Title: Shock Wave in Strongly Coupled Plasmas

Speakers: Martin Kruczenski

Collection: Exploring AdS/CFT Dualities in Dynamical Settings - 2012

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Shock waves in strongly coupled plasmas

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arXiv:1105.1355 (JHEP)
(S. Khlebnikov, G. Michalogiorgakis, M.K.)

Perimeter 2012

Summary

- **Introduction**

  Shock waves (fluids, plasmas)

  String / gauge theory duality (**AdS/CFT**)

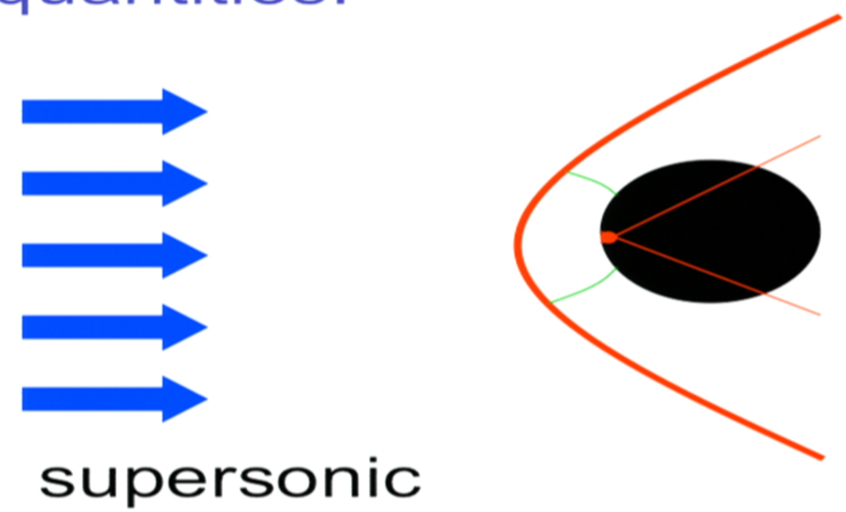
  Strongly coupled plasmas in AdS/CFT

- **Shock waves in AdS/CFT**

  Dual description and main properties

Shock waves in fluids (Landau-Lifshitz)

When an object moves supersonically in a fluid generically creates shock waves. These shocks are perturbations that propagate supersonically and (usually) seen as discontinuities in the hydrodynamic quantities.



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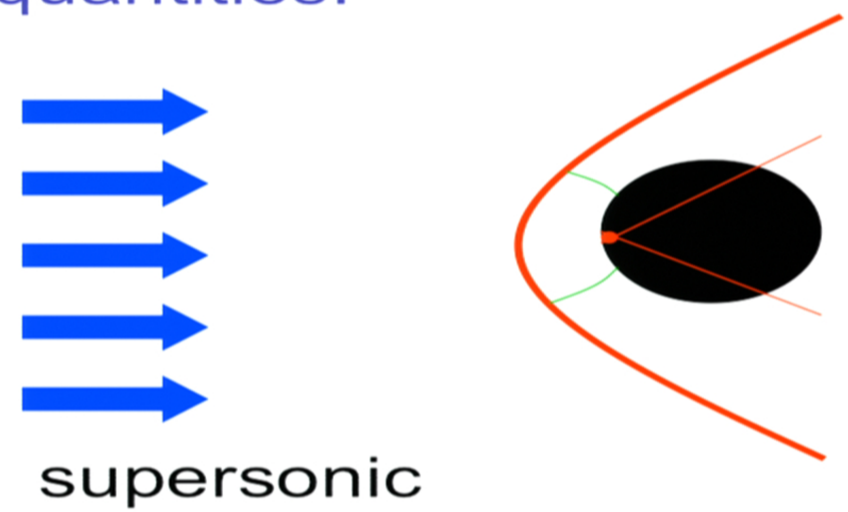
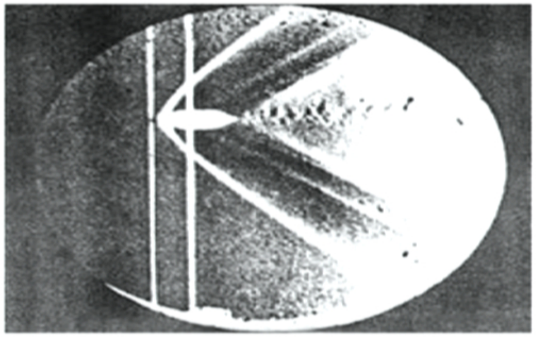


Examples



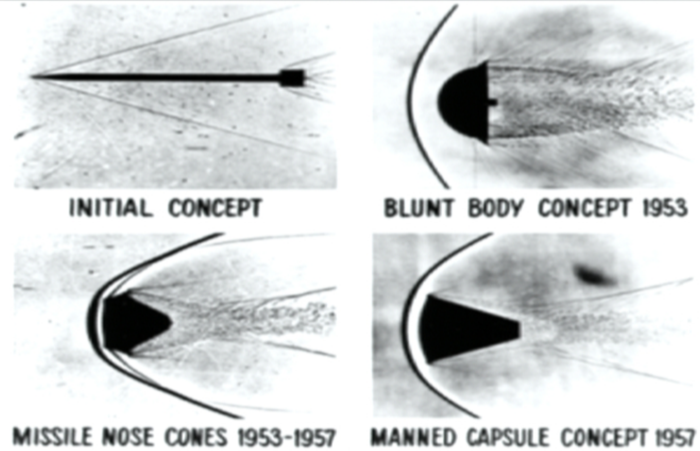


Photo by John Gay

Photo by spacex

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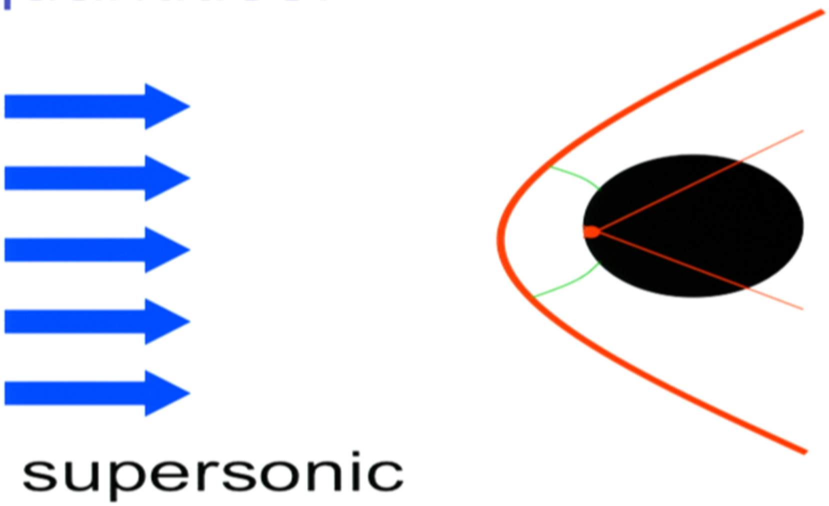


AdS/CFT correspondence (Maldacena)

Gives a precise example of the relation between strings and gauge theory.

Gauge theory	String theory
$\mathcal{N} = 4$ SYM SU(N) on R^4	IIB on $AdS_5 x S^5$
A_μ , Φ^i, Ψ^a	radius R
Operators w/ conf. dim. Δ	String states w/ $E = \frac{\Delta}{R}$

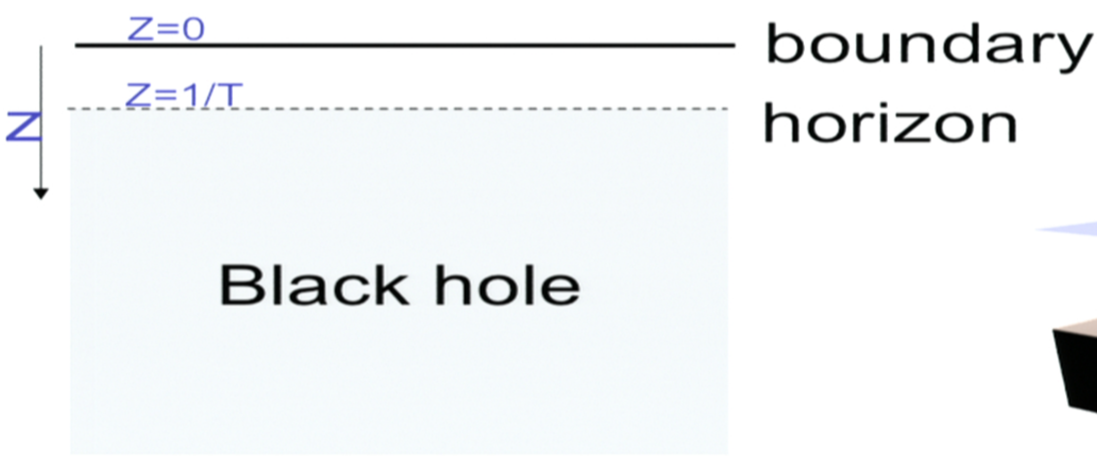
$$g_s = g_{YM}^2; \qquad R / l_s = (g_{YM}^2 N)^{1/4}$$

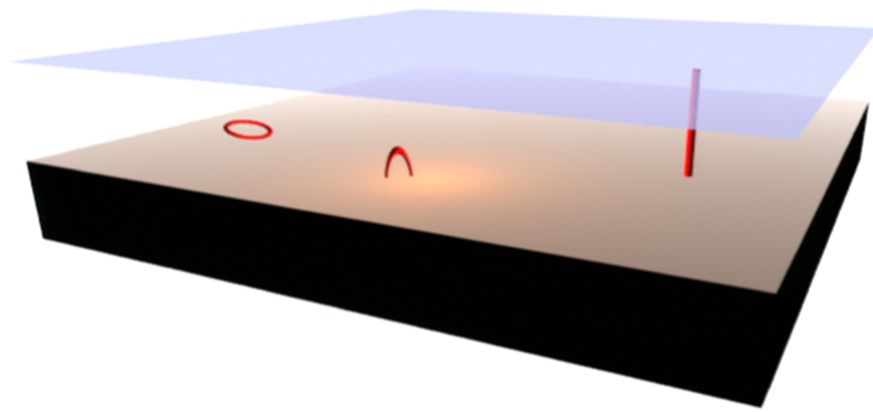
$N \rightarrow \infty, \lambda = g_{YM}^2 N$ fixed ➡

λ large ⟶ string th.
λ small ⟶ field th.

AdS/CFT correspondence and plasma physics

$\mathcal{N}$=4 SYM at finite temperature is a conformal plasma which is dual to a black hole in AdS space.





Hydrodynamics of the $\mathcal{N}$=4 conformal plasma

Low energy excitations of a plasma are given by temperature variations and displacements characterized by T and u_μ (4 variables). We define T and u_μ through:

$$T^{\mu\nu} u_\nu = -3(\pi T)^4 u^\mu$$

$T_{\mu\nu}$ has nine indep.comp. Hydrodynamics determines these 9 variables in terms of u_μ and T.
Then we can use 4 equations:

$$\partial_\mu T^{\mu\nu} = 0$$

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Hydrodynamics from gravity (Battacharyya, Hubeny, Minwalla, Rangamani)

For any conserved $T_{\mu\nu}$ we can find a dual metric $g_{\mu\nu}$ (asymptotically AdS + $\delta g_{\mu\nu} \rightarrow T_{\mu\nu}$). However those metrics are generically singular. For long wavelengths (along the boundary direction) we can systematically find the $T_{\mu\nu}$ that gives rise to a non-singular metric.

$$T^{\mu\nu} = (\pi T)^4 \left(\eta^{\mu\nu} + 4u^\mu u^\nu\right) - 2(\pi T)^3 \sigma^{\mu\nu}$$
$$+ (\pi T)^2 \left((\ln 2) T^{\mu\nu}_{2a} + 2T^{\mu\nu}_{2b} + (2 - \ln 2) \left(\frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e} \right) \right)$$

Policastro, Son, Starinets

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \partial_{(\alpha} u_{\beta)} - \frac{1}{3} P^{\mu\nu} \partial_\alpha u^\alpha$$

$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

BHMR construction (more detail)

Eddington-Finkelstein coordinates (infalling)

$$ds^2 = -2u_\mu dx^\mu dr - r^2 f(br) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu$$

$$f(r) = 1 - \frac{1}{r^4}, \quad u^0 = \frac{1}{\sqrt{1-\beta^2}}, \quad u^i = \frac{\beta^i}{\sqrt{1-\beta^2}}$$

"tube wise approximation":

$$u^\mu \longrightarrow u_\mu(x^\alpha), \qquad b \longrightarrow b(x^\alpha)$$

$$g = g^{(0)}(\beta_i, b) + \epsilon g^{(1)}(\beta_i, b) + \epsilon^2 g^{(2)}(\beta_i, b) + \cdots$$

$$\beta_i = \beta_i^{(0)} + \epsilon \beta_i^{(0)} + \cdots$$

$$b_i = b_i^{(0)} + \epsilon b_i^{(0)} + \cdots$$

$$g_{rr} = 0, \quad g_{r\mu} \propto u_\mu, \quad \mathrm{Tr}\left((g^{(0)})^{-1} g^{(n)} \right) = 0, \ (n > 0)$$

Second order differential eqn. at each order:

$$\mathcal{L}\left[g^{(0)}(\beta_i^{(0)}, b^{(0)})\right]\left(g^{(n)}(x^\alpha)\right) = S_n$$

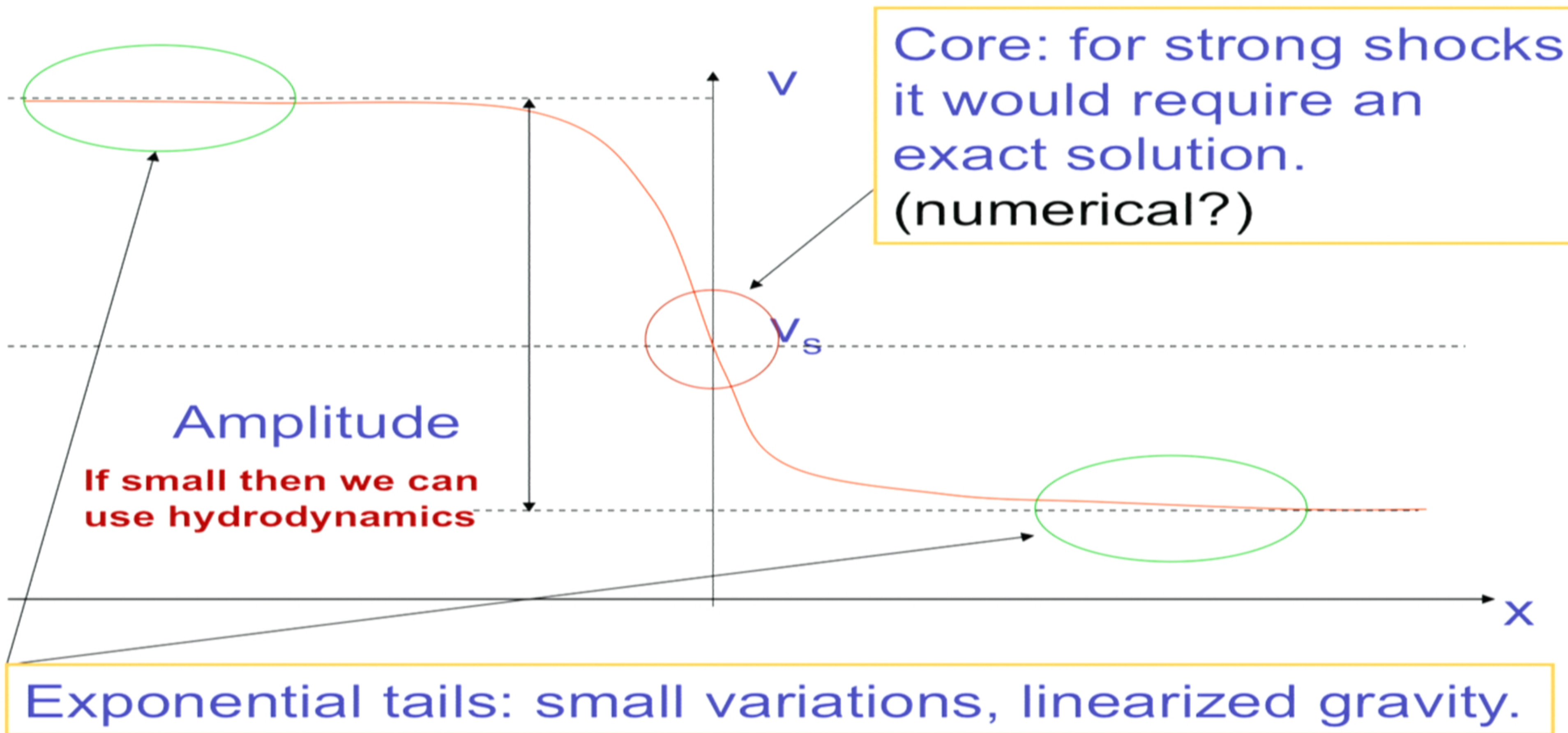
No derivatives of $g^{(0)}$

But always the same!.

We choose non-singular solution at the horizon and normalizable at infinity.

This is the concrete implementation of the procedure.

Gravitational resolution of the shock waves



Weak Shock waves in (gravitational) hydrodynamics

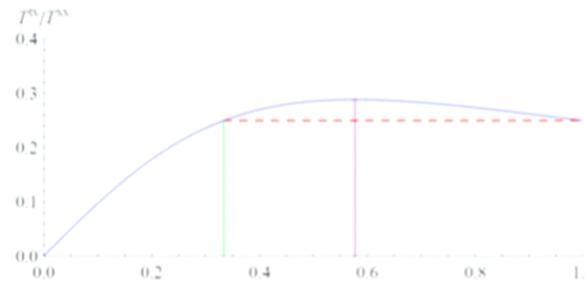
Ideal hydro $T^{\mu\nu} = (\pi T)^4 \left(\eta^{\mu\nu} + 4u^\mu u^\nu\right)$

$$v_s = \frac{1}{\sqrt{3}}$$

Matching

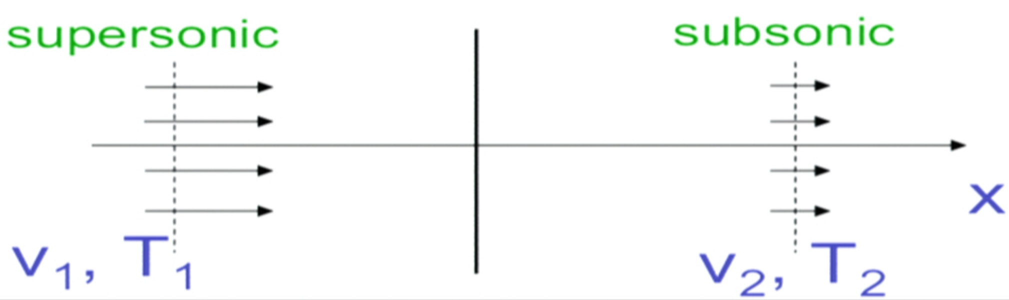
$$T^{tx} = 4pu^t u^x = 4p\frac{v}{1-v^2}\ ,$$

$$T^{xx} = p\left(1+4u_x^2\right) = p\frac{1+3v^2}{1-v^2}\ .$$



gives

$$v_2 = \frac{1}{3v_1}\ , \quad p_2 = p_1\frac{9v_1^2-1}{3(1-v_1^2)}\ , \quad T_2 = T_1\left(\frac{9v_1^2-1}{3(1-v_1^2)}\right)^{1/4}$$



Higher order hydrodynamics: (v breaks time rev.)

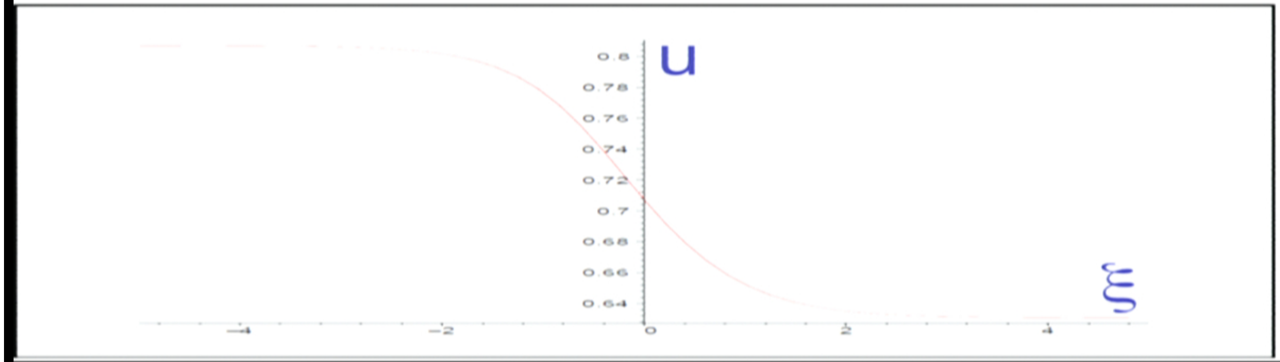
We perform a systematic expansion in the amplitude of the shock: $u = u_{(0)} + u_{(1)} + u_{(2)}$

$$u_{(0)} = \frac{1}{\sqrt{2}} ,$$

$$u_{(1)} = -u_\infty \tanh \xi ,$$

$$u_{(2)} = \frac{u_\infty^2}{6} \left[4\sqrt{2}(1 - \ln 2) \frac{\ln \cosh \xi}{\cosh^2 \xi} + 5\sqrt{2} \left(\tanh^2 \xi + \tanh \xi + \frac{\xi}{\cosh^2 \xi} \right) \right]$$

$$T = T_{(0)} - \frac{\sqrt{2}}{3} T_{(0)} u_{(1)} - \frac{\sqrt{2}}{3} T_{(0)} u_{(2)} + \frac{1}{3\pi} u'_{(1)}$$



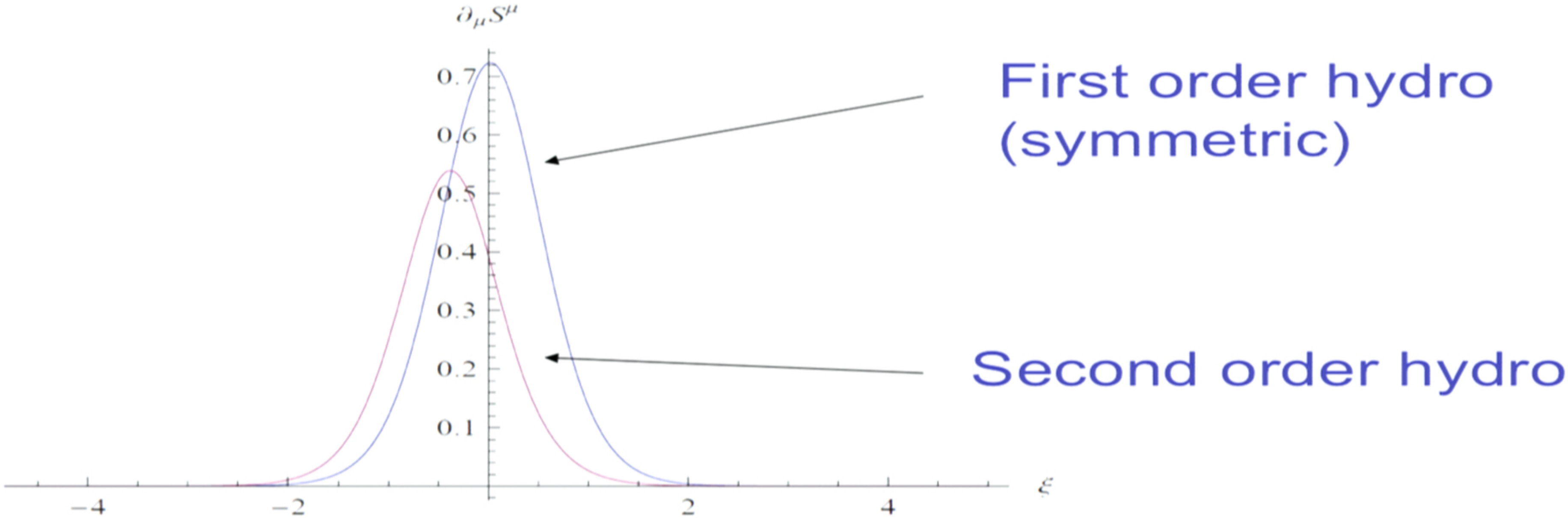
$$u = \frac{v}{\sqrt{1 - v^2}}$$

$$\xi = \frac{4\pi T_{(0)} u_\infty}{3} x$$

Entropy generation:

Current: $s^\mu = 4\pi\eta u^\mu - \frac{\tau_\pi \eta}{4T}\sigma^{\kappa\nu}\sigma_{\kappa\nu}u^\mu$ (Loganayagam)

$$\partial_\mu s^\mu = \frac{\eta}{2T}\sigma^{\mu\nu}\sigma_{\mu\nu} \ .$$



Gravity dual: (using BHMR construction)

$$\begin{aligned} ds^2 &= -2u_\mu dx^\mu dr + \frac{1}{r^2}((\pi T)^4 + k)u_\mu u_\nu dx^\mu dx^\nu + r^2\eta_{\mu\nu}dx^\mu dx^\nu + \frac{1}{r^2}j\tilde{u}_\mu u_\nu dx^\mu dx^\nu \\ &\quad + \alpha r^2(\tilde{u}_\mu \tilde{u}_\nu dx^\mu dx^\nu - \frac{1}{2}(dy^2 + dz^2)) \ . \end{aligned}$$

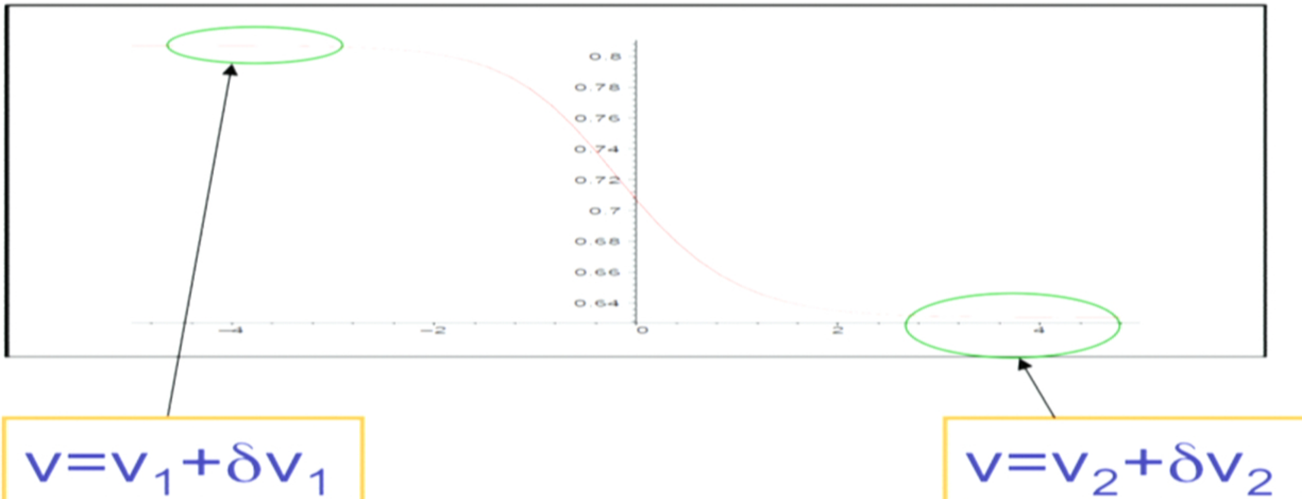
$$\tilde{u}^\mu = (u(x), u^0(x), 0, 0)$$

$$\begin{aligned} k &= \frac{2}{3}r^3(u'_{(1)} + u'_{(2)}) - \frac{\sqrt{2}}{3}r^2 u''_{(1)} \ , \\ j &= -\frac{2}{\sqrt{3}}r^3(u'_{(1)} + u'_{(2)}) + \frac{4\sqrt{2}}{3\sqrt{3}}u_{(1)}u'_{(1)}(\pi T_{(0)})^3 F_2\left(\frac{r}{\pi T_{(0)}}\right) \ , \\ \alpha &= \frac{u'_{(1)} + u'_{(2)}}{3\pi T_{(0)}}F_1\left(\frac{r}{\pi T_{(0)}}\right) - \frac{4\sqrt{2}}{9\pi T_{(0)}}u_{(1)}u'_{(1)}F_3\left(\frac{r}{\pi T_{(0)}}\right) \ . \end{aligned}$$

$$F_1(y) = \ln\left(\frac{(1+y^2)(1+y)^2}{y^4}\right) - 2\arctan y + \pi$$

$$\begin{aligned} F_2(y) &= \frac{1}{2}(y^4 - 1)\left(2\arctan y + \ln\left(\frac{1+y^2}{(1+y)^2}\right)\right) - \frac{1}{2}\pi y^4 + y^3 + y^2 - \frac{25}{12} \ . \\ F_3'(y) &= \frac{1}{y^5 - y}\left\{2(1 - y^3)\left(\arctan y - \frac{\pi}{2} - \ln(1+y)\right) + (1 + y^3)\ln(1 + y^2)\right. \\ &\quad \left. -4y^3\ln y + 7 - 2\ln 2 - \frac{2(1+y)}{1+y^2} - \frac{2}{1+y} - 4y^2\right\} \ . \end{aligned}$$

Strong shock waves (linearized gravity)



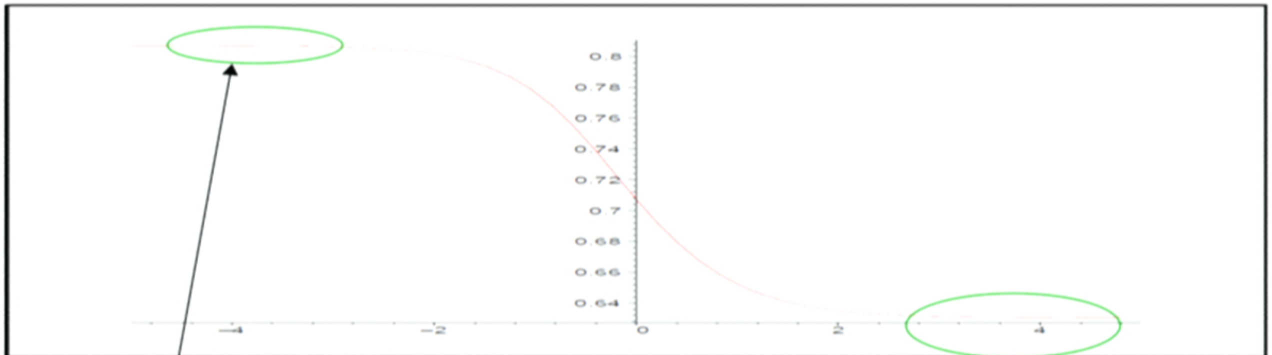
Even for strong shocks $\delta v_1/v_1 << 1$ $(\delta v_2/v_2 << 1)$
We can use linearized gravity.

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu}(t,x,r) = r^2 H_{\mu\nu}(r) e^{-i\omega t + iqx}$$

Boosted black hole

$\omega=0$, q imag.

Strong shock waves (linearized gravity)



$v=v_1+\delta v_1$

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Boundary conditions:

Infalling boundary conditions at the horizon are analytically continued to give the correct b.c. breaking the time reversal symmetry leading to the correct shock wave. Equivalently we ask regularity in Eddington-Filkenstein coordinates.

$$ds_0^2 = r^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{r_0^4}{r^2}(dt \cosh\beta - dx \sinh\beta)^2 + \frac{dr^2}{r^2(1 - r_0^4/r^4)}$$

$$ds_1^2 = r^2 \left[H_{00} dt^2 + H_{11} dx^2 + 2H_{01} dt dx + H(dy^2 + dz^2) \right] e^{iqx}$$

$$Z(r) = H_{00}(r) + \left(1 + \frac{r_0^2}{r^4}\gamma^2\right) H(r)$$

$$Z'' + P(u)Z' + Q(u)Z = 0$$

$$P(u) = \frac{3 + 3u^2 - 5\gamma^2 u^2 + 3\gamma^2 u^4}{u f(u)(\gamma^2 u^2 - 3)},$$

$$Q(u) = -\frac{4\gamma^2 u^2}{f(u)(\gamma^2 u^2 - 3)} + q^2 \frac{\gamma^2 u^2 - 1}{4 u f^2(u)}$$

$$u = r_0^2/r^2 \qquad \gamma = \cosh\beta$$

$$f(u) = 1 - u^2$$

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Results:

$v = 0$ **(fluid at rest)** $iq = 2.3361$

$v = 1/\sqrt{3}$ **(the speed of sound)** $q = 0,\ Z(u) = u^2$

$v = \sqrt{2/3}$ **(the singular point)** $iq = \sqrt{2}$

$v \to 1$ **(ultrarelativistic limit)** $iq_0(v) \ = \ 1.895\sqrt{\gamma}$

$$x = \frac{u^2}{u_1^2} = \frac{1}{3}u^2 \cosh^2\beta$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

Scaling

$$Z'' + \frac{2}{1-x}Z' + \frac{p^2}{16}\left(3 - \frac{1}{x}\right)\frac{1}{\sqrt{x}}Z = 0$$

$$p^2 \equiv q^2 u_1 = \frac{q^2\sqrt{3}}{\cosh\beta}$$

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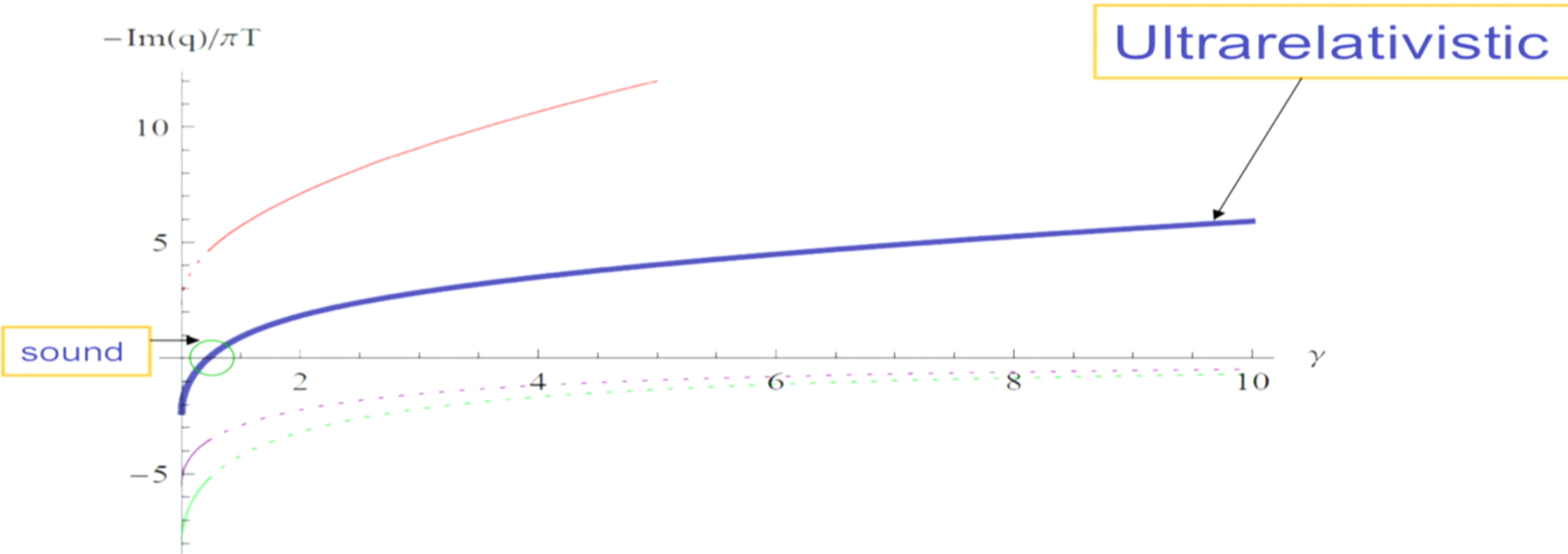
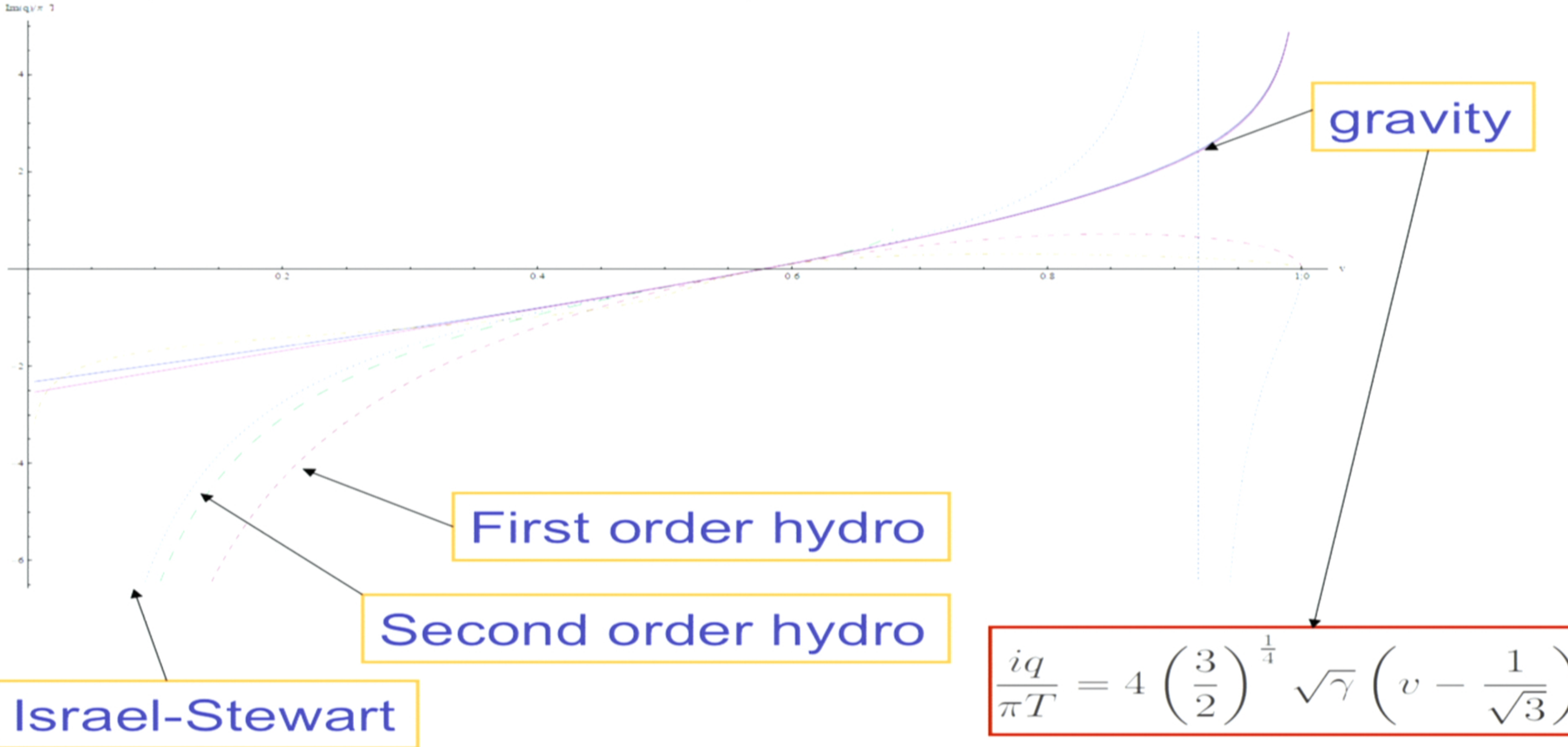
Numerically:



Comparison with hydrodynamics:



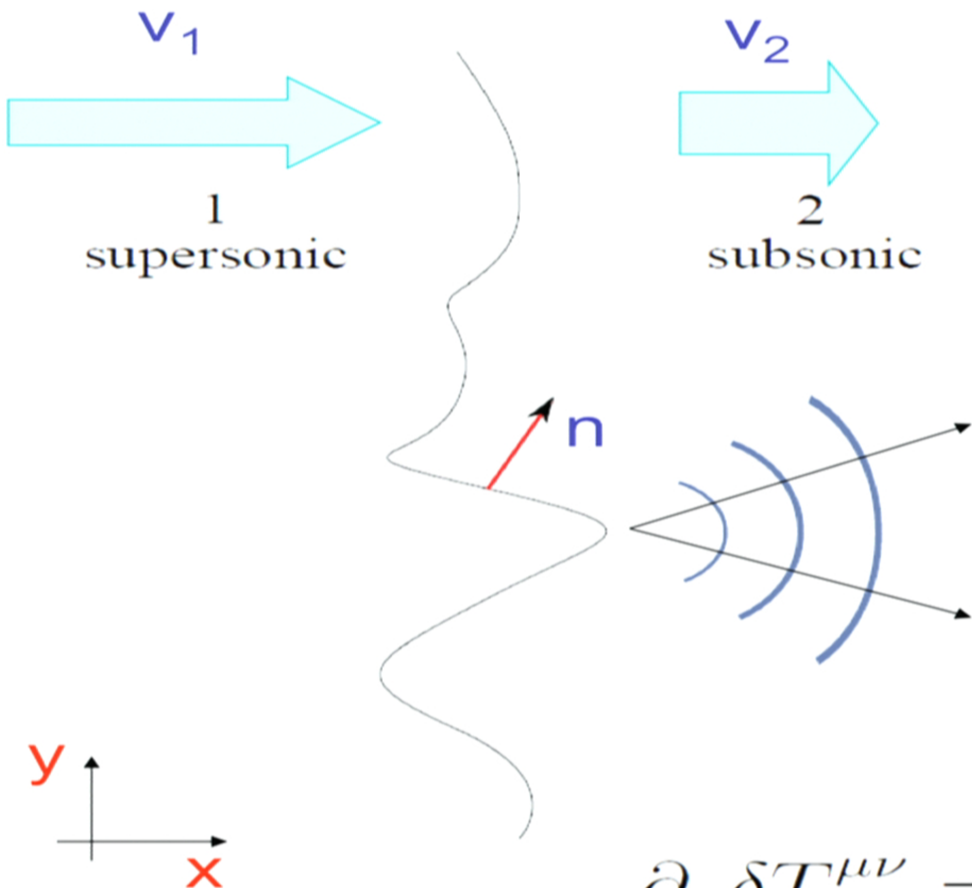
$$\frac{iq}{\pi T} = 4\left(\frac{3}{2}\right)^{\frac{1}{4}}\sqrt{\gamma}\left(v - \frac{1}{\sqrt{3}}\right)$$

Further results:

The scaling of $\mathbf{q}$ for $\gamma \longrightarrow \infty$ probes the ultraviolet of the theory and should be interesting to study in other cases. Even in the conformal case the exponent depends on the dimension as $\mathbf{q} \sim \gamma^{2/d}$

For a stationary solution the surface gravity should be constant? This would mean the temperature is uniform. However the very definition of surface gravity requires the existence of a Killing vector becoming light-like at the horizon. We do not seem to have any.

Numerical metrics.



Matching conditions

$$n_\mu T_1^{\mu\nu} = n_\mu T_2^{\mu\nu}$$

$$T^{\mu\nu} = T^4 \left(\eta^{\mu\nu} + 4u^\mu u^\nu\right)$$

$$x = \zeta = \zeta_0 e^{ik_y y - i\omega t}$$

$$\delta n^\mu = -i\zeta(\omega, 0, k_y, 0)$$

$n^\mu = (0,1,0,0)$

$$\partial_\mu \delta T_2^{\mu\nu} = 0, \qquad \text{subsonic side,}$$

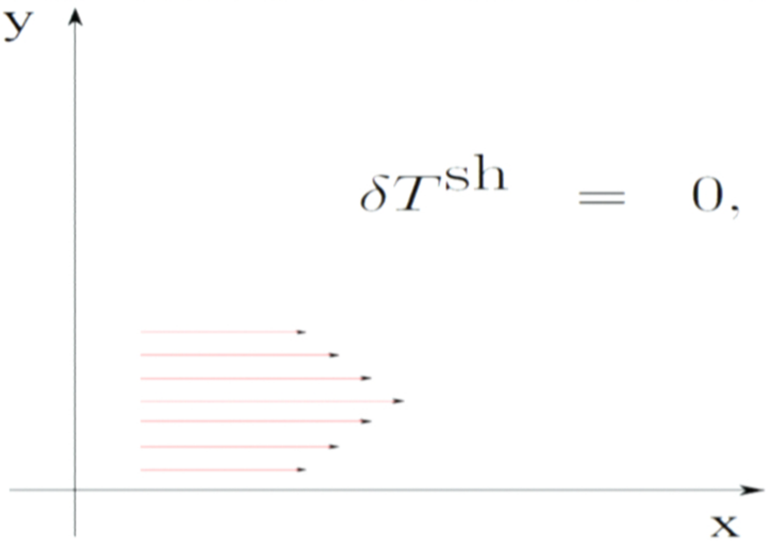
$$\delta n_\nu \left(T_1^{\mu\nu} - T_2^{\mu\nu}\right) = n_\mu \delta T_2^{\mu\nu}, \qquad \text{interface}$$

Generation of sound waves:
(corrugation instability)

Similarly as in the case of domain walls, the surface of the shock can oscillate. These oscillations generate sound-waves. In the case of viscous fluids the oscillations are damped but can persist for a long time for large wave-lengths.

In the subsonic side we have a superposition of a sound and a shear wave.

Shear wave: Fluid at rest $v_x(y)$ is a solution (ideal fluid)



$$\delta T^{\rm sh} = 0, \quad \delta v_x^{\rm sh} = -\frac{v_2 k_y}{\omega}\delta v_y^{\rm sh}, \quad k_x^{\rm sh} = \frac{\omega}{v_2}$$

Sound: pressure wave (c_s^2=1/3)

$$\delta v_x^{\rm S} = \frac{\delta T^{\rm S}}{T_2}(1-v_2^2)\frac{k_x - \omega v_2}{\omega - k_x v_2}, \quad \delta v_y^{\rm S} = \frac{\delta T^{\rm S}}{T_2}(1-v_2^2)\frac{k_y}{\omega - k_x v_2}$$

Conclusions

We performed a systematic study of how gravity resolves shocks in AdS/CFT for the $\mathcal{N}=4$ conformal plasma.

For weak shocks we solve for the hydrodynamic shock and reconstructed the metric. fluid $\longrightarrow$ gravity

For strong shocks we computed the exponential tails and found an interesting scaling for large γ factor:
$i\,q \sim \gamma^{1/2}$ gravity $\longrightarrow$ fluid

Shock waves are important probes of the microscopic description of the theory and should be carefully studied.