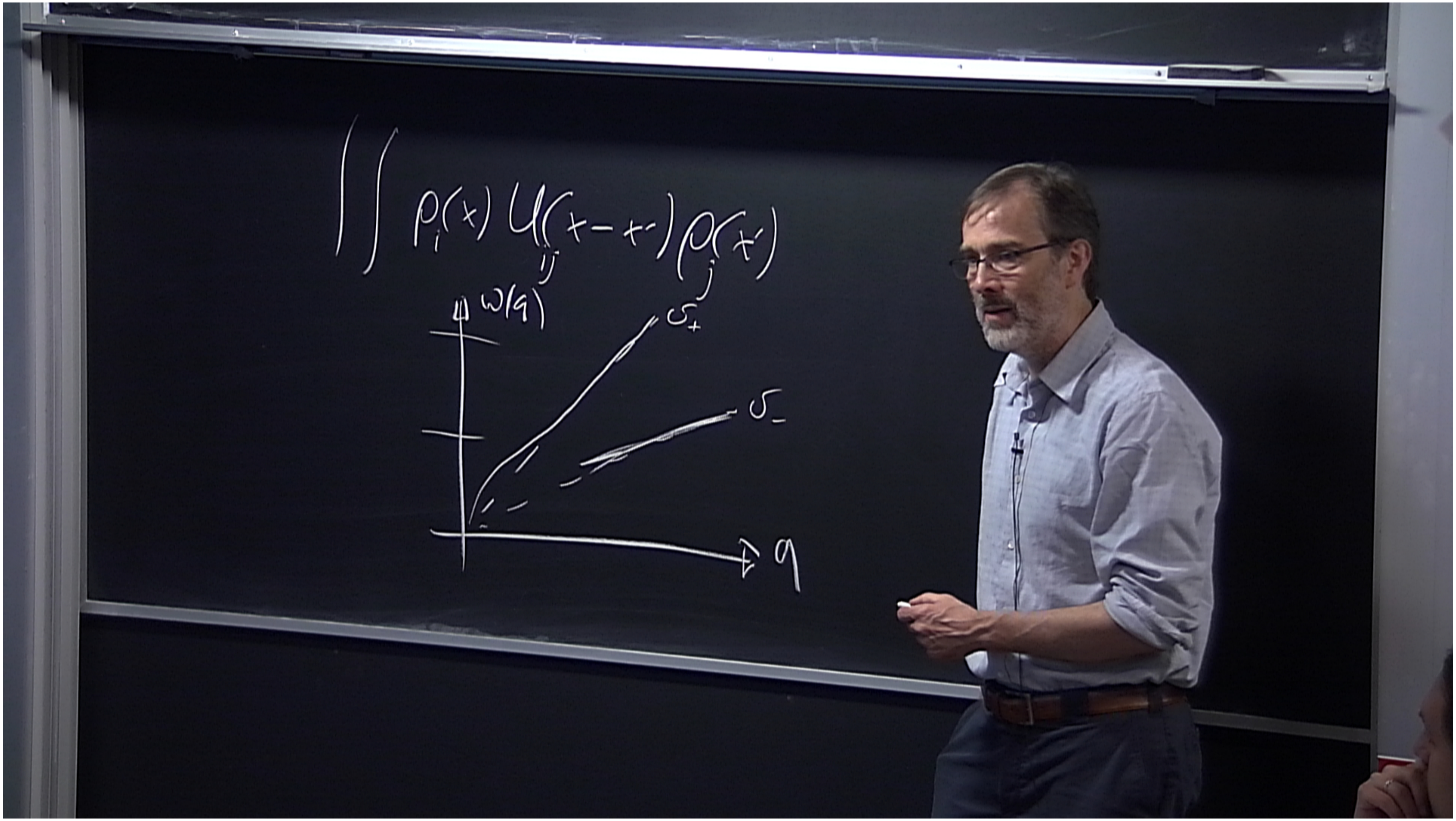
Title: Relaxation in Driven Integer Quantum Hall Edge States

Date: Jun 01, 2012 02:30 PM

URL: http://pirsa.org/12060000

Abstract: This talk will be about non-equilibrium many-body physics in integer quantum Hall edge states far from equilibrium. Recent experiments have generated a highly non-thermal electron distribution by bringing together at a point contact two quantum Hall edge states originating from sources at different potentials. The relaxation of this distribution to a stationary form is observed as a function of distance downstream from the contact [Phys. Rev. Lett. 105, 056803 (2010)]. I will discuss the broader context for the experiments and a physical picture of the equilibration process. I will also present an exact treatment of a minimal model for the experiment with results that account well for the observations. [Joint work with Dmitry Kovrizhin]


ω(q)
c₊
c₋
q

Relaxation in driven integer quantum Hall edge states

John Chalker

Physics Department, Oxford University

Work with Dmitry Kovrizhin

Phys. Rev. B 84, 085105 (2011) and arXiv:1111.3914

Related papers: D. Kovrizhin + JTC: PRB 81 (2010), PRB 80 (2009)

JTC + Y. Gefen and M. Veillette, PRB 76 (2007)

Outline

Motivation

Experiments on evolution of non-equilibrium

electron distribution in QHE edge states

Theoretical Approaches

Quantum quench as idealisation

Exact treatment via bosonization + refermionisation

Results

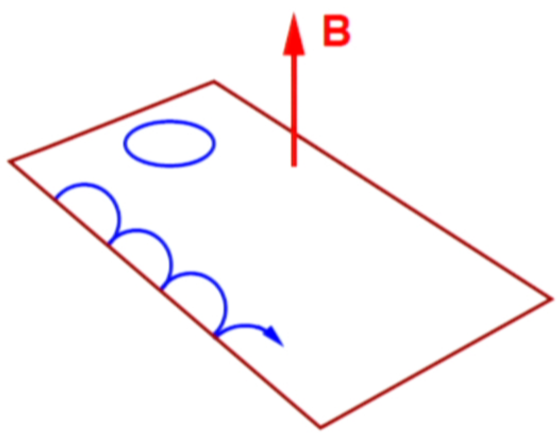
Relaxation in an integrable system

Non-thermal steady state

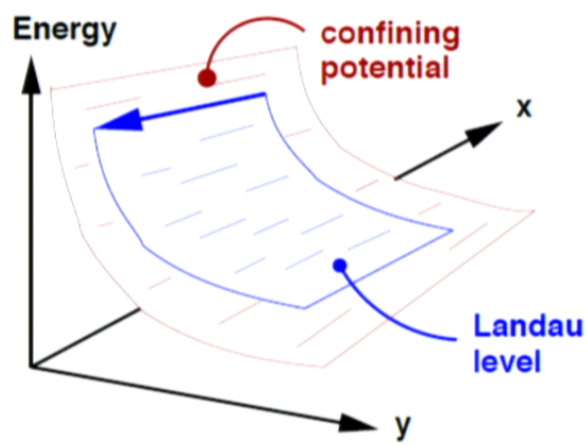
Quantum Hall Edge States

Two-dimensional electron gas in magnetic field

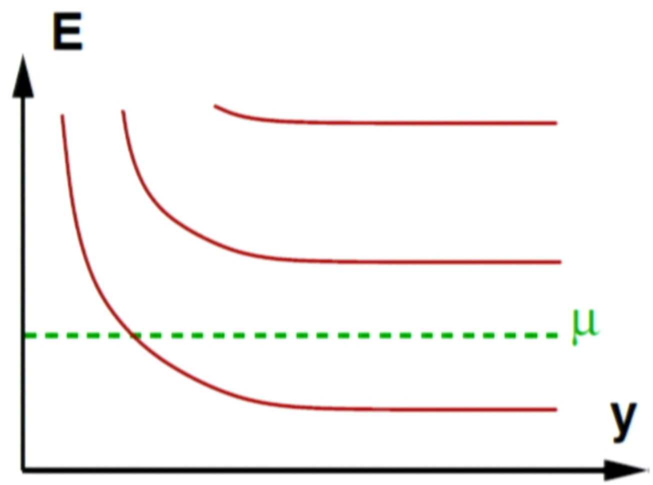
Classical skipping orbits



Quantum edge states



Edge states as Ideal Waveguides

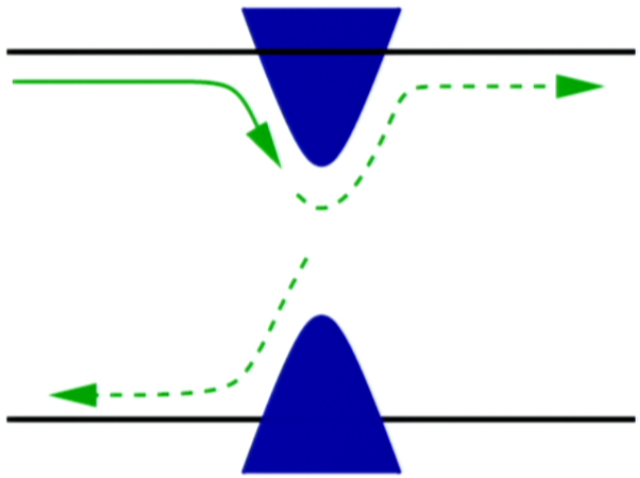


Chiral motion

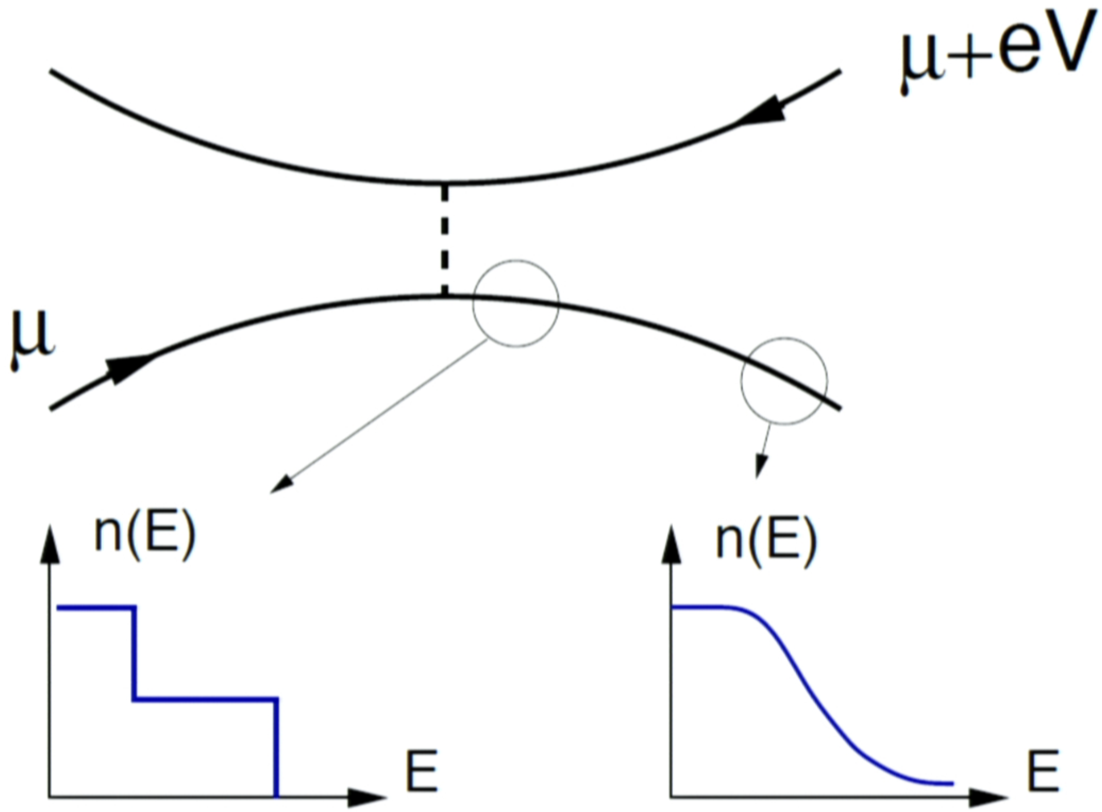
Only possible scattering is in forward direction

Manipulating Edge States

Quantum point contacts as beam splitters

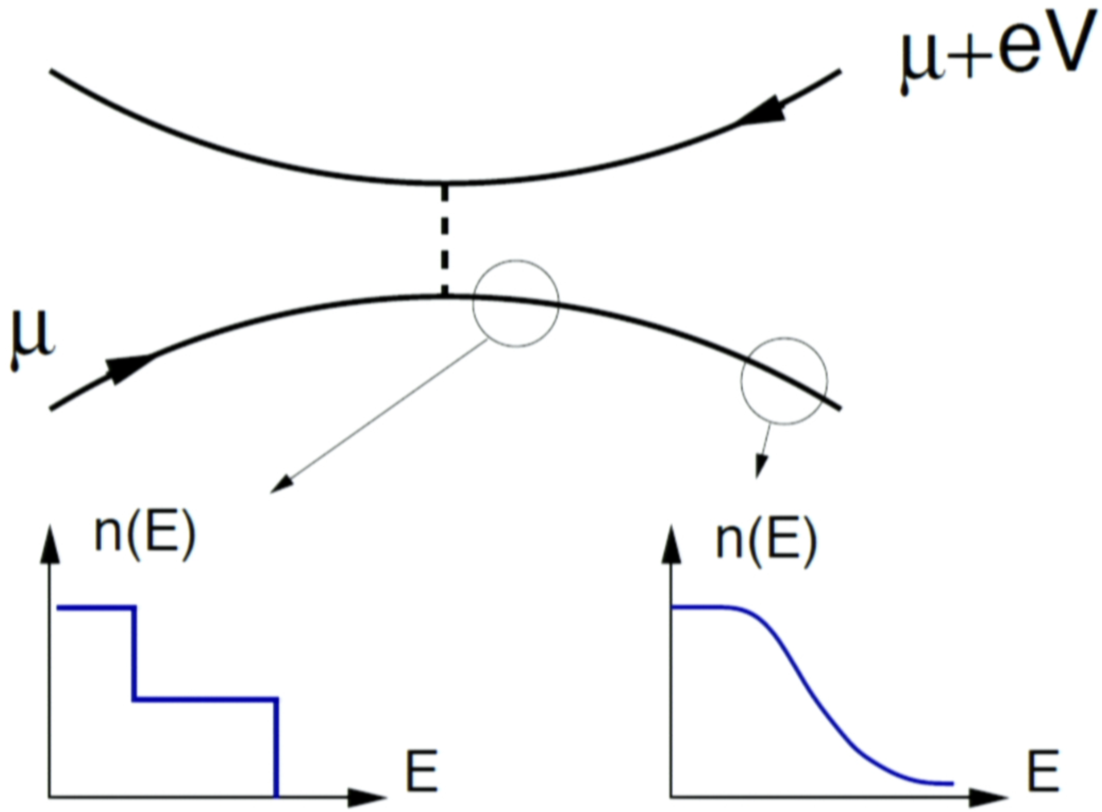


Schematic view of experiment



le Sueur, Altimiras, Gennser, Cavanna, Mailly & Pierre, PRL (2010)

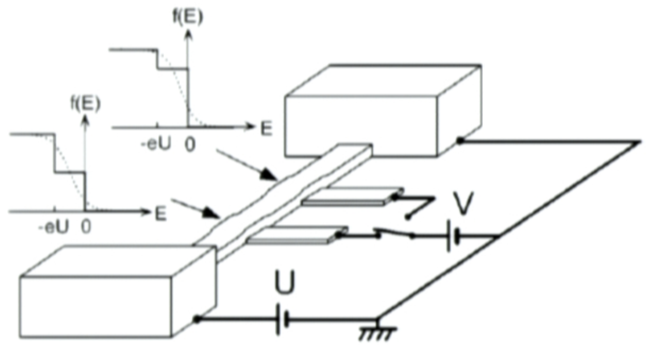
Schematic view of experiment

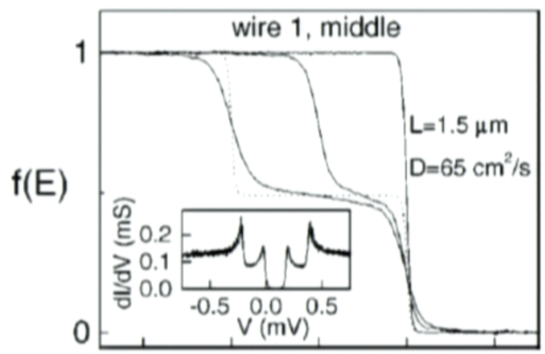


le Sueur, Altimiras, Gennser, Cavanna, Mailly & Pierre, PRL (2010)

Analogue of earlier experiment in wires

Electron distribution in biased diffusive wires

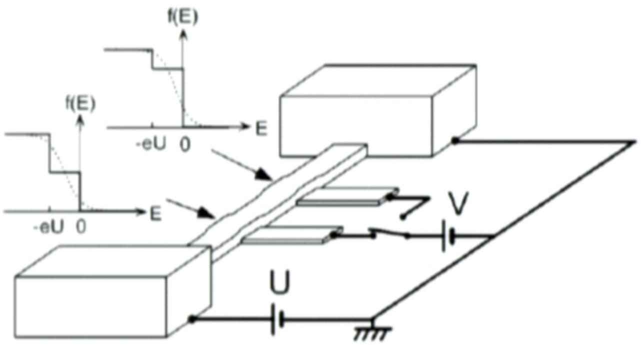


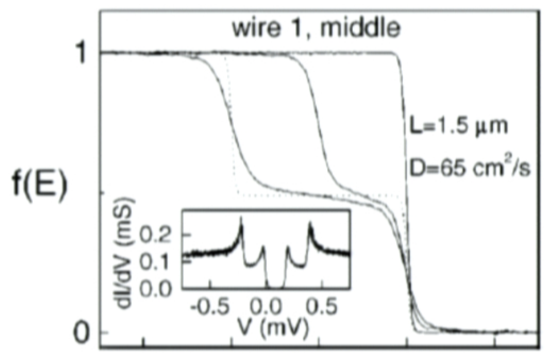


Poithier, Guzron, Birge, Esteve, & Devoret, PRL (1997)

Analogue of earlier experiment in wires

Electron distribution in biased diffusive wires

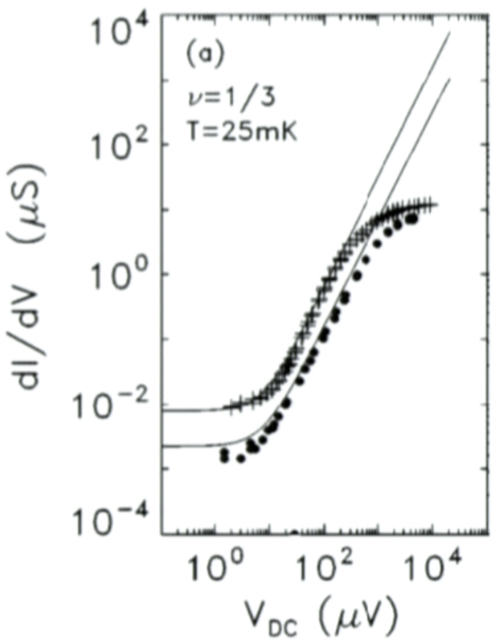




Poithier, Guzron, Birge, Esteve, & Devoret, PRL (1997)

Contrast with tunnelling probe of fractional quantum Hall edge states

Voltage-dependent differential conductance from electron correlations

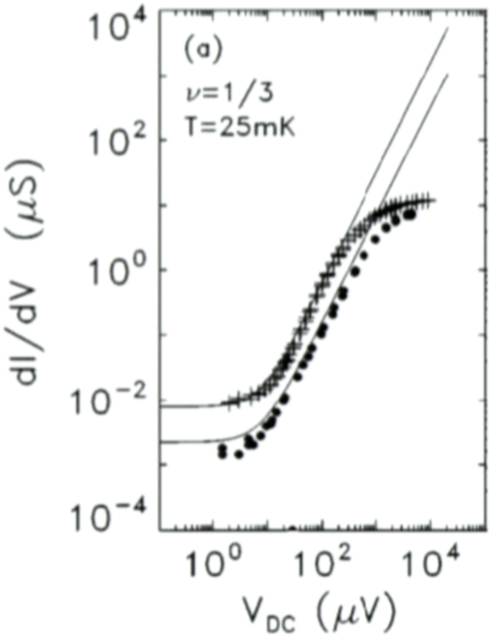


Theory: Kane & Fisher (1992)

Expt: Chang, Pfeiffer & West (1996)

Contrast with tunnelling probe of fractional quantum Hall edge states

Voltage-dependent differential conductance from electron correlations

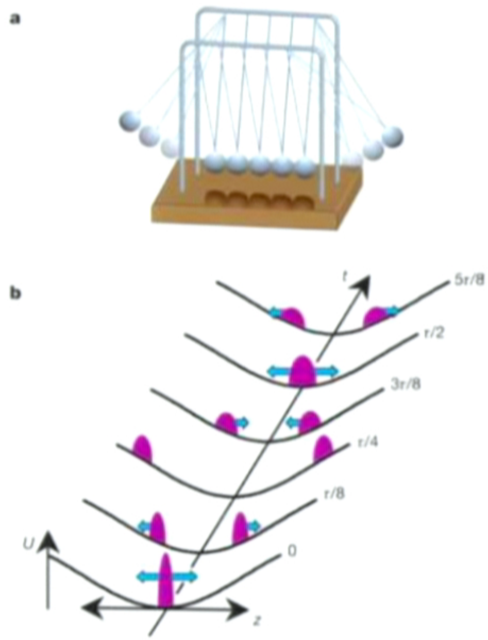


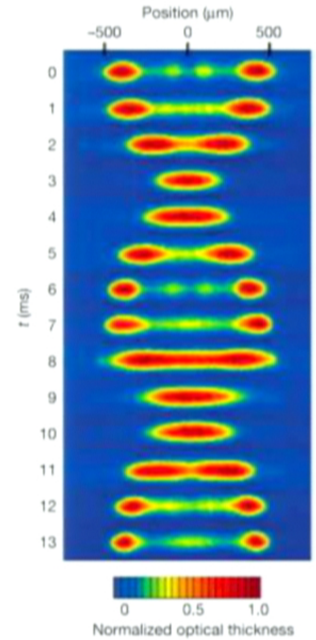
Theory: Kane & Fisher (1992)

Expt: Chang, Pfeiffer & West (1996)

Analogies with driven cold atom systems

Quantum Newton's Cradle



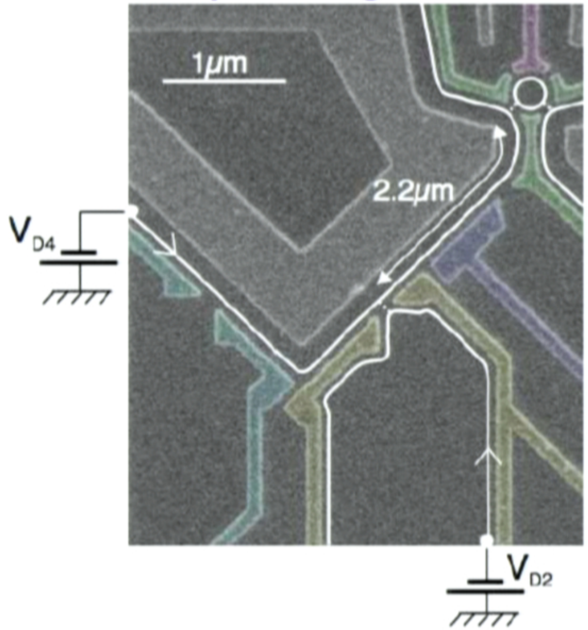


Kinoshita, Wenger & Weiss, Nature (2006)

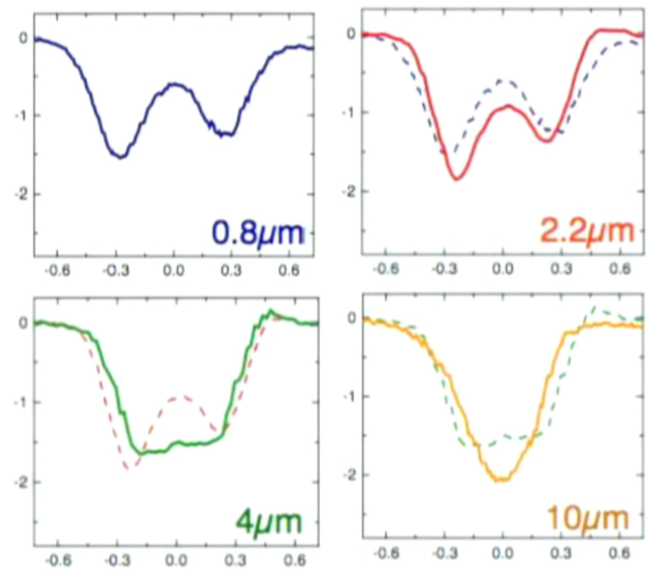
Experiment – Actual

le Sueur, Altimiras, Gennser, Cavanna, Mailly & Pierre, PRL (2010)

Sample Design



Evolution of Distribution



$\partial n(E)/\partial E$ **vs.** E

Theoretical Description of Edge States

Project from 2D to 1D

Classical Hamiltonian:

drift at constant speed

$$\mathcal{H} = v p_x \qquad \dot{x} = \partial_p \mathcal{H} = v$$

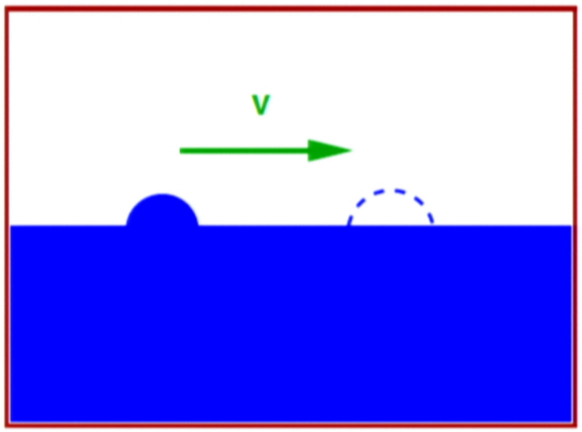
Single-particle quantum Hamiltonian:

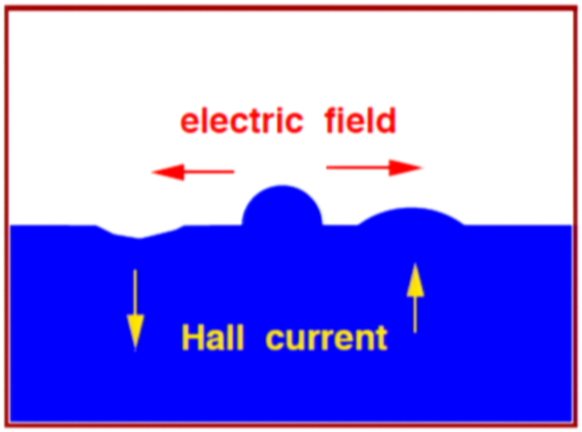
$$\mathcal{H} = \int \psi^\dagger(x)(-i\hbar v \partial_x)\psi(x) dx$$

Edge state dynamics with interactions

Free propagation

Charge flow in and out of bulk





Interactions make collective modes dispersive

Edge state Hamiltonian

As electrons:

$$H = -i\hbar v \int dx \psi^\dagger(x)\partial_x\psi(x) + \int dx \int dx'\, U(x-x')\rho(x)\rho(x')$$

$$\rho(x) = \psi^\dagger(x)\psi(x)$$

As collective modes:

$$H = \sum_q \hbar\omega(q) b_q^\dagger b_q$$

$$\omega(q) = [v + u(q)]\, q \qquad u(q) = (2\pi\hbar)^{-1} \int dx\; e^{iqx} U(x)$$

– related via bosonization

Edge state Hamiltonian

As electrons:

$$H = -i\hbar v \int dx \psi^\dagger(x)\partial_x\psi(x) + \int dx \int dx'\, U(x-x')\rho(x)\rho(x')$$

$$\rho(x) = \psi^\dagger(x)\psi(x)$$

As collective modes:

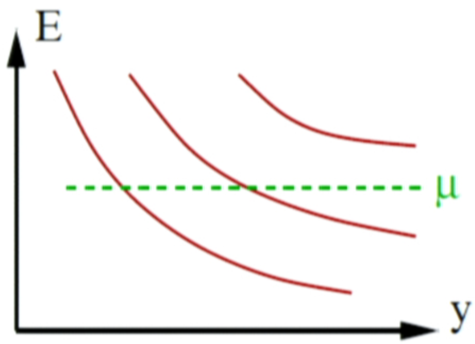
$$H = \sum_q \hbar\omega(q) b_q^\dagger b_q$$

$$\omega(q) = [v + u(q)]\, q \qquad u(q) = (2\pi\hbar)^{-1} \int dx\, e^{iqx} U(x)$$

– related via bosonization

Fractionalisation at $\nu = 2$

Two filled Landau levels

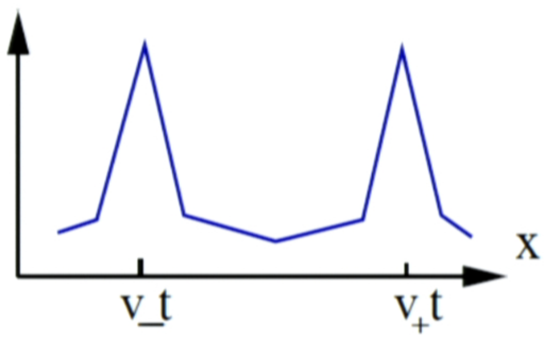


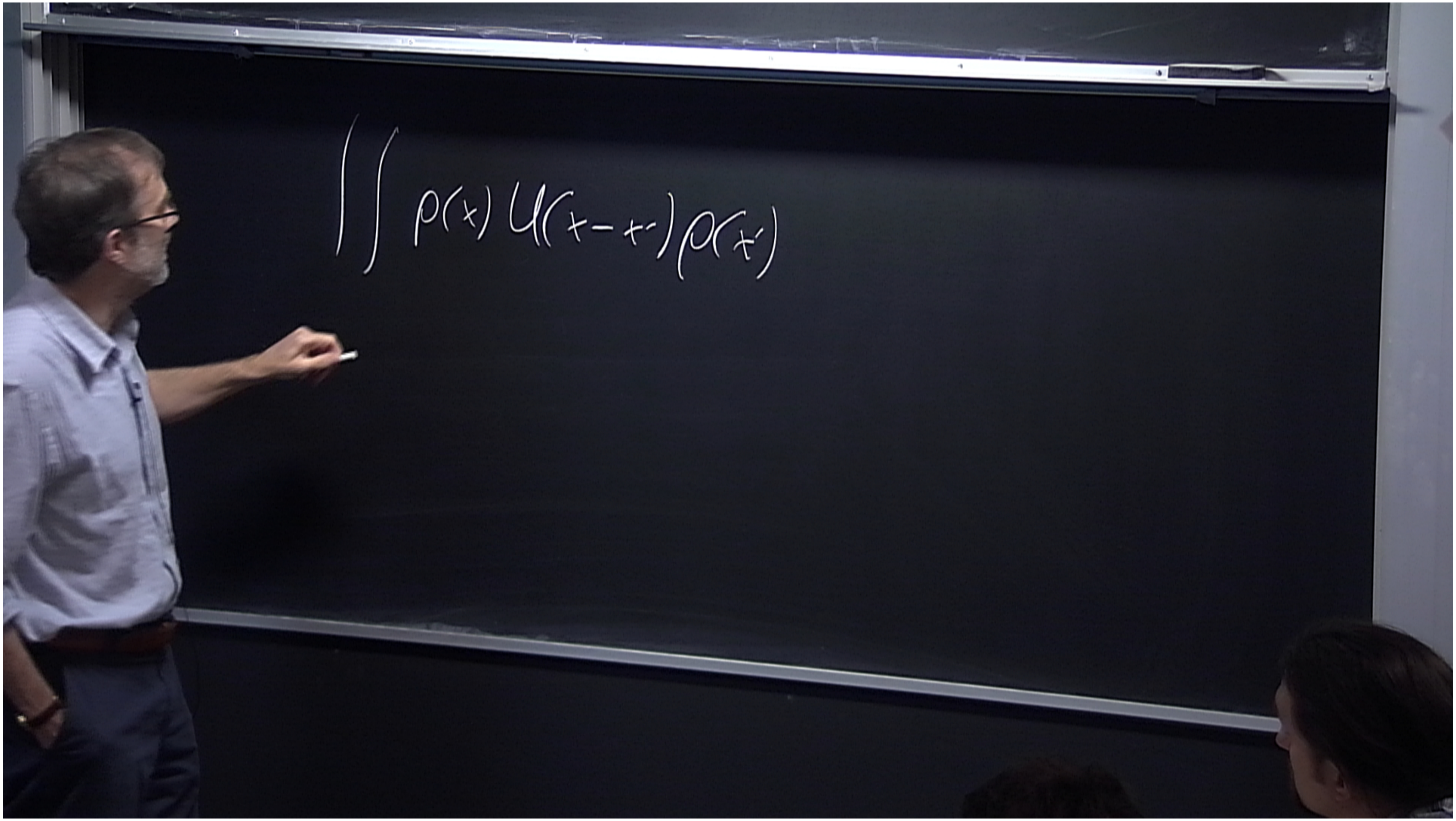
Modes mixed by interaction g

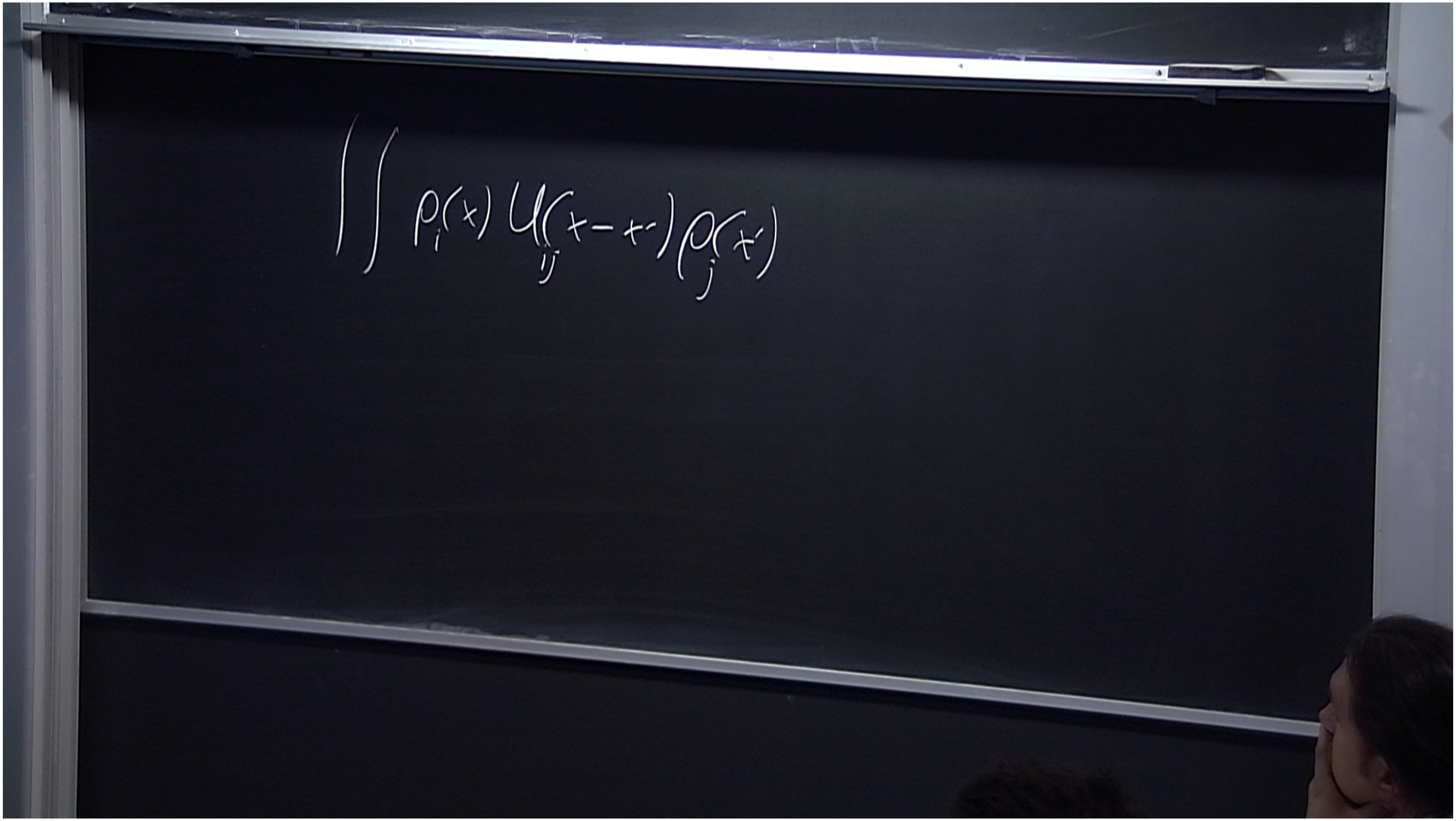
$$v_{\pm} = v \pm g$$

Injected electron fractionalises

$$|\langle \psi_1^{\dagger}(x,t)\psi_1(0,0)\rangle|^2 \sim$$




∫∫ ρ(x) U(x − x′) ρ(x′)


$\iint \rho_i(x)\, U_{ij}(x - x')\, \rho_j(x')$

Edge states far from equilibrium

Difficulties

Interactions treated most simply via bosonization

Tunneling at QPC simplest in fermionic language

Previous work on theory for relaxation expt

Boltzmann Eqn: Lunde *et al* (2010)

QPC as source of plasmon noise: Degiovanni *et al* (2010)

Weak tunnelling at QPC: Levkivskyi & Sukhorukov (2012)

Approaches here

Quantum quench as idealisation

Exact treatment via bosonization + refermionisation

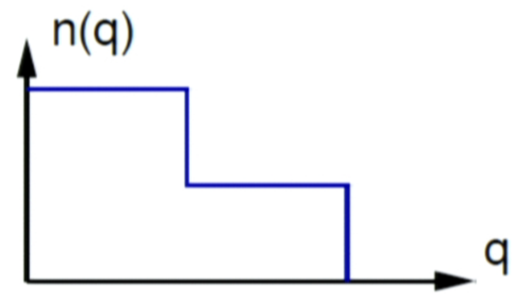
Theoretical Idealisation: Quantum quench

Evade treatment of point contact

– study time evolution in translationally-invariant edge

Initial state

$|\Psi_0\rangle$ **with**



Time evolution

$$|\Psi(t)\rangle = \mathrm{e}^{\mathrm{i}\mathcal{H}t}|\Psi_0\rangle$$

Properties of $|\Psi(t)\rangle$ **?**

Energies of collective modes conserved

– consequences for equilibration?

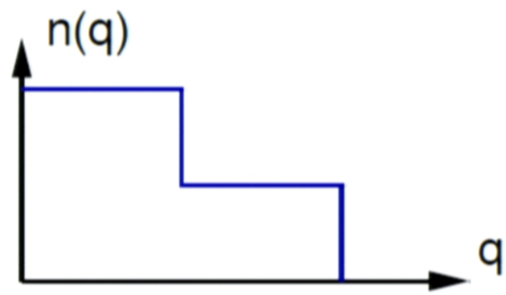
Theoretical Idealisation: Quantum quench

Evade treatment of point contact

– study time evolution in translationally-invariant edge

Initial state

$|\Psi_0\rangle$ **with**



Time evolution

$$|\Psi(t)\rangle = e^{i\mathcal{H}t}|\Psi_0\rangle$$

Properties of $|\Psi(t)\rangle$ **?**

Energies of collective modes conserved

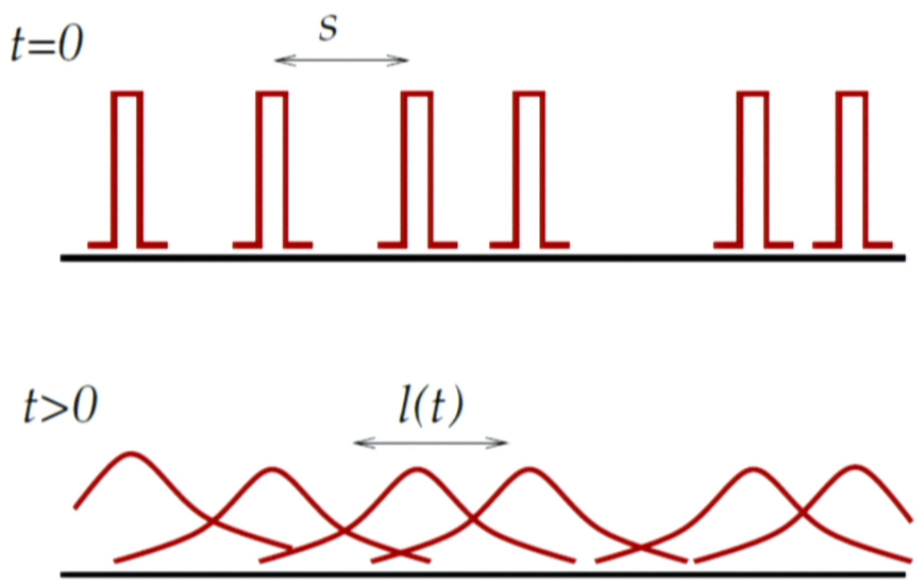
– consequences for equilibration?

Physical picture of equilibration

Collective mode Hamiltonian $\mathcal{H} = \sum_{nq} \hbar\omega_n(q) b^\dagger_{nq} b_{nq}$

Edge magnetoplasmon dispersion $\longrightarrow$ electron equilibration?

Initial quasi-particle separation $s = \hbar v / eV$



Equilibration when wavepacket spread $l(t) \gtrsim s$

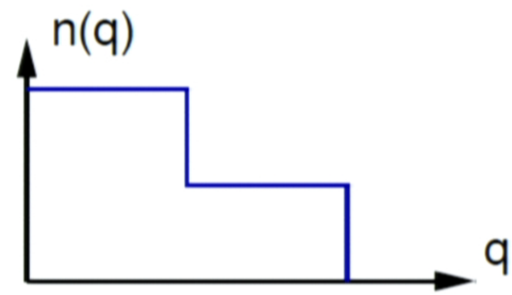
Theoretical Idealisation: Quantum quench

Evade treatment of point contact

– study time evolution in translationally-invariant edge

Initial state

$|\Psi_0\rangle$ with



Time evolution

$$|\Psi(t)\rangle = \mathrm{e}^{\mathrm{i}\mathcal{H}t}|\Psi_0\rangle$$

Properties of $|\Psi(t)\rangle$?

Energies of collective modes conserved

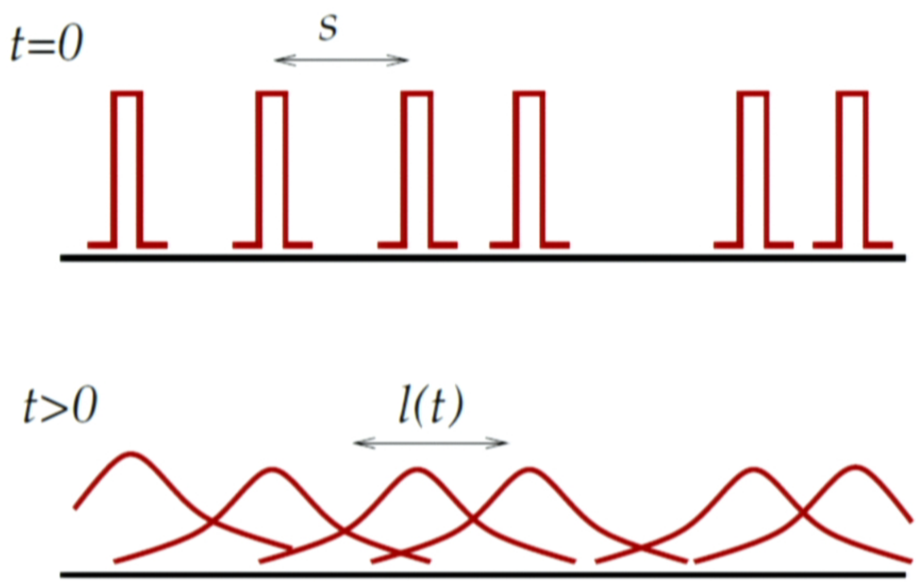
– consequences for equilibration?

Physical picture of equilibration

Collective mode Hamiltonian $\mathcal{H} = \sum_{nq} \hbar\omega_n(q) b^\dagger_{nq} b_{nq}$

Edge magnetoplasmon dispersion $\rightarrow$ electron equilibration?

Initial quasi-particle separation $s = \hbar v / eV$



Equilibration when wavepacket spread $l(t) \gtrsim s$

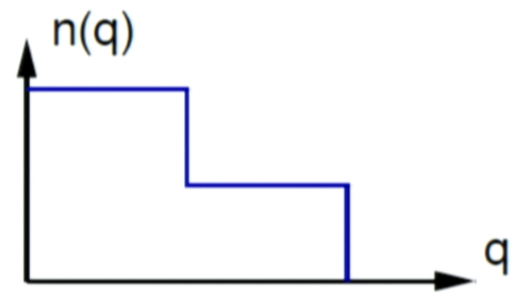
Theoretical Idealisation: Quantum quench

Evade treatment of point contact

– study time evolution in translationally-invariant edge

Initial state

$|\Psi_0\rangle$ **with**



Time evolution

$$|\Psi(t)\rangle = \mathrm{e}^{\mathrm{i}\mathcal{H}t}|\Psi_0\rangle$$

Properties of $|\Psi(t)\rangle$ **?**

Energies of collective modes conserved

– consequences for equilibration?

Equilibration from two mode velocities

Two edge modes with short-range interactions

Two linearly dispersing modes $\omega_1(q) = v_+ q$ & $\omega_2(q) = v_- q$

Initial quasi-particle separation $s = \hbar v / eV$

Equilibration when wavepacket spread $l(t) \gtrsim s$

Spread $l(t) = [v_+ - v_-]t$

Equilibration time: $t_{\text{eq}} \sim \frac{\hbar}{eV} \cdot \frac{v_+ + v_-}{v_+ - v_-}$

What is the equilibrium state?

Characterise via one-electron correlations

Calculate $G(x,t) = \langle \psi^\dagger(x,t)\psi(0,t)\rangle$

in thermal state $G(x,t) = [-2i\beta\hbar v \sinh(\pi[x+i0]/\beta\hbar v)]^{-1}$

What is the equilibrium state?

Characterise via one-electron correlations

Calculate $G(x,t) = \langle \psi^\dagger(x,t)\psi(0,t)\rangle$

in thermal state $G(x,t) = [-2i\beta\hbar v \sinh(\pi[x+i0]/\beta\hbar v)]^{-1}$

Approach to calculations:

Alternate between fermionic and bosonic descriptions

$$\Psi(x) \sim \mathrm{e}^{-\mathrm{i}\varphi(x)}$$

$$\varphi(x) \propto \sum q^{-1/2}\left[b_q^\dagger \mathrm{e}^{-\mathrm{i}qx} + \mathrm{h.c}\right]$$

$$b_q^\dagger \propto \sum c_{k+q}^\dagger c_k$$

Initial state

simple for fermions

Time evolution

simple for bosons

Observables

fermion fields

Comparison with thermal state

Short-distance correlations

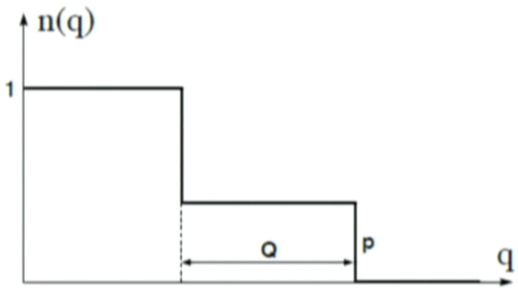
As in thermal state at same energy density

Long-distance correlations

$G(x,t) \sim \exp(-\alpha|x|)$ with α **not** fixed by energy density

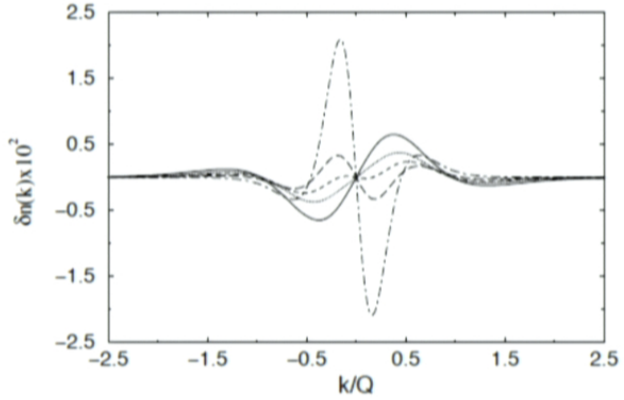
Example

Initial momentum distribution

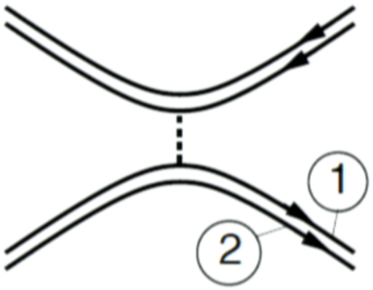


Difference from thermal in steady state

$p = 0.1, 0.2, 0.25, 0.3, 0.5$



Equilibration with two edge modes



Each edge

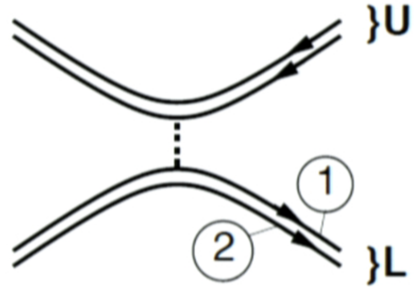
$$\mathcal{H} = \mathcal{H}_{\rm kin} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm kin} = -i\hbar v \int \Big(\Psi_1^\dagger(x)\partial_x \Psi_1(x) + \Psi_2^\dagger(x)\partial_x \Psi_2(x) \Big)\, dx$$

$$\mathcal{H}_{\rm int} = 2\pi\hbar g \int \rho_1(x)\rho_2(x) dx\,, \qquad \rho_n(x) = \Psi_n^\dagger(x)\Psi_n(x)$$

Refermionize

Combine bosons from opposite edges



$$\varphi_{S\pm} = \frac{1}{\sqrt{2}}[\varphi_{U\pm}(x) + \varphi_{L\pm}(x)]$$

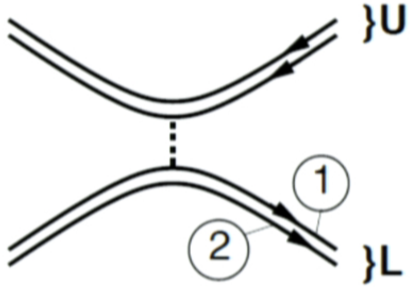
$$\varphi_{A\pm} = \pm\frac{1}{\sqrt{2}}[\varphi_{U\pm}(x) - \varphi_{L\pm}(x)]$$

Tunneling $\mathcal{H}_{\mathrm{tun}} = t_{\mathrm{QPC}}[\Psi^{\dagger}_{U1}(0)\Psi_{L1}(0) + \Psi^{\dagger}_{L1}(0)\Psi_{U1}(0)]$

$$\begin{aligned}\Psi_1^{\dagger}(0)\Psi_2(0) &\sim \mathrm{e}^{\mathrm{i}[\varphi_{U1}(0)-\varphi_{L1}(0)]}\\ &\sim \mathrm{e}^{\frac{\mathrm{i}}{\sqrt{2}}[\varphi_{U+}(0)+\varphi_{U-}(0)-\varphi_{L+}(0)-\varphi_{L-}(0)]}\\ &\sim \mathrm{e}^{\mathrm{i}[\varphi_{A+}(0)-\varphi_{A-}(0)]}\end{aligned}$$

Refermionize

Combine bosons from opposite edges



$$\varphi_{S\pm} = \frac{1}{\sqrt{2}}[\varphi_{U\pm}(x) + \varphi_{L\pm}(x)]$$

$$\varphi_{A\pm} = \pm\frac{1}{\sqrt{2}}[\varphi_{U\pm}(x) - \varphi_{L\pm}(x)]$$

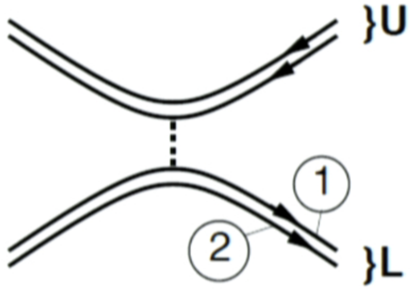
Tunneling $\mathcal{H}_{\text{tun}} = t_{\text{QPC}}[\Psi^\dagger_{U1}(0)\Psi_{L1}(0) + \Psi^\dagger_{L1}(0)\Psi_{U1}(0)]$

$$\Psi^\dagger_1(0)\Psi_2(0) \sim \mathrm{e}^{\mathrm{i}[\varphi_{U1}(0)-\varphi_{L1}(0)]} \sim \mathrm{e}^{\mathrm{i}[\varphi_{A+}(0)-\varphi_{A-}(0)]} \sim \Psi^\dagger_{A+}(0)\Psi_{A-}(0)$$

Edges

$$\mathcal{H}_{\text{kin}}+\mathcal{H}_{\text{int}} = -\mathrm{i}\hbar\int[v_+\Psi^\dagger_{A+}(x)\partial_x\Psi_{A+}(x)+v_-\Psi^\dagger_{A-}(x)\partial_x\Psi_{A-}(x)]\mathrm{d}x + [\mathrm{A}\leftrightarrow\mathrm{S}]$$

Observables



Electron energy distribution at x from

$$\langle \Psi^\dagger_{L1}(x,t)\Psi_{L1}(x,0)\rangle$$

or $$\langle \Psi^\dagger_{L2}(x,t)\Psi_{L2}(x,0)\rangle$$

Transforms to $\langle \mathrm{e}^{\mathrm{i}\pi(n_- - n_+)}\rangle$ **or** $\langle \mathrm{e}^{\mathrm{i}\pi(n_- + n_+)}\rangle$

with $n_\pm = \int_x^{x+v_\pm t} :\Psi^\dagger_{A\pm}(y)\Psi_{A\pm}(y): \,\mathrm{d}y$

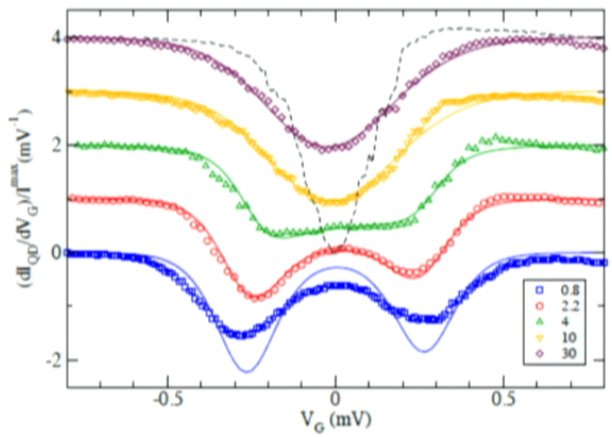
Large x – recover results from quantum quench

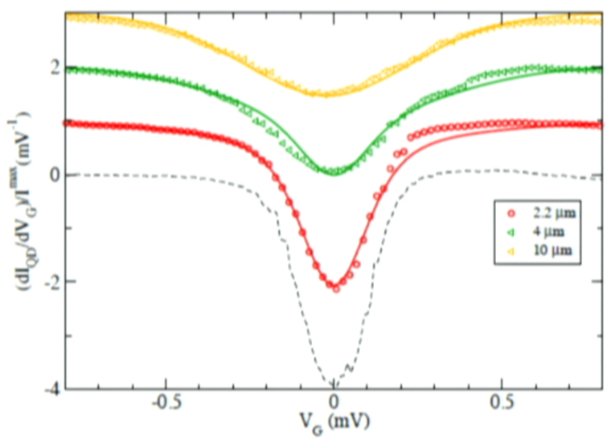
General x – evaluate free fermion averages numerically

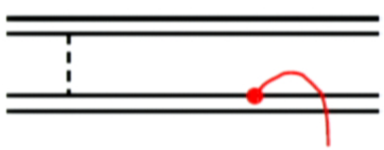
Comparison with experiment

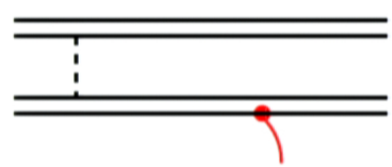
Measurement in channel coupled at QPC

Measurement in channel coupled by interactions









Fitting parameter: interaction strength $(v^2 + g^2)/2g = 6.5 \times 10^4 \mathrm{ms}^{-1}$

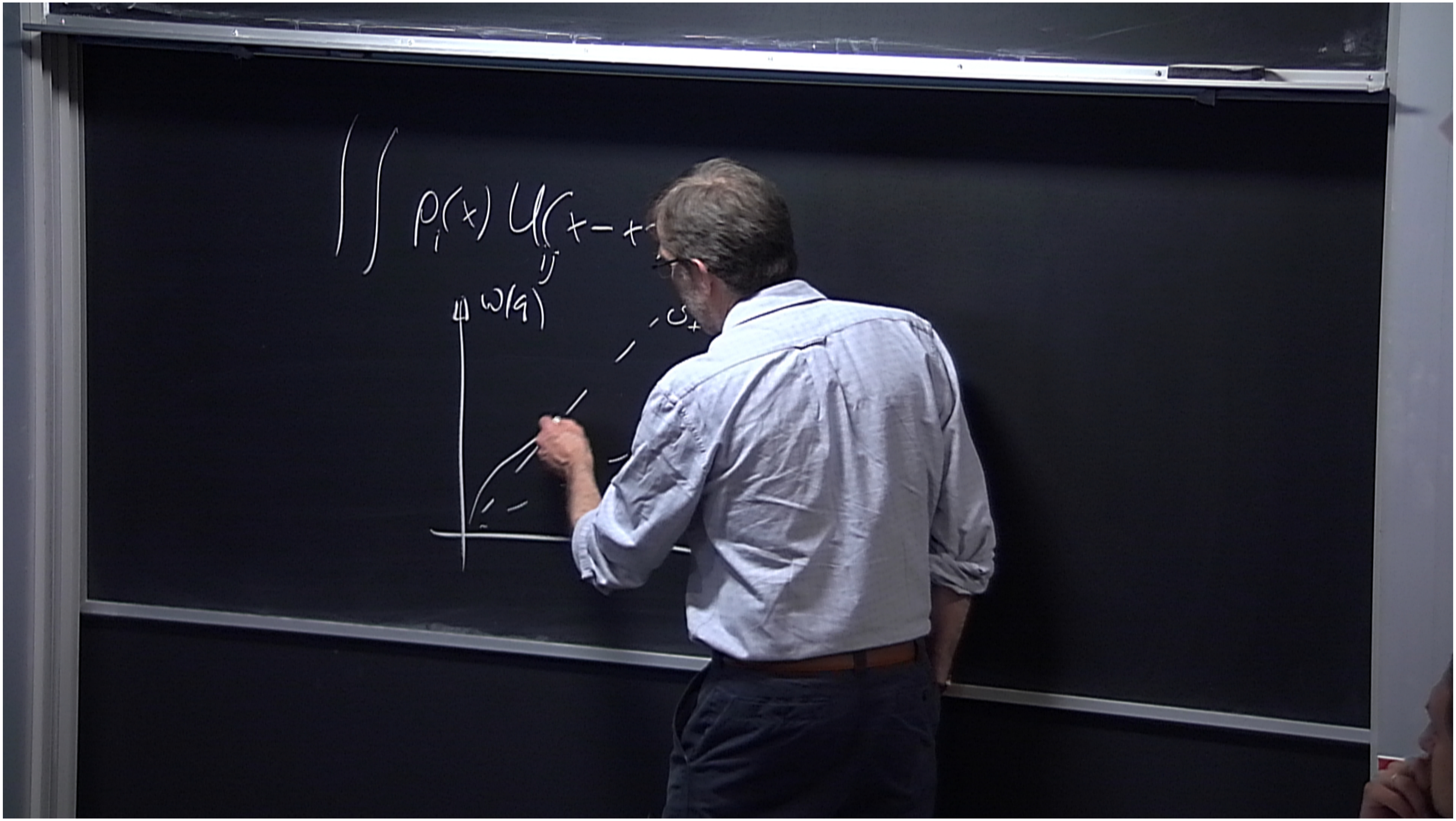
Summary

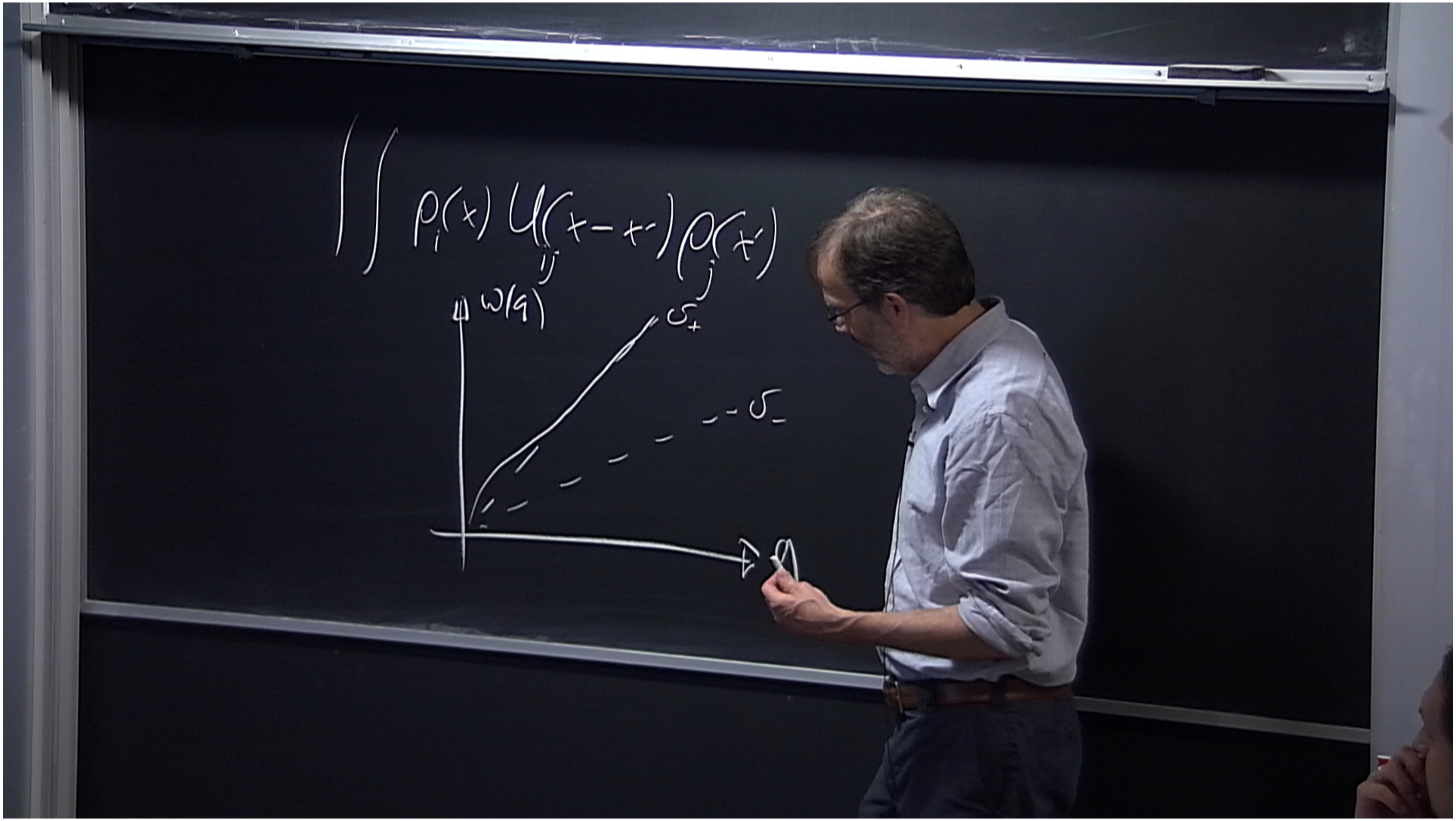
'Quantum quench' on isolated edge is useful caricature of experiment with two edges coupled at QPC

- Interactions bring system into non-thermal steady state
- At $\nu = 1$ steady state is indept of interactions

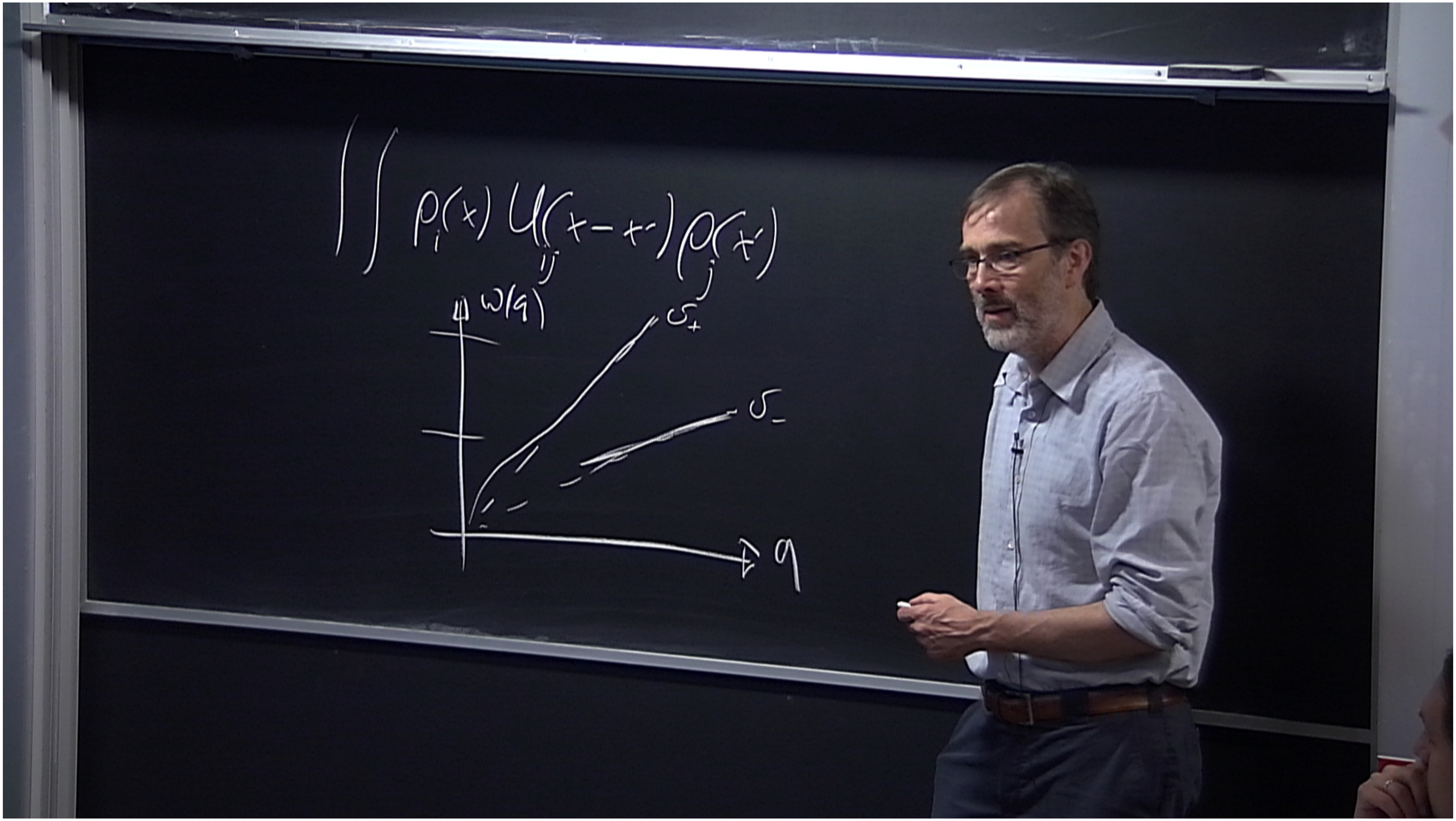
Full problem with QPC solvable at $\nu = 2$

- Asymptotic state is same as for quench, and non-thermal
- Calculated evolution of tunneling density of state with distance matches experiment well


ω(q)



∬ ρ_i(x) U_ij(x − x') ρ_j(x')
ω(q)
c_+
c_−
q



$\iint \rho_i(x)\, U_{ij}(x-x')\, \rho_j(x')$
$\omega(q)$
v_+
v_-
q