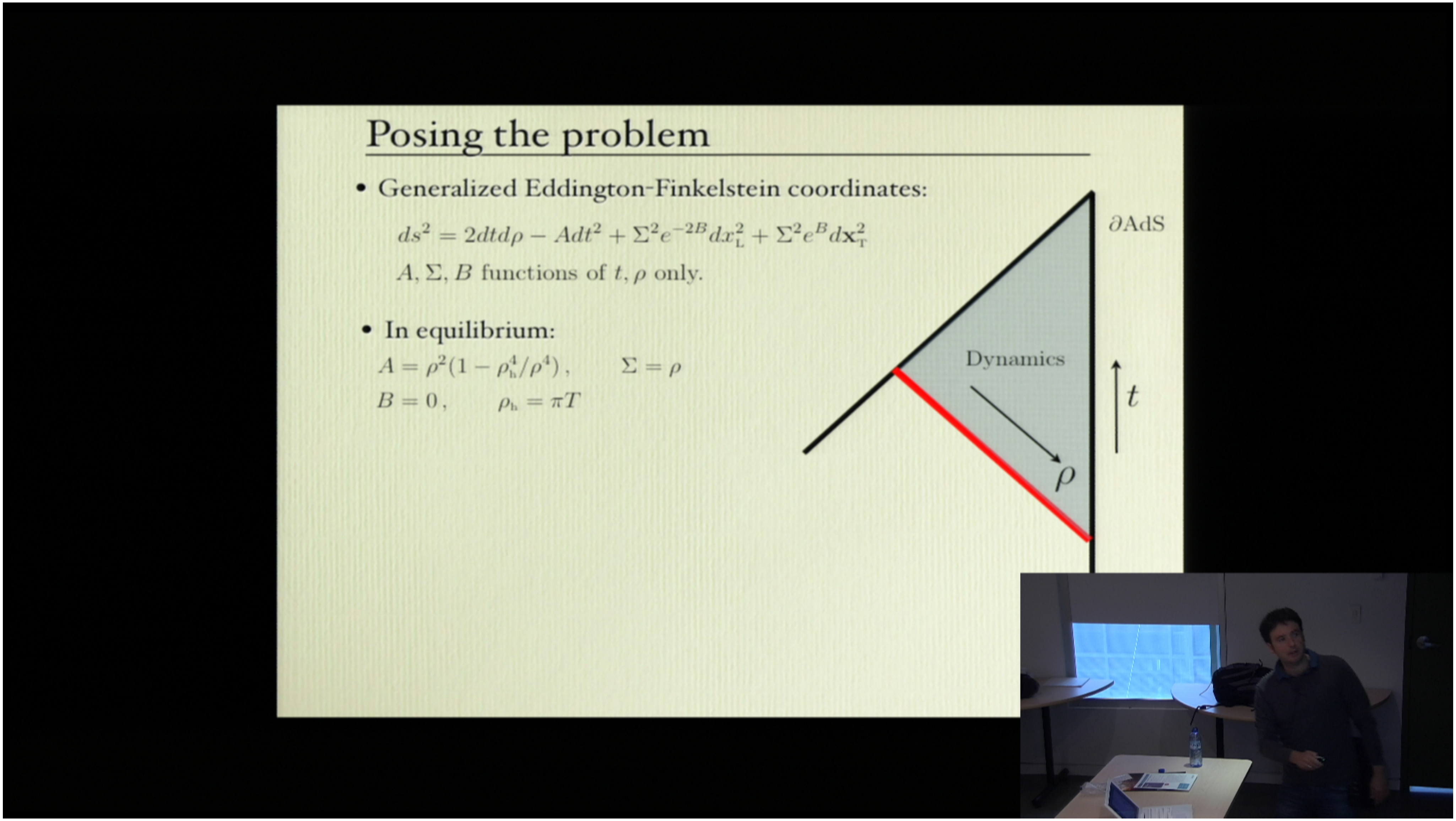
- Generalized Eddington-Finkelstein coordinates:

$$ds^2 = 2dtd\rho - Adt^2 + \Sigma^2 e^{-2B} dx_{\mathrm{L}}^2 + \Sigma^2 e^{B} d\mathbf{x}_{\mathrm{T}}^2$$

A, Σ, B functions of t, ρ only.





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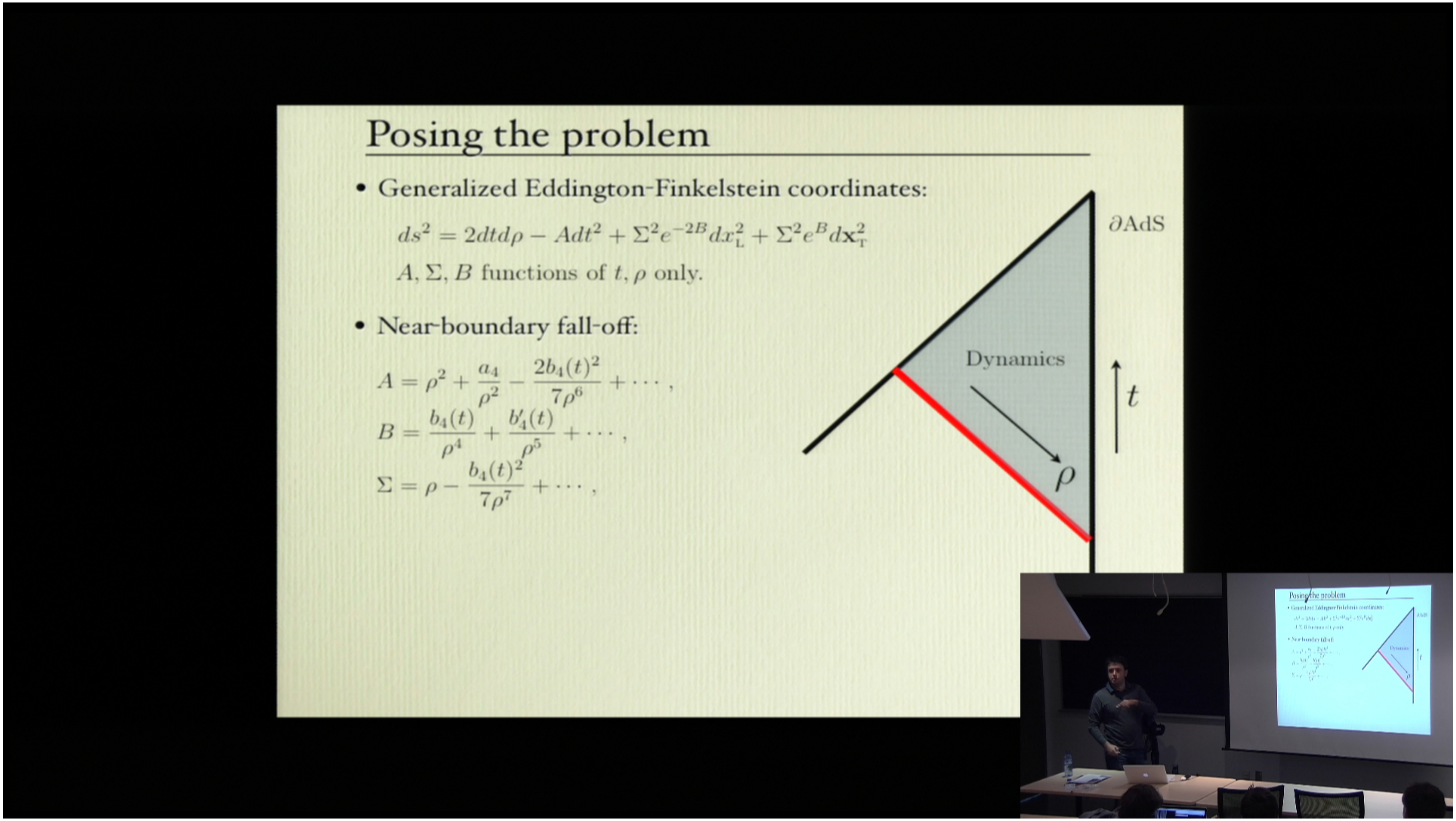

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- Near-boundary fall-off:

$$A = \rho^2 + \frac{a_4}{\rho^2} - \frac{2b_4(t)^2}{7\rho^6} + \cdots ,$$
$$B = \frac{b_4(t)}{\rho^4} + \frac{b_4'(t)}{\rho^5} + \cdots ,$$
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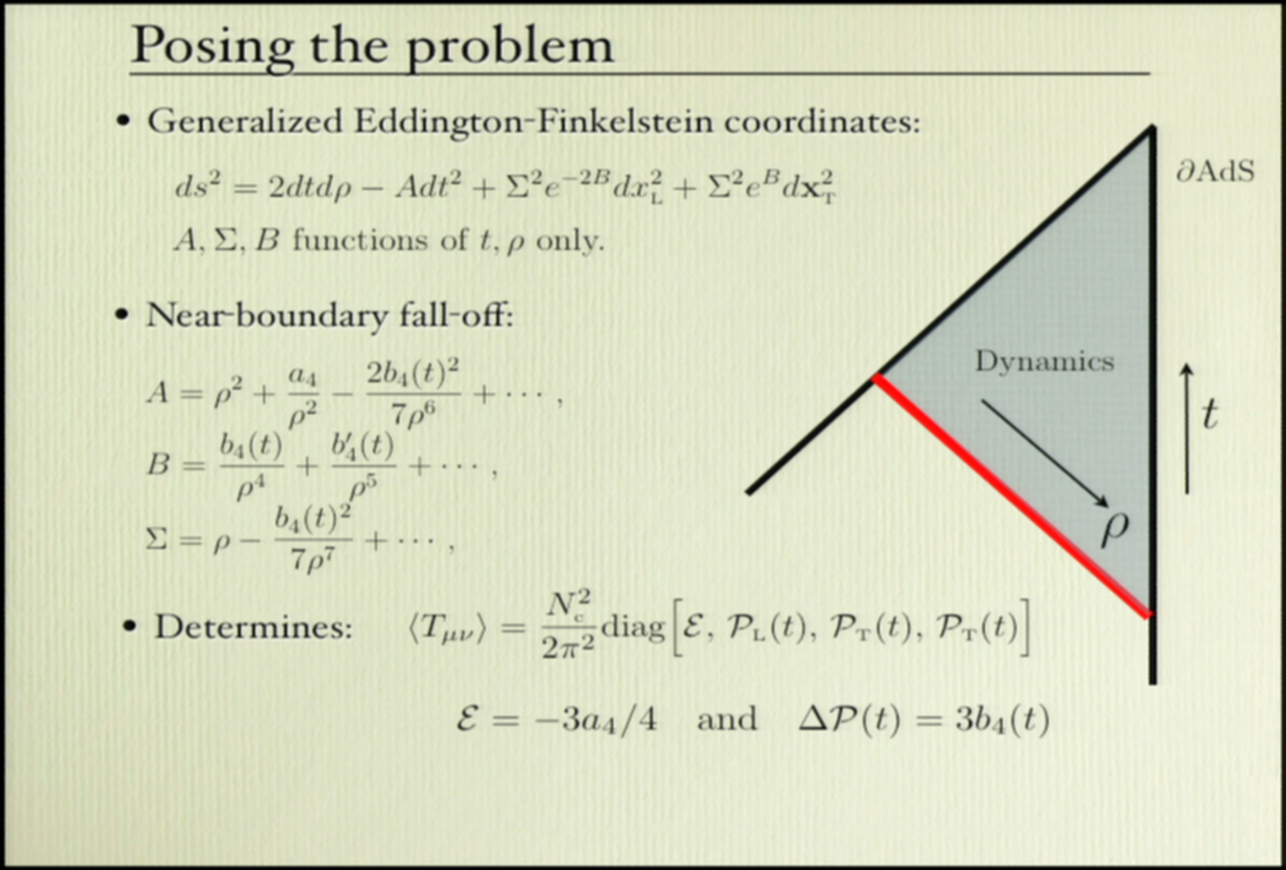

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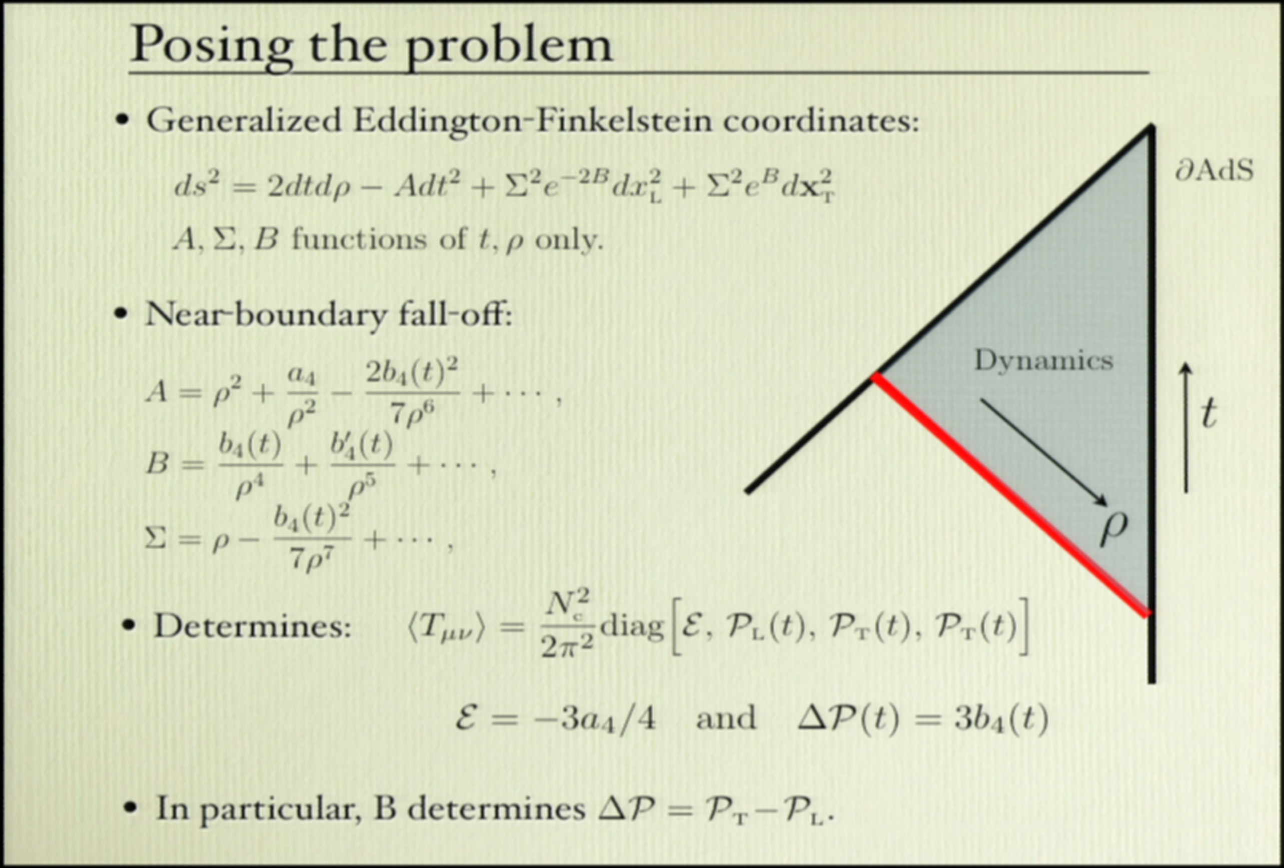
- Determines: $\langle T_{\mu\nu} \rangle = \frac{N_c^2}{2\pi^2} \mathrm{diag}\Big[\mathcal{E},\, \mathcal{P}_{\mathrm{L}}(t),\, \mathcal{P}_{\mathrm{T}}(t),\, \mathcal{P}_{\mathrm{T}}(t)\Big]$

$$\mathcal{E} = -3a_4/4 \quad \text{and} \quad \Delta\mathcal{P}(t) = 3b_4(t)$$



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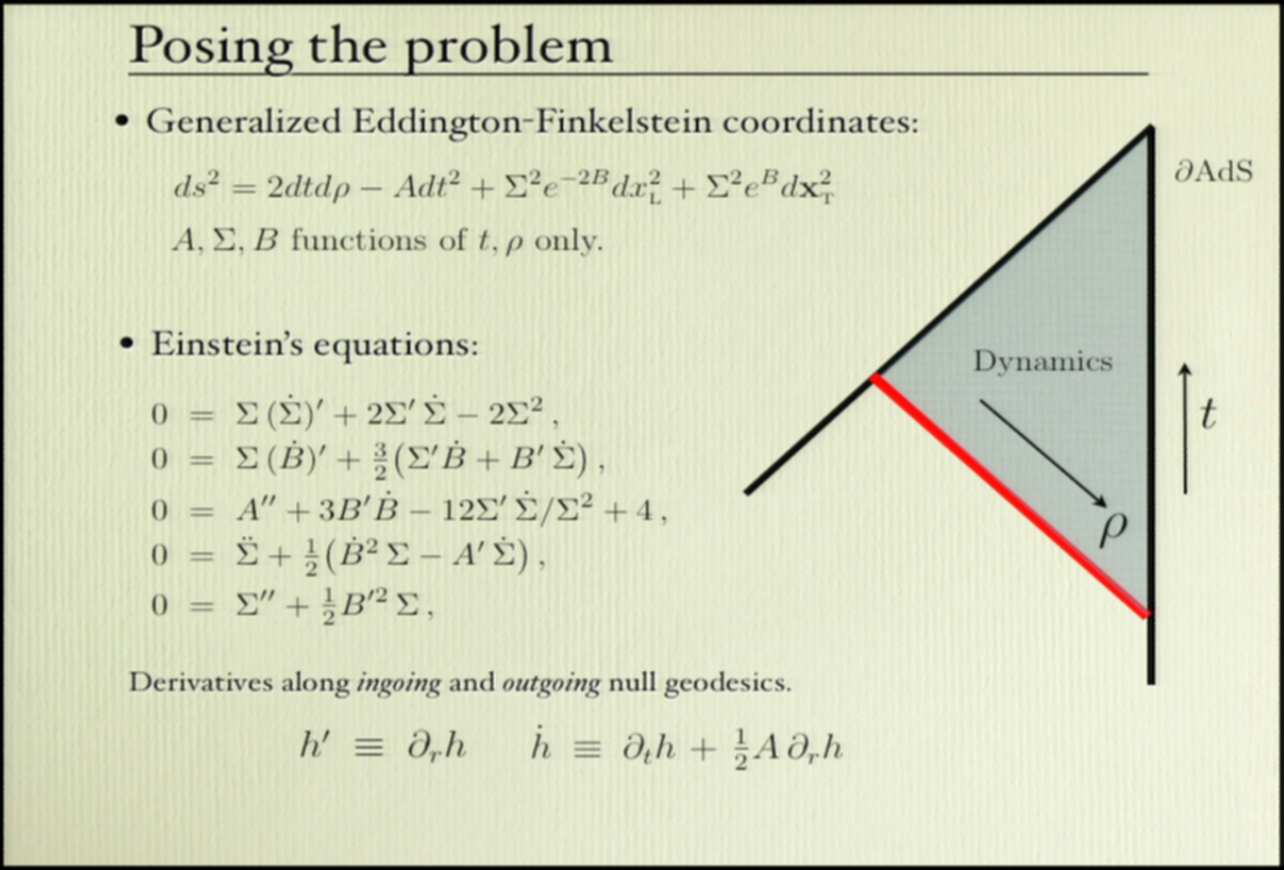
- In particular, B determines $\Delta\mathcal{P} = \mathcal{P}_{\mathrm{T}} - \mathcal{P}_{\mathrm{L}}$.



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$$\begin{aligned}
0 &= \Sigma\,(\dot{\Sigma})' + 2\Sigma'\,\dot{\Sigma} - 2\Sigma^2\,,\\
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\end{aligned}$$

Derivatives along *ingoing* and *outgoing* null geodesics.

$$h' \equiv \partial_r h \qquad \dot{h} \equiv \partial_t h + \tfrac{1}{2} A\,\partial_r h$$

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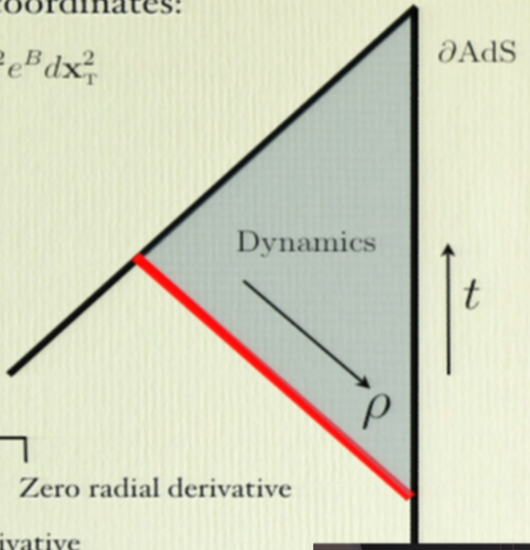

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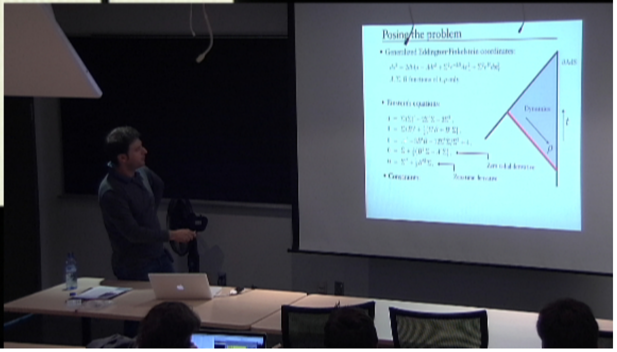
Zero radial derivative

Zero time derivative

- Constraints:



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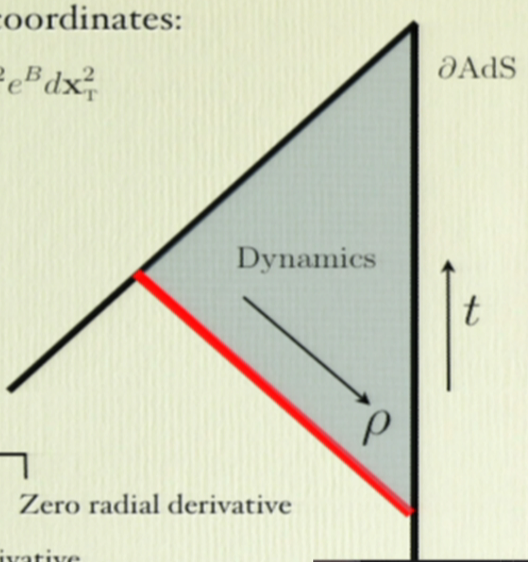

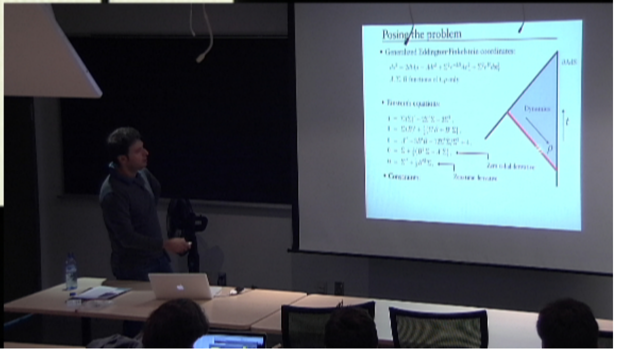

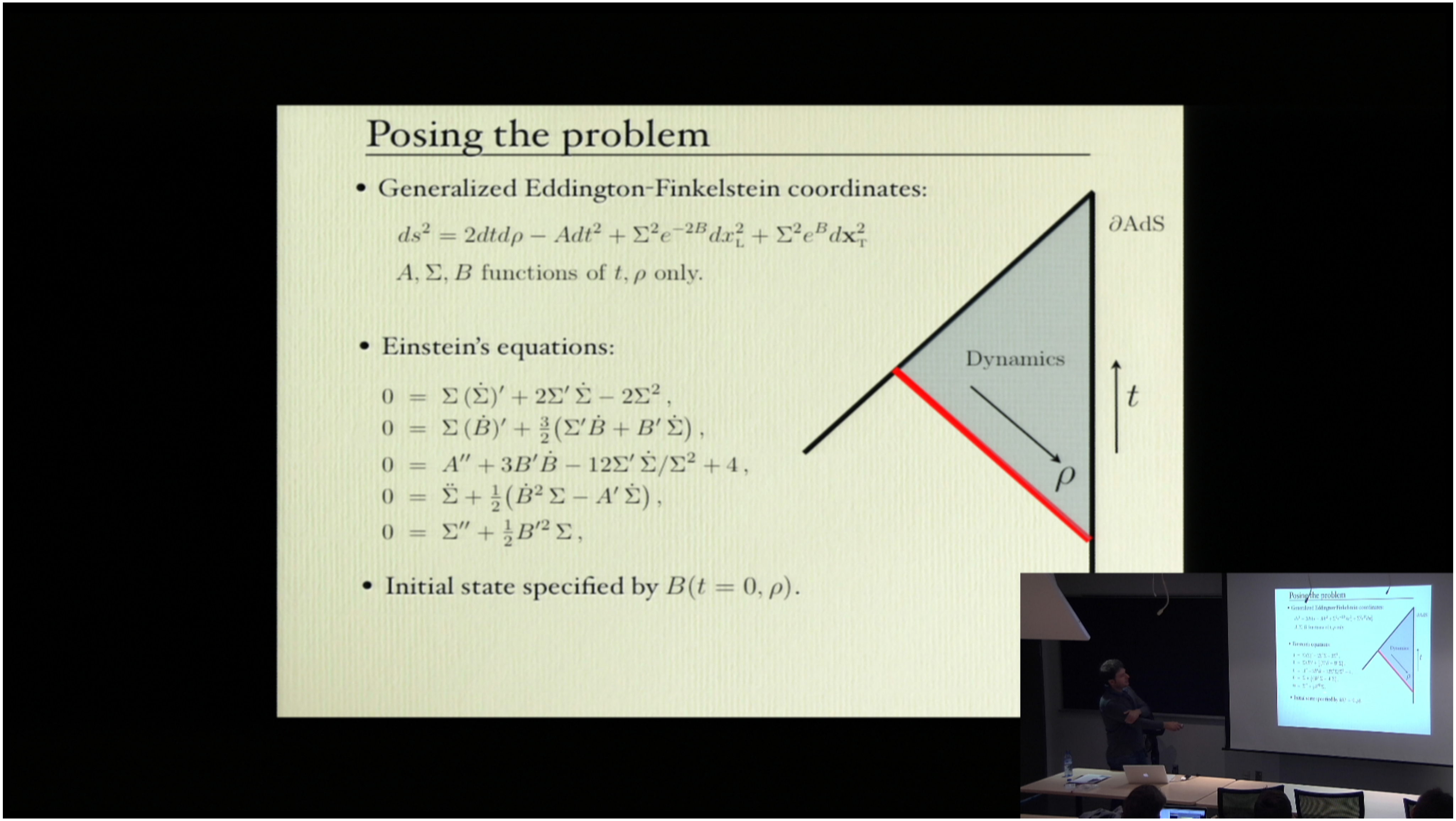

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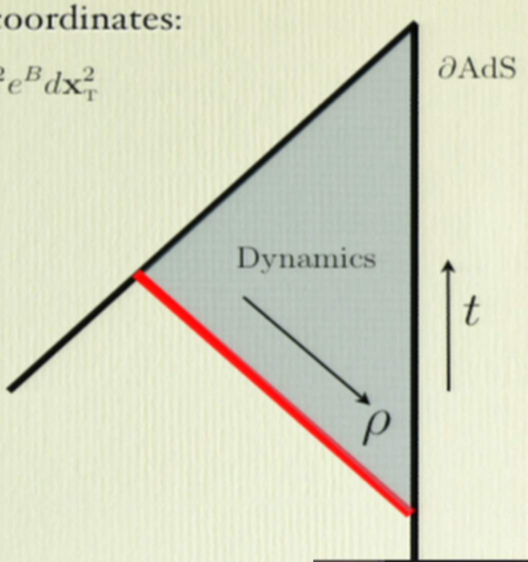

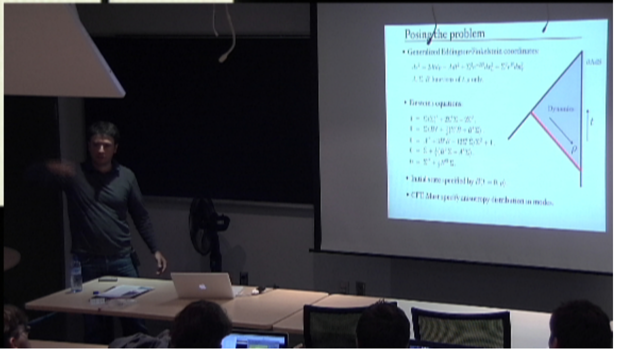

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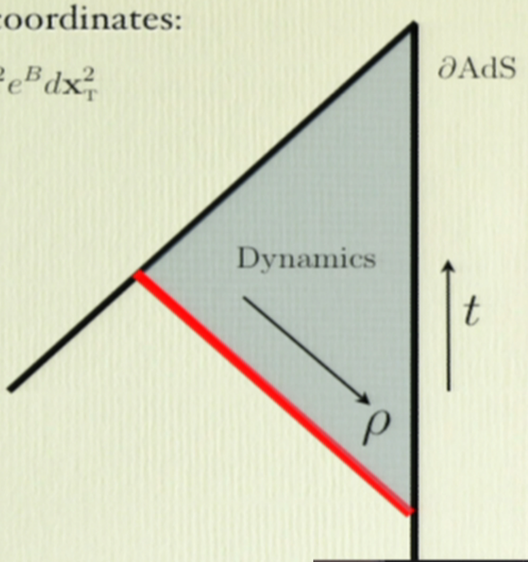
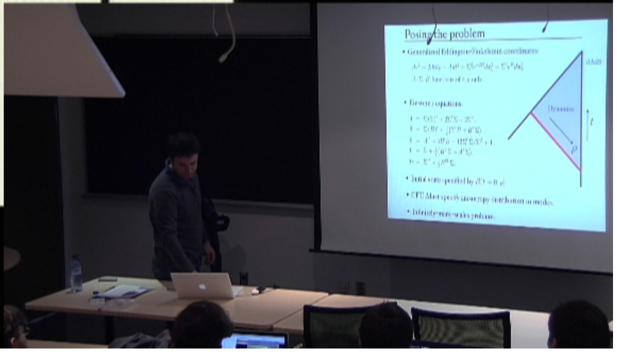
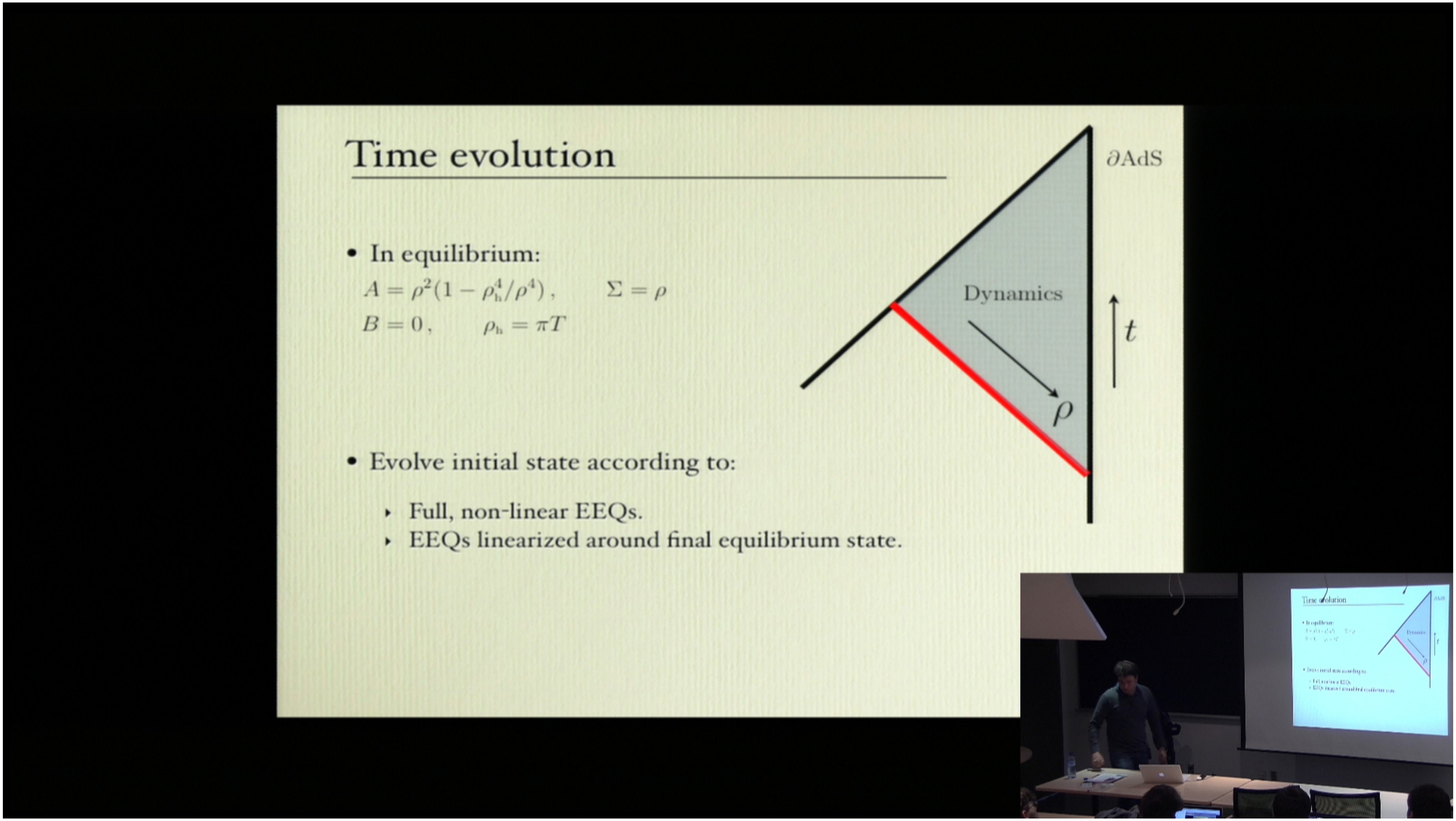
- CFT: Must specify anisotropy distribution in modes.
- Infinitely-many-scales problem.







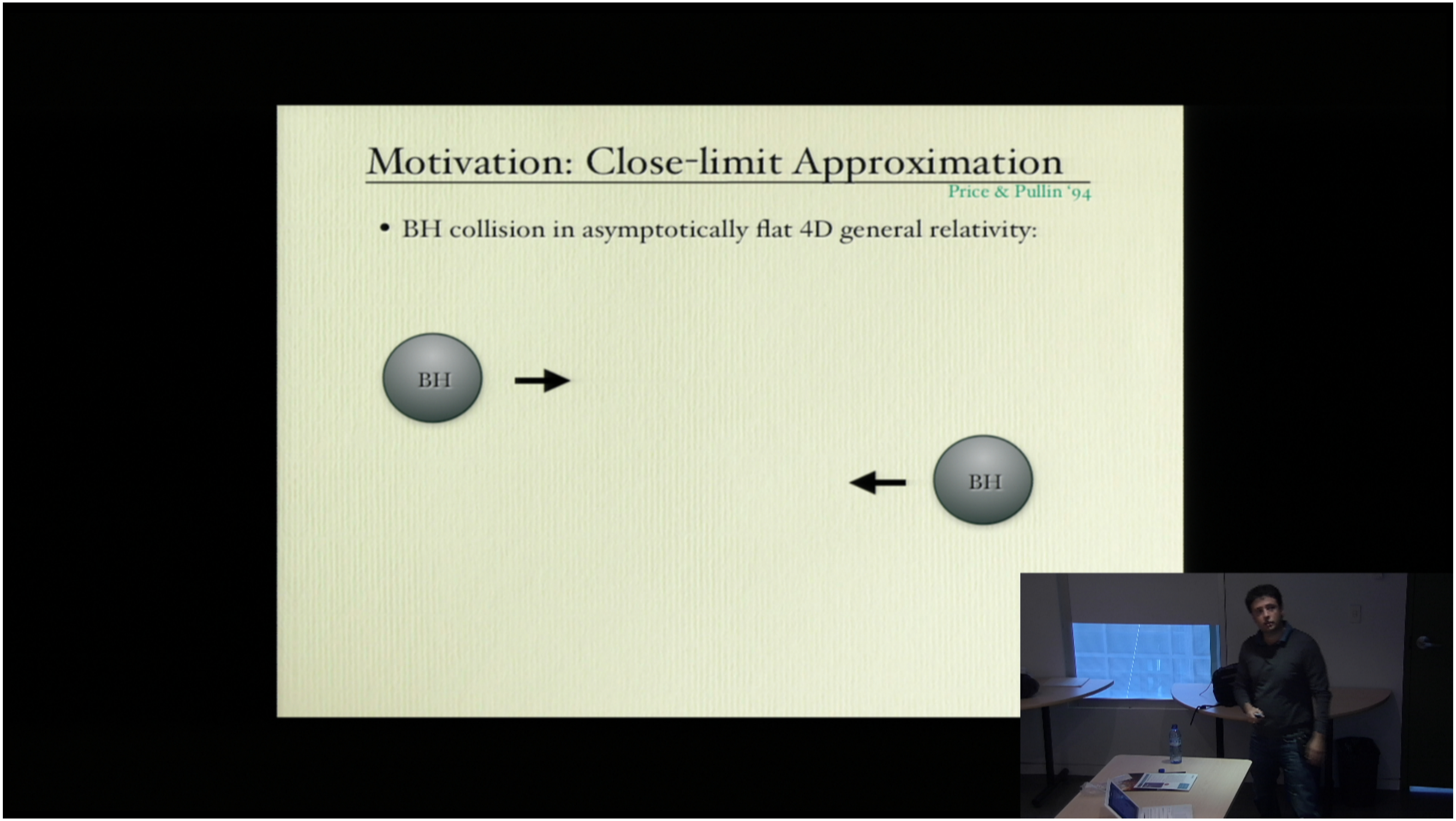
Time evolution

- In equilibrium:

$$A = \rho^2(1-\rho_{\rm h}^4/\rho^4)\,, \qquad \Sigma = \rho$$

$$B = 0\,, \qquad \rho_{\rm h} = \pi T$$

- Evolve initial state according to:
  - Full, non-linear EEQs.
  - EEQs linearized around final equilibrium state.



Motivation: Close-limit Approximation

Price & Pullin '94

- BH collision in asymptotically flat 4D general relativity:

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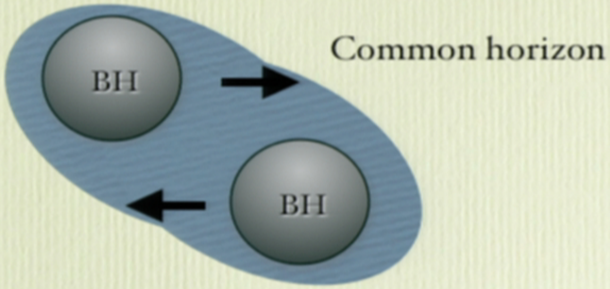

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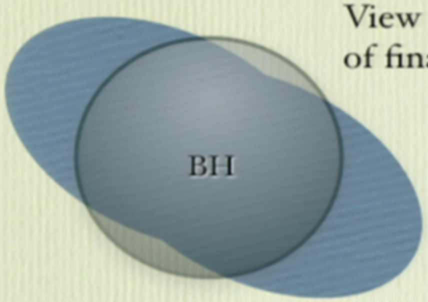

View as perturbation
of final BH

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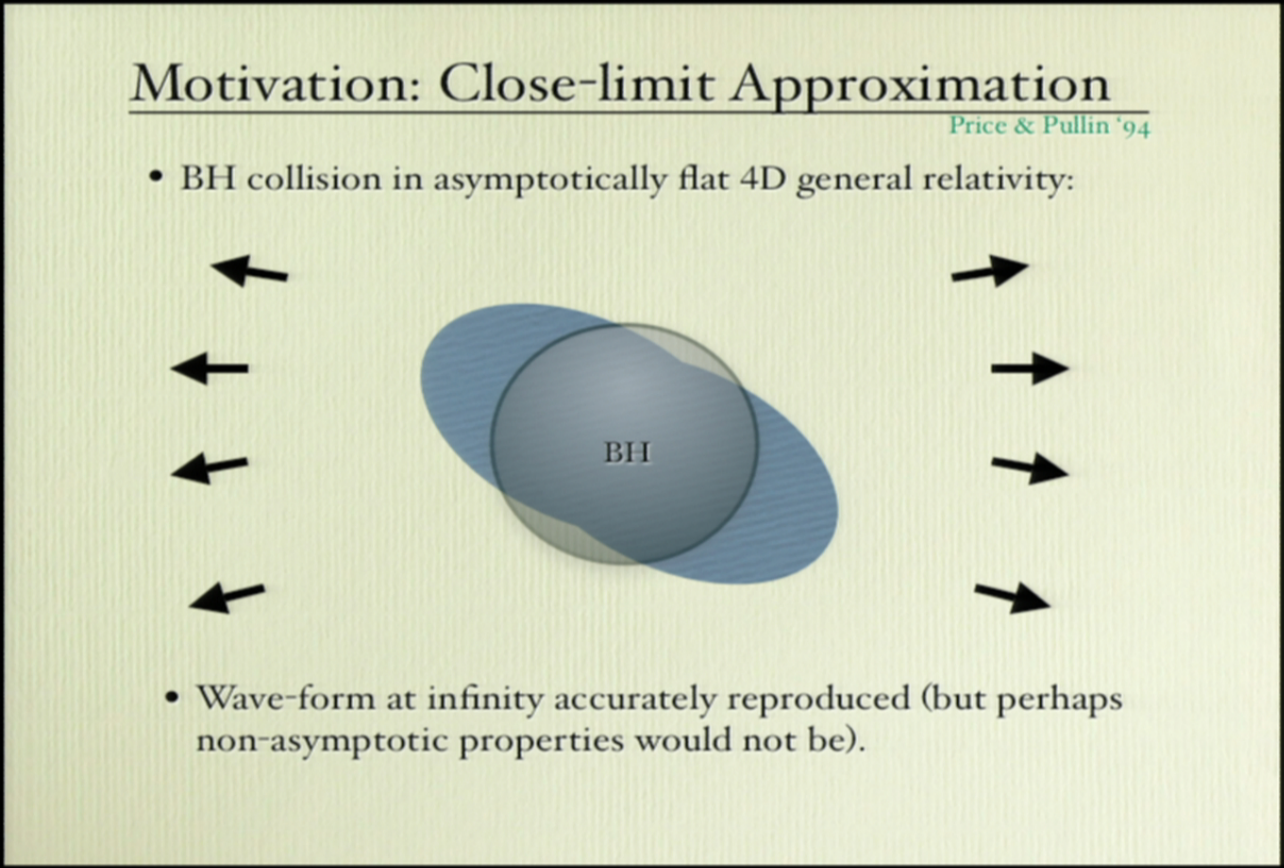
Evolve linearized EEQs



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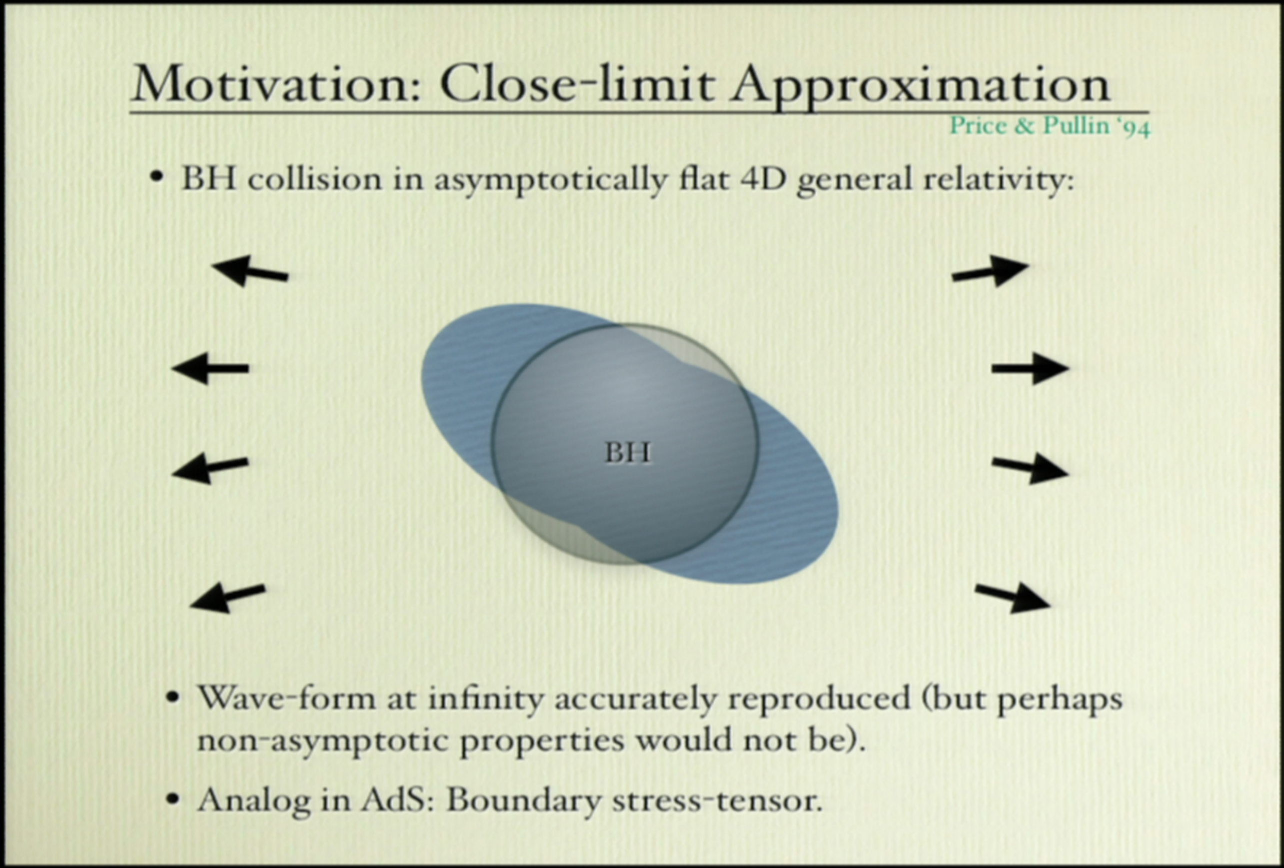
- BH collision in asymptotically flat 4D general relativity:

- Wave-form at infinity accurately reproduced (but perhaps non-asymptotic properties would not be).



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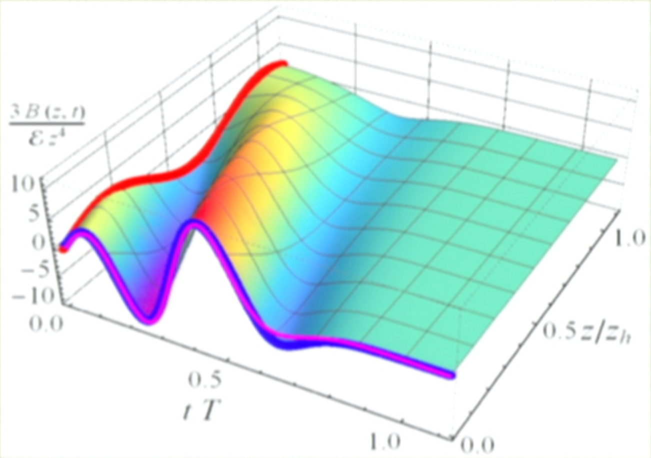
Price & Pullin '94

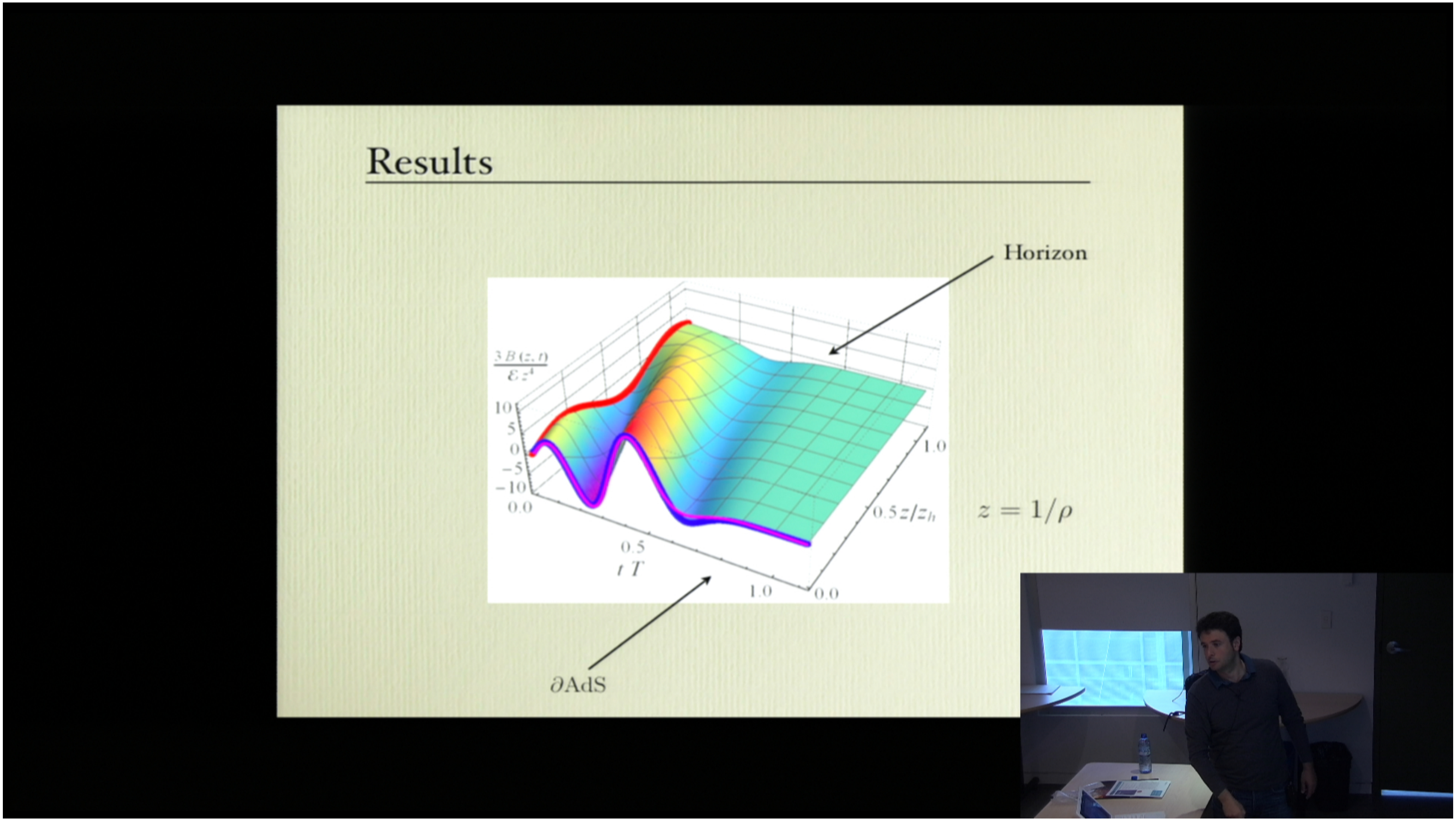
- BH collision in asymptotically flat 4D general relativity:

- Wave-form at infinity accurately reproduced (but perhaps non-asymptotic properties would not be).
- Analog in AdS: Boundary stress-tensor.

Results



$z = 1/\rho$



Results

Horizon

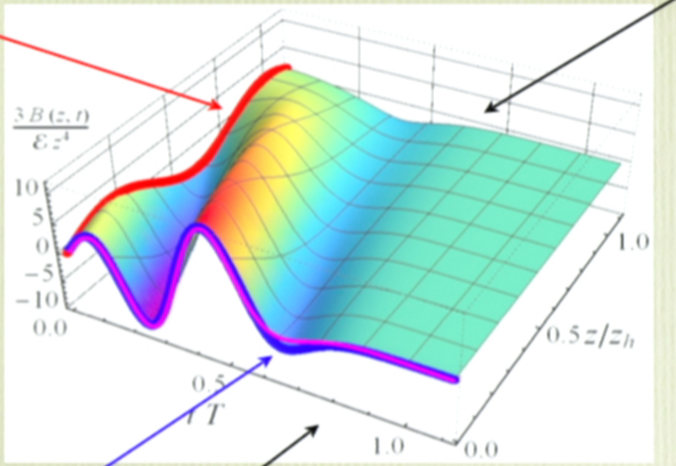
$z = 1/\rho$

∂AdS

Results

Horizon

Initial state



$z = 1/\rho$

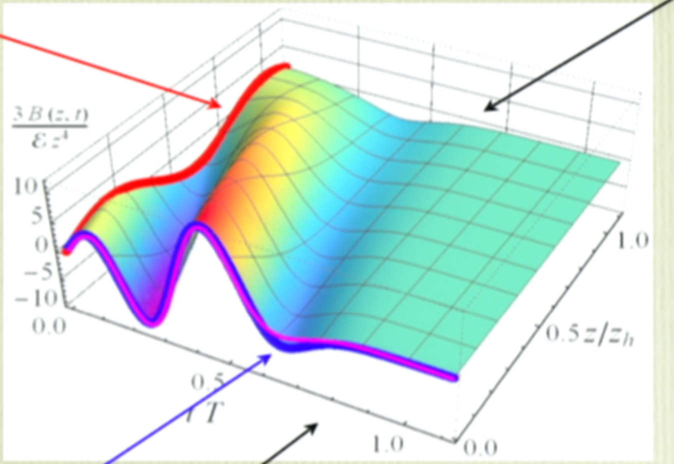
Full $\Delta\mathcal{P}(t)/\mathcal{E}$

∂AdS

Results

Horizon

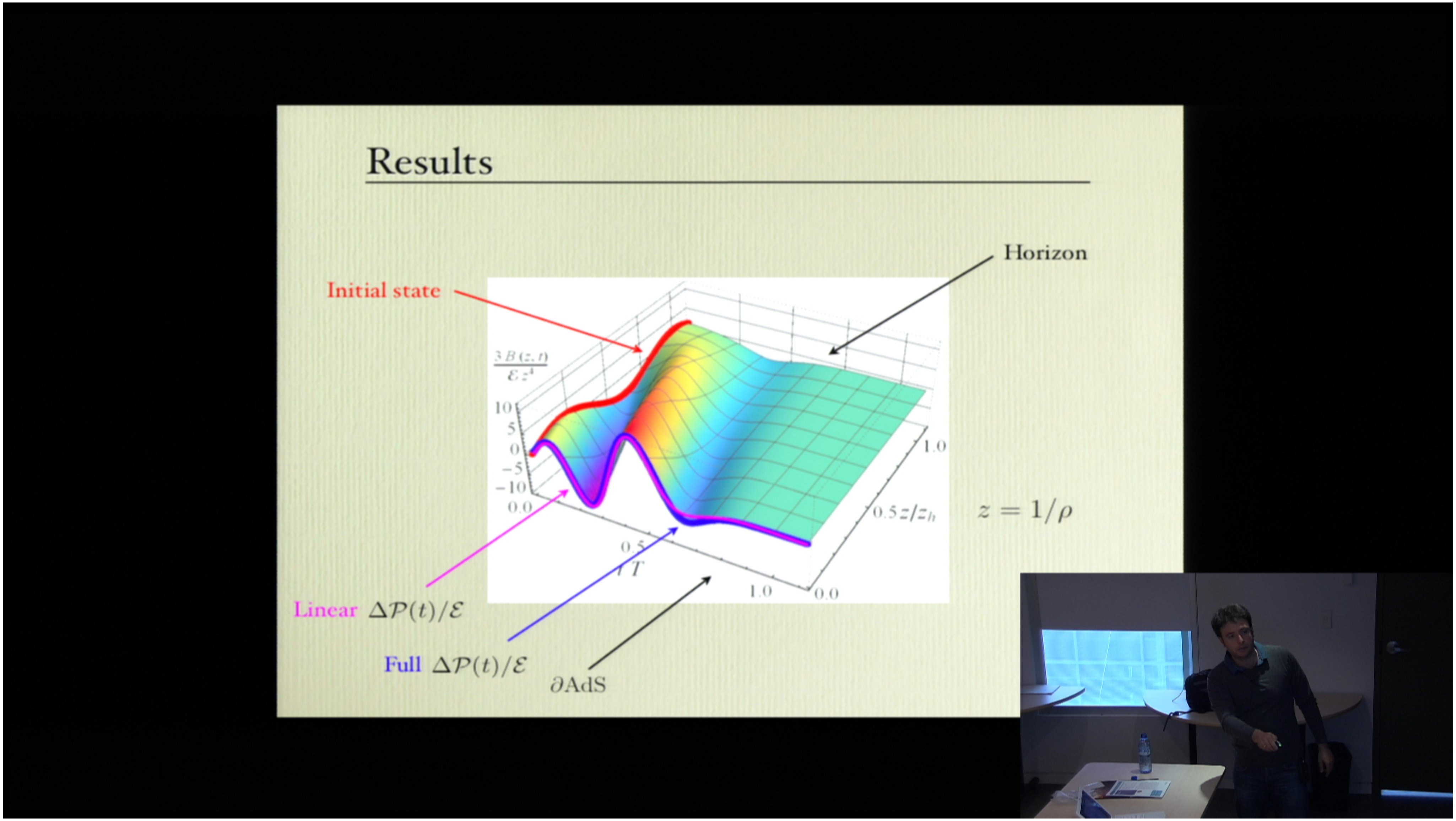
Initial state



$z = 1/\rho$

Full $\Delta\mathcal{P}(t)/\mathcal{E}$

∂AdS



Results

Initial state

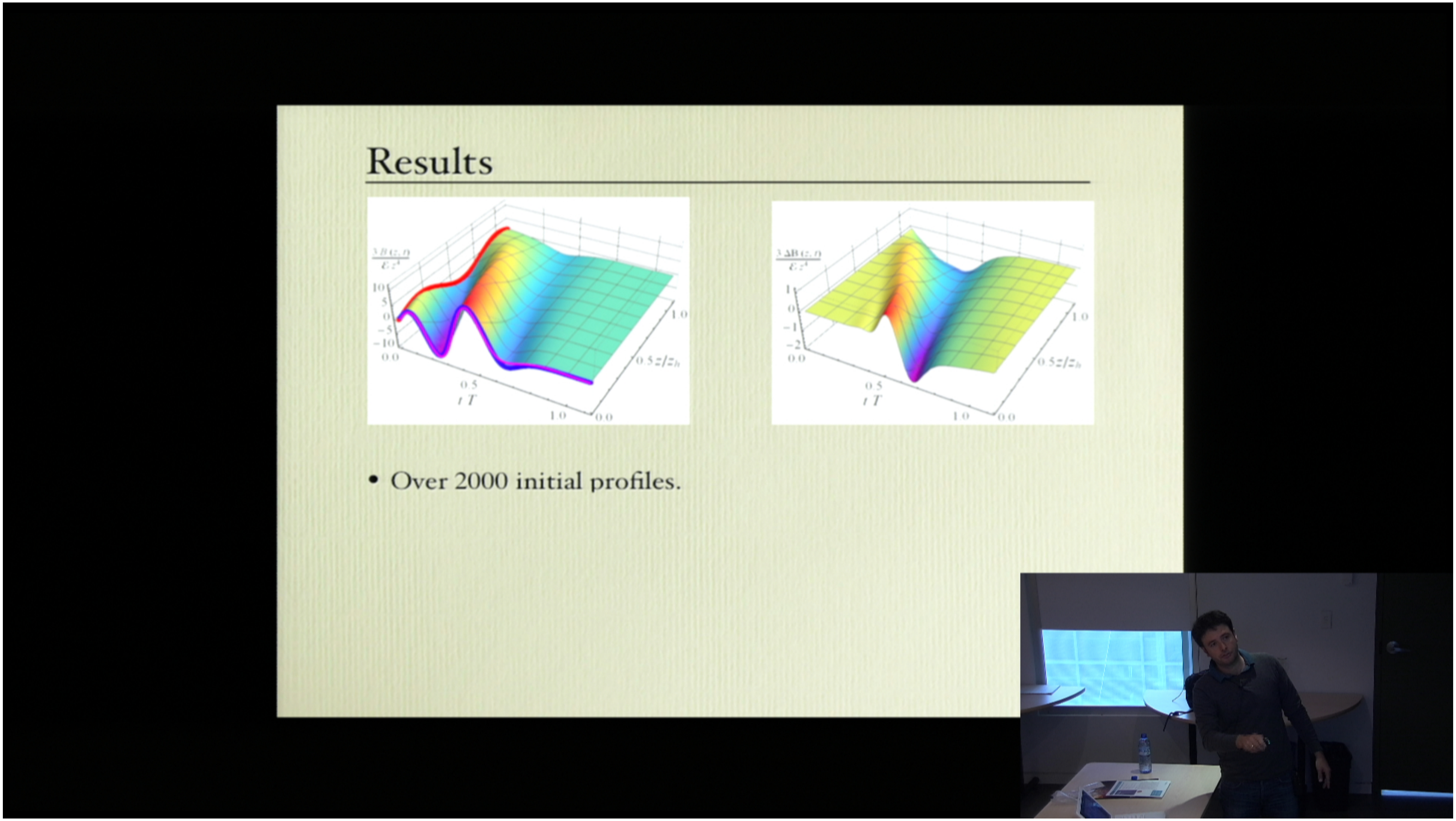
Horizon

Linear $\Delta\mathcal{P}(t)/\mathcal{E}$

Full $\Delta\mathcal{P}(t)/\mathcal{E}$

∂AdS

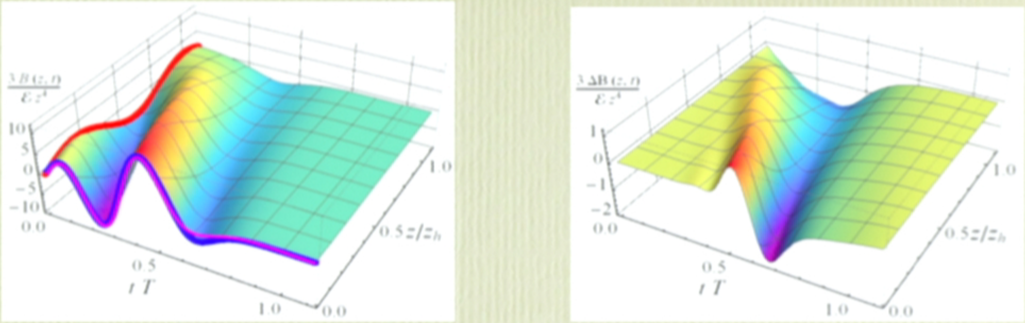
$z = 1/\rho$



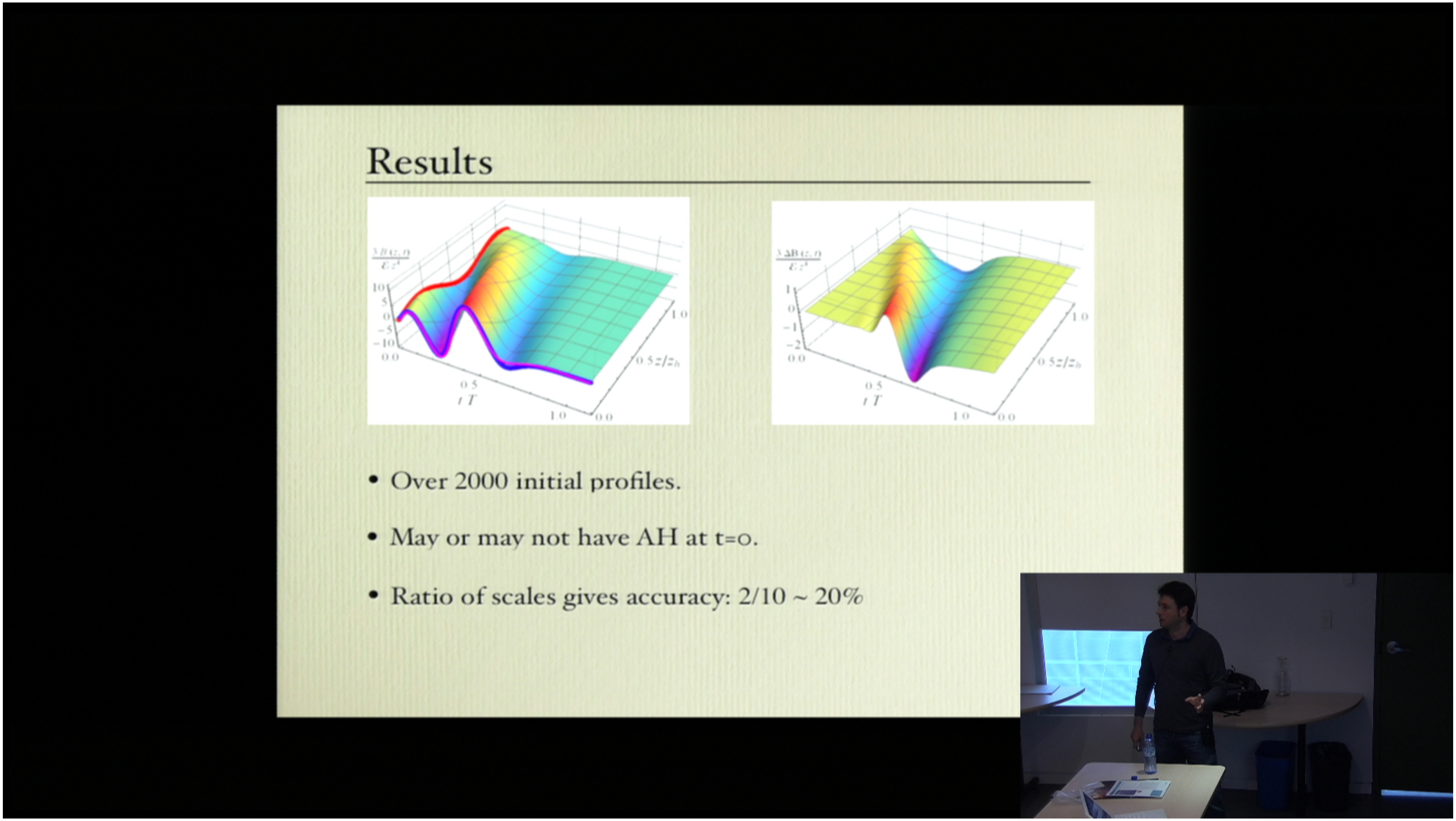
Results

- Over 2000 initial profiles.

Results



- Over 2000 initial profiles.
- May or may not have AH at t=o.
- Ratio of scales gives accuracy: 2/10 ~ 20%



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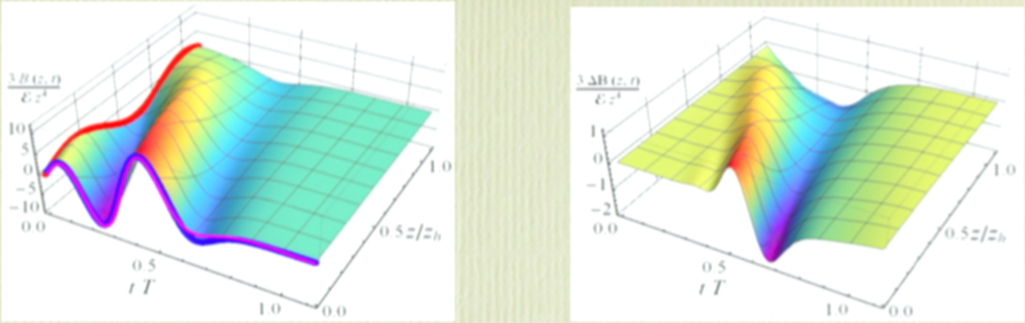
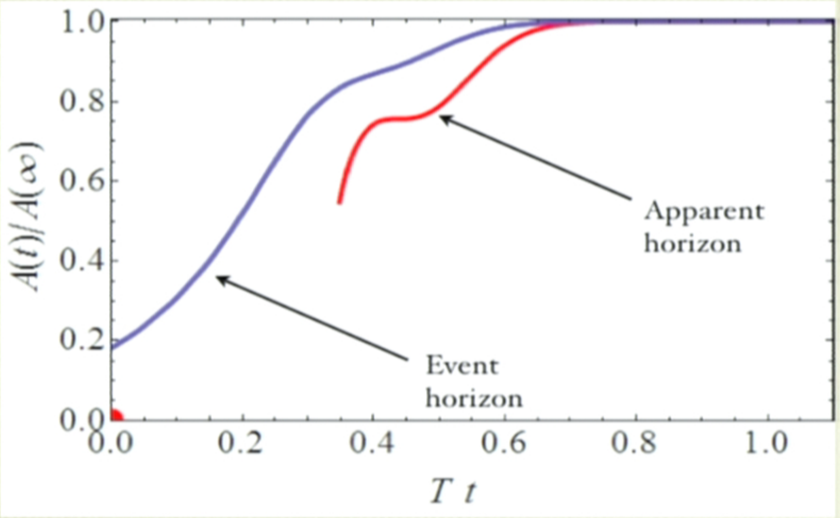
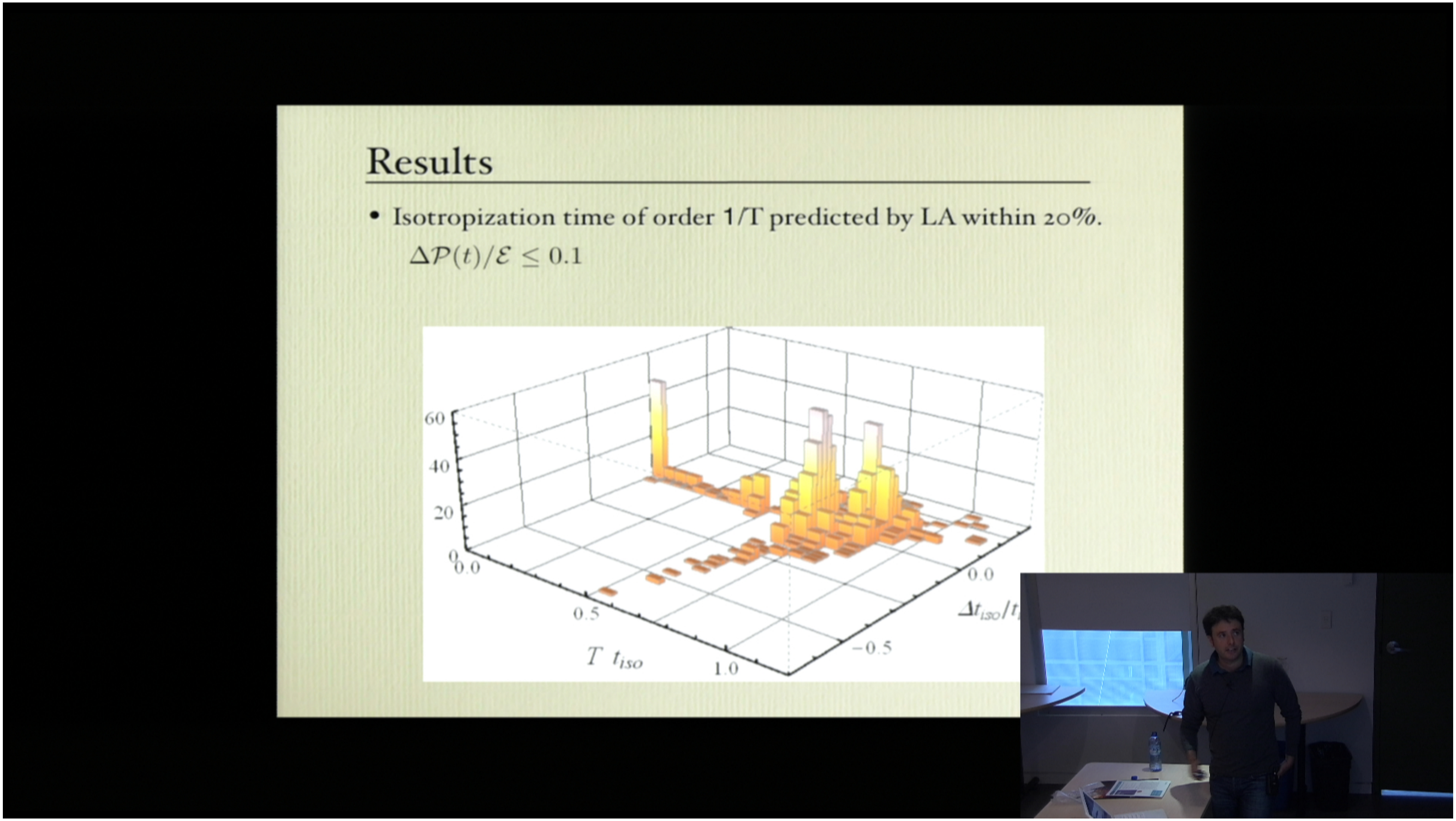


- Over 2000 initial profiles.
- May or may not have AH at t=0.
- Ratio of scales gives accuracy: 2/10 ~ 20%
- $\Delta\mathcal{P}(t)/\mathcal{E} \sim 10$ implies far from equilibrium.

Results

- "Entropy" increases during isotropization.

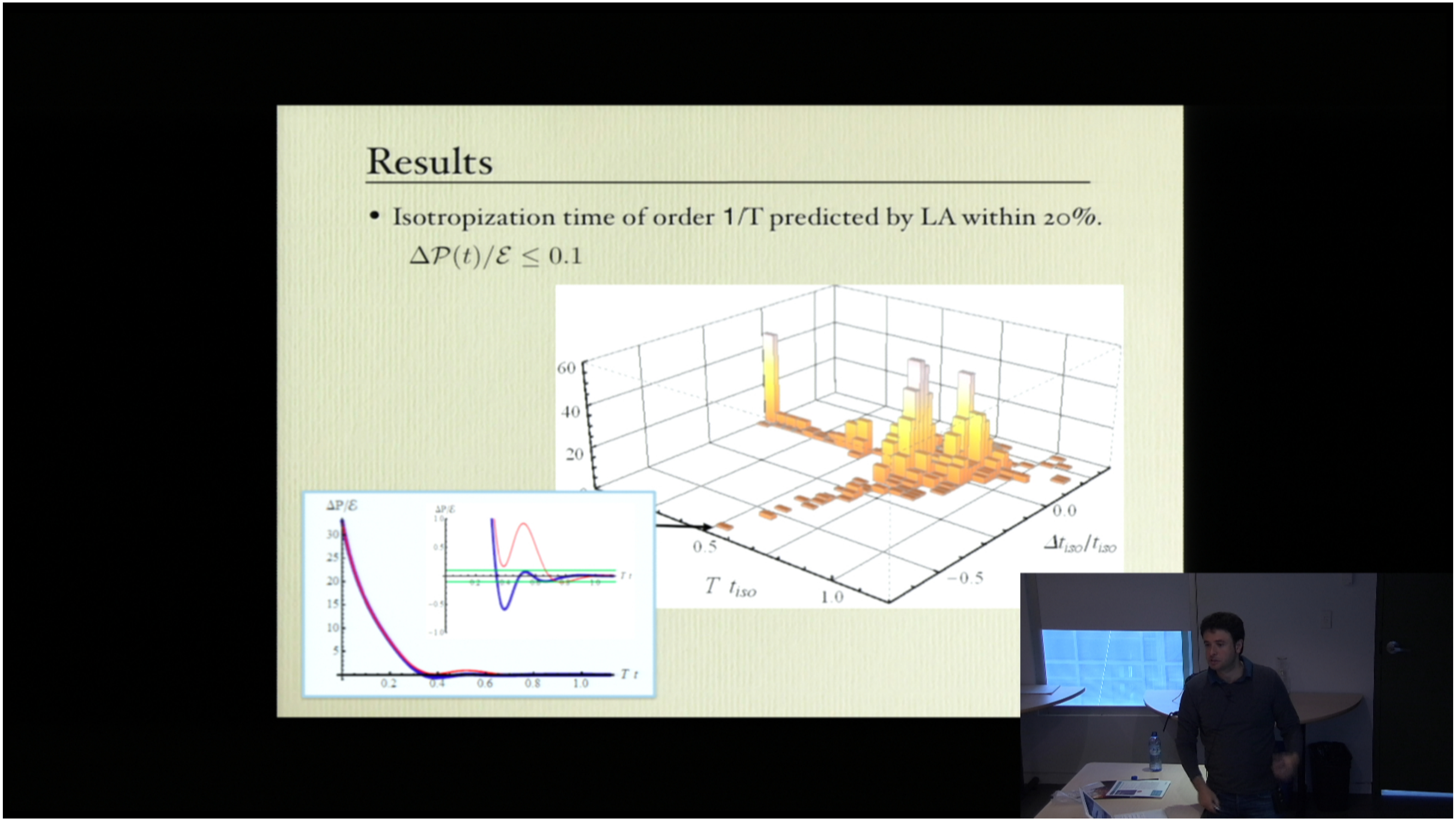




Results

- Isotropization time of order 1/T predicted by LA within 20%.

  $\Delta\mathcal{P}(t)/\mathcal{E} \leq 0.1$



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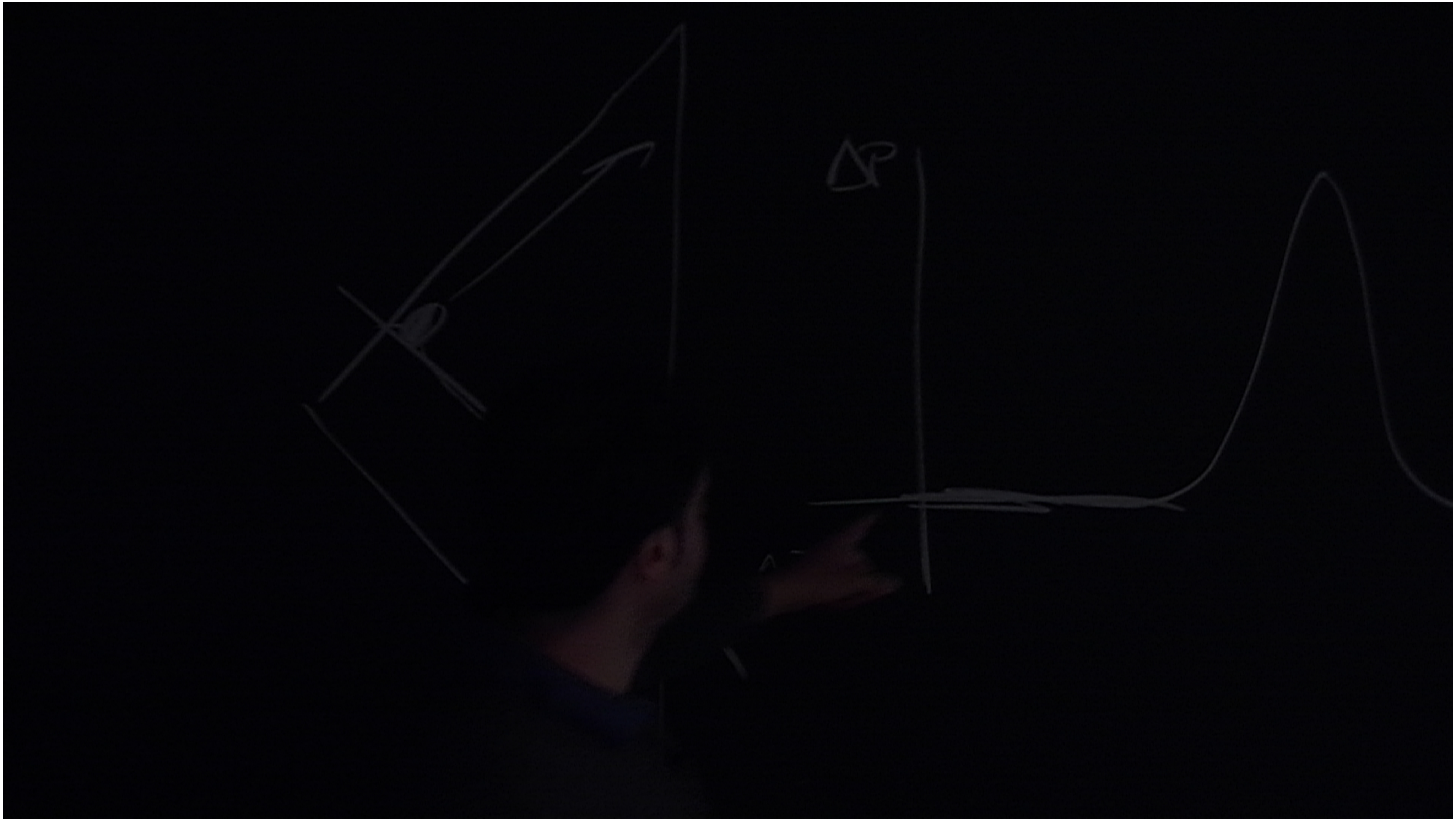
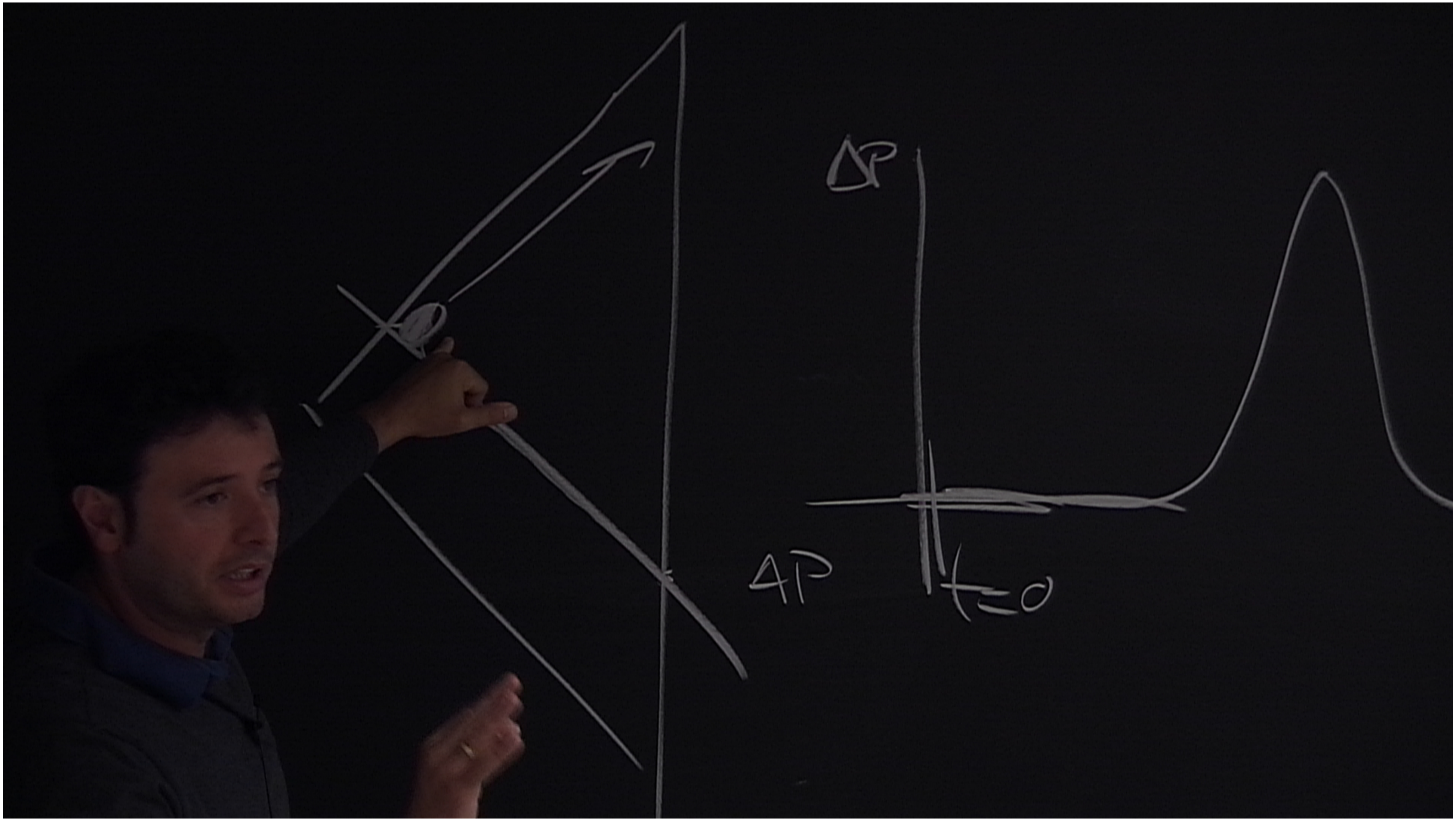
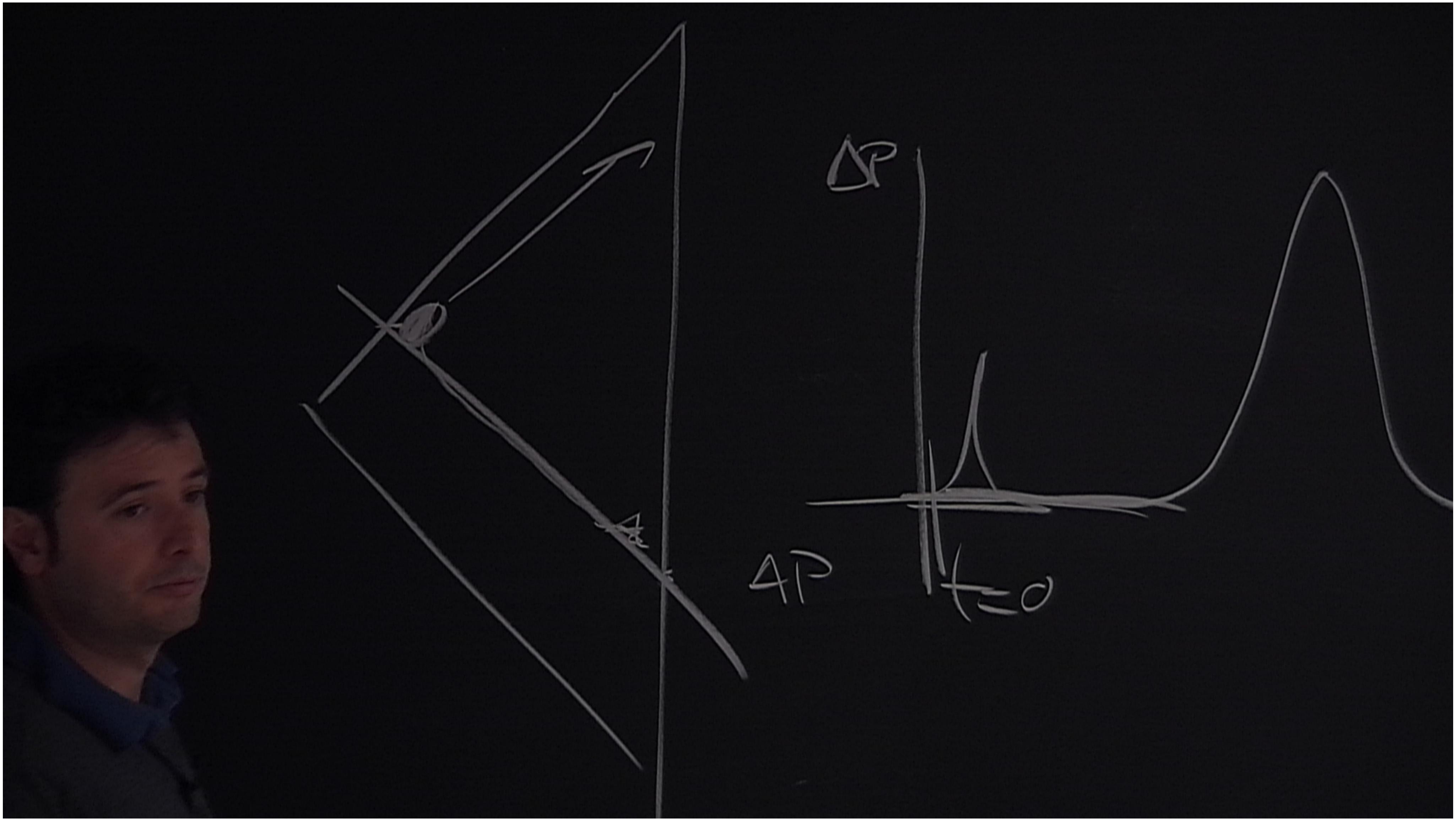

Δp


ΔP
4P
t=0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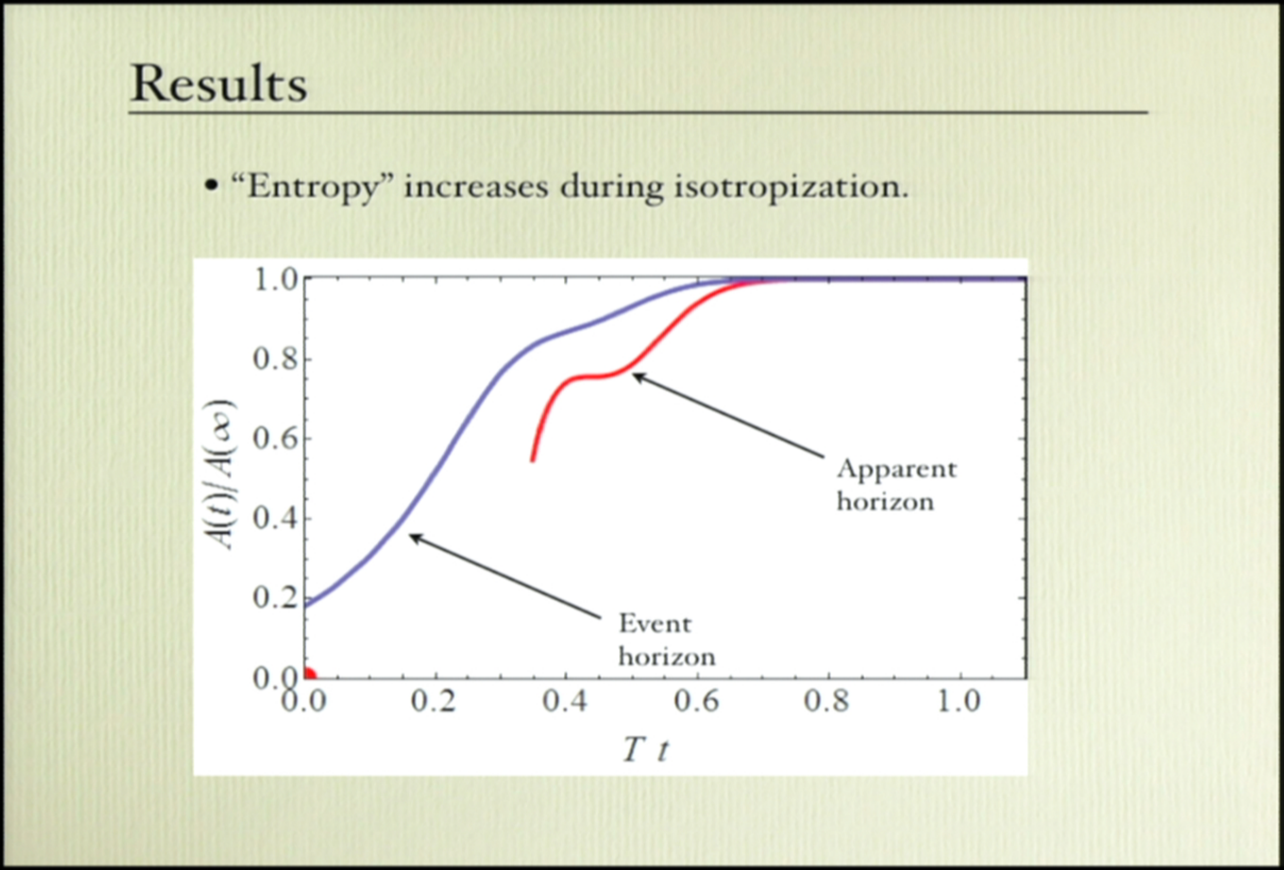

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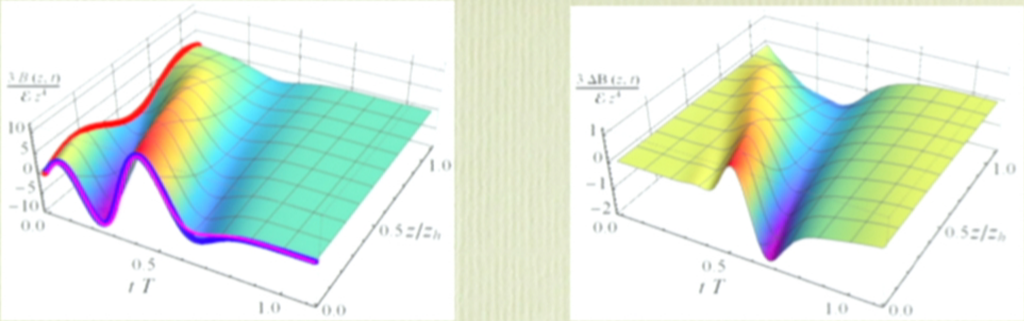

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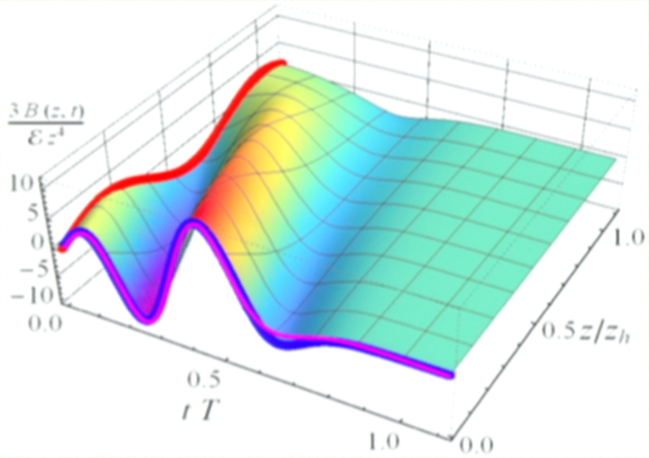

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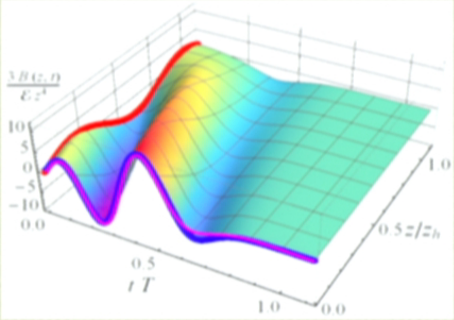

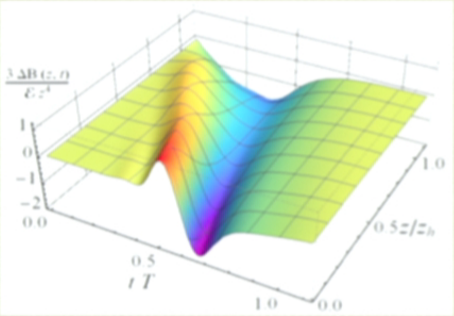

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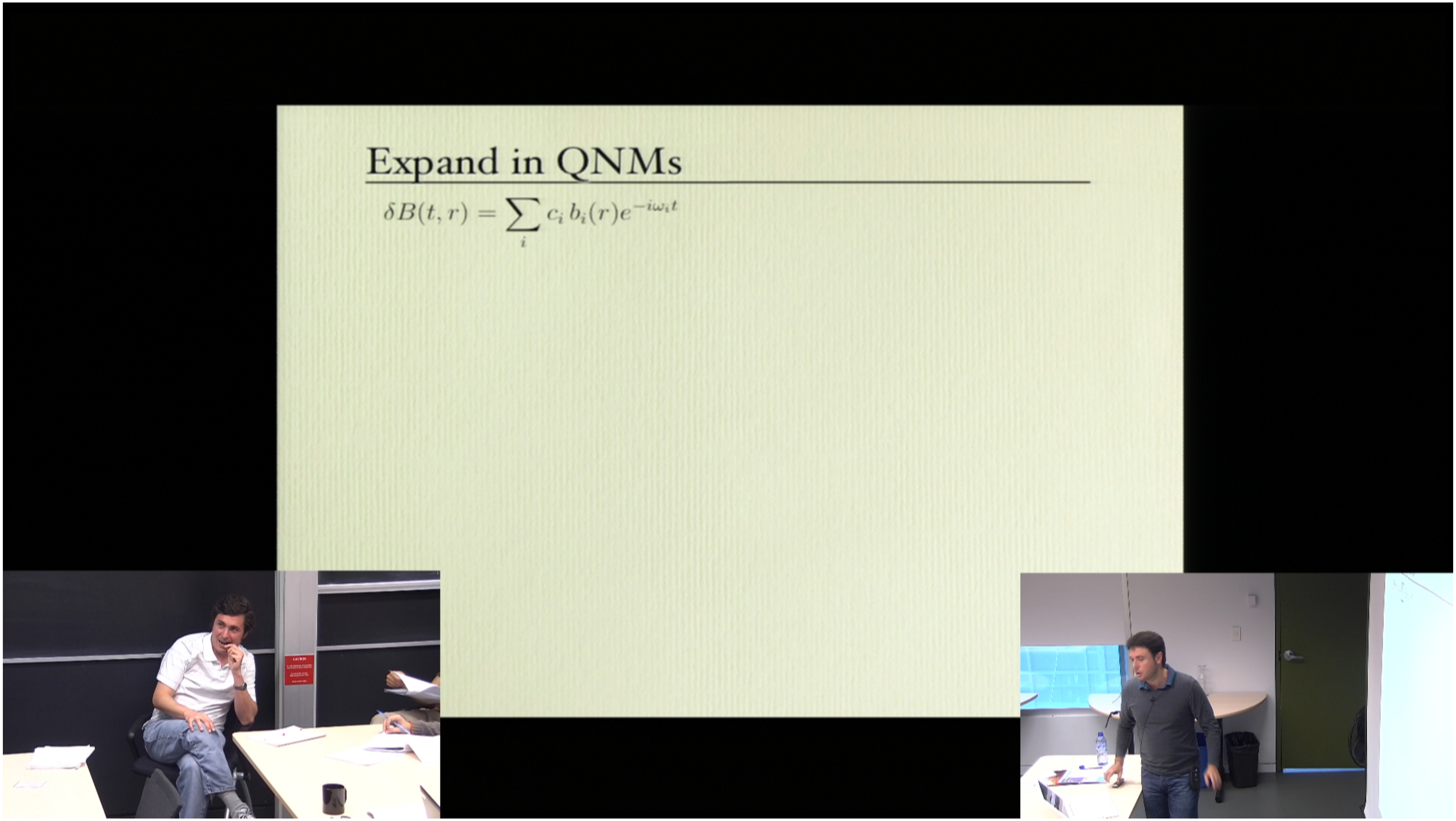
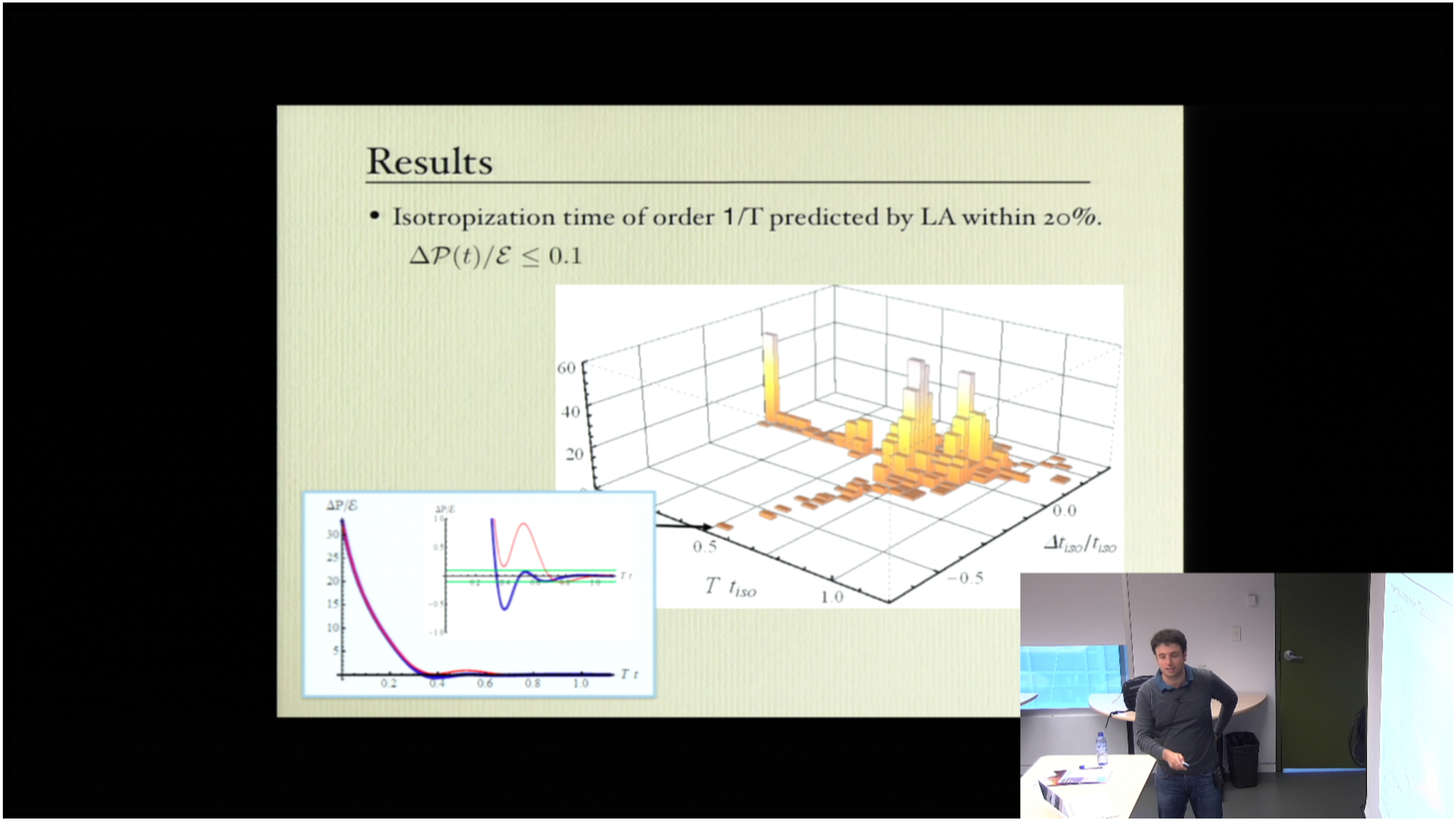


Expand in QNMs

$$\delta B(t,r) = \sum_i c_i\, b_i(r) e^{-i\omega_i t}$$



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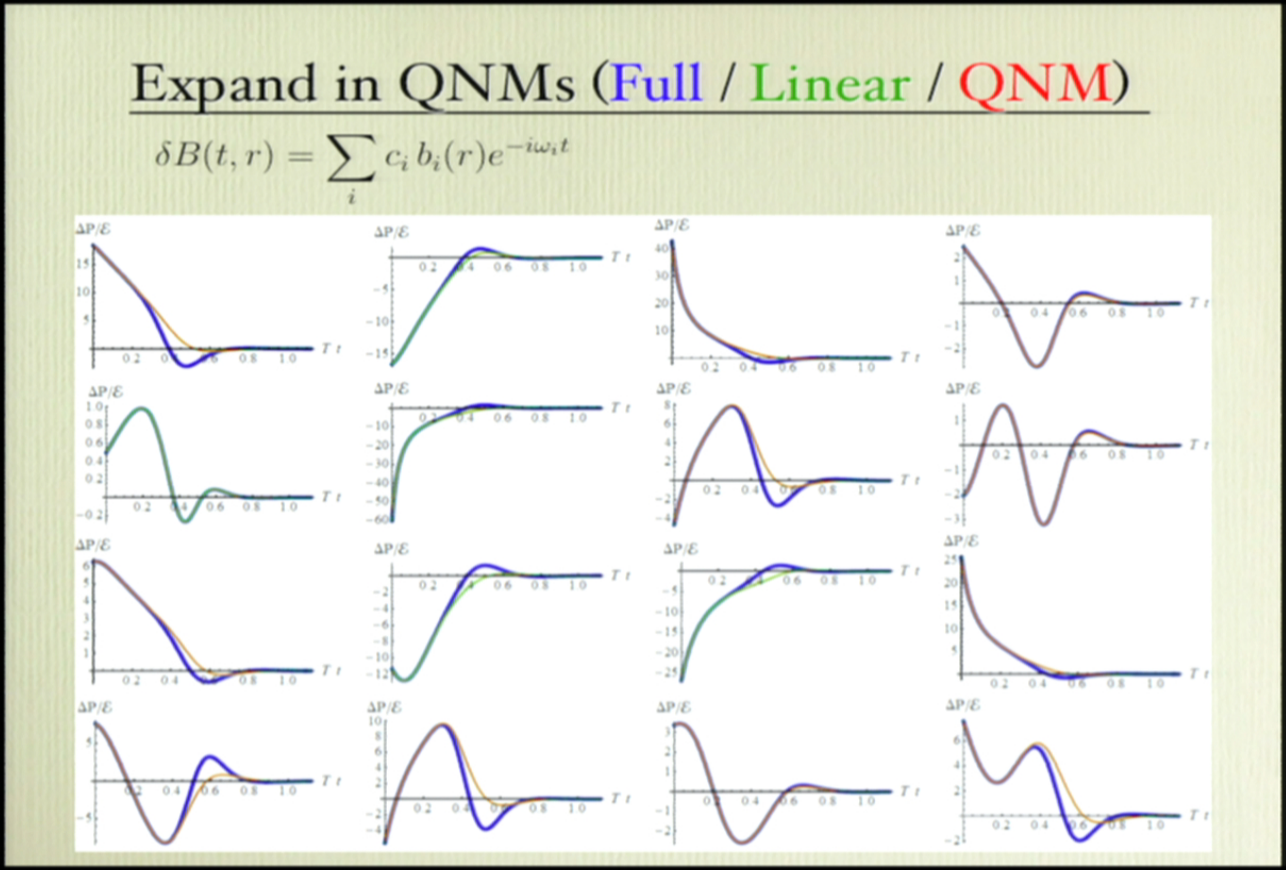

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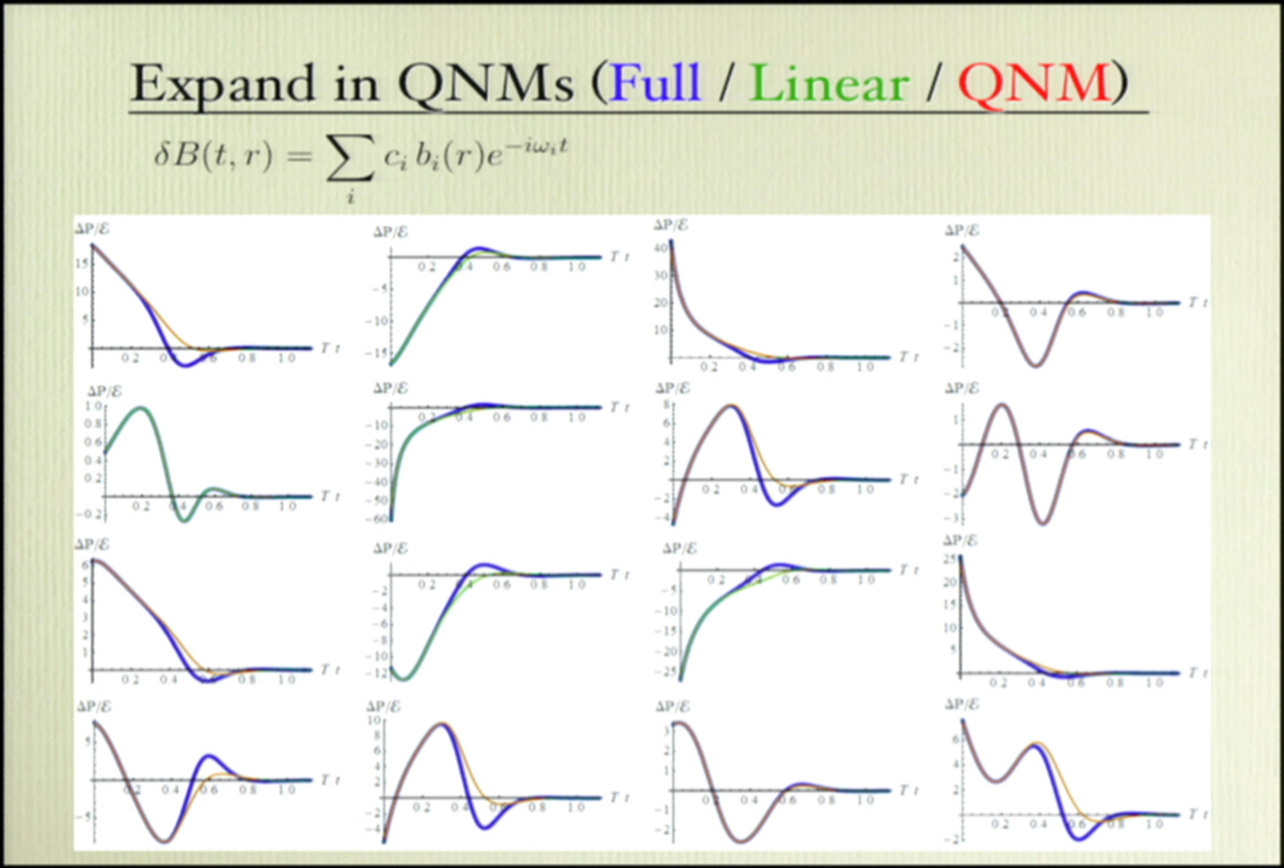

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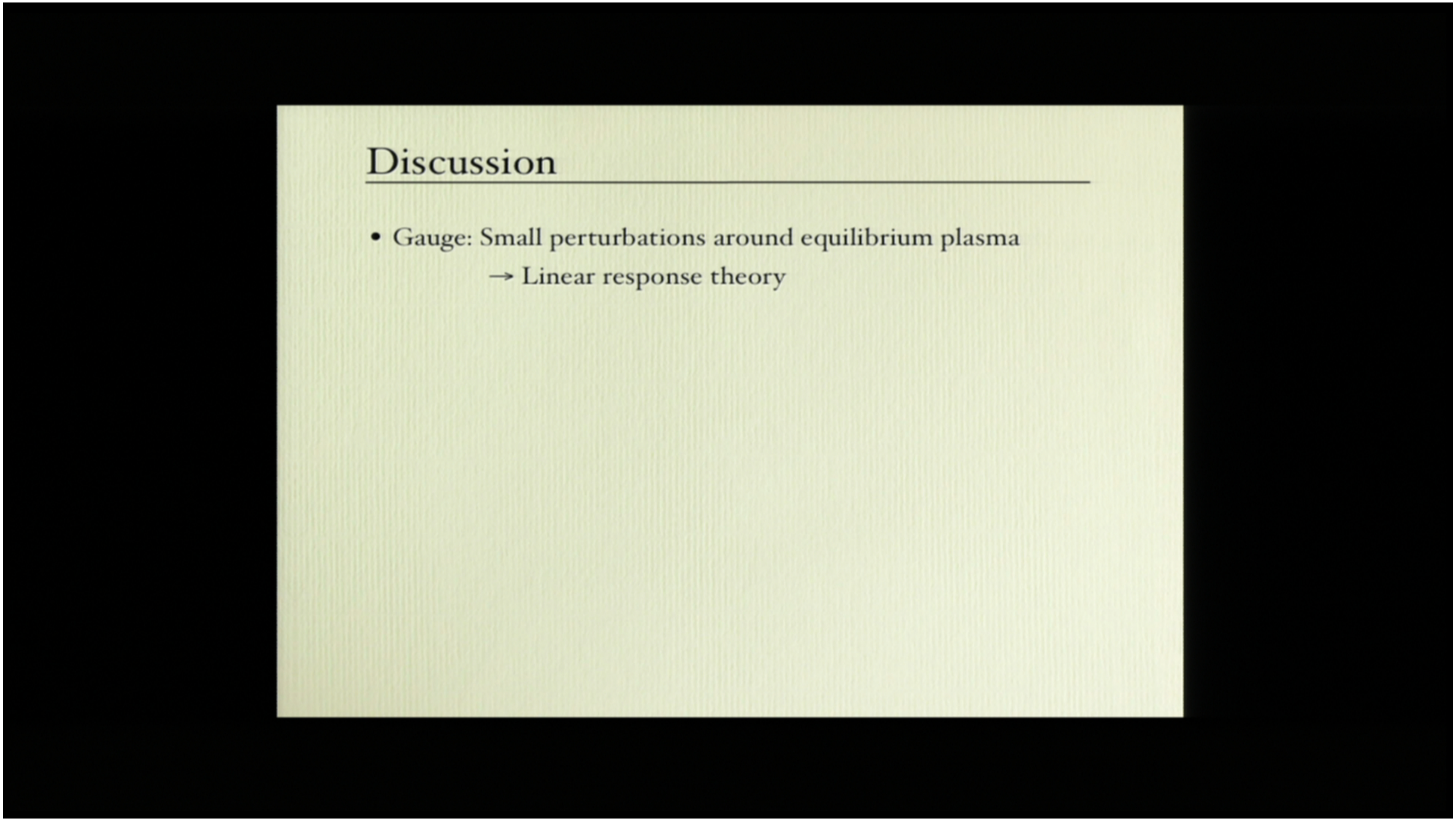
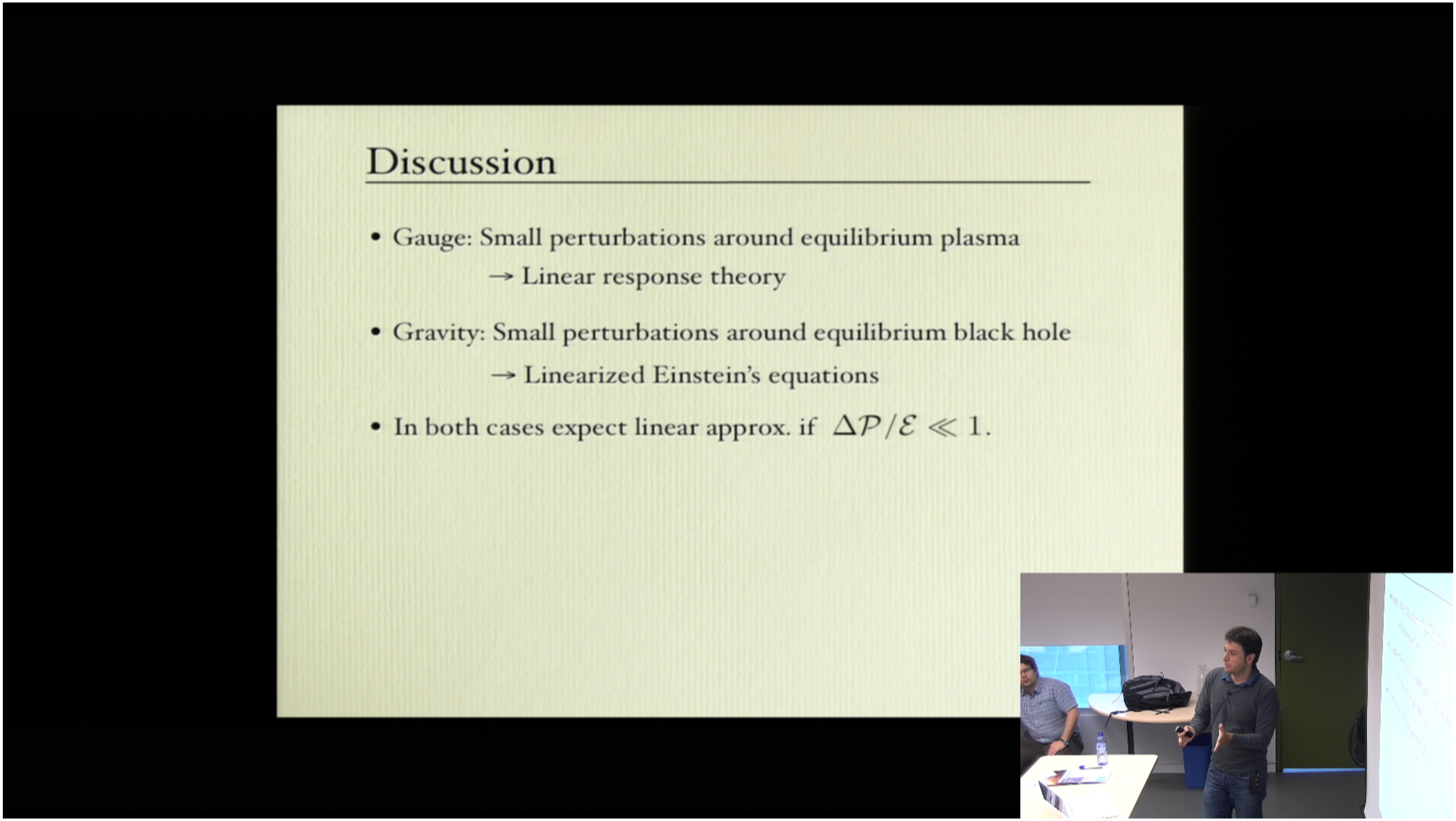


Discussion

- Gauge: Small perturbations around equilibrium plasma
  → Linear response theory



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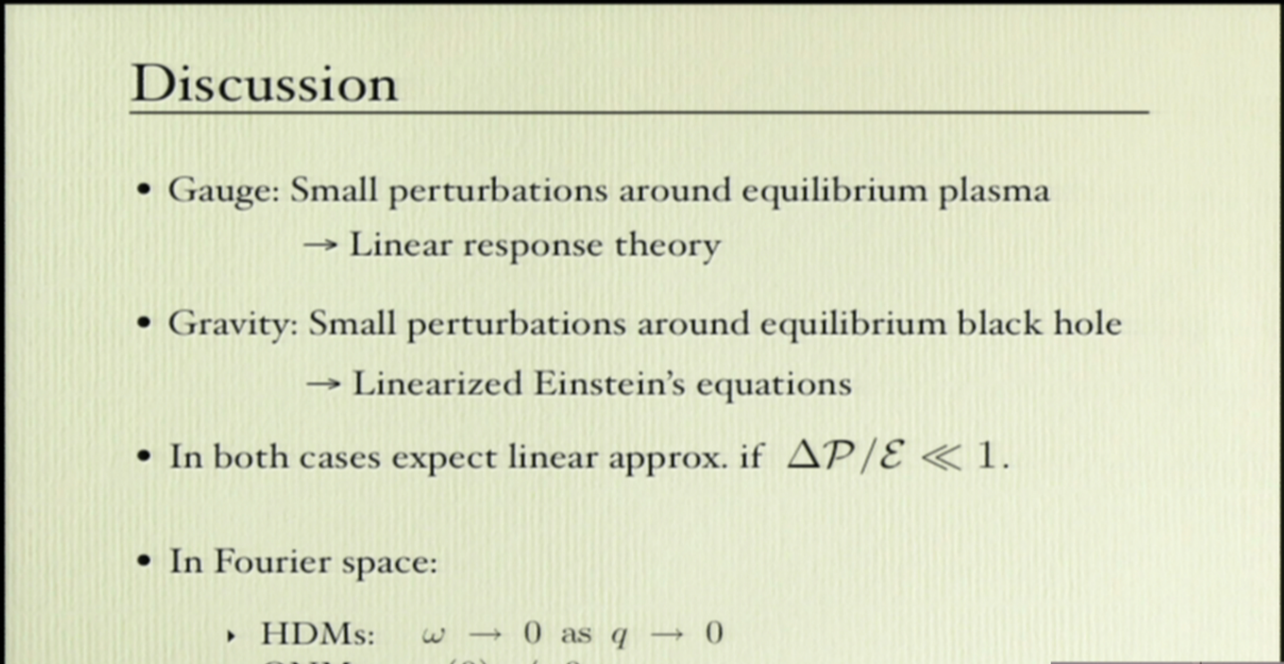
- Gravity: Small perturbations around equilibrium black hole
  → Linearized Einstein's equations
- In both cases expect linear approx. if $\Delta\mathcal{P}/\mathcal{E} \ll 1$.



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  - HDMs: $\omega \to 0$ as $q \to 0$
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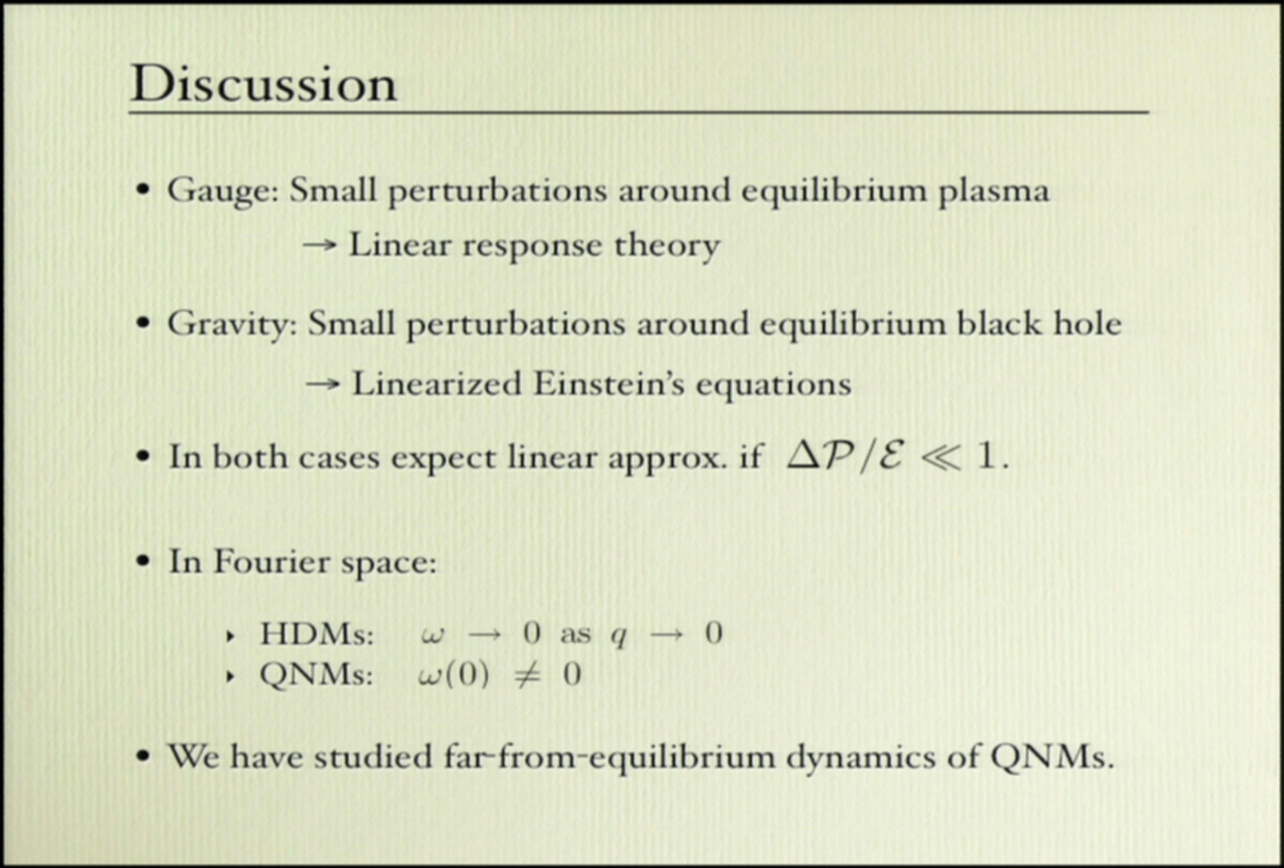

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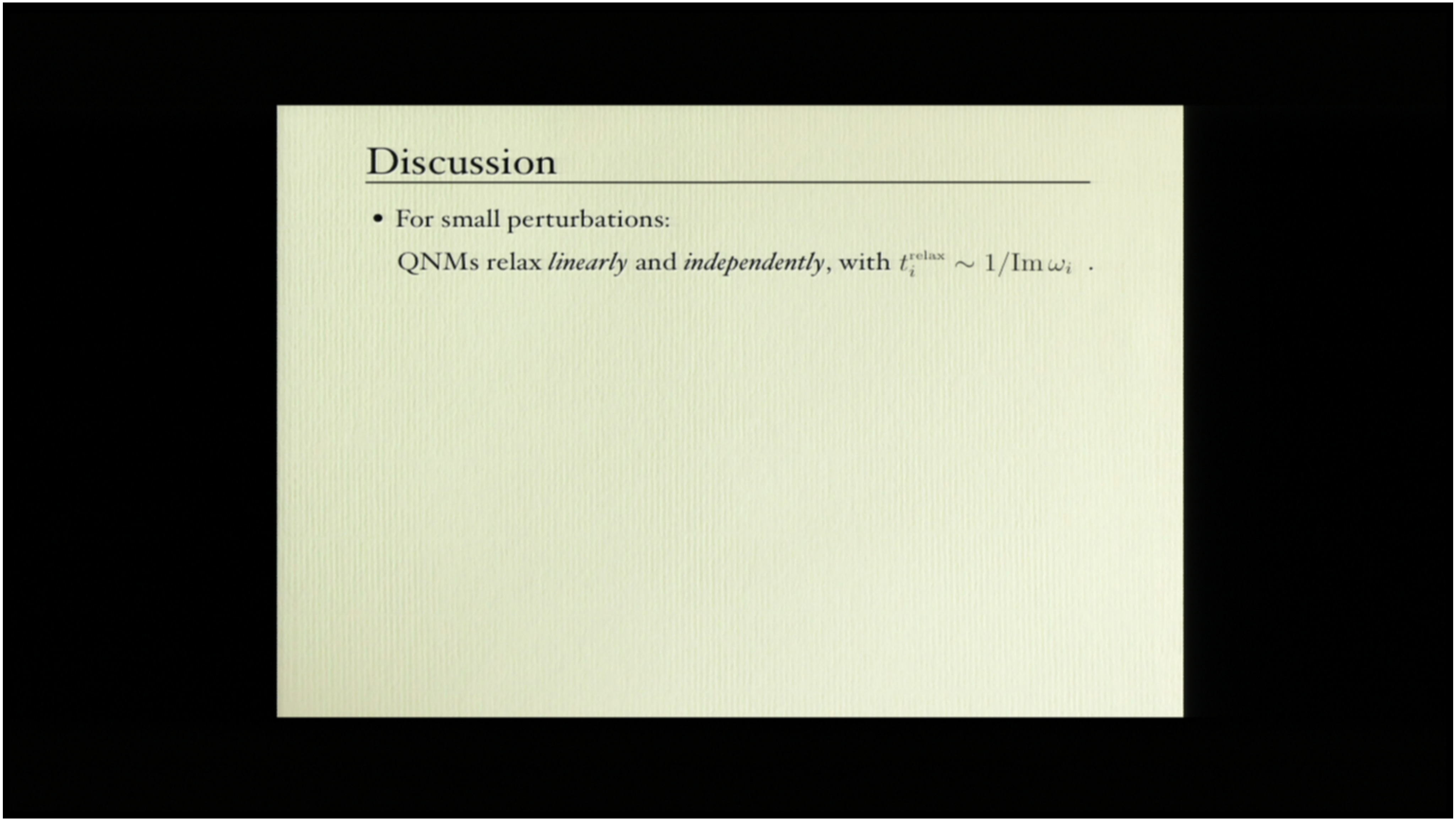
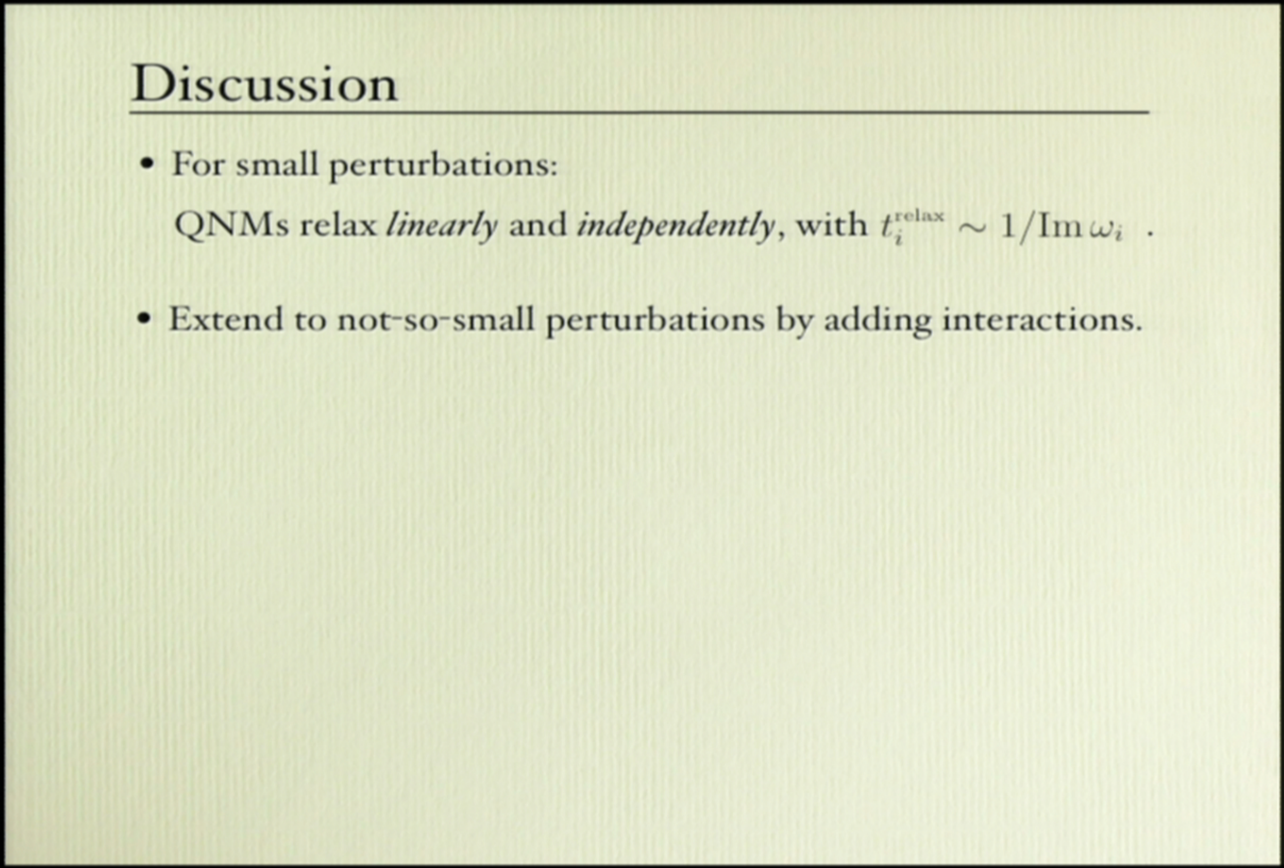
- We have studied far-from-equilibrium dynamics of QNMs.



Discussion

- For small perturbations:

  QNMs relax *linearly* and *independently*, with $t_i^{\text{relax}} \sim 1/\text{Im}\,\omega_i$.



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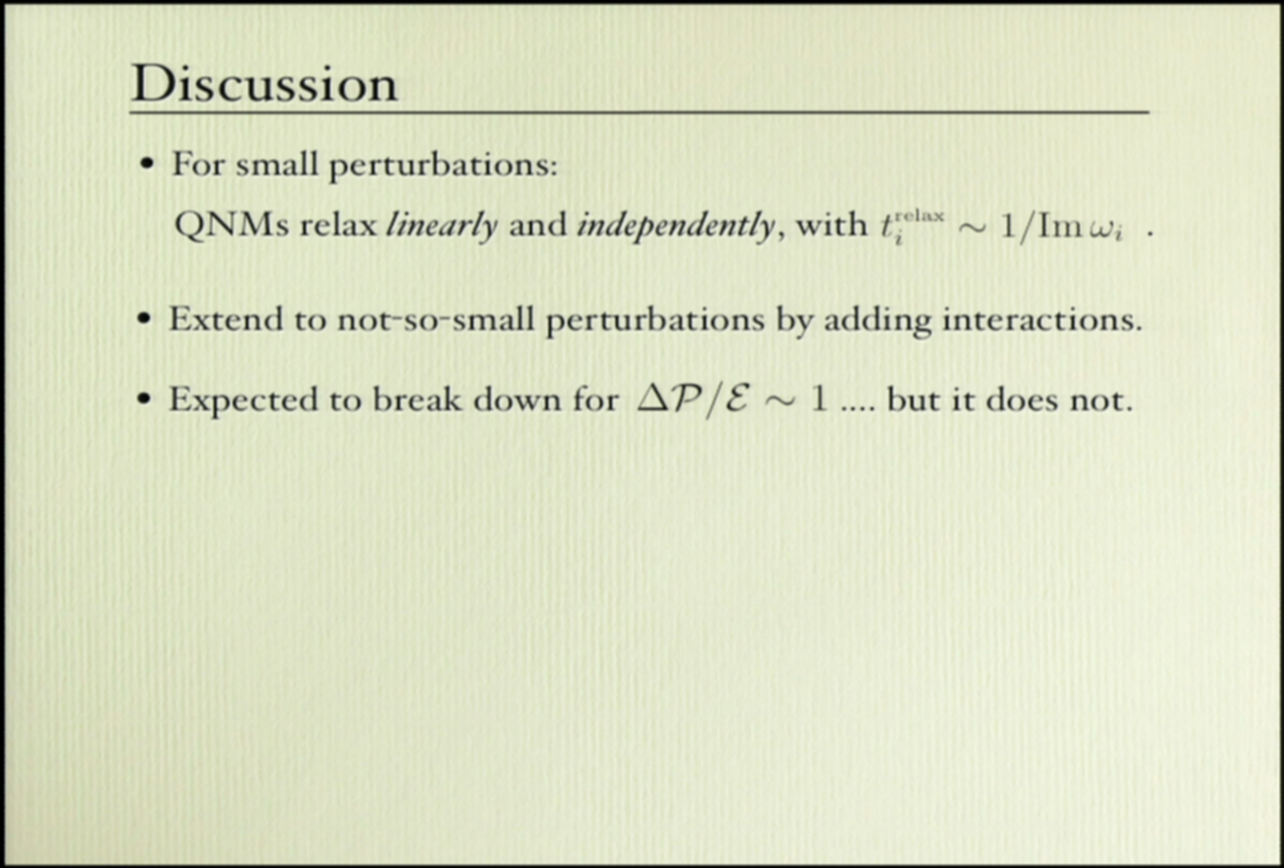

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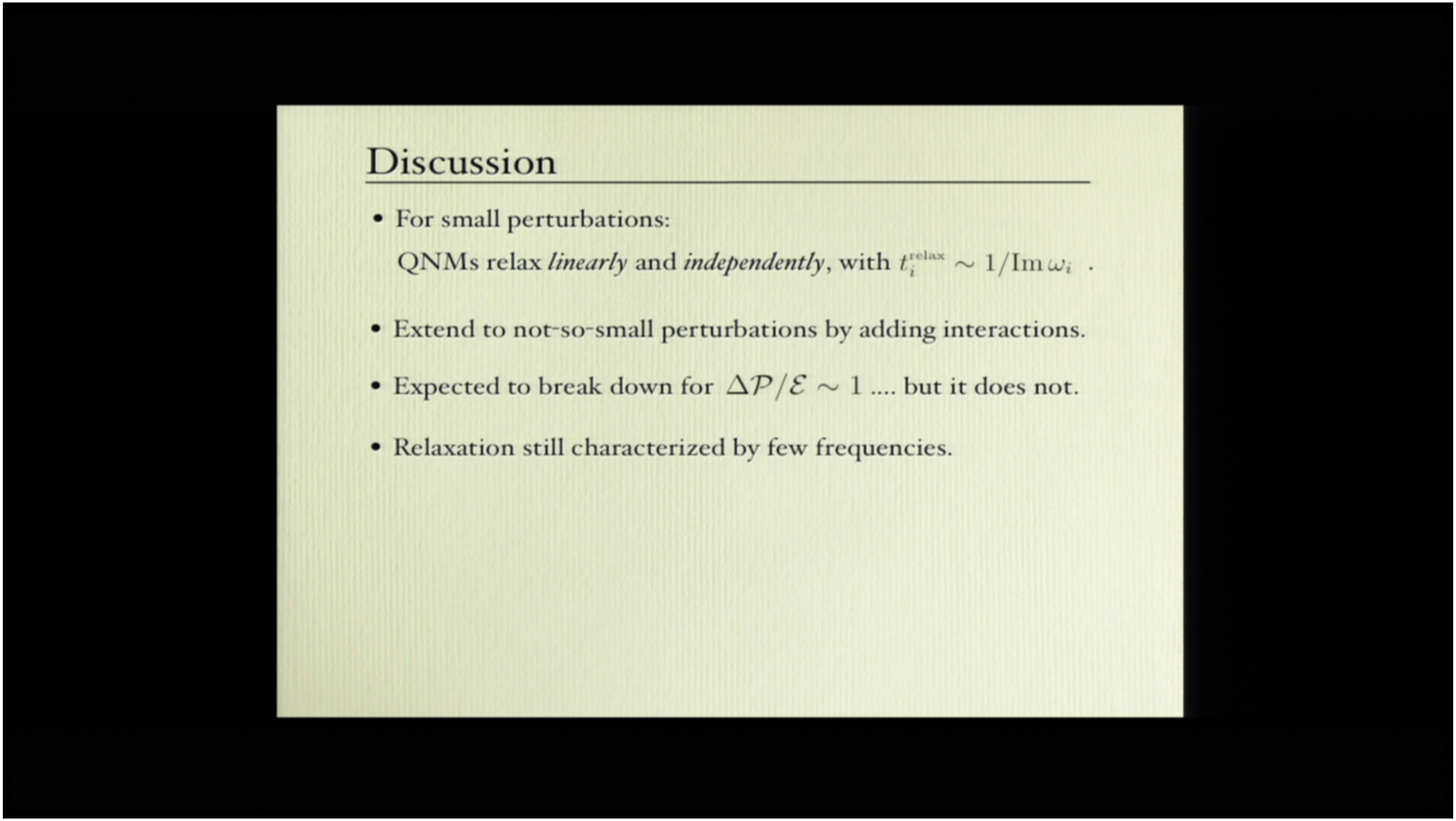

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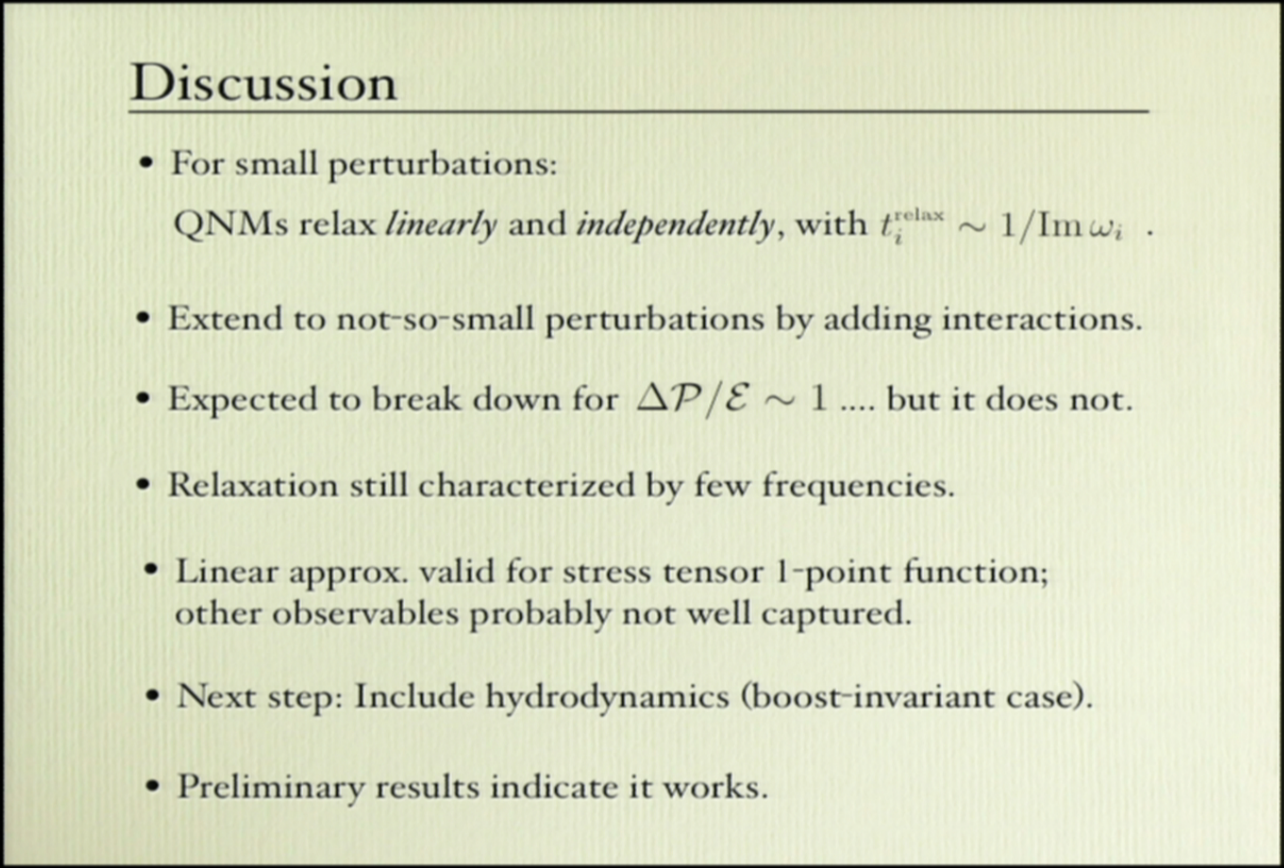

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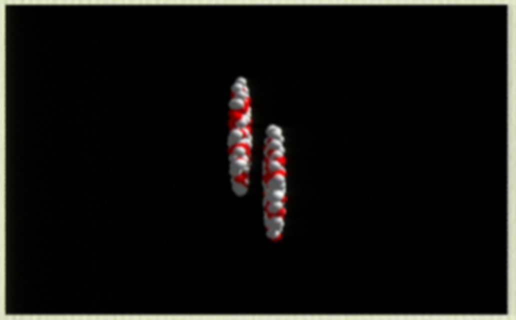
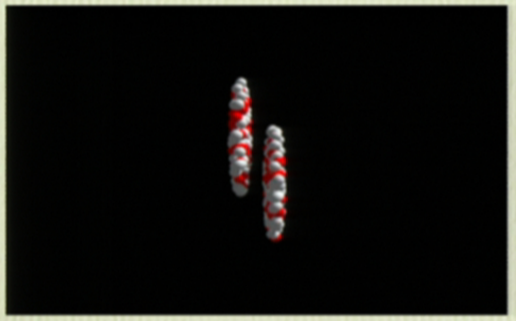
- Next step: Include hydrodynamics (boost-invariant case).
- Preliminary results indicate it works.

Potential implication

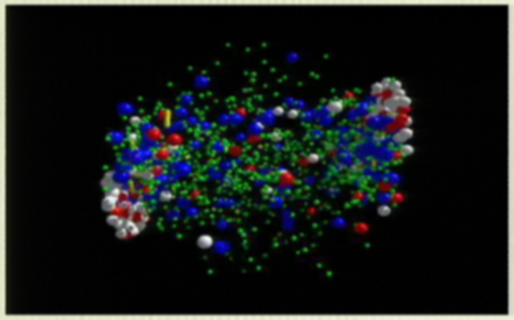


Initial state

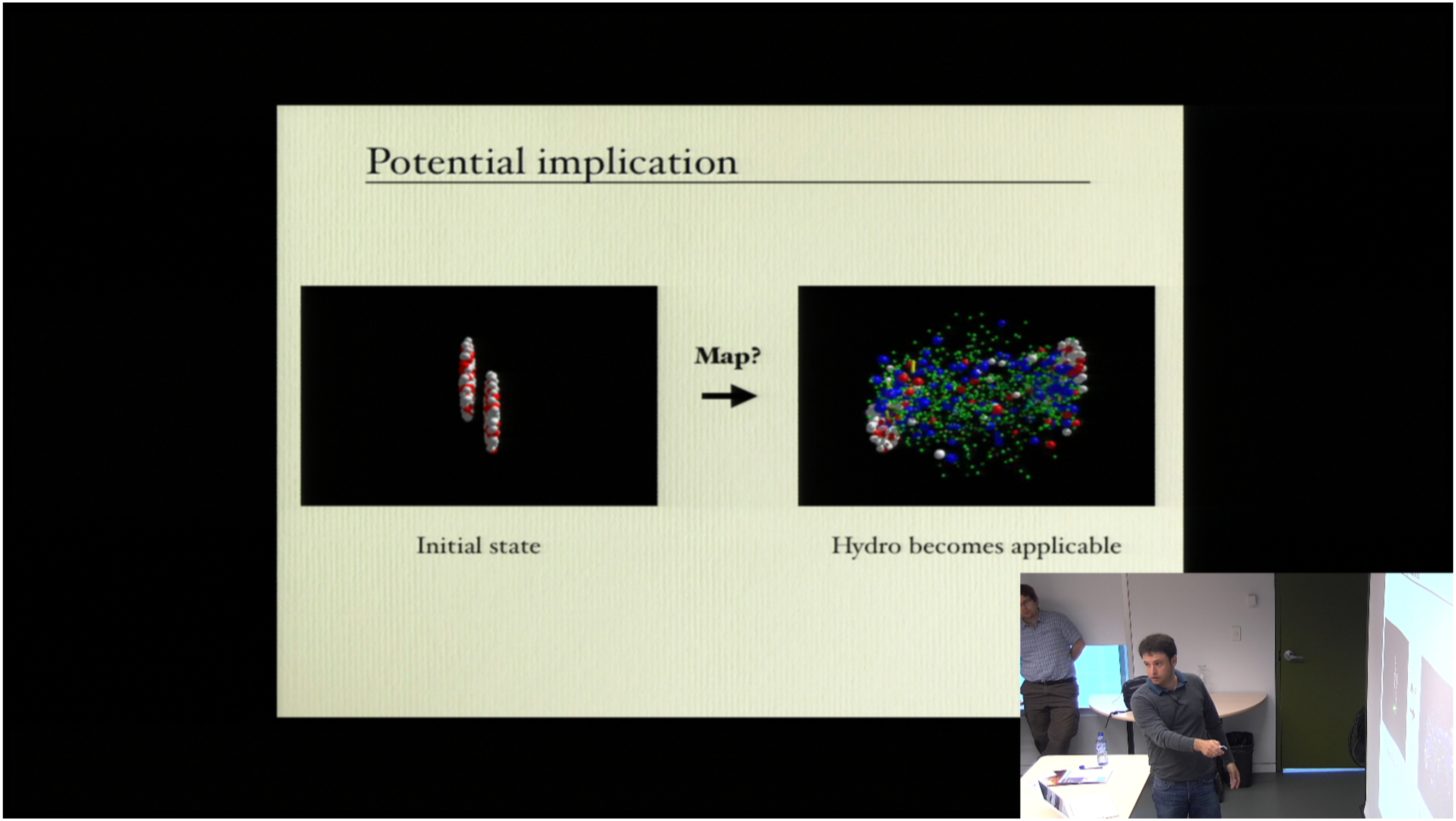
Potential implication



Initial state



Hydro becomes applicable



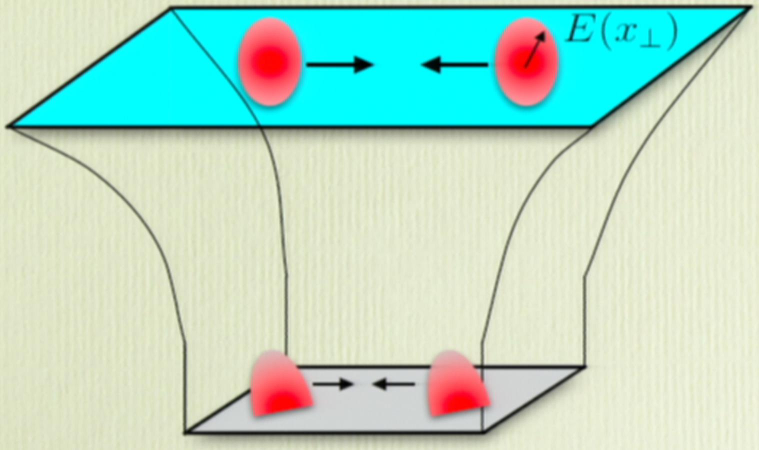
Potential implication

Map?

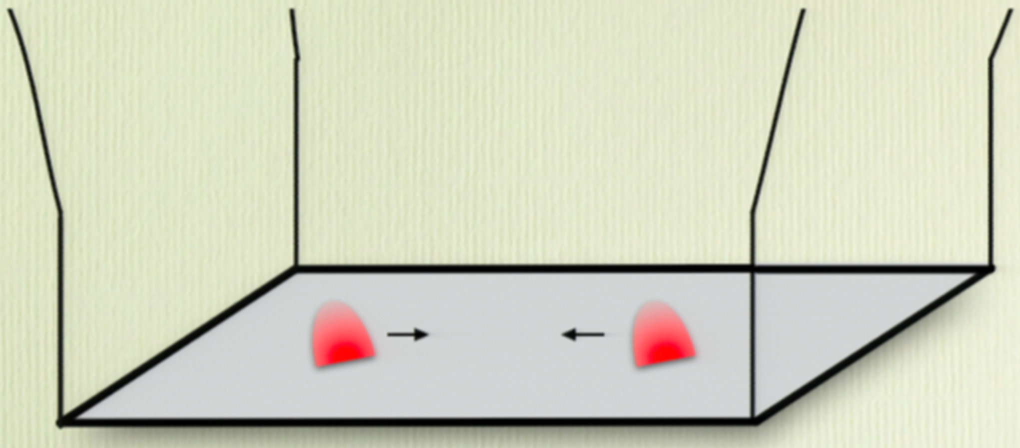
Initial state

Hydro becomes applicable

Potential implication



Potential implication



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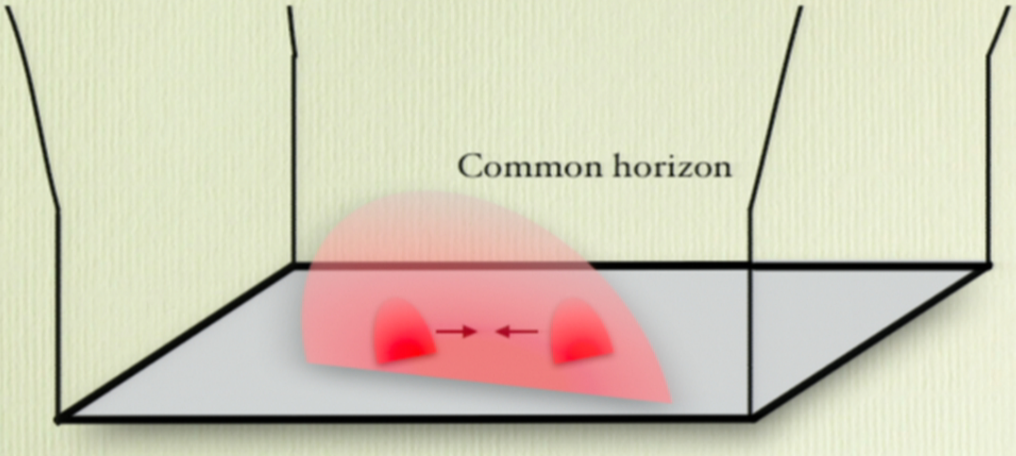

Full non-linearity of gravity encoded in the initial horizon.

Thank you.