Title: Looking for Quantum Gravity in the Space of Conformal Field Theories

Date: Jun 13, 2012 02:00 PM

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Abstract: Conformal field theories have many applications ranging from continuous phase transitions in Statistical Mechanics to models of beyond the Standard Model physics in Particle Physics.

In this talk, I will explain another remarkable application: some conformal field theories can be used to define and study Quantum Gravity.

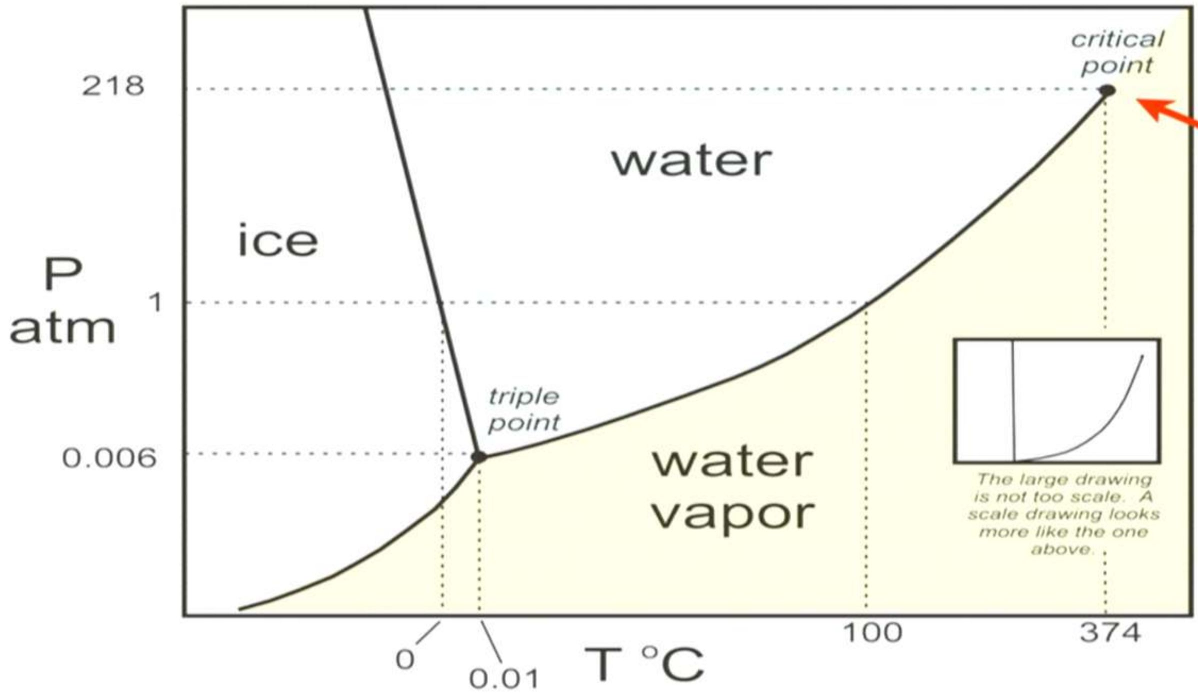
I will also try to give a brief summary of some of the main ideas being discussed at the conference "Back to the Bootstrap II".

Back to the
Bootstrap II
$T_{\mu\nu}$
ε
a_{UV}
a_{IR}

Outline

- Why study Conformal Field Theory?
  - Continuous Phase Transitions
  - Quantum Gravity
- What we are doing
  - c-Theorems
  - Supersymmetry and Integrability
  - Numerical Bootstrap
- What we would like to do

Continuous Phase Transitions



Critical
Opalescence

Scale
invariant
system

Critical Exponents

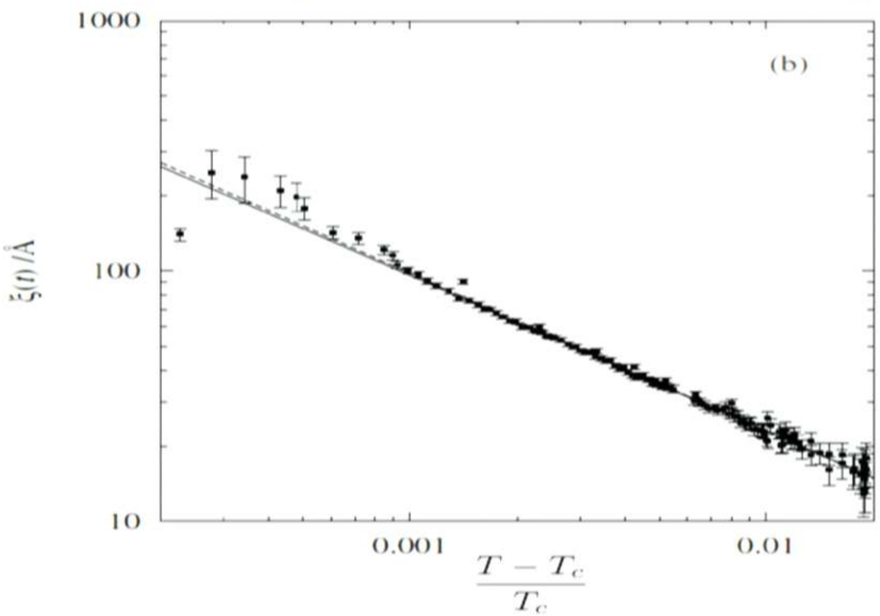
Correlation length of D_2O near the critical point

[Sullivan, Neilson, Fischer, Rennie '00]

$$\xi \sim \left(\frac{T - T_c}{T_c}\right)^{-\nu}$$

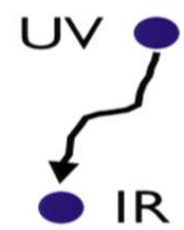
$$\nu = 0.62 \pm 0.03$$

3d Ising model
universality class



Particle Physics

Conformal field theories describe UV and IR fixed points.
They are the stepping stone for our description of dynamics in the continuum: Quantum Field Theory.



Could the TeV scale of Particle Physics require a strongly coupled CFT description?

Watch the PI colloquium of Slava Rychkov to know more about the idea of Conformal Technicolor.

[Luty, Okui '04]

[Rattazzi, Rychkov, Tonni, Vichi '08]

Quantum Gravity

General Relativity is a low energy effective field theory that requires UV completion.

$$G_N = M_{Pl}^{-2} = L_{Pl}^2$$

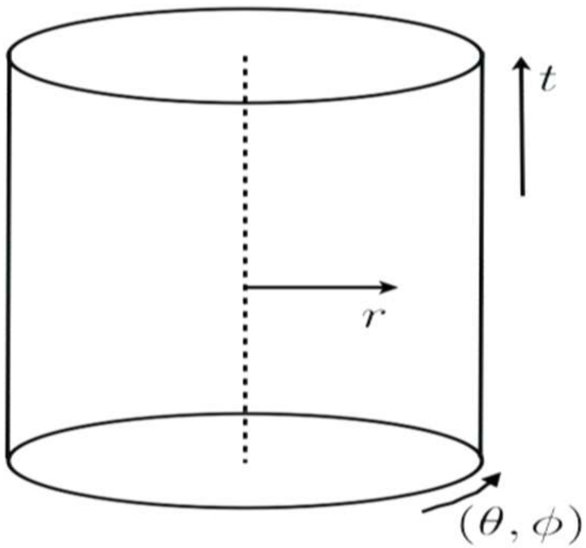
Definition: a quantum theory of gravity is a quantum mechanical theory (unitary time evolution) whose low energy dynamics is well described by General Relativity.

In particular, it should have the same low lying energy spectrum.

QG in a box: Anti-de Sitter

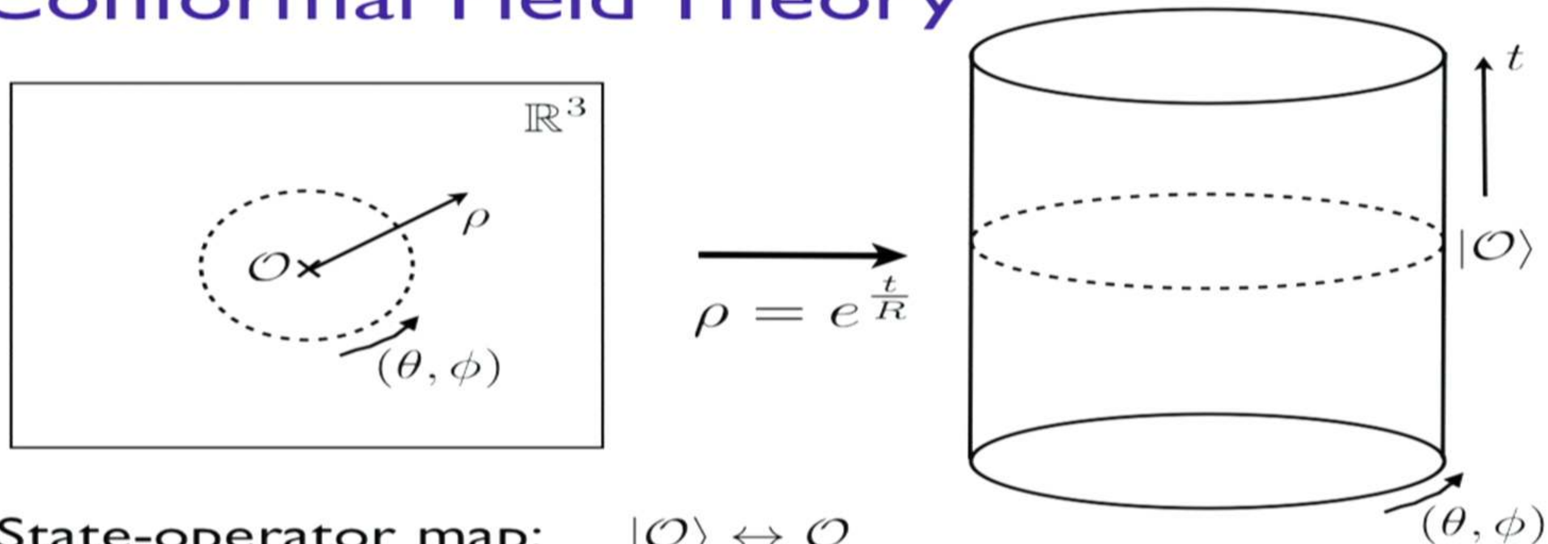
Anti-de Sitter spacetime of radius R

$$ds^2 = -\cosh^2\left(\frac{r}{R}\right)dt^2 + dr^2 + R^2\sinh^2\left(\frac{r}{R}\right)(d\theta^2 + \sin^2\theta d\phi^2)$$



Do we know any quantum system with this low-energy spectrum?

Conformal Field Theory

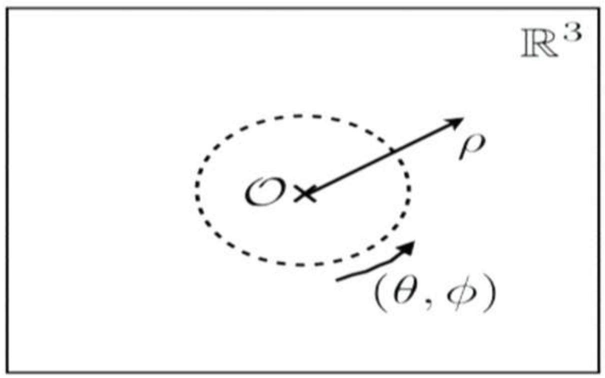


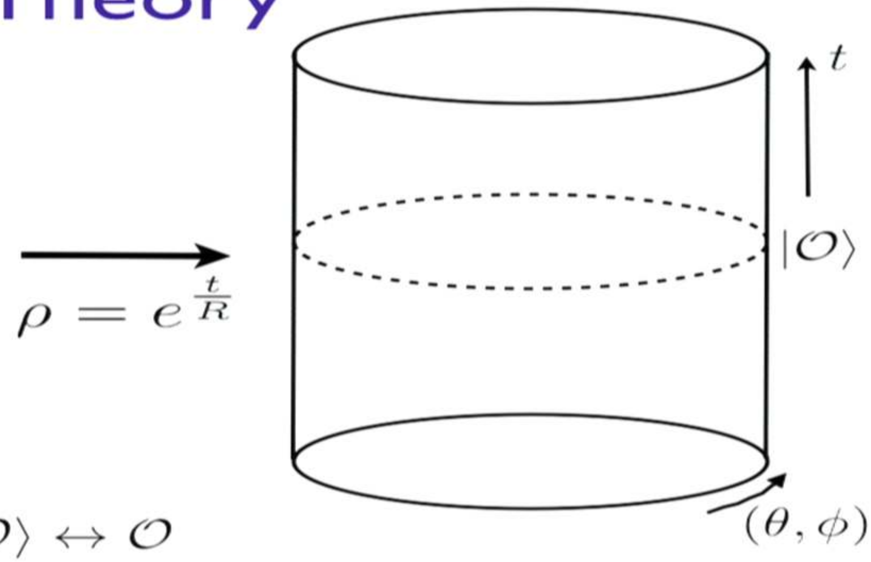
State-operator map: $|\mathcal{O}\rangle \leftrightarrow \mathcal{O}$

Dilatation operator in $\mathbb{R}^3$ maps to the Hamiltonian on $S^2 \times \mathbb{R}_t$

$$\Delta = ER$$

Conformal Field Theory





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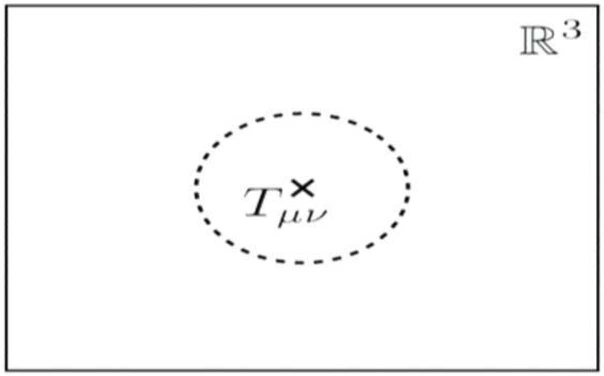
CFT Spectrum

States created by insertions of the stress-energy tensor have a discrete spectrum

$$\Delta(T_{\mu\nu}) = 3$$
$$\Delta(\partial_{\alpha_1} \ldots \partial_{\alpha_n} T_{\mu\nu}) = 3 + n$$



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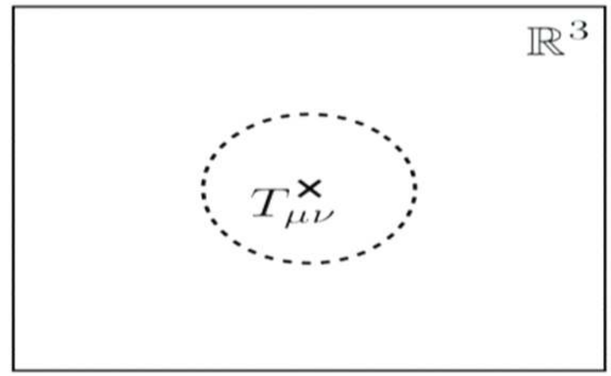
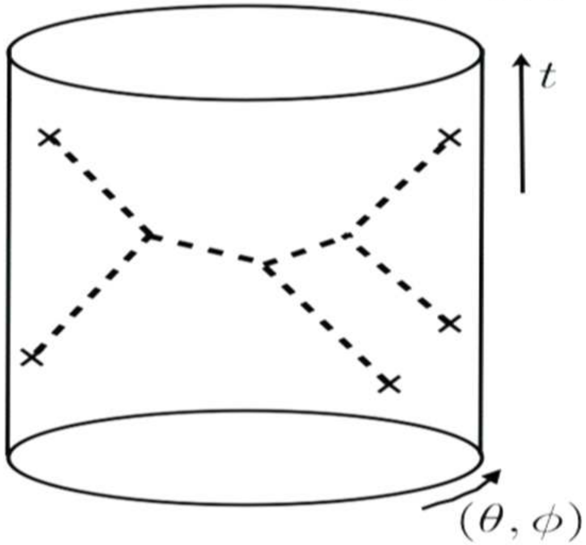


Observables

Correlation functions are natural observables of the CFT

$$\langle T_{\mu_1\nu_1}(x_1)\ldots T_{\mu_n\nu_n}(x_n)\rangle$$

$x = (t, \theta, \phi)$



These correspond to graviton "scattering amplitudes" and form a complete set of observables.
No local observables in quantum gravity. [See Nima's Colloquium]

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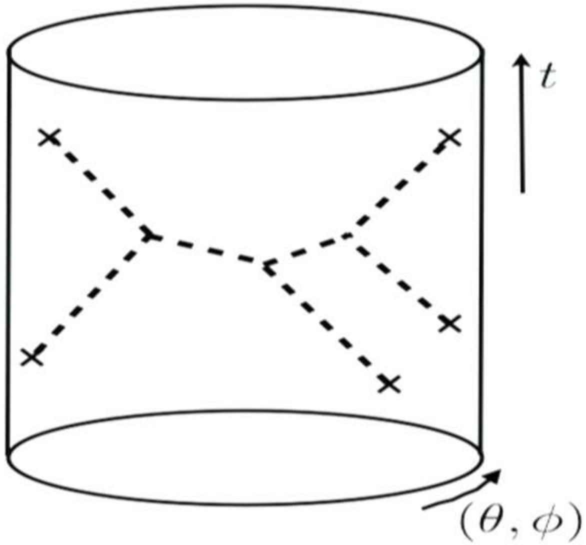

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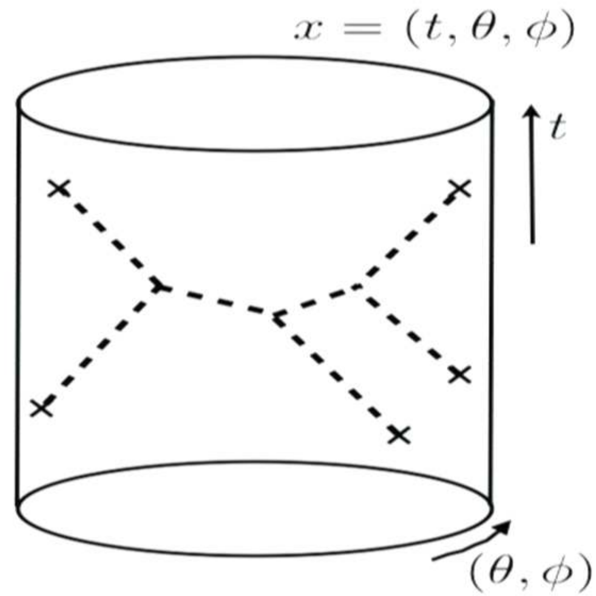

Weak coupling implies factorization

$$\langle T_{\mu_1\nu_1}(x_1)\dots T_{\mu_n\nu_n}(x_n)\rangle_c \sim \left(\frac{L_{Pl}}{R}\right)^{n-2}$$

Quantum Gravity = CFT

Definition: quantum gravity with AdS boundary conditions is a 3-dimensional CFT obeying:

- Large gap in its spectrum of dimensions

$$\Delta(\mathcal{O} \neq T_{\mu\nu}) > \Lambda_{UV} R \gg 1$$

- Factorization

$$\langle T_{\mu_1\nu_1}(x_1) \dots T_{\mu_n\nu_n}(x_n)\rangle_c \sim (M_{Pl}R)^{2-n}$$

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Can we find such a CFT?

- Conformal QFTs in 3 dimensions are not hard to find - there are many examples based on Chern-Simons gauge fields plus charged matter (bosons and/or fermions).

- Factorization is easy - it follows from the large N expansion of SU(N) gauge theories.

$$T_{\mu\nu} = \frac{1}{N} Tr\left(\partial_\mu \Phi \partial_\nu \Phi\right) + \ldots \qquad N \sim \frac{R}{L_{Pl}}$$

- Large gap in the spectrum of dimensions is hard - this requires strong coupling!

$$\mathcal{O} = \frac{1}{N} Tr\left(\Phi^2\right) \qquad \Delta(\mathcal{O}) = 1 + \gamma > \Lambda_{UV} R \gg 1$$

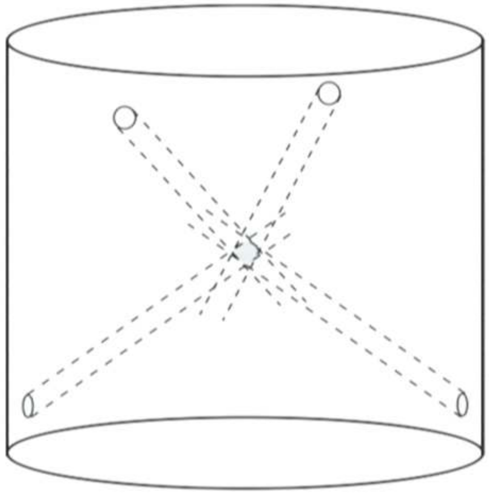
anomalous dimension

There are supersymmetric examples (ABJM).

QG in flat space

Take the zero curvature limit of AdS: $R \to \infty$

Main difficulty: local bulk physics is encoded in CFT correlation functions in a non-trivial way.
How to extract the S-matrix?



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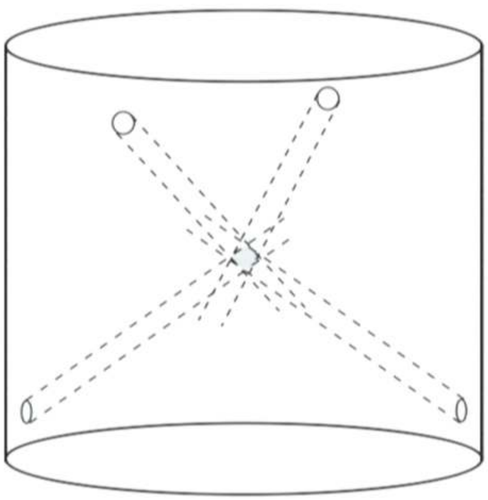
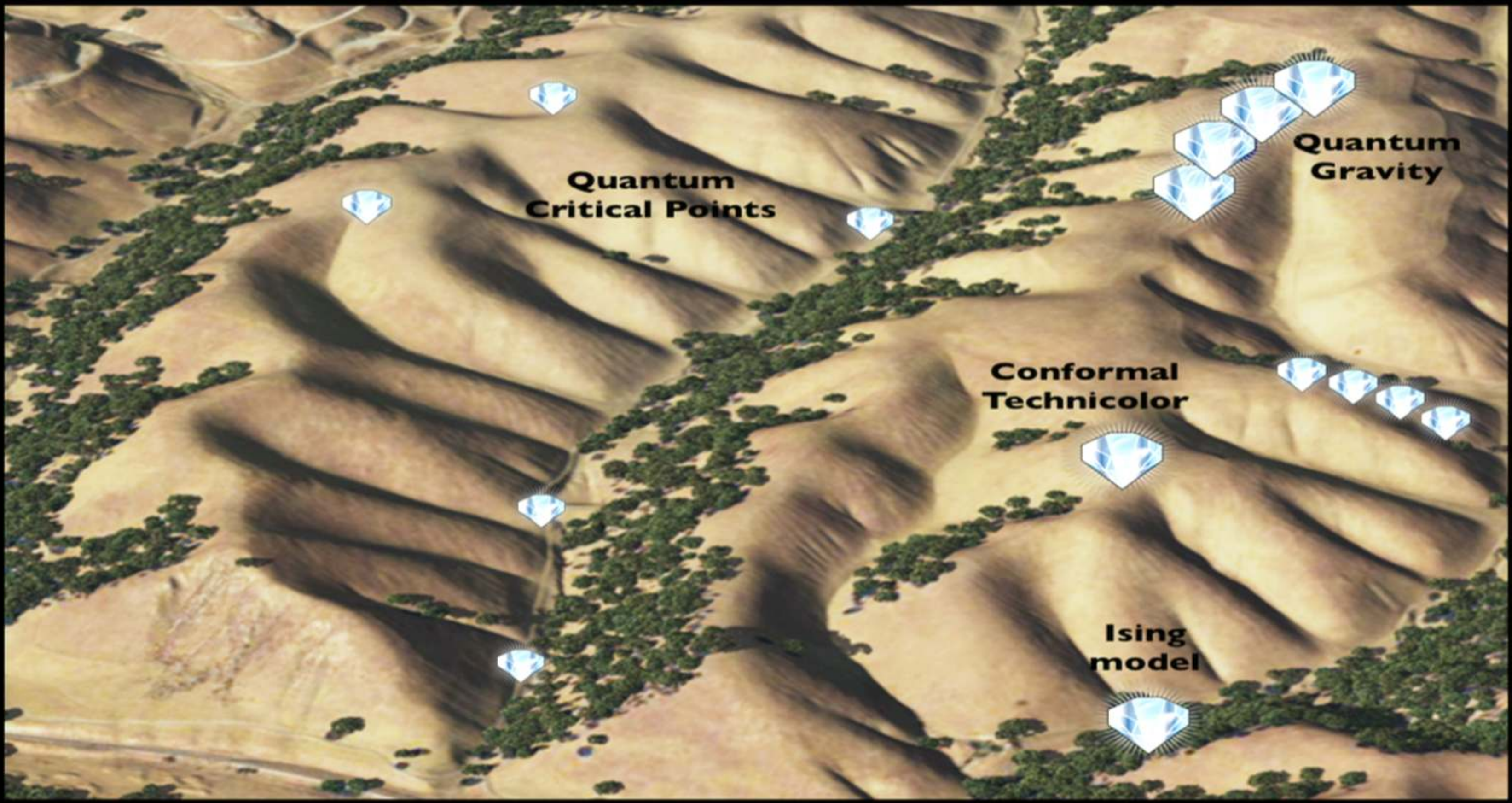


Idea: prepare wave-packets that scatter in small region of AdS
[Polchinski '99] [Susskind '99] [Gary, Giddings, JP '09]

Best language: Mellin amplitudes
[Mack '09]

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \int_{-i\infty}^{i\infty}[d\delta]\boxed{M(\delta_{ij})}\prod_{i<j}^{n}\Gamma(\delta_{ij})\left(x_{ij}^2\right)^{-\delta_{ij}}$$

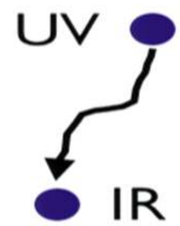

Quantum
Critical Points
Quantum
Gravity
Conformal
Technicolor
Ising
model

c-Theorems

The space of CFTs is very large and complex.

Weakly coupled CFTs are only a small part of the landscape.

Can we use Renormalization Group (RG) flows to organize the landscape?

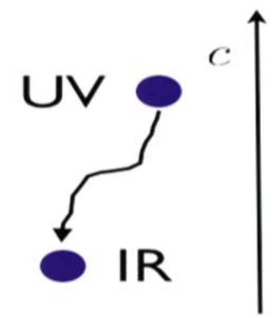


c-Theorem in d=2

[Zamolodchikov 86']

In a Lorentz invariant and unitarity 2d QFT, there is a function c(g) that decreases under RG flow.

At RG fixed points (CFT), $\beta_i(g) = 0$, this function is stationary and equal to the central charge of the CFT.



$$c_{IR} < c_{UV}$$

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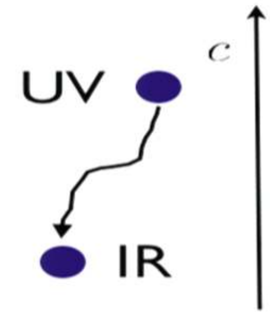

$$c_{IR} < c_{UV}$$

$$s = \frac{\pi}{3} c\, T \qquad \langle T_{\mu\nu}(x) T_{\alpha\beta}(0) \rangle = \frac{c}{x^4} I_{\mu\nu,\alpha\beta}(x)$$

$$-\frac{c}{6} = \int_{S^2} \langle T^{\mu}_{\mu} \rangle \qquad T^{\mu}_{\mu} = -\frac{c}{12} \mathcal{R}$$

c-Theorem in d=4?

$$s = \tilde{c}\, T^3$$

$$\langle T_{\mu\nu}(x) T_{\alpha\beta}(0) \rangle = \frac{c}{x^4} I_{\mu\nu,\alpha\beta}(x)$$

$$-2a = \int_{S^4} \langle T^\mu_\mu \rangle$$

$$T^\mu_\mu = -a\, E_4 - c\, W^2$$

Euler density

Weyl tensor

a-Theorem in d=4

[Cardy '88]
[Komargodski, Schwimmer '11]

$$\cancel{s = cT^3}$$

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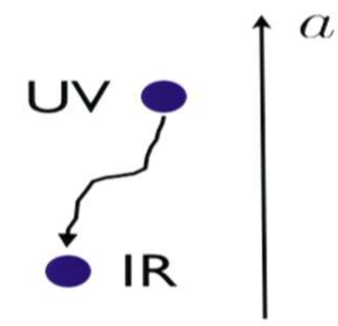
Two CFTs connected by
an RG flow must satisfy

$$\boxed{a_{IR} < a_{UV}}$$



c-Theorem in d=3?

No conformal anomaly

$$T^{\mu}_{\mu} = 0$$

F-Theorem in d=3

[Myers, Sinha '10]
[Jafferis, Klebanov, Pufu, Safdi '11]
[Casini, Huerta '12]

No conformal anomaly

$$T^{\mu}_{\mu} = 0$$

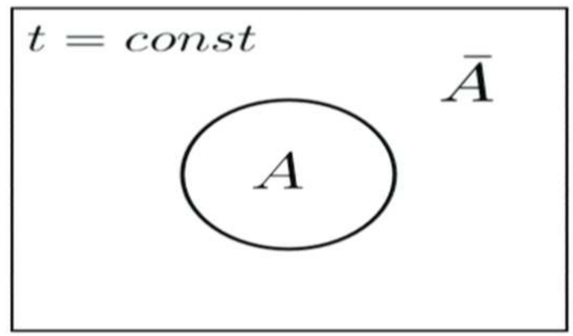
Free energy on 3-sphere

$$F = -\log Z_{S^3}|_{\text{finite part}}$$

Entanglement Entropy of a circle

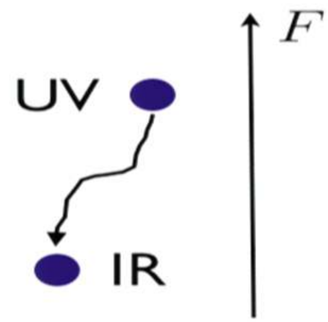
$$S_{EE}(A) = -Tr(\rho_A \log \rho_A)$$

$$F = S_{EE}(A)|_{\text{finite part}}$$



Two CFTs connected by an RG flow must satisfy

$$\boxed{F_{IR} < F_{UV}}$$



Supersymmetry and Integrability

Supersymmetry and Integrability

In planar N=4 SYM, we can compute the spectrum of the dilatation operator for all values of the 't Hooft coupling. The current focus is on studying higher point functions.

There is a very large class of (calculable) Superconformal Field Theories (N=2), some of which are not continuously connected to free theories. [Gaiotto '09]

Strategy I: These special theories are the perfect laboratory to develop new techniques for describing CFTs.

Strategy II: These special theories are calculable and closer to interesting theories than free theories.

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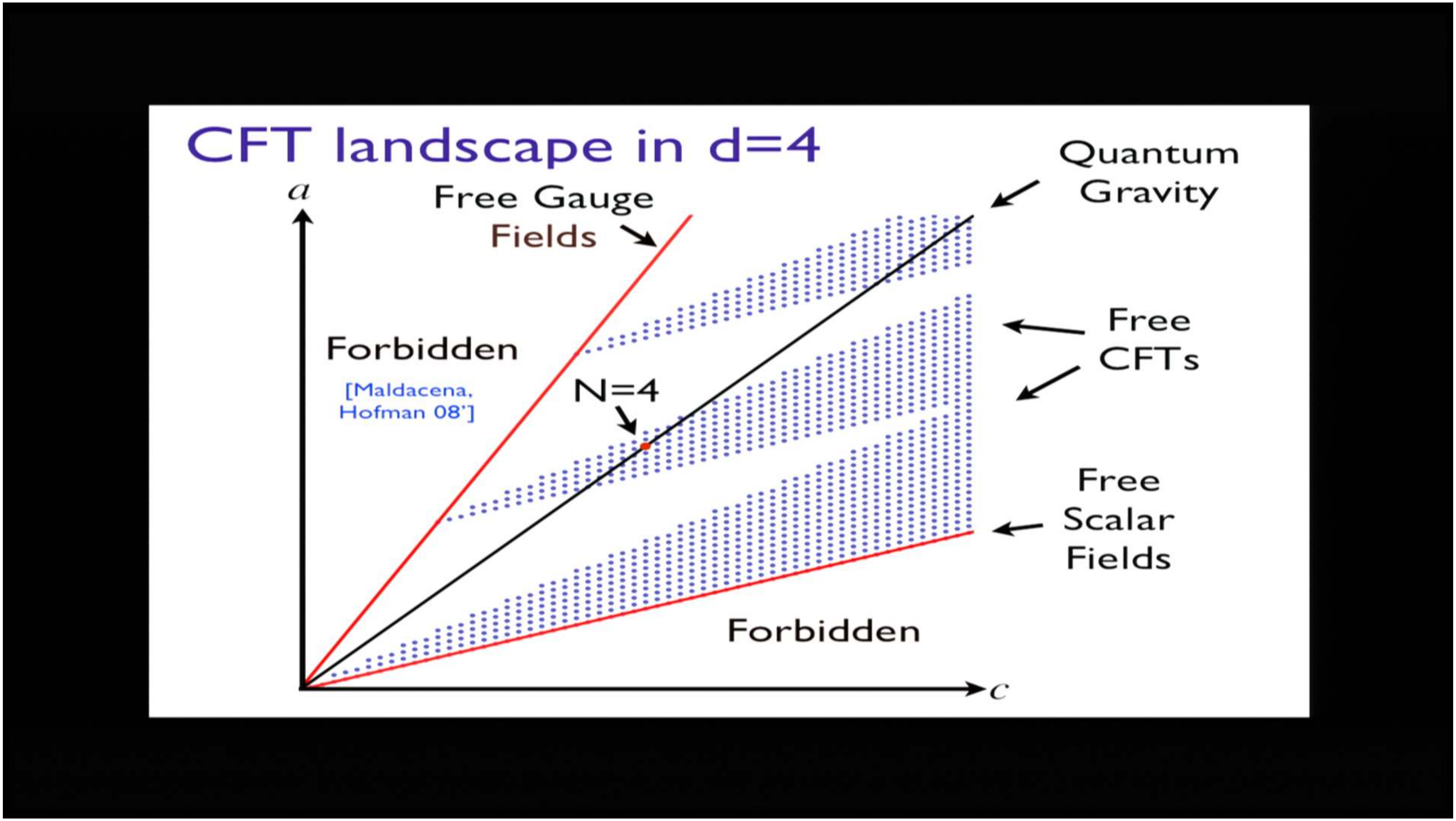
CFT landscape in d=4
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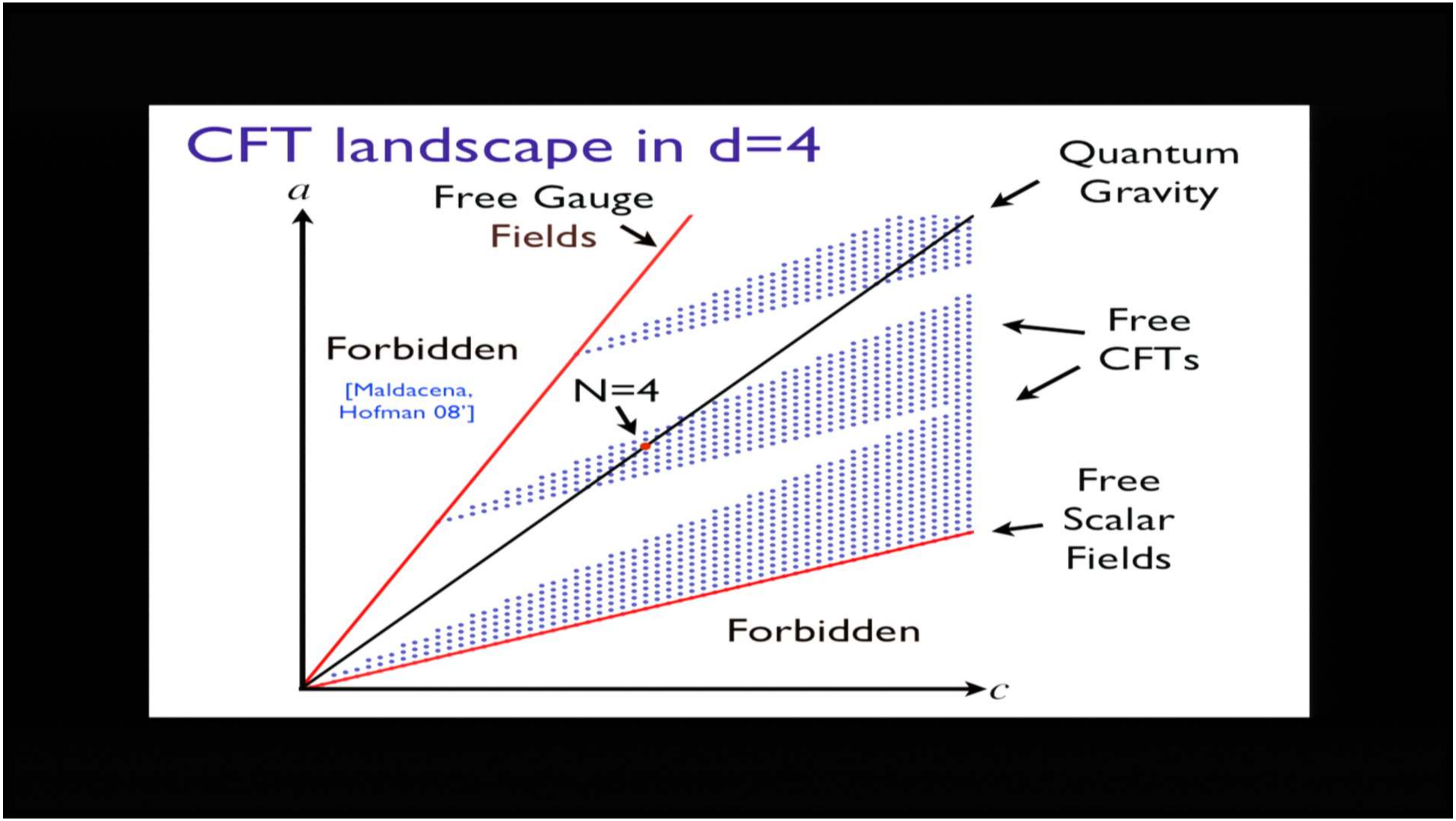
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Conformal Bootstrap

A CFT is a consistent set of correlation functions.

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Operator Product Expansion (OPE)

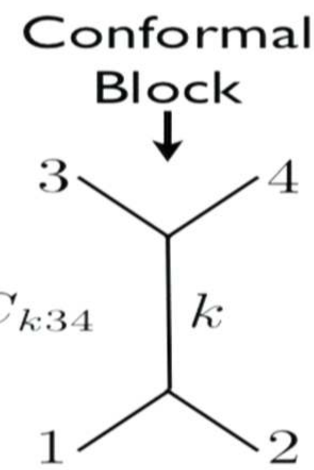
$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \frac{C_{12k}}{x^{\Delta_1+\Delta_2-\Delta_k}}\mathcal{O}_k(0)$$

The CFT data $\{\Delta_i, C_{ijk}\}$ defines all correlations functions by successive use of the OPE. The CFT data is constrained by unitarity and associativity of the OPE.

$$\text{Unitarity} \quad \Longrightarrow \quad \begin{array}{l} \Delta_i \geq \dfrac{d-2}{2} \\ C_{ijk} \in \mathbb{R} \end{array}$$

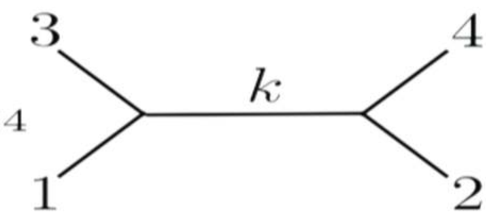
Bootstrap Equations

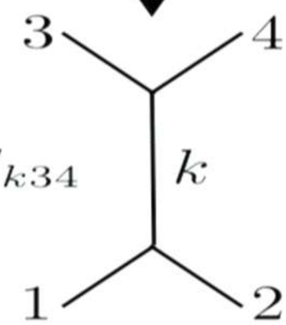
$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \sum_k C_{12k}C_{k34}$$



Bootstrap Equations

Conformal Block

$$\sum_k C_{13k} C_{k24}$$ 

$$= \sum_k C_{12k} C_{k34}$$ 

Associativity of the OPE gives rise to an infinite set of equations for the CFT data $\{\Delta_i, C_{ijk}\}$ [Polyakov '74]

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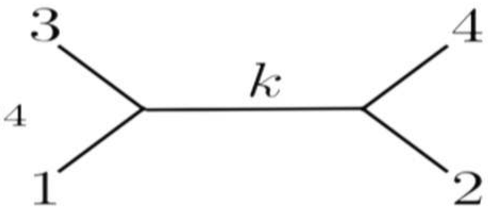

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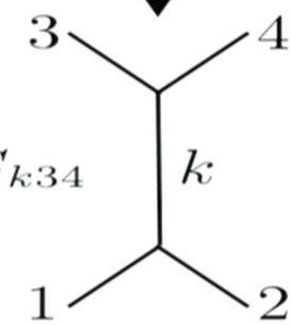

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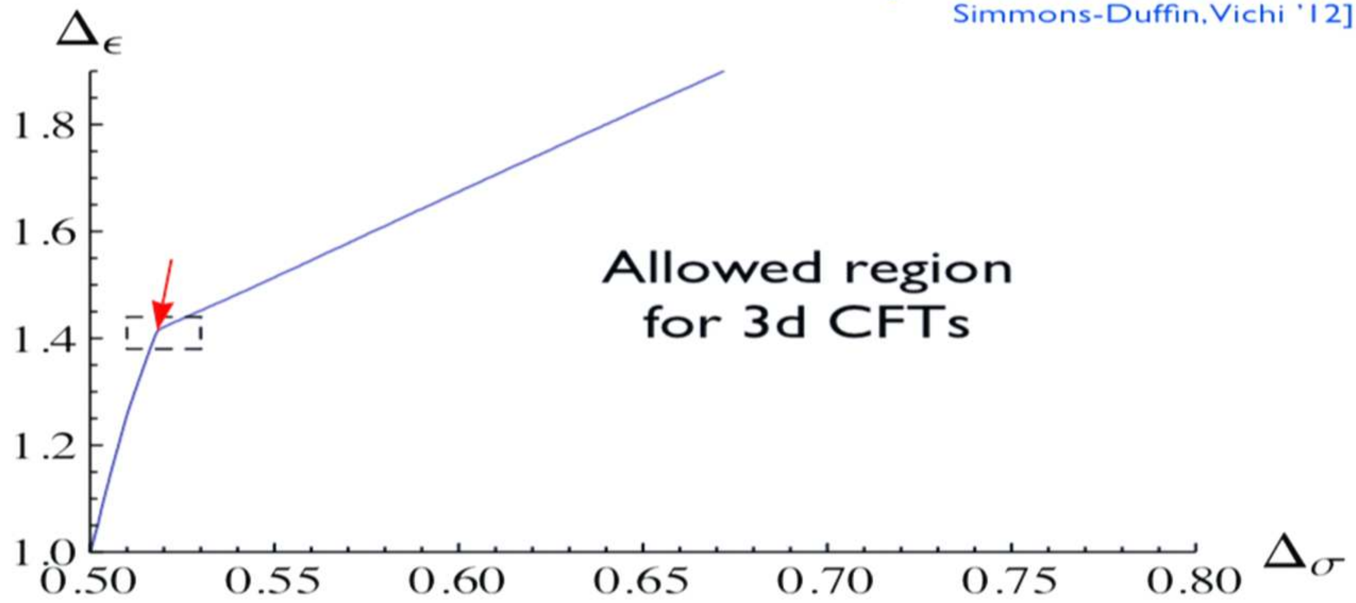
Bounds from Bootstrap equations

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi '12]



$$\sigma \times \sigma \sim \epsilon + \ldots$$

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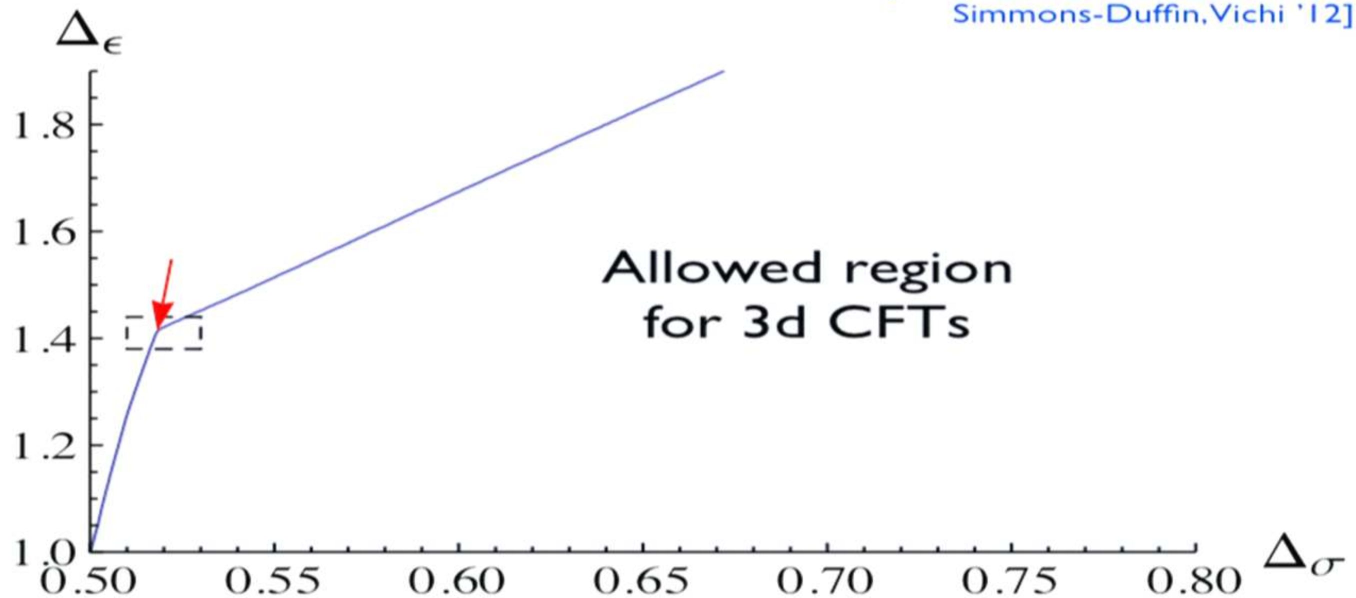

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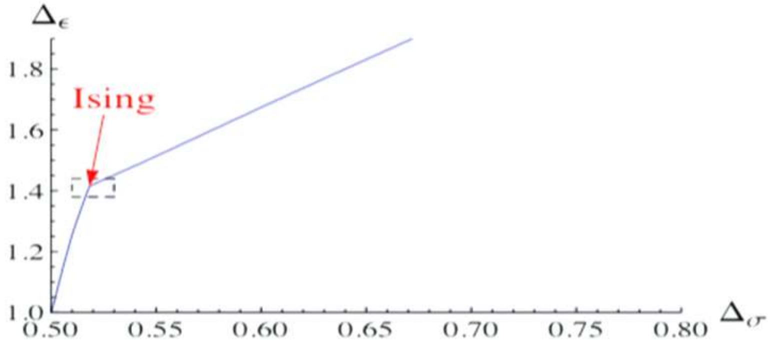
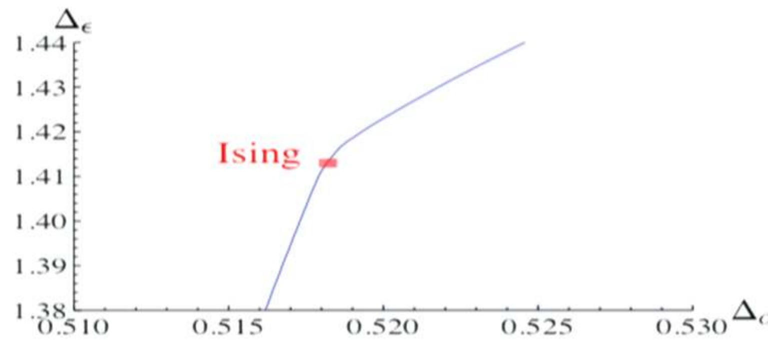
Example: 3d Ising model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi '12]





Operator	Spin l	$\mathbb{Z}_2$	Δ	Exponent
σ	0	$-$	0.5182(3)	$\Delta = 1/2 + \eta/2$
σ'	0	$-$	$\gtrsim 4.5$	$\Delta = 3 + \omega_A$
ε	0	$+$	1.413(1)	$\Delta = 3 - 1/\nu$
ε'	0	$+$	3.84(4)	$\Delta = 3 + \omega$
ε''	0	$+$	4.67(11)	$\Delta = 3 + \omega_2$
$T_{\mu\nu}$	2	$+$	3	n/a
$C_{\mu\nu\kappa\lambda}$	4	$+$	5.0208(12)	$\Delta = 3 + \omega_{\mathrm{NR}}$

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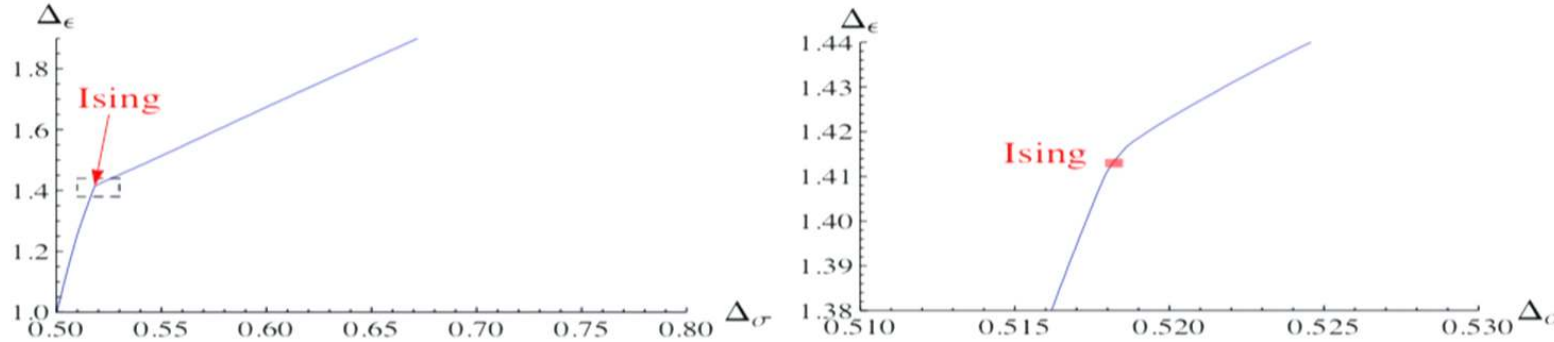

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What we would like to do

Problems for the future

- Formulate the numerical bootstrap program as an algorithm that finds (all) CFTs consistent with a given set of assumptions
- Interacting CFTs in d>6?
- Find a CFT that UV completes pure Einstein gravity (or show that it does not exist)

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Problems for the future

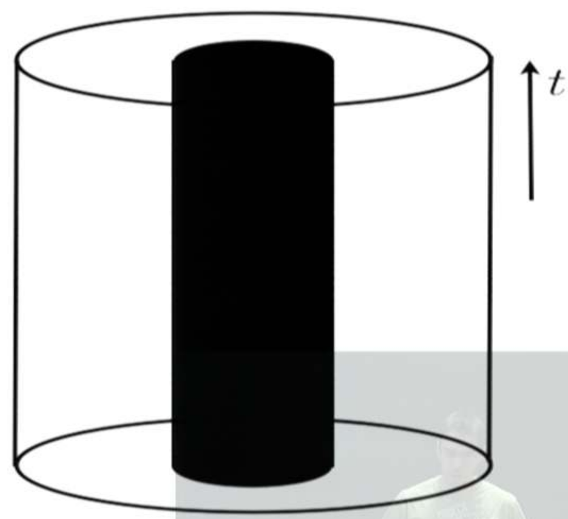
- Show that all CFTs that provide UV completions of GR give rise to the exact area law of black hole entropy

CFT$_d$ entropy density: $s = \tilde{c}\, T^{d-1}$

$$\langle T_{\mu\nu}(x) T_{\alpha\beta}(0)\rangle = \frac{c}{x^4} I_{\mu\nu,\alpha\beta}(x)$$

General Relativity

$$S = \frac{A}{4G_N} \qquad \Rightarrow \qquad \boxed{c = \tilde{c}}$$

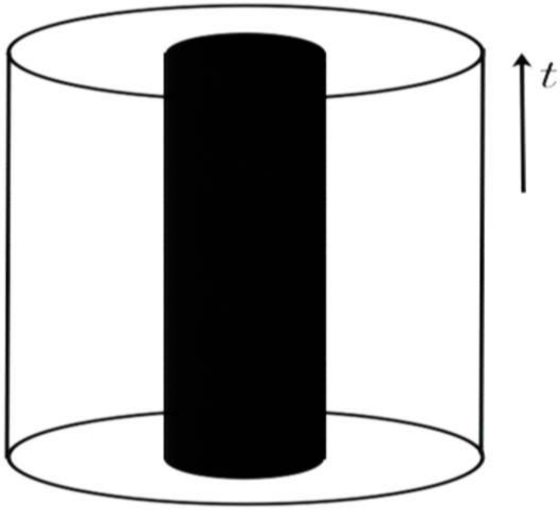


Problems for the future

- Show that all CFTs that provide UV completions of GR give rise to the exact area law of black hole entropy

CFT$_d$ entropy density: $s = \tilde{c}\, T^{d-1}$

$$\langle T_{\mu\nu}(x) T_{\alpha\beta}(0) \rangle = \frac{c}{x^4} I_{\mu\nu,\alpha\beta}(x)$$



General Relativity

$$S = \frac{A}{4G_N} \qquad \Rightarrow \qquad \boxed{c = \tilde{c}}$$


Let's keep mining the
CFT space!