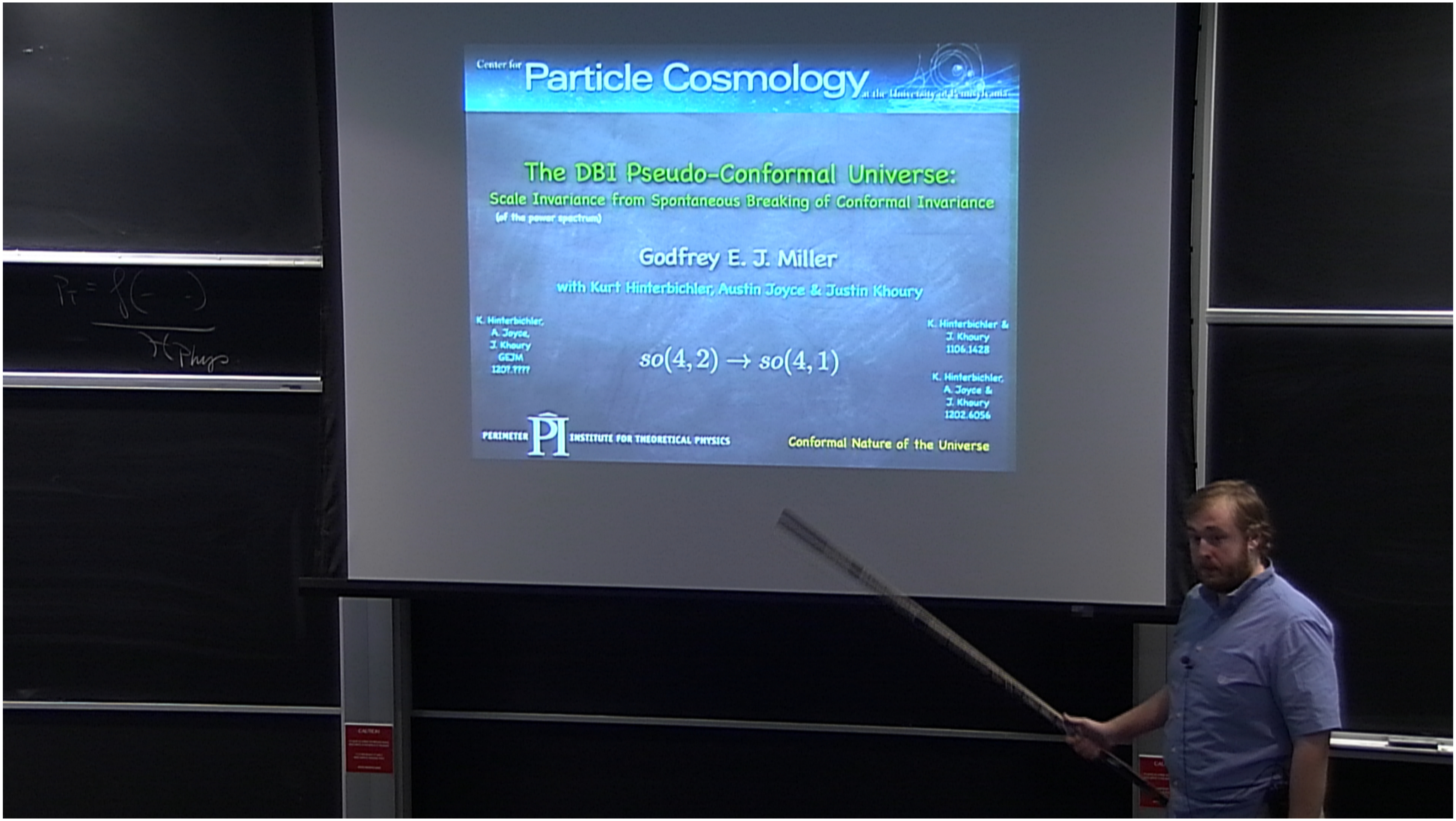
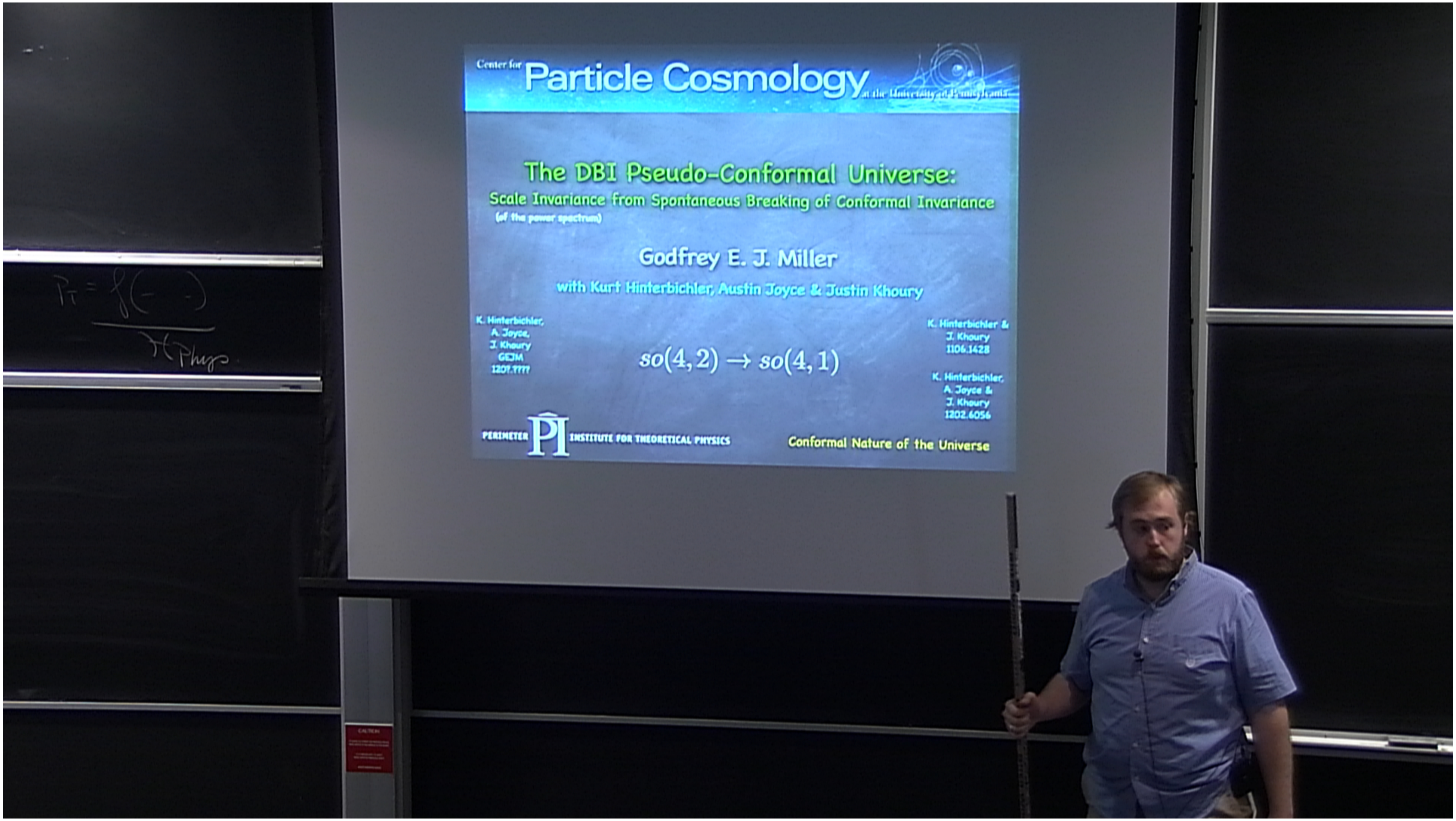
Title: The DBI Pseudo-Conformal Universe: Scale Invariance from Spontaneous Breaking of Conformal Symmetry

Date: May 12, 2012 02:30 PM

URL: http://pirsa.org/12050078

Abstract: The pseudo-conformal scenario is an alternative to inflation in which the early universe is approximately described by a conformal field theory in Minkowski space. Crucially, the cosmological background spontaneously breaks the flat space so(4,2) conformal algebra down to its so(4,1) de Sitter subalgebra, causing conformal-weight-0 fields to acquire a scale invariant spectrum of perturbations. This framework is very general, and its essential features are determined by the symmetry breaking pattern, irrespective of the details of the underlying microphysics. After reviewing the salient features of the model, I will describe a DBI realization of the pseudo-conformal scenario, in which scale-invariance is further protected by an additional shift symmetry acting on the weight-0 field.


Center for Particle Cosmology at the University of Pennsylvania
The DBI Pseudo-Conformal Universe:
Scale Invariance from Spontaneous Breaking of Conformal Invariance
(of the power spectrum)
Godfrey E. J. Miller
with Kurt Hinterbichler, Austin Joyce & Justin Khoury
K. Hinterbichler, A. Joyce, J. Khoury GEJM 1207.????
$so(4,2) \to so(4,1)$
K. Hinterbichler & J. Khoury 1106.1428
K. Hinterbichler, A. Joyce & J. Khoury 1202.6056
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS
Conformal Nature of the Universe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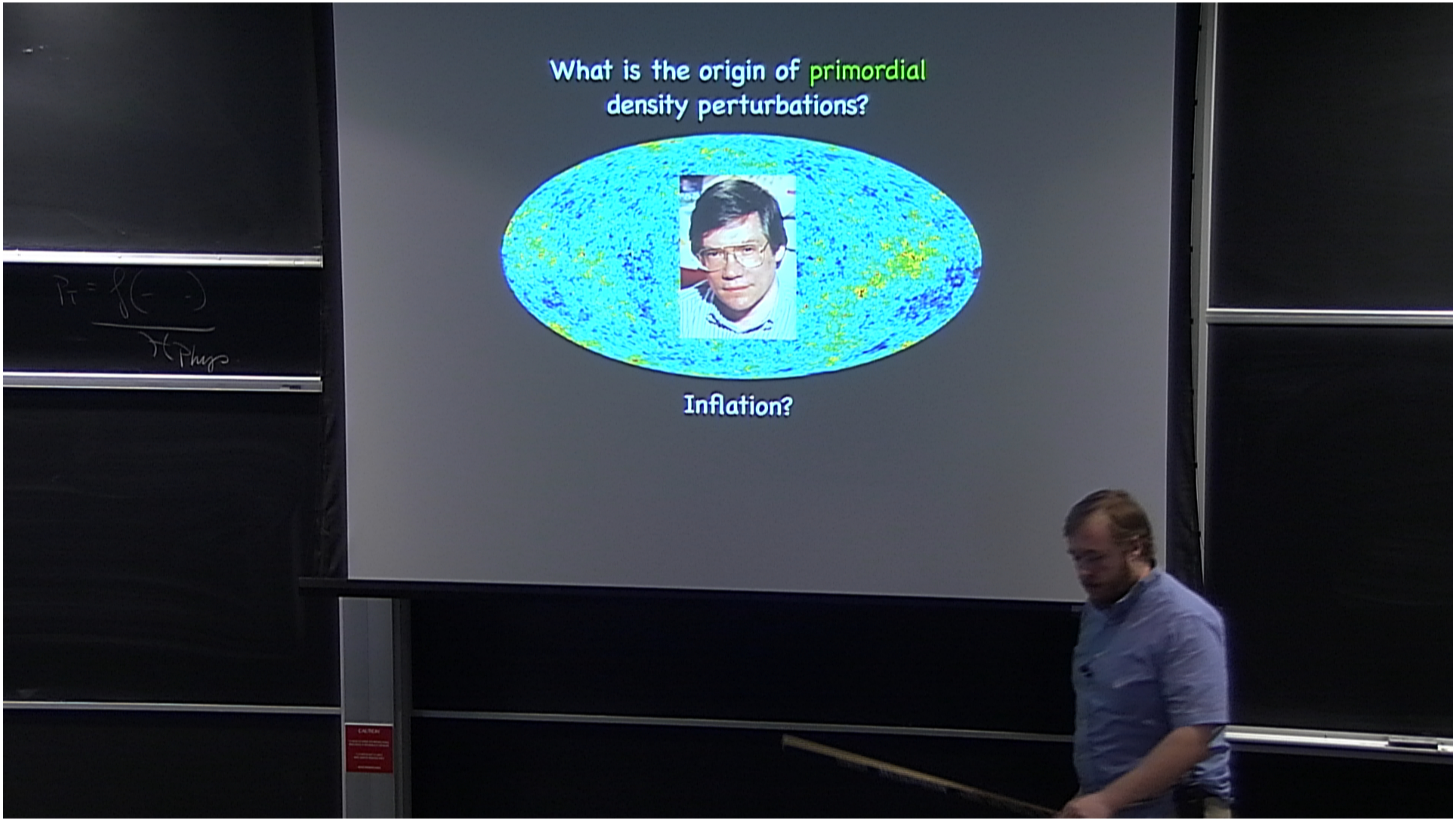
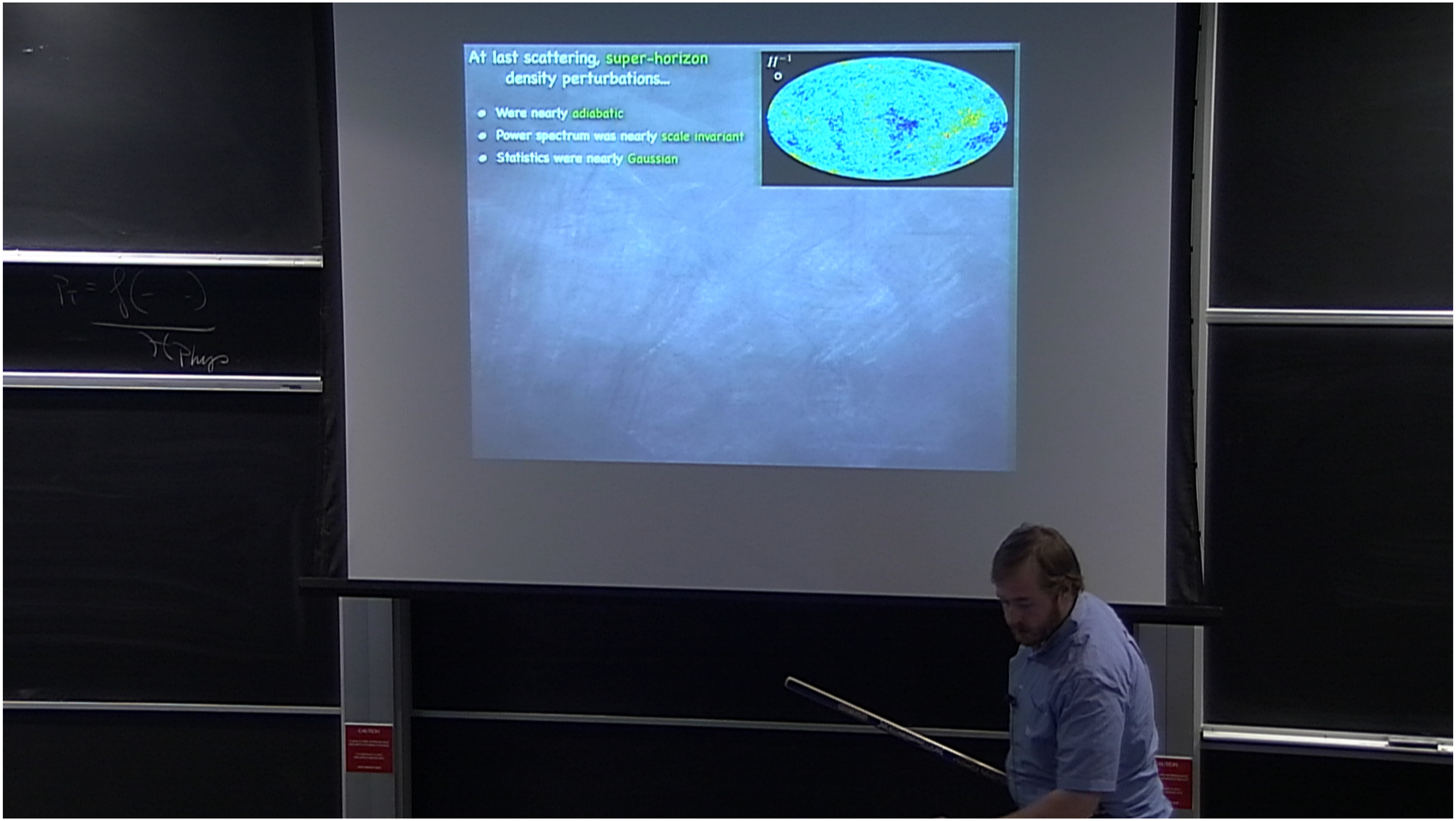
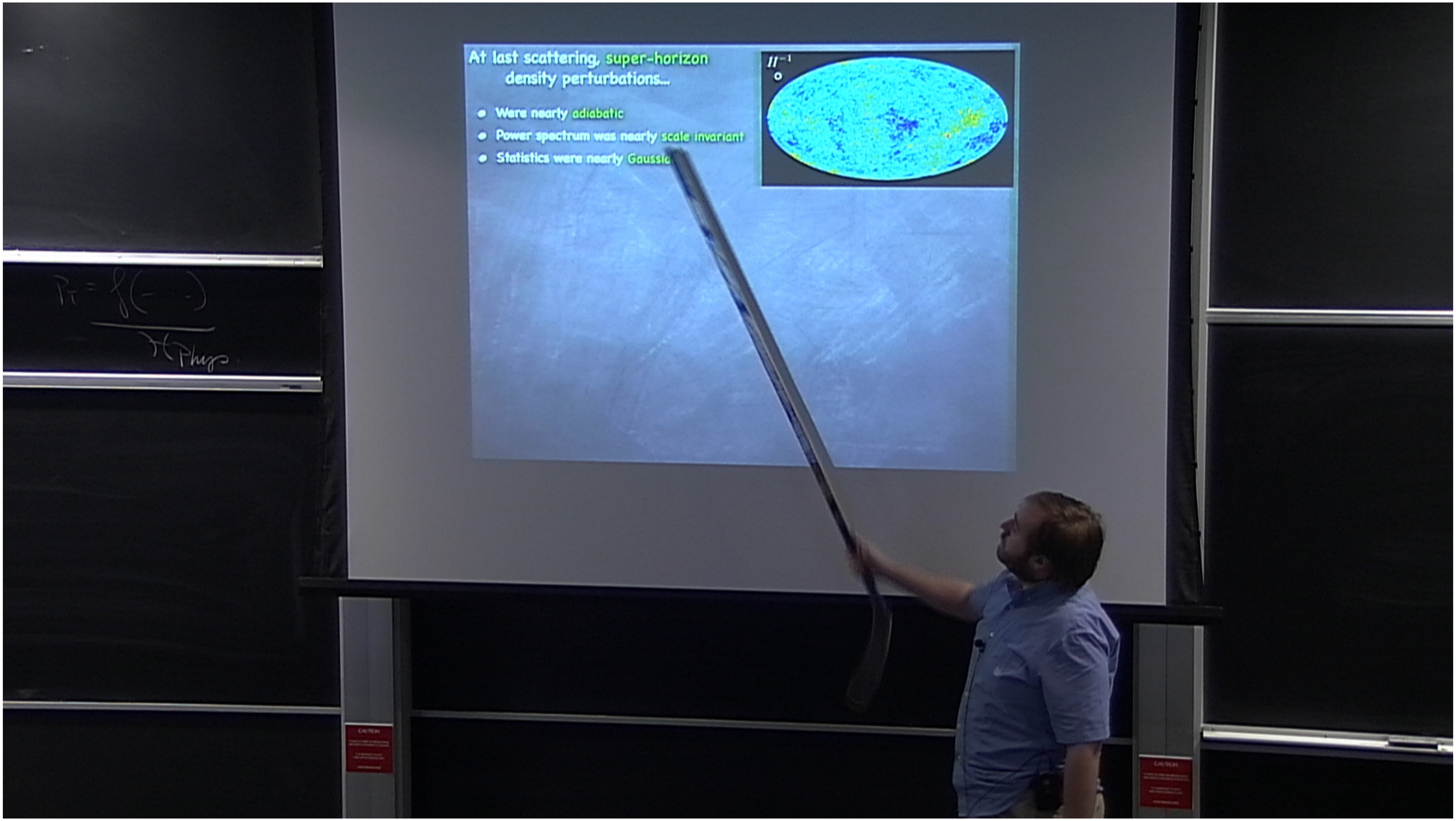

What is the origin of primordial
density perturbations?
Inflation?


At last scattering, super-horizon density perturbations...
Were nearly adiabatic
Power spectrum was nearly scale invariant
Statistics were nearly Gaussian
H^-1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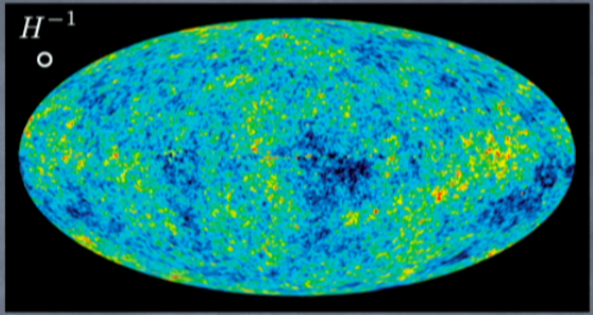

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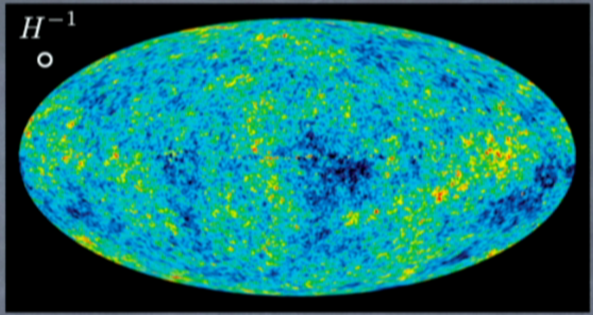


How do these observations constrain models of the early universe?

(Minimal Coupling to)

Paradigm: Matter + Einstein Gravity

Adiabatic models

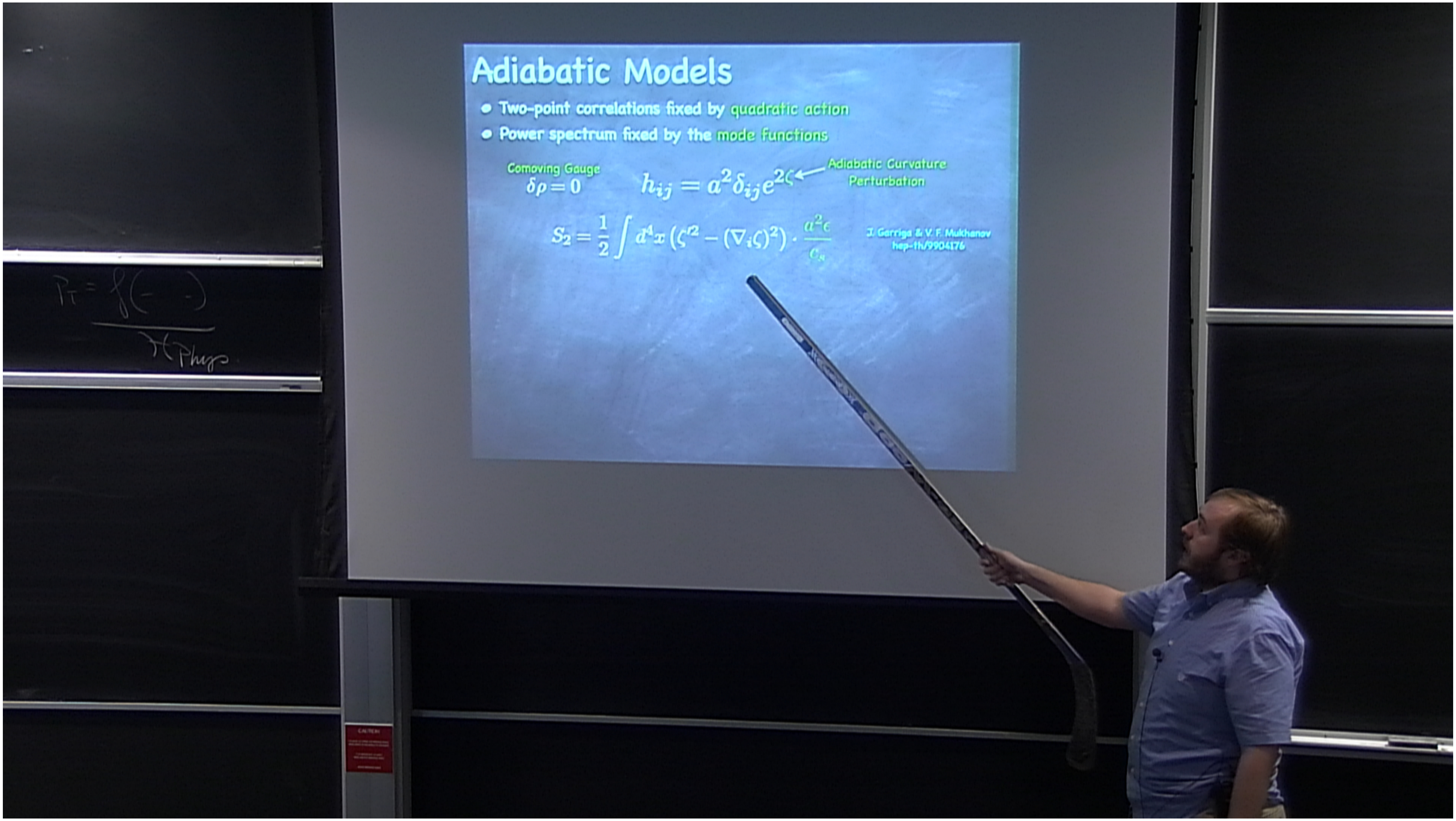
Cosmological evolution directly generates adiabatic super-horizon perturbations

One active field

Entropic models

Background evolution generates entropic super-horizon perturbations, which subsequently become adiabatic

Two or more active fields


Adiabatic Models
Two-point correlations fixed by quadratic action
Power spectrum fixed by the mode functions
Comoving Gauge
$\delta\rho = 0$
$h_{ij} = a^2 \delta_{ij} e^{2\zeta}$
Adiabatic Curvature Perturbation
$S_2 = \frac{1}{2}\int d^4x \left(\zeta'^2 - (\nabla_i \zeta)^2\right) \cdot \frac{a^2\epsilon}{c_s}$
J. Garriga & V. F. Mukhanov
hep-th/9904176

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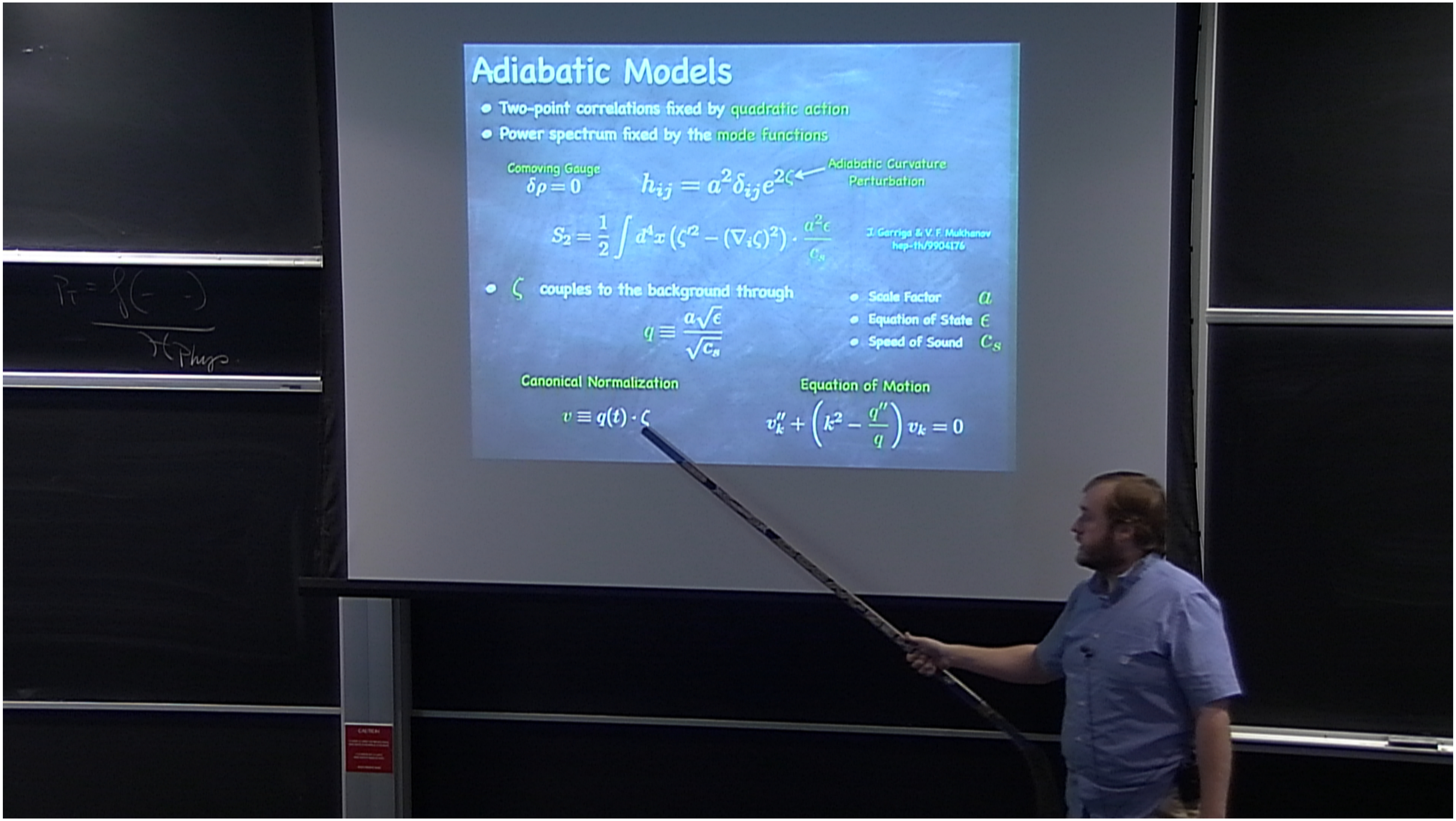
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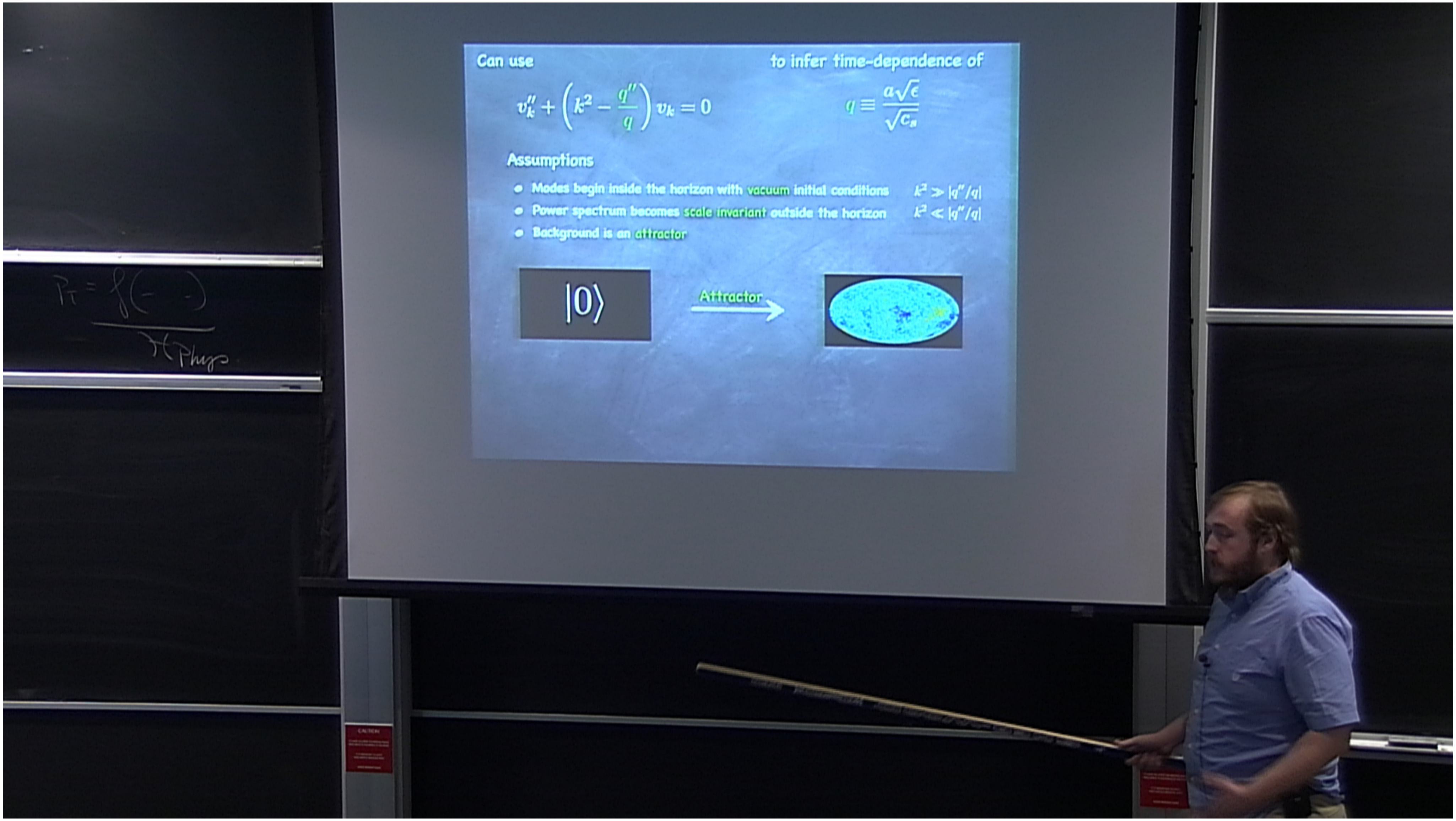
ζ couples to the background through
$q \equiv \frac{a\sqrt{\epsilon}}{\sqrt{c_s}}$
Scale Factor a
Equation of State ϵ
Speed of Sound c_s
Canonical Normalization
$v \equiv q(t) \cdot \zeta$
Equation of Motion
$v_k'' + \left(k^2 - \frac{q''}{q}\right) v_k = 0$

Can use to infer time-dependence of

$$v_k'' + \left(k^2 - \frac{q''}{q}\right) v_k = 0$$

$$q \equiv \frac{a\sqrt{\epsilon}}{\sqrt{c_s}}$$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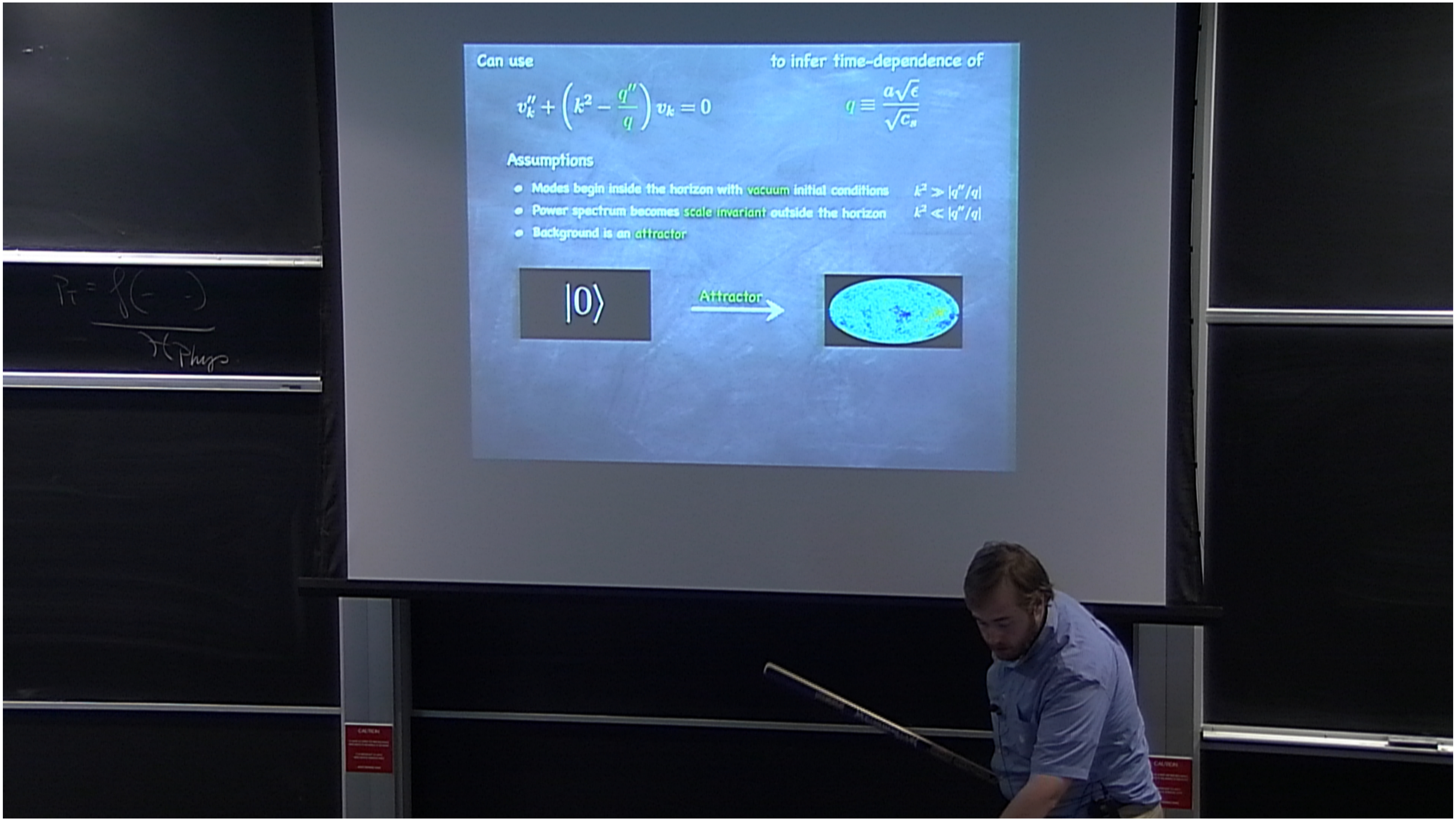
$v_k'' + \left(k^2 - \frac{q''}{q}\right) v_k = 0$
$q \equiv \frac{a\sqrt{\epsilon}}{\sqrt{c_s}}$
Assumptions
Modes begin inside the horizon with vacuum initial conditions $k^2 \gg |q''/q|$
Power spectrum becomes scale invariant outside the horizon $k^2 \ll |q''/q|$
Background is an attractor
$|0\rangle$
Attractor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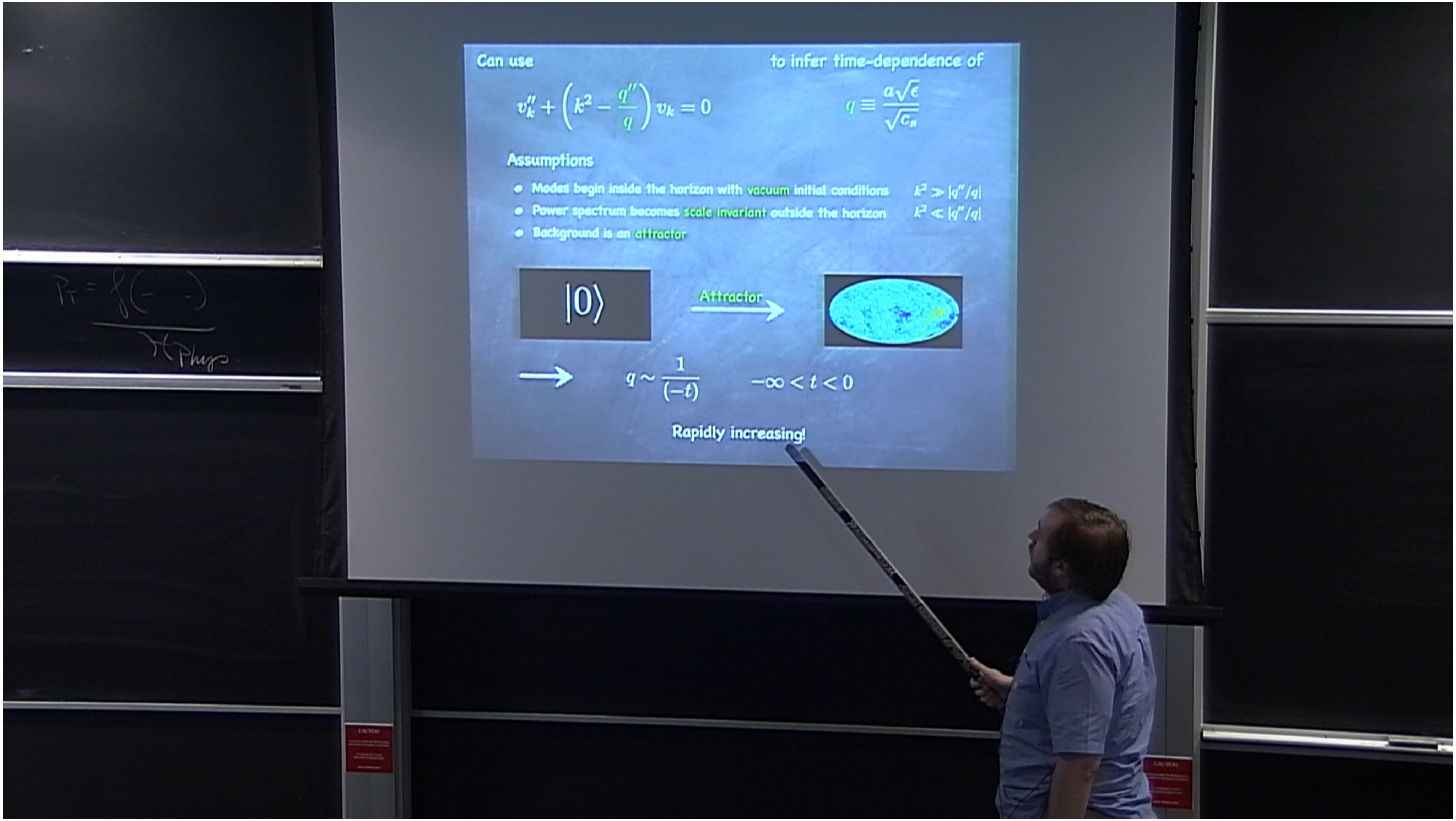
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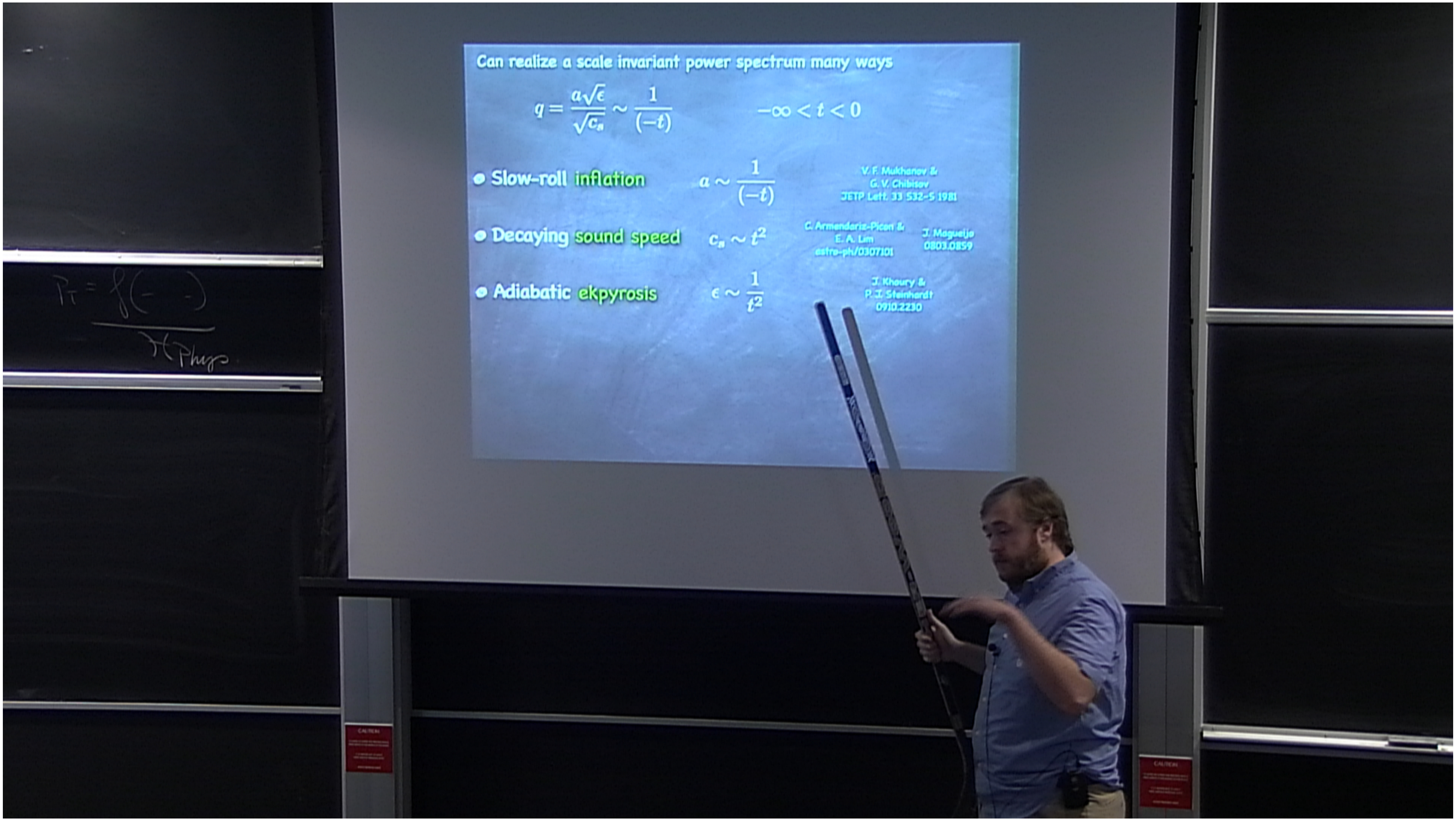
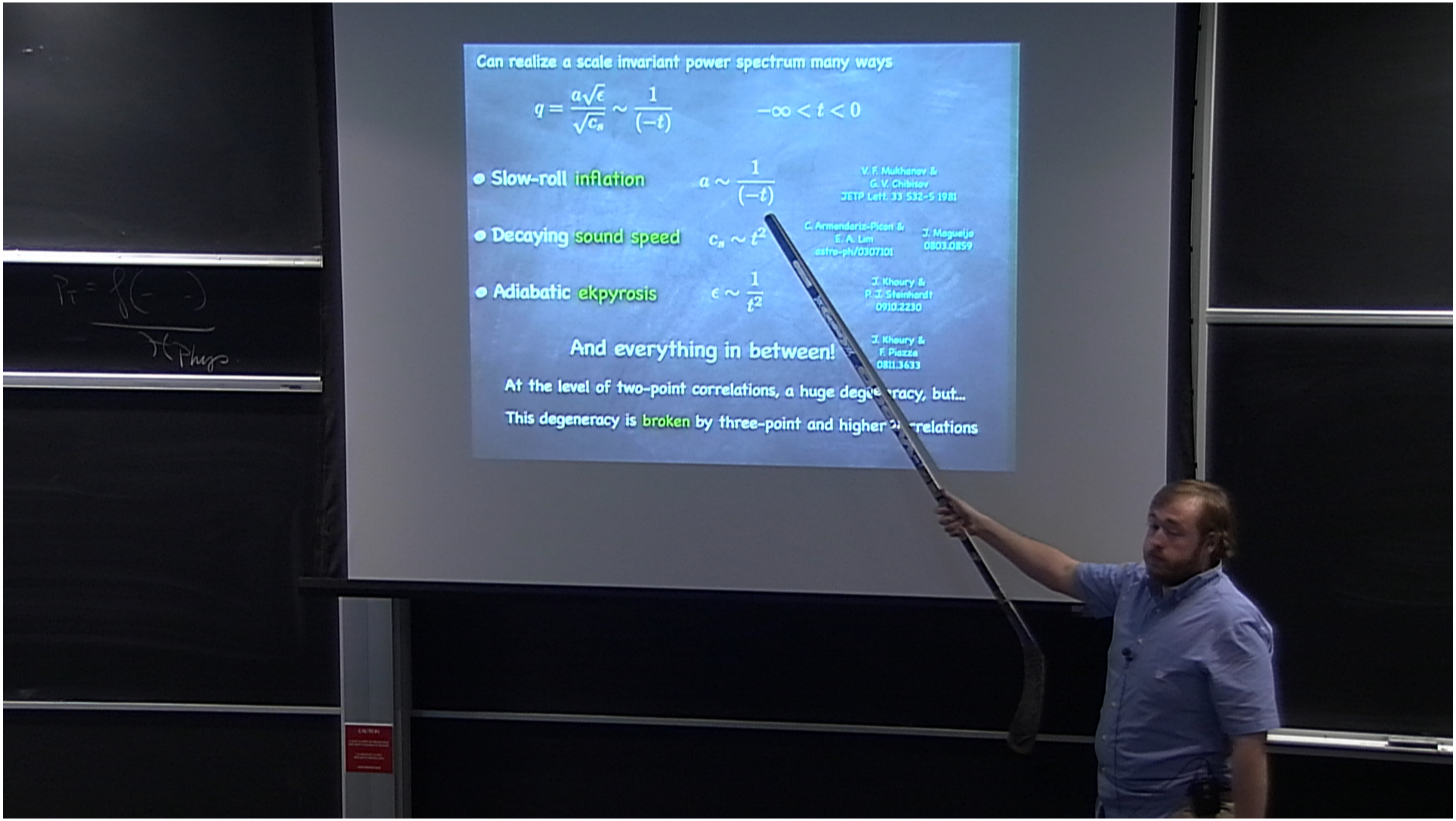
$q \sim \frac{1}{(-t)}$
$-\infty < t < 0$
Rapidly increasing!


Can realize a scale invariant power spectrum many ways
$q = \frac{a\sqrt{\epsilon}}{\sqrt{c_s}} \sim \frac{1}{(-t)}$
$-\infty < t < 0$
Slow-roll inflation
$a \sim \frac{1}{(-t)}$
V. F. Mukhanov & G. V. Chibisov JETP Lett. 33 532-5 1981
Decaying sound speed
$c_s \sim t^2$
C. Armendariz-Picon & E. A. Lim astro-ph/0307101
J. Magueijo 0803.0859
Adiabatic ekpyrosis
$\epsilon \sim \frac{1}{t^2}$
J. Khoury & P. J. Steinhardt 0910.2230


Rapidly varying ϵ, c_s lead to
Scale dependent non-gaussianities
Breakdown of perturbation theory due to strong coupling
Super-Planckian energy densities
Adiabatic alternatives to inflation can generate only a finite window of modes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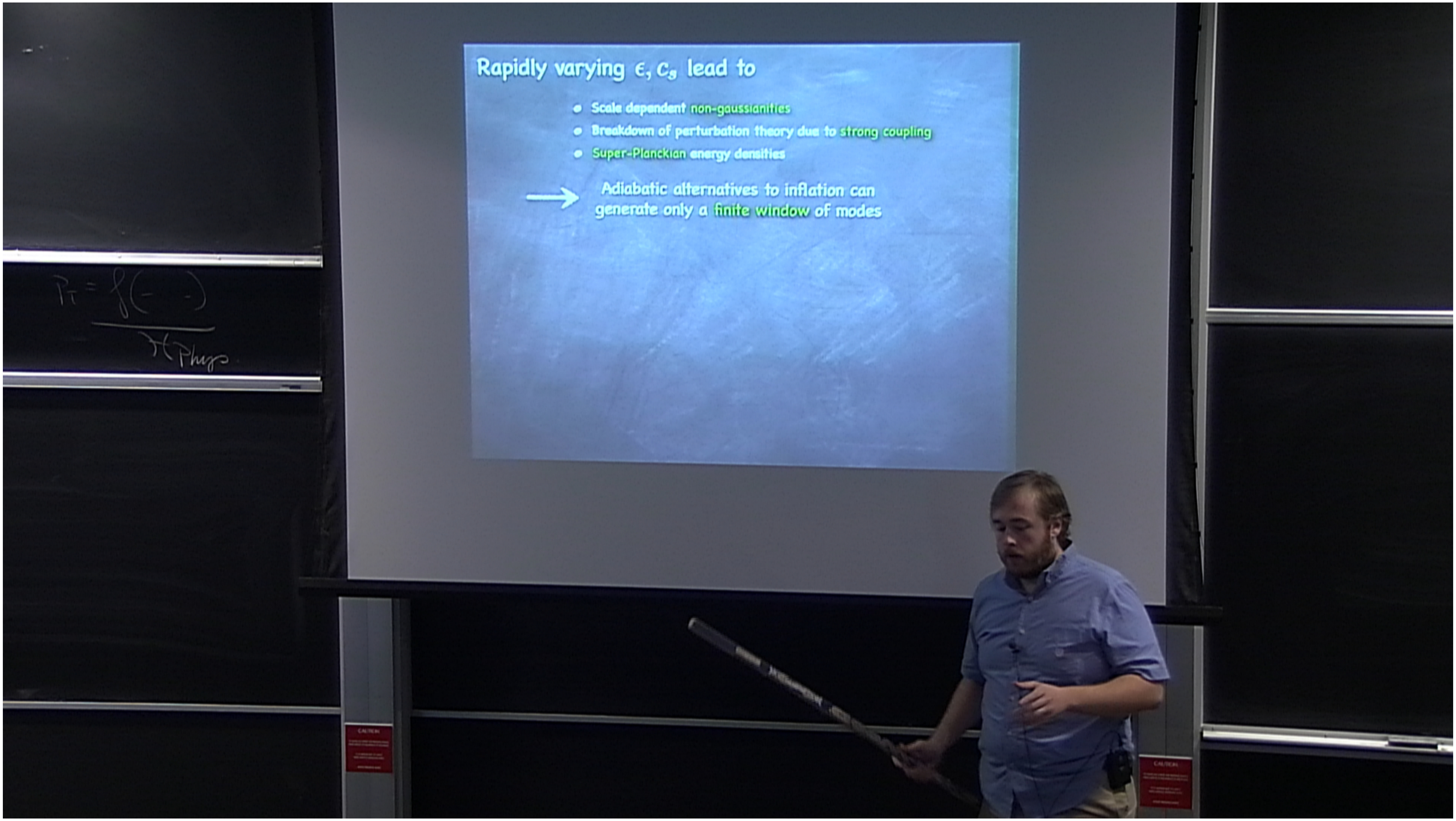
And everything in between!
J. Khoury &
F. Piazza
0811.3633
At the level of two-point correlations, a huge deg racy, but...
This degeneracy is broken by three-point and higher rrelations



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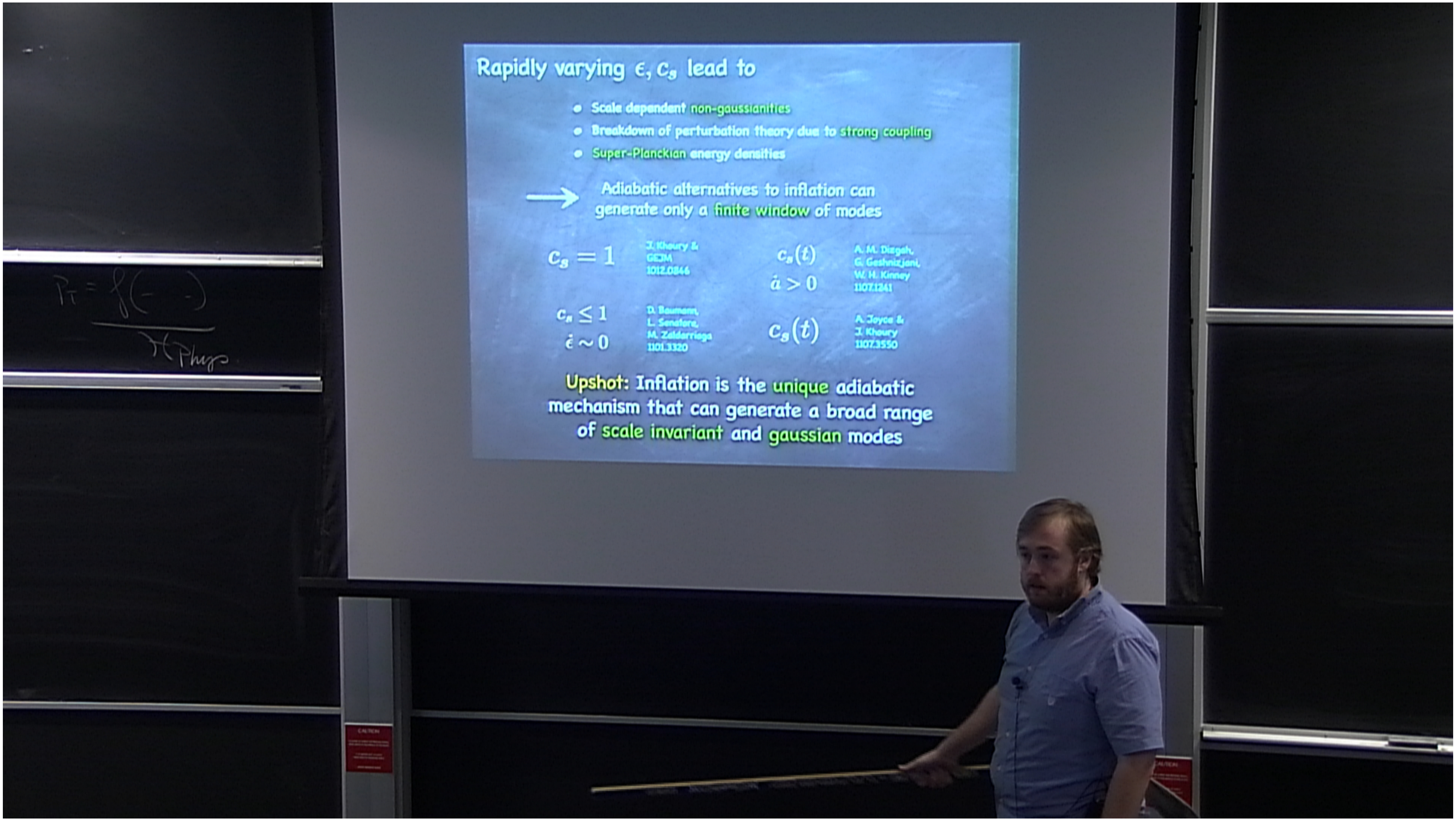
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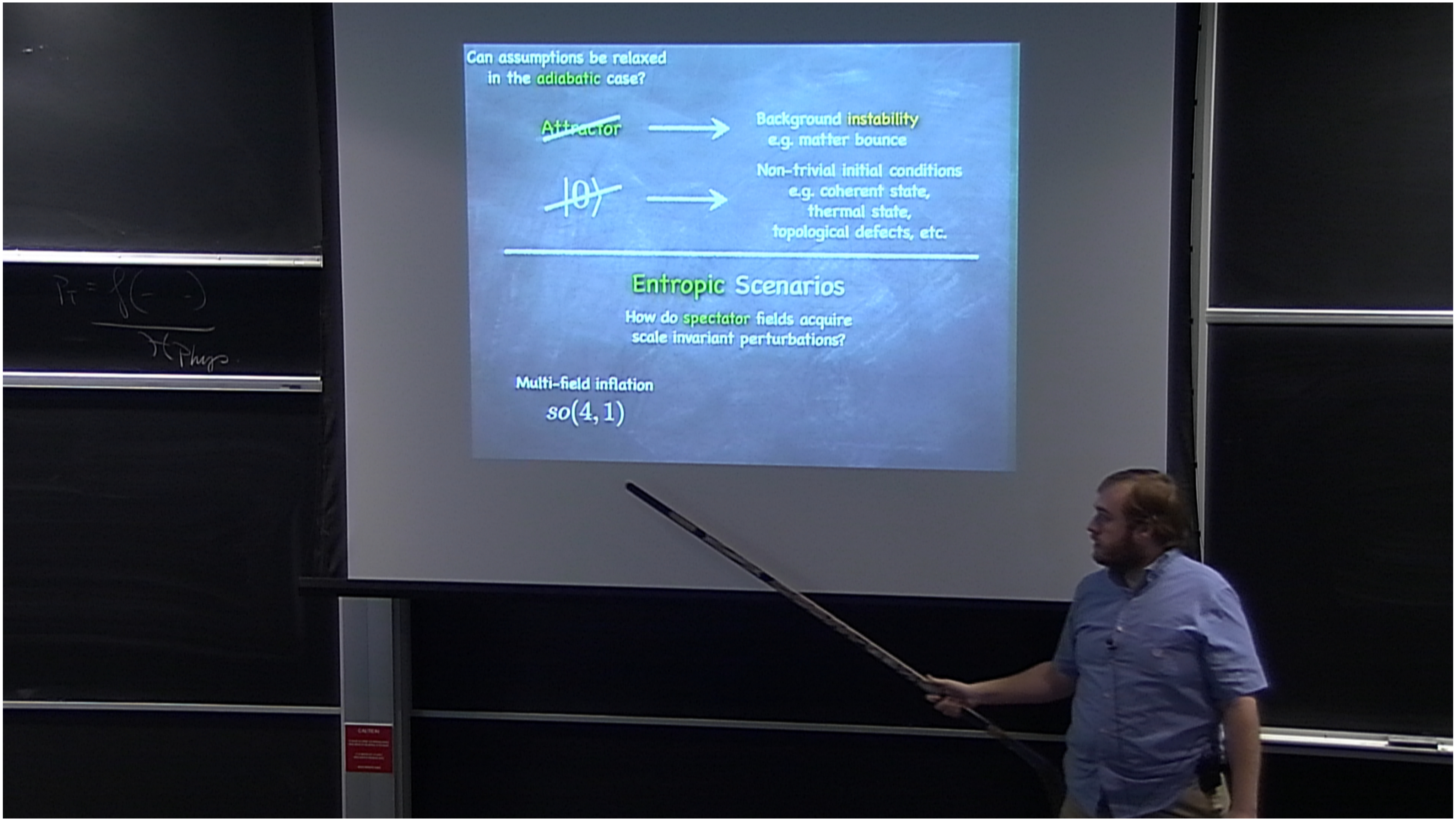
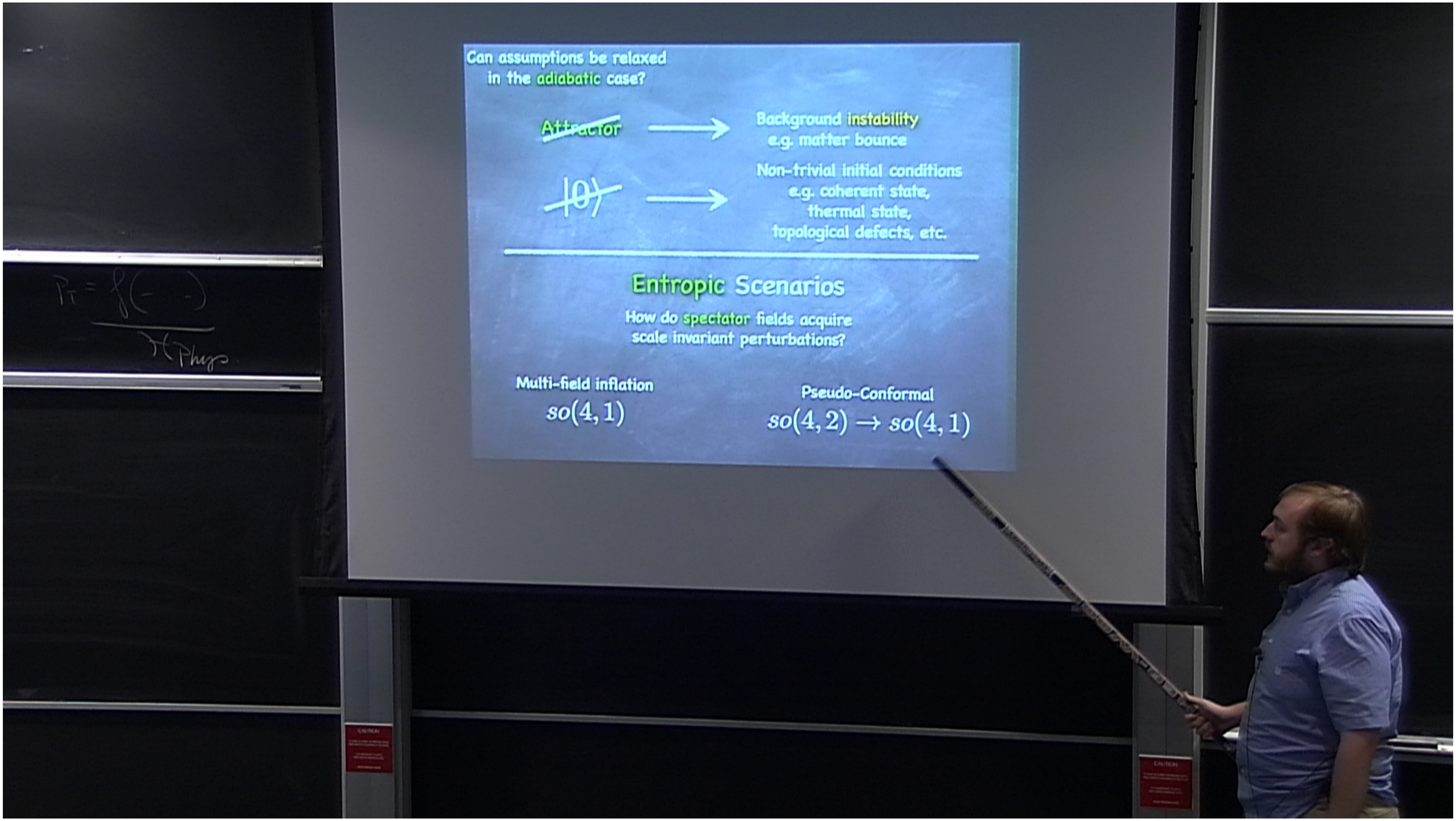
$c_s = 1$
J. Khoury & GEJM 1012.0846
$c_s(t)$
$\dot{a} > 0$
A. M. Dizgah, G. Geshnizjani, W. H. Kinney 1107.1241
$c_s \leq 1$
$\dot{\epsilon} \sim 0$
D. Baumann, L. Senatore, M. Zaldarriaga 1101.3320
$c_s(t)$
A. Joyce & J. Khoury 1107.3550
Upshot: Inflation is the unique adiabatic mechanism that can generate a broad range of scale invariant and gaussian modes


Can assumptions be relaxed
in the adiabatic case?
Attractor
Background instability
e.g. matter bounce
$|0\rangle$
Non-trivial initial conditions
e.g. coherent state,
thermal state,
topological defects, etc.
Entropic Scenarios
How do spectator fields acquire
scale invariant perturbations?
Multi-field inflation
$so(4,1)$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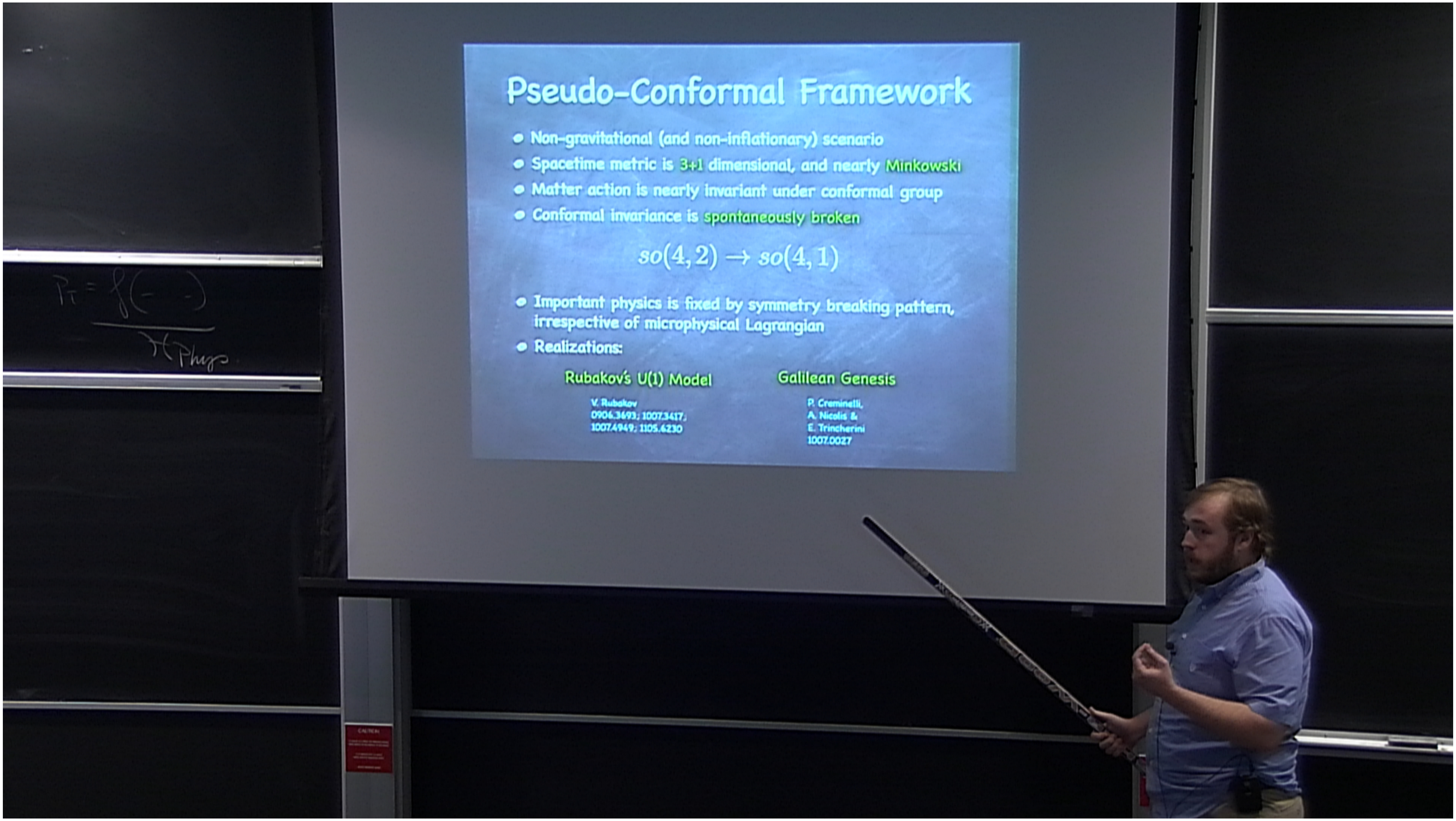
so(4, 1)
Pseudo-Conformal
so(4, 2) → so(4, 1)


Pseudo-Conformal Framework
Non-gravitational (and non-inflationary) scenario
Spacetime metric is 3+1 dimensional, and nearly Minkowski
Matter action is nearly invariant under conformal group
Conformal invariance is spontaneously broken
$so(4,2) \to so(4,1)$
Important physics is fixed by symmetry breaking pattern, irrespective of microphysical Lagrangian
Realizations:
Rubakov's U(1) Model
V. Rubakov
0906.3693; 1007.3417;
1007.4949; 1105.6230
Galilean Genesis
P. Creminelli,
A. Nicolis &
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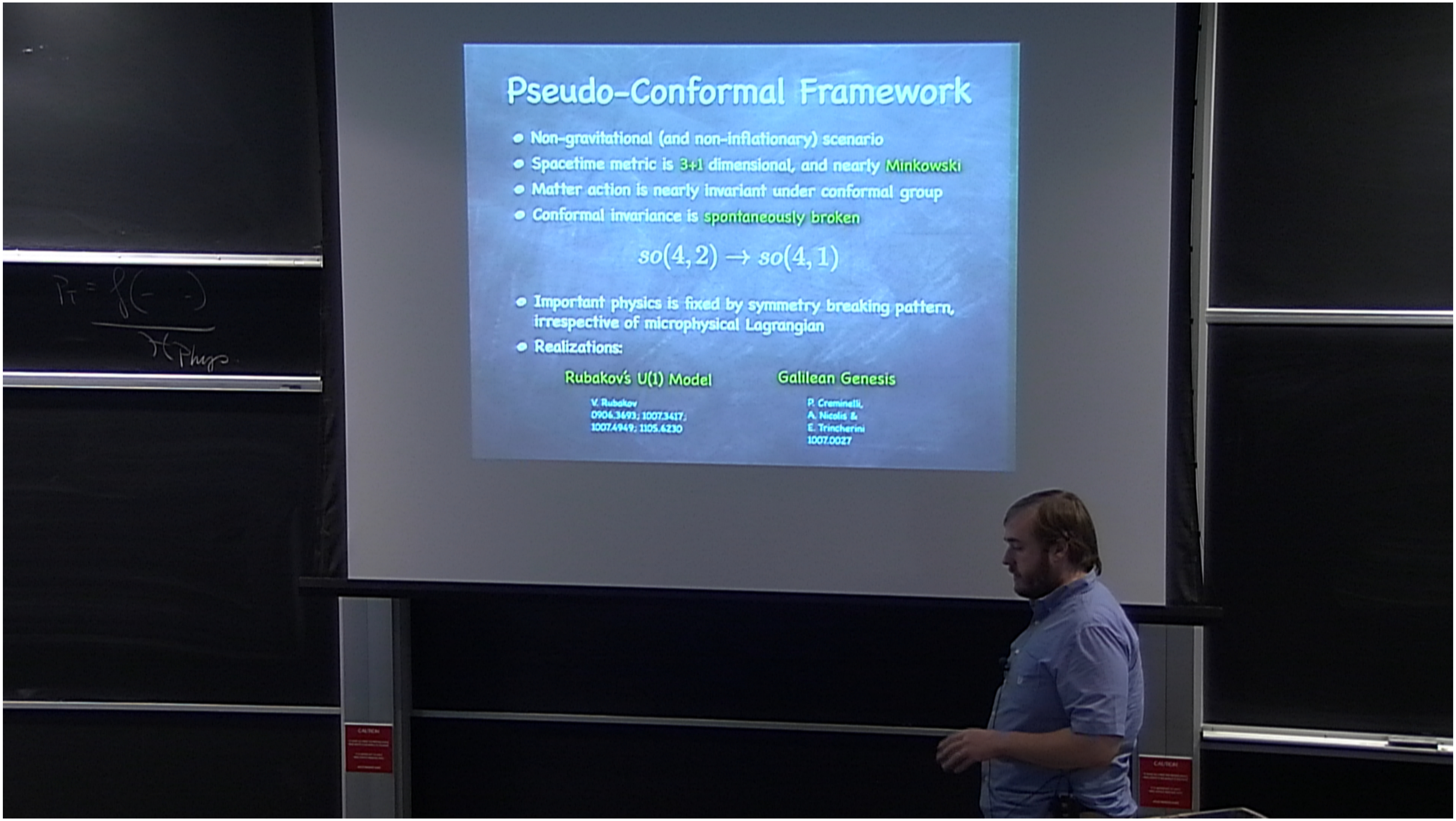
1007.0027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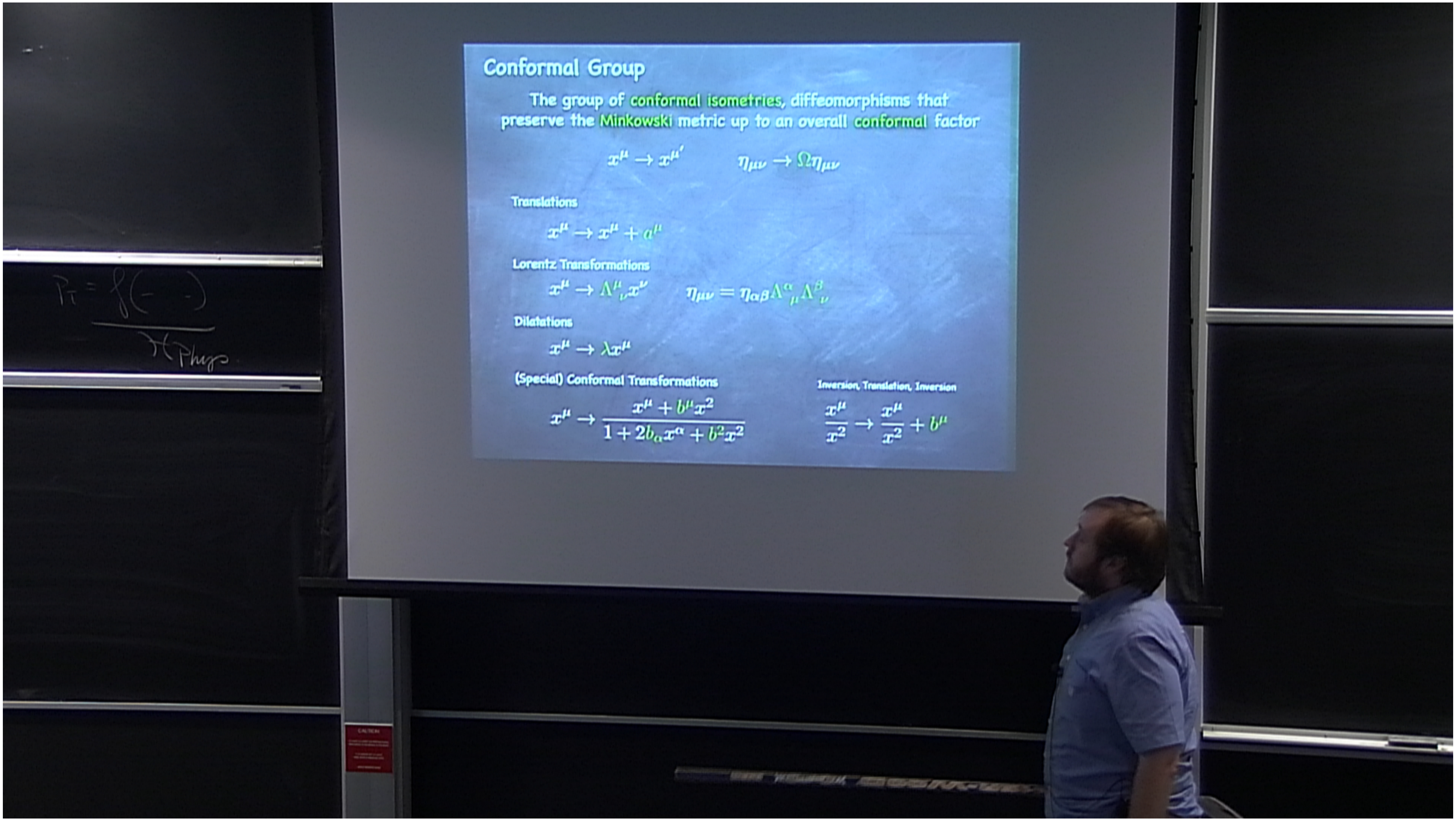
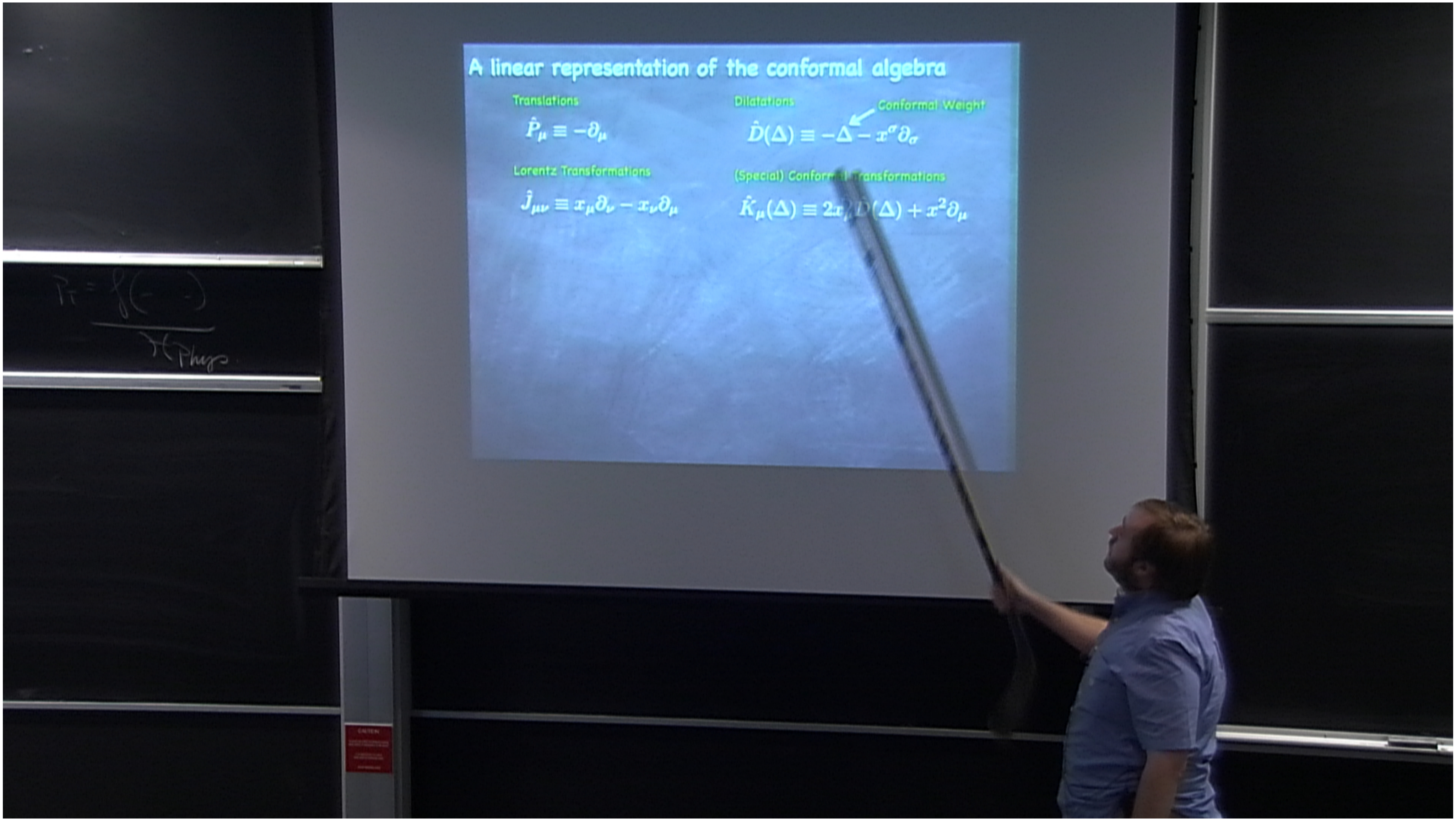
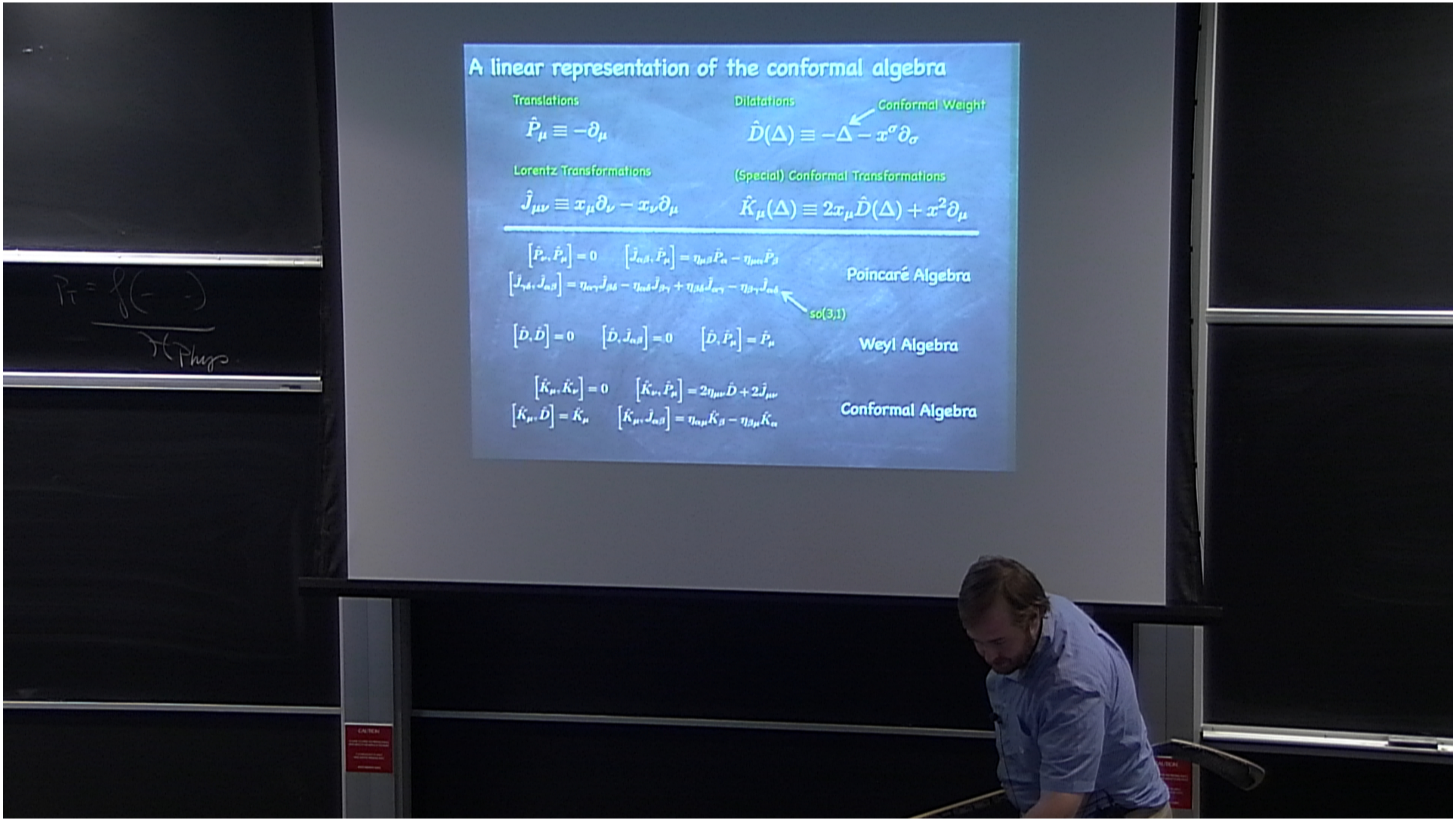

Conformal Group
The group of conformal isometries, diffeomorphisms that preserve the Minkowski metric up to an overall conformal factor
$x^\mu \to x^{\mu'}$
$\eta_{\mu\nu} \to \Omega \eta_{\mu\nu}$
Translations
$x^\mu \to x^\mu + a^\mu$
Lorentz Transformations
$x^\mu \to \Lambda^\mu{}_\nu x^\nu$
$\eta_{\mu\nu} = \eta_{\alpha\beta} \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu$
Dilatations
$x^\mu \to \lambda x^\mu$
(Special) Conformal Transformations
$x^\mu \to \frac{x^\mu + b^\mu x^2}{1 + 2 b_\alpha x^\alpha + b^2 x^2}$
Inversion, Translation, Inversion
$\frac{x^\mu}{x^2} \to \frac{x^\mu}{x^2} + b^\mu$


A linear representation of the conformal algebra
Translations
$\hat{P}_\mu \equiv -\partial_\mu$
Dilatations
Conformal Weight
$\hat{D}(\Delta) \equiv -\Delta - x^\sigma \partial_\sigma$
Lorentz Transformations
$\hat{J}_{\mu\nu} \equiv x_\mu \partial_\nu - x_\nu \partial_\mu$
(Special) Conformal Transformations
$\hat{K}_\mu(\Delta) \equiv 2x_\mu \hat{D}(\Delta) + x^2 \partial_\mu$



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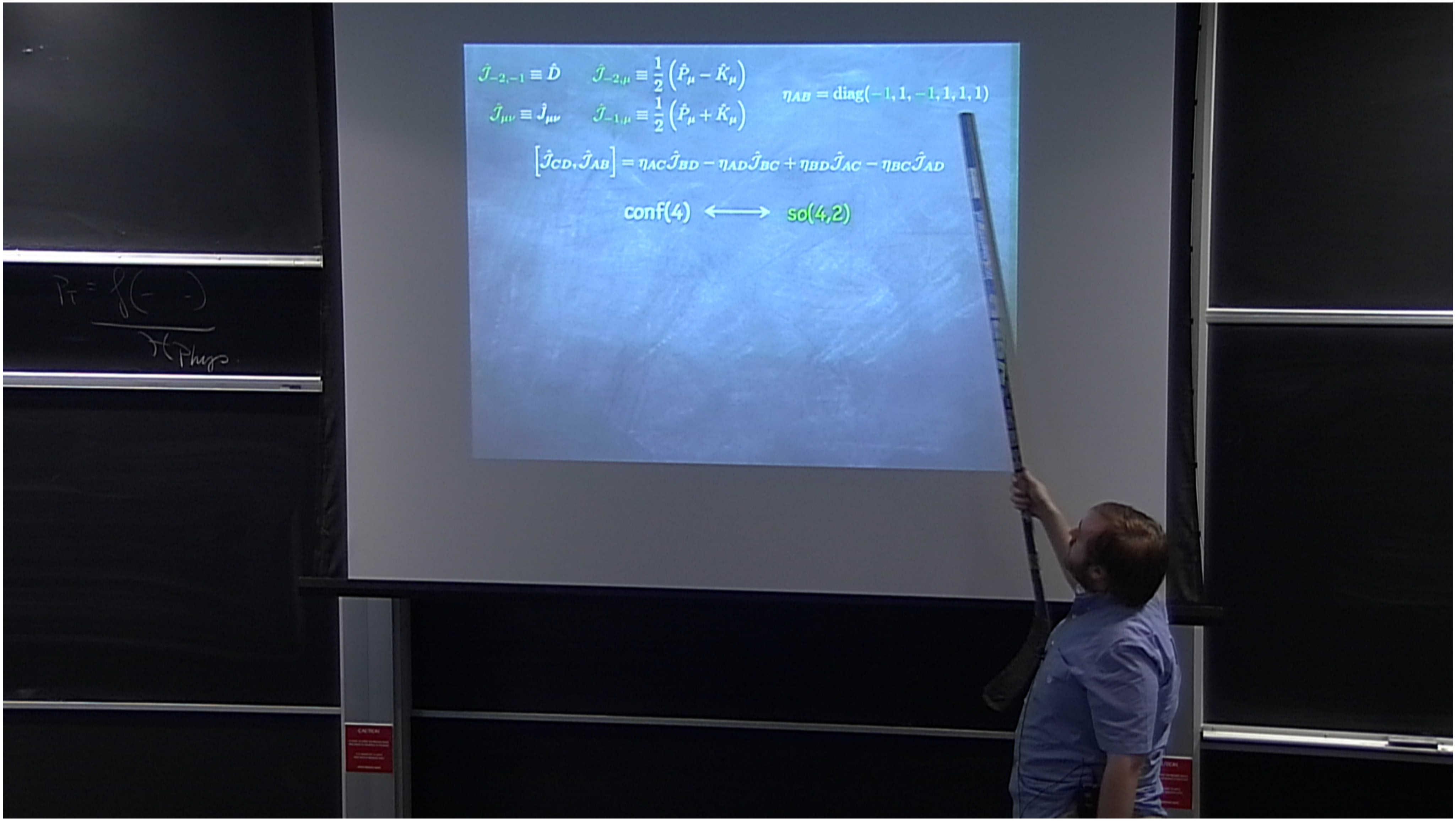
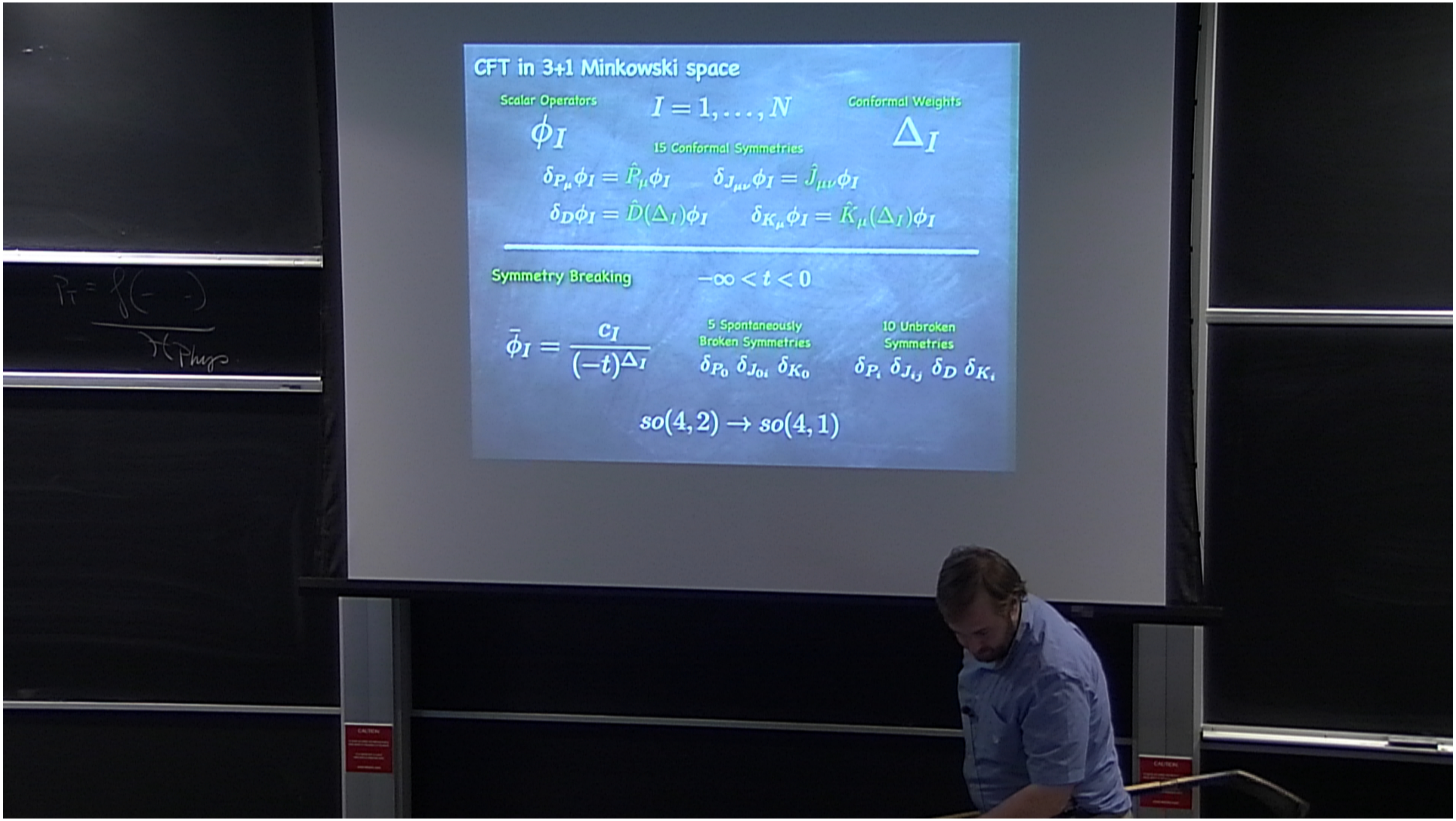
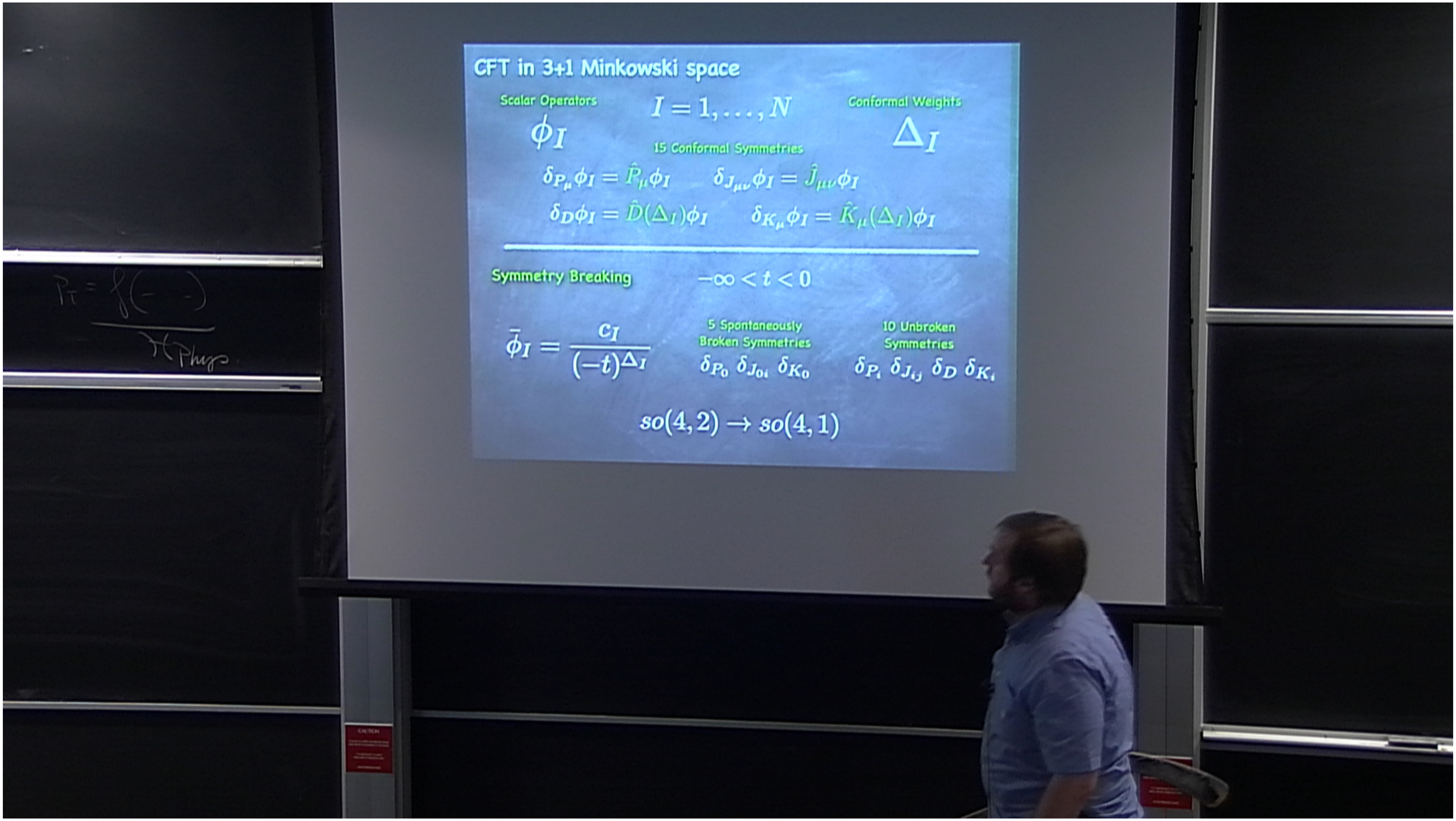
(Special) Conformal Transformations
$\hat{K}_\mu(\Delta) \equiv 2x_\mu \hat{D}(\Delta) + x^2 \partial_\mu$
$[\hat{P}_\nu, \hat{P}_\mu] = 0 \quad [\hat{J}_{\alpha\beta}, \hat{P}_\mu] = \eta_{\mu\beta}\hat{P}_\alpha - \eta_{\mu\alpha}\hat{P}_\beta$
$[\hat{J}_{\gamma\delta}, \hat{J}_{\alpha\beta}] = \eta_{\alpha\gamma}\hat{J}_{\beta\delta} - \eta_{\alpha\delta}\hat{J}_{\beta\gamma} + \eta_{\beta\delta}\hat{J}_{\alpha\gamma} - \eta_{\beta\gamma}\hat{J}_{\alpha\delta}$
so(3,1)
Poincaré Algebra
$[\hat{D}, \hat{D}] = 0 \quad [\hat{D}, \hat{J}_{\alpha\beta}] = 0 \quad [\hat{D}, \hat{P}_\mu] = \hat{P}_\mu$
Weyl Algebra
$[\hat{K}_\mu, \hat{K}_\nu] = 0 \quad [\hat{K}_\nu, \hat{P}_\mu] = 2\eta_{\mu\nu}\hat{D} + 2\hat{J}_{\mu\nu}$
$[\hat{K}_\mu, \hat{D}] = \hat{K}_\mu \quad [\hat{K}_\mu, \hat{J}_{\alpha\beta}] = \eta_{\alpha\mu}\hat{K}_\beta - \eta_{\beta\mu}\hat{K}_\alpha$
Conformal Algebra


$\hat{\mathcal{J}}_{-2,-1} \equiv \hat{D}$ $\quad \hat{\mathcal{J}}_{-2,\mu} \equiv \frac{1}{2}\left(\hat{P}_\mu - \hat{K}_\mu\right)$
$\hat{\mathcal{J}}_{\mu\nu} \equiv \hat{J}_{\mu\nu}$ $\quad \hat{\mathcal{J}}_{-1,\mu} \equiv \frac{1}{2}\left(\hat{P}_\mu + \hat{K}_\mu\right)$
$\eta_{AB} = \mathrm{diag}(-1, 1, -1, 1, 1, 1)$
$\left[\hat{\mathcal{J}}_{CD}, \hat{\mathcal{J}}_{AB}\right] = \eta_{AC}\hat{\mathcal{J}}_{BD} - \eta_{AD}\hat{\mathcal{J}}_{BC} + \eta_{BD}\hat{\mathcal{J}}_{AC} - \eta_{BC}\hat{\mathcal{J}}_{AD}$
conf(4) ⟷ so(4,2)


CFT in 3+1 Minkowski space
Scalar Operators
ϕ_I
$I = 1, \ldots, N$
Conformal Weights
Δ_I
15 Conformal Symmetries
$\delta_{P_\mu}\phi_I = \hat{P}_\mu\phi_I$ $\delta_{J_{\mu\nu}}\phi_I = \hat{J}_{\mu\nu}\phi_I$
$\delta_D\phi_I = \hat{D}(\Delta_I)\phi_I$ $\delta_{K_\mu}\phi_I = \hat{K}_\mu(\Delta_I)\phi_I$
Symmetry Breaking
$-\infty < t < 0$
$\bar{\phi}_I = \frac{c_I}{(-t)^{\Delta_I}}$
5 Spontaneously Broken Symmetries
$\delta_{P_0}\ \delta_{J_{0i}}\ \delta_{K_0}$
10 Unbroken Symmetries
$\delta_{P_i}\ \delta_{J_{ij}}\ \delta_D\ \delta_{K_i}$
$so(4,2) \to so(4,1)$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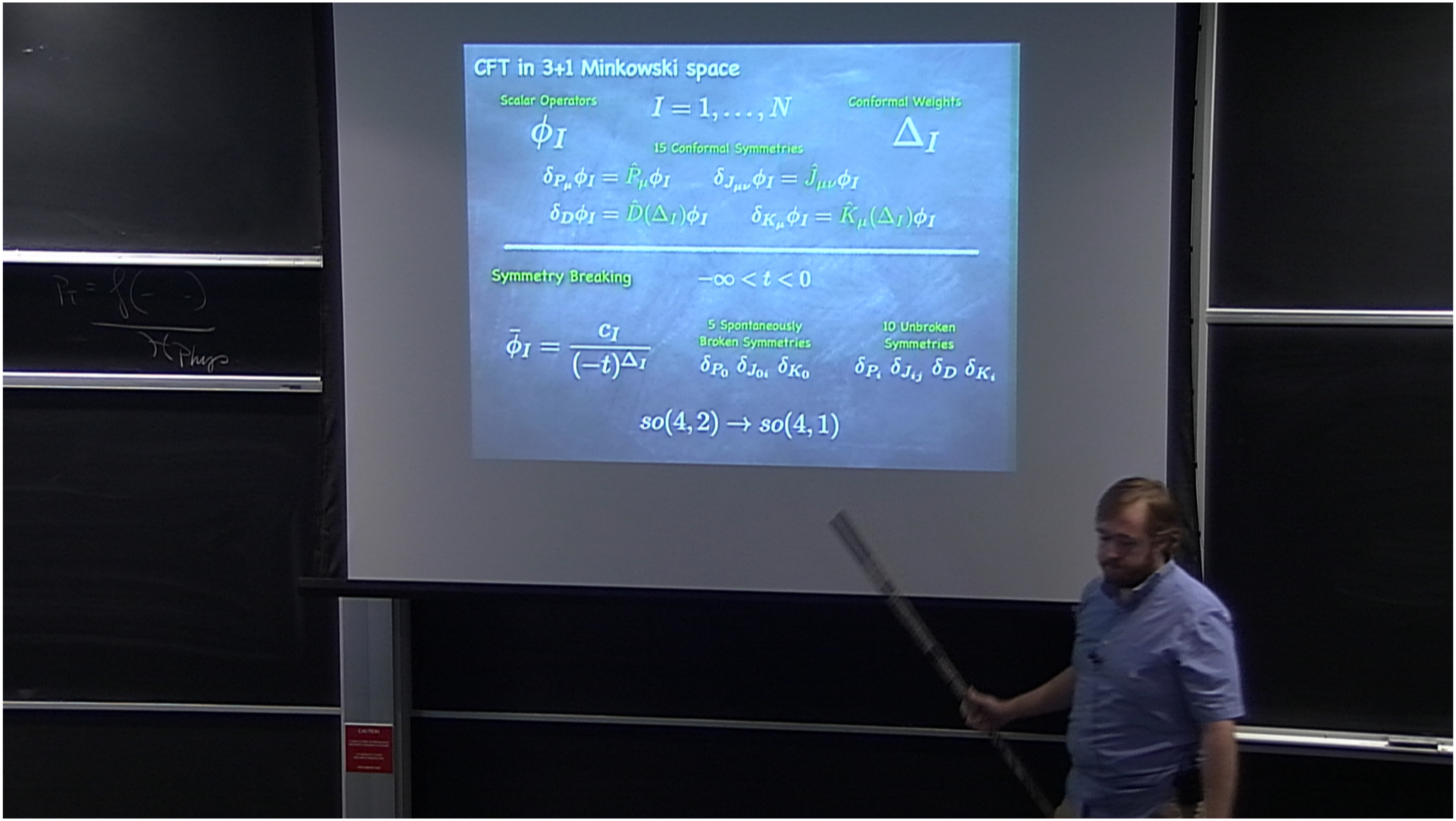
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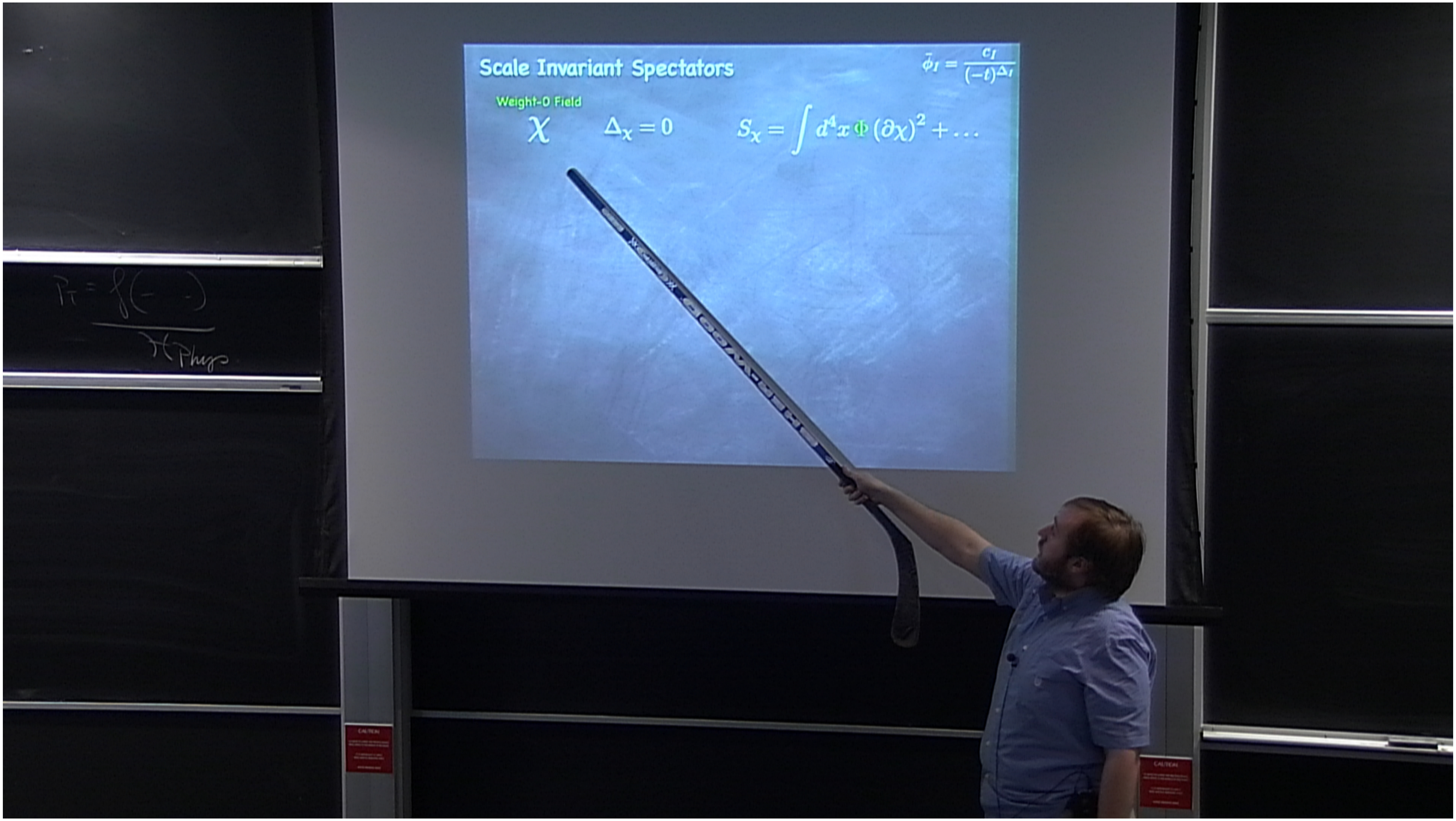
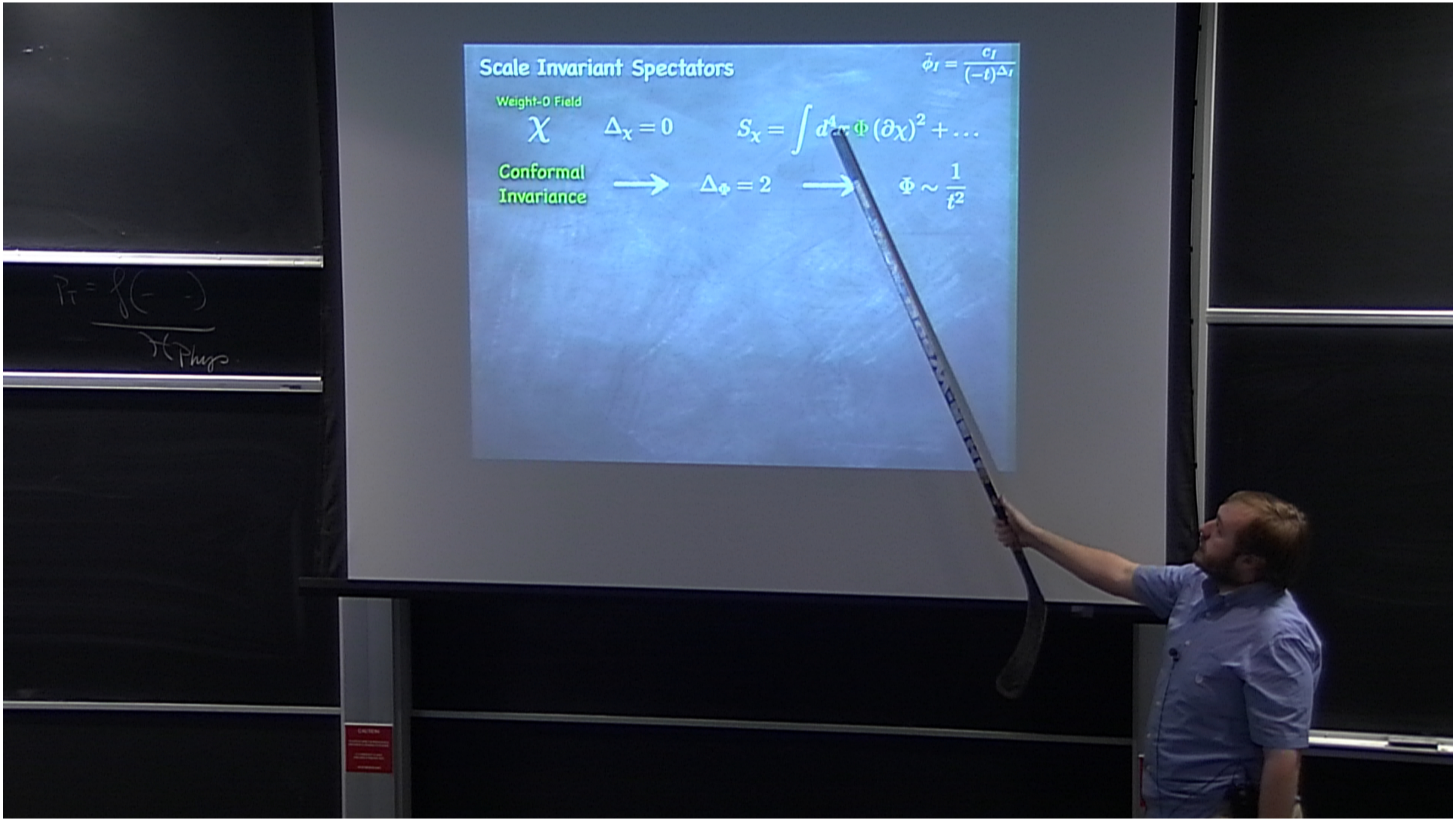


Scale Invariant Spectators
$\bar{\phi}_I = \frac{c_I}{(-t)^{\Delta_I}}$
Weight-0 Field
χ
$\Delta_\chi = 0$
$S_\chi = \int d^4x\, \Phi\, (\partial\chi)^2 + \dots$


Scale Invariant Spectators
$\bar{\phi}_I = \frac{c_I}{(-t)^{\Delta_I}}$
Weight-0 Field
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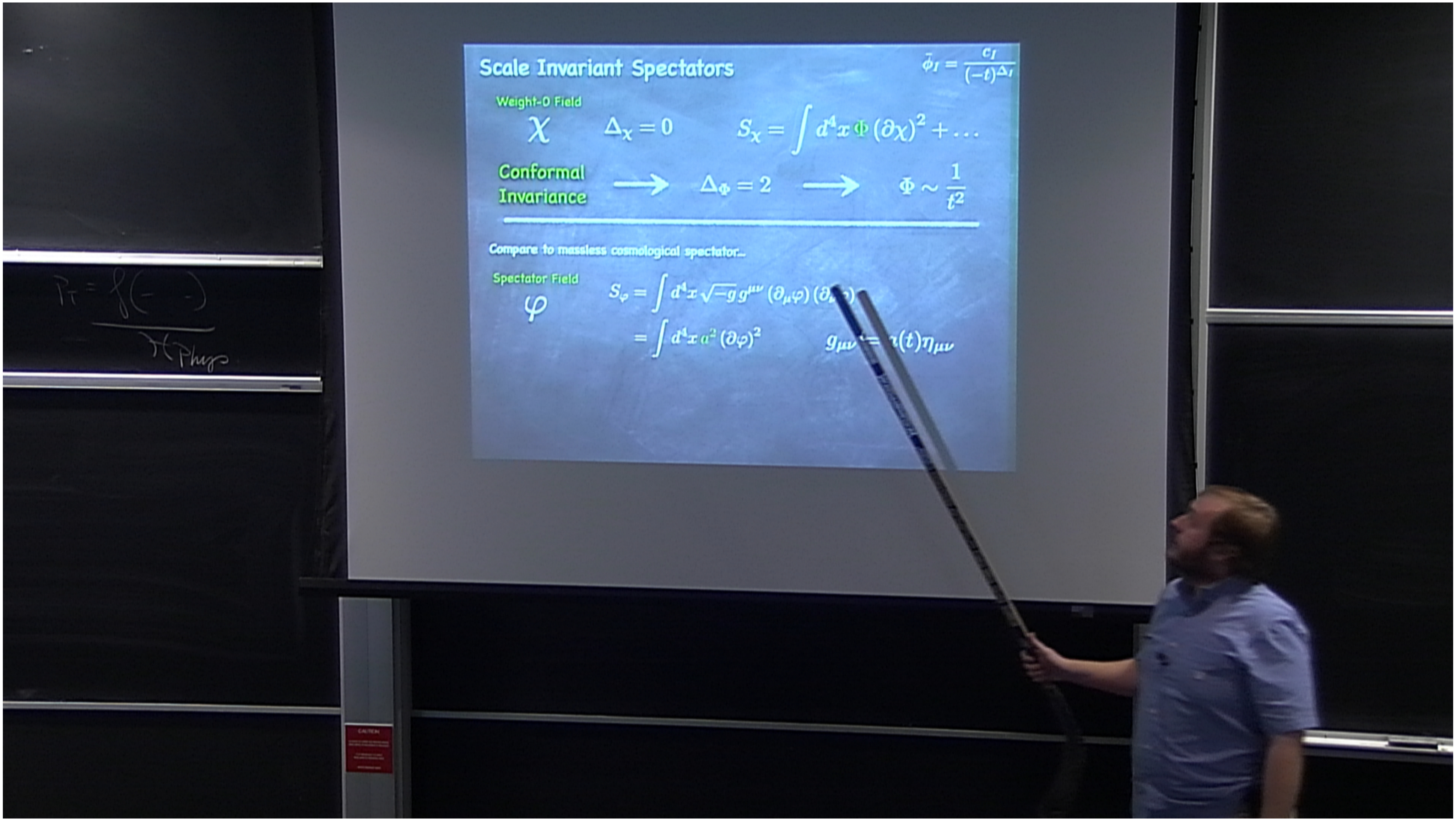
Conformal Invariance
$\Delta_\Phi = 2$
$\Phi \sim \frac{1}{t^2}$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Compare to massless cosmological spectator...
Spectator Field
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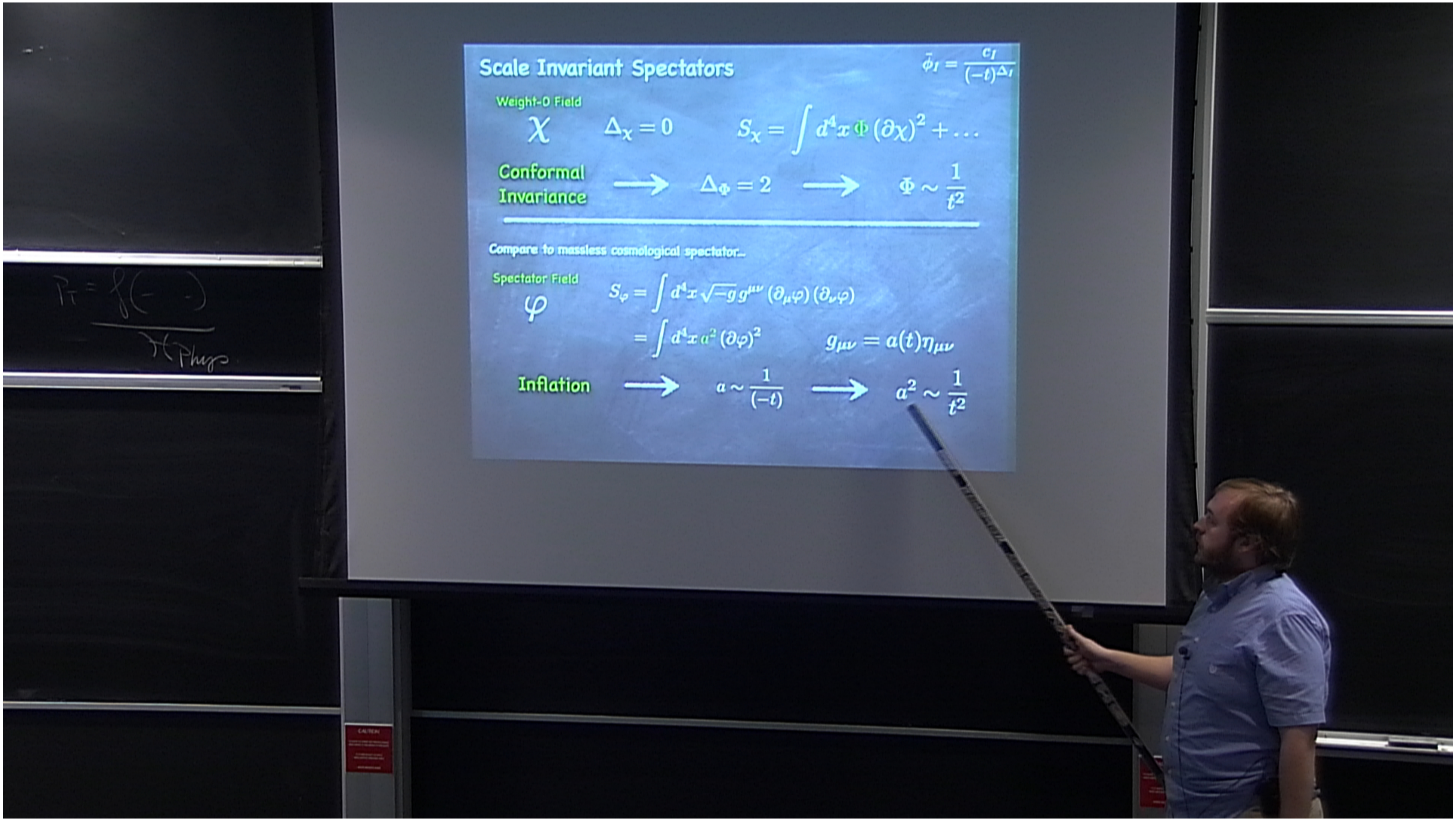
$S_\varphi = \int d^4x \sqrt{-g}\, g^{\mu\nu} (\partial_\mu\varphi)$
$= \int d^4x\, a^2 (\partial\varphi)^2$
$g_{\mu\nu}$



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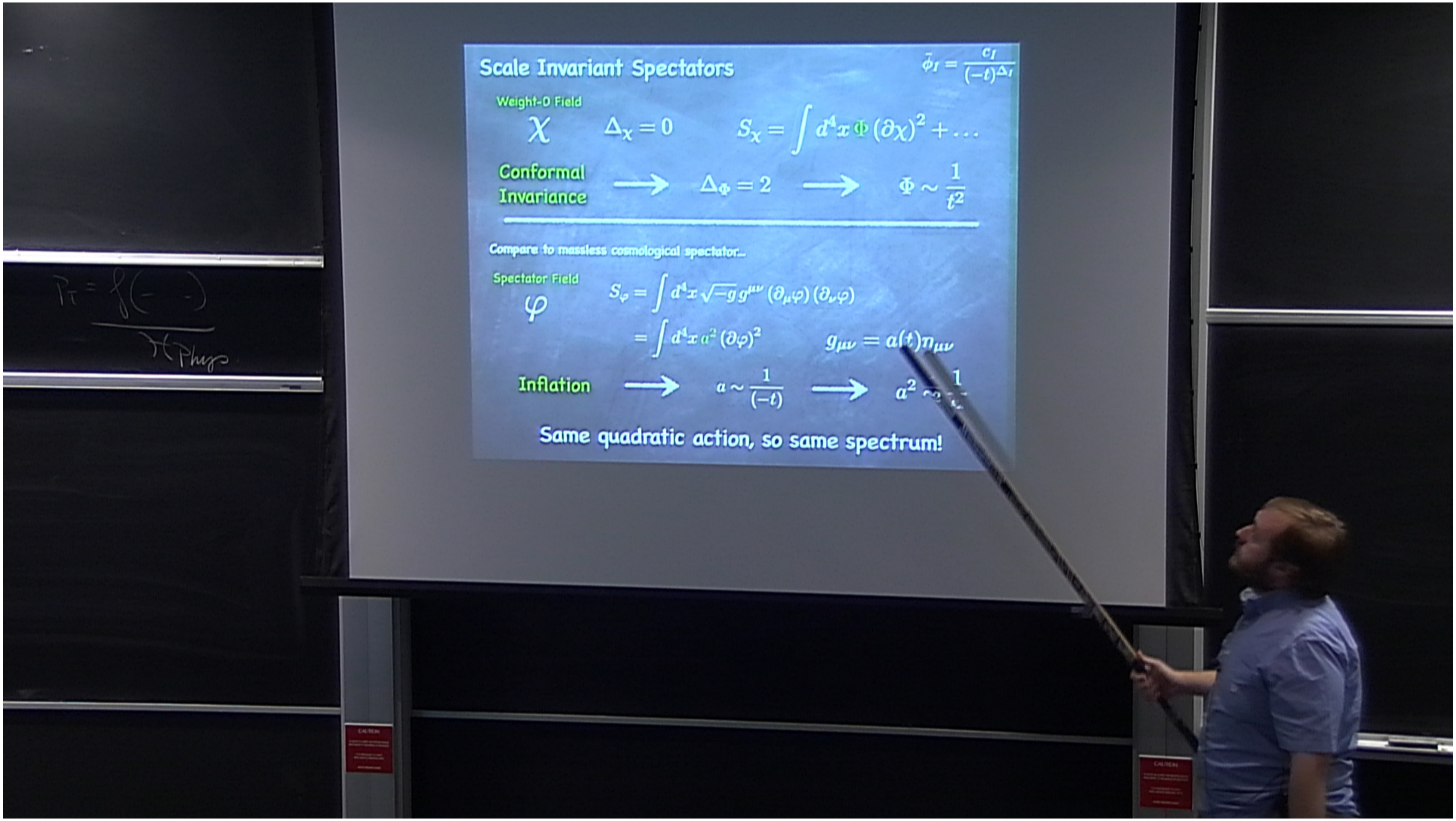
$S_\varphi = \int d^4x \sqrt{-g}\, g^{\mu\nu} (\partial_\mu\varphi)(\partial_\nu\varphi)$
$= \int d^4x\, a^2 (\partial\varphi)^2$
$g_{\mu\nu} = a(t)\eta_{\mu\nu}$
Inflation
$a \sim \frac{1}{(-t)}$
$a^2 \sim \frac{1}{t^2}$



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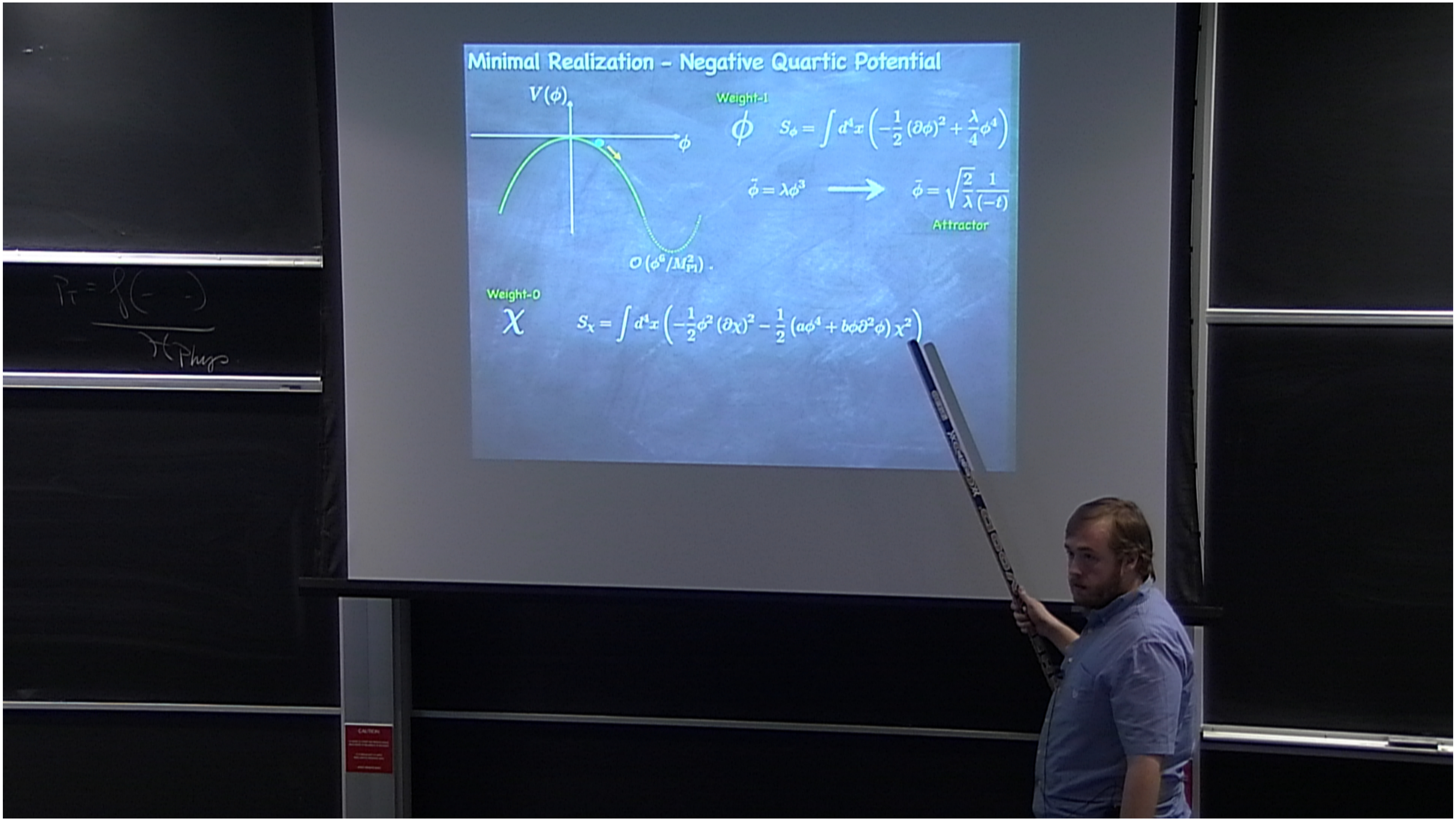
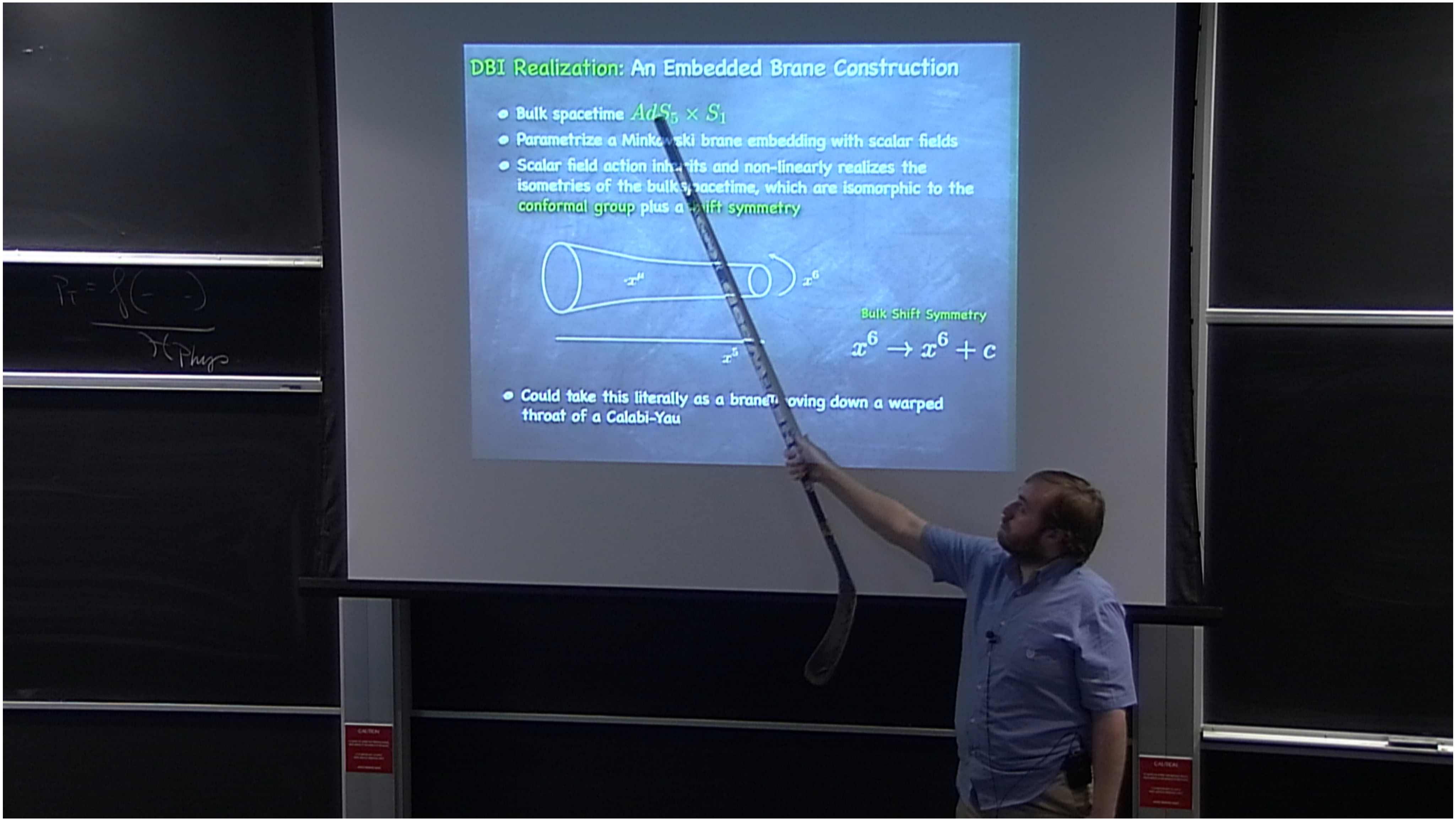
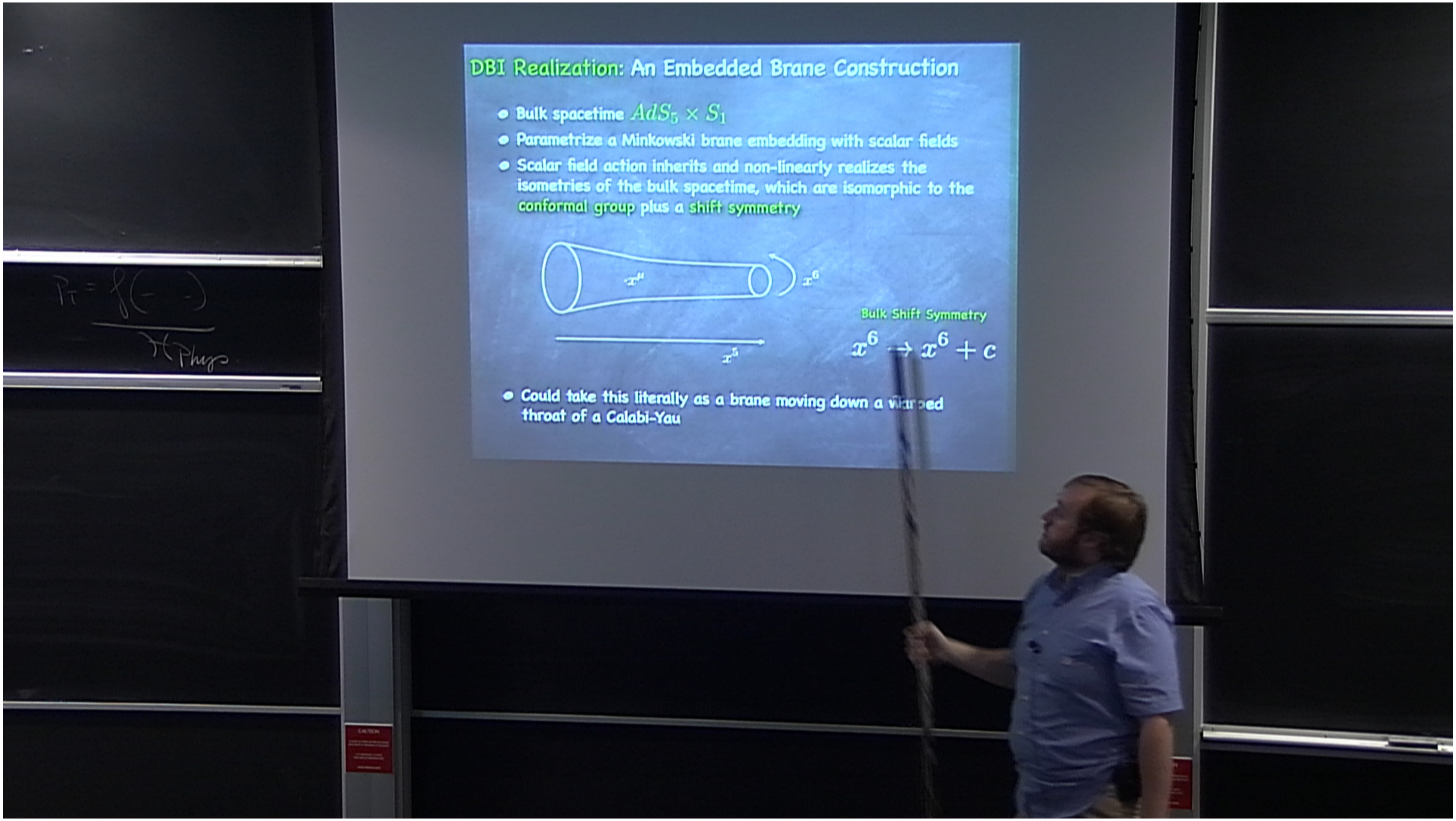
Same quadratic action, so same spectrum!


Minimal Realization – Negative Quartic Potential
$V(\phi)$
ϕ
Weight-1
ϕ
$S_\phi = \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4}\phi^4\right)$
$\ddot{\phi} = \lambda\phi^3$
$\bar{\phi} = \sqrt{\frac{2}{\lambda}}\frac{1}{(-t)}$
Attractor
$\mathcal{O}(\phi^6/M_{Pl}^2)$.
Weight-0
χ
$S_\chi = \int d^4x \left(-\frac{1}{2}\phi^2(\partial\chi)^2 - \frac{1}{2}(a\phi^4 + b\phi\partial^2\phi)\chi^2\right)$


DBI Realization: An Embedded Brane Construction
Bulk spacetime $AdS_5 \times S_1$
Parametrize a Minkowski brane embedding with scalar fields
Scalar field action inherits and non-linearly realizes the isometries of the bulk spacetime, which are isomorphic to the conformal group plus a shift symmetry
x^μ
x^6
x^5
Bulk Shift Symmetry
$x^6 \to x^6 + c$
Could take this literally as a brane moving down a warped throat of a Calabi-Yau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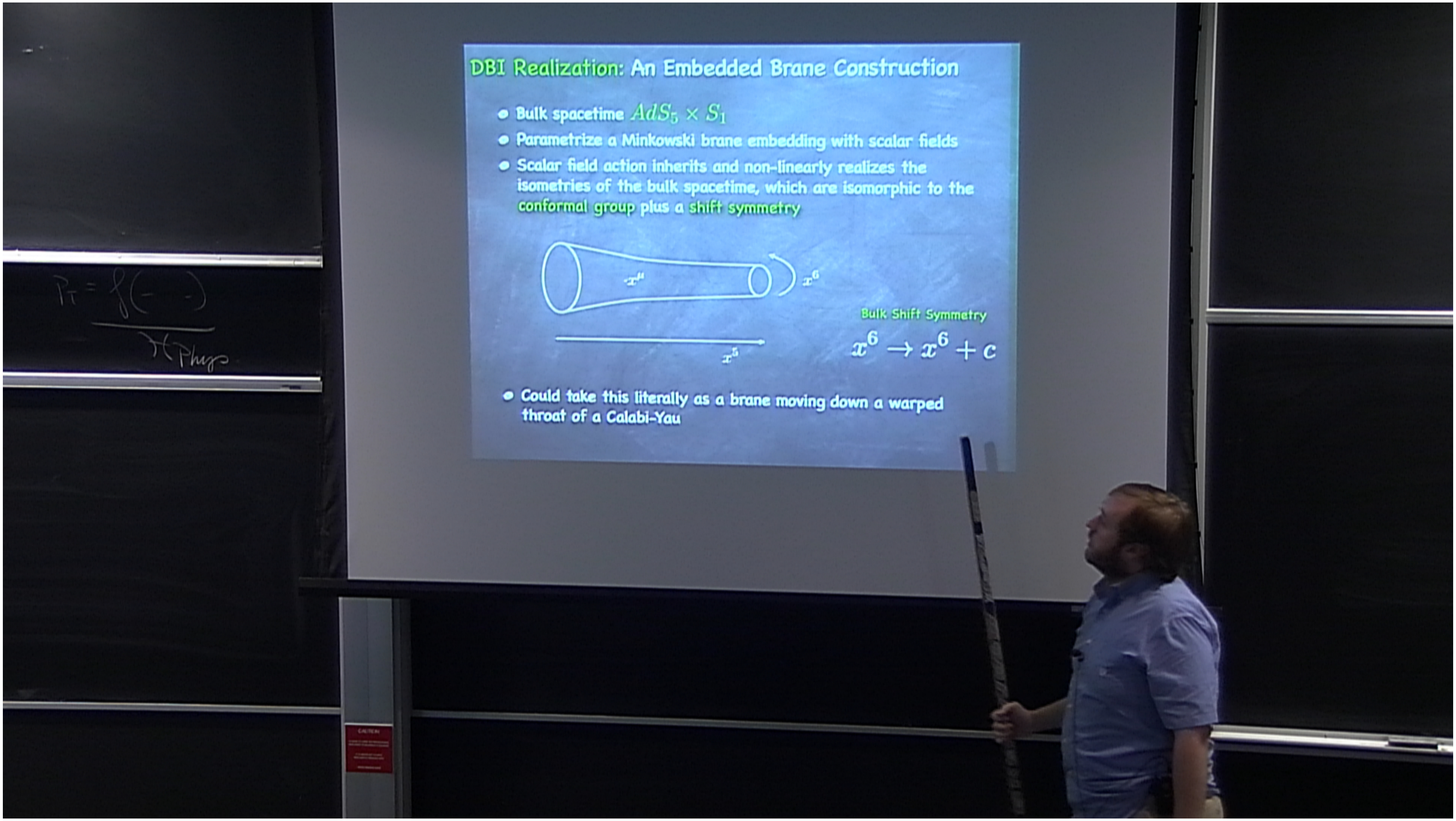
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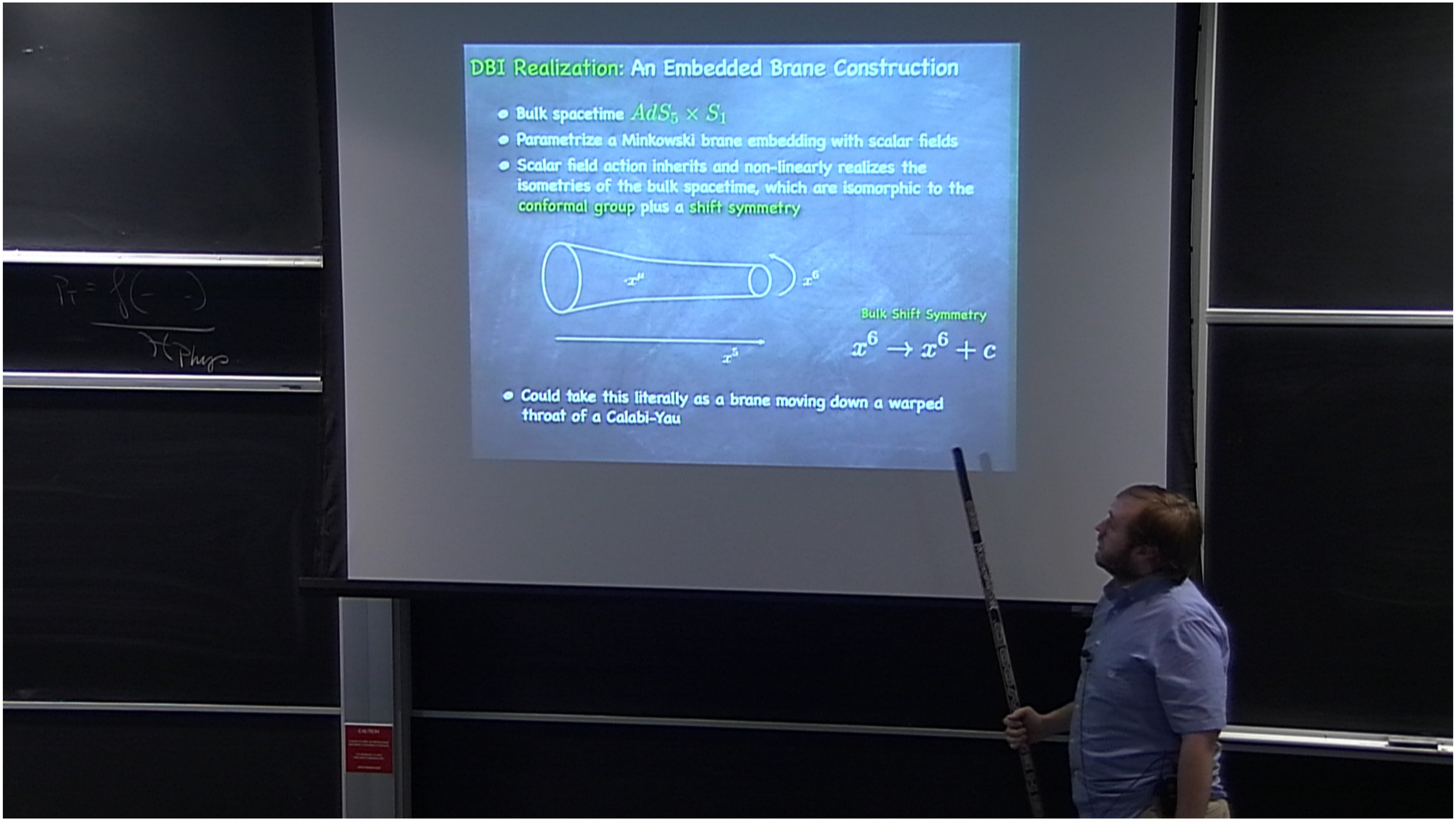
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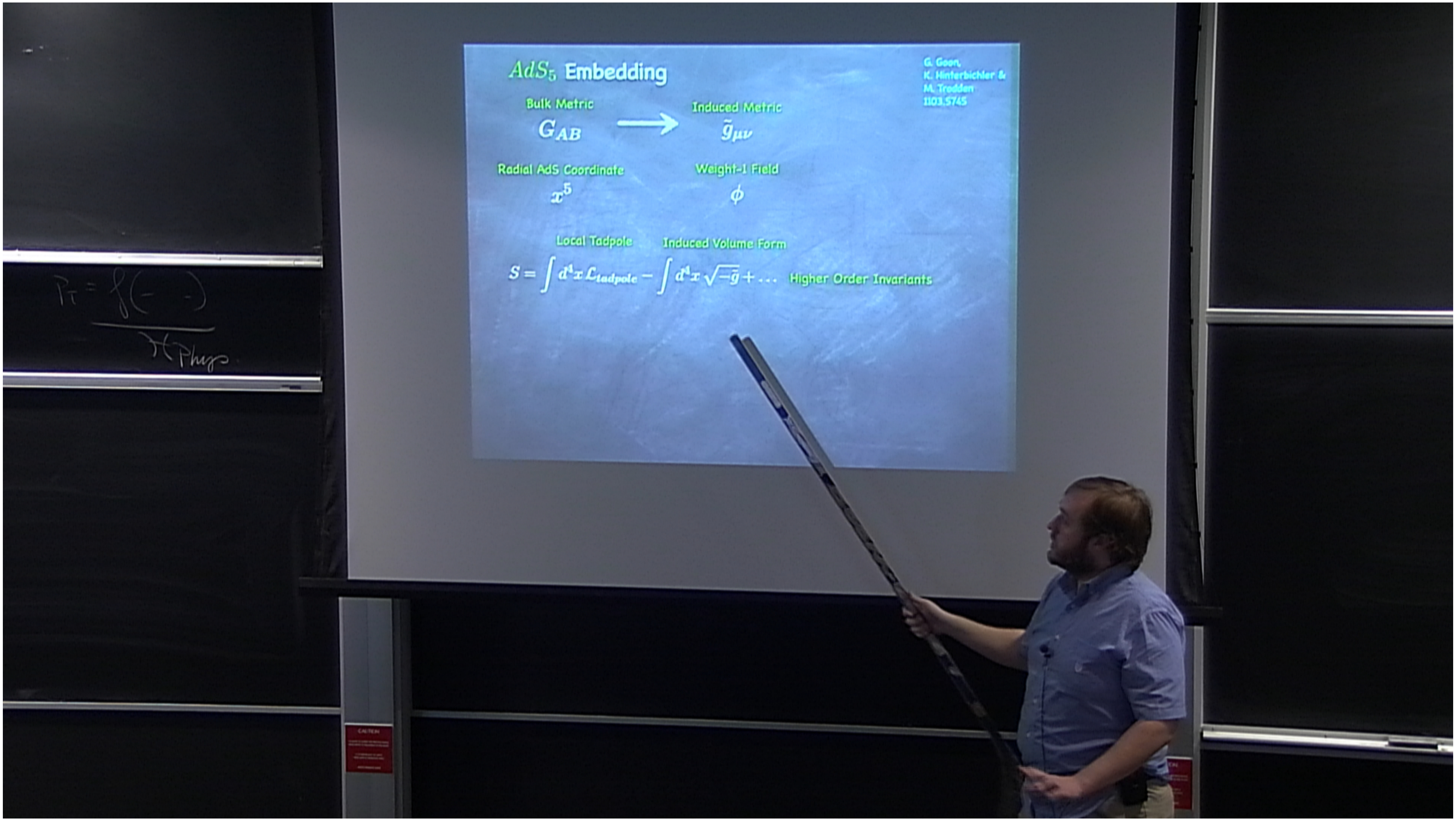
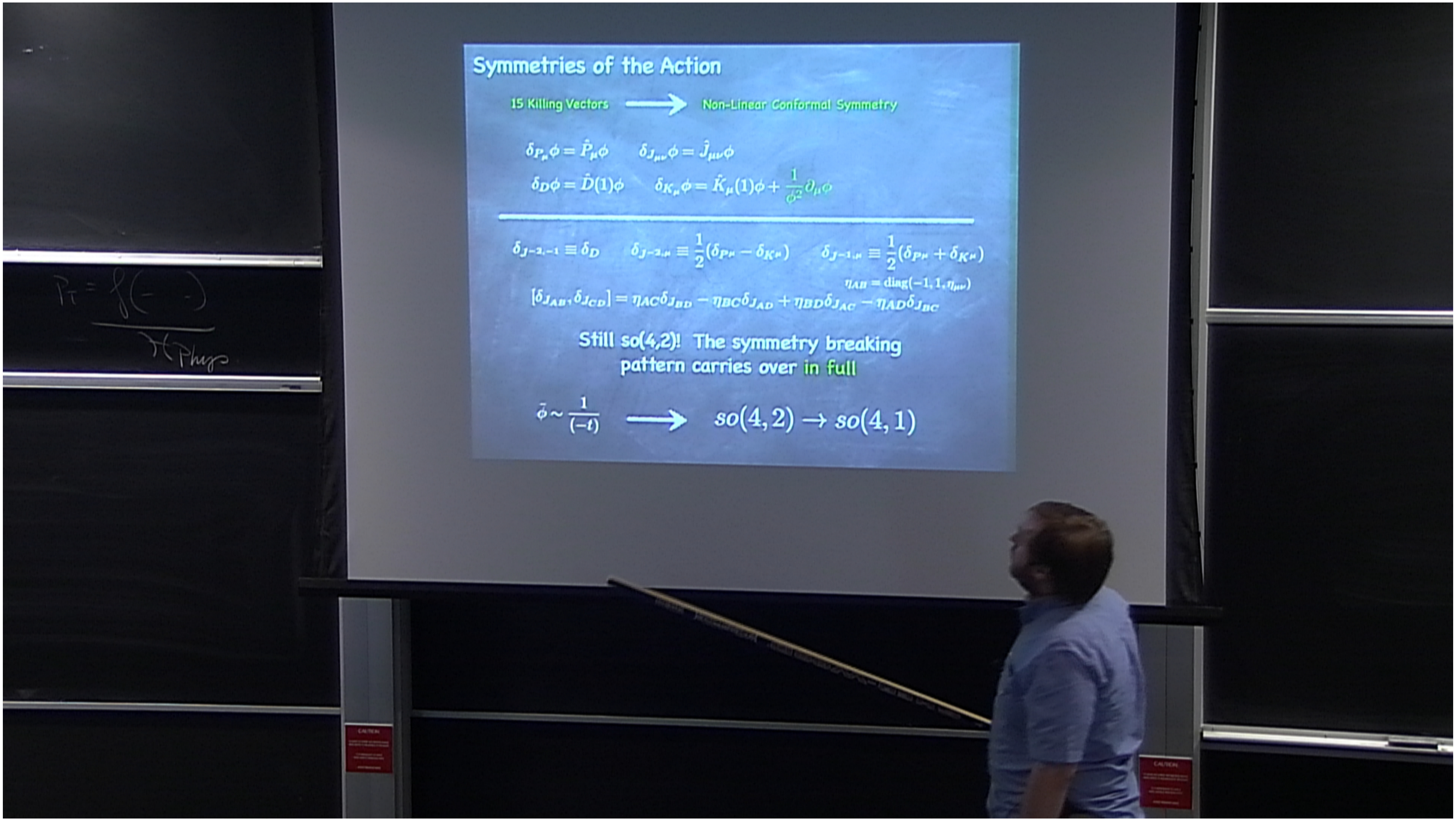
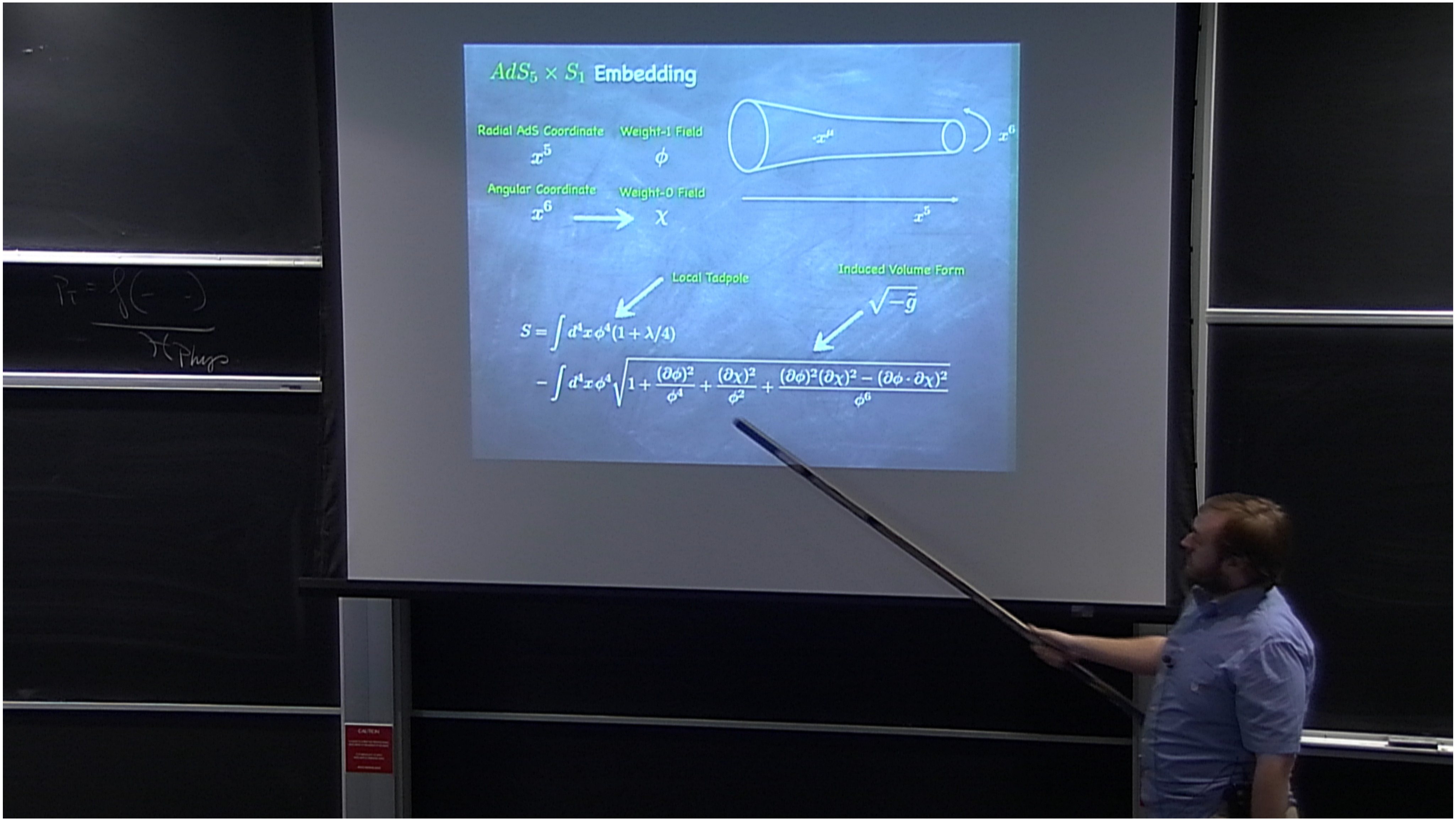
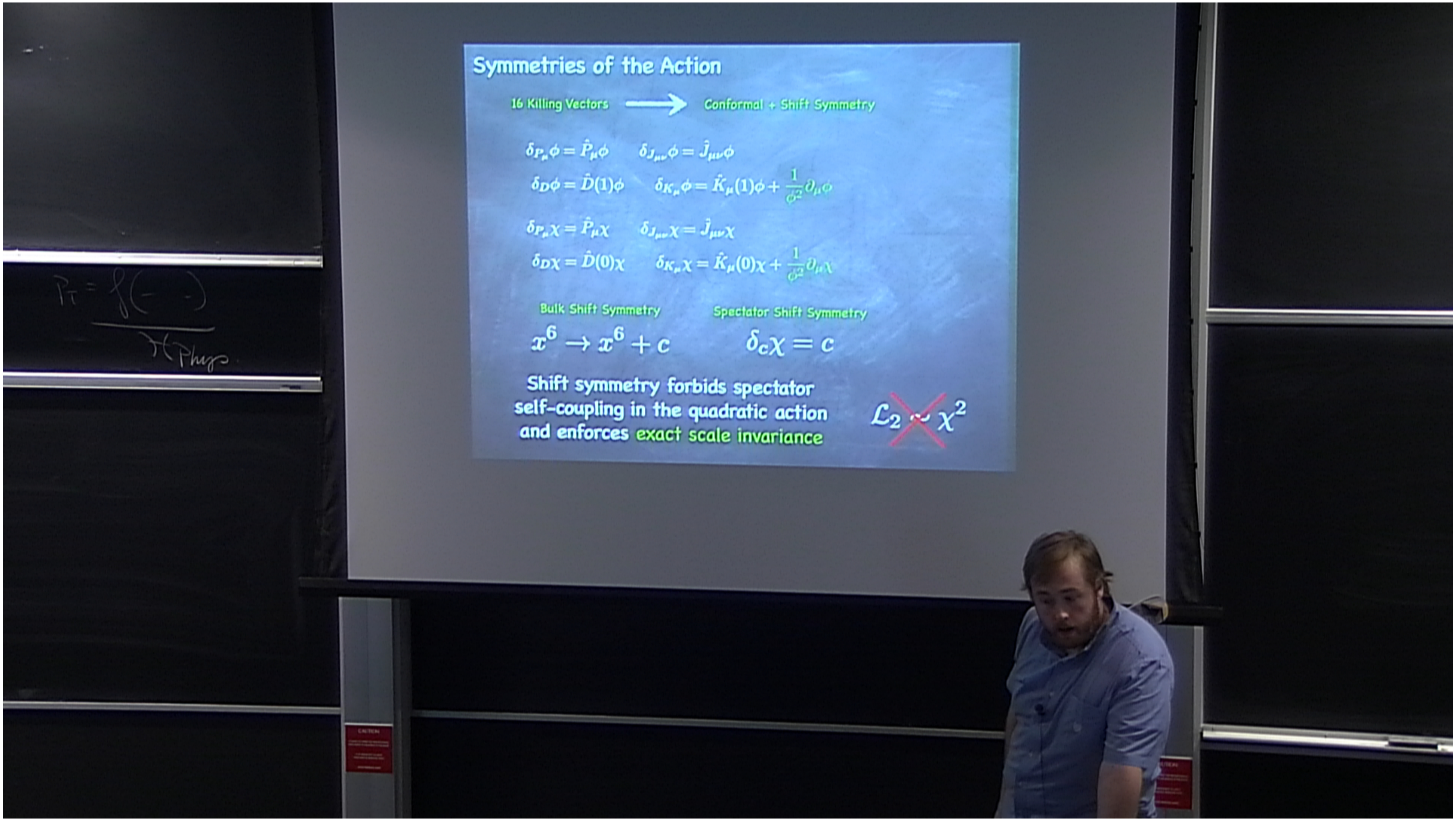

AdS_5 Embedding
G. Goon, K. Hinterbichler & M. Trodden 1103.5745
Bulk Metric
G_{AB}
Induced Metric
$\tilde{g}_{\mu\nu}$
Radial AdS Coordinate
x^5
Weight-1 Field
ϕ
Local Tadpole
Induced Volume Form
$S = \int d^4x \, \mathcal{L}_{tadpole} - \int d^4x \sqrt{-\tilde{g}} + \ldots$
Higher Order Invariants


Symmetries of the Action
15 Killing Vectors → Non-Linear Conformal Symmetry
$\delta_{P_\mu}\phi = \hat{P}_\mu\phi$ $\delta_{J_{\mu\nu}}\phi = \hat{J}_{\mu\nu}\phi$
$\delta_D\phi = \hat{D}(1)\phi$ $\delta_{K_\mu}\phi = \hat{K}_\mu(1)\phi + \frac{1}{\phi^2}\partial_\mu\phi$
$\delta_{J_{-2,-1}} \equiv \delta_D$ $\delta_{J_{-2,\mu}} \equiv \frac{1}{2}(\delta_{P^\mu} - \delta_{K^\mu})$ $\delta_{J_{-1,\mu}} \equiv \frac{1}{2}(\delta_{P^\mu} + \delta_{K^\mu})$
$\eta_{AB} = \mathrm{diag}(-1, 1, \eta_{\mu\nu})$
$[\delta_{J_{AB}}, \delta_{J_{CD}}] = \eta_{AC}\delta_{J_{BD}} - \eta_{BC}\delta_{J_{AD}} + \eta_{BD}\delta_{J_{AC}} - \eta_{AD}\delta_{J_{BC}}$
Still so(4,2)! The symmetry breaking pattern carries over in full
$\bar{\phi} \sim \frac{1}{(-t)}$ → $so(4,2) \to so(4,1)$



$AdS_5 \times S_1$ Embedding
Radial AdS Coordinate
x^5
Weight-1 Field
ϕ
Angular Coordinate
x^6
Weight-0 Field
χ
x^μ
x^6
x^5
Local Tadpole
Induced Volume Form
$\sqrt{-\tilde{g}}$
$S = \int d^4x\, \phi^4 (1 + \lambda/4)$
$- \int d^4x\, \phi^4 \sqrt{1 + \frac{(\partial\phi)^2}{\phi^4} + \frac{(\partial\chi)^2}{\phi^2} + \frac{(\partial\phi)^2(\partial\chi)^2 - (\partial\phi\cdot\partial\chi)^2}{\phi^6}}$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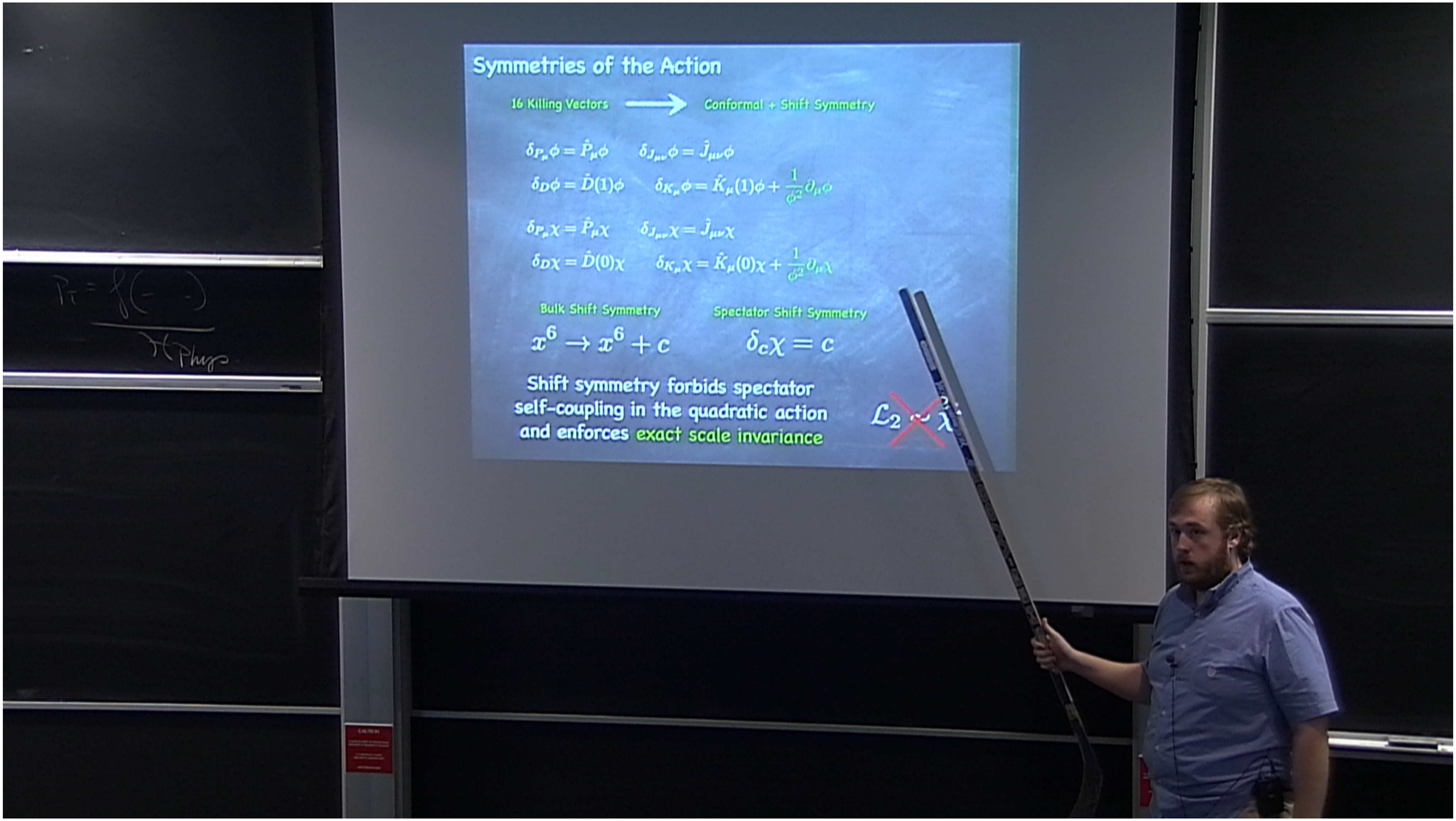
Bulk Shift Symmetry
$x^6 \to x^6 + c$
Spectator Shift Symmetry
$\delta_c\chi = c$
Shift symmetry forbids spectator self-coupling in the quadratic action and enforces exact scale invariance
$\mathcal{L}_2 \sim \chi^2$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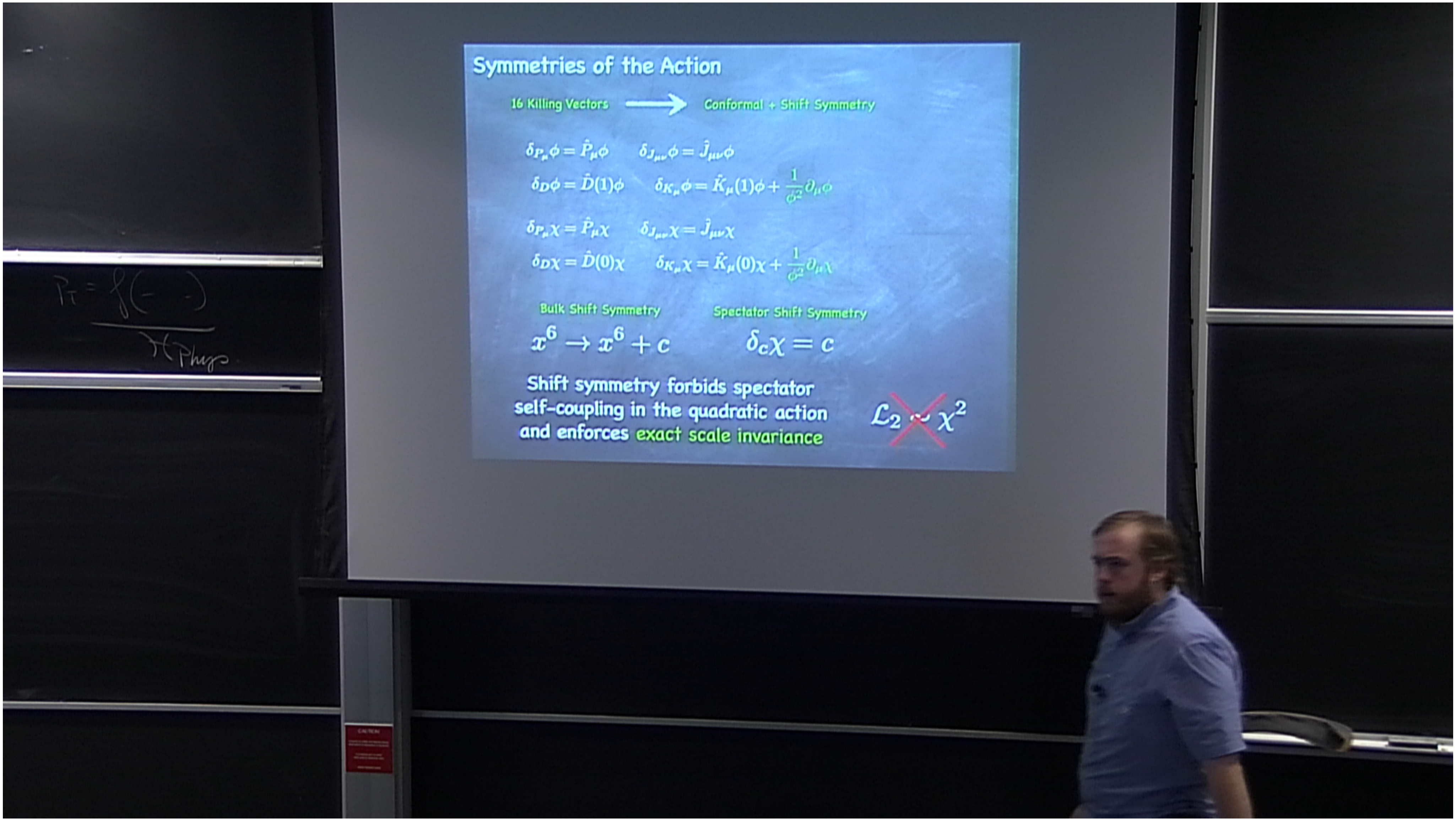
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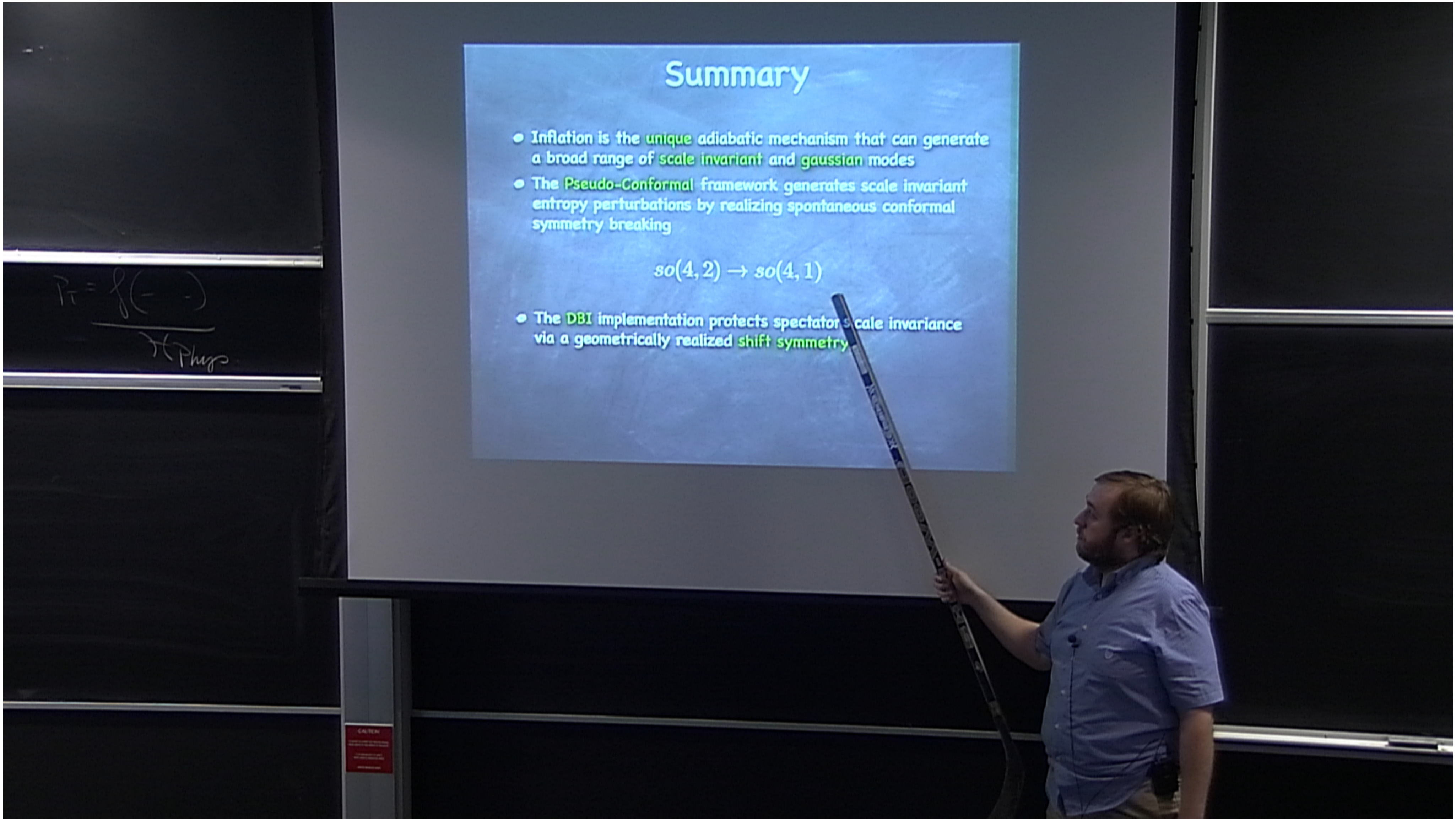
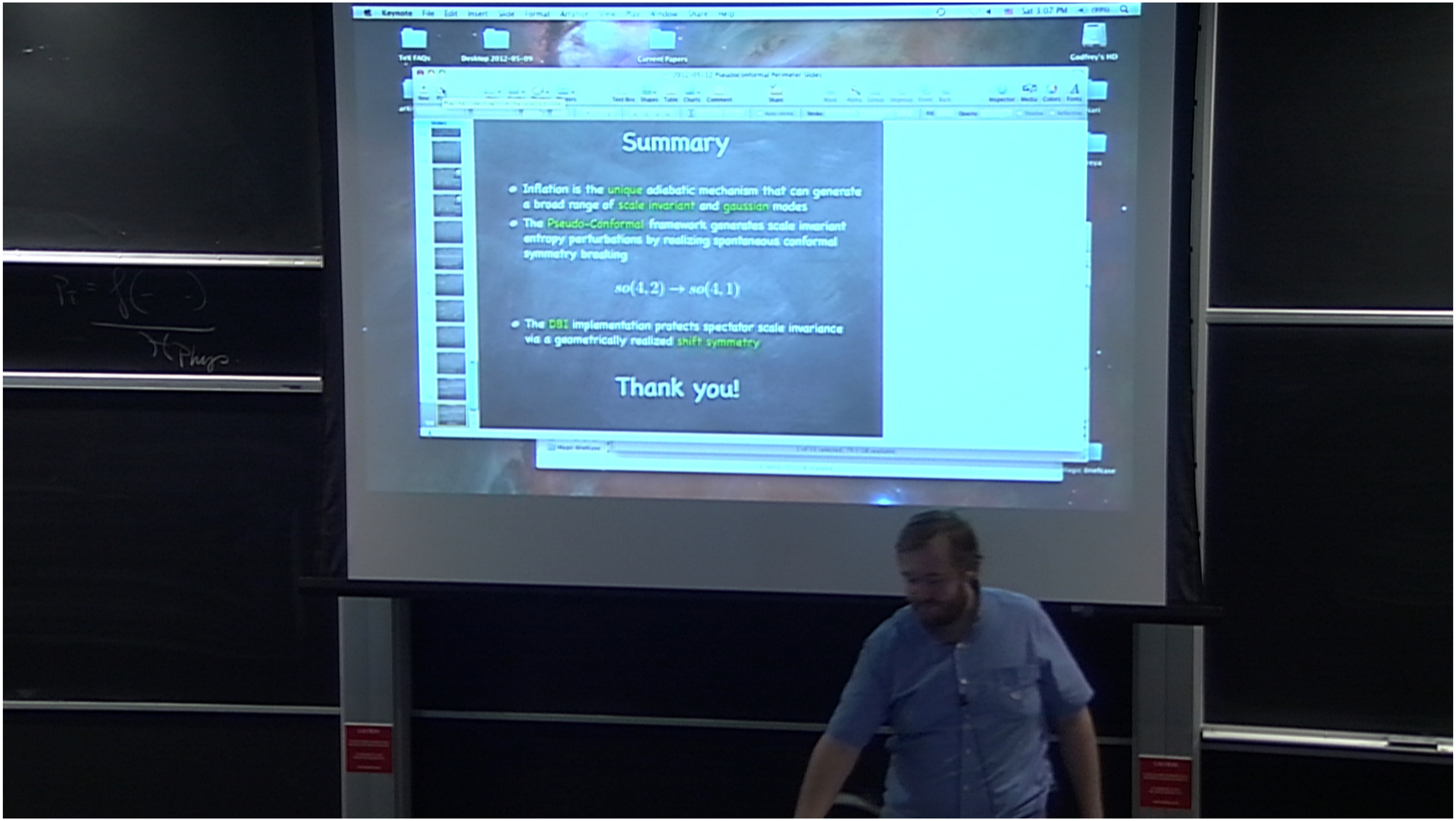

Summary
Inflation is the unique adiabatic mechanism that can generate a broad range of scale invariant and gaussian modes
The Pseudo-Conformal framework generates scale invariant entropy perturbations by realizing spontaneous conformal symmetry breaking
$so(4,2) \to so(4,1)$
The DBI implementation protects spectator scale invariance via a geometrically realized shift symmetry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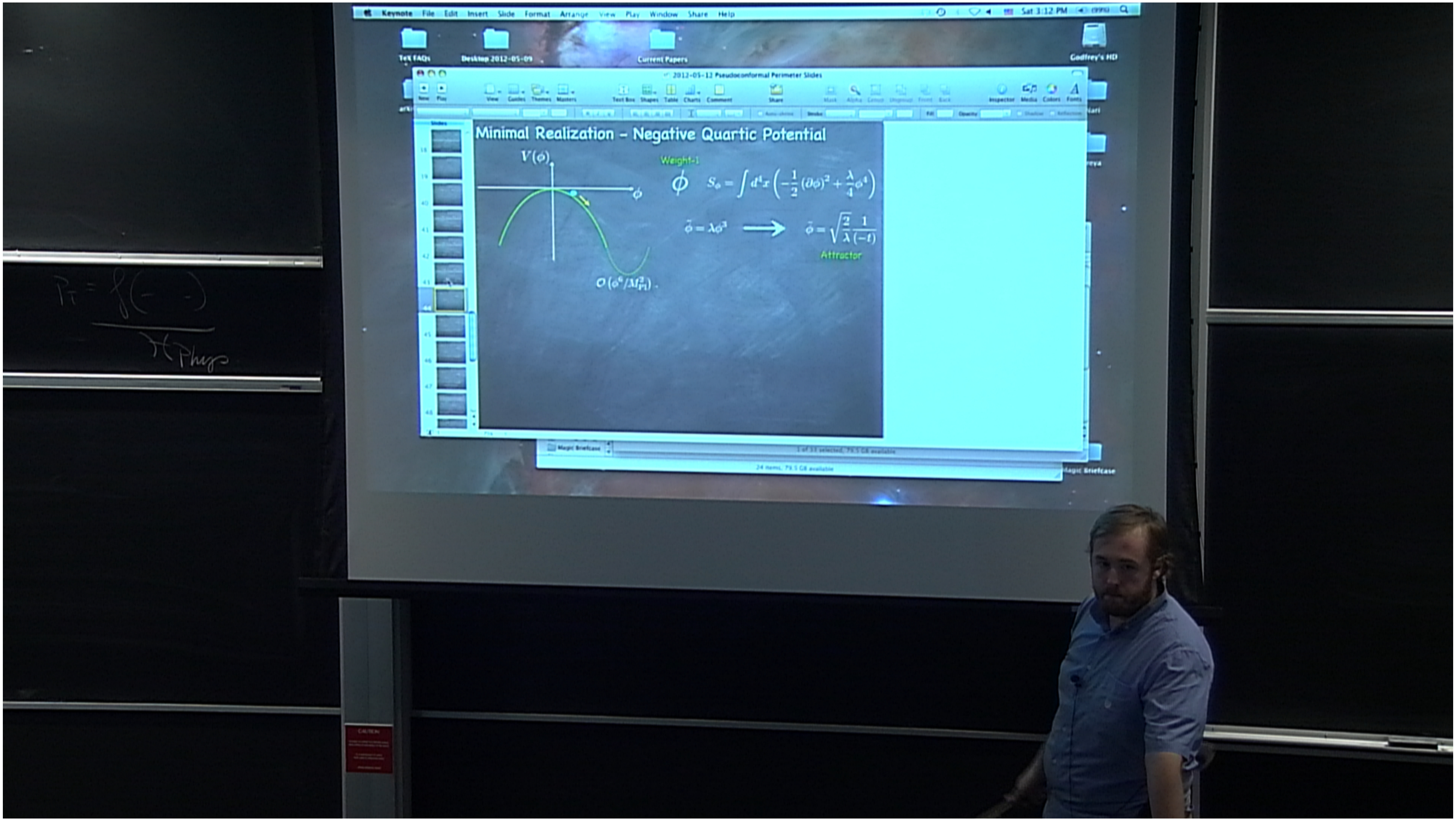
Thank you!



Minimal Realization – Negative Quartic Potential
$V(\phi)$
ϕ
Weight-1
ϕ
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